Design of a Novel Compliant Sensor for Series Elastic Actuation and Control of a Flexible Cable Conduit Transmission

by

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ABSTRACT

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While robotic rehabilitation following neurological injury is gaining traction, traditional rigid systems are confined to use in a clinic and mechanical designs can limit their portability for use in an assistive mode. Soft robotic wearable exoskeletons offer potential solutions, yet the cable-based actuation systems commonly used introduce non-linear dynamics and friction, increasing control challenges. This thesis presents a novel compliant sensor design for use in flexible cable conduit transmissions that leverages the natural transmission compliance and utilizes series elastic actuation (SEA), a method of control previously shown effective for dynamic compensation. Dynamic simulations and static models are used to inform the analysis of physical experiments using the sensor in the transmission. The sensor is validated for use in force feedback for both force and impedance control scenarios. Experimental results provide insight to the design of soft exoskeleton devices regarding the effects of sensor location and the challenges of non-collocation of sensor and user interface.
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Chapter 1

Introduction

1.1 Motivation

In the United States alone millions of people are living with decreased motor function due to stroke, traumatic brain injury (TBI), or spinal cord injury (SCI). Each year around 700,000 individuals experience a new or recurring stroke incident. Meanwhile 50% of the over 7 million stroke survivors will have some amount of hemiparesis and 26% will be dependent on someone else in activities of daily living (ADL) [1]. For SCI, there are approximately 12,500 new cases per year [2], while 80% of TBI survivors needing acute rehabilitation require further rehabilitation to regain gross motor function [3]. With the growing economic and social cost of combating the long term effects of these diseases, effective and efficient tools for sensorimotor rehabilitation are needed.

Robotic solutions for rehabilitation have been proposed to improve the consistency and efficacy of treatment and allow for quantitative evaluation of progress [4, 5, 6]. A variety of devices have been developed and implemented to address rehabilitation for stroke [7, 8] and SCI [9]. While there has not previously been significant effort to address robotic rehabilitation of TBI, it is likely that similar devices will be applicable to the needs for rehabilitation of motor deficits in TBI as well. The main advantage of robotic rehabilitation devices is in leveraging the repeatability of the devices to deliver high intensity, high repetition movements, successfully matching non-robotic
rehabilitation outcomes [10, 11]. To meet these goals, robotic exoskeleton designs have used well characterized actuators and hardware joints designed to be actuated in parallel to the target human joints [12]. Though some exoskeleton designs have used pneumatic cylinders to provide actuation [13], a majority of designs have used power dense DC motors.

One such robotic exoskeleton for rehabilitation, the CADEN-7, provides actuation to all seven joints of the arm through DC motors and pulley transmission [14]. While an example of successful design implementation, the CADEN-7 does represent the limitations of traditional exoskeleton design: pin joint approximations of human joints, large heavy hardware supported by external structures, and mechanical limitations such as singularities. In general, rehabilitation robot designs have been bulky and require large structures to support actuators or transmissions. These structures restrict motion [15], may require passive degrees of freedom to accommodate different users and natural movements [16, 17], and add inertia to the user that the control schemes must overcome. In recent years, an alternative to traditional exoskeleton
design has been proposed, namely soft wearable exoskeletons. These soft exoskeletons eliminate the need for large bulky structures by relying on the human joints to form the joints of the device and by either using remote actuation systems with soft transmissions or making use of non-traditional actuators. Fig. 1.1 shows a concept image for the soft exoskeleton device for upper arm motor rehab after TBI in design by NASA JSC which the analysis of this thesis will focus on.

1.2 Soft Wearable Exoskeletons

Soft wearable exoskeletons are a fairly recent trend in exoskeleton design. The motivation of these designs is to provide rehabilitation that is 1) less physically intrusive to the user and therapist, 2) usable in more natural environments, outside of the clinic, 3) able to provide more engaging therapy through practice of activities of daily living, and 4) able to implement therapy that targets more of the body simultaneously through natural, coordinated joint movements. By replacing traditional mechanical structures with the user’s body, soft wearable devices attempt to address these goals.

Previously, exoskeletons for the arm and leg have focused on providing capabilities above human natural strength, either for augmentation purposes or to safely cover the forces needed for rehabilitation. This explains the existing traditional exoskeleton designs, as hard structures are needed to provide hardstops for safety purposes when it is possible to actuate the human joints past normal human capabilities. As a result, a majority of early soft wearable devices were designed for applications where traditional exoskeletons were difficult to implement due to physical size constraints and large forces were not needed, making soft designs more feasible. Specifically, a multitude of robotic gloves have been designed as rehabilitative or assistive devices both for strengthening grasp and opening the hand in the case of spasticity after
Figure 1.2 : Soft robotic glove examples. (a) Cable actuated Robo Glove designed by NASA JSC [18]. (b) Hydraulic glove for stroke rehabilitation designed at Harvard [19].

Some examples can be seen in Fig. 1.2. Another early design utilizing soft wearable robotic techniques is the soft orthosis for stabilization and rejection of tremors [23]. These range of devices display the actuators commonly used in soft wearable robotics: remote actuated tendon drives [18] and pressurized actuators like Mckibben artificial muscles and, recently, soft pressurizable elastomeric cylinders [19].

Extensions into higher force applications, such as upper arm and gait rehabilitation, have been fueled by the successful implementation of these soft actuators. Though some pressurized actuator based designs have been made [25], the majority of these designs for both upper and lower limb leverage the ability to use remote actuation transmitted through cable-conduit systems [26, 27]. In this way, applied forces are increased by increasing actuator size without the negative effects this usually brings in traditional exoskeleton design. One of the most successful recent examples is a tendon-driven assistive walking exosuit designed by the Harvard Biodesign
Figure 1.3: Soft robotic exosuit for actuation of the ankle and hip designed by Harvard Biodesign lab [24].

lab, shown in Fig. 1.3 [28]. While initially designed for the purpose of reducing the metabolic cost of loaded walking in healthy users, some initial studies have been made to test the feasibility for use with stroke patients [29]. The exosuit demonstrates implementation of a Bowden cable, or cable-conduit, transmission for actuating human joints at high force ranges with a soft, garment-like attachment to the human body. Remote cable-conduit actuation allows for the use of actuators that are well understood in robotic design, DC motors, without impeding the design principles of soft robotic exoskeletons. A similar tendon based system could be designed to address the application of upper limb motor rehabilitation.
1.3 Tendon Based Actuation

Bowden cable or tendon based actuation schemes have been used in multiple applications to transfer force from an actuator to a remote or otherwise inaccessible location. Besides the soft exoskeleton devices discussed in Section 1.2, cable-conduit transmissions have been used in traditional exoskeleton devices, such as LOPES [30], robotic surgery tools [31], and other humanoid or bio-inspired robot designs, like the Robonaut 2 hand [32]. These examples can be seen in Fig. 1.4. Between these three applications, analysis and testing of Bowden cables has shown a wide range of capabilities from the precision actuation needed for surgical applications to the gross motor power needed for actuation of a knee during walking.

The design flexibility demonstrated above comes at the expense of non-linear transmission dynamics. The general behavior of these dynamics are summarized by Kaneko et al. as apparent tendon stiffness and friction backlash [33]. The more significant of these dynamics are the effects due to Coulomb friction between the cable and conduit. Some models describe and treat this friction as a simple backlash across the transmission [34, 35]. Others develop more complex models of the tension loss due to friction by including two cable systems with slack [36] or different models of friction force [37, 38]. Many of the exoskeleton designs forgo modeling the friction at all, relying instead on measurement of the tension loss and closed loop control to cancel out the undesirable dynamics [39].

It is important to make note of the applications for these different methods of addressing the conduit-cable dynamics. For example, a dynamic model proposed by Phee et al. is intended to provide insight into the elongation and end effector forces for surgical tendon sheath robots [40]. This is a situation where small package size and other considerations restrict end effector sensing, making the achieved max force
error of 7% of full state output quite reasonable. These methods, though, may not be sufficient for force or impedance control where complete knowledge of the interaction with the environment is often required [41]. Ideally, a force sensor could be placed between the user and exoskeleton to measure the interaction directly, but this is likely to conflict with a soft exoskeleton design. Taking into consideration the sensing needs, non-linear dynamics, and desired control, the various control methods developed for DC motor systems suggest a solution: series elastic actuation.

1.4 Series Elastic Actuation

Series elastic actuation is a method of force control proposed by Pratt and Williamson that places intentional compliance in the transmission to provide safety to user interaction, shock tolerance for the actuator, and reduced reflected inertia [42]. This way of thinking about compliant transmissions is the antithesis of the traditional approach of making actuators and transmissions as stiff as possible. When dealing with almost
guaranteed transmission or actuator compliance, series elastic actuation is a plausible method of controlling Bowden cable transmissions and soft wearable robots in general. Though the modeling of friction is highlighted in the literature in the analysis of cable-conduit dynamics, there is also significant non-linear compliance present in the transmission. Additionally, compliance may even exist in soft wearable robotics at the interface point to the user. Other tendon based transmissions of non-soft exoskeletons have previously made use of SEA methods to implement force control [30, 43], and the reasons to use series elastic actuation strategies—transmission compliance, desire for accurate force control—are even more present for soft robotic exoskeletons.

Though the control is not a focus of this thesis, it is important to understand. Control of series elastic actuators (SEAs) is primarily based off idealized dynamics as illustrated in Fig. 1.5. This model includes actuator dynamics, \( A \), and load dynamics, \( L \), connected by the compliant element, \( k_c \). The SEA measures difference between actuator and load positions in the transmission. Therefore, if the position or velocity of the actuator can be accurately controlled, an realistic task for DC motors, the spring force can be controlled using a position input [44]. This line of thought leads to a cascaded control strategy, with force control wrapped around accurate velocity control [45]. Impedance control of series elastic actuators is generally derived from the force control architecture, such that the desired force is calculated based on measured user position input and the desired impedance to be displayed [46].

Some limitations do exist for series elastic actuation in general and as it is applied to soft cable-conduit systems. Compliance lowers system force bandwidth which in turn limits the impedances that can be passively rendered to the user, capped by the stiffness of the SEA [47]. Though some stiffnesses above this limit can be rendered, they can only be rendered to the user in a stable manner, not a passive manner,
Figure 1.5: Traditional block diagram model used to analyze series elastic actuation based on the dynamics of the interacting user [48]. Additionally, for soft robotics, the assumptions made in the model of Fig. 1.5 do not hold true for the non-linear cable-conduit, namely that the sensor will be a measure of the position difference between load and actuator. Since actuator position sensing is often integrated, it will be most important for a measure of the user/load side position and force to be made. This again suggests placement of a sensor near the interaction point. For best results, a compliant sensor will need to be designed that does not conflict with soft robotic exoskeleton design or the requirements of series elastic actuation.

1.5 Thesis Outline

The thesis is structured as follows: Chapter 2 gives an in-depth analysis of problem definition, design, modeling, prototyping, and validation for a compliant force sensor for series elastic actuation. Chapter 3 introduces the specific Bowden cable model used for system identification and transmission analysis as well as providing simulations to give an idea of what dynamics should be expected. Chapter 4 covers the design and use of a Bowden cable testbed for validation of the compliant force sensor in its intended application.
Chapter 2

Design of a Compliant Force Sensor for Series Elastic Actuation

The process of fully integrating a compliant force sensor in series with the standard Bowden cable-conduit transmission requires consciously examining the constraints imposed by the transmission and the requirements of the intended application. In this work the sensor was designed to act as part of an SEA in a soft robotic exoskeleton for the rehabilitation of elbow and shoulder movements in design by the Warrior Web group at NASA Johnson Space Center. This chapter will explain the specifications driving the design of the sensor, the process undertaken to reach the final design, and the validation of the final design as a force and position sensor.

