Design and Control of a Cable-Driven Actuation System for Soft Robotic Exoskeletons

by

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ABSTRACT

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A robotic exoskeleton is a powered wearable device that is capable of actuating one or more of a person’s joints to physically interact with the user in a significant and controlled manner. The work presented in this thesis is a collaboration with NASA’s Wearable Robotics Laboratory to design a soft, wearable exoskeleton for neuro-rehabilitation, an application that requires accurate sensing and control of the interaction forces and torques between the robot and the human to be successfully implemented. To achieve accurate interaction control, a well-established method of cascaded control with an inner velocity loop is applied to a series elastic actuator (SEA) on an experimental test bed. The test bed is made modular and reconfigurable to closely study the effect of the location of a custom-designed compliant force sensor placed within the conduit transmission. Proper implementation of the control scheme is verified through system identification techniques with comparison to model predictions. The choice between desirable force feedback sensor location and controller accuracy is identified as an engineering tradeoff that must be considered by soft exoskeleton designers in the context of the requirements specific to their application.
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Chapter 1

Introduction

A robotic exoskeleton is a powered wearable device that is capable of actuating one or more of a person’s joints to physically interact with the user in a significant and controlled manner. Robotic exoskeletons come in many forms, and are designed for nearly all parts of the body. This work focuses on devices made for the upper limbs, specifically targeting the shoulder and elbow joints. The major open challenges to creating such a device include the design of the physical interface with the human body, and the design of actuators that are safe, lightweight, compact, powerful, and very well controlled. This thesis considers the actuation problem, and seeks to illuminate the design and control issues that arise when exploring the specific approach of using a cable routed through a flexible conduit transmission to transfer power from a fixed DC motor to a location on the moving human arm.

The work presented in this thesis is a collaboration with NASA’s Wearable Robotics Laboratory to design a soft, wearable exoskeleton for neuro-rehabilitation. Individuals who have experienced a stroke or a traumatic brain injury (TBI) often suffer damage to the motor cortex, leading to lack of motor function in various parts of the body. A recent development in the physical rehabilitation of these patients is the introduction of robots that usually help move the patient through repetitive motions that were once the tedious job of a physical therapist. There is evidence that people must be mentally engaged in a therapeutic task to regain motor function [1]; in response, one strategy of robotic rehabilitation is to apply assistive force to the
user only when they fall behind a predefined motion trajectory. This method is appropriately titled assist-as-needed (AAN) control, and requires accurate sensing and control of the interaction forces and torques between the robot and the human to be successfully implemented [2].

1.1 Robot Interaction Control

Early industrial robots were designed to perform tasks that involved precise motion; to execute these tasks, robots were outfitted with actuators (DC motors) and position sensors that enabled closed-loop position control. Here, position can be considered synonymous with orientation, and velocity control can also be considered to be a simple variant of position control. Similarly, strain-gauge-based force sensors, or “load cells,” were used in feedback to control the forces and torques at a robot’s end effector or joints, though stability of closed-loop force controlled systems when making and breaking contact with the environment was notoriously problematic [3, 4]. However, standard practice was for position/velocity control and force control to always be carried out on orthogonal degrees of freedom in the robot’s workspace. Robots with this “hybrid” position-force control scheme are only suited to complete tasks where the robot does not exchange energy with its environment in a significant way, such as movement through free space, or sliding contact.

Network models provide a theoretical framework for thinking about the interaction between the robot and its environment, as well as how energy is exchanged between them [5]. Since power is defined as the rate of energy transfer, we will define what are called power conjugate variables for the mechanical transfer of energy between objects in contact. Power can always be described as a product of two time-varying physical quantities – the effort variable and the flow variable; in the mechanical
domain these are force and velocity respectively. To obey causality, a system may control one of these variables over time – corresponding to position/velocity control or force control modes – or it may control the dynamic relationship between these two variables. Specifically, we call a system that accepts velocity as its input and provides force as its output an impedance; conversely, a system that accepts force as its input and provide a velocity as its output an admittance. As an example, for the linear time-invariant mass-spring-damper system that is depicted in Fig. 1.1 and described by the following differential equation:

\[ M \ddot{x}(t) + B \dot{x}(t) + Kx(t) = f(t), \]

where \( f(t) \) is the exogenous force input and \( x(t) \) is the resultant position, we use the Laplace transform of this equation to derive the complex valued admittance relationship \( Y(s) \) between the input and output variables in the frequency domain as follows:

\[ (Ms^2 + Bs + K)X(s) = F(s) \]
\[ Y(s) = \frac{V(s)}{F(s)} = \frac{sX(s)}{F(s)} = \frac{1}{Ms + B + K/s} \]

The impedance \( Z(s) \) for this example, which would be non-causal, is defined as the reciprocal of \( Y(s) \). In [6], Hogan makes the case for the use of interaction control
– a generalization meaning impedance or admittance control – over force control when there is significant interaction between the robot and the environment. For a detailed explanation of the theoretical underpinnings of interaction control, the reader is referred to the Ph.D. thesis by Colgate [7]. More recent examples of fully realized impedance control schemes on multi-degree-of-freedom (DOF) humanoid robot arms were published by Ott, Albu-Schaffer, et al. where they made use of the robot’s joint flexibility to achieve better performance [8,9]. In the following section I will explain in greater detail how joint flexibility is related to force and impedance control of robots.

1.2 Series Elastic Actuation

Gill A. Pratt and his colleagues introduced series elastic actuators (SEAs) in their work at the MIT Artificial Intelligence Laboratory and Laboratory for Computer Science in the mid-nineties [10], [11]. The concept of the SEA, illustrated in Fig. 1.2, is to introduce a linear elastic element, such as a spring, into the transmission of a robot joint between the actuator and the load. In addition to altering the dynamic behavior of the joint, the elastic element can act as a force sensor by measuring its deflection.

![Series Elastic Actuator](image)

Figure 1.2 : Series Elastic Actuator

Deliberately introducing elasticity went against current standards in robot design, which were to design the interface from actuator to load to be as stiff as possible.
In industrial applications, robots were most commonly designed to accurately and precisely control the position of the end-effector or load. It was understood that increasing the resonant frequency of the actuator inertia and interface compliance allows the position control bandwidth to be safely increased. Looking to biology for inspiration, Pratt suggested in [10] that when using robots to execute “natural tasks” we should make use of elasticity. By “natural tasks,” it is inferred that he is describing human and animal activities of daily living, which can be said to consist of lower-frequency motions than those performed by industrial robots. Specifically, natural human motions are largely bounded to be below a frequency of about 10 Hz [12]. The excessive position tracking bandwidth of industrial robots when compared to biological organisms suggests an excellent opportunity for an engineering trade-off. Pratt identified that SEAs could have a host of potential advantages over traditional actuators – reduced reflected inertia, shock tolerance, accurate and stable force control. The key disadvantage associated with SEAs is reduced zero-motion force bandwidth; however, the SEA can have higher force bandwidth than stiff actuators in tasks involving load motion. SEAs are particularly advantageous when used with a gear train, as they alleviate most of the problems to force control introduced by a gear train (e.g. friction, backlash, noise, gear teeth shock loading) [10], while still allowing for the gear train’s significant contribution to increased power density. Furthermore, the introduction of the spring converts the force control problem into a position control problem, and it is much easier to accurately control position through a gear train than force. We can conclude that for cable-driven robots, where series elasticity is unavoidable, we should use it to our advantage.

Following the work of Pratt et al., Robinson studied the use of SEAs in closed-loop force control; the major effects one should expect to see from increased joint elasticity
are improved small-force bandwidth, reduced large-force bandwidth, and low output
impedance [13]. Robinson also highlights that SEAs tend to have comparatively good
power density and force density, which is of particular importance to the design of
upper-limb exoskeletons. Power density is defined as the ratio of maximum actuator
output power to actuator mass or volume, and force density is defined as the ratio of
maximum actuator force output to actuator mass or volume. Robinson also provides
key intuitive explanations for how force control accuracy and stability are improved.
As is stated in the thesis, the stiffness of a force sensor can be thought of as a gain
applied to a force signal just like the gain used in a force feedback controller. Ignoring
motor dynamics momentarily, the product of these two gains would yield the loop
gain that governs closed-loop performance and stability. By decreasing one, we may
increase the other to recover the same performance. Restated, the reduction of force
sensor stiffness in an SEA allows for a proportional increase in controller gain, where
the noisiness of motors, gear trains, and other transmission elements is not present.
This is a major boon for the force controller’s accuracy and stability. [13]

Pratt et al. expanded the exploration of controlling systems with SEAs [14]. Af-
fter noting that their original control scheme suffered from issues of non-collocation
between the sensor and the actuator – no motor position or velocity sensors were
used – they designed a custom motor amplifier for motor voltage control that allowed
them to implement back-EMF based velocity control. This strategy was effective for
rejecting disturbances due to brush and gear stiction. An impedance control scheme
was created using sensed load force and motor position in feedback. A high-gain
proportional-derivative (PD) motor position controller was used with feedforward
cancellation of the motor mass and the sensed load force. The desired motor po-
sition was then commanded based on sensed load force and the desired impedance
parameters $F_v$, $B_v$, and $K_v$.

Building on this development in effective SEA control, Wyeth advocated for the treatment of the motor as a velocity source rather than a torque source [15]. He proposed that with an additional torque control loop wrapped around the motor velocity control loop, with the spring deflection used to provide torque feedback, the resulting torque-controlled SEA can be easily tuned to have sufficiently high fidelity and bandwidth while still remaining safe for human interaction. Cascaded proportional-integral controllers are designed using linear control theory to yield torque control with a fast response and no steady-state error.

