Characterization of a Submersible In-line Pump as a Thrust Generator for Swimming Robots

by

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Abstract

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Throughout history, machines have replaced human labor in many industries. Our machines have made it possible to transcend human limitations. In the present day, underwater robots have stretched the boundaries of what was previously thought impossible in terms of exploration, security and scientific discovery. These robots have become prevalent in a number of sectors and fields including the military, marine biology, oceanography and documentary production. As their applications become increasingly diverse, establishing precise control over the movement of these robots become more and more crucial.

DC motors as force/torque generators for robots are well understood. We know exactly how much voltage must be applied to generate precise movement to achieve specific tasks. On the other hand, this is not the case for thrust generators for swimming robots due to the complexity of the dynamics of the combination of motor action with fluid/solid interactions. An in-line pump consists of a DC motor pumping fluid via a centrifugal pump through a tight space to generate thrust. Understanding of the dynamics of these interactions are vital if we are to establish accurate control over underwater robots. Through the mathematical modeling of ducted thrusters, computational analysis of fluid dynamics of ideal centrifugal pumps and a pendulum arm experimental setup, this research seeks to improve our understanding of the submersible in-line pumps as thrust generators for swimming robots.
Acknowledgements

I dedicate this work to my beloved wife Ozlem Karalomlu and to my son Kaan Karalomlu who will hopefully be born in two months. This journey would not be that much lovely and funny without her.

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Chapter 1

Introduction

Since the invention of fully controlled robotic arms, machines have replaced manpower. These machines first became useful for industry, since factory work is recursive and most of the time includes heavy lifting. Developments in these machines made it possible to go beyond the human limitations. Establishing a control over the robots widened different fields of vision on them. Within this scope, swimming robots have been first built by the military for different missions such as mine cleaning, deep sea rescues. Also some fields such as marine biology, oceanography and documentary production are interested in using swimming robots to perform studies with them.

1.1 Motivation

Robotic control basically consists of three elements: the input (voltage), the controlled system and the output. For instance, a robotic arm consists of joints and DC motors. Motion is the output and making these arms move precisely requires a control system to achieve specific tasks. DC motors as torque generators for robots are well understood so that it is known how much voltage should be applied to achieve a specific task. On the other hand, thrust generators(input) for swimming robots are not fully understood to generate precise movement because of the complexity of the dynamics of the combination of motor action with fluid/solid interaction. In case of understanding the controllability of the thrusters, when a propeller of a ship is taken as an example, the angular velocity of the propeller is increased to make the ship move faster. It is obvious that there is a relationship between the angular velocity of the propeller and the thrust force. In that
manner, thrust force equations are derived with respect to the physical laws. The issue with these equations for swimming robots is that calculation requires flow rate value. Although flow rate measurement is feasible via flow-meters for pumps, these sensors/instruments implementation to the thrusters is not applicable. For this reason, in literature flow rate is related to the angular velocity to simplify the model under some assumptions, but the complexity of the water column around the propeller which contribute to the flow rate is found much more complex. So the thrust force is estimated via the axial velocity on the propeller. Besides that, for the relatively small pumps, adding an instrument to measure the flow rate changes the flow path and optimization. So difficulties of the flow rate measurement motivated us to find another way to predict the flow rate. In this thesis, an innovative way of propulsion: submersible in-line pump as a thrust generator is studied.

1.2 Problem Definition

From a control standpoint, establishing an accurate control over the thrusters requires understanding the dynamics of the working system. To simplify and understand the submersible in-line pump dynamics, ducted thrusters are selected as a threshold matter to study. It is found in literature that after physical laws are applied, the thrust force equation is formed in terms of the flow rate and angular velocity. The Robotics and Intelligence Systems Laboratory (RiSYS) aims to determine the angular velocity via the back EMF voltage of the DC motor, but the flow rate measurement needs to be addressed. As adding an instrument to measure the flow rate changes the environment of the system, an assumption was made to simplify the flow rate in terms of angular velocity in literature. But after several studies it is understood that this assumption is not necessarily true for all the thrusters. At that point, as our pump is a type of centrifugal pump, the centrifugal pump theory is questioned to be suitable to provide the relationship between flow rate and angular velocity. The main problem is the necessity of measuring the flow rate without using an instrument to control the submersible in-line pump accurately.
1.3 Contributions

The previous section addressed the main necessities of the flow rate measurement for an accurate control over the submersible in-line pump. This section identifies the four contributions of this study, that will help to understand under which conditions the centrifugal pump theory assumption is applicable.

1. The first contribution of this thesis is to present the ducted thruster modeling under certain assumptions and how physical laws are applied to these thrusters.

2. The second contribution of this study is to model the pump in two cases. The first one is the iL500P pump and the second one is the ideal centrifugal pump.

3. The third contribution of this study is to verify the centrifugal pump theory with an ideally designed centrifugal pump and then to apply the theory to the iL500P pump with a commercial finite element method (FEM) based software. The results showed that flow rate is proportional to the angular velocity which is extremely important for deriving dynamical equations amenable to the closed loop feedback control in case of an ideally designed in-line pumps.

4. The fourth contribution of this thesis is that a pendulum based apparatus is devised to accurately measure the steady state values of the iL500P pump thrust force under different input voltage conditions.

1.4 Outline of this Thesis

Chapter 2 presents the ducted thrusters modeling. These models are introduced by derivations from physical laws to final assumption. In Chapter 3, submersible in-line pump modeling is presented in two ways (with and without the centrifugal pump theory). In Chapter 4, the centrifugal pump theory is verified with the ideal centrifugal pump. After the verification, the theory is applied to the iL500P pump to show the applicability of
the theory. In Chapter 5, to characterize the iL500P pump the pendulum arm structure is used as the experimental setup to measure the steady state responses under different input voltage conditions.
Chapter 2

Submersible In-line Pump

Using submersible in-line pumps as thrusters is an innovative way of propulsion. For this reason, the pump’s components are introduced and explained how they contribute to the thrust generation part by part in this chapter. After the parts are shown, related literature is explained. Within this scope, ducted thrusters are used as a starting point to understand the pump. One state and two state models are also presented in this chapter.

2.1 Submersible In-Line Pumps

As submersible in-line pumps are normally built to transfer fluids from one point to another point, using them as thrust generators for swimming robots is a completely new idea. This pump is a type of centrifugal pump as it transfers its kinetic energy to the fluids by a rotating impeller which absorbs water from an inlet and pushes that fluid through an outlet. Rule iL500P submersible in-line pump is shown in Figure 2.1 and its specifications are tabulated in Table 2.1.

Figure 2.1: Rule iL500P Submersible In-line Pump
Table 2.1: Rule iL500P Submersible In-line Pump Specifications [1]

<table>
<thead>
<tr>
<th>Part No.</th>
<th>iL500P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current</td>
<td>12V (6amp)</td>
</tr>
<tr>
<td>Head</td>
<td>9.7 m (32ft)</td>
</tr>
<tr>
<td>Pressure</td>
<td>0.96 bar(14psi)</td>
</tr>
<tr>
<td>Flow Rate</td>
<td>1920 lph (500 gph)</td>
</tr>
<tr>
<td>Length</td>
<td>165 mm (6.5”)</td>
</tr>
<tr>
<td>Diameter</td>
<td>38 mm (1.5”)</td>
</tr>
<tr>
<td>Weight</td>
<td>500 g (1.1lbs)</td>
</tr>
</tbody>
</table>

2.2 Pump Parts

There are six parts which form the pump as they are assembled. Some parts are connected by close-fit and others are threaded. The parts are the impeller, a stator, a DC motor, an inner case, a rear part and an outer case. All components are modeled in SolidWorks 2015 with the exact dimensions. Fully assembled iL500P pump is illustrated in Figure 2.2

Figure 2.2: iL500P Pump
**Impeller**

The impeller is made of plastic and has a smooth body. When we compare it to larger centrifugal pumps, there is not many blades on it and these blades are not twisted. The blade height decreases linearly from the leading edge to the trailing edge. The DC motor shaft and the impeller is connected by close-fit. The impeller is shown in Figure 2.3

![Figure 2.3: Impeller](image)

**Stator**

The stator shown in Figure 2.4 overcomes two main functions thanks to its shape. The first one, keeping impeller in line and the second one is setting a path for the fluid. This stator has four backward waterways around it. The fluids flow in the direction of these ways.

![Figure 2.4: Stator](image)
**DC Motor**

DC motor’s brand is MABUCHI FS-390PH MOTORS and it is illustrated in Figure 2.5. It works up to 12 V voltage and provides rotation to the impeller. DC motor’s properties will be mentioned in the mathematical modeling section.

![Figure 2.5: MABUCHI FS-390PH DC motor](image)

**Inner Case**

The inner case is made of stainless steel in order to protect the material from corrosion. The fluid moves outside of that part (Figure 2.6). This case is connected by close-fit from both ends. There are two rubber o-rings on these ends to seal the DC motor and protect it from facing the fluid.

![Figure 2.6: Inner Case](image)
Rear Part

There are two holes on the rear part. The first one is the outlet for the flow and the second one is for the DC motor’s cables. The outer case begins from this rear part (Figure 2.7) and the upper surface of the rear part touches the inner case DC holders underside surface. The rear part to the outer case connection is threaded and the rear part to the inner case connection is close-fit.

![Rear Part](image)

Figure 2.7: Rear Part

Outer Case

This outer case part is made of plastic and has two main functions. First, it covers every part around, and the second one is that it pushes the fluid all throughout the rear part via waterways on it. There are eight equally spaced waterways inside of that outer case. The outer case is represented in Figure 2.8 and Figure 2.9.

![Outer Case Back](image)  ![Outer Case Front](image)

Figure 2.8: Outer Case Back  Figure 2.9: Outer Case Front
In contrast with conventional centrifugal pumps, there is no shroud surrounding the impeller in this pump. The impeller is a type of semi-closed impeller and covered by the stator which has guiding rails on it. Although the fluid is pushed radially by the impeller to the case walls, guiding rails on the stator and flow channels on the outer case makes the fluid go through the pump between the outer case and inner case. The moving fluid comes to a chamber before going out from the outlet. Then the fluid is pushed through the outlet in the rear part.

2.3 Review of Related Literature

There are different types of propulsion systems for underwater vehicles. As the pump is used as a thrust generator for a swimming robot, remotely operated vehicle thruster’s dynamic modeling is searched in literature. A submersible in-line pump is a type of ducted thrusters which are the most commonly used ones. The objective of searching the literature is to understand how to apply physical laws to ducted thrusters and characterize them to establish a precise control. These ducted thrusters are studied and modeled by Cooke [2], Cody [3], Mclean [4] and Blanke [5]. In these studies, a flow rate-angular velocity relationship is derived to establish a control over the thruster.

In Cooke’s thesis [2], an energy-based physical system modeling is shown and the flow rate-angular velocity relationship is derived from the slip ratio of the propeller. With respect to this derivation, a one state model is proposed.

Cody [3] and Mclean [4] proposed a two-state model as a mass-damper system including hydrodynamic load in torque modeling. Cooke’s [2] flow rate-angular velocity derivation is used to express the first state. The second state is that a first order lag is assumed to exist between the angular velocity of the propeller and the acceleration of the water column.

Cooke One State Model

Cooke [2] concentrated on the thruster dynamics and assumes the electric motor operates as a torque source when defining a control model. Applicability of the energy-based physical system modeling to an open system is explained. Parameters that are used to define a typical thruster are torque source ($\tau$), angular velocity ($\omega_p$), thruster shroud’s cross sectional area ($A$), enclosed volume ($V$), fluid density ($\rho$) and volumetric flow rate ($Q$). Diagram of the model variables are shown in Figure 2.10.

Some assumptions are made to simplify the model:

- The energy which is stored inside the duct is purely kinetic energy,
- External ambient fluid’s kinetic energy is negligible,
- Friction losses are negligible,
- Fluid is incompressible,
- Fluid flow at the inlet and the outlet of the thruster is parallel and at ambient pressure,
- Gravity effects are negligible.
- Rotational flow effects are negligible.
Geometry is completely axially symmetrical with respect to the flow direction. The kinetic co-energy ($T^*$) is expressed as a state of volumetric flow rate ($Q$):

\[ T^*(Q) = \frac{1}{2} \rho V \left(\frac{Q}{A}\right)^2. \]  

(2.1)

Generalized momentum is defined as \[ \Gamma = \frac{dT^*}{dQ} = \rho V \frac{Q}{A^2}. \]  

(2.2)

($\Gamma$) is referred to as the pressure momentum in the units of momentum/area. After definition of the pressure momentum kinetic energy is represented as a state function of the pressure momentum:

\[ T(\Gamma) = \frac{A^2}{2\rho V} \Gamma^2. \]  

(2.3)

As power balance is the time rate of change in kinetic energy of the fluid inside the thruster, it is rewritten below:

\[ \frac{dT}{dt} = \frac{A^2}{\rho V} \Gamma \dot{\Gamma} = \omega_p \tau_m - KQ, \]  

(2.4)

$\omega_p$, $\tau_m$ – The power input coming from the propeller.

$K$ – Exiting kinetic energy per volume.

$\dot{\Gamma}$ – The time rate of change of the pressure momentum.

Fluid momentum per volume within the thruster is defined as $\gamma = \frac{A \Gamma}{V}$.

$K$; the exiting kinetic energy is expressed in terms of fluid momentum per volume as,

\[ K = \frac{A^2 \Gamma^2}{2\rho V^2} = \frac{\gamma^2}{2\rho}. \]  

(2.5)

After exiting kinetic energy is defined, the developed thrust force is generated by the convected linear momentum.

\[ Thrust = \gamma Q \]  

(2.6)
Flow rate - angular velocity transition is formulated in terms of propeller area \((A)\), angular velocity \((\omega_p)\) and pitch \((p)\) but as there is a difference between the theory and the reality related to propeller characteristics, slip ratio is defined.

\[
\sigma = \frac{\omega_p p A - Q}{\omega_p p A}.
\] (2.7)

Pitch \((p)\) represents the axial distance traveled with blades unit of rotation (1 rad). Propeller efficiency is referred as \((\eta=1 - \sigma)\). When propeller efficiency and Equation (2.7) are combined, the flow rate equation is represented:

\[
Q = \eta p A \omega_p
\] (2.8)

By combining equations from Equation (2.2) to Equation (2.8), the time rate of change of the pressure momentum is represented as:

\[
\dot{\Gamma} = \frac{\tau}{\eta p A} - K
\] (2.9)

In Cooke’s thesis [2], the propeller efficiency \((\eta)\), pitch \((p)\) and duct area \((A)\) are assumed constants so the thruster dynamic state and output equations are represented with the propeller angular velocity \(\omega_p\) as the thruster dynamic state variable:

\[
\dot{\omega}_p = \frac{\tau}{\eta^2 p^2 \rho V} - \frac{\eta p A}{2V} \omega_p |\omega_p|
\] (2.10)

\[
Thrust = A \eta p^2 \omega_p |\omega_p|
\] (2.11)

In this one-state model, thrust force is represented as a function of the angular velocity.