2.1 Problem Statement

A compliant force sensor involves two main components: the compliant element and the sensor to measure displacement. Design choices for one impact the constraints for the other and vice versa. The overall design objective for the compliant sensor is to make a force sensor that can leverage the advantages of Series Elastic Actuation in a system with the dynamics associated with a flexible cable-conduit transmission. This requires the compliant sensor to both accurately measure forces within the appropriate range for the application while at the same time having low stiffness for use as an SEA (see Section 1.4). Looking specifically at the compliant element, this feature of an SEA tends to define the design and behavior of the full robotic actuator in which it
resides. The placement and size of the spring used can drastically change the design of the transmission or human interface. This is especially true for soft robotics, where the rigid structures used to support SEAs in traditional exoskeletons may not be part of the existing soft design. Additionally, with existing SEA control strategies, the stiffness of the compliant element also inherently limits the maximum stiffness that it is possible to render to the user [48]. Meanwhile, the combination of the resolution of the position sensor, the stiffness of the spring, and the maximum allowable spring displacement give the force sensor range and resolution. The following sections will touch on the design process used when examining each of these design considerations.

2.1.1 Placement

In series elastic actuation, the location of the elastic element has an effect on the whole system design as well as on the system behavior and our ability to control it. For a Bowden cable transmission, like that used in the soft exoskeleton, there are four distinct choices for placement that should be considered (Figure 2.1): at the motor, in the cable, in the conduit, and at the load end/attachment to user. Previous designs using Bowden cable transmissions and SEAs have mostly focused on placement of the compliant sensor either at the motor or as close to the user attachment as possible [30, 39, 43]. These choices, however, were derived from SEAs implemented in systems using more traditional forms of transmissions. The addition of the Bowden cable transmission yields more possible placement options which should be explored more fully. After carefully considering the effects that placement of the SEA has on system design, I will show that placement of the compliant element in the Bowden conduit becomes an equal if not better choice compared to the other options.
Figure 2.1: (a) Top view of SEA cable conduit testbed with locations within considered for SEA placement. (b) Isometric view with relevant components highlighted.
Beginning with the considerations that affect force sensors in general, the nonlinear dynamics of the transmission create a trade-off between measurement accuracy and system stability. An accurate measurement is best made when the sensor is placed at the point of interest or with negligible dynamics in between the sensor and point of interest. This would suggest that the sensor be placed at the interface between the Bowden cable and the user, or at the very least on the user side of the conduit. Stability, however, is negatively affected by the non-collocation of the sensor and actuator that this creates [49, 50]. The backlash caused by the Bowden cable friction causes a delay between the motor commands and the force sensor detecting the affect of that command [35], which, while leading to an increased region of stability, can make designing a stable controller more challenging for accurate force or impedance control [51, 52].

The desired application and design define the other major consideration for placement of the compliant sensor. Since the desired application is a soft robotic exoskeleton, the sensor should not oppose that goal by impeding the natural movement of the user. The nature of a soft exoskeleton is to make use of the human body as the frame for the robot, generally guaranteeing that a user can make natural movements. Therefore, the sensor should not place unnecessary mass on the user’s limb or restrict the range of motion of the user in any way. Complexity of the compliant element or sensor also becomes a concern when thinking about placement. Lower complexity is especially important mechanically, as a simpler design tends to be more robust and can be more easily be modified for similar systems.

Balancing these considerations, the choice to place the sensor in the conduit makes the most sense from a qualitative design perspective. Placement at the end of the conduit allows flexibility to place the sensor near the user or near the actuator. This
gives a choice in the control of the system, needing either to compensate for the sensor inaccuracy or the sensor non-collocation. A spring within the conduit will also be stationary relative to the user and generally away from the user’s joints. The hardware of the sensor may have some interference with natural movements when placed on the user side of the conduit, but any interference should generally be on the level as what already appears from the need to restrain the end of the conduit to the user. A major concern with the conduit placement is that the accuracy of the measurements may be compromised in some conduit configurations, since the sensor would not be directly measuring cable tension. A sensor in the conduit would measure no force when the conduit is straight. However, the error due to this issue reduces quickly as the conduit is bent, and it would be fairly easy to keep a minimum bend in the conduit for measurement purposes.

While the other possible placement locations had their own advantages, the main concerns for each were enough to rule out their selection. First, placement between the user and the cable or in the cable itself can be easily seen to diminish the range of motion of the joint being measured. Since the sensor is compliant, it will be larger than most non-compliant sensors. This size inherently limits the joint range. While placement at the user interface gets the sensor the closest to a direct measurement of the user’s joint torques, this placement creates sufficient design problems to be essentially unfeasible in a soft robotic exoskeleton. Placement of the sensor at the motor is ruled out due to complexity. Rotational springs are not generally available in the same range of stiffness values and force ranges as compression or extension springs are. This leads to many designs with custom compliant elements [30, 53, 54]. Even in designs with standard springs, allowing the SEA to freely rotate while still getting accurate measurements can be challenging. While a direct deflection measurement of
the elastic element is preferred to reduce error due to noise [55], this leads to overly complex systems. While placement in the conduit does not necessarily fully eliminate these problems, it does give rise to a healthy balance of the considerations, with no one issue derailing the usefulness of the sensor. As well, an SEA within the conduit can be designed with a compression spring alone, eliminating a need for hardstops to protect the sensor.

2.1.2 Specifications of Compliant Element

With the location of the compliant element determined, the specifications can now be defined. To fully define the desired spring, I will focus on the maximum force and desired stiffness. Designing with these parameters in mind ensures that the system will be able to measure the full range of forces needed for the application and that it will at the same time have a stiffness appropriately matched to that of the human user. The other possible consideration for designing or choosing the spring is a focus on the length or footprint of the spring. However, it is only possible to choose two of these three—force, stiffness, or size—as the choice of any two will automatically define the third. Since force and stiffness are of higher concern, the focus will remain on those at the possible expense of a compact spring.

Placement of the sensor in-line with the conduit means that only a compression spring is necessary for compliance. For the application of upper limb rehabilitation, the sensor will need a sufficient force range to measure the torques applied during rehabilitation, while maintaining a stiffness large enough to passively render a range of stiffness values but not so large as to negatively impact the accuracy of the sensor. To determine rehabilitation torques, a common method of analysis is through the Activities of Daily Living (ADL) [56]. These show the range of motion and torque necessary
for a set of movements that approximate what is required to live independently. The torques and joint speeds for a range of movements are specified in Table 2.1 [57]. Taking these ADL movements as the benchmark, the torques necessary for normal elbow and shoulder movements are below 4 N-m and 10 N-m respectively. It can be seen that in general the shoulder requires higher torques than the elbow for the same movement. Since the SEA design should work for both joints, the max force must be based on the higher shoulder torques. The joint speeds are also summarized. Though

Table 2.1: Activities of Daily Living

<table>
<thead>
<tr>
<th>Activity</th>
<th>Elbow Torque (N-m)</th>
<th>Elbow Velocity (deg/s)</th>
<th>Shoulder Torque (N-m)</th>
<th>Shoulder Velocity (deg/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arm Reach to Head Level</td>
<td>3.5</td>
<td>146</td>
<td>10.1</td>
<td>171</td>
</tr>
<tr>
<td>Moving Object at Waist Level</td>
<td>3.8</td>
<td>53.2</td>
<td>9.6</td>
<td>83.7</td>
</tr>
<tr>
<td>Picking and Hanging Wall-Mounted Phone</td>
<td>3.1</td>
<td>173</td>
<td>9.0</td>
<td>137</td>
</tr>
<tr>
<td>Eating with Spoon (Normal Grasp)</td>
<td>1.1</td>
<td>35.5</td>
<td>4.5</td>
<td>84.5</td>
</tr>
<tr>
<td>Eating with Spoon (Power Grasp)</td>
<td>0.5</td>
<td>68.6</td>
<td>4.9</td>
<td>70.6</td>
</tr>
</tbody>
</table>
Table 2.2: Anthropometry Measurements *where no formula is given, values for 5th percentile female and 95th percentile male are taken directly from [58]

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Formula</th>
<th>5th% Female</th>
<th>95th% Male</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forearm-to-Hand (A)</td>
<td>*</td>
<td>37.3cm</td>
<td>55.5cm</td>
</tr>
<tr>
<td>Hand Length (B)</td>
<td>*</td>
<td>15.8cm</td>
<td>20.6cm</td>
</tr>
<tr>
<td>Bicep Circum. (C)</td>
<td>*</td>
<td>21.8cm</td>
<td>35.1cm</td>
</tr>
<tr>
<td>Bicep Radius (D)</td>
<td>((C)/(2 \times \pi))</td>
<td>3.5cm</td>
<td>5.6cm</td>
</tr>
<tr>
<td>Joint-to-Wrist (E)</td>
<td>((A) - (B) - 2 \times (D))</td>
<td>14.5cm</td>
<td>23.7cm</td>
</tr>
<tr>
<td>Forearm Circum. (F)</td>
<td>*</td>
<td>19.9cm</td>
<td>32.7cm</td>
</tr>
<tr>
<td>Forearm Radius (G)</td>
<td>((F)/(2 \times \pi))</td>
<td>3.2cm</td>
<td>5.2cm</td>
</tr>
</tbody>
</table>

these do not directly impact the design of the SEA, they do have an effect on the practical design of the exoskeleton as discussed below.

Since the exact design of the soft exoskeleton is not known, some assumptions must be made to insure the device will meet the requirements given any design. These assumptions can be made based on knowledge of anthropometry data and ADL movements. Some basic arm measurements are summarized in Table 2.2 and the corresponding positions on the arm are shown in Fig. 2.2 [58]. These measurements give the range of placement locations for the cable attachment point on the forearm. The measurement of interest to relate the ADL torques to the compliant element forces is the lever arm length. This lever arm length will determine the cable tensions.
Figure 2.2: Anthropometric measurements of the arm. Corresponding values for 5th percentile female and 95th percentile male can be found in Table 2.2. Measurements in red correspond to the minimum and maximum possible lever arms for the elbow joint using a tendon actuation system.

and speeds necessary to match ADLs. For tension, the lever arm length and cable tensions are inversely related:

$$T = \frac{M_{ADL}}{X}$$  \hspace{1cm} (2.1)

where $T$ is the cable tension, $M_{ADL}$ is the ADL joint torque, and $X$ is the lever arm length. So the shorter the lever arm, the larger the forces required from the cable,
Figure 2.3: Example calculations for two elbow attachment locations. The example torque and joint speed represent the max torque and an average joint speed for the combined shoulder and elbow values. The cable forces (a) and speeds (b) are shown for attachment at 4cm and 20cm from the point of rotation along the forearm.
a situation which can grow uncomfortable to the user. However, increasing the lever arm indefinitely does not work practically either. To match the joint speeds for ADLs, the cable speed will grow as the lever arms grows. The cable speed, while not directly related to the SEA design, will be limited by the motor properties. These tradeoffs in force and speed can be seen for two choices of elbow attachment in Fig. 2.3.

An additional complication arises in this analysis when considering the elbow joint. While the lever arm for the shoulder remains constant for a large fraction of the range of motion, the elbow lever arm changes rapidly. When the elbow is fully flexed, the lever arm is approximately the length to the attachment point along the forearm. From there, the lever arm decreases rapidly until the elbow is fully extended, at which point it is equal to the radius of the forearm. This concept is illustrated in Fig. 2.4(a). The equation describing this change is:

$$X = \begin{cases} 
L \cos\left(\frac{\theta}{2}\right) + \frac{r}{\cos\left(\frac{\theta}{2}\right)} & \theta \leq \frac{\pi}{2} \\
L \cos\left(\frac{\theta}{2}\right) + \frac{r}{\sin\left(\frac{\theta}{2}\right)} & \theta > \frac{\pi}{2}
\end{cases} \quad (2.2)$$

where $L$ is the length from the elbow joint to the attachment location, $r$ is the radius of the forearm, and $\theta$ is the joint angle. When plotted for various attachment points, it can be seen that the value decreases slowly or remains fairly constant for joint angles below 90 deg and decreases more rapidly between 90 deg and 180 deg (Fig. 2.4(b)). As well, closer attachment locations experience less severe changes in lever arm. Regardless of the attachment location, the lever arm length at full extension (180 deg) diminishes to the forearm radius. Additionally, attachment far from the joint in order to increase the average lever arm causes more area on the interior of the joint to be impeded by the unsupported cable length. While the best exoskeleton design may not necessarily have an attachment point close to the joint, these considerations suggest that the attachment point may be brought close without
imparting stricter sensor design restrictions. The design of the sensor must meet the requirement of ADLs even when the lever arm is reduced to its minimum at full extension, which results in the same minimum lever arm for any design. Therefore, it is a reasonable assumption that any soft robotic exoskeleton design will have as small a lever arm as possible at the joint while remaining comfortable to the user and feasible for the actuator. Taking the 5th percentile anthropometry measurements, the minimum lever arm should never fall below 3 cm. Using (2.1), this gives the maximum force necessary for the sensor of 330 N.