The cascaded control scheme was then demonstrated for SEAs with an inner velocity loop implemented on a lower-limb exoskeleton [16]. In this work, the authors stressed that the implementation of the cascaded linear controller is very straightforward, as it can be designed sequentially - inner velocity loop first, then outer torque loop. Using a simple model of the elastic joint as a mass-spring-mass in series, they define the impedance of the closed-loop controlled device in terms of the model parameters and controller gains. Using inner-loop proportional-integral (PI) and outer-loop proportional-integral-derivative (PID) controllers, conditions for asymptotic stability and passivity are calculated using Routh-Hurwitz like criteria. Insights into the control design process include making the time delay introduced by filter of velocity feedback as small as possible, and dropping the derivative term on the outer-loop torque controller due to its amplification of noise.
1.3 Design of Actuation Systems for Wearable Robotic Exoskeletons

Typically, rehabilitation robot designs are rigid, with DC motors driving joints either directly or through some rigid transmission. This approach has worked well for lower-limb exoskeletons that assist in walking, and the result has been a plethora of devices that can alter the human gait for the purposes of rehabilitation, basic research into the biomechanics of walking, and now for augmentation of human capabilities, as in Fig. 1.3 [17, 18]. These devices can be untethered, containing and supporting their own power supplies, and with a limited battery life they can be worn in very unstructured environments [19]. By contrast, fundamental challenges remain in the

Figure 1.3: Berkeley lower extremity exoskeleton (BLEEX) [18]
design of upper-limb exoskeletons – namely, the interference of rigid actuation systems with the workspace of the human arm, limiting range of motion. Furthermore, the weight the actuators add to the robot arm itself can also be limit the usefulness of a device in dynamic tasks. The weight, size, and complexity of current upper-limb exoskeleton designs restricts them from being worn and used outside of fixed laboratory or clinical settings, meaning that there is no opportunity to apply therapy or augmentation to a patient or user in daily life. Devices such as the MAHI Exo-II, shown in Fig. 1.4 developed in our lab can provide accurate sensing and control of joint-level interaction torques between the robot and the user [20], but a shift to a more lightweight and flexible design is necessary, in the author’s opinion, to achieve portability with limited restrictions.

Figure 1.4 : MAHI Exo-II upper extremity exoskeleton for neurorehabilitation [20]

In response, our collaborators at NASA have taken the approach of moving the actuators and sensors from the portion of the robot worn by the user to a stationary, well-supported location (either a side-bench or a “backpack”). The resulting interface is a more comfortable, semi-soft garment that is pulled by biologically-inspired cables, which are routed to the stationary motors through flexible conduit transmissions.
This style of transmission is sometimes called a Bowden cable, and it is commonly seen in bicycle braking and gear-shifting systems. In later chapters I will simply refer to a cable-conduit transmission in an effort to reduce confusion over the word “cable,” but the name Bowden cable will also be used here as it appears frequently in the literature. The conduit is usually constructed of tightly wound steel springs or lines of solid beads, often with a Teflon or other plastic lining to reduce friction. This structure, while stiff under axial loading (tension or compression), is rather flexible in response to transverse loads. As a result, the Bowden cable can be thought of as a flexible, or “soft,” transmission due to its ability to transmit cable tension while freely changing its shape. The soft garment design being developed at NASA’s Wearable Robotics Lab is part of a larger effort known as the Warrior Web project, funded by the Defense Advanced Research Projects Agency (DARPA), that is aimed at reducing soldier fatigue and injury from carrying heavy loads.

Many upper-limb exoskeletons are cable driven by way of rigid pulleys and capstans, including the previously mentioned MAHI Exo-II, the CAREX [21], and a 7-DOF design by Perry and Rosen [22]. Schiele et al. present a Bowden cable actuation design intended for robotic teleoperation with force feedback [23]. In this work they identify the complex non-linear friction introduced by the Bowden cable, as well as what I will refer to as the capstan equation, which governs the relationship
between the cable tension on either end of the conduit. Agrawal et al. have provided a more detailed analytic model of the Bowden cable dynamics [24, 25]. The authors simulate discrete cable elements in series, each with its own frictional interaction with the conduit and stick-slip behavior. This model aligns with experiments very well, and captures the effect of adding loops into the Bowden cable path that increase friction.

The Bowden cable transmission was combined with the concept of series elastic actuation by Veneman et al. in the lower-limb exoskeleton LOPES [26, 27] shown in Fig. 1.7. In this design, a pair of compression springs are combined to make an effective torsional spring at the knee joint that measures torque at the load, which is driven by a pair of Bowden cables coming from motors that are mounted away from the device on a stationary platform. The resulting actuation system is a sufficiently accurate torque source that adds very little weight or volume to the rigid exoskeleton’s
human interface. Most recently, Agarwal et al. have used the same combination of Bowden cable and load-side SEA in a drastically smaller index finger exoskeleton for rehabilitation, Fig. 1.8. The authors show high-fidelity torque tracking at the two proximal finger joints and low minimum realizable impedance, also known as device transparency [28].

The actuation system presented in this work also makes use of a Bowden cable transmission. The decision to also use series elasticity arose not only from successful examples such as the LOPES, but also from consideration of the Bowden cable’s mechanical properties. The cable itself has some intrinsic compliance, which will result in more stretch under loading the longer the cable is. Once the cable is routed through the conduit, complex conformations of the conduit under loading lead to even
more motion effectively between the two ends of the cable. This means that there is already a non-negligible amount of compliance between the actuator and the load before the addition of any springs. Control methods that ignore system compliance can lead to unstable designs, while control schemes designed for series elastic actuators can lead to improved force control accuracy and stability when compared to their rigid counterparts [10]. Therefore, the SEA is a natural choice to be used with a Bowden cable transmission.

The Bowden cable, or cable-conduit, transmission introduces an appreciable amount of compliance between the actuator and the load along the direction of the loading path; however, measuring the deflection between actuator and load positions does not work well as a force sensing strategy for many applications due to cable and conduit nonlinearities. Like Veneman [29], we therefore insert a linear elastic spring element into the transmission to be used as a force sensor as well as to better control the stiffness of the actuation system. Since the spring is placed in series with the Bowden cable, a spring with significantly lower stiffness should dominate the compliance of the full transmission.
1.4 Thesis Outline

This thesis begins with a detailed description of the experimental test bed and its components in Chapter 2 that was used in all of the experiments that follow. The test bed is made modular and reconfigurable to closely study the effect of the location of a custom-designed elastic force sensor placed within the conduit transmission. This chapter also introduces the reader to the mathematical modeling used to describe the system dynamics.

Next, to achieve accurate interaction control on the test bed, a well-established method of cascaded torque control with an inner velocity loop is applied to the system in Chapter 3. Proper implementation of the control scheme is verified through system identification techniques with comparison to model predictions. The actuation system is evaluated as a torque source under zero load motion, and its accuracy is assessed under varying conditions, including changing the elastic force sensor location.

In Chapter 4, the system is evaluated as a variable mechanical impedance when interacting with a user through a handle. With the load now free to move, different “virtual” stiffnesses are rendered to the user. Again, the accuracy of the impedance controllers are evaluated when the location of the elastic force sensor is changed and the results are compared.

Finally, Chapter 5 considers the choice between desirable sensor location and controller accuracy as an engineering tradeoff that must be considered by soft exoskeleton designers in the context of the requirements specific to the application, and recommendations of future work are made.
Chapter 2

Design and Modeling of an Experimental Test Bed for a SEA with Cable-Conduit Transmission

As detailed in Section 1.3, the design of a soft, wearable robotic exoskeleton for the upper limb is extremely complex, meriting the isolation of the actuation system onto a stationary experimental test bed for controlled study. The test bed, pictured in Fig. 2.1a, was designed to be modular, so that the choices of actuation, transmission, sensing, and output could be made independently. The 2D diagram below in Fig. 2.1b illustrates the basic concept of the system: torque is transmitted between the handle and the motor through the cable-conduit transmission, where the spring is compressed when the cable is in tension.

Section 2.1 will detail the individual components of the test bed and how they were integrated so that the experimental results presented in this thesis can be reproduced. Section 2.2 covers the derivation of the complete mathematical model of this system that will be used to predict the experimental results in subsequent chapters.

2.1 Test Bed Components and Integration

As pictured in Fig. 2.2, all of the test bed components were rigidly mounted to a mechanical breadboard, which was in turn rigidly mounted to a fixed table. The actuator chosen was a DC brushless EC-4 pole 200 Watt Maxon motor (P/N 305015), with a planetary gearhead attached to the output with a reduction ratio of 246:1 (P/N 324942.) This highly non-backdrivable actuator has large torque output capa-
Figure 2.1: Experimental test bed for an actuated rotary joint with human interaction through a handle. (a) A user is gripping the load-side handle while the device is in impedance control mode. (b) The basic concept of the system illustrated as a 2D system where torque is transmitted between the handle and the motor through the cable-conduit transmission, and the spring is compressed when the cable is in tension.

...ibilities, and consequently it can reach only modest speeds. This has been deemed an appropriate trade-off for the intended application, where subject movements will be generally slow but regularly call for high-torque assistance. The motor is also packaged with an Avago 500-count optical disk encoder and line driver HEDL 5540 (P/N 110514) that can be read with quadrature for a resolution of $7.317 \times 10^{-4}$ degrees of rotation of the motor shaft after gearing. A 36 mm diameter pulley is attached to the output shaft of the gearbox, and the pulley allows cables with eye-spliced (see Fig. 2.3d) ends to be attached rigidly to it. The full motor assembly is shown in Fig. 2.3a.
Figure 2.2: This image shows the experimental test bed components all mounted onto the mechanical breadboard where they can be rearranged as needed. All the force/torque sensors are labeled, along with the cable-conduit transmission, the DC brushless motor, and the rotating load assembly with handle for human interaction.