**Cody Two State Model**

Thruster is divided into three simple physical subsystems (Electrical, Mechanical and Hydrodynamic) in Cody’s thesis [3]. Conservation of linear momentum theorem is used to define the thrust force generated by the propeller and conservation of energy theorem is used to define the torque generated by the propeller. There is a reduction gear to connect
the propeller to the DC motor so this reduction is included in the equations. The two-state model diagram is represented in Figure 2.11.

![Two-State Model Diagram](image.png)

**Figure 2.11: Two-State Model Diagram**

\[ V = \text{input voltage} \]
\[ R = \text{armature resistance} \]
\[ i = \text{current} \]
\[ \tau_m = \text{motor torque} \]
\[ \tau_{APPL} = \text{applied torque} \]
\[ \tau_P = \text{propeller torque} \]
\[ \omega_m = \text{motor shaft angular velocity} \]
\[ \omega_P = \text{propeller angular velocity} \]
\[ J_m = \text{motor inertia} \]
\[ J_{DG} = \text{reduction gear inertia} \]
\[ J_P = \text{propeller inertia} \]
\[ \tau_H = \text{hydrodynamic load} \]
Electrical Model of a DC Motor

The motor law is used to express the electrical portion of the model. Motor torque-applied current relation is expressed as:

\[ \tau_m = K_T i \]  

The generator law is mentioned to show the back EMF voltage-motor angular velocity relation.

\[ e = K_m \omega_m \]

Kirchhoff’s voltage law is applied to include back EMF voltage in Equation (2.14) motor model as:

\[ i = \frac{V_s - K_m \omega_m}{R} \]

The current in Equation (2.14) is substituted into Equation (2.12) to get the complete electrical model.

\[ \tau_m = \frac{K_T V_s}{R} - \frac{K_T K_m}{R} \omega_m \]

Mechanical Model

In the mechanical model there are three components in Cody’s [3] work; the motor shaft, the propeller shaft and the connecting gear box. The mechanical torque equals to the inertial, damping and applied loads:

\[ \tau_m = (J_m + J_{DG}) \dot{\omega}_m + C_m \omega_m + \tau_{APPL} \]

In this mechanical model \( \tau_{APPL} \) is representing the hydrodynamic load that comes from the propeller. This term consists of the inertial, damping and hydrodynamic loads on the propeller. \( \tau_{APPL} = \frac{\tau_{H} + \tau_{N}}{N} \) and propeller mass damper model is \( \tau_P = J_P \dot{\omega}_P + C_P \omega_P \)

Combining all the equations together gives:

\[ \tau_m = (J_m + J_{DG} + \frac{J_P}{N^2}) \dot{\omega}_m + (C_m + \frac{C_P}{N^2}) \omega_m + \frac{\tau_H}{N} \]
Hydrodynamic Load Model

Cody used a hydrodynamic model which was solved by McLean[4] under some assumptions regarding the conservation of momentum and energy laws.

- Adiabatic control volume (No heat transfer into the system),
- Inviscid fluid flow (Fluid viscosity),
- Incompressible fluid flow (Density does not change),
- Potential energy is negligible (No height difference between inlet and outlet),
- Enthalpy is negligible,
- Internal energy negligible,
- $W_v$ is negligible (Viscous work),
- Inlet and outlet of control volume are at constant pressure,
- Control volume is non-deformable.

First law of thermodynamics is applied to get the torque equation. This torque is used as a propeller load in $\tau_{APPL}$.

$$\dot{Q} - \dot{W}_s - \dot{W}_v = \frac{\partial}{\partial t} \iint_{cv} (u + \frac{1}{2}|\vartheta|^2 + gz) \rho dV + \iint_{cs} (h + \frac{1}{2}|\vartheta|^2 + gz) \rho (\vartheta \cdot n) dA$$

(2.18)

where shaft work can be defined as $\dot{W}_s = -\tau_p \Omega_p$. The negative sign represents that a work is done on the control volume. For this equation, horizontal velocity is represented by $\vartheta$ as $u$ is representing the internal energy. Assuming that $\dot{W}_v = z = u = h = Q = 0$, Equation (2.18) is simplified:

$$\tau_p \Omega_p = \frac{\partial}{\partial t} \iint_{cv} \frac{1}{2}|\vartheta|^2 \rho dV + \iint_{cs} \frac{1}{2}|\vartheta|^2 \rho (\vartheta \cdot n) dA$$

(2.19)

The same steps are followed to get the simplified torque equation. Firstly Equation (2.19) is divided into two components. For the unsteady part, triple integral is transformed into double integral over flow cross-sectional area by extracting length out of the integral and fluid density ($\rho$) is removed from the inside of the integral as fluid is
assumed to be incompressible.

\[
\frac{\partial}{\partial t} \iint_{cv} \frac{1}{2} |\vartheta|^2 \rho dV = \frac{L}{2} \frac{\partial}{\partial t} \iint_{cs} \vartheta^2 dA \tag{2.20}
\]

Replacing double integral with Equation (2.32), the unsteady part can be transformed into an expression as follows:

\[
\frac{\partial}{\partial t} \iint_{cv} \frac{1}{2} |\vartheta|^2 \rho dV = \frac{\rho L \beta}{A} Q \dot{Q} \tag{2.21}
\]

The steady part of the Equation (2.19) is transformed into an algebraic equation in terms of kinetic energy correction factor (\(\alpha\)):

\[
\alpha_O = \frac{\iint S_{\text{outlet}} \vartheta^3 dA}{A \vartheta_{\text{avg}}^3}, \quad \alpha_i = \frac{\iint S_{\text{inlet}} \vartheta^3 dA}{A \vartheta_{\text{avg}}^3} \tag{2.22}
\]

Substituting the integral with Equation (2.22) yields to:

\[
\iint_{cs} \frac{1}{2} |\vartheta|^2 \rho(\vartheta \cdot n) dA = \frac{\rho}{2A^2} (\alpha_O - \alpha_i) Q^3 \tag{2.23}
\]

Adding Equation (2.21) and (2.23) together gives the hydrodynamic torque as follows:

\[
\tau_H \omega_p = \frac{\rho L \beta}{A} Q \dot{Q} + \frac{\rho}{2A^2} (\alpha_O - \alpha_i) Q^3 \tag{2.24}
\]

By applying Equation (2.8) to the hydrodynamic equation, it becomes:

\[
\tau_H = (\eta p)^2 \rho A L (K_a + 1) \Delta \beta \omega_p + \frac{(\eta p)^3 \rho A}{2} \Delta \alpha \omega_p |\omega_p| \tag{2.25}
\]

**Thrust Force Equation**

Cody used the conservation of linear momentum law to derive the thrust equation. As only the horizontal velocity component is effecting the thrust force, velocity is represented as \(u\) for following equations. Flow is assumed incompressible so \(\rho\) is constant and can be removed from the integral.

\[
\sum F = \frac{\partial}{\partial t} \iint_{cv} \vartheta \rho dV + \iint_{cs} \vartheta \rho (\vartheta \cdot n) dA \tag{2.26}
\]
THRUST = \rho \frac{\partial}{\partial t} \left[ \int \int \int_{cv} u dV \right] + \rho \int \int_{cs} u(k \cdot n) dA, \quad (2.27)

“u” is set to “u(r, t)” to emphasize the velocity’s dependence on radial position and time. The unsteady term is named as A and the steady term is named as B;

\[ A = \rho \frac{\partial}{\partial t} \left[ \int \int \int u(r, t) dV \right] \quad (2.28) \]

As flow-rate means the velocity on the cross sectional area:

\[ Q(t) = \int \int_{cs} u(r, t) dA \quad (2.29) \]

Since the volume’s length is the same, effective length is included into the equation and the triple integral becomes a double integral by removing length from the integral.

\[ \rho \frac{\partial}{\partial t} \left[ \int \int \int u(r, t) dV \right] = \rho Le \frac{\partial}{\partial t} \left[ \int \int_{cs} u(r, t) dA \right] = \rho Le \dot{Q}(t) \quad (2.30) \]

\[ B = \rho \int \int_{cs} u(r, t)[u(r, t)k \cdot n] dA \quad (2.31) \]

Although fluid viscosity is assumed negligible, the velocity at the boundaries is less than the velocity at a distance from the boundaries. So a momentum correction factor is used. Flux term has two components: \( V \cdot n = [+\beta_o u] \) for outlet and \([-\beta_i u] \) for inlet flow. Momentum-flux correction factor (\( \beta_O \) for outflow and \( \beta_i \) for inflow) can be defined as follows:

\[ \beta_O = \frac{\int S_{outlet} u^2 dA}{Au_{avg}^2}, \quad \beta_i = \frac{\int S_{inlet} u^2 dA}{Au_{avg}^2}, \quad (2.32) \]

where average velocity can be defined as \( u_{avg} = \frac{Q}{A} \).

Substituting Equation (2.32) into the second component of thrust Equation (2.26) yields the following expression:

\[ \int \int_{cs} u\rho(u \cdot n) dA = \frac{\rho}{A} (\beta_O - \beta_i)Q|Q| \quad (2.33) \]
Absolute term is put not to miss the direction of the fluid. Because sign is related to normal (n) vector. So if the absolute term is not used then flow goes in the positive direction either way. By combining “A” and “B” together:

\[ THRUST = \rho Le\dot{Q}(t) + \frac{\rho}{A}(\beta_{O} - \beta_{i})Q(t)|Q(t)| \]  

(2.34)

For this model “Q” is replaced with Equation (2.8) afterwards as there is a relation between angular velocity and flow-rate. It is convenient to recall Equation (2.8) as the second state is also related to that equation.

\[ Q = \eta p A \omega_p \]
\[ \frac{Q}{A} = U = \eta p \omega_p \]

Cody assumed that there is lag between the propeller angular velocity and the water column acceleration. \( t_c \) term in the denominator is a time constant. He expressed it as:

\[ \dot{U} = \frac{-U + (\eta p)\omega_m}{t_c} \]  

(2.35)

Hydrodynamic load \((\tau_H)\) is related to a thrust on the propeller blades by some efficiency factor \((\sigma)\) at a specific length \((L_B)\).

\[ \tau_H = \sigma L_B T \]  

(2.36)

Then the thrust force is written in terms of “U” (Acceleration of the water column around propeller and its momentum). Thruster governing equation and dynamic states are represented as:
\[ K_I = (\eta p)^2 \rho A L (K_a + 1) \Delta \beta \]
\[ K_{II} = \frac{(\eta p)}{2} \rho A \Delta \alpha \]
\[ K_{III} = (\eta p) \rho A L (K_a + 1) \]
\[ K_{IV} = (\eta p)^2 \rho A \Delta \beta \]
\[ K_0 = C_m + \frac{C_p}{N^2} + \frac{K_T K_m}{R} \]
\[ K_1 = \frac{K_T}{R} \]
\[ K_2 = J_m + J_{DG} + \frac{J_p + K_I}{N^2} \]
\[ \dot{\omega}_m = \left(-\frac{K_0}{K_2}\right) \omega_m - \frac{K_{II}}{K_2 N^3} \omega_m |\omega_m| + \frac{K_1}{K_2} V_s \]
\[ T = -\frac{K_{III} K_0}{N K_2} \omega_m + \left(\frac{K_{IV}}{N^2} - \frac{K_{III} K_{III}}{K_2 N^4}\right) \omega_m |\omega_m| + \frac{K_{III} K_1}{N K_2} V_s \]

For the two state model,
\[ \dot{\omega}_m = \left(-\frac{C_{pm} + \frac{K_T K_m}{R}}{J_{pm}}\right) \omega_m + \frac{K_T}{R J_{pm}} V_s - \frac{\tau_H}{N J_{pm}} \]
\[ \dot{U} = \frac{U - (\eta p) \omega_m}{t_c} \]

where
\[ T = \rho A L \dot{U} + \rho A \Delta \beta U |U| \]
Chapter 3

Submersible In-line Pump Modeling

Related literature was mentioned in Chapter 2. Congruently, conservation of momentum is used to derive the thrust force equation. The DC motor electrical model and the conservation of energy law are used to derive the states in this chapter. After equations and states are represented, centrifugal pump theory is applied to show the relationship between flow rate-angular velocity for an ideal centrifugal in-line pump.

3.1 Motor Torque Modeling

Mathematical Modeling of DC Motors

Since control systems use DC motors extensively, a mathematical model is necessarily established for control applications and represented as it is mentioned in (Kuo,2010 [6]). Conventional circuit diagram of a DC motor is presented in Figure 3.1.

![DC Motor Circuit Diagram](image)

Figure 3.1: DC Motor Circuit Diagram
The motor variables and parameters are defined as follows:

- \( i_a(t) \) = armature current
- \( R_a \) = armature resistance
- \( e_b(t) \) = back emf
- \( \tau_L(t) \) = load torque
- \( \tau_m(t) \) = motor torque
- \( \theta_m(t) \) = rotor displacement
- \( K_i \) = torque constant
- \( L_a \) = armature inductance
- \( e_a(t) \) = applied voltage
- \( K_b \) = back-emf constant
- \( \phi \) = magnetic flux
- \( \omega_m(t) \) = rotor angular velocity
- \( J_m \) = rotor inertia
- \( B_m \) = viscous-friction coefficient

With respect to the circuit diagram, the torque developed by the motor is proportional to the air-gap flux and the armature current.

\[
\tau_m = K_m \phi i_a(t) \quad (3.1)
\]

As \( \phi \) is constant, Equation (3.1) becomes:

\[
\tau_m = K_i i_a(t) \quad (3.2)
\]

Governing equations are derived based on Newton’s 2nd law and Kirchhoff’s voltage law in according to Figure 3.1.

\[
\frac{d}{dt} i_a(t) = \frac{1}{L_a} V(t) - \frac{R_a}{L_a} i_a(t) - \frac{1}{L_a} e_b(t) \quad (3.3)
\]

\[
\tau_m(t) = K_i i_a(t) \quad (3.4)
\]

\[
e_b(t) = K_b \frac{d\omega_m(t)}{dt} = K_b \omega_m(t) \quad (3.5)
\]

\[
\frac{d^2\omega_m(t)}{dt^2} = \frac{1}{J_m} \tau_m(t) - \frac{1}{J_m} \tau_L(t) - \frac{B_m}{J_m} \frac{d\omega_m(t)}{dt} \quad (3.6)
\]

As \( \frac{d\omega_m(t)}{dt} = \omega_m(t) \), equations (3.3)-(3.6) are represented in state space form below,

\[
\begin{bmatrix}
\frac{di}{dt} \\
\frac{d\omega_m}{dt}
\end{bmatrix}
= \begin{bmatrix}
\frac{-R_a}{L_a} & \frac{-K_b}{L_a} \\
\frac{K_i}{J_m} & \frac{-B_m}{J_m}
\end{bmatrix}
\begin{bmatrix}
i \\
\omega_m
\end{bmatrix}
+ \begin{bmatrix}
\frac{1}{L_a} \\
0
\end{bmatrix}
V(t)
+ \begin{bmatrix}
0 \\
-\frac{1}{J_m}
\end{bmatrix}
\tau_L(t)
\]

\( \tau_L(t) \) represents hydrodynamic load(\( \tau_H \)) in our application.
Hydrodynamic Load

Hydrodynamic load \( (\tau_H) \) is found via the first law of thermodynamics, also known as the conservation of energy principle, is applied to calculate the power needed to move the fluid through the pump.

\[
\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}} = \frac{d}{dt} \int_{\text{sys}} e \rho \, dV \tag{3.7}
\]

\[
e = u + ke + pe = u + \frac{V_r^2}{2} + gz \tag{3.8}
\]

There is no added heat transfer to the system so \( \dot{Q} = 0 \). As a velocity term \( V_r \) is used.