Taking a similar biologically inspired approach to stiffness, the rehabilitation device should be able to render stiffness values up to and above natural joint stiffness, in order to accommodate a range of uses. The highest stiffness that can be displayed to the user in a guaranteed passive manner is limited by the stiffness of the transmission, in this case the compliant element [48]. Joint stiffness is not constant and even varies based on joint angle during motion. For the elbow joint the stiffness reaches a maximum output at $10 - 15 \text{ Nm/rad}$, with the exact max stiffness varying person-to-person and between tasks [59]. Again, there exists a relationship to relate joint stiffness and stiffness of the compliant element using the lever arm:

$$K_{SEA} = \frac{K_{Joint}}{X^2}$$

(2.3)

where $K_{SEA}$ is the stiffness of the compliant element, $K_{Joint}$ is the joint stiffness, and $X$ is the lever arm length. Similar to the calculation of max force, the required linear stiffness grows as the lever arm is minimized. Therefore, the analysis performed to find the restricting lever arm for max force applies to stiffness as well. Considering the smallest elbow lever arm of 3 cm, the corresponding linear stiffness for 10 Nm/rad is 166.7 N/cm.

With these requirements for force and stiffness, a steel compression spring with
Figure 2.4: (a) Change in lever arm (red) during elbow extension/flexion. (b) Lever arm length versus elbow joint angle for various forearm attachment locations (L).
spring rate of 169.7 N/cm (96.88 lbf/in) and maximum load of 365.96 N (82.27 lbf) was chosen for use in the design (McMaster PN: 9657K395). These specifications match well with the biologically inspired requirements. This spring does not, however, give much room for error in either force or stiffness. Since the requirements for both are based on conservative measures of minimum lever arm, the 5th percentile female measurements, these values will be sufficient in the majority of users, while increasing the values could negatively impact sensor resolution and the benefits of Series Elastic Actuation.

2.1.3 Measuring Displacement

The last component to be specified before beginning the design is the method for measuring displacement of the compliant element. The main concerns for the sensor are the range and resolution. Since the compliant element has already been specified by the desired force range and stiffness, the sensor should have a measurement range as close as possible to the maximum displacement of the spring. Given the properties of the spring, this equates to 21.6 mm of displacement. While the sensor does not need to match this range, since the maximum force of the spring was chosen conservatively for the system and the full range may not always be in use, the closer it is to matching the range the better.

The resolution of the sensor is also of concern. Since digital and analog sensors are being considered, a measure of resolution common between both types of sensors should be used. While a simple measure of resolution can be defined for a digital sensor as the difference between any two consecutive measurements, this definition leads to the erroneous proposition that analog sensors have "infinite" resolution. Instead, error between the sensor measurement and true value captures this idea for
any sensor. One last adjustment is needed to compare choices with different ranges. Therefore, the measure of resolution chosen is max error as a percentage of full scale output (FSO). While the linearity of the spring can also affect this error measurement, we can expect most of the error to lie in the inherent resolution of the sensor. This is a measure that has also been used previously to describe series elastic actuators. Other similar SEA designs have achieved 3 – 5% FSO error [60]. It is difficult to determine the exact FSO error of the sensor before prototyping, but the sensor chosen should at least theoretically meet this requirement. In addition to these requirements, the sensor should also ideally be robust during use and should not increase the size of the series elastic sensor significantly.

With these basic requirements in mind, a few common types of position sensors can be examined. Beyond the categories of digital and analog, position sensors can generally be broken down into two categories, contact and non-contact sensors. Contact sensors include encoders and potentionmeters while non-contact sensors are Hall effect sensors or optical reflection sensors which react to changes in an energy field or beam. While contact sensors are generally simpler and easier to decode, non-contact sensors do not require any physical contact, which reduces wear and friction, and the design of non-contact sensors is more flexible. These properties make non-contact sensors appealing for use. Another dichotomy between sensor options is absolute versus incremental sensors. Incremental sensors, like many encoders, only have knowledge of a position relative to an index or start point, and can start to build error if the sensor misses increments. For this reason, absolute sensors, which have a unique reading for each point in the range, are required to avoid performing a start-up calibration, which may not always be possible if the system is pre-tensioned. Potential sensor choices are summarized in Table 2.3.
Table 2.3: Possible Displacement Sensors: Commercial and Custom

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Digital or Analog</th>
<th>Contact or Non-contact</th>
<th>Absolute or Incremental</th>
<th>Range (mm)</th>
<th>Resolution (% FSO)</th>
<th>Size: LxWxH (mm)</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Digital Encoder</td>
<td>Digital</td>
<td>Contact</td>
<td>Incremental</td>
<td>&gt; 21.6</td>
<td>0.06</td>
<td>34.9 x 23.4 x 10.2</td>
<td>$79.40</td>
</tr>
<tr>
<td>Renishaw Encoder</td>
<td>Digital</td>
<td>Contact</td>
<td>Absolute</td>
<td>&gt; 21.6</td>
<td>&lt; 0.01</td>
<td>39.6 x 14.9 x 21.7</td>
<td>$1000</td>
</tr>
<tr>
<td>String Pot</td>
<td>Analog</td>
<td>Contact</td>
<td>Absolute</td>
<td>&gt; 21.6</td>
<td>1.7</td>
<td>40.6 x 10.0 x 19.0</td>
<td>$100—$500</td>
</tr>
<tr>
<td>Laser Position Sensor</td>
<td>Analog</td>
<td>Non-Contact</td>
<td>Absolute</td>
<td>20</td>
<td>&lt; 0.01</td>
<td>100 x 65 x 20</td>
<td>$400—$800</td>
</tr>
<tr>
<td>LVDT</td>
<td>Analog</td>
<td>Non-Contact</td>
<td>Absolute</td>
<td>25</td>
<td>0.25</td>
<td>108.8 x 9.5 x 9.5</td>
<td>$400—$800</td>
</tr>
<tr>
<td>Hall Effect Sensor [60]</td>
<td>Analog</td>
<td>Non-Contact</td>
<td>Absolute</td>
<td>8</td>
<td>4.5</td>
<td>13.7 x 6.0 x 5.73</td>
<td>$3</td>
</tr>
</tbody>
</table>
The sensors described in Table 2.3 are commercially available sensors, with the exception of the Hall effect sensor, which was designed by Sergi et. al. for use in a series elastic actuator [60]. Comparing across sensor types, we can see that while encoders have the most flexibility when it comes to range, going from incremental to absolute encoders results in a large cost increase. On the other hand, the prepackaged non-contact sensors require a package length over triple the measurement range, which will increase the size of the full SEA significantly. For the remaining choices, the string pot and Hall effect sensor have comparable resolutions that meet the resolution requirement, though it should be noted the Hall effect sensor resolution is inflated due to including the hysteresis of the compliant element. Each sensor does have its own downsides though. The string pot is a contact sensor, which can add friction to the sensor operation. But more importantly, the method of operation for a string pot requires a restoring force to re-spool the string, which, while only on the order of 1 – 5% of the force range, can have some effect on sensor operation. The Hall effect sensor, on the other hand, does not have the range sufficient in the designed configuration to cover the full spring compression. This is partly due to the design requirements considered in [60], so it would be possible to redesign to extend the range. The Hall effect sensor is also very cheap and flexible in design, making a custom sensor design feasible.

2.2 Single Hall Force Sensor

With the problem statement and general design of the series elastic sensor set, the hardware design and analysis of the displacement sensor can be done. The design is shown in Figure 2.5. The design of this sensor is an extension of the SEA design used in [60].
2.2.1 Non-contact Linear Displacement Sensor

Based on the requirements and discussion in Section 2.1.3, a non-contact analog displacement sensor was selected for use in the compliant sensor design. Since commercial non-contact displacement sensors have large package sizes, a custom Hall effect sensor was chosen. Hall effect sensors are analog transducers that measure magnetic flux in a direction perpendicular to the sensor at a point. While they are often used in proximity switches or tachometers, where the sensor consists of an on/off signal triggered by a threshold flux, they can also be used to measure linear position over short ranges. The commonly used design for a linear position sensor consists of a Hall
Figure 2.6: Hall effect based linear position sensor. Relative positioning of magnets (orange) and Hall effect sensor (red) are shown. Sensor translates at a fixed distance $h$ above the magnets, which are positioned a distance $d_{mag}$ apart.

effect sensor measuring at a set height above oppositely polarized magnets separated by a small distance. Using this setup, a linear range of varying magnetic flux exists between the magnets and this linear range can easily be translated into a position sensor.

To design the custom Hall effect sensor, 2D simulations were completed using the freeware magnetic simulator FEMM (http://www.femm.info/wiki/HomePage) to determine flux values for a range of available magnets and designs. The design values that were examined were distance between the magnets, $d_{mag}$ and height of the sensor above the magnets, $h$, as shown in Fig. 2.6. A single cylindrical magnet was simulated and the results were shifted and superimposed to create the full magnetic flux field for the multi-magnet system. The design was optimized to provide the largest range while still retaining a highly linear measurement region. As will be discussed in Section 2.2.3, this analysis was carried out multiple times for different configurations.
2.2.2 Hardware

A detailed view of the sensor can be seen in Fig 2.7. The sensor consists of two pieces connected to either side of the spring. Each half of the hardware attaches to one component of the displacement sensor, either the magnets or the Hall effect sensor. These halves of the sensor are contained on separate rapidly prototyped pieces to easily modify the value of $d_{mag}$, for maximum flexibility in designing the displacement sensor. Additionally the sensor pieces contain slots to vary the value of $h$ and the zero position of the displacement sensor relative to the force range. If the range of the sensor is large enough, this will allow the position sensor range to be matched properly over the necessary force range.

To insure the proper functioning of the sensor, all force on the spring must be translated into pure linear compression. In addition to the support of the spring, a pair of round linear bearings (McMaster PN: 626K81) and quarter-inch aluminum shafts act to resist side forces and moments on the spring. These bearings ideally increase the lateral stiffness of the device. The round bearings were chosen for ease of installation. The last features of the design are interface points to the testbed and to the conduit.

2.2.3 Modeling

The displacement sensor configuration includes an off-the-shelf Halls sensor with a range of 100 mT and neodymium magnets, grade N52, from K & J Magnetics Inc. with diameter 4.8 mm and height 1.59 mm. Using the FEMM magnetic simulator, an axisymmetric magnetostatic finite element method simulation of the N52 magnet was carried out. The results of this simulation are shown in Fig. 2.8. For a magnetostatic problem, defined as a problem where the fields are time-invariant, Gauss’s law of
magnetism and Ampère’s law define the behavior of the magnetic flux density $B$ and magnetic field intensity $H$ respectively:

$$\nabla \cdot B = 0 \quad (2.4)$$

$$\nabla \times H = J \quad (2.5)$$

where $J$ is the current density. For each material being simulated, these equations are subjected to an additional constraint:

$$B = \mu \ast H \quad (2.6)$$

where $\mu$ is permeability, which in a general problem may be a function of the magnetic flux $B$. FEMM attempts to find a field satisfying (2.4)-(2.6) by finding a solution to the equation:

$$\nabla \times \left( \frac{1}{\mu(B)} \nabla \times A \right) = J \quad (2.7)$$
Figure 2.8: Magnetic flux simulation results for a single cylindrical neodymium magnet, grade N52 (generated by FEMM)
where \( A \) is a magnetic vector potential defined by:

\[
B = \nabla \times A
\]  \hspace{1cm} (2.8)

For an axisymmetric problem, the solutions of (2.7) reduce from three dimensional to two dimensional, which the methods employed by FEMM can solve. The solution for the magnetic flux, \( B \), is shown in Fig. 2.8. From this solution, we can determine the perpendicular flux that the Hall effect sensor would be subjected to given a height above the magnet. Though the solution generated by FEMM only accounts for a single magnet, through superposition the solution for multiple magnets can be found.