The cables are a proprietary braid of Teflon and Vectran obtained from our collaborators at NASA, and are routed through spiral steel conduit, which has a thin inner coating to reduce friction and a thick outer coating for safety. The conduit has an inner diameter of 2.5 mm and an outer diameter of 5 mm, with the plastic coatings included in these measurements. The two ends of the conduit are held in place at either a plastic mounting block or the compliant compression sensor (see Fig. 2.3b), each of which is fixed to an adjustable height aluminum stand that attaches to the breadboard. Cables are always fed in a straight path out of the conduit to wrap onto the pulleys.
Figure 2.3: (a) Motor assembly including DC brushless motor, planetary gearbox, optical encoder, and pulley with cable attached. (b) Conduit mounting into termination point on compliant compression force sensor. (c) Tension load cell with hooks in series with two cables. (d) Braided blend of Vectran and Teflon terminated in an eyesplice knot for convenient attachment.

At the load side the cable connects to a larger pulley, which is attached to a keyed shaft that is sitting in bearings raised above the mechanical breadboard. Attached directly to the load pulley is a torsional load cell, which is attached on its other end to the load handle. The handle also sits on a rotary precision ball-bearing, so that nearly the entirety of the moments applied through the handle are measured by the load cell. Also attached to the load shaft is a US Digital optical encoder (P/N DISK-2-2500-1000-IE) that provides 10,000 cycles per revolution after quadrature for a resolution of 0.036 degrees of rotation of the load shaft. The angular positions and velocities of both the motor and the load are needed as inputs to the interaction controller, and they will also be used in measures of controller performance.
An array of sensors of both torque and force were outfitted on the testbed to measure the interaction force at all relevant locations: two tension sensors were used in series with the cable, a torque sensor was mounted between the load handle and pulley, and a compliant compression force sensor was used at the rigid mounting locations for the conduit ends. Sensors such as these that convert mechanical force into electrical signals are often called load cells, and they will be referred to as such throughout this thesis. The first is the Futek model TFF350 reaction torque load cell, which measures torques felt at the load handle as accurately as possible up to a capacity of 60 Nm. This is used as a ground truth measurement of torque felt by the user. The second is the Futek model LSB200 S-Beam Junior load cell designed for inline loading in tension and compression, with a capacity of 445 N. This sensor comes with two mountable hooks that allow it to be suspended between two looped ends of cable, as shown in Fig. 2.3c. The third is the compliant conduit compression sensor, which was custom-designed in our lab to measure cable tension via the compression in the conduit. Fig. 2.4 shows the custom sensor outside of the test bed and without a cable running through it. The design and calibration of the compliant conduit compression sensor is described in great detail in the thesis by Blumenschein [30]. The basic operating principle is as follows. The steel spring has a stiffness of 16,970 N/m, and it converts conduit compression force linearly into motion of the bottom slider. Two hall effect sensors on the top mount detect changing magnetic fields generated by the three magnets on the bottom slider. The voltages output by the two hall effect sensors are passed through an algorithm that can extract a position of the bottom slider relative to the top mount. This position is then used as a measure of spring deflection and, consequently, compression force.

I am using an Advance Motor Controls model BE15A8 analog servo drive to
Figure 2.4 : The compliant conduit compression sensor that was custom designed in our lab to measure cable tension by way of measuring the equal and opposite conduit compression force at one end of the conduit. Details of the sensor design and calibration can be found in the thesis by Blumenschein [30].

...
breadboard. This feature is necessary to isolate the system’s torque/force tracking performance from the disturbance of load motion. Second, the compliant conduit compression sensor can be located at either the load or motor end of the conduit, which is a very important choice for control design.

2.2 System Model

In this section, I present the derivation of a lumped parameter model of the SEA combined with the cable-conduit transmission that is similar to what was used by Palli et al. for the cable-conduit transmission without a linear elastic element [31]. The model has been carefully chosen to only include lumped parameters with a clear physical interpretation to avoid modeling errors. The major difference between the dynamic model derived here and what was presented in [31] is that I have not explicitly modeled cable stretching, since the compression of the softer steel spring will dominate when the two are in series. The resulting model is well-suited to the experimental test bed described in section 2.1, where the elastic spring can be moved to either side of the conduit.

Figure 2.5 : Reproduction of Fig. 2.1b for clarity.

The 2D representation of the test bed is reproduced here in Fig. 2.5. The torque
measured by the torque load cell is $\tau_L$, which we are considering to be equivalent to the torque felt at the cable, ignoring inertial effects of the load assembly. The torque generated by the motor after gearing is denoted $\tau_A$; throughout this thesis the subscript $A$ will denote the actuator side of the conduit and the subscript $L$ will denote the load side of the conduit. The angles $\theta_A$ and $\theta_L$ are the angular positions of the motor shaft after gearing and the load shaft respectively. Their rigid relationship is defined by the radii of the two pulleys, $r_A$ and $r_L$, to be $\eta = r_L/r_A$. The compression spring’s stiffness in N/m is labeled as $\kappa$. In Fig. 2.6a the cable-conduit transmission is isolated as a subsystem, with translational effort and flow variables defined on either end of the cable. The relationships between the variables in Figs. 2.5 and 2.6a are as follows:

\[
\begin{align*}
    x_A &= r_A\theta_A & x_L &= r_L\theta_L \\
    f_A &= \frac{\tau_A}{r_A} & f_L &= \frac{\tau_L}{r_L}
\end{align*}
\]

where $\tau_A$ and $f_A$ have a more complicated relationship due to the included motor dynamics in the model.

Because the motion of the compression spring corresponds directly to changing the cable path length, we can define the force in the spring as

\[
    f_{spring} = \kappa(x_A - x_L). \tag{2.1}
\]

The frictional force acting on the cable due to the conduit is also defined as

\[
    f_F = f_A - f_L. \tag{2.2}
\]

The cable-conduit system with the spring can be redrawn as a cable wrapped over a fixed arc, with a spring between the conduit arc and ground; the two equivalent representations are shown side-by-side in Fig. 2.6. Once again, in contrast to the
Figure 2.6: (a) Isolated cable-conduit transmission with compression spring of stiffness $\kappa$ at one end of the conduit. Each end of the cable has a tension force $f$ and translational position $x$, with subscript $A$ denoting actuator side and subscript $L$ denoting load side. (b) The same cable-conduit transmission in (a) is redrawn as a cable wrapped around an arc with friction, with a compressive spring between the arc and ground. The arc is labeled $C$ as it represents the conduit.

model by Palli et al. [31], the cable is assumed to be rigid and all of the stiffness in the system is brought into one location. This is considered to be a valid assumption because the cable is much stiffer than the spring, assuming the cable is always in tension. Friction between the cable and conduit is represented by the x’s drawn between the two. The use of a factor of four times the stiffness allows the original relationship between cable motion and spring force defined in Eq. 2.1 to remain true at the end of the derivation.

In this new representation, it becomes clear that the static relationship between the cable forces and the spring compression is

$$\frac{f_A + f_L}{2} = \kappa (x_A - x_L).$$  

(2.3)

Now, for simplicity, we first look at the dynamics of the transmission system with the load position locked, which has a single input $f_A$ and a single output $x_A$. As it was mentioned in Section 2.1, locking the load position corresponds to a real experimental condition on the test bed that is utilized when studying torque/force control. The
diagram of the system with the load position locked is shown in Fig. 2.7, and the relationship between the forces and the cable motion according to Newton’s Second Law are given in Eq. 2.4, where the friction force’s functional dependence on the relative motion between the cable and the conduit is highlighted so that the following is recognized as a dynamic equation of motion.

\[ f_A = \frac{f_F(\dot{x}_A)}{2} + \kappa x_A \]  \hspace{1cm} (2.4)

Figure 2.7 : Model of the cable-conduit transmission with the load position locked

The next step in this derivation is to relocate the spring so that it is in series with the cable, as would be expected for a system described as a series elastic actuator. However, if this system has only a single lumped parameter stiffness, then the symmetry between the two sides can no longer exist, giving us two distinct choices for a model – a spring on the actuator side or a spring on the load side. Both options are explored in the sequel.

First, the spring is included on the actuator side of the cable in Fig. 2.8a. Since Eqs. 2.2 and 2.3 both still apply, we can conclude that the compression spring at the conduit mounting block is in fact acting as a spring in series with the cable. However, now that an asymmetry has been introduced into the model, the definitions of the cable tensions and cable frictions need to be handled with care for the two separate
cases. For the actuator-side series spring model in Fig. 2.8a, the system dynamics are described simply as

\[ f_A = \kappa x_A, \quad (2.5) \]

where the frictional term is not seen because the cable is rigid and cannot move relative to the conduit.

![Diagram of actuator-side spring](a)

![Diagram of load-side spring](b)

Figure 2.8: Two series elastic models of the cable-conduit transmission derived from Fig. 2.7 with two different placements of the spring: (a) actuator-side spring (b) load-side spring

By contrast, the system model when the spring is included in series with the cable on the load side of the conduit is depicted in Fig. 2.8b, and has the following equation of motion:

\[ f_A = f_F(\dot{x}_A) + \kappa x_A. \quad (2.6) \]

To complete the system, a model of the motor dynamics must be included to relate the actuator-generated torque to the actuator side cable tension and motion. The motor mechanical model is given in Fig. 2.9, and by combining it with the two models of the cable-conduit transmission for the two different spring locations, two full-system models result. These combined models are shown in Fig. 2.10. A class one lever is used to represent the pulley ratio, meaning that rotation about the fulcrum
in these drawings represents translation of the cable. The spring is represented as a torsional spring as experienced at either the motor shaft or the load shaft, depending on the model. The torsional stiffness relate to the original spring stiffness as follows:

\[ K_A = \kappa r_A^2 \]

\[ K_L = \kappa r_L^2 \]

Figure 2.9: Mechanical model of the motor with inertia \( J_A \), damping \( B_A \), the externally applied actuator torque \( \tau_A \), the backdriving cable torque \( r_A f_A \), and the angular positions of actuator \( \theta_A \)

Figure 2.10: Two series elastic models of the full SEA system with the load position locked for two different placements of the spring: (a) actuator-side spring with torsional stiffness \( K_A = \kappa r_A^2 \) (b) load-side spring with torsional stiffness \( K_L = \kappa r_L^2 \)
The equation of motion for the locked-load system with the actuator-side spring shown in Fig. 2.10a is

\[ J_A \ddot{\theta}_A + B_A \dot{\theta}_A + K_A \theta_A = \tau_A. \]  

(2.7)

This version of the plant is not affected by cable-conduit friction, and would therefore lend itself to control and analysis techniques intended for linear systems. This is the approach that will be taken in Chapter 3 to the design of the torque control scheme. Conversely, the locked-load system with the load-side spring shown in Fig. 2.11a has the following equation of motion:

\[ J_A \ddot{\theta}_A + B_A \dot{\theta}_A + K_L \theta_A / \eta^2 = \tau_A - r_A f_F. \]  

(2.8)

The conduit friction appears in this version of the system dynamics, illustrating that the motor and the dynamics are now coupled with the conduit friction dynamics. Just as the linearity of the equation of motion for the actuator-side spring model suggests linear control techniques can be applied there, the addition of the nonlinear friction to the equation of motion for the load-side spring model has implications for our ability to control the torque output of the actuator easily.