Total work in the system is expressed as

\[
W_{\text{total}} = W_{\text{shaft}} + W_{\text{pressure}} + W_{\text{viscous}} + W_{\text{other}} \tag{3.9}
\]

The power transmitted via a rotating shaft is proportional to the shaft torque \( (T_{shaft}) \) and angular velocity \( (\omega_m) \). \( W_{shaft} = \omega_m T_{shaft} \). \( T_{shaft} \) represents hydrodynamic load \( (\tau_H) \) that is needed to be found.

Pressure work is shown in Equation (3.10)

\[
W_{\text{pressure,net in}} = - \int_A P(\vec{V}_r \cdot \vec{n}) dA = - \int_A P \rho (\vec{V}_r \cdot \vec{n}) dA \tag{3.10}
\]

Since \( W_v \) and \( W_{\text{other}} \) are assumed to be negligible, general form of the energy equation becomes;

\[
\dot{W}_{\text{shaft,net in}} + \dot{W}_{\text{pressure,net in}} = \frac{d}{dt} \iiint_{cv} e \rho \, dV + \iint_{cs} e \rho (\vec{V}_r \cdot \vec{n}) \, dA \tag{3.11}
\]

By replacing \( \dot{W}_{\text{pressure,net in}} \) with Equation (3.11) in Equation (3.10),

\[
\dot{W}_{\text{shaft,net in}} = \frac{d}{dt} \iiint_{cv} e \rho \, dV + \iint_{cs} (\frac{P}{\rho} + e) \rho (\vec{V}_r \cdot \vec{n}) \, dA \tag{3.12}
\]

Potential and internal energies are assumed negligible. So Equation (3.8) becomes \( u = 0, pe = 0 \) and \( e = \frac{V_r^2}{2} \). By putting modified \( e \) into general energy integral equation,

\[
\dot{W}_{\text{shaft,net in}} = \frac{d}{dt} \iiint_{cv} \frac{V_r^2}{2} \rho \, dV + \iint_{cs} (\frac{P}{\rho} + \frac{V_r^2}{2}) \rho (\vec{V}_r \cdot \vec{n}) \, dA \tag{3.13}
\]
Equation (3.13) is divided into two components. For the unsteady part, triple integral is transformed into double integral over flow cross-sectional area by extracting length out of the integral and fluid density ($\rho$) is removed from the inside of integral as fluid is assumed to be incompressible.

$$\frac{d}{dt} \int_\text{cv} \int_\text{cv} \int_\text{cv} \frac{V^2}{2} \rho \, dV = \frac{\rho L}{2} \frac{d}{dt} \int_\text{cs} \int_\text{cs} V_r^2 \, dA$$  \hspace{1cm} (3.14)

Replacing double integral in the same way with [4], unsteady part can be transformed into an expression as follows:

$$\frac{d}{dt} \int_\text{cv} \int_\text{cv} \int_\text{cv} \frac{V^2}{2} \rho \, dV = \rho \beta \dot{Q}(t)|Q(t)| \left( \frac{1}{A_{\text{out}}} - \frac{1}{A_{\text{in}}} \right)$$  \hspace{1cm} (3.15)

For the steady part of the Equation (3.13)

$$\int_\text{cs} \int_\text{cs} \left( \frac{P}{\rho} + \frac{V^2}{2} \right) \rho (\vec{V}_r \cdot \vec{n}) \, dA = \int_\text{cs} \int_\text{cs} PV_r \, dA + \frac{\rho}{2} \int_\text{cs} \int_\text{cs} V_r^3 \, dA$$

$$= (P_{\text{out}}Q(t) - P_{\text{in}}Q(t)) + \rho \alpha Q(t)^3 \left( \frac{1}{A_{\text{out}}^2} - \frac{1}{A_{\text{in}}^2} \right)$$

$$= \Delta PQ(t) + \frac{\rho \alpha Q(t)^3}{4} \left( \frac{1}{A_{\text{out}}^2} - \frac{1}{A_{\text{in}}^2} \right)$$  \hspace{1cm} (3.16)

To get the complete shaft work rate; unsteady and steady parts are combined:

$$\dot{W}_{\text{shaft,net in}} = \rho \beta \dot{Q}(t)|Q(t)| \left( \frac{1}{A_{\text{out}}} - \frac{1}{A_{\text{in}}} \right) + \Delta PQ(t) + \frac{\rho \alpha Q(t)^3}{4} \left( \frac{1}{A_{\text{out}}^2} - \frac{1}{A_{\text{in}}^2} \right)$$  \hspace{1cm} (3.17)

As $\dot{W}_{\text{shaft,net in}} = \omega_m \tau_H$

$$\omega_m \tau_H = \rho \beta \dot{Q}(t)|Q(t)| \left( \frac{1}{A_{\text{out}}} - \frac{1}{A_{\text{in}}} \right) + \Delta PQ(t) + \frac{\rho \alpha Q(t)^3}{4} \left( \frac{1}{A_{\text{out}}^2} - \frac{1}{A_{\text{in}}^2} \right)$$  \hspace{1cm} (3.18)

$$\tau_H = \frac{\rho \beta \dot{Q}(t)|Q(t)|}{\omega_m} \left( \frac{1}{A_{\text{out}}} - \frac{1}{A_{\text{in}}} \right) + \frac{\Delta PQ(t)}{\omega_m} + \frac{\rho \alpha Q(t)^3}{4\omega_m} \left( \frac{1}{A_{\text{out}}^2} - \frac{1}{A_{\text{in}}^2} \right)$$  \hspace{1cm} (3.19)

### 3.2 Thrust Force Modeling

Ducted thrusters and pump theory are mentioned in Chapter 2. These theorems are needed to be combined to model this pump. The conservation of linear momentum theory
is used to derive the thrust force. Before beginning the derivation, some assumptions are made to simplify the model.

- Fluid is incompressible \((\rho = \text{constant})\),
- Inviscid fluid flow (Fluid viscosity = 0),
- Adiabatic control volume (No heat transfer),
- Potential energy change is negligible \((z_{\text{inlet}} = z_{\text{outlet}})\),
- Enthalpy is negligible,
- Internal energy negligible,
- Viscous work is negligible \((W_v)\),
- Fixed control volume.

\[
\frac{d}{dt}(mV)_{\text{sys}} = \sum F_{\text{sys}} \tag{3.20}
\]

\[
\sum F_{\text{sys}} = \frac{\partial}{\partial t} \int \int \int_{cv} V_r \rho dV + \int \int_{cs} V_r \rho (V_r \cdot n) dA \tag{3.21}
\]

Horizontal component of the velocity “\(V_r\)” is used to represent velocity as fluid moves in and out of the pump horizontally. As fluid is assumed incompressible, density is removed from the integral.

\[
\sum F_{\text{sys}} = \rho \frac{\partial}{\partial t} \left[ \int \int \int_{cv} V_r dV \right] + \rho \int \int_{cs} V_r (V_r \cdot n) dA \tag{3.22}
\]

The unsteady term is named as A and the steady term is named as B;

\[
A = \rho \frac{\partial}{\partial t} \left[ \int \int \int_{cv} V_r dV \right] \tag{3.23}
\]

The flow-rate equals to the integrated velocity on a cross sectional area,

\[
Q(t) = \int \int_{cs} V_r dA \tag{3.24}
\]

Control volume is assumed to be fixed and to simplify the triple integral, pump’s length can be removed from integral.

\[
\rho \frac{\partial}{\partial t} \left[ \int \int \int_{cv} V_r dV \right] = \rho L \frac{\partial}{\partial t} \left[ \int \int_{cs} V_r dA \right] = \rho L \dot{Q}(t) \tag{3.25}
\]
Although fluid viscosity is assumed negligible, the velocity at the boundaries is less than the velocity at a distance from the boundaries. So a momentum correction factor is used. The flux term has two components: \( V \cdot n = [+\beta_o u] \) for outlet and \([-\beta_i u]\) for inlet flow. Momentum-flux correction factor \((\beta_o\) for outflow and \(\beta_i\) for inflow) can be defined as follows:

\[
\beta_o = \frac{\iint V_r^2 dA}{AV_{r,\text{avg}}^2} \quad \beta_i = \frac{\iint V_r^2 dA}{AV_{r,\text{avg}}^2}
\]  

(3.27)

where average velocity can be defined as \(V_{r,\text{avg}} = \frac{Q}{A}\).

Substituting Equation (3.27) into second component of thrust Equation (3.26) after considering momentum correction factor as a constant for inlet and outlet so this yields the following expression:

\[
\int\int_{cs} V_r \rho (V_r \cdot n) dA = \rho \beta Q |Q|(\frac{1}{A_{out}} - \frac{1}{A_{in}})
\]  

(3.28)

When equations are expressed, it is understood that an absolute term must be put to one of the flow-rate term to make sure that the flow direction is not missed. Otherwise flow would be considered to move in the positive direction every time. For the left hand side of the part,

\[
\sum F_{sys} = \sum \vec{F}_{\text{body}} + \sum \vec{F}_{\text{pressure}} + \sum \vec{F}_{\text{viscous}} + \sum \vec{F}_{\text{other}}
\]  

(3.29)

\(\vec{F}_{\text{viscous}}\) and \(\sum \vec{F}_{\text{other}}\) equal “0” as assumed. Also \(\vec{F}_{\text{body}}\) is the thrust force that is tried to be modeled. After force, steady and unsteady terms are combined:

\[
\vec{F}_{\text{thrust}} + \vec{F}_{\text{pressure}} = \rho L \dot{Q}(t) + \rho \beta Q(t)|Q(t)|(\frac{1}{A_{out}} - \frac{1}{A_{in}})
\]  

(3.30)

Pressure force is represented in (Stephanoff,1948 [7]) as acting on the fluid facing impeller area. \(P_{out}\) stands for the pressure leaving the impeller area and goes through outlet and \(P_{in}\) stands for the pressure on the impeller from the inlet.

\[
\vec{F}_{\text{pressure}} = (P_{out} - P_{in}) A_{\text{impeller}} = \Delta P A_{\text{impeller}}
\]  

(3.31)
By substituting Equation (3.31) into Equation (3.30), final thrust force model is represented in Equation (3.32),

\[
\vec{F}_{\text{thrust}} = -\Delta PA_{\text{impeller}} + \rho L\dot{Q}(t) + \rho\beta Q(t)\frac{1}{A_{\text{out}}} - \frac{1}{A_{\text{in}}} \tag{3.32}
\]

A pump is characterized by its net head \((H)\), defined as the change in Bernoulli head between the inlet and outlet of the pump as;

\[
H = \left(\frac{P}{\rho g} + \frac{V^2}{2g} + z\right)_{\text{out}} - \left(\frac{P}{\rho g} + \frac{V^2}{2g} + z\right)_{\text{in}} \tag{3.33}
\]

According to assumptions \(z_1 = z_2\) and with respect to the Equation (3.33), \(\Delta P\) is expressed as

\[
\Delta P = H\langle Q \rangle \rho g - \frac{\rho Q^2(t)}{2} \left(\frac{1}{A_{\text{out}}^2} - \frac{1}{A_{\text{in}}^2}\right) \tag{3.34}
\]

Complete thrust force equation is written in Equation (3.35) by combining Equation (3.32) and 3.34 in terms of \(Q\) (Flow Rate) after state representation is rewritten. Note that \(Q(t) = Q(\omega(t))\) because it is already known that flow rate is somehow proportional to the angular velocity. Dynamic equations are:

\[
\begin{bmatrix}
\frac{di}{dt} \\
\frac{d\omega_{im}}{dt}
\end{bmatrix} = \begin{bmatrix}
\frac{K_a}{L_a} & -\frac{K_b}{L_a} \\
\frac{K_{ia}}{L_a} & -\frac{B_{ia}}{J_m}
\end{bmatrix} \begin{bmatrix}
i \\
\omega_{im}
\end{bmatrix} + \begin{bmatrix}
\frac{1}{L_a} \\
0
\end{bmatrix} V(t) + \begin{bmatrix}
0 \\
-\frac{1}{J_m}
\end{bmatrix} \tau_H(t)
\]

\[
\tau_H = \rho\beta\hat{Q}(\omega_m)Q(\omega_m)\frac{1}{A_{\text{out}}} - \frac{1}{A_{\text{in}}} + \frac{Q(\omega_m)H\rho g}{\omega_m} - \frac{\rho Q^2(\omega_m)}{2\omega_m} \left(\frac{1}{A_{\text{out}}^2} - \frac{1}{A_{\text{in}}^2}\right) - \frac{\rho\alpha Q(\omega_m)^3}{4\omega_m} \left(\frac{1}{A_{\text{out}}^2} - \frac{1}{A_{\text{in}}^2}\right)
\]

Output equation:

\[
\vec{F}_{\text{thrust}} = \rho L\dot{Q}(\omega_m) + \left[\frac{A_{\text{impeller}}}{2} (\frac{1}{A_{\text{out}}^2} - \frac{1}{A_{\text{in}}^2}) + \beta (\frac{1}{A_{\text{out}}} - \frac{1}{A_{\text{in}}})\right] \rho Q(\omega_m) |Q(\omega_m)| - H\rho g A_{\text{impeller}} \tag{3.35}
\]

As it was explained in Section 2.3; the thrust force equation is derived from the conservation of linear momentum principle and expressed in terms of \(Q\) flow rate. Since the studied pump is a type of centrifugal pump, the relationship between angular velocity and the flow rate is tried to be expressed via centrifugal pump theory in the next section under certain assumptions.
3.3 Centrifugal Pump Assumption

Pump theory is explained, and then used to establish a relationship between flow rate and the angular velocity in this section.

Pump Theory

A pump’s main purpose is to add energy to a fluid. This added energy does not necessarily make fluids go faster but increases fluid pressure (Cengel and Cimbala, 2006 [8]). Although pumps can be classified in a wide range of types, basically it is possible to categorize them for two different kinds (Positive Displacement and Centrifugal). This categorization is related to how the pump transfers its mechanical energy to the working fluid. Positive displacement squeezes the working fluid by pistons or shaft rotation and it transfers the fluid by that way. On the other hand centrifugal pumps transfer their kinetic energy to the working fluid by a rotating impeller. Working fluids kinetic energy converts to pressure in pump passages and outlet (U.S.Hydraulic Institute, 2006 [9]).

As the studied pump is a type of centrifugal pump, centrifugal pump theory is expressed in the rest of this section. In an ideal centrifugal pump, flow comes into the pump from an inlet and spreads out from the eye of the impeller through the blades. The impeller is surrounded by a shroud. The impeller pushes this fluid radially throughout the case and water exits the pump from the outlet. Schematic of the pump is illustrated in Figure 3.2 [8].