Solutions for the full two magnet system are found varying magnet height above the sensor, \( h \), and distance between the magnets, \( d_{\text{mag}} \). For values of \( h \) between 2.5 mm to 4.25 mm and values of \( d_{\text{mag}} \) between 7.5 mm and 10.5 mm, the magnetic flux is examined. The results for three values of \( d_{\text{mag}} \) can be seen in Fig. 2.9: the extremes of the range and the chosen value of \( d_{\text{mag}} \). As the distance between the magnets increases and the height of the sensor above the magnets decreases, the non-linearity of the measurement increase within the desired range (between the peaks). However, increasing \( d_{\text{mag}} \) also increase the peak to peak distance, therefore increasing the range of the sensor. And decreasing \( h \) leads to a higher sensor sensitivity, so each change in displacement leads to a larger change in voltage from the Halls sensor. These three qualities—linearity, sensor range, and sensitivity—must be balanced to achieve a sensor with the high range and low error desired. Reexamining the requirements in Section 2.1.3, the range of 21.6 mm will be the much more difficult to achieve fully. Therefore, in this instance, range was prioritized in order to reach close to 50% of the full spring displacement. At a \( d_{\text{mag}} \) of 9.4 mm, good linearity is maintained for the larger values of \( h \), though choosing higher values of \( h \) does lower the sensitivity. To check if a combination is acceptable in terms of linearity, the maximum percent
Figure 2.9: Magnetic flux versus sensor position for two magnets separated by distance $d_{mag}$ of (a) 7.5mm, (b) 9.4mm, and (c) 10.5mm for values of $h$ between 2.58 and 4.25mm
error and $R^2$ values for a linear fit was calculated, which corresponds to the highest theoretically achievable resolution and linearity for that set of parameters. For $d_{mag} = 9.4$ these results can be found in Table 2.4. These calculations show that the $R^2$ is maximized between $h = 3.97\text{mm}$ and $h = 4.25\text{mm}$ while the maximum percent error is minimized between $h = 3.41\text{mm}$ and $h = 4.25\text{mm}$. It should be noted, though, that all values of maximum percent error fall within the requirements set in Section 2.1.3. So while the range of $h$ values between 3.69mm and 4.25mm will be examined most closely and the sensor will be tested near these configurations, any value for $h$ should lead to acceptable behavior.

Table 2.4: Resolution for Two Magnet Sensor Simulations with $d_{mag} = 9.4\text{mm}$

<table>
<thead>
<tr>
<th>Magnet Height $h$ (mm)</th>
<th>$R^2$</th>
<th>Resolution (% FSO)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.58</td>
<td>0.9918</td>
<td>3.83%</td>
</tr>
<tr>
<td>2.86</td>
<td>0.9957</td>
<td>2.85%</td>
</tr>
<tr>
<td>3.13</td>
<td>0.9978</td>
<td>2.11%</td>
</tr>
<tr>
<td>3.41</td>
<td>0.9990</td>
<td>1.50%</td>
</tr>
<tr>
<td>3.69</td>
<td>0.9996</td>
<td>0.99%</td>
</tr>
<tr>
<td>3.97</td>
<td>0.9999</td>
<td>0.62%</td>
</tr>
<tr>
<td>4.25</td>
<td>0.9999</td>
<td>1.17%</td>
</tr>
</tbody>
</table>
2.2.4 Design Limitations

After construction of the compliant sensor according to the above design (Figure 2.5), the sensor was installed and tested to see its performance against the design specifications. However, before the sensor could be calibrated and fully tested, some design issues were found that required a full redesign of the hardware and Hall effect sensor set-up. The largest issue was the range of the Hall effect sensor. The measurement range with high linearity covered approximately 7.25 mm, 34% of the full sensor range of 21.6 mm. This was known from the FEMM simulations. While the hypothesis that the full force range would not be needed was found true, the tensions experienced in the testbed during interaction with a user would still consistently reach outside the linear range of the sensor. Since measurements on either side of the peaks are indistinguishable from each other, it is vital for the displacement sensor to read the force range used when interaction with a user. This tension was not found to exceed 200 N or 55% of the max spring force.

The remaining issue dealt with the hardware. During attempts to calibrate the Hall effect sensor, repeated calibrations would yield different results and the measurement error in the sensor was significantly higher than expected. It was determined that these effects were caused by off-axis movements resulting from moments induced by the conduit and lower than expected lateral stiffness from the cylindrical bearings. The relative twisting of the sensor halves changed the orientation of the Hall effect sensor relative to the magnetic flux field, leading to error in the measurements. Even with a proper calibration, this inconsistency rendered the sensor difficult to use. The solution to these design issues and the final design of the compliant sensor is addressed in Section 2.3.
2.3 Dual Hall Force Sensor

After examining the results of the first compliant sensor prototype, modifications were made to the hardware and displacement sensor design to increase the sensor range and repeatability. The final compliant sensor design is shown in Fig. 2.10.

2.3.1 Extending Linear Range

To extend the linear range of the displacement sensor, a novel Hall effect sensor design was developed and tested using three magnets of alternating polarity and two Hall effect sensors offset by a short distance, $d_{\text{hall}}$, shown in Fig. 2.11. Similar to how the A and B channels on an optical encoder take advantage of quadrature, or separation of signals by 90 deg, to be able to distinguish the direction of the movement, the use of two Hall effect sensors should allow for the sensor to distinguish between sections of the magnetic field with positive and negative slopes. This allows for an instant doubling of the range as the sections of the magnetic flux distribution on either side of the peaks are now measurable in Fig. 2.9. The addition of a third magnet helps linearize these outside regions of the flux. Additionally, as the use of two channels on an encoder allows the resolution of the encoder to quadruple, using two Hall effect sensors should lead to some improvement in accuracy of measurements.

2.3.2 Hardware

The design of the hardware is very similar to that described in Section 2.2.2, with some modifications to accommodate a replacement linear bearing and fix minor issues to improve sensor robustness. The bearing chosen for the redesign is a Nippon slide guide type bearing (pn: $SEBS_{7WB}$). While not as easy to install or as compact as the round bearings used in the initial design, the Nippon slides are significantly
Figure 2.10: (a) Rendering of compliant sensor with color emphasizing magnets (orange), Hall effect sensors (red), and compression spring (blue); (b) physical prototype
Figure 2.11: Modified Hall effect based linear position sensor using a novel three magnet, two sensor design. Relative positioning of the magnets (orange) and Hall effect sensors (red) are shown. Parameter $d_{\text{hall}}$ corresponds to the offset between the sensors.

more resistant to moments and off axis movements. Also, by virtue of the design of the bearing, a single slide is necessary, compared to two round bearings in the initial design.

The conduit interface piece (Fig. 2.12) was modified to mount the bearing slider. For proper alignment, one side of the mounting location was used as a reference and 4 – 40 set screws were used to clamp the bearing slide to this wall while the bearing is mounted. The testbed interface (Fig. 2.13) was also modified, this time to mount the bearing rail. Again, one wall was used as the reference to ensure proper alignment, however here an external clamp is needed to clamp the rail while it is attached. The last pieces of the compliant sensor (Fig. 2.14) which hold the magnets and Hall effect sensors also required slight modification. In addition to increasing the attachment locations for magnets and Hall effect sensors, the thickness of both
Figure 2.12: Rendering of one half of sensor with conduit interface. The bearing slider is mounted here.

Figure 2.13: Rendering of second half of sensor with interface to testbed. The bearing rail is mounted here.
Figure 2.14: Parts for mounting Hall effect sensors and magnets to the two halves of the sensor.

Figure 2.15: Exploded detail of compliant sensor rendering
pieces was increased to eliminate some cantilever effects over time that were seen in the initial design. Additionally, the Hall effect sensor mount was given wings for easier method of alignment with the direction of compression. The relative positions of these pieces can be seen in the exploded view of the sensor (Fig. 2.15).

### 2.3.3 Modeling

The basic set-up of the Hall effect sensor modeling for the two sensor, three magnet design is the same as 2.2.3, using the same magnets, sensors, and modeling software to model a single axisymmetric magnet. Adding the additional magnet does not change the original distribution significantly, as can be seen in Fig. 2.16. Comparing Fig. 2.16 to Fig. 2.9 (b), there is little noticeable difference between the magnetic flux distributions besides the additional peak for the third magnet. The additional peak to peak range spans the same linear displacement as the original two magnet system, allowing for the possibility of doubling the usable sensor range from the initial design. When the resolution metrics described in Section 2.2.3 are recalculated for the new sensor, the additional magnet does have a small detrimental impact on the linearity and resolution (Table 2.5), but not as large as the difference generated by increasing the magnet separation. As well, the impact was greatest for smaller values of \( h \). So the methods that led to a value for \( d_{mag} \) and a range of acceptable \( h \) values in Section 2.2.3 applies here as well, with only slight modification. The combination of linearity and resolution for the three magnet simulation leads to a slightly reduced range of \( h \) between 3.69mm and 4.25mm.

With the addition of a second Hall effect sensor the distance between the sensors, \( d_{hall} \), must also be considered during the design. While both sensors will read the same magnetic flux distribution, the offset between them allows the sensor to read
Figure 2.16: Magnetic flux versus sensor position for three magnets separated by a distance $d_{mag}$ of 9.4mm for values of $h$ between 2.58mm and 4.25mm. Both linear segments of the flux distribution as explained in Section 2.3.1. Changing this distance does not have a significant effect on the range the sensor can read. To make an absolute sensor, each reading must be unique, which by definition means that the sensor range will be around one period of the signal or two peak-to-peak distance. However, to make use of the full range, at least one of the Hall effect sensors must be reading a linear range at any given time. This sets a range of values for $d_{hall}$ between 3mm and 6.4mm. This range is further restricted by the physical size of the Hall effect sensor, which is 4.1mm wide. A gap between the sensors of 1mm is also needed to be able to accurately 3D print the Hall sensor attachment point. This gives the final allowable range for $d_{hall}$ of 5 to 6.4mm. Simulations for the limits of the $d_{hall}$ range can be seen in Fig. 2.17.

In parts of the range where both signals are linear (shown striped), the readings from both sensors can be used to decrease the measurement error. As with the sensor
Figure 2.17: Magnetic flux readings for two Hall effect sensors offset by a distance $d_{\text{hall}}$ of (a) 5mm and (b) 6.4mm and with $d_{\text{mag}}$ held constant at 4.25mm. Striped regions show the sections of the range where both sensors read a linear region of the magnetic flux.
range, changing the value of $d_{hall}$ within this range does not significantly change the percentage of the sensor range where both sensors provide a reading, it simply shifts the areas that range covers. Having the regions spread out more evenly across the range as in Fig. 2.17 (a) could have greater benefits to sensor resolution whereas the situation in Fig. 2.17 (b) has half the affected areas concentrated on the outer edges of the range where they are less likely to have an impact. These regions can also be thought of as buffer zones, showing the amount of change necessary in $d_{hall}$ to put the sensor in a configuration where some areas are not covered by a linear region of the magnetic flux distribution. In the configuration where $d_{hall} = 6.4$mm, the slightest
error could cause the sensor to rely on more non-linear regions. For these reasons, a value of $d_{hall} = 5\text{mm}$ is chosen for the two Hall effect sensor design. With this, the compliant sensor, as shown in Fig. 2.10, is fully designed with $d_{mag} = 9.4\text{mm}$ and $d_{hall} = 5\text{mm}$, and with an adjustment of $h$ between 3.69 and 4.25mm.