Now, both models are extended to the unlocked load position condition, which requires only a slight modification. The unlocked load models are shown side by side in Fig. 2.11. The system equations for the actuator-side spring model are

\[ J_A \ddot{\theta}_A + B_A \dot{\theta}_A + K_A \theta_A = \tau_A + K_A \eta \dot{\theta}_L \]  

(2.9)\[ \tau_L = \eta K_A (\theta_A - \eta \dot{\theta}_L) - r_L f_F. \]  

(2.10)

From these equations it is clear that the frictional force \( f_F \) is present in the load-side torque felt by the user.

The system equations for the load-side spring model with unlocked load position
Figure 2.11: Two series elastic models of the full SEA system with the load position unlocked to allow interaction with the load. Again, the models are derived for two different placements of the spring: (a) actuator-side spring with torsional stiffness $K_A = \kappa r_A^2$ (b) load-side spring with torsional stiffness $K_L = \kappa r_L^2$

are

\[
J_A\ddot{\theta}_A + B_A\dot{\theta}_A + K_L\theta_A/\eta^2 = \tau_A - r_Af_F + K_L\theta_L/\eta \tag{2.11}
\]

\[
\tau_L = K_L(\theta_A/\eta - \theta_L). \tag{2.12}
\]

The load-side torque $\tau_L$ is now equal to the torque in the spring, and the actuator dynamics are still coupled to the conduit friction dynamics. The differences between these two models in both the locked and unlocked load position cases will be born out in the experimental results detailed in Chapters 3 and 4.

Thus far, two mechanical models have been derived for the system – one corresponding to each choice of conduit compression spring sensor location. The equations of motion describing the two models predict that the effect of cable-conduit friction will manifest itself differently for each case. Next, I will introduce a model for the friction interaction between the cable and conduit that will allow us to define the friction force in terms of already specified state variables – cable velocities and cable
tensions. With the friction force specified, the behavior of the system over time subject to certain inputs can be predicted in simulation and compared to experimental results.

![Cable-conduit coefficient of friction](image)

Figure 2.12: Cable-conduit coefficient of friction $\mu$ as a function of relative velocity. The model includes static, Coulomb, and viscous friction components. The discontinuity can be interpreted as the cable being in either a "stuck" or "sliding" state, depending on relative velocity and the forces applied to the cable.

Considering the locked-load condition with the spring on the load side, as in Fig. 2.8b, the dynamics of the system are

\[
 f_A = f_F(\dot{x}_A) + \kappa x_A ,
\]  

(2.13)

where the friction force can be rewritten as a product of a friction coefficient and a normal force:

\[
 f_F = \mu(\dot{x}_A)(f_A + f_L) .
\]

(2.14)

For this system, the normal force happens to be the sum of the cable tensions on either side of the conduit. Next, we can use a simple model of friction between rigid bodies that includes static, Coulomb, and viscous components as shown in Fig. 2.12. Combining these two equations and plugging in $f_L = \kappa x_A$ yields the dynamic equation that governs the motion of this system in response to an input $f_A(t)$. We cannot
solve for $x_A(t)$ because the function $\mu(\dot{x}_A)$ is not invertible. Indeed, the existence of solutions to a differential equation is predicated on there being no such discontinuities in the state derivatives as we see introduced by the friction function. However, approximations to this model can be simulated.

The system modeling can now be validated by comparing simulation with experimental data. On the experimental test bed, a sinusoidal input was provided as the actuator-side cable tension $f_A$, and the load-side cable tension $f_L$ was recorded during the experiment. The same input signal $f_A$ recorded from the experiment was then run through a simulation of the cable-conduit transmission dynamics to generate a simulated load-side cable tension $f_L$. In Figs. 2.13 and 2.14, the results of the experiment are shown for two different-sized input waveforms.
Figure 2.13: (a) Experimental and simulated load-side cable tension in response to the same sinusoidally varying actuator-side tension, input signal mean of 40 N (b) Functional relationship between cable tensions on either side of conduit illustrated by plotting load-side cable tension against actuator-side cable tension for the same trial.
Figure 2.14: (a) Experimental and simulated load-side cable tension in response to the same sinusoidally varying actuator-side tension, input signal mean of 90 N (b) Functional relationship between cable tensions on either side of conduit illustrated by plotting load-side cable tension against actuator-side cable tension for the same trial.
Since the stiffness of the spring $\kappa$ is already known, the only physical parameter that needs to be found for the model is the coefficient of friction. Looking at previous work on modeling cable-conduit friction provides guidance on how this parameter can be easily found. According to Agrawal et al., the relationship between the tensions on the two sides of the conduit is described by the capstan equation [24]. The equation is normally written

$$f_L = e^{-\mu \theta} f_A.$$  

(2.15)

This equation says that if you apply a tension $f_A$ to a cable on one side of a capstan pulley that has friction coefficient $\mu$ and total wrap angle $\theta$, then the tension of the cable on the after frictional losses is described by Eq. 2.15. However, clearly this equation should be symmetric if we were to apply the tension to the cable on the other side in the same way. The basic capstan equation is therefore amended to reflect this symmetry and illustrated in Fig. 2.15a.

$$f_L = \begin{cases} e^{\mu \theta} f_A, \\ e^{-\mu \theta} f_A \end{cases}$$  

(2.16)

When writing this updated version of the capstan equation it becomes apparent that this is still not sufficient because we have been given no information about the conditions that determine which case should be chosen. The solution is that dynamic model of the conduit system must include some memory of how the cables have been loaded in order to describe the relationship between $f_A$ and $f_L$ at any given time. The true path that the function will follow in the case of loading up and the loading down the actuator side tension is shown in Fig. 2.15b.

Applying this knowledge to our existing model, we now know that the product of the friction coefficient and the total wrap angle $\mu \theta$ appears in the slopes of the
Figure 2.15: (a) Illustration of the capstan equation relating actuator-side and load-side cable tensions when loading up or relaxing one end of the cable. (b) The curve predicted by the capstan equation for one loading cycle of the cable from the actuator side.

Since this product is a constant, it can be extracted from experimental data one time and used in all simulations. This is how the simulation results were obtained.
Chapter 3
Design and Implementation of Cascaded Torque-Velocity Control for an SEA

The linear cascaded controller selected to control the individual joint impedance of the device was modeled after the design popularized by Vallery [32], and discussed in Chapter 1. This design leads to a fast and simple implementation of a force and/or impedance controller that is stable while in contact with any environment. When considering stability of interaction controllers, we look at the passivity of the controlled system. If this proves to be passive, then the coupled system of controller and user can also be assumed to be passive [7].

Controlling the torque at the joint of a robot using tendon actuators requires the ability to control tendon tension. This chapter is addressed towards controlling joint torque, but since the problem of torque control can be reduced to controlling tension, much of the discussion will revolve around force control; specifically, the ability to control the tension in the tendon before or after it has been routed through the conduit. The problem of force/torque control will be defined differently for the actuator-side and load-side force sensor locations. The force to be controlled will be defined to be the same as the cable tension being sensed directly by the spring, making it $f_A$ for the actuator-side spring configuration and $f_L$ for the load-side spring configuration.

When the load position is locked at its zero position and the sensor is mounted at the actuator side of the conduit, then our system model developed in Chapter 2 is
nearly identical to the one used by [16]. The model is shown here as a block diagram in Fig. 3.1. In this system configuration, load motion has been eliminated and the nonlinear friction introduced by the conduit is not measured by the sensor; the spring force that would be sensed is represented in the block diagram by the force feedback path coming from the spring impedance block $K_A/s$. In Sections 3.1 and 3.2 the linear cascaded torque-velocity controller will be designed and implemented for the system in Fig. 3.1 ignoring the feed-forward path of the conduit friction, as it should not appear in this version of the force control problem. Therefore, we can use linear control theory to design a force controller and accurately predict the experimental results. Subsequently, I will apply the same controller to system configuration with the sensor placed on the load side of the conduit, with load position still locked. The conduit friction now acts as a disturbance to the force controller, and I characterize the resulting controller performance.

Looking at our model of the system from a modern control perspective, the four states of the system are the load and actuator positions and velocities, or equivalently the actuator position and velocity and the spring deflection and its time derivative. Common practice would be to use full-state feedback to place the poles of the system
where we desire. However, designing the full state feedback controller makes the assumption that you can accurately control the torque applied by the motor, while the cascaded controller makes no such assumption. Instead, the cascaded controller assumes that the inner velocity loop will accurately control the velocity of the motor, which is a much better practical assumption.