![Centrifugal Pump Schematic](image)

Figure 3.2: Centrifugal Pump Schematic
Centrifugal pumps are also classified in terms of the blade geometry. These are backward-inclined, forward-inclined and straight blades. Backward inclined blades type is the most common as it yields the highest efficiency of the three. In terms of blade geometry, centrifugal pump impellers are depicted in Figure 3.3

Figure 3.3: a) Backward-Inclined, b) Forward-Inclined, c) Straight

Simply, the fluid flow passing the impeller eye moves in blade passages from the leading edge to the trailing edge. Most of the time, the impeller blade height decreases from the leading edge to the trailing edge. The flow-rate passing an imaginary circular area for the selected radius and the height must be the same according to the conservation of mass theorem. This relation is shown in Figure 3.4.

\[ Q = 2\pi r_1 b_1 V_{1,n} = 2\pi r_2 b_2 V_{2,n} \]  

(3.36)

Figure 3.4: Conservation of Mass on an Impeller
“It is assumed that the flow is everywhere tangent to the blade surface when viewed from a reference frame rotating with the blade. This assumption is also known as shockless entry condition which implies that there is no flow separation anywhere along the blade surface.” (Cengel and Cimbala, 2006 [8])

\[
V_{2,n} = V_{2,\text{relative}} \cdot \sin \beta_2 \quad V_{2,\text{relative}} = \frac{V_{2,n}}{\sin \beta_2} \quad (3.37)
\]

\[
V_{2,t} = \omega r_2 - V_{2,\text{relative}} \cdot \cos \beta_2 \quad (3.38)
\]
When Equation (3.38) is substituted with Equation (3.37);

$$V_{2,t} = \omega r_2 - \frac{V_{2,n}}{\tan \beta_2}$$  
(3.39)

As it is the same geometry for the leading edge;

$$V_{1,t} = \omega r_1 - \frac{V_{1,n}}{\tan \beta_1}$$  
$$V_{1,n} = (\omega r_1 - V_{1,t}) \cdot \tan \beta_1$$  
(3.40)

By replacing $V_{1,n}$ into conservation of mass formula represented in Equation (3.36), flow-rate is defined in terms of angular velocity;

$$Q = 2\pi r_1 b_1 (\omega r_1 - V_{1,t}) \cdot \tan \beta_1$$  
(3.41)

At this point Equation (3.41) is considered as a design flow-rate (Cengel and Cimbala, 2006 [8]), (White 1999 [10]), (Hughes and Brighton, 1999 [11]) and within this scope $V_{1,t}$ is taken “0” in this equation. Equation (3.41) is going to applied as it was done in Mehmet’s thesis [12]. This means flow enters exactly normal to the impeller from the eye. So $V_{1,n} = V_1$ and it yields to:

$$Q = 2\pi b_1 \omega r_1^2 \cdot \tan \beta_1$$  
(3.42)

Within this scope, Section 3.3 is recalled. It is assumed that the flow is everywhere tangent to the blade surface when viewed from a reference frame rotating with the blade. This assumption is also known as shockless entry condition which implies that there is no flow separation anywhere along the blade surface.

At this point Equation (3.41) is considered as a design flow-rate [10],[8],[11] and within this scope $V_{1,t}$ is taken “0” in this equation. This means flow enters exactly normal to the impeller from the eye. So Equation (3.42) is recalled to be convenient as

$$Q = 2\pi b_1 \omega_m r_1^2 \cdot \tan \beta_1$$

In order to simplify the Equation (3.42), $K = 2\pi b_1 r_1^2 \cdot \tan \beta_1$ as

$$Q = K \omega_m$$
According to this assumption and Equation (3.42), differential equations describing the in-line pump thrust generators are given by:

\[
\frac{di}{dt} = -\frac{R_a}{L_a} i(t) - \frac{K_b}{L_a} \omega_m(t) + \frac{1}{L_a} V(t) \tag{3.43}
\]

\[
\frac{d\omega_m}{dt} = -\frac{K_i}{J_m + \rho \beta K^2 \left( \frac{1}{A_{out}} - \frac{1}{A_{in}} \right)} i(t) - \frac{B_m}{J_m + \rho \beta K^2 \left( \frac{1}{A_{out}} - \frac{1}{A_{in}} \right)} \omega_m(t) - \frac{\rho K^3 \left( \frac{1}{A_{out}} - \frac{1}{A_{in}} \right) (2 - \alpha)}{4 (J_m + \rho \beta K^2 \left( \frac{1}{A_{out}} - \frac{1}{A_{in}} \right))} |\omega_m(t)| \omega_m(t) + \frac{KH \rho g}{J_m + \rho \beta K^2 \left( \frac{1}{A_{out}} - \frac{1}{A_{in}} \right)} \tag{3.44}
\]

where thrust force equation becomes

\[
\vec{F}_{thrust} = \rho L K \dot{\omega}_m(t) + \left( \frac{\rho K^2 A_{impeller}}{2} \left( \frac{1}{A_{out}^2} - \frac{1}{A_{in}^2} \right) + \rho \beta K^2 \left( \frac{1}{A_{out}^2} - \frac{1}{A_{in}^2} \right) |\omega_m(t)| \right) \omega_m(t) - H \rho g A_{impeller} \tag{3.45}
\]

In the state space formulation which will be suitable for applying control methods, we can write the equations as:
\[ \dot{x} = f(x, u) \]
\[ y = g(x, \dot{x}) \]

where

\[ x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} i \\ \omega_m \end{bmatrix} \]
\[ u = V \]

\[ f = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \]
\[ f_1 = -\frac{R_a}{L_a} x_1 - \frac{K_b}{L_a} x_2 + \frac{1}{L_a} u \]
\[ f_2 = \frac{K_i}{J_m + \rho \beta K^2 (\frac{1}{A_{out}} - \frac{1}{A_{in}})} x_1 - \frac{B_m}{J_m + \rho \beta K^2 (\frac{1}{A_{out}} - \frac{1}{A_{in}})} x_2 - \frac{\rho K^3 (\frac{1}{A_{out}} - \frac{1}{A_{in}}) (2 - \alpha)}{4(J_m + \rho \beta K^2 (\frac{1}{A_{out}} - \frac{1}{A_{in}}))} |x_2| x_2 + \frac{K_{H\rho g}}{J_m + \rho \beta K^2 (\frac{1}{A_{out}} - \frac{1}{A_{in}})} \]
\[ g = \rho L K \dot{x}_2 + [\frac{\rho K^2 A_{impeller}}{2} (\frac{1}{A_{out}^2} - \frac{1}{A_{in}^2}) + \rho \beta K^2 (\frac{1}{A_{out}} - \frac{1}{A_{in}})] |x_2| x_2 - H_{\rho g A_{impeller}} \]

The variables are recalled and listed below for convenience:

\[ x_1 = i_a(t) = \text{armature current} \quad K = 2\pi b_1 r_1 \tan \beta_1 = \text{flow rate constant} \]
\[ x_2 = \omega_m(t) = \text{rotor angular velocity} \quad b_1 = \text{impeller’s blade leading edge height} \]
\[ u = V(t) = \text{applied voltage} \quad r_1 = \text{impeller’s blade leading edge radius} \]
\[ R_a = \text{armature resistance} \quad \beta_1 = \text{impeller’s blade leading edge angle} \]
\[ L_a = \text{armature inductance} \quad \beta = \text{momentum correction factor} \]
\[ K_b = \text{back EMF constant} \quad \alpha = \text{energy correction factor} \]
\[ K_i = \text{torque constant} \quad A_{out} = \text{outlet area} \]
\[ B_m = \text{viscous-friction coefficient} \quad A_{in} = \text{inlet area} \]
\[ \rho = \text{density} \quad A_{\text{impeller}} = \text{impeller area} \]

\[ L = \text{pump length} \quad H = \text{pump net head} \]

The block diagram of the governing system of equations is drawn with respect to the state space formulation and shown in Figure 4.11.
Figure 3.6: The Block Diagram of the Governing Equations

\[ C_0 = \frac{R_a}{L_a} \]
\[ C_1 = \frac{K_b}{L_a} \]
\[ C_2 = \frac{1}{L_a} \]
\[ C_3 = \frac{K_i}{J_m + \rho \beta K^2 (\frac{1}{A_{out}} - \frac{1}{A_{in}})} \]
\[ C_4 = \frac{B_m}{J_m + \rho \beta K^2 (\frac{1}{A_{out}} - \frac{1}{A_{in}})} \]
\[ C_5 = \frac{\rho K^3 (\frac{1}{A_{out}} - \frac{1}{A_{in}})(2 - \alpha)}{4(J_m + \rho \beta K^2 (\frac{1}{A_{out}} - \frac{1}{A_{in}}))} \]
\[ C_6 = \frac{K}{J_m + \rho \beta K^2 (\frac{1}{A_{out}} - \frac{1}{A_{in}})} \]
\[ C_7 = \rho L K \]
\[ C_8 = \left[ \frac{\rho K^2 A_{impeller}}{2} (\frac{1}{A_{out}^2} - \frac{1}{A_{in}^2}) + \rho \beta K^2 (\frac{1}{A_{out}} - \frac{1}{A_{in}}) \right] \]
\[ C_9 = A_{impeller} \]
Comment about consequences of using the centrifugal pump assumption:

- If true, there is no need to use a sensor to measure flow rate for feedback since the flow rate is truly proportional to pump’s angular velocity.

- Not all in-line centrifugal pumps (such as ours) necessarily satisfy the assumption as it is studied in the next chapter.

- It follows that it is important to design in-line pumps with respect to centrifugal pump theory in order to obtain dynamics amenable to closed loop feedback control.
Chapter 4

Flow Simulation

The centrifugal pump theory is applied to our pump in this chapter. To verify the theory and question its applicability to the in-line pump, two different models are studied. The ideal centrifugal pump and the commercial iL500P pump are shown in Figure 4.1.

The most important difference between the iL500P pump and ideal centrifugal pump is that the iL500P pump’s impeller is not surrounded by a shroud. Because of that, an inlet flow comes onto the impeller instead of spreading out from an impeller eye. An inlet tube is designed to get rid of this condition in the ideal centrifugal pump. Flow diagrams for both the ideal centrifugal pump and iL500P pump are represented in Figure 4.1.

In this section one commercial CFD solver is used to model the fluid flow and calculate the velocities on the impeller. SolidWorks Flow Simulation 2015 is selected as a solver because its limitations (Technical Reference[13]) allow for rotational part studies.

First of all; after the in-line pump parts are measured, all six parts are modeled in SolidWorks 2015. To avoid irregular meshing, some parts are modified slightly. For example, the rear part’s threaded section is flattened and mated with the outer case as a close fit. Also, the threaded part of the outer case is removed. This modification made the design smoother for meshing. After some modifications, the parts are assembled together. In addition to these parts, one more part is designed to encapsulate the impeller. It will be called “capsule” for the rest of this section. In flow simulation, rotation is simulated by a rotating area around the real rotating part, and for this purpose the capsule is designed around the impeller.
Figure 4.1: Centrifugal In-line Pump
Before running the studies, boundary conditions need to be set. One of these boundary conditions is the rotation of the impeller. As there is no way to predict the rotations for different voltages, a rotation measurement setup is constructed. A DC motor is extracted from a Rule 500P submersible in-line pump and fixed on a table with a C-clamp. The extracted DC motor is connected to a power supply that is also used during the pendulum arm experiments. During initial experiments, it is recorded that flowing amperage does not show the same amount of amperage that was recorded in the pendulum arm experiments. Therefore, rotations had to be measured under certain loads. For this reason, two leather rubbers are used to brake the shaft and add a load onto the impeller. These leather brake pads are squeezed by a needle nose until the current reaches the same amperage as the pendulum arm experiments. At that time, impeller revolution is measured by a tachometer. The revolution approximation experiment picture is shown in Figure 4.2.

Figure 4.2: Revolution Approximation Experiment
Impeller revolutions measured by a tachometer shown in Figure 4.3 are recorded and averaged after several experiments. The boundary condition in simulations is approximated. The tachometer’s work principle is defined in the instrument’s manual [14] as it measures the rate at which light is reflected back onto the instrument by a reflective tape stuck on the impeller blade. Measurements are tabulated in Table 4.1.

![Monarch Pocket Tach 100](image)

**Figure 4.3: Monarch Pocket Tach 100**

**Table 4.1: Measured Impeller Revolutions**

<table>
<thead>
<tr>
<th>Voltages (V)</th>
<th>Rotation (RPM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>10,125</td>
</tr>
<tr>
<td>11</td>
<td>9,185</td>
</tr>
<tr>
<td>10</td>
<td>8,495</td>
</tr>
<tr>
<td>9</td>
<td>6,990</td>
</tr>
<tr>
<td>8</td>
<td>6,650</td>
</tr>
<tr>
<td>7</td>
<td>5,200</td>
</tr>
<tr>
<td>6</td>
<td>3,980</td>
</tr>
</tbody>
</table>

For both studies, the ideal centrifugal and iL500P pumps, 3 different revolutions per minute (RPM) values (10,000, 7,000 and 4,000) are used, and regression analysis is done with respect to these revolutions.
4.1 Ideal Centrifugal Pump Model

The ideal centrifugal pump is designed to verify the centrifugal pump theory. The iL500P pump was shown in Section 2.1. Design differences will be illustrated by model pictures in Section 4.1 and ideal centrifugal pump studies are discussed in Section 4.1. The fully assembled ideal centrifugal pump is represented in Figure 4.4.

**Figure 4.4: Ideal Centrifugal Pump**

**Ideal Centrifugal Pump Parts**

**Inlet Tube of the Ideal Centrifugal Pump**

By definition, a flow that comes in from the inlet spreads out from the eye of the impeller through blades in an ideal centrifugal pump. As a result, one inlet tube is designed and placed on top of the impeller so as to provide the mentioned flow path for the inlet flow. This inlet tube is illustrated in Figure 4.5.
Outer Case of the Ideal Centrifugal Pump

The outer case needs to be modified as the inlet tube radius differs from the iL500P pump measurement. Also the threaded part in the outer case is flattened to make the model smoother for meshing. The outer case of ideal centrifugal pump is shown in Figure 4.6 and Figure 4.7.
Impeller of the Ideal Centrifugal Pump

In the ideal centrifugal pump, the impeller is designed as the same blade counts as the iL500P pump impeller. Blades are modeled thinner to reduce the blade effect on fluid area calculation for this design. Blade height decreases linearly just as the iL500P pump impeller does.

![Figure 4.8: Impeller of the Ideal Centrifugal Pump](image)

Stator of the Ideal Centrifugal Pump

The stator in the iL500P pump was shown in Figure 2.4. The stator blocks the flow path after the impeller in the this pump. For this reason, the ideal centrifugal pump stator around the impeller is designed and represented in Figure 4.9.

![Figure 4.9: Stator of the Ideal Centrifugal Pump](image)
Rear Part of Ideal Centrifugal Pump

During the initial studies, the threaded part on the rear part caused some irregular mesh. For this reason mesh generation abruptly terminated. To avoid irregular meshing, the rear part of the ideal centrifugal pump is designed and shown in Figure 4.10.