### 2.3.4 Sensor Validation

After prototyping, the compliant force sensor described in Sections 2.3.1-2.3.3 was validated in three separate stages. First, the sensor readings were recorded at set points in the motion range to calculate the magnetic flux distribution and compare to simulation. From this distribution, the linear and non-linear regions were distinguished, transition points were found, and linear fits calculated in order to find the piece-wise linear function relating sensor reading and displacement. Lastly, the spring was installed in the compliant sensor and the force sensing abilities of the sensor were verified. These validation steps also acted as the calibration process, first calibrating the displacement from the magnetic flux and then calibrating the force from the displacement measurements. This two step calibration was done to allow for the stiffness to be found along the way, rather than directly calculating force from the magnetic distribution and having the stiffness obscured in the total calibration constants.

As discussed, the first step in the validation and calibration process involved finding the actual magnetic flux distribution corresponding to that simulated in Section 2.3.3. This was accomplished by first removing the compliant element from the full sensor, so the magnetic flux could be measured at constant known displacements. The flux was recorded at 75 points between the mechanical displacement limits and plotted versus the sensor position measured externally using calipers. The results are shown in Fig. 2.18. The distribution closely resembles the simulated one seen in
Figure 2.18: Experimentally obtained magnetic flux readings for two Hall effect sensors offset by a distance $d_{\text{hall}}$ of (a) 5mm and (b) 6.4mm and with $d_{\text{mag}}$ held constant at 4.25mm. Sensor position is measured externally by calipers and does not correspond to spring displacement.

Fig. 2.17 (a) on a qualitative level, with two linear regions per sensor and a sufficient offset between the sensor readings to properly stagger the linear and non-linear regions. In Fig. 2.19, the experimentally obtained distributions were compared to the simulated magnetic flux distributions. However, to make this comparison, the simulated data must be converted from magnetic flux to sensor voltage. Two conversions are shown in Fig. 2.19, one where the nominal sensor output voltage value of 14V/T and voltage offset value 2.5V were used and one where a best fit values for the simulated data were found. The fit values, summarized in Table 2.6, differed slightly between the sensors, with Sensor 1 using output voltage 14.6V/T and offset 2.52V and Sensor 2 using output voltage 14.3V/T and offset 2.54V. These best fit values differ from the nominal by less than 5%, well within the operating conditions of the
Figure 2.19: Comparison of experimentally collected sensor readings to simulated magnetic flux found in Section 2.3.3. Simulated values are plotted for nominal output voltage (14V/T) and offset voltage (2.5V) and for values chosen to most closely fit simulated curves to experimental curves.

sensors. To compare the simulated best fit to the experimental data, the Root Mean Squared Error (RMSE) for each sensor was calculated. These values, shown in Table 2.6, are 1–3% of the voltage range of the sensor, indicating that the experimental data matches the simulated data fairly well. Examining Fig. 2.19 more closely, a few discrepancies in slopes and spacing can be seen, indicating that the actual values of $d_{mag}$, $d_{hall}$, and $h$ may differ slightly from the simulation.

After validating that the sensor output closely matches the simulated output, validation of the experimental data was continued by developing a piece-wise linear
Figure 2.20: (a) Sensor reading versus position. Linear fits for position calibration are shown with labels corresponding to Table 2.7. (b) Difference in sensor reading versus position. Dashed lines indicate the upper and lower limits of the linear regions. In striped regions both sensors occupy a linear region.
Table 2.6: Constants for fitted simulated curves and RMSE values

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Output Voltage (V/T)</th>
<th>Offset Voltage (V)</th>
<th>RMSE (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal</td>
<td>14</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>Sensor 1</td>
<td>14.6</td>
<td>2.52</td>
<td>0.0199</td>
</tr>
<tr>
<td>Sensor 2</td>
<td>14.3</td>
<td>2.54</td>
<td>0.0284</td>
</tr>
</tbody>
</table>

calibration function relating Hall sensor voltage and sensor position. Looking at the simulated data in Fig. 2.17, each sensor has three linear regions separated by two non-linear regions. Therefore, to construct the calibration, three sets of parameters must be found: the slopes and intercepts of all six linear regions, and the upper and lower cutoff voltages delineating the transition between the linear and non-linear regions. For each linear region, this was accomplished by the following process:

1. Assume the full peak-to-peak region is linear.
2. Calculate a linear fit for the region.
3. Check the error to the linear fit at the endpoints against the desired threshold.
4. If the threshold is not met, remove a set number of points from the boundary of the linear region and repeat steps 2 – 3.
5. Else, the maximized linear region has been found. The linear fit gives the values for slope m and intercept b and the endpoints correspond to the upper and lower cutoff values.
Fig. 2.20 shows the results of this process graphically. In Fig. 2.20 (a), the six linear regions are shown and their linear fits with endpoints marked. The process successfully separates the linear and non-linear regions, such that the processed data resembles the simulated linear and non-linear regions in Fig. 2.17. The difference between in the sensor readings is shown in Fig. 2.20 (b). A band on each peak between the upper and lower limits shows the sections of the distribution where only one sensor can be used, corresponding to the four non-linear regions around the peaks of the sensor distributions. As in Fig. 2.17, the regions using two sensors are marked by striped boxes. Completing this process with the experimental data yielded the calibration coefficients shown in Table 2.7. These coefficients are used in the piece-wise linear equation as follows:

\[
m_1, b_1, c_1 = \begin{cases} 
\frac{1}{m_A}, \frac{-b_A}{m_A}, 1 & \text{if } s2(x) < s1(x) < U_A \text{ and } s2(x) < \frac{V_p}{2} \\
\frac{1}{m_B}, \frac{-b_B}{m_B}, 1 & \text{if } L_B < s1(x) < U_B \text{ and } s2(x) > \frac{V_p}{2} \\
\frac{1}{m_C}, \frac{-b_C}{m_C}, 1 & \text{if } L_C < s1(x) < s2(x) \text{ and } s2(x) < \frac{V_p}{2} \\
0, 0, 0 & \text{else}
\end{cases}
\]

\[m_2, b_2, c_2 = \begin{cases} 
\frac{1}{m_D}, \frac{-b_D}{m_D}, 1 & \text{if } L_D < s2(x) < s1(x) \text{ and } s1(x) < \frac{V_p}{2} \\
\frac{1}{m_E}, \frac{-b_E}{m_E}, 1 & \text{if } L_5 < s2(x) < U_E \text{ and } s1(x) > \frac{V_p}{2} \\
\frac{1}{m_F}, \frac{-b_F}{m_F}, 1 & \text{if } s1(x) < s2(x) < U_F \text{ and } s1(x) < \frac{V_p}{2} \\
0, 0, 0 & \text{else}
\end{cases}
\]

\[x = \frac{c_1 * (m_1 * s1(x) + b_1) + c_2 * (m_2 * s2(x) + b_2)}{c_1 + c_2}
\]

where \(s1(x)\) and \(s2(x)\) are the sensor outputs, \(V_p\) is the voltage powering the sensor, and \(c_1\) and \(c_2\) are binary variables indicating whether the given sensor is
Table 2.7: Sensor voltage to sensor displacement calibration constants. * For regions on the outside of the distribution only a upper or lower bound is provided.

<table>
<thead>
<tr>
<th>Linear Region</th>
<th>Slope, m (V/mm)</th>
<th>Intercept, b (V)</th>
<th>Upper Limit, U (V)</th>
<th>Lower Limit, L (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1426</td>
<td>2.233</td>
<td>3.0427</td>
<td>*</td>
</tr>
<tr>
<td>2</td>
<td>−0.1454</td>
<td>4.201</td>
<td>3.0563</td>
<td>2.0053</td>
</tr>
<tr>
<td>3</td>
<td>0.1365</td>
<td>−0.365</td>
<td>*</td>
<td>1.9967</td>
</tr>
<tr>
<td>4</td>
<td>−0.1211</td>
<td>2.201</td>
<td>*</td>
<td>2.0128</td>
</tr>
<tr>
<td>5</td>
<td>0.1426</td>
<td>1.517</td>
<td>3.0542</td>
<td>2.0237</td>
</tr>
<tr>
<td>6</td>
<td>−0.1490</td>
<td>5.002</td>
<td>3.0335</td>
<td>*</td>
</tr>
</tbody>
</table>

contributing to the linear measurement. Applying (2.9)-(2.11) and the calibration constants in Table 2.7 to the experimental data, the measured displacement of the sensor is calculated. The error to the actual displacement is shown in Fig. 2.21. Across the position range of 18.57mm there is an average error of 0.055mm and a max error of 0.192mm or 1.04% FSO. This resolution is within the theoretical range based on Table 2.5 and the range covers 86% of the full displacement of the spring, well exceeding the minimum requirement of 50%. This validates the two Hall effect sensor design as a displacement sensor meeting the necessary requirements for the compliant sensor.

The last step in validating and calibrating the compliant sensor was to test its
Figure 2.21: Sensor error in measured displacement versus actual displacement. Dashed lines show the average error.

capabilities as a force sensor. Using the previously established position calibration and an external tension sensor (Futek LSB200), the force versus position relationship of the sensor, shown in Fig. 2.22(a), was found. The data appears very linear as expected. A linear fit between the sensor displacement and measured force was calculated with an $R^2$ value of 0.9998. The slope of this line corresponds to the stiffness of the compliant sensor and was found to be 177.1N/cm. This is within 4.5% of the stiffness of the compression spring. To find the sensor resolution, the error to the linear fit was found, plotted in Fig. 2.22(b). Over the full range of the sensor of 329N, the average error was 1.29N while the max error was 6.60N or 2.01% FSO. With this the compliant force sensor has been fully validated to meet the requirements in terms of force range, stiffness, and resolution as set out in Section 2.1.
Figure 2.22: (a) Force versus position for the compliant sensor and linear fit to the data. (b) Force error from linear fit. Dashed lines show average error.

2.4 Summary and Conclusions

In this chapter I have presented the design process for a compliant force sensor for use in Series Elastic Actuation in a system with Bowden cable transmission. The requirements for the sensor, presented in Section 2.1, arose from biologically inspired considerations or practical design limitations of the intended soft robotic use. An
initial design for the compliant sensor, Section 2.2, successfully integrated all of the pieces theoretically needed to meet these requirements, but failed to have the mechanical robustness or sensor range necessary to act as a force sensor. The mechanical improvements made to this design in Section 2.3 improved the consistency of the sensor while the redesign of the displacement sensor to use a two Hall effect sensor design increased the sensor range from 34% to 86% of the full compression of the spring while keeping the sensor error low. The final sensor design fully meets the requirements laid out in Section 2.1.

All the validation in this chapter applies to the compliant force sensor as a stand alone force sensor. For further validation of the sensor for its intended use, it must be placed in a Bowden conduit transmission. This will be carried out in Chapter 4 for the purpose of validating the strategy of sensing conduit compression forces, performing system analysis to find the dynamic effects of the sensor, and verifying the compliant force sensors performance in force and impedance control.
Chapter 3

Bowden Cable Model

A model to facilitate an understanding of the cable-conduit dynamics and behavior was developed as a precursor to experimentation on the Bowden cable testbed. Two main approaches were undertaken: a lumped parameter dynamic model and simulation showing the elasticity and non-linear damping of the conduit-cable transmission, and a simplified quasi-static mathematical model with the conduit held to a shape. This modeling gives predictions as to what behavior can be expected from the transmission based off initial shape, friction coefficient, and material properties, as well as an answer as to what extent the measure of the conduit compression matches the cable tension.

3.1 Lumped Parameter Dynamic Model

A lumped parameter model of the Bowden cable transmission was developed and simulated.