The plant is described by the second-order differential equation of motion derived in Chapter 2, reproduced here for convenience.

\[ J_A \ddot{\theta}_A + B_A \dot{\theta}_A + K_A \theta_A = \tau_A + K_A \eta L \theta_L \]  

(3.1)

As the load includes the human, we have no dynamic model for its behavior and instead consider the load position as an exogenous input to the system. Fig. 3.2 shows the plant augmented by our selected impedance control scheme. The dashed box surrounds the force control subsystem which I will refer to as \( Q_f(s) \). Simple scalar multiplication by the radius of the load pulley \( r_L \) and its inverse at the output and input respectively transform this subsystem into a torque controller. It is plain to see that the reference force \( f_r \) and the load position both serve as inputs to the force controller. Since this model is an LTI system, the output of the force controller \( f_L \) can be thought of as a superposition of the contributions of the two inputs considered one at a time. Consequently, the cascaded torque-velocity controller presented in this chapter was designed and tested first with the load position locked at \( \theta_L = 0 \). Next, the ability to reject the disturbance introduced by the load position was evaluated with \( f_r = 0 \) to obtain the full system characterization.
Figure 3.2: Full cascaded impedance control with impedance “reference” \( Z_r(s) \), force control block \( C_f(s) \), and velocity control blocks \( C_\omega(s) \) and \( H_\omega(s) \) all part of the digital control system.
The cascaded controller design allows for the engineer to design the controller in stages. First, the linear current amplifier in the motor driver is set up to control current using pulse-width modulation at a switching frequency of $33 \, kHz$, a fast enough rate that it is considered to be perfect current source. Second, the velocity controller can be tuned to approximate a velocity source limited by a first-order low pass filter. This makes the plant to be force controlled a simple first-order low pass filter in series with a pole at the origin and a static gain equal to the spring stiffness, which is shown in Fig. 3.13. Finally, the force controller is tuned to have the fastest response as possible while minimizing overshoot. The force controller having overshoot suggests that it could become non-passive in impedance control mode.

Fig. 3.3 depicts the same system as in Fig. 3.2, but with the input $\omega_L$ removed. Standard block diagram transformations have been applied to arrange the feedback loops in an entirely nested way. This configuration allows us to redraw the system as it is depicted in Fig. 3.4, where the block $G_\omega(s)$ contains the feedback loop caused by the actuator spring $K_A$, and represents the plant to be controlled by the velocity controller given in equation 3.2

\[
G_\omega(s) = \frac{s}{J_As^2 + B_As + K_A},
\]

where The dashed box in Fig. 3.4 surrounds the velocity control subsystem represented by the transfer function $Q_\omega(s)$. This subsystem has been redrawn with the detailed contents of each block in the left half of Fig. 3.5, where it now looks more like the standard feedback control problem. As was alluded to earlier, the velocity controller can be tuned to approximate a low pass filter, which is why an equivalent block is drawn in the right half of Fig. 3.5. This block is governed by a single parameter, the time constant $T_\omega$. 
Figure 3.3: Cascaded torque-velocity controller redrawn so that all loops are fully nested, with the feedback loops (from outermost to innermost) corresponding to the force sensor, velocity sensor, and spring reaction force.
3.1 Velocity Control

Tight velocity control of the motor can be achieved through proportional plus integral (PI) control with the velocity measurement in feedback. In our system, the motor encoder measures the rotor angle with 2000 counts per revolution of the rotor, corresponding to a resolution of $7.317 \times 10^{-4}$ degrees of rotation of the motor shaft after gearing. The motor velocity is generated from the motor encoder signal using analog circuitry on the Quanser data acquisition (DAQ) board to minimize the negative effects of the discrete differentiation and filtering that are necessary. The velocity controller is represented by the blocks $H_\omega(s)$ and $C_\omega(s)$ in Fig. 3.4. Each of these blocks contains a proportional and integral gain, as is illustrated in Fig. 3.5 and expressed in the following equations.

\begin{align*}
H_\omega(s) &= K_{P,\omega} + K_{I,\omega}/s \quad (3.3) \\
C_\omega(s) &= K_{P,\omega r} + K_{I,\omega r}/s \quad (3.4)
\end{align*}

The resulting closed-loop transfer function for velocity tracking is

\[ Q_\omega = \frac{K_{P,\omega} s + K_{I,\omega r}}{J_A s^2 + (B_A + K_{P,\omega}) s + (K_A + K_{I,\omega})} \quad (3.5) \]
The time constant $T_\omega$ for the equivalent representation is found from the faster of the two poles of $Q_\omega(s)$, calculated in MATLAB as $T_\omega = 1/(-\min(\text{roots}([J_A, B_A + K_{P,\omega}, K_A + K_{I,\omega}, \omega])))$.

![Diagram of velocity tracking subsystem](image)

Figure 3.5: Velocity tracking subsystem $Q_\omega(s)$ drawn in full detail and in its equivalent representation as a first-order low-pass filter with time constant $T_\omega$.

In practice, the controller is implemented by multiplying the encoder position signal by $K_{I,\omega}$ and the DAQ-produced velocity signal by $K_{P,\omega}$, still yielding a PI controller in velocity. The gains used in $C_\omega(s)$ for the incoming velocity reference signal need not be the same as those used in feedback. I will now explain the process used to select the four gains used in this controller.

If the inner velocity control loop is designed independently of the outer force loop, with the assumption that it can be treated as a velocity source by the outer loop, then the best choice is for $Q_\omega(s)$ to have a flat response with a high bandwidth. To achieve this we must first choose the feedback gains in $H_\omega(s)$ to place the poles of the closed-loop transfer function as far to the left of the origin in the complex plane as possible. This is illustrated by the root locus plot in Fig. 3.6, with the poles predicted by the gains I selected marked. The choice of $K_{P,\omega}$ is limited by the noise present in the feedback velocity signal that becomes larger as the proportional gain is increased, eventually leading to a rumbling present in the motor’s torque output. Not only is this noise a threat to system stability, but it is also highly undesirable in a haptic device. Obtaining the velocity measurement is a step that has received much attention
Figure 3.6: Root locus plot for velocity control of the plant $G_\omega(s)$ with a fixed value of proportional gain $K_{P,\omega} = 35$ and varying the integral gain $K_{I,\omega}$. The closed-loop poles corresponding to the selected gain $K_{I,\omega} = 1200$ are shown as large black x’s.

in the realm of haptics [33]. With the presumption that there is a digital encoder measuring position, the velocity must be approximated in some way that minimizes distortions and discontinuities introduced by discretization and quantization while also minimizing time delay. The traditional approach is to combine a finite difference method with a linear filter. For our system, we make use of the Quanser DAQ velocity measurement, which is obtained directly from analog encoder inputs at a faster rate before the signals enter the 1 kHz control loop, yielding better results. If the same gains were used in $C_\omega(s)$ as in $H_\omega(s)$, the predicted frequency response of the closed-loop system would be that shown in Fig. 3.7. To flatten the frequency response, $K_{P,\omega}$ and $K_{I,\omega}$ are instead chosen so that the zero they introduce to the closed-
loop system cancels the smaller of the two poles, and the system looks like a first order low-pass filter governed by a single time constant. The final constraint is to set \( K_{I,w} = K_{I,w} + K_A \) to yield a unity DC gain, i.e. when \( s = 0 \). The new predicted frequency response is plotted in Fig. 3.8.

![Bode plot](image)

Figure 3.7: Bode plot for closed-loop velocity tracking system \( Q(\omega)(s) \) without independent tuning of the reference gains in \( C_\omega(s) \); peak magnitude of 0.48 dB at frequency 8.17 Hz.

To validate the model of the system and controller design, I used a linear system identification technique to obtain the estimated magnitude of the frequency response of the closed-loop motor velocity control. The reference input chosen was the Schroeder multi-sine because it has a flat power spectrum, which tends to yield a more accurate transfer function estimate [34]. The Schroeder multi-sine was applied as an excitatory input to the velocity-controlled motor while the tendon was kept in
Figure 3.8: Bode plot for closed-loop velocity tracking system $Q_\omega(s)$ with independent tuning of the reference gains in $C_\omega(s)$ to be $K_{P,\omega_r} = 31$ and $K_{I,\omega_r} = 1200 + K_A$, and bandwidth = 35.9 Hz.

tension; a plot of the reference velocity and actual motor velocity over the first five seconds of the one hundred-second test is shown in Fig. 3.9. For velocity tracking, I examined a range of 0 to 40 Hz. The resulting transfer function magnitude estimates for three individual trials are shown in Fig. 3.11. In the same figure, the average of these trials is overlaid after apply a smoothing filter to the frequency responses, under the assumption that the true frequency response has a certain degree of smoothness. The ripple effect seen in the estimates after about 10 Hz can be attributed to distortions in the time domain attributed to the motor current amplifier, which can be seen clearly in Fig. 3.10. Fig. 3.12 shows the transfer function estimate compared with the model estimated transfer function, demonstrating our ability to accurately predict
the bandwidth of the velocity controller.