![Figure 4.10: Rear Part of the Ideal Centrifugal Pump](image)

Ideal Centrifugal Pump Studies

After ideal centrifugal pump parts are designed and assembled together, boundary conditions are set for the simulation. Initially, both the inlet and outlet are set to atmospheric pressure. An averaging mesh is not applicable as inlet flow-rate is not set as a boundary condition for the inlet area. For this reason, sliding mesh is selected as a mesh generation type to be sure about inlet-outlet flow rate equality (conservation of mass). A rotating part is selected as the capsule part shown in Figure 4.12. Since the capsule touches both the stator and inlet tube, these touched parts are defined as real walls. The aim of this study is not only to verify the centrifugal pump theory and question the flow-rate with respect to the angular velocity, but also to question the iL500P pump in the same fashion. For this reason, instead of no slip walls (ideal walls), adiabatic walls are selected as a wall condition. Each study result is represented in related sections. With respect to these
results, calculations and comparison between different flow-rate values are represented in Section 4.1.

**Ideal Centrifugal Pump Study : 10,000 RPM**

All of the boundary conditions are represented in Figure 4.11. The computational domain is shown as a tetragonal prism surrounding the pump. Square spotted areas (the inlet tube and stator) represent real walls. The green rotating arrow shows the rotation direction. Environment pressures are also shown on both the inlet and outlet. The angular velocity is automatically converted and written in rad/s.

Figure 4.11: Ideal Centrifugal Pump Study : 10,000 RPM Boundary Conditions

To run a rotational study in SolidWorks Flow Simulation 2015, the capsule is defined as a rotating area that is blue-highlighted in Figure 4.12.

Figure 4.12: Ideal Centrifugal Pump 10,000 RPM Study Rotating Boundary Condition
In SolidWorks 2015, flow rate is measured by selecting an area. Therefore, two rings are designed to measure the flow rate on desired points. The first ring is an inner circle, which touches the leading edges of the blades. The second one is an outer circle, which touches the trailing edges of the blades. Measurement rings are shown in Figure 4.17. During initial studies, an error occurred on the inner circle related to flow-rate. The flow-rate on the inner circle is calculated different than the inlet flow-rate. This error rate is recorded around 7%. This error takes its source from interpolation error. As our mesh cells are tetragonal, when circular area is observed, cells are cut from one side irregularly. For this reason, calculation depends on an interpolation on that curve. To improve the resolution on the impeller and reduce the error rate, a local initial mesh is assigned on the impeller and its level is set to 8 out of 8. After the local initial mesh level on the impeller is increased, the error rate is recorded at less than 4%. The complete fluid mesh of the ideal centrifugal pump is represented in Figure 4.14 and the local initial mesh is illustrated in Figure 4.13.

![Figure 4.13: Local Initial Mesh On Impeller](image-url)
Table 4.2: Ideal Centrifugal Pump Study: 10,000 RPM Analysis Mesh Settings

<table>
<thead>
<tr>
<th>Setting</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cell count</td>
<td>542,901</td>
</tr>
<tr>
<td>Fluid Cells</td>
<td>266,125</td>
</tr>
<tr>
<td>Solid Cells</td>
<td>112,872</td>
</tr>
<tr>
<td>Partial Cells</td>
<td>163,904</td>
</tr>
<tr>
<td>Trimmed Cells</td>
<td>0</td>
</tr>
</tbody>
</table>

The flow study of the 10,000 RPM is run and completed after mesh generation that its settings are tabulated in Table 4.2. Flow trajectory is shown in Figure 4.15.
A cut plot is captured on the impeller to observe the flow velocity vectors and magnitudes. In Figure 4.16, contours show velocity magnitude while vectors represent velocity direction, which has three components (Axial, Circumferential and Radial). The centrifugal pump theory was explained in Section 3.3 and one assumption was made: $V_{t,1} = 0$. It is seen in Figure 4.16 that the tangential velocity contribution over the leading edge is very small when it is compared to the radial velocity. For this reason velocity vectors are radial on that area.

![Figure 4.16: Ideal Centrifugal Pump Study : 10,000 RPM Cut Plot](image)

Two rings are used to measure flow rate over leading and trailing edges. These rings are blue highlighted areas shown in Figure 4.17.

![Figure 4.17: Measurement Rings](image)
Measurement results are tabulated in Table 4.3.

Table 4.3: Ideal Centrifugal Pump Study : 10,000 RPM Flow Simulation Results

<table>
<thead>
<tr>
<th></th>
<th>Inner Circle ((r_1 = 3.8\text{mm}))</th>
<th>Outer Circle ((r_2 = 12.5\text{mm}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity ([m/s])</td>
<td>8.4231</td>
<td>6.9634</td>
</tr>
<tr>
<td>Axial Velocity ([m/s])</td>
<td>6.1648</td>
<td>0.2602</td>
</tr>
<tr>
<td>Radial Velocity ([m/s])</td>
<td>4.4881</td>
<td>1.5078</td>
</tr>
<tr>
<td>Circumferential Velocity ([m/s])</td>
<td>0.4513</td>
<td>6.5042</td>
</tr>
<tr>
<td>Volume Flow Rate ([m^3/s])</td>
<td>0.000491</td>
<td>0.000504</td>
</tr>
<tr>
<td>Area (Fluid) ([m^2])</td>
<td>0.00010949</td>
<td>0.00033476</td>
</tr>
</tbody>
</table>

**Ideal Centrifugal Pump Study : 7,000 RPM**

With the exception of the angular velocity, boundary conditions are set in the same way as the ideal centrifugal pump 10,000 RPM study (shown in Figure 4.11). Therefore only angular velocity \((\text{rad/s})\) is shown in Figure 4.18.

Figure 4.18: Ideal Centrifugal Pump Study : 7,000 RPM Rotating Boundary Condition

The fluid mesh and local initial mesh conditions are preserved for the ideal centrifugal pump 7,000 RPM study. The mesh settings are presented in Table 4.4
Table 4.4: Ideal Centrifugal Pump Study : 7,000 RPM Analysis Mesh Settings

<table>
<thead>
<tr>
<th>Total Cell count</th>
<th>542,768</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid Cells</td>
<td>265,997</td>
</tr>
<tr>
<td>Solid Cells</td>
<td>112,842</td>
</tr>
<tr>
<td>Partial Cells</td>
<td>163,929</td>
</tr>
<tr>
<td>Trimmed Cells</td>
<td>0</td>
</tr>
</tbody>
</table>

After the 7,000 RPM ideal centrifugal pump flow study is completed, flow trajectory is shown in Figure 4.19.

In the same fashion as with the 10,000 RPM observation, a cut plot on the impeller is taken from the results and represented in Figure 4.20. Vectors on the inner circle also seemed normal to the circular sectional area. The plotted data palette maintains the same values to make comparison easier. Therefore there is no red contour on the Figure 4.20.
The same measurement ring (Figure 4.17) is used for measurements, and results are tabulated in Table 4.5.

Table 4.5: Ideal Centrifugal Pump Study : 7,000 RPM Flow Simulation Results

<table>
<thead>
<tr>
<th></th>
<th>Inner Circle ($r_1 = 3.8mm$)</th>
<th>Outer Circle ($r_2 = 12.5mm$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity [m/s]</td>
<td>5.8673</td>
<td>4.8781</td>
</tr>
<tr>
<td>Axial Velocity [m/s]</td>
<td>4.2921</td>
<td>0.1694</td>
</tr>
<tr>
<td>Radial Velocity [m/s]</td>
<td>3.1342</td>
<td>1.0545</td>
</tr>
<tr>
<td>Circumferential Velocity [m/s]</td>
<td>0.3142</td>
<td>4.5619</td>
</tr>
<tr>
<td>Volume Flow Rate [m$^3$/s]</td>
<td>0.000343</td>
<td>0.000353</td>
</tr>
<tr>
<td>Area (Fluid) [m$^2$]</td>
<td>0.00010955</td>
<td>0.00033500</td>
</tr>
</tbody>
</table>
Ideal Centrifugal Pump Study : 4,000 RPM

Only the angular velocity is changed from the ideal centrifugal pump 10,000 RPM study boundary conditions (represented in Figure 4.11). The angular velocity is set to 4,000 RPM and the software automatically changed it to \( \text{rad/s} \) and it is shown in Figure 4.21.

![Angular Velocity](image)

Figure 4.21: Ideal Centrifugal Pump Study : 4,000 RPM Rotating Boundary Condition

The local initial and regular mesh levels are not changed from the previous studies. Mesh properties for this study are tabulated in Table 4.6

<table>
<thead>
<tr>
<th>Mesh Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
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<td>Total Cell count</td>
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</tr>
<tr>
<td>Fluid Cells</td>
<td>265,997</td>
</tr>
<tr>
<td>Solid Cells</td>
<td>112,842</td>
</tr>
<tr>
<td>Partial Cells</td>
<td>163,929</td>
</tr>
<tr>
<td>Trimmed Cells</td>
<td>0</td>
</tr>
</tbody>
</table>
Flow trajectory of the ideal centrifugal pump 4,000 RPM study is depicted in Figure 4.22.

An ideal centrifugal pump 4,000 RPM cut plot is depicted in Figure 4.23. As rotation counts decrease, velocity values also behaved in the same fashion. This can be seen from the cut plot contour color changes. There is no red contour in both 7,000 and 4,000 RPM studies. Even the yellow contour is not seen in the ideal centrifugal pump 4,000 RPM study cut plot.
The measurement ring (Figure 4.17) is used to collect data over the ring, and results are tabulated in Table 4.7.

### Table 4.7: Ideal Centrifugal Pump Study: 4,000 RPM Flow Simulation Results

<table>
<thead>
<tr>
<th></th>
<th>Inner Circle ((r_1 = 3.8 mm))</th>
<th>Outer Circle ((r_2 = 12.5 mm))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity ([m/s])</td>
<td>3.3110</td>
<td>2.8026</td>
</tr>
<tr>
<td>Axial Velocity ([m/s])</td>
<td>2.4127</td>
<td>0.0745</td>
</tr>
<tr>
<td>Radial Velocity ([m/s])</td>
<td>1.7790</td>
<td>0.6009</td>
</tr>
<tr>
<td>Circumferential Velocity ([m/s])</td>
<td>0.1810</td>
<td>2.6292</td>
</tr>
<tr>
<td>Volume Flow Rate ([m^3/s])</td>
<td>0.000194</td>
<td>0.000201</td>
</tr>
<tr>
<td>Area (Fluid) ([m^2])</td>
<td>0.00010955</td>
<td>0.00033500</td>
</tr>
</tbody>
</table>

### Calculations

A flow rate refers to a volume of fluid that passes per unit time. In other words, flow rate is the integration of velocities over an area. In the mean time, the conservation of mass principle states that flow-rate must be the same on cross sectional areas in a ducted volume. Therefore, pump theory is applied to the ideal centrifugal pump and results are compared by table in this section.

Two different circular areas are defined to be used as an area in the flow-rate equation. One is the conventional circular area, and the second one is a three blade thickness’ subtracted area. Blade thickness and height are illustrated in Figure 4.24 as thickness \((t = 0.5 \text{ mm})\) and height \((b_1 = 4.9 \text{ mm}, b_2 = 4 \text{ mm})\).
Before defining how to calculate 6 flow-rate values, the flow-rate’s conventional equation is recalled for convenience ($V_n =$ Normal Velocity).

\[ Q = V_n \cdot A \] (4.1)

True flow-rate is identified as $Q_1$. $Q_2$ is derived from Equation (4.1). $Q_{2,A}$ includes 3 blade thickness’ but in $Q_{2,B}$ blade thickness’ are extracted.

\[ Q_{2,A} = (2\pi r_n b_n)V_n \] (4.2)
\[ Q_{2,B} = (2\pi r_n b_n - 3b_n t_n)V_n \] (4.3)

$Q_3$ is derived from the pump theory without assuming “$V_t = 0$” in the same fashion as Equation (3.41). Blades thickness’ are included in $Q_{3,A}$ but extracted in $Q_{3,B}$ as

\[ Q_{3,A} = 2\pi r_n b_n(\omega r_n - V_{n,t}) \tan \beta_n \] (4.4)
\[ Q_{3,B} = (2\pi r_n b_n - 3b_n t_n)(\omega r_n - V_{n,t}) \tan \beta_n \] (4.5)

$Q_4$ is derived from the Equation (3.42). $V_t$ is assumed to be equal to “0”. Blades thickness’ are included in $Q_{4,A}$ but extracted in $Q_{4,B}$ as

\[ Q_{4,A} = 2\pi r_n b_n \omega r_n \tan \beta_n \] (4.6)
\[ Q_{4,B} = (2\pi r_n b_n - 3b_n t_n)\omega r_n \tan \beta_n \] (4.7)

Blades angles are shown in Figure 4.25 as $\beta_1 = 46.3^\circ$ and $\beta_2 = 38.9^\circ$. Each flow-rate equation is calculated and tabulated in Table 4.8 and Table 4.9 by comparison to the true flow rate.
Table 4.8: Ideal Centrifugal Pump Inner Ring Calculations with Blade Angles

<table>
<thead>
<tr>
<th>$\omega$ (RPM)</th>
<th>$Q_1$ (m³/s)</th>
<th>$Q_{2,A}$ (m³/s)</th>
<th>% Error Rate</th>
<th>$Q_{2,B}$ (m³/s)</th>
<th>% Error Rate</th>
<th>$Q_{3,A}$ (m³/s)</th>
<th>% Error Rate</th>
<th>$Q_{3,B}$ (m³/s)</th>
<th>% Error Rate</th>
<th>$Q_{4,A}$ (m³/s)</th>
<th>% Error Rate</th>
<th>$Q_{4,B}$ (m³/s)</th>
<th>% Error Rate</th>
<th>$V_n$ (m/s)</th>
<th>$V_t$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000</td>
<td>0.000511</td>
<td>0.000525</td>
<td>-2.8</td>
<td>0.000492</td>
<td>3.6</td>
<td>0.000432</td>
<td>15.4</td>
<td>0.000405</td>
<td>20.7</td>
<td>0.000487</td>
<td>4.6</td>
<td>0.000457</td>
<td>10.6</td>
<td>4.4882</td>
<td>0.4513</td>
</tr>
<tr>
<td>7,000</td>
<td>0.000357</td>
<td>0.000367</td>
<td>-2.6</td>
<td>0.000344</td>
<td>3.8</td>
<td>0.000303</td>
<td>15.3</td>
<td>0.000284</td>
<td>20.6</td>
<td>0.000341</td>
<td>4.5</td>
<td>0.000320</td>
<td>10.5</td>
<td>3.1343</td>
<td>0.3143</td>
</tr>
<tr>
<td>4,000</td>
<td>0.000203</td>
<td>0.000208</td>
<td>-2.6</td>
<td>0.000195</td>
<td>3.8</td>
<td>0.000173</td>
<td>14.8</td>
<td>0.000162</td>
<td>20.2</td>
<td>0.000195</td>
<td>3.9</td>
<td>0.000183</td>
<td>9.9</td>
<td>1.7791</td>
<td>0.1810</td>
</tr>
</tbody>
</table>