3.1.1 Derivation of Model Equations

The lumped parameter model adapted from [33] is illustrated in Fig. 3.1. Within an individual element, shown in Fig. 3.1(a), the cable is assumed to have a constant radius of curvature, \( R \), and therefore constant curvature, \( C \). The combination of the cable tension, \( T(x,t) \), and curvature creates a normal force, \( N(x,t) \) between the conduit wall and cable, which in turn causes a non-linear damping between the
Figure 3.1: (a) Single cable-conduit element with forces and dimensions. Cable element (grey) interacts with the conduit element (black) through a normal force, $N(x,t)$ and friction force, $f(x,t)$. Tension, $T(x,t)$, and compression, $C(x,t)$, forces represent the interaction between neighboring cable or conduit elements respectively. The geometry of the element is illustrated by the element length, $dx$, the radius of curvature, $R$, and subtended angle, $d\theta$. (b) Dynamic model of cable conduit transmission with actuator and load. Subscripts differentiate between actuator (A), conduit (C), and load (L) portions of the model. Bowden cable is discretized into $n$ elements, each with stiffness, $k_c$, Coulomb friction, $f_i$, and viscous damping, $b_c$. 
conduit and cable. This normal force is equal to:

\[ N(x, t) = \sin\left(\frac{d\theta}{2}\right) \ast [T(x, t) + T(x + dx, t)] \]  

(3.1)

where \( T(x, t) \) and \( T(x + dx, t) \) are the tensions on either side of the cable element, \( d\theta \) is the angle subtended by the arc, and \( N(x, t) \) is the normal force. A few simplifications can be made to this equation. First, the angle is equal to the reciprocal of the curvature, which can be written \( C = R/dx \). Additionally, since the conduit is broken into small elements, a small angle approximation can be made in place of the \( \sin(d\theta) \) term. When these approximations are made, (3.1) becomes:

\[ N(x, t) = \frac{dx}{R} \ast \frac{T(x, t) + T(x + dx, t)}{2} \]  

(3.2)

This equation shows that, for a small cable element, the normal force is approximately equal to the average tension across the element divided by the curvature. So, as the curvature increases and the transmission becomes straighter, the normal force in the cable will decrease.

Using (3.2), the Coulomb friction in the conduit can be generalized with the addition of static and dynamic friction coefficients, \( \mu_s \) and \( \mu_d \). The static friction coefficient determines the tension needed to begin movement, while the dynamic friction coefficient determines the friction force during motion. The full model of friction used is:

\[ f(x, t) = \begin{cases} 
T(x + dx, t) - T(x, t) & f_s(x, t) \leq |T(x, t) - T(x + dx, t)| \\
\text{sign}(v) \frac{\mu_d dx}{R} \ast \frac{T(x, t) + T(x + dx, t)}{2} & f_s(x, t) > |T(x, t) - T(x + dx, t)| 
\end{cases} \]  

(3.3)

where \( v \) is the cable velocity and

\[ f_s(x, t) = \frac{\mu_s dx}{R} \frac{T(x, t) + T(x + dx, t)}{2} \]  

(3.4)
Pure Coulomb friction acts when $\mu_s = \mu_d$, while $\mu_s > \mu_d$ causes Coulomb friction plus stiction. These equations, (3.3) and (3.4), show the defining non-linearity of the Bowden cable transmission.

An illustration of the remainder of the modeled dynamics in the lumped model can be seen in Fig. 3.1(b). The cable tensions, $T(x, t)$, from Fig. 3.1(a) are the interaction forces between the cable elements and are represented as linear springs. As well, a viscous damping term, $b_c$, between the cable element and the conduit is added to fully generalize the nonlinear friction term and each element is given a non-negligible mass, $m_c$. While many analyses of cable conduit dynamics transition to a continuum model using the derived equations for normal force and friction [36, 37, 40, 61], the lumped model will be sufficient to determine the major components of the underlying dynamics in simulation when an adequate number of elements are used.

### 3.1.2 Lumped Model Simulation

The dynamic model shown in Fig. 3.1 was simulated in Simulink using ten cable elements. For the $i$th element, the equation of motion can be written:

$$m_c \dddot{x}_{c,i} + b_c \dot{x}_{c,i} + f_i(x_{c,i}, t) + k_c(2x_{c,i} - x_{c,i-1} - x_{c,i+1}) = 0$$  \hspace{1cm} (3.5)

where $x_{c,i}$, $\dot{x}_{c,i}$, and $\dddot{x}_{c,i}$ are the position, velocity, and acceleration of the $i$th cable element. The values used for the dynamic properties are summarized in Table 3.1. The cable stiffness, $k_c$, is chosen to be significantly higher than the stiffness of the compliant conduit sensor while the mass, $m_c$, is small but non-negligible. While a smaller mass will lead to a more difficult to stabilize simulation, it is also necessary to keep mass low to properly represent a small cable element. To ensure stability of the simulation, the viscous damping coefficient, $b_c$, is increased until the dynamic model
is stabilized. Dynamic friction coefficients for Teflon™ range between 0.05 to 0.2 so a value of 0.1 is chosen for $\mu_d$. The value for $\mu_s$ was selected above the upper end of the range as, for many material combinations, Teflon™ has a marked difference between static and dynamic friction coefficients [62]. The cable-conduit geometry was selected based off the analysis for conduit length and curvature presented in Section 4.1.1.

Table 3.1: Parameter values used in ten element simulation

<table>
<thead>
<tr>
<th>Linear Dynamic Properties</th>
<th></th>
<th>Coulomb Friction Characteristics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_c$ (kg)</td>
<td>0.001</td>
<td>$\mu_s$</td>
<td>0.25</td>
</tr>
<tr>
<td>$b_c$ (Ns/cm)</td>
<td>5.5</td>
<td>$\mu_d$</td>
<td>0.1</td>
</tr>
<tr>
<td>$k_c$ (N/cm)</td>
<td>1500</td>
<td>$dx$ (cm)</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$R$ (cm)</td>
<td>25</td>
</tr>
</tbody>
</table>

Simulations were first carried out with the motor dynamics modeled as a pure torque source and the load position fixed. The modeled motor applied a sinusoidal tension with initial tension of 60N, amplitude of 50N, and frequency of 0.08rad/s. Both the case of pure Coulomb friction and friction with stiction were tested. For the case of pure Coulomb friction, $\mu_s$ was set equal to $\mu_d$. The results for pure Coulomb friction are shown in Fig. 3.2. The cable element displacement and inter-element tensions are plotted for a subset of the cable elements (CE). Fig. 3.2(a) shows that the cable displacement increases from the locked load side to the location where the
Figure 3.2: Results of ten cable element simulation with $\mu_s = \mu_d$, pure tension source, and blocked load side. (a) Cable displacement for subset of conduit elements (CE is conduit element). (b) Cable tension for subset of conduit element and input tension. The tension is applied. This is expected as each new element, and therefore cable length addition, decreases the overall stiffness. Similarly in Fig. 3.2(b), the cable tension decreases as it propagates from the motor input to the locked load. This is indicative of the backlash effects expected due to nonlinear friction. However, this backlash effect is dependent on the input tension, as there is a lower difference present across the cable when the input tension is at a minimum. This tension difference will also be partly responsible for the decreasing displacement, as the relative displacement of neighboring cable elements is directly dependent on the tension between those elements.

Similar trends can be seen within the friction with stiction results shown in Fig. 3.3. The addition of stiction causes these results to be more extreme, which
Figure 3.3: Results of ten cable element simulation with $\mu_s > \mu_d$, pure tension source, and blocked load side. (a) Cable displacement for subset of conduit elements (CE is conduit element). (b) Cable tension for subset of conduit element and input tension.

is especially visible in the cable tension. The tenth cable element for Coulomb friction loses 18% of the input tension while the stiction result shows a tension loss of 33%. Since the value of $\mu_s$ determines the transition point in (3.3) between static and dynamic regimes, increasing that value increases the necessary tension difference for movement, thereby increasing tension loss across the cable. The stiction also results in a stair step or start and stop behavior, likely due to the sudden decrease in friction force as the elements begin motion followed by a sudden increase when they are stopped again. Based on this analysis, the dynamics displayed in both Fig. 3.2 and Fig. 3.3 show the expected behavior.
3.2 Quasi-static Model

To simplify the Bowden cable model and highlight the defining characteristics of the transmission, a quasi-static model was derived from the lumped parameter equations and results in Section 3.1.

3.2.1 Capstan Equation

The Capstan equation relates the load force to the hold or output force for a flexible line wound around a cylinder. This equation takes the general form:

\[
T_{out} = \begin{cases} 
T_{in}e^{-\mu \phi} & v_{in} > 0 \\
T_{in}e^{\mu \phi} & v_{in} < 0 
\end{cases} 
\] (3.6)

where \( \mu \) is the friction against the Capstan drive and \( \phi \) is the wrap angle. Though many differences exist between this condition and the Bowden cable dynamics, the Capstan equation does accurately describe the tension loss across the transmission.

To visualize the simulated tension loss due to the dynamics, the input tension is plotted against the transmission output tension in Fig. 3.4. The results show a triangular backlash profile is present. As input tension is increased, the output tension lags below the input, while the opposite is true as input is decreased. Additionally, horizontal lines show states in the simulation where a change in the input tension results in no change in the output, which occur when the input changes from increasing to decreasing or vice versa. The length of this horizontal transition increases as input tension increases, so the tension loss is proportional to the input tension. These are all traits of the Capstan equation, indicating the tension loss is visually similar to the profile described by the Capstan equation.

This same conclusion can be reached using the dynamic part of (3.3). By redefi-
Figure 3.4: Input versus output tension for ten element simulation. Both pure coulomb friction and friction with stiction are shown, along with the output equal to input line in shown in black.

In the friction as the tension loss across the element, replacing the average tension with the input tension, and reorganizing the equation, the differential equation:

\[
\frac{T(x + dx, t) - T(x, t)}{dx} = \text{sign}(v) \frac{\mu_d}{\mu_s} T(x, t) \tag{3.7}
\]

\[
\frac{dT(x, t)}{dx} = \text{sign}(v) \frac{\mu_d}{\mu_s} T(x, t) \tag{3.8}
\]

can be derived. The boundary condition for this equation is the input tension, \( T(0, t) = T_{in} \). Therefore, the solution to (3.8) is:

\[
T(x, t) = \begin{cases} 
T_{in} e^{-\mu_d \frac{x}{\mu_s}} & v_{in} > 0 \\
T_{in} e^{\mu_d \frac{x}{\mu_s}} & v_{in} < 0 
\end{cases} \tag{3.9}
\]
Figure 3.5: Simulated tension transmission and Capstan equation based on Coulomb friction characteristics in Table 3.1

which is the Capstan equation in (3.6) substituting conduit geometry for the wrap angle using the curvature relationship in 3.1.1. Using the values of $\mu_d$, $dx$, and $R$ in Table 3.1, the related Capstan equation described by (3.9) at the full length of the cable is shown in Fig. 3.5.

Based on this derivation and relationship, the Capstan equation is proposed as a model for the tension transmission. It is consistent that the Capstan equation describes the general dynamics of the cable-conduit transmission. The relatively high stiffness of the transmission means that, in the quasi-static case, the conduit appears as a pulley of varying curvature that the cable is wrapping around. There is some small discrepancy between the quasi-static model described by the Capstan
equation lines and the simulated relationship in Fig. 3.5, indicating that the mass-
spring-damper dynamics of the cable elements have a noticeable effect. As these linear
dynamics are based only on estimates of the dynamics and the simulation stability,
there may be more or less discrepancies present between the Capstan equation model
and the physical system.