Figure 3.9: Velocity tracking for a Schroeder multi-sine input waveform with amplitude $30 \, \text{deg/s}$ and frequency range $0 - 40 \, \text{Hz}$; initial 5 seconds of experiment.
Figure 3.10: Velocity tracking for a Schroeder multi-sine input waveform with amplitude $30 \, \text{deg/s}$ and frequency range $0 - 40 \, Hz$; full 100 seconds of experiment with decreasing amplitude of $\omega_A$ as frequency increases from left to right.
Figure 3.11: Transfer function magnitude estimate of closed-loop velocity tracking system $Q_\omega(s)$ obtained experimentally for three individual trials, with the smoothed average of all three trials in solid black.
Figure 3.12: Transfer function magnitude estimate of closed-loop velocity tracking system $Q_\omega(s)$ obtained experimentally and averaged across trials compared with the linear model prediction for $Q_\omega(s)$. 
3.2 Force Control

Fig. 3.13 shows the block diagram of the force tracking system, which contains the velocity tracking block \( Q_\omega(s) \) designed in the previous section. The actuator velocity \( \omega_A \) is transformed into the force in the spring \( f_L \) by the block \( K_A/sr_A \). The sensed spring force, \( f_L \), is shown in negative feedback to the controller, but it is also shown feeding into the velocity tracking system \( Q_\omega(s) \) by a dashed line. This connection indicates the presence of the spring force applied to the actuator mass; though the connection is already contained within the transfer function \( Q_\omega(s) \), the relationship between the variables is being highlighted for the reader.

\[
G_f(s) = \frac{K_A/(r_A T_\omega)}{s^2 + s/T_\omega}. \tag{3.6}
\]

Figure 3.13: Force tracking system \( Q_f(s) \) with sensed spring force in feedback and linear force controller block \( C_f(s) \) wrapped around closed-loop velocity tracking subsystem \( Q_\omega(s) \).

To design a force tracking system, we first make note that there is a pole at the origin in the open-loop system due to the \( K_A/sr_A \) block. This means that there will be zero steady-state force tracking error, assuming no disturbances, and there is no need for integral action. The open-loop transfer function for the system minus the force controller is written

\[
Q_f(s) = \frac{(K_{D,f}+K_{P,f})K_A/(r_A T_\omega)}{s^2 + (K_{D,f} K_A/r_A + 1)s/T_\omega + K_{P,f} K_A/(r_A T_\omega)} \tag{3.7}
\]
In [16] the authors warn against the use of a derivative feedback term in the force control loop because the error signal will be excessively noisy, causing instability. Therefore, we will examine the performance of the closed-loop force controlled system with only proportional feedback. Using the root locus technique we can see the effect of the proportional feedback gain $K_{P,f}$ on the locations of the close-loop poles. First, we note the effect that the placement of the zero in the inner velocity loop has on the shape of the root locus plot for the outer force loop; root loci are shown for six different values of $K_{P,\omega}$, in Fig. 3.14.
Figure 3.14: Effect of varying the gain $K_{P_{\omega r}}$ in the velocity controller on the design of the force control loop shown through root locus plots of the force control plant $\frac{K_{F}Q_{\omega}(s)}{s}$, (a) $K_{P_{\omega r}} = 18$, (b) $K_{P_{\omega r}} = 25$, (c) $K_{P_{\omega r}} = 26$, (d) $K_{P_{\omega r}} = 30$, (e) $K_{P_{\omega r}} = 31$, (f) $K_{P_{\omega r}} = 45$. 
The other parameter in the velocity controller we would like to vary is the time constant $T_\omega$, which sets the bandwidth of velocity tracking. We will assume first that we have achieved perfect cancellation of the pole so that the transfer function in Eq. 3.6 is exact. Changing $T_\omega$ now has a more intuitive effect on the force controller. From Eq. 3.6, we see that the open-loop system has poles at the origin and at $-1/T_\omega$. Since there are no open-loop zeros, the theory of root locus says that the poles will tend toward $\pm \infty$ and will depart from the real axis at $-1/2T_\omega$.

The high variability shown in Fig. 3.14 in response to relatively small changes in tuning demonstrates the importance of the inner-loop characteristics in the overall system response. The root locus plot for $K_{P,\omega R} = 31$ is what we have used in our controller design; it has been redrawn in Fig. 3.15, showing pole locations for our chosen value of $K_{P,f}$. The root locus plot suggests that the controlled system will always be stable for any choice of $K_{P,f}$; however, we know that our actual choice of $K_{P,f}$ for stable systems will be limited due to sensor noise, time discretization, and actuator saturation. Since $Q_f(s)$ with only proportional feedback can be rewritten in the standard form of a second-order system as

$$Q_f(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2},$$

(3.8)

the value of $K_{P,f}$ to achieve a certain damping ratio $\zeta$ can be written as a function of model parameters.

$$K_{P,f} = \frac{r_A}{4\zeta^2 K_AT_\omega}$$

(3.9)

For a fast but smooth response of the force controller, we set $K_{P,f}$ to be in the range of values that produces a slightly underdamped response. A slightly underdamped response is defined to be $0.707 < \zeta < 1$, which corresponds to $21.21 > K_{P,f} > 10.60$. The predicted frequency response of the force tracking system $Q_f(s)$ for $K_{P,f} = 16$
is shown in Fig. 3.16, which has a predicted bandwidth, defined to be the frequency at which the magnitude reaches $-3dB$, of $18.9Hz$. We can also note that the phase delay of this system always remains in the range $0 - 180$ degrees.

![Root locus plot for control of plant $G_f(s)$ with the closed-loop poles corresponding to the selected gain $K_{P,f} = 16$ marked as large black x's.]

Figure 3.15: Root locus plot for control of plant $G_f(s)$ with the closed-loop poles corresponding to the selected gain $K_{P,f} = 16$ marked as large black x's.

### 3.2.1 Force Control with Sensor at Actuator Side

With the load position locked and the series elastic force sensor at the motor-side location, we expect the frictional effects of the conduit to not be present in the force control loop. Therefore, we will design the controller and make performance predictions using our linear system model, and then we will compare the model predictions
to our experimental results. Once the force sensor is moved to the load-side location, cable-conduit frictional forces will appear as a disturbance to the

Once again, we make use of the Schroeder multi-sine excitation input to estimate the magnitude of the frequency response of the closed-loop force controlled system, $Q_f(s)$. This time, the frequency range of estimation chosen is $0 – 20Hz$. The transfer function was estimated while varying the value of $K_{P,f}$ from 6 to 16 in order to validate the model predictions. Fig. 3.17 shows the experimental results and model generated results overlaid. The model-predicted bandwidth is consistently higher than the actual force tracking bandwidth.

Because this system is actuating through a tendon, only positive force can be generated. Therefore, the input must have a mean value larger than its amplitude, so
that the commanded force never becomes negative. Since the friction in the conduit varies proportionally with the value of the tension, excitation with a large non-zero mean signal will exaggerate the frictional effect. For this reason, I will vary the mean and amplitude of the input signal in an attempt to capture its effect while the sensor is in both locations to compare the results.

We will assess force tracking accuracy by looking at the root-mean-square error (RMSE) between the reference force signal and the measured force signal. In order to compare directly between the conditions of sensor at load side and sensor at motor side, a single proportional force feedback gain of $K_{P,f} = 12$ was used for every trial. The force was commanded to track a sinusoid of the form

\begin{equation}
    f_r = m + A \sin 2\pi ft,
\end{equation}

while varying the following parameters: $m =$ reference force mean, $A =$ reference
force amplitude, $f = \text{reference force frequency}$. Fig. 3.18 shows the consistent force tracking accuracy for the motor-side sensor when the mean and amplitude of the sinusoid are varied largely. Fig. 3.19 shows the same conditions with the frequency of the sinusoid increased; once again, tracking performance is very consistent.

Figure 3.18 : Accuracy of closed-loop force tracking system $Q_f(s)$ while tracking sinusoids of different mean and amplitude at a frequency of 0.1 $Hz$, with the force sensor on the actuator side measuring $f_A$. Top row has input mean of 90 $N$ and bottom row has input mean of 30 $N$. Left column has input amplitude of 20 $N$ and right column has input amplitude of 5 $N$. 
Figure 3.19: Accuracy of closed-loop force tracking system $Q_f(s)$ while tracking sinusoids of different mean and amplitude at a frequency of 0.5 Hz, with the force sensor on the actuator side measuring $f_A$. Top row has input mean of 90 N and bottom row has input mean of 30 N. Left column has input amplitude of 20 N and right column has input amplitude of 5 N.

The RMSE was calculated for all of the conditions plotted in Figs. 3.18 and 3.19, as well as for two additional choices of reference force mean not plotted. The main source of error is the phase delay that we would predict due to the finite bandwidth of our force tracking system. The phase delay is larger for the higher frequency, and so the errors for 0.5 Hz are noticeably larger.
Table 3.1: RMSE Actuator side spring placement

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3.2.2 Force Control with Sensor at Load Side

When moving the feedback sensor to the load side of the cable-conduit transmission, a new disturbance has been introduced into the feedforward path of the force controller. Our model of the cable-conduit friction presented in Chapter 2 demonstrates how this frictional force is dependent on both the cable tension and the velocity of the cable. This is illustrated in the block diagram in Fig. 3.20. The first step is to see how our previous controller fares with this modification to the plant. Using the same inner velocity loop, we apply the same proportional force feedback control, varying the gain $K_{P,f}$. Figs. 3.21 and 3.22 show the force tracking capability of the proportional controller for three different values of the gain $K_{P,f}$ with force reference signals of low and high means respectively. It is immediately apparent that the static friction introduced by the conduit does not allow for smooth tracking of a time-varying reference force. Instead, a certain amount of force error must be accumulated before the force controller action can cause gross motion of the cable and effect a change in load-side force. Depending on the tuning of the force controller gain and the rate at which the reference force signal is changing, the controller can overshoot its target

Figure 3.20: Force tracking system $Q_f(s)$ with the addition of the conduit friction coupled to the actuator dynamics.
and initiate “discrete” oscillations around the reference signal. As the proportional gain is increased, the tendency of the oscillations to occur increases until the actual force signal comes to resemble a quantized version of the reference force.

Figure 3.21 : Accuracy of closed-loop force tracking system $Q_f(s)$ with load-side sensor placement for three values of $K_{P,f} = [8, 12, 16]$ while tracking a sinusoid with a mean of $30 \, N$ at a frequency of $0.1 \, Hz$.