Table 4.9: Ideal Centrifugal Pump Outer Ring Calculations with Blade Angles

<table>
<thead>
<tr>
<th>$\omega$ (RPM)</th>
<th>$Q_1$ (m³/s)</th>
<th>$Q_{2,A}$ (m³/s)</th>
<th>% Error Rate</th>
<th>$Q_{2,B}$ (m³/s)</th>
<th>% Error Rate</th>
<th>$Q_{3,A}$ (m³/s)</th>
<th>% Error Rate</th>
<th>$Q_{3,B}$ (m³/s)</th>
<th>% Error Rate</th>
<th>$Q_{4,A}$ (m³/s)</th>
<th>% Error Rate</th>
<th>$Q_{4,B}$ (m³/s)</th>
<th>% Error Rate</th>
<th>$V_n$ (m/s)</th>
<th>$V_t$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000</td>
<td>0.000511</td>
<td>0.000515</td>
<td>-0.9</td>
<td>0.000505</td>
<td>1.0</td>
<td>0.001816</td>
<td>-255.6</td>
<td>0.001781</td>
<td>-248.8</td>
<td>0.003609</td>
<td>-606.7</td>
<td>0.003540</td>
<td>-593.2</td>
<td>1.5078</td>
<td>6.5042</td>
</tr>
<tr>
<td>7,000</td>
<td>0.000357</td>
<td>0.000360</td>
<td>-0.9</td>
<td>0.000353</td>
<td>1.1</td>
<td>0.001268</td>
<td>-255.1</td>
<td>0.001244</td>
<td>-248.3</td>
<td>0.002526</td>
<td>-607.1</td>
<td>0.002478</td>
<td>-593.6</td>
<td>1.0546</td>
<td>4.5619</td>
</tr>
<tr>
<td>4,000</td>
<td>0.000203</td>
<td>0.000205</td>
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<td>0.000201</td>
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<td>0.000719</td>
<td>-254.4</td>
<td>0.000705</td>
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<td>0.001443</td>
<td>-611.8</td>
<td>0.001416</td>
<td>-598.2</td>
<td>0.6009</td>
<td>2.6292</td>
</tr>
</tbody>
</table>

After flow rate calculations are completed, it is seen that blade angle is very sensitive and especially for the outer ring it must be redefined with respect to the deviation angle (Brennen, 2011 [15]).
Table 4.10: Ideal Centrifugal Pump Inner Ring Calculations with Deviation Angle Modified Blade Angles

<table>
<thead>
<tr>
<th>$\omega$ (RPM)</th>
<th>$Q_1$ (m$^3$/s)</th>
<th>$Q_{2,A}$ (m$^3$/s)</th>
<th>% Error Rate</th>
<th>$Q_{2,B}$ (m$^3$/s)</th>
<th>% Error Rate</th>
<th>$Q_{3,A}$ (m$^3$/s)</th>
<th>% Error Rate</th>
<th>$Q_{3,B}$ (m$^3$/s)</th>
<th>% Error Rate</th>
<th>$Q_{4,A}$ (m$^3$/s)</th>
<th>% Error Rate</th>
<th>$Q_{4,B}$ (m$^3$/s)</th>
<th>% Error Rate</th>
<th>$V_n$ (m/s)</th>
<th>$V_t$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000</td>
<td>0.000511</td>
<td>0.000525</td>
<td>-2.8</td>
<td>0.000492</td>
<td>3.6</td>
<td>0.000525</td>
<td>-2.7</td>
<td>0.000492</td>
<td>3.7</td>
<td>0.000592</td>
<td>-15.9</td>
<td>0.000554</td>
<td>-8.6</td>
<td>4.4882</td>
<td>0.4513</td>
</tr>
<tr>
<td>7,000</td>
<td>0.000357</td>
<td>0.000367</td>
<td>-2.6</td>
<td>0.000344</td>
<td>3.8</td>
<td>0.000367</td>
<td>-2.8</td>
<td>0.000344</td>
<td>3.6</td>
<td>0.000414</td>
<td>-15.9</td>
<td>0.000388</td>
<td>-8.6</td>
<td>3.1343</td>
<td>0.3143</td>
</tr>
<tr>
<td>4,000</td>
<td>0.000203</td>
<td>0.000208</td>
<td>-2.6</td>
<td>0.000195</td>
<td>3.8</td>
<td>0.000210</td>
<td>-3.4</td>
<td>0.000197</td>
<td>3.1</td>
<td>0.000237</td>
<td>-16.7</td>
<td>0.000222</td>
<td>-9.4</td>
<td>1.7791</td>
<td>0.1810</td>
</tr>
</tbody>
</table>

Table 4.11: Ideal Centrifugal Pump Outer Ring Calculations with Deviation Angle Modified Blade Angles

<table>
<thead>
<tr>
<th>$\omega$ (RPM)</th>
<th>$Q_1$ (m$^3$/s)</th>
<th>$Q_{2,A}$ (m$^3$/s)</th>
<th>% Error Rate</th>
<th>$Q_{2,B}$ (m$^3$/s)</th>
<th>% Error Rate</th>
<th>$Q_{3,A}$ (m$^3$/s)</th>
<th>% Error Rate</th>
<th>$Q_{3,B}$ (m$^3$/s)</th>
<th>% Error Rate</th>
<th>$Q_{4,A}$ (m$^3$/s)</th>
<th>% Error Rate</th>
<th>$Q_{4,B}$ (m$^3$/s)</th>
<th>% Error Rate</th>
<th>$V_n$ (m/s)</th>
<th>$V_t$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000</td>
<td>0.000511</td>
<td>0.000515</td>
<td>-0.9</td>
<td>0.000515</td>
<td>1.0</td>
<td>0.000515</td>
<td>-0.9</td>
<td>0.000515</td>
<td>1.0</td>
<td>0.001024</td>
<td>-100.6</td>
<td>0.001005</td>
<td>-96.8</td>
<td>1.5078</td>
<td>6.5042</td>
</tr>
<tr>
<td>7,000</td>
<td>0.000357</td>
<td>0.000360</td>
<td>-0.9</td>
<td>0.000353</td>
<td>1.1</td>
<td>0.000360</td>
<td>-0.8</td>
<td>0.000353</td>
<td>1.1</td>
<td>0.000717</td>
<td>-100.7</td>
<td>0.000703</td>
<td>-96.9</td>
<td>1.0546</td>
<td>4.5619</td>
</tr>
<tr>
<td>4,000</td>
<td>0.000203</td>
<td>0.000205</td>
<td>-1.2</td>
<td>0.000201</td>
<td>-0.6</td>
<td>0.000204</td>
<td>-0.6</td>
<td>0.000200</td>
<td>1.3</td>
<td>0.000410</td>
<td>-102.0</td>
<td>0.000402</td>
<td>-98.2</td>
<td>0.6009</td>
<td>2.6292</td>
</tr>
</tbody>
</table>

After several studies in which the deviation angle changed, trailing and leading edge angles are found: $\beta_1 = 51.8^\circ$ and $\beta_2 = 12.9^\circ$. With respect to $Q_{4,B}$, flow rate is found by angular velocity with error rate of less than 9%.
Conclusion of the Ideal Centrifugal Pump Studies

The ideal centrifugal pump is designed and studied to verify the centrifugal pump theory. Results are collected for three different revolutions (10,000, 7,000 and 4,000 RPM). As the centrifugal pump theory states, the tangential velocity contribution to the fluid velocity is very small on the inner ring (encircling blade leading edges) when it is compared to the radial velocity at the same point. This condition is observed for all the revolutions. Within this scope, the velocity on the leading edge (inner ring) seems 100% radial as it was represented in Figure 4.16, Figure 4.20 and Figure 4.23. After studies are run and completed, flow-rates are calculated with respect to these study results. The most valuable flow rate \(Q_{4B}\) calculations are showed as red colored cells in Table 4.10 for inner circle. These results mean that the flow-rate is measured with an error rate less than 9% via the angular velocity. The same conditions are applied to the iL500P pump in next section to question the applicability of the centrifugal pump theory on it.

4.2 iL500P Pump Model

iL500P Pump Studies

Rule iL500P submersible in-line pump is used as it is modeled and shown in Section 2.1 after two modifications. The threaded sections for both the rear part and outer case are flattened since they caused mesh generation termination during initial studies. So the modified rear part (Figure 4.10) and outer case’s back side (Figure 4.7) are used in these studies.

iL500P Pump Study : 10,000 RPM

Atmospheric pressure is selected as boundary conditions for both the inlet and outlet. As there is no inlet tube in the iL500P pump, only the stator is selected as a real wall boundary condition illustrated as square spotted in Figure 4.26. The pump and boundary conditions are represented in Figure 4.26.
The capsule is used to simulate the impeller rotation. It is placed on the impeller in the same fashion as the ideal centrifugal pump studies. Blue-highlighted area (shown in Figure 4.27) represents the capsule. 10,000 RPM is converted to \( \text{rad/s} \) value by the software.

A local initial mesh is not used because the mesh resolution is not considered as an error factor for this study. The mesh level is set to 6 out of 8. The regular mesh level properties of this study are tabulated in Table 4.6 and the fluid mesh is shown in Figure 4.28.
Table 4.12: iL500P Pump Study: 10,000 RPM Analysis Mesh Settings

<table>
<thead>
<tr>
<th>Total Cell count</th>
<th>1,304,500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid Cells</td>
<td>529,264</td>
</tr>
<tr>
<td>Solid Cells</td>
<td>361,330</td>
</tr>
<tr>
<td>Partial Cells</td>
<td>413,906</td>
</tr>
<tr>
<td>Trimmed Cells</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 4.28: iL500P Pump Fluid Mesh

Flow trajectory of the iL500P pump 10,000 RPM study is demonstrated in Figure 4.29.

Figure 4.29: iL500P Pump Study: 10,000 RPM Flow Trajectory
The iL500P pump 10,000 RPM study is completed to question the flow rate on leading and trailing edges. A cut plot on the impeller is shown in Figure 4.30. On the cut plot, contours represent velocity magnitudes and vectors stand for velocity directions.

Figure 4.30: iL500P Pump Study : 10,000 RPM RPM Cut Plot

The flow rate is measured by two rings that are put as imaginary rings touch leading and trailing edges. These rings are shown in Figure 4.31 as blue-highlighted.

Figure 4.31: Flow Rate Measurement Rings
The iL500P pump 10,000 RPM study flow simulation results are tabulated in Table 4.13.

Table 4.13: iL500P Pump Study : 10,000 RPM Flow Simulation Results

<table>
<thead>
<tr>
<th></th>
<th>Inner Circle ($r_1 = 5.81 mm$)</th>
<th>Outer Circle ($r_2 = 12.5 mm$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity [m/s]</td>
<td>3.8269</td>
<td>5.5869</td>
</tr>
<tr>
<td>Axial Velocity [m/s]</td>
<td>2.7474</td>
<td>2.6443</td>
</tr>
<tr>
<td>Radial Velocity [m/s]</td>
<td>1.5999</td>
<td>1.6784</td>
</tr>
<tr>
<td>Circumferential Velocity [m/s]</td>
<td>0.7638</td>
<td>4.2925</td>
</tr>
<tr>
<td>Volume Flow Rate [m$^3$/s]</td>
<td>0.000207</td>
<td>0.000417</td>
</tr>
<tr>
<td>Area (Fluid) [m$^2$]</td>
<td>0.00012919</td>
<td>0.00024864</td>
</tr>
</tbody>
</table>

iL500P Pump Study : 7,000 RPM

The angular velocity is the only boundary condition that is changed from the iL500P pump 10,000 RPM study (shown in Figure 4.26). For this reason only the angular velocity (rad/s) is shown in Figure 4.32.

Figure 4.32: iL500P Pump Study : 7,000 RPM Rotating Boundary Condition
The fluid mesh condition level is increased for the iL500P pump 7,000 RPM study. Mesh properties are tabulated in Table 4.14.

<p>| | |</p>
<table>
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<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cell count</td>
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<td>Fluid Cells</td>
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<tr>
<td>Solid Cells</td>
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<tr>
<td>Partial Cells</td>
<td>1,629,737</td>
</tr>
<tr>
<td>Trimmed Cells</td>
<td>15</td>
</tr>
</tbody>
</table>

Flow trajectory of the iL500P pump 7,000 RPM study is shown in Figure 4.33.

A cut plot is used to observe the flow path on the impeller and see the velocities’ magnitudes. In Figure 4.34 contours represents velocity magnitudes and vectors are representing velocity directions.
Figure 4.34: iL500P Pump Study : 7,000 RPM Cut Plot

The same measurement ring (Figure 4.31) is used for measurement and the results are tabulated in Table 4.15.

Table 4.15: iL500P Pump Study : 7,000 RPM Flow Simulation Results

<table>
<thead>
<tr>
<th></th>
<th>Inner Circle ($r_1 = 5.81mm$)</th>
<th>Outer Circle ($r_2 = 12.5mm$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity [m/s]</td>
<td>2.8821</td>
<td>4.0121</td>
</tr>
<tr>
<td>Axial Velocity [m/s]</td>
<td>2.1373</td>
<td>1.8483</td>
</tr>
<tr>
<td>Radial Velocity [m/s]</td>
<td>1.1453</td>
<td>1.1950</td>
</tr>
<tr>
<td>Circumferential Velocity [m/s]</td>
<td>0.5018</td>
<td>3.0853</td>
</tr>
<tr>
<td>Volume Flow Rate [m³/s]</td>
<td>0.000146</td>
<td>0.000297</td>
</tr>
<tr>
<td>Area (Fluid) [m²]</td>
<td>0.0001276</td>
<td>0.00024864</td>
</tr>
</tbody>
</table>
iL500P Pump Study : 4,000 RPM

The angular velocity is changed from 10,000 RPM boundary conditions represented in Figure 4.26 for 4,000 RPM study. The angular velocity is set to 4,000 RPM. It is shown in Figure 4.35 and written in unit of (rad/s).

Figure 4.35: iL500P Pump Study : 4,000 RPM Rotating Boundary Condition

After several studies with 4,000 RPM angular velocity, the mesh level is decided to set 6 out of 8 for this study. Mesh properties are tabulated in Table 4.16

Table 4.16: iL500P Pump Study : 4,000 RPM Analysis Mesh Settings

<table>
<thead>
<tr>
<th>Total Cell count</th>
<th>1,304,500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid Cells</td>
<td>529,264</td>
</tr>
<tr>
<td>Solid Cells</td>
<td>361,330</td>
</tr>
<tr>
<td>Partial Cells</td>
<td>413,906</td>
</tr>
<tr>
<td>Trimmed Cells</td>
<td>0</td>
</tr>
</tbody>
</table>
Flow trajectory of the iL500P pump 4,000 RPM study is illustrated in Figure 4.36.

The iL500P pump 4,000 RPM study impeller cut plot is shown in Figure 4.37. Velocity magnitudes are seemed to decrease with respect to the change in the angular velocity of the impeller. This velocity decrease can be easily identified by comparing velocity contours with respect to the color palette.

Figure 4.36: iL500P Pump Study : 4,000 RPM Flow Trajectory

Figure 4.37: iL500P Pump Study : 4,000 RPM Cut Plot
Measurement ring (Figure 4.31) is used to collect data over the ring and results are tabulated in Table 4.17.