3.2.2 Conduit Compression

Additional analysis was made of the cable-conduit interaction to determine the va-
lidity of a conduit compression measurement, and to check the effect the compliant
conduit sensor has on this interaction. Examining Fig. 3.1(a) again for the conduit
dynamics, a normal force equal to (3.2) with opposite sign will act on the conduit. A
similar equation to (3.2) can be written relating the conduit compression and normal
force:

\[ N(x, t) = \frac{dx}{R_{conduit}} \ast \frac{C(x, t) + C(x + dx, t)}{2} \]  

(3.10)

The notable difference here is, though the same value can be used for \( dx \), the value
\( R_{conduit} \) indicates the radius to the center of the conduit, not the cable. The relation-
ship between the cable tension and conduit compression can therefore be written:

\[ \frac{C(x, t) + C(x + dx, t)}{2} = \frac{R_{conduit}}{R_{cable}} \frac{T(x, t) + t(x + dx, t)}{2} \]  

(3.11)

The conduit chosen in Section 4.1.1 has a radius of 2.5mm, so, using the conduit
geometry in Table 3.1, we can expect the conduit reading to be within 1% of the cable
reading. It will likely be much more accurate than this as the cable center is closer
to the center of the conduit than the edge. One limitation is highlighted by (3.11),
that when the cable-conduit transmission becomes straight, \( R_{cable}, R_{conduit} \to \infty \) so
the normal forces between the conduit and cable will go to zero.
3.3 Summary and Conclusions

The Bowden cable dynamics were successfully simulated using discretization of the conduit and a resulting lumped parameter model. Through intelligent estimates of model parameters, the friction was found to have an overwhelming effect on the dynamics compared to the linear dynamics of the cable elements. Based on this, a quasi-static model was derived for the key relationship of interest, the tension transmission, and was shown to follow the Capstan equation. Meanwhile the accuracy of the conduit compression measurement was quantified. These results will be used in Chapter 4 to choose the proper model parameters and better quantify the transmission dynamics found through the use of the Bowden cable testbed.
Chapter 4

Sensor Integration in Bowden Cable Testbed

To fully test and characterize the compliant force sensor designed in Chapter 2 and the Bowden cable transmission modeled in Chapter 3, a representative system with motor, transmission, and user joint was constructed with redundant force sensing and interchangeable sensor blocks to test different sensor locations and configurations. This chapter will briefly discuss the testbed design before moving on to explain the experiments needed to validate the compliant sensor design in the Bowden cable and quantify the effect of the compliant sensor on the control and dynamics of the transmission.

4.1 Testbed Overview

The Bowden cable testbed was designed to closely match the transmission present in the Warrior Web soft robotic exoskeleton. As shown in Fig. 4.1, the main components consist of a DC motor, cable-conduit transmission, and user interface in the form of a load handle. While sensors in the Warrior Web design are limited to the motor side of the transmission, force and position sensing occurs across the testbed to give the most choices possible for collecting experimental data. The choice of hardware and sensors will be covered in further depth in Section 4.1.1 and Section 4.1.2.
4.1.1 Hardware

Hardware choices for the testbed were made to allow for an accurate representation of the transmission of the soft robotic exoskeleton. This involved matching the desired torques and joint speeds, the transmission dynamics, and the user interaction capabilities. The motor, Bowden conduit and cable, and load interface were chosen to meet each of these specifications.

The testbed actuator defines the torque-speed capabilities of the testbed. For the testbed to accurately model rehabilitation of the user’s joints, the actuator must be able to provide torques equal to or greater than the human joint torques used during Activities of Daily Living (ADLs). For the elbow and shoulder, the process for identifying the required joint torques and joint speeds based on ADLs are summarized
in Section 2.1.2. This analysis gives max torques of 10 N-m and max angular velocities of 170 deg/s. It may not be feasible to meet both requirements simultaneously, so of these two, it is more important for the motor to meet the max torque requirement. While it is possible to instruct users to move more slowly, thereby limiting joint velocities, limiting the joint torques below what is necessary to complete ADLs will lead to insufficient capabilities for rehabilitation. In addition to these requirements, consideration should be given to the fact that, even though the actuators are removed from the user’s limb using the Bowden cables, to make the system fully mobile the actuators will need to be relocated to another point on the user, such as a backpack. This actuator location will still load the user. Ideally an actuator with high power to weight ratio can be chosen to minimize design weight.

High voltage brushless DC motors have the high power to weight ratios desired, and this choice of actuator is consistent with the actuators used in Robonaut 2 and the X1 exoskeleton designed at NASA Johnson Space Center [54, 63]. DC motors in general produce high speed and low torque, so a gearbox-motor combination is needed to meet the torque and speed specifications. The selected combination is a 200 W, 48 V EC-4Pole motor from Maxon Motors (pn: 305015) with a 246:1 planetary gearhead (pn: 324942) and 500 count per revolution optical encoder (pn: HEDL 5540). With this gear ratio, the nominal speed and torque of the motor become 32 N-m and 385 deg/s. However, this motor performance is further limited by the strength characteristics of the gearbox. The total performance of the chosen actuator is summarized in Table 4.1. These specifications nearly meet the requirements based on ADLs. An additional gear reduction can be applied between the motor pulley and load pulley to bring the continuous torque performance to meet the max shoulder torques, through this will be at the expense of the continuous speed.
Table 4.1: Performance Specifications of EC-4Pole Maxon Motor and 246:1 Planetary Gearhead Combination

<table>
<thead>
<tr>
<th>Continuous Torque (N-m)</th>
<th>Intermittent Torque (N-m)</th>
<th>Continuous Speed (deg/s)</th>
<th>Intermittent Speed (deg/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>12</td>
<td>195</td>
<td>385</td>
</tr>
</tbody>
</table>

To match the transmission dynamics, the cable conduit system between the motor and load should match the length and bend angle expected, as well as matching the friction characteristics and other material properties. To achieve the proper material properties, a cable conduit system matching that previously analyzed and used in the design of the Robonaut 2 hand was implemented [32]. This consisted of a Vectran™ tendon paired with a coiled steel conduit with Teflon™ lining. This combination was previously selected for its low friction as well as the strength and resistance to creep of the Vectran™ cable. For the configuration of the cable conduit transmission, an estimate of the conduit path can be developed from anthropometry data. If the actuators are relocated to a backpack, the conduit guiding the tendon for elbow flexion will traverse from the center of the back to the front side of the bicep. This requires at most a bend angle of 180 deg. Additionally, the conduit length can be approximated by the sum of the shoulder-to-elbow length [58] and the bicep radius calculated in Table 2.2. This gives lengths between 34.2 and 50.6 cm. Since, as discussed in Section 3.2, the conduit length affects the transmission dynamics mainly in its relationship to the curvature or bend angle, any conduit length on this order of lengths is acceptable.
The last hardware choice involves picking the load interface to represent the user joint. As discussed above for the actuator choice, an additional pulley ratio between the motor and load is needed allow the actuator to fully meet the ADL torque requirements. A pulley ratio of 2:1 accomplishes and provides a factor of safety of 1.6 on continuous torque without significantly reducing continuous speed. For the design of the load interface itself, a handle connected directly to the load pulley was selected. The handle length of 16.2 cm gives ample mechanical advantage for a user to generate the full range of joint torques and speeds needed for testing.

4.1.2 Sensors

An extensive set of force and position sensing options were selected to allow for a full analysis of the transmission and control capabilities. These sensors were positioned to read the state at the motor, conduit, and load interface.

To sense position, optical encoders were positioned to read motor and load state. The conduit position was indirectly measured through the compliant conduit force sensor. In total, options for three different position measurements were built into the design: load position, motor position pre-gearbox, and motor position post-gearbox. While the pre-gearbox measurement was built into the actuator, the addition of the post-gearbox measurement provides information on the gearbox backlash, which averages 1 deg under zero load. The load and post-gearbox motor measurements were provided by US Digital encoders with 10000 counts per revolution after quadrature (pn: EM2-1-2500-I).

The remaining sensors on the testbed measured the applied force at various stages. Five distinct measurements of force could be made: load torque, load-side cable tension, load-side conduit compression, motor-side conduit compression, and motor-
Figure 4.2: Force and torque sensing options present in Bowden cable testbed

side cable tension. The different sensors used and their placement in the testbed can be seen in Fig. 4.2. Unlike the position sensors, the majority of the force sensors can be removed or re-positioned from load to motor side of the conduit, allowing for reconfiguration of the sensor design scheme. The one exception to this is the torque load cell which is built into the load handle design to get a direct measurement of the user interaction. As well, with the exception of the compliant conduit force sensor, all sensors selected for the testbed are highly stiff strain gauge based load cells available from Futek. These stiff force sensors in the cable (pn: LSB200), conduit (pn: LTH350), and load handle (pn: TFF350) have sensing ranges of 445 N, 2224 N, and 60 N-m respectively. This well covers the expected range of forces to be applied to the testbed.
4.1.3 Set-Up

The full integration of hardware and sensors into a functional testbed followed the goals of the hardware and sensor designs: to create an accurate reflection of the soft exoskeleton transmission while using a variety of sensors to measure position and force in as many locations as possible on the testbed for the purposes of testing and system identification. Mounting pieces were designed for the motor and load interface to attach to a mechanical breadboard with 1/4 – 20 threaded holes spaced in a grid at 1 inch intervals. This mounting method for the testbed hardware allows for conduit configurations with bend angles of 0 deg, 90 deg, or 180 deg to be assembled and tested. The conduit terminates in one of three places, the compliant conduit force sensor, the Futek donut load cell, or a sensorless conduit termination block. These blocks are interchangeable, having identical interfaces to the conduit and to the 80/20 extrusions that serves as the mounting surface. Mounting to 80/20 also allows the conduit termination to be lined up vertically with the cable termination at motor or load, reducing cable wear. Similarly, the attachment method between the cable and the motor pulley, load pulley, or tension load cell is identical, allowing for the tension load cell to be easily added or removed from any location in the free cable. One possible set-up for the fully assembled testbed is illustrated in Fig. 4.1.

4.2 Overview of Experiments

The compliant conduit force sensor from Chapter 2 and testbed designed in Section 4.1 were tested through a series of experiments designed to analyze the transmission dynamics, the compliant force sensor measurement capabilities and control capabilities, and the effect of conduit compliance on dynamics.
4.2.1 Measurement of Bowden Conduit Compression

Before the full testbed was constructed, a comparison of the cable and conduit measurements was made. Applying forces manually, the cable tension was measured at the input while the conduit compression was measured at the near and/or far side of the conduit. Measurements of conduit compression were made both with the compliant conduit force sensor and with the donut load cell. The forces were applied to testbed set-ups with a variety of compliance values and compliant element placements in the conduit as shown in Fig. 4.3. Comparison between the cable tension measurement and near side conduit compression measurement will validate the conduit measurement strategy, while comparison to the far side measurement will confirm the Capstan model discussed in Section 3.2 in the case of quasi-static loading. For some tests, in exchange for or addition to the spring selected in Section 2.1.2, two significantly softer springs, \( k_s = 33.2 \text{ N/cm} \) and \( k_s = 13.48 \text{ N/cm} \), were used in combination to provide compliance to the ends of the conduit. The stiffness values tested are listed in Table 4.2. This was done to determine the effect sensor stiffness had on tension transmission or measurement.

<table>
<thead>
<tr>
<th>Stiffness 1</th>
<th>Stiffness 2</th>
<th>Stiffness 3</th>
<th>Stiffness 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.59</td>
<td>33.2</td>
<td>84.8</td>
<td>169.7</td>
</tr>
</tbody>
</table>

Table 4.2 : Stiffness Values Added in Series to Conduit (N/cm)
Figure 4.3: Testbed set-ups for quasi-static Bowden cable measurements with (a) no compliant elements, (b) single compliant element, (c) double compliant element.

4.2.2 System Identification

Further experiments were conducted to identify the open loop dynamics of the transmission. For these, the full testbed in Fig. 4.1 was used. To get precise measurements to compare to the lumped parameter simulations in Section 3.1, the load handle is kept in at a constant position using a clamp constructed of 80/20 and attached directly to the mechanical breadboard. This matches the conditions under which the conduit simulations were performed. Though it would be preferable to use the motor as a pure torque source, DC motors on their own are more accurately controlled as
velocity sources. For this reason, the motor input to the system in this experiment is a velocity signal, with the controlled velocity profile described by a Schroeder multisine with variable amplitude and flat frequency ranging between 0.01 and 40 Hz. This velocity input will excite the system at the full range of frequencies, allowing a Bode plot and transfer function estimate to be extracted from the data using Welch’s method. The control architecture used to control the velocity of the motor is illustrated in Fig. 4.4.