According to [16], we have the option of introducing integral and/or derivative action to the force controller without necessarily jeopardizing stability or passivity. The problem introduced by the conduit friction can be broken down into two parts, one of which can be addressed with integral action and the other with derivative. [35] Firstly, the proportional controller cannot alter the load-side force until the force error is large enough to generate a control action that can overcome static friction in the conduit. This means that the proportional force controller will have steady-state error. Integral action would serve to eliminate this error in a certain amount of time.
Figure 3.22: Accuracy of closed-loop force tracking system $Q_f(s)$ with load-side sensor placement for three values of $K_{P,f} = [8, 12, 16]$ while tracking a sinusoid with a mean of $90\, N$ at a frequency of $0.1\, Hz$.

determined by the integral gain $K_{I,f}$ if the second problem did not exist. Secondly, a proportional controller tuned to have a moderately fast response time will overshoot the reference signal when static friction (and especially the phenomenon known as "stiction") are present. In fact, introducing the integral action will exacerbate this problem that leads to limit cycles. Derivative action serves to limit or remove overshoot, and indeed the application of a nonzero derivative gain $K_{D,f}$ will remove limit cycle behavior in some situations. However, the behavior of the PID controlled system changes depending on both the amount of tension in the cable and the rate of change of the force reference signal. This makes arriving at one set of gains that provides satisfactory behavior at all times practically impossible. This opens up the possibility for an adaptive scheme, where the gains are time-varying depending on the state variables of the system as is suggested in [35], but this was not considered for this thesis. Instead, I chose to pick a single value of the proportional gain $K_{P,f}$ that yielded acceptable force tracking accuracy under varying conditions of cable tension and reference signal frequency. This “optimal” value was chosen to minimize the
RMSE as best it could while conditions varied. Fig. 3.23 shows the functions to be minimized; the gain chosen was $K_{P,f} = 12$.

![Graph showing RMSE as a function of force feedback gain]

**Figure 3.23**: Plotting force tracking error as a function of the force feedback gain in order to identify the gain that produces the minimum error across all force ranges.

Figs. 3.24 and 3.25 are the load-side companions to Figs. 3.18 and 3.19, showing the higher degree of force tracking error at the load side. The RMSE calculated for all conditions with the sensor on the load side is detailed in Table .3.2.
Figure 3.24: Accuracy of closed-loop force tracking system $Q_f(s)$ while tracking sinusoids of different mean and amplitude at a frequency of 0.1 Hz, with the force sensor on the load side measuring $f_L$. Top row has input mean of 90 N and bottom row has input mean of 30 N. Left column has input amplitude of 20 N and right column has input amplitude of 5 N.
Figure 3.25: Accuracy of closed-loop force tracking system $Q_f(s)$ while tracking sinusoids of different mean and amplitude at a frequency of 0.5 Hz, with the force sensor on the load side measuring $f_L$. Top row has input mean of 90 N and bottom row has input mean of 30 N. Left column has input amplitude of 20 N and right column has input amplitude of 5 N.
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3.2.3 Float Mode

When the load position is unlocked so that the device can be backdriven by the user, we can then command a reference force of zero at all times in order to render zero impedance. This very basic mode of impedance control is often called float mode, and it can also be thought of as the ability of the force controller to reject disturbances. Fig. 3.26 is a block diagram of the force controller including the disturbance introduced by the load motion.

![Close-loop force tracking system](image)

Figure 3.26: Close-loop force tracking system $Q_f(s)$ with the additional disturbance input from load motion introduced by load velocity $\omega_L$.

The linear model predicts that the float mode impedance will be equivalent to a constant damping coefficient over a certain frequency range, and transparent outside of that range. The effect of including the integral gain in the force controller is to remove the damper at low frequency. Both of these results are shown in Fig. 3.27.

$$Z_{L,fi} = -\frac{\tau_L}{s\theta_L} = \frac{(s^2 + s/T_v)K_A}{(s^2 + s/T_v + K_{P,f}K_A/T_v)s}$$ (3.11)
Figure 3.27: Bode plot for float mode impedance with and without integral gain feedback; the effect of integral feedback is to reduce the impedance to zero at steady state (0 Hz).

3.3 Conclusion

To achieve accurate interaction control, a well-established method of cascaded control with an inner velocity loop is applied to the series elastic actuator (SEA) on the experimental test bed. The inner velocity loop is characterized using system identification techniques in the frequency domain. The actuation system is evaluated as a torque source under zero load motion, using system identification techniques in the frequency domain and error metrics in the time domain. Having the feedback force sensor at the actuator side results in more accurate and stable force control. Having the feedback force sensor at the load side introduces steady state error and limit cycling behavior.
Chapter 4

Impedance Control for SEA

Using the torque controlled system designed in Chapter 3, the extension to impedance control is fairly straightforward, as can be seen in Fig. 4.1. The force tracking system that is detail in Chapter 3 is represented by the $Q_f(s)$ block. The block $Z_r(s)$ is the reference impedance, and it contains the desired relationship between load velocity and torque, which is expressed in the form of a virtual spring and damper as

$$\frac{\tau_L(s)}{-\omega_L(s)} = K/s. \quad (4.1)$$

The reference impedance block $Z_r(s)$ is set to equal the desired impedance exactly, so that

$$\tau_r = -Z_r(s)\omega_L = -(K/s)\omega_L = -K\theta_L \quad (4.2)$$

If the torque controller had perfect tracking and perfect disturbance rejection, so that $\tau_L = \tau_r$ for all time, then the relationship between $\tau_L$ and $\omega_L$ would be the exact impedance relationship desired. However, our torque controller has been tuned so
that in the ideal case of our linear model it only behaves as a perfect torque source up to a certain cutoff frequency. This is illustrated in Fig. 4.2, where we see that the impedance of the device $Z_L$ matches the reference $Z_r$ up to the bandwidth of the torque controller. For frequencies higher than the torque control bandwidth, the impedance of the device approaches asymptotically the impedance of the spring in the SEA.

![Graph showing impedance control](image)

**Figure 4.2 : Model-predicted impedance control**

Fig. 4.2 shows the device impedance when the desired virtual stiffness is below the natural stiffness of the spring in the SEA. When the desired virtual stiffness is set in $Z_r(s)$ to be above that of the spring stiffness, the device impedance becomes what is shown in Fig. 4.3. The key difference between these two virtual impedances, as explained by Colgate [4], is that setting the desired stiffness above the natural stiffness causes the closed-loop device impedance $Z_L$ to have phase that lies outside of the range $[-90^\circ, 90^\circ]$. This condition corresponds to the device being non-passive,
meaning that it is capable of generating more energy than is put in to it by the user over time. Passivity is a strict condition to require for a robotic device, but it is highly desirable; when a system is passive, it will always be stable when it is coupled to another passive system [4]. In summary, if we restrict the device to be passive, and assume that the human will behave passively – a ubiquitous assumption in the haptics literature – then our device will tend to interact with humans in a stable manner.

Figure 4.3: Model-predicted impedance control for non-passive choice of K

The implementation of impedance control does require joint position and/or velocity sensing as an input to the system. I have chosen to use the digital encoder at the load shaft to accurately measure joint position during the impedance control experiments in this chapter, for the purpose of isolating the study of force control within an impedance loop.
4.1 Experimental Evaluation of Pure Stiffness Rendering

Using the experimental test bed, I compared the ability to render a virtual stiffness to the user while using the compliant conduit force sensor at both load-side and motor-side locations. The purpose of this experiment is to demonstrate and quantify the expected degradation in performance of the impedance controller when the feedback force sensor is moved further away from interaction to be controlled at the load handle. This question has relevance to the design of a soft wearable robot, where having a force sensor near the stationary actuation system is more attractive. When the force feedback sensor is on the load side of the conduit the virtual impedance should be accurately rendered, meaning that the relationship between the position of the load and the torque felt at the load will be linear with a slope equal to the desired virtual spring constant. When the force feedback sensor is on the actuator side of the conduit the user should feel the friction between the cable and the conduit in addition to the desired virtual stiffness, because we are no longer “closing the loop” around the force errors introduced by the conduit.

For both sensor locations, the controller has been tuned to control force at the location of the sensor. For the sensor at the load side, this is essentially equivalent to controlling the load torque felt by the user directly. For the sensor at the actuator side, the torque felt by the user will be the controlled system output plus whatever contribution is made by the interaction force between the cable and the conduit.

Data was taken in one single session for the sensor at each location. To investigate the performance of the impedance controller, I had the system render a range of pure virtual stiffnesses varying from 0.05 to 0.80 Nm/deg. The physical stiffness of the series elastic sensor is 169.7 N/cm, which is equivalent to 0.7643 Nm/deg at the load. I chose a range of virtual stiffnesses that exceeds the actual stiffness of the spring
because in [32], Vallery et al. show that under this control scheme the system should become non-passive for commanded virtual stiffnesses above the spring stiffness. For each virtual stiffness rendered, the user grasping the handle made natural movements along the one rotational degree of freedom, varying their velocity largely over time. This ensured a good characterization of the impedance relationship between velocity and torque.

4.1.1 Sensor at Load Side

Fig. 4.4 shows the force tracking performance while the smallest and largest virtual impedances of the range are being rendered, with the force feedback sensor at the load side. Interestingly, there seems to be significant lead for the low value of stiffness in Fig. 4.4a, while the delay for the high stiffness in Fig. 4.4b is hardly visible for the same time scale. This might be counterintuitive, but it is easily explained by the fact that rendering a stiffness in impedance mode equal to the natural stiffness of the spring, 0.7643 $Nm/deg$, should require the motor to stay almost entirely still. On the other hand, the further a value of desired virtual stiffness is from the natural stiffness, the more motion is required from the actuator in response to load motion.