Table 4.17: iL500P Pump Study: 4,000 RPM Flow Simulation Results

<table>
<thead>
<tr>
<th></th>
<th>Inner Circle ($r_1 = 5.81mm$)</th>
<th>Outer Circle ($r_2 = 12.5mm$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity [$m/s$]</td>
<td>1.5394</td>
<td>2.2458</td>
</tr>
<tr>
<td>Axial Velocity [$m/s$]</td>
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</tr>
<tr>
<td>Radial Velocity [$m/s$]</td>
<td>0.6398</td>
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<tr>
<td>Circumferential Velocity [$m/s$]</td>
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<tr>
<td>Volume Flow Rate [$m^3/s$]</td>
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<tr>
<td>Area (Fluid) [$m^2$]</td>
<td>0.0001276</td>
<td>0.00024864</td>
</tr>
</tbody>
</table>

Calculations

As the pump theory was explained in the Section 3.3, it is applied to the iL500P pump in the same fashion as ideal centrifugal pump studies presented in Section 4.1. Each iL500P pump result was tabulated in the end of each study. They are gathered, flow rates are calculated with respect to these gathered results and then tabulated in the end of this section.

Two different circular areas are defined to be used as an area in flow-rate equation. One is direct circular area and the second one is 3 blade thickness’ extracted area. Blade thickness’ and heights are illustrated in Figure 4.38 as thickness ($t_1 = 3.15\ mm$, $t_2 = 4.3\ mm$) and height ($b_1 = 4.78\ mm$, $b_2 = 3.78\ mm$).
Equations being used to define flow rates are recalled below to be convenient. Flow rate subscript “A” means including the 3 blade thickness’, flow rate subscript “B” means blades thickness’ are subtracted from the related area.

- \( Q = V_n.A \)
- \( Q_1 = \) True Flow Rate
- \( Q_{2,A} = (2\pi r_n b_n)V_n \)
- \( Q_{2,B} = (2\pi r_n b_n - 3b_n t_n)V_n \)
- \( Q_{3,A} = 2\pi r_n b_n(\omega r_n - V_{n,t}).\tan \beta_n \)
- \( Q_{3,B} = (2\pi r_n b_n - 3b_n t_n)(\omega r_n - V_{n,t}).\tan \beta_n \)
- \( Q_{4,A} = 2\pi r_n b_n \omega r_n \tan \beta_n \)
- \( Q_{4,B} = (2\pi r_n b_n - 3b_n t_n)\omega r_n .\tan \beta_n \)

Figure 4.39: Leading Edge and Trailing Edge Blade Angles of the iL500P Pump

The blade angles are shown in Figure 4.39 as \( \beta_1 = 20.48^\circ \) and \( \beta_2 = 21.17^\circ \). Each flow-rate equation is calculated and tabulated in Table 4.18 and Table 4.19 by comparison to the true flow rate.
Table 4.18: iL500P Pump Inner Ring Calculations with Blade Angles

<table>
<thead>
<tr>
<th>$\omega$ (RPM)</th>
<th>$Q_1$ (m$^3$/s)</th>
<th>$Q_{2,A}$ (m$^3$/s)</th>
<th>$%$ Error Rate</th>
<th>$Q_{2,B}$ (m$^3$/s)</th>
<th>$%$ Error Rate</th>
<th>$Q_{3,A}$ (m$^3$/s)</th>
<th>$%$ Error Rate</th>
<th>$Q_{3,B}$ (m$^3$/s)</th>
<th>$%$ Error Rate</th>
<th>$Q_{4,A}$ (m$^3$/s)</th>
<th>$%$ Error Rate</th>
<th>$Q_{4,B}$ (m$^3$/s)</th>
<th>$%$ Error Rate</th>
<th>$V_n$ (m/s)</th>
<th>$V_t$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000</td>
<td>0.000410</td>
<td>0.000279</td>
<td>31.9</td>
<td>0.000207</td>
<td>49.5</td>
<td>0.000347</td>
<td>15.3</td>
<td>0.000257</td>
<td>37.3</td>
<td>0.000397</td>
<td>3.2</td>
<td>0.000294</td>
<td>28.2</td>
<td>1.600</td>
<td>0.764</td>
</tr>
<tr>
<td>7,000</td>
<td>0.000289</td>
<td>0.000200</td>
<td>30.8</td>
<td>0.000148</td>
<td>48.7</td>
<td>0.000245</td>
<td>15.2</td>
<td>0.000182</td>
<td>37.1</td>
<td>0.000278</td>
<td>3.9</td>
<td>0.000206</td>
<td>28.7</td>
<td>1.145</td>
<td>0.502</td>
</tr>
<tr>
<td>4,000</td>
<td>0.000161</td>
<td>0.000117</td>
<td>27.4</td>
<td>0.000087</td>
<td>46.2</td>
<td>0.000140</td>
<td>13.3</td>
<td>0.000104</td>
<td>35.7</td>
<td>0.000159</td>
<td>1.4</td>
<td>0.000118</td>
<td>26.9</td>
<td>0.669</td>
<td>0.293</td>
</tr>
</tbody>
</table>

Table 4.19: iL500P Pump Outer Ring Calculations with Blade Angles

<table>
<thead>
<tr>
<th>$\omega$ (RPM)</th>
<th>$Q_1$ (m$^3$/s)</th>
<th>$Q_{2,A}$ (m$^3$/s)</th>
<th>$%$ Error Rate</th>
<th>$Q_{2,B}$ (m$^3$/s)</th>
<th>$%$ Error Rate</th>
<th>$Q_{3,A}$ (m$^3$/s)</th>
<th>$%$ Error Rate</th>
<th>$Q_{3,B}$ (m$^3$/s)</th>
<th>$%$ Error Rate</th>
<th>$Q_{4,A}$ (m$^3$/s)</th>
<th>$%$ Error Rate</th>
<th>$Q_{4,B}$ (m$^3$/s)</th>
<th>$%$ Error Rate</th>
<th>$V_n$ (m/s)</th>
<th>$V_t$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000</td>
<td>0.000410</td>
<td>0.000498</td>
<td>-21.5</td>
<td>0.000416</td>
<td>-1.6</td>
<td>0.001013</td>
<td>-147.1</td>
<td>0.000847</td>
<td>-106.5</td>
<td>0.001507</td>
<td>-267.6</td>
<td>0.001260</td>
<td>-207.3</td>
<td>1.678</td>
<td>4.293</td>
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<tr>
<td>7,000</td>
<td>0.000289</td>
<td>0.000355</td>
<td>-22.8</td>
<td>0.000297</td>
<td>-2.6</td>
<td>0.000700</td>
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<td>0.000585</td>
<td>-102.4</td>
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<td>-205.1</td>
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<td>3.085</td>
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<td>0.000161</td>
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<td>0.000334</td>
<td>-107.3</td>
<td>0.000603</td>
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<td>0.000504</td>
<td>-212.5</td>
<td>0.627</td>
<td>1.768</td>
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</tbody>
</table>

Flow rate calculations are completed in the same way as ideal centrifugal pump calculations. Deviation angle is defined and calculations are done with these new blade angles($\beta_1 = 16.75^o$ and $\beta_2 = 10.8^o$). New calculations are tabulated in Table 4.20 and in Table 4.21.
Table 4.20: iL500P Pump Inner Ring Calculations with Deviation Angle Modified Blade Angles

<table>
<thead>
<tr>
<th>$\omega$ (RPM)</th>
<th>$Q_1$ (m³/s)</th>
<th>$Q_{2,A}$ (m³/s)</th>
<th>$Q_{2,B}$ (m³/s)</th>
<th>$Q_{3,A}$ (m³/s)</th>
<th>$Q_{3,B}$ (m³/s)</th>
<th>$Q_{4,A}$ (m³/s)</th>
<th>$Q_{4,B}$ (m³/s)</th>
<th>$V_n$ (m/s)</th>
<th>$V_t$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000</td>
<td>0.000410</td>
<td>0.000279</td>
<td>31.9</td>
<td>0.000207</td>
<td>49.5</td>
<td>0.000279</td>
<td>31.9</td>
<td>0.000320</td>
<td>42.2</td>
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<td></td>
<td>1.600</td>
<td>0.764</td>
</tr>
<tr>
<td>7,000</td>
<td>0.000289</td>
<td>0.000200</td>
<td>30.8</td>
<td>0.000148</td>
<td>48.7</td>
<td>0.000197</td>
<td>31.7</td>
<td>0.000224</td>
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<td></td>
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<td>1.145</td>
<td>0.502</td>
</tr>
<tr>
<td>4,000</td>
<td>0.000161</td>
<td>0.000117</td>
<td>27.4</td>
<td>0.000087</td>
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<td>0.000112</td>
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<td></td>
<td>0.669</td>
<td>0.293</td>
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</tbody>
</table>

Table 4.21: iL500P Pump Outer Ring Calculations with Deviation Angle Modified Blade Angles

<table>
<thead>
<tr>
<th>$\omega$ (RPM)</th>
<th>$Q_1$ (m³/s)</th>
<th>$Q_{2,A}$ (m³/s)</th>
<th>$Q_{2,B}$ (m³/s)</th>
<th>$Q_{3,A}$ (m³/s)</th>
<th>$Q_{3,B}$ (m³/s)</th>
<th>$Q_{4,A}$ (m³/s)</th>
<th>$Q_{4,B}$ (m³/s)</th>
<th>$V_n$ (m/s)</th>
<th>$V_t$ (m/s)</th>
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</thead>
<tbody>
<tr>
<td>10,000</td>
<td>0.000410</td>
<td>0.000498</td>
<td>-21.5</td>
<td>0.000416</td>
<td>-1.6</td>
<td>0.000498</td>
<td>-21.5</td>
<td>0.000741</td>
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<td>0.000620</td>
<td>-51.1</td>
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<td>7,000</td>
<td>0.000289</td>
<td>0.000355</td>
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<td>0.000297</td>
<td>-2.6</td>
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<td></td>
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<td>0.000248</td>
<td>-53.9</td>
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</tbody>
</table>
Conclusion of the iL500P Pump Studies

The centrifugal pump theory was verified in Section 4.1. The theory is applied to the iL500P pump in the same fashion as it was applied to the ideal centrifugal pump. Results are gathered for three different revolutions (10,000, 7,000 and 4,000 RPM). Flow does not interact with the impeller in the same way as the ideal centrifugal pump because of the iL500P pump’s design condition. For this reason, velocity vectors do not show the intended direction on the leading edge of blades and this situation is represented in Figure 4.30, Figure 4.34 and Figure 4.37. After flow-rates are calculated with respect to the flow simulation studies, they are tabulated in the end of Section 4.2. Red colored cells in Table 4.20 indicate that flow-rates are calculated with an error rate of about 42%. In other words, application of the centrifugal pump theory to the iL500P pump does not satisfy the requirements for measuring flow-rate via the angular velocity.

4.3 Discussion

Because our pump is a type of centrifugal pump, the flow rate-angular velocity relationship is studied via the centrifugal pump theory in the way described in this chapter. In addition to the Rule the iL500P submersible in-line pump, the ideal centrifugal in-line pump is designed and studied to verify this theory. The most important design change was adding the inlet tube. Since there is no shroud around the impeller and the inlet flow does not spread out from the eye of the impeller, the inlet tube deals with these two issues to make the pump an ideal centrifugal pump.

The following boundary conditions are set: a rotation, pressures on both the inlet and outlet, wall condition. 10,000, 7,000 and 4,000 RPM are selected as rotation counts after the revolution approximation experiment. Atmospheric pressure is chosen as pressure conditions. Walls are defined as adiabatic walls.

After studies are run with respect to the mentioned boundary conditions, flow rates are calculated and tabulated at the end of ideal centrifugal pump and iL500P pump studies.
In these tabulated results; “$Q_1$” represents true flow rate, and the error rate is calculated with respect to this flow rate. “$Q_2$” flow rates are calculated with the multiplication of the ring areas and the normal velocities, which are collected from the software and tabulated in tables after each study ends. “$Q_3$” flow rates are calculated via the centrifugal pump theory including $V_t$ (Tangential Velocity). “$Q_4$” flow rates are calculated by the centrifugal pump theory under the assumption that $V_t = 0$.

In conclusion, the flow rate can be measured via the angular velocity for ideally designed pumps. This can be seen from red colored cells in the Table 4.10. Flow rate is measured with an error rate less than 9%. On the other hand, although our pump is a type of centrifugal pump, the result does not satisfy the flow rate measurement because of its design condition. So if an ideal in-line pump is constructed, thrust force can be expressed in terms of angular velocity. Therefore, this pump can be controlled via the angular velocity.
Chapter 5

Thrust Force Measurement

The Rule submersible in-line pumps’s thrust force equation was derived in Chapter 3. The governing equations of both the iL500P pump and the ideal centrifugal pump are recalled for convenience.

The dynamic states of the iL500P pump was expressed as:

$$\begin{bmatrix} \frac{di}{dt} \\ \frac{d\omega_m}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_b}{L_a} \\ \frac{K_i}{J_m} & \frac{B_m}{J_m} \end{bmatrix} \begin{bmatrix} i \\ \omega_m \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} \\ 0 \end{bmatrix} V(t) + \begin{bmatrix} 0 \\ -\frac{1}{J_m} \end{bmatrix} \tau_H(t)$$

$$\tau_H = \rho \beta Q(\omega_m)Q(\omega_m) \left( \frac{1}{A_{out}} - \frac{1}{A_{in}} \right) + \frac{Q(\omega_m)H \rho g}{\omega_m} - \rho Q^3(\omega_m) \left( \frac{1}{A_{out}^2} - \frac{1}{A_{in}^2} \right) + \frac{\rho \alpha Q(\omega_m)^3}{4 \omega_m} \left( \frac{1}{A_{out}^2} - \frac{1}{A_{in}^2} \right)$$

Output equation:

$$\vec{F}_{thrust} = \rho L \dot{Q}(\omega_m) + \left[ \frac{A_{impeller}}{2} \left( \frac{1}{A_{out}^2} - \frac{1}{A_{in}^2} \right) + \beta \left( \frac{1}{A_{out}} - \frac{1}{A_{in}} \right) \right] \rho Q(\omega_m)Q(\omega_m) - H \rho g A_{impeller}$$

Moreover, the governing equations for the ideal centrifugal pump was also expressed in terms of angular velocity with respect to the centrifugal pump theory in Chapter 3. After derivation of the governing equations, naturally experiments are done to validate the expression. Within this scope, a pendulum arm structure is constructed as an experimental setup to measure the steady state responses of the iL500P pump and to characterize it. Three different lengths of pendulum arms are used to check the accuracy of the measurements. During initial tests, two major issues observed. First of all, a metal-metal surfaces impact is seen on the measurement point between the force gauge and pendulum arm.
A damper is used to eliminate these oscillations. Second issue is the verticality of the pendulum arm. The force gauge measures only the horizontal forces so that pendulum arm must be 100% vertical. As the generated thrust force is relatively small, in case of an angled position of the pendulum, the gravitational and buoyancy forces contribute to the measured thrust force. A digital inclinometer is used to ensure the verticality. Verticality analysis is shown in this chapter. After solving two major problems, experiments are done by each different pendulum arms under two different conditions. These conditions differed by the way of the input voltage. At first, the voltage is applied incrementally and then it is applied from “0” to a given voltage by a sudden increase. All of these studies are plotted and regression analysis of the steady state responses is shown in this chapter. Also transient responses are collected and shown in the end of the chapter.