I have selected a subset of the virtual stiffnesses rendered to plot in Figs. 4.5 through 4.12. The left subfigure shows in blue load torque versus simultaneous load displacement for a contiguous time segment with constant desired virtual stiffness. The solid red line indicates the desired torque, with its slope being equal to the corresponding desired stiffness. This is a time domain representation of a pure virtual stiffness. The green dashed line is a linear fit to the measured data, indicating the approximate virtual stiffness experienced by the user. When looking at small values of desired stiffness in particular, a relatively large amount of hysteresis can be seen;
hysteresis being the tendency for the measured torque to loop around the desired torque instead of tracing back and forth along the red line. The hysteresis indicates that there is a dependence of the load torque not just on displacement, but also on some other time-dependent variable. I have defined the torque error as

$$\tau_{err} = \tau_L - \tau_r$$  \hspace{1cm} (4.3)

The torque error can be seen as the vertical distance between the measured torque and the desired torque in the plot on the left. The right subfigure shows the torque error versus the load angular velocity; It is immediately apparent that for low values of desired virtual stiffness, up to about 0.250 Nm/deg, the torque error is positively correlated with the velocity, while above 0.250 Nm/deg virtual stiffness the correlation disappears. Then, at the highest values of virtual stiffness the correlation between error and velocity starts to become negative.

Figure 4.4 : Force tracking while rendering pure stiffness with sensor at load side
Figure 4.5: Rendering virtual stiffness of $K = 0.050 \text{ Nm/deg}$, sensor at load side
Figure 4.6: Rendering virtual stiffness of $K = 0.100 \, Nm/\text{deg}$, sensor at load side
Figure 4.7: Rendering virtual stiffness of $K = 0.150 \, Nm/deg$, sensor at load side
Figure 4.8: Rendering virtual stiffness of $K = 0.200 \, Nm/deg$, sensor at load side
Figure 4.9: Rendering virtual stiffness of $K = 0.300 \, \text{Nm/deg}$, sensor at load side.
Figure 4.10: Rendering virtual stiffness of $K = 0.600 \text{Nm/deg}$, sensor at load side
Figure 4.11: Rendering virtual stiffness of $K = 0.750 \text{ Nm/deg}$, sensor at load side.
Figure 4.12: Rendering virtual stiffness of $K = 0.800 \text{ Nm/deg}$, sensor at load side
One way to quantify the error is to compare the slope of the linear fit with the slope of the desired torque. Fig. 4.25 represents this error metric as a ratio of the rendered virtual stiffness to the desired virtual stiffness, with a ratio of one representing zero error. However, this is really just the controller’s performance on average; therefore, a large amount of hysteresis could yield no error in rendered virtual stiffness calculated this way. To solve this, I also look at the root-mean-square error (RMSE) of the torque, which is plotted in Fig. 4.14 as a function of desired stiffness. This metric reflects the worse performance at low values of desired stiffness due to hysteresis.

Figure 4.13: Accuracy in rendering virtual stiffness

There is, however, another way to look at the torque error that makes use of our insight into the mechanical properties of the system. We know that there is velocity-dependent friction in our system that cannot be completely removed by the closed-loop force controller; thus, plotting the torque error against the load velocity can allow us to think of the error as an undesired virtual damping rendered to the user. This
undesired virtual damping that appears will be referred to as the minimum virtual damping, because the impedance controller is being implicitly commanded to render zero damping at the time. In Fig. 4.15, the calculated minimum virtual damping is plotted versus the desired stiffness. From this plot, we see that minimum damping decreases as the natural stiffness of the spring is approached, which is accounted for by the smaller and smaller motions needed by actuator. At desired virtual stiffnesses above the natural spring stiffness, the minimum damping becomes negative. This is due to non-passivity of the controlled system, as it was discussed at the beginning of this chapter; a negative damping coefficient is automatically non-passive.

### 4.1.2 Sensor at Actuator Side

Fig. 4.16 shows the force tracking performance while the smallest and largest virtual impedances of the range are being rendered, with the force feedback sensor at the
actuator side. The effect of changing desired virtual stiffness is similar to what was seen when the sensor was at the load side – the accuracy of force tracking is worse for smaller values of desired stiffness due to an increase in the required actuator motion.

In Figs. 4.17 through 4.23 the data for individual values of desired stiffness are plotted in the same format as the previous section, with virtual stiffness plotted as load torque versus load deflection on the left and torque error versus load velocity plotted on the right. The stiffness plots now resemble the relationship between cable tensions predicted by the capstan equation in Chapter 2, but with the load-side cable tension \( f_L \) as the input so that the transitions between loading up and loading down conditions are vertical. The reason the plot of virtual stiffness encountered by the user, a force versus position relationship, and the capstan equation, a force versus force relationship, are similar in shape is that the force in the cable on the actuator side is \( f_A \) is equal to \(- (K/r_L)Q_f(s)\theta_L\). Since \(|Q_f(s)| \approx 1\) below the force controller
Figure 4.16: Force tracking while rendering pure stiffness with sensor at actuator side

bandwidth, $f_A$ is proportional to $\theta_L$ plus some small time delay introduced by the phase delay of $Q_f(s)$. To verify this, the force measured at the compression spring is multiplied by $r_L$ to convert it to a torque at the load handle and plotted versus load position in Fig. 4.24.
Figure 4.17: Rendering virtual stiffness of $K = 0.050 \text{Nm/deg}$, sensor at motor side
Figure 4.18: Rendering virtual stiffness of $K = 0.100 \text{ Nm/deg}$, sensor at motor side
Figure 4.19: Rendering virtual stiffness of $K = 0.150 \, Nm/deg$, sensor at motor side
Figure 4.20: Rendering virtual stiffness of $K = 0.200 \, Nm/deg$, sensor at motor side
Figure 4.21: Rendering virtual stiffness of $K = 0.250 \, Nm/deg$, sensor at motor side
Figure 4.22: Rendering virtual stiffness of $K = 0.750 \, Nm/deg$, sensor at motor side
Figure 4.23: Rendering virtual stiffness of $K = 0.800 \, Nm/deg$, sensor at motor side

Figure 4.24: Rendering virtual stiffness of $K = 0.400 \, Nm/deg$ as measured by the sensor at the motor side
In the virtual stiffness plots, instead of fitting a single line to the data and calling this the effective stiffness, I have fit two lines – one corresponding to each direction of load motion. The solid red line still represents desired spring stiffness as a function of load displacement. The dashed green line corresponds to the user moving away from the spring’s equilibrium, loading up the cable. The dashed pink line corresponds to the user moving toward the spring’s equilibrium, returning the cable load to its minimum value. Fig. 4.25 shows the accuracy of these two virtual stiffnesses as compared to the one desired virtual stiffness as a ratio.

Figure 4.25 : Error in rendering virtual stiffness, sensor at motor side

The green line always lies above the red line, and the pink line always lies below the red line, as is predicted by our capstan friction model. The model can be easily fit to the data because the slope of the green and pink lines should be $e^\mu K$ and $e^{-\mu \theta} K$ respectively. Therefore, taking the natural log of the ratios calculated for Fig. 4.25 of the fitted slopes to the desired stiffnesses, we get estimates of the product $\mu \theta$ for each
desired stiffness, shown in Fig. 4.26. We can think of the product $\mu \theta$ as the friction coefficient of the conduit and cable for the one shape of conduit that was tested, a semicircle corresponding to $\theta = 180^\circ$. Since this value is a physical parameter that should remain constant, the average of the two estimates is taken to yield a single estimate for each set of data. The downward trend as a function of desired stiffness is most likely a function of the algorithm used to separate the data, and will be corrected in future work.

![Figure 4.26: Estimating the friction coefficient](image)

Finally, looking at the torque error in the plots on the right, the velocity dependent variation in the coefficient of friction of the conduit as predicted in Fig. 2.12 is seen prominently in the data for low desired virtual stiffness. Just like the damping seen when the sensor was on the load side, this effect dominates at low desired stiffnesses because they require large actuator motions. When the desired stiffness is large, the velocity sign-dependent effect of switching between loading up and loading down...
conditions dominates as the source of torque error. Overall, the error in torque tracking during impedance rendering is measurably larger when the sensor is moved to the actuator side. The amount of error is a direct function of the friction between the cable and the conduit, and the effect of the capstan equation is to cause the user to feel larger stiffness when moving away from the spring equilibrium and lower stiffness when moving toward the equilibrium.
Chapter 5

Conclusion

The goal of this thesis was stated to be to illuminate the design and control issues that arise when exploring the specific approach of using a cable routed through a flexible conduit transmission to transfer power from a fixed DC motor to a location on the moving human arm. I have presented a design of a series elastic actuator that incorporates a cable conduit transmission with two possible locations for the elastic force sensing element.

To achieve accurate interaction control, a well-established method of cascaded control with an inner velocity loop was applied to the SEA design on an experimental test bed. The test bed was made modular and reconfigurable to closely study the effect of the location of a custom-designed elastic force sensor placed within the conduit transmission. Proper implementation of the control scheme was verified through system identification techniques with comparison to model predictions. The actuation system was evaluated as a torque source under zero load motion, and as a variable mechanical impedance when interacting with a user through a handle. The accuracy of the torque and impedance controllers were evaluated under varying conditions, including changing the elastic force sensor location. Having the force sensor as close to the user interaction point as possible predictably resulted in a more accurate rendering of a virtual impedance to the user, despite the no load motion torque controller showing issues of steady-state error when the sensor was in this location. When the force sensor was located close to the stationary DC motor, a choice that is attractive
for designers of the human interface to a soft exoskeleton, the degradation in the device’s ability to render virtual impedance was characterized using a model of the nonlinear friction interaction between the cable and the conduit transmission. The choice between desirable sensor location and controller accuracy has been identified as an engineering tradeoff that must be considered by soft exoskeleton designers in the context of the requirements specific to the application.

The next step for this work is to expand the experimental test bed to have two series elastic actuation units driving the same rotary joint with interaction handle, so that a bidirectional joint can be studied. After studying the bidirectional joint, a more realistic human interface could be designed. Ideally, the pair of SEAs would drive the elbow of a wearable soft exoskeleton prototype. This would allow testing under more realistic dynamic loading.
Bibliography