5.1 Experimental Setup

A pendulum arm structure is used as an experimental setup. This setup is commonly used in the literature to measure the thrust force that the pump provides. In this structure, equality of moments is created by moment arms. Pump force is causing a moment on the pivot point depending on the pump arm. The reaction force on the force gauge is causing a reverse moment on the same point by the gauge arm to equal the moment. The pendulum arm structure is illustrated in Figure 5.1
Before modeling a main structure, an underground pool is decided to be used as a test bed. By that pool, experiments can be considered as open impeller tests as there is no bounced wave effect on the force.

**Main Structure**

The main structure is made of seven steel tubes of different lengths (2.5 cm width, 2.5 cm height, 2 x 0.91 m, 2 x 0.39 m and 3 x 0.43 m length). These tubes are combined with each other by welding. Besides that, one steel plate is welded on three vertical tubes to hold the force gauge. Four screw paths are cut on the steel plate by laser cut to adjust the force gauge’s place by screws. The main structure is shown in Figure 5.2.
Pendulum Arms

Three different length pump arms are used to check on the force gauge accuracy. Arms are made of Low Carbon Steel, 2.5 cm Wide, 2.5 cm Height, 0.15 cm Wall Thickness. Arms lengths are 2 ft, 4 ft and 6 ft but measurement is made from the hole that is drilled to fix the pump mount to the arms beginning. Lengths that have been used for calculations are 22.75” (0.57785 m), 46.75” (1.18745 m) and 70.7” (1.79705 m). The pendulum arms are shown in Figure 5.3.

Figure 5.2: Main Structure

Figure 5.3: 3 Different Lengths of Pendulum Arms
Extension Arm and Joint

An extension arm which goes above and below the pivot point is used to make the experimental setup portable, and the pump arms convertible. The joint between the extension arm and pump arms is provided by an aluminum bar which is fixed by close fit and screws to the extension arm. This aluminum bar is made of Multipurpose 6061 Aluminum, and dimensions are 2.22 cm width x 2.22 cm thick x 15.2 cm length. Extension arm and joint are shown in Figure 5.4

![Figure 5.4: Extension Arm and Joint](image)

The extension arm is attached to the main structure by a rotatable shaft and 2 shaft bearings. These bearings are fixed to the structure by welding. 4 shaft collars are used to avoid vibration and make the assembly a rigid body. Figure 5.5 shows this connection.

![Figure 5.5: Extension Arm - Main Structure Connection](image)
Pump Mount

One 3D printed mount is used to attach the submersible pump to the experimental setup. After the pumps placement to the mount; the mount is fixed to the pump arm by an M3 threaded 8 cm length screw. The pump mount is presented in Figure 5.6.

![Pump Mount](image)

Figure 5.6: Pump Mount

DC Regulated Power Supply

TENMA 72 7655 DC regulated power supply shown in Figure 5.7 is used to supply different voltages to the pump in experiments. Specifications of the device are written in Table 5.1.

![DC Regulated Power Supply](image)

Figure 5.7: TENMA 72 7655 DC Regulated Power Supply
Table 5.1: TENMA 72 7655 DC Regulated Power Supply Specifications

<table>
<thead>
<tr>
<th>Specifications</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Voltage</td>
<td>15 Volt</td>
</tr>
<tr>
<td>Output Current</td>
<td>60 A</td>
</tr>
<tr>
<td>Load Regulation</td>
<td>0.1% +5mV</td>
</tr>
<tr>
<td>Line Regulation</td>
<td>0.05% +3mV</td>
</tr>
<tr>
<td>Ripple and Noise</td>
<td>40mV (p-p)</td>
</tr>
<tr>
<td>Efficiency</td>
<td>85%</td>
</tr>
<tr>
<td>Meter Accuracy</td>
<td>1% +1 count</td>
</tr>
<tr>
<td>Power requirements</td>
<td>120VAC, 60Hz</td>
</tr>
<tr>
<td>Dimensions</td>
<td>4-1/4” (H) x 8-3/4” (W) x 14-1/4” (D)</td>
</tr>
</tbody>
</table>

**Force Gauge**

A Digital HF-20 Push Pull Force Gauge is used as a force sensor. Specifications are written in Table 5.2. The data acquisition system is provided by the manufacturer. This software collected data during experiments (Force Gauge Manual [16]). Force gauge is shown in Figure 5.8.

Table 5.2: Digital HF-20 Push Pull Force Gauge Specifications

<table>
<thead>
<tr>
<th>Specifications</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity</td>
<td>20 N</td>
</tr>
<tr>
<td>Min Unit</td>
<td>0.1 N</td>
</tr>
<tr>
<td>Value Error</td>
<td>0.5%</td>
</tr>
<tr>
<td>Units</td>
<td>N</td>
</tr>
<tr>
<td>Sampling Rate</td>
<td>10 Hz</td>
</tr>
<tr>
<td>Power Supply</td>
<td>110 V</td>
</tr>
<tr>
<td>Dimensions</td>
<td>12” (L) x 7.3” (W) x 2.7” (H)</td>
</tr>
</tbody>
</table>
During the initial experiments, metal-metal impact was seen because of the voltage that was applied to the pump. Applying 12 V directly to the pump caused a big oscillation in the readings. Because force sensor lost contact under these conditions, the experiments were not valid. In order to avoid that impact and zero contacts, voltage was applied by step function to the pump periodically. Step function voltage was decided to go between 6 V to 12 V for 10-second periods. The Pololu VNH5019 motor driver and the Arduino Uno microcontroller are used to supply those certain voltages and to protect the pump from electrical damage. A wiring diagram that is used for connecting Arduino Uno to the VNH5019 motor driver is shown in Figure 5.9.
5.2 Experiments

Experiments are done for pump arms of three different lengths. Data are collected and seemed to change linearly depending on pump arm length as expected. Five different materials are tested as a damper to avoid losing contact between the force sensor - gauge arm and for collecting less noisy data. For each material, two tests with each pendulum arm are represented. By taking pump arms lengths and weights into consideration, these arms are carefully placed vertically in each test to collect accurate horizontal force, and to neglect the gravity force effect. Gravitational and buoyancy forces are represented in Figure 5.10 under an angled position.
$F_r = \text{Reaction Force}$

$F_{pump} = \text{Pump Force}$

$GA = \text{Arm Gravity Force}$

$GP = \text{Pump Gravity Force}$

$\bullet = \text{Pivot Point}$

$BA = \text{Arm’s Buoyancy Force}$

$BP = \text{Pump’s Buoyancy Force}$

$a = \text{Gauge Arm}$

$b = \text{Pivot - Arms Center of Gravity}$

$c = \text{Pivot - Pumps Center of Gravity}$

$d = \text{Pivot - Buoyancy Center of Arm}$

\[
\frac{Fr}{\cos(\theta)} \cdot a + (GA \cdot \sin(\theta)) + (GP \cdot \sin(\theta)) \cdot c = (F_{pump} \cdot c) + (BA \cdot \sin(\theta)) \cdot d + (BP \cdot \sin(\theta)) \cdot c \quad (5.2)
\]

As represented in Figure 5.10 and Equation (5.2), placing pump arms vertically is crucial for this experiment. Even 1° change effects measured thrust force considerably. Experiments are done to measure steady state responses, but transient responses are also collected. In transient responses, response is effected by this vertical angled position and water drag force.
Without Damping Material

Initial experiments are done without using any damping material. Two different experiments for each pendulum arms are plotted as Figure 5.11 and Figure 5.12.
White foam is used as the 1st material. Although it reduced oscillation in shorter arms, it did not produce clear accurate transient data. Material - 1 is shown in Figure 5.13.

Figure 5.13: Material - 1

Figure 5.14: Experiments With Material-1 1

Figure 5.15: Experiments With Material-1 2
Figure 5.16: Experiment Picture With Material-1 1

Figure 5.17: Experiment Picture With Material-1 2
Material - 2

Another type of foam is used as a 2\textsuperscript{nd} material shown in Figure 5.18. This material is the most flexible one. Experiments are done by taking this flexibility into consideration and plotted in Figure 5.19 and Figure 5.20.

Figure 5.18: Material - 2

Figure 5.19: Experiments With Material-2 1

Figure 5.20: Experiments With Material-2 2
Figure 5.21: Experiment Picture With Material-2 1

Figure 5.22: Experiment Picture With Material-2 2
Material - 3

Material 3 shown in Figure 5.23 is the thinnest material tried as a damper. Its thickness is not able to avoid oscillation caused by metal-metal impact effectively. Experiment results for Material 3 are plotted in the Figure 5.24 and Figure 5.25. As the pump arm gets longer, oscillation becomes significant and the collected data gets noisier.

Figure 5.23: Material - 3

Figure 5.24: Experiments With Material-3 1

Figure 5.25: Experiments With Material-3 2
Figure 5.26: Experiment Picture With Material-3 1

Figure 5.27: Experiment Picture With Material-3 2
Material - 4

Material 4 is a kind of soft plastic represented in Figure 5.28. In terms of oscillation and noise in data it produced similar data with the 1st and 3rd materials. For this experiment Material 4 is used as a force gauge tip. Experiment results for Material 4 is shown in Figure 5.29 and Figure 5.30.
Figure 5.31: Experiment Picture With Material-4
Material - 5

Material 5, shown in Figure 5.32, is a kind of soft foam. It is normally used as an ear plug. When data noise and oscillation values are taken into consideration, Material 5 is the best option to be used as a damper for the experiment. The experiment results are shown in Figures 5.33 and Figure 5.34.
Figure 5.35: Experiment Picture With Material-5 1

Figure 5.36: Experiment Picture With Material-5 2
Thrust Force Experiments

Material 5 is determined to be used as a damping material for the follow up experiments after each material has been tried.

After a damping material is selected, a series of experiments are performed to collect responses to different given voltages. By using the motor driver and microcontroller, voltages are applied incrementally from 4 V to 12 V. The results are plotted in Figure 5.37.

![Figure 5.37: Step by Step Voltages 6 V to 12 V](image)

Before completing thrust regression analysis, overshoot percentages for each voltage, data are taken from Figure 5.37 and plotted in Figure 5.38
Figure 5.38: Overshoot Percentages Detailed Plot
Steady state responses of each voltage are collected and used in a regression analysis. The thrust regression analysis is plotted in Figure 5.39. Curve fitting showed a linear relationship as expected and an Empirical Equation (5.3) is written below.

\[ \text{Thrust}(N) = 0.0603 \cdot \text{Voltage Value (V)} - 0.2341 \]  

(5.3)

In contrast with the conventional ducted thrusters, this submersible in-line pump provides a same direction thrust force for both negative and positive voltages. Positive voltage responses are shown in Figure 5.39. Beyond that, both negative and positive voltage responses are plotted in Figure 5.40.
After step by step experiments are done, different step responses plotted from 0 V to a given voltage. These voltages are applied instantaneously rather than incrementally. As this pump is not going to work under specific step by step voltages in reality, responses for different voltages are observed and plotted in following Figures. Each given voltage experiment is done for three different pendulum arms.
Figure 5.41: 0 - 5V Step Response

Figure 5.42: 0 - 6V Step Response
Figure 5.43: 0 - 7V Step Response

Figure 5.44: 0 - 8V Step Response
Figure 5.45: 0 - 9V Step Response

Figure 5.46: 0 - 10V Step Response
This experiment showed that the transient response of the force gauge is not only coming from the pump, but also the structural vibration and water drag force.
Chapter 6

Conclusion

The thrusters deliver thrust force to the swimming objects. When a propeller of a ship is taken as a thruster example, it is obvious that propeller’s rotation is increased to move faster by generating more thrust force. This thrust force increase means that there is a relationship between the thrust and angular velocity of the propeller. To establish a precise control over these thrusters, dynamics of the system need to be explained. These dynamics and the thrust force equation is derived with respect to the physical laws but the thrust force calculation requires a flow rate value. Difficulties of the flow rate measurement and setting up an accurate control depending on that measurement motivated this study for our pump (Rule iL500P submersible in-line pump).

In this case, the Rule iL500P submersible in-line pump’s working principle is explained and its components are illustrated via computer aided design (CAD) models in Chapter 2. The angular velocity-flow rate relationship is searched from the previous studies on ducted thrusters in literature. Also literature review guided us how to derive the thrust force of the iL500P pump. Within this scope, the thrust force equation is, similarly to related literature, derived from the conservation of linear momentum principle. Conventional DC motor modeling is used to derive the states \((i, \omega_m)\) which include hydrodynamic load just as the previous studies do. These equations include flow rate which needs to be calculated. For this reason, the iL500P pump is examined for being suitable for the application of the centrifugal pump theory or not.

Before applying the centrifugal pump theory to the iL500P, the theory is verified under the ideal centrifugal pump conditions. Within this framework, the ideal centrifugal pump,
which means the inlet flow spreads out from the eye of the impeller that is surrounded by a shroud, is designed. The finite element method (FEM) based commercial software (SolidWorks Flow Simulation 2015) is used to verify the theory. Flow simulations are completed for three different revolutions. Six different flow rate calculations are showed in Chapter 4 to validate the flow rate. After flow simulations and calculations are completed, the flow rate is measured with an error rate less than 9% via the angular velocity. Theory verification is followed by the application of the centrifugal pump theory to the iL500P pump. Studies and data collections are done exactly the same way. The results of the iL500P pump indicate that the theory is not applicable for our pump. Flow rate is calculated with an error rate about 42% which is unsatisfactory. This error rate takes vast majority of its source from the design of the iL500P pump.

Besides the flow simulation studies, the pendulum arm structure is used as the experimental setup to measure the steady state thrust force of the iL500P pump. During initial experiment, two major issues occurred. The first one was the verticality of the pendulum arm. The second one was the oscillation that was caused by the metal-metal surfaces impact. A digital inclinometer is used to ensure the verticality of the pendulum arm, and the ear plug is used as a damper material to get rid of the oscillations. In these circumstances, the experiments are done for two different conditions in terms of applying the input voltage. For the first condition, the input voltage is applied incrementally. For the second condition, input voltage is applied instantaneously from “0 V” to the given voltage. These experiment results are plotted in Chapter 5.

In conclusion, in this thesis two very valuable results are achieved:

- Through careful mathematical modeling and an extensive finite element simulation, we showed that specially built in-line pumps could provide flow rate, which is responsible for thrust generation, proportional to the angular velocity of the motor driving the pump. This has extremely important consequences resulting in dynamical equations amenable to control of the in-line pump to generate accurately controlled thrust force,
• We carefully devised a pendulum-based apparatus to accurately measure steady state values of the thrust force. This was not a trivial task given the nature of the pendulum system. The metal impact effect on the measurement system and the verticality of the pendulum arm based issues were dissolved.
Bibliography