Figure 4.4: Block diagram for closed loop PI control of motor velocity. Within the plant $J_A$, $B_A$, and $K_A$ are the actuator inertia, damping, and stiffness. The remaining blocks show the PI controller with separate proportional, $K_P$, and integral, $K_I$, gains tuned for desired velocity, $\omega_r$, and actuator velocity in feedback, $\omega_A$.

4.2.3 Locked Load Force Control

With the load handle remaining locked, experiments were performed to validate the compliant conduit force sensor for use in force feedback when placed on either load or motor side of the conduit. Taking the velocity control loop in Fig. 4.4, an additional feedback loop is closed around the velocity controller to control force as illustrated in Fig. 4.5. Measurements were made with sinusoidal force tracking on either side.
of the conduit to visualize the transmission dynamics and friction with a controlled dynamic input. The amplitude, mean, and frequency of the desired force sine wave were modulated to see the how the non-linear dynamics effect the tracking. Depending on the placement of the compliant conduit force sensor on load or motor side of the conduit, either the in-cable tension load cell or torque load cell was used to measure the opposite side force.

Figure 4.5 : Block diagram for closed loop PI control of cable tension. The $Q_\omega$ block is the velocity controller found in Fig. 4.4, while $C_f$ contains the force loop controller. For locked load position the value of the load position input, $\omega_L$, is always zero.

4.2.4 Impedance Control

Implementation of a accurate impedance controller was the final validation of the compliant conduit force sensor and Bowden cable testbed. For this experiment the load handle was unlocked and a user interacted with the testbed, inputting a handle position. The impedance controller, illustrated in Fig. 4.6, makes use of the force controller in Fig. 4.5. While the impedance control is not explicitly a feedback loop, there is an implicit link between the load torque, $\tau_L$, and the load position, $\omega_L$, through the dynamics of the user. The desired impedance relationship is located in
the $Z_r$ block which takes in load position and outputs desired torque. The impedances of interest were virtual springs. Lower values of desired stiffness test the ability of the actuator and control to eliminate the existing non-linear transmission dynamics and actuator mass as perceived by the user. While the larger values of stiffness test the theorized boundaries of the impedances that can be rendered. The impedance control is more successful when the non-linear dynamics are masked from the user and replaced with the desired linear impedance.

Figure 4.6: Block diagram for closed loop impedance control. The $Q_f$ block contains the force control block diagram from Fig. 4.5. Desired impedance relationship exists within $Z_r$.

4.3 Results and Discussion

4.3.1 Measurement of Bowden Conduit Compression

Using the testbed set-ups shown in Fig. 4.3, the far-side conduit compression measurement was found without the compliant sensor in the conduit, with the compliant sensor placed on the near-side of the conduit, with the compliant sensor placed on the far-side of the conduit, and with compliant sensors placed on both sides of the conduit. The results are plotted in Fig. 4.7 for conduit bend angles of 90° and 180°. With the exception of sensor noise and some drifting of the sensor zero, there is no
Figure 4.7: Static testing with conduit bend angle of (a) 90° and (b) 180°
Figure 4.8: Comparison of tension loss for different levels of added stiffness, static testing

visible difference between the tension relationships found when the compliant element location is changed. Similarly, the results obtained by modulating conduit stiffness between 9.59 N/cm and 169.7 N/cm, shown in Fig. 4.8, indicate that the change in tension transmission caused by adding the compliant element is negligible.

Additionally, both of these results, varying sensor placement and varying stiffness, qualitatively show the expected tension transmission based on the Capstan equation, as predicted in Section 3.2.1. Plotting the combined results from Fig. 4.7 in Fig. 4.9, we can see that the increasing conduit bend from 90° to 180° increases the tension
loss for any given input tension. Slopes for pull and release phase for both conduit bends were found, and, using the relationship in (3.6), the friction coefficients were calculated as:

$$\mu = \text{sign}(v) \frac{\ln(m)}{\phi}$$  \hspace{1cm} (4.1)

where \text{sign}(v) indicates pull phase or release phase, $m$ is the slope of the input tension versus conduit compression, and $\phi$ is the conduit bend angle in radians. The average friction coefficient was $\mu = 0.0989$. This value is within the expected range for Teflon$^\text{TM}$, and near the value used for simulation in Section 3.1.2. The linear fits
shown in Fig. 4.9 are calculated using (3.6) with the average friction coefficient, the bend angle, and the input tension. The Capstan fitted lines indicate there may be some decrease in friction coefficient as bend angle increases. Most likely, this may be due to differences between estimated bend angle and actual bend angle. The bend angle was enforced by setting an angle between the attachment points of the conduit and not measured directly. The discrepancy between the friction expected based on bend angle and the actual friction indicates the friction may be sensitive to the conduit routing path beyond simply start and end point location, and the assumption of constant conduit curvature may be limited. The location and orientation of the conduit ends define a minimum conduit bend needed to fit the constraints. However, the conduit can make additional bends that counter each other in terms of total path but add friction along the path. This is an important consideration as position of motor and load may not be enough to accurately estimate conduit path.

In addition to confirming the Capstan model of the Bowden cable transmission, the static testing can be used to confirm the strategy of measuring the conduit compression in exchange for the cable tension. Both the donut load cell and compliant conduit force sensor were placed on the near-side of the conduit and their measurement errors are plotted in Fig. 4.10. Values for the average error, max error, and % FSO are summarized in Table 4.3. While the compliant conduit force sensor shows higher max error than the donut load cell, the average error is about the same between the sensors. The compliant conduit force sensor’s max error of 1.99% FSO matches the force error when the sensor was used in a direct force measurement in Section 2.3.4. This indicates that the conduit measurement is on par with direct force measurement in terms of error and that conduit measurement is valid for bend angles of at least 90%.
Figure 4.10: Error in near-side conduit compression measurement versus input tension. Error is calculated for two conduit bend angles, 90 deg and 180 deg, and both the compliant conduit force sensor and donut load cell.

Table 4.3: Accuracy of Near-Side Conduit Compression Measurement

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Conduit Bend Angle (deg)</th>
<th>Average Error (N)</th>
<th>Max Error (N)</th>
<th>%FSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Donut Load Cell</td>
<td>90</td>
<td>0.9837</td>
<td>3.59</td>
<td>1.09%</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>0.4759</td>
<td>3.29</td>
<td>1.0%</td>
</tr>
<tr>
<td>Compliant Conduit Sensor</td>
<td>90</td>
<td>0.9134</td>
<td>6.56</td>
<td>1.99%</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>0.9612</td>
<td>4.91</td>
<td>1.49%</td>
</tr>
</tbody>
</table>
4.3.2 System Identification

Though the Bowden cable transmission has non-linear dynamics, an estimate of the linear properties can be made using a frequency response. This response can give some insight to the transfer function, but the frequency response will only be valid at the specific amplitude and mean used. The Schroeder multisine signal used as the system velocity input produced a motor-side cable tension with the same range of excitation frequencies, 0 to 40 Hz. The derived transfer function estimate is shown in Fig. 4.11. The magnitude plot resembles an over-damped second order linear system. To find an estimate of the corresponding linear parameters, the motor-side force was used as an input force for the lumped parameter model in Section 3.1. The model fit shown in Fig. 4.11 was found using the friction coefficient from Section 4.3.1 and the model parameters: $m_c = 0.015 \text{ kg}$, $b_c = 2.7 \text{ Ns/cm}$, and $k_c = 1500 \text{ N/cm}$.

4.3.3 Load Side Sensor Placement

Force control and impedance control experiments were carried out as described in Section 4.2.3 and Section 4.2.4 with the compliant conduit sensor placed on the load side of the conduit.

Sinusoidal force tracking was tested for two frequencies of desired force, 0.1 Hz and 0.5 Hz, as shown in Fig. 4.12 and Fig. 4.13 respectively. At each frequency, mean values of 90 N and 30 N and amplitude values of 20 N and 5 N are shown for a 10 second period. Across the data, the effect of the Coulomb friction can be seen in the effect of “quantization” in feedback force near the desired force. Looking at Fig. 4.12, the size of the steps increase as the mean force increases from 30 N to 90 N, while the amplitude, on the other hand, affects the amount of time held on a step before switching. When the frequency is increased, the step effect can only be seen when
Figure 4.11: Bode plot of open loop force transfer function. Transfer functions for experimental data, smoothed and not smoothed, and simulated data are shown.

The signal changes direction from pull to release and vice versa. The affects of mean force and amplitude are also seen for the higher frequency on the single visible step in Fig. 4.13. Taken together, the “quantization” is exacerbated by increasing the value of the desired force and/or by decreasing the derivative of the force. The derivative of the force is decreased either by decreasing the frequency or decreasing the amplitude.
Figure 4.12: Force tracking for 1 Hz sinusoidal input with (a) 90 N mean and 20 N amplitude, (b) 90 N mean and 5 N amplitude, (c) 30 N mean and 20 N amplitude, (d) 30 N mean and 5 N amplitude.
Figure 4.13: Force tracking for 5 Hz sinusoidal input with (a) 90 N mean and 20 N amplitude, (b) 90 N mean and 5 N amplitude, (c) 30 N mean and 20 N amplitude, (d) 30 N mean and 5 N amplitude.
This response to changing inputs matches the behavior of Coulomb friction present in the Capstan equation, where there is a minimum tension difference that must be covered to change the direction and a friction deadband that increases with the value of the input tension. For a rapidly changing force signal, the necessary tension difference is quickly reached when switching from pull to release and back again. The additional steps present in the lower frequency response are due to the cable switching between the static and dynamic friction regimes in (3.3). This is another effect of slowly changing input signal, where the friction can more easily stop the cable motion.

Figure 4.14: Root Mean Square Error (RMSE) for load side force tracking using the compliant conduit force sensor varying amplitude, mean, and frequency.
Examining the performance of the force tracking, aside from this non-linear effect, the measured load force curve closely tracks the period and amplitude of the reference force. The overshoot present at the peaks is due to the “quantization” steps not lining up with the peak. The root mean square error (RMSE) of the force tracking is plotted in Fig. 4.14 for the distributions seen in Fig. 4.12 and Fig. 4.13 with two additional values for mean. Without friction, we would expect the error to increase with amplitude and frequency and remain fairly constant with mean, within the actuator performance limits. However, the effects of friction cause the error to increase with increasing mean, which correlates with the increasing “quantization” step size. As well, for 0.1 Hz, some signals with lower amplitude have greater or equal error compared to the higher amplitude signals, indicating that the detrimental effects of friction on the smaller signal has canceled the benefits normally netted by decreasing the amplitude. At higher frequency, this effect is not large enough to be visible.

For impedance control, the system was tested at a range of desired stiffness values from a zero stiffness float mode to stiffness values just above that of the compliant conduit force sensor, 0.76 Nm/deg at the load handle. Results for the low, middle, and high end of the range are plotted in Fig. 4.15. For each value of desired stiffness, the relationship between load displacement and load torque is shown. The slope of this relationship defines the stiffness rendered to the user. While the slope of the linear fit for each desired stiffness value is relatively close, there is error present, which is especially evident in Fig. 4.15(a). This torque error, when plotted against the load velocity, shows a linear relationship. This indicates that, while no damping was prescribed by the impedance controller, a linear virtual damping is rendered to the user. Across the example values, this damping coefficient decreases and even becomes negative. It is important to note that this damping is linear, indicating
Figure 4.15: Rendered stiffness and damping for impedance control with compliant conduit sensor placed on the load side for desired stiffness (a) $k = 0.05 \text{ Nm/deg}$, (b) $k = 0.40 \text{ Nm/deg}$, (c) $k = 0.75 \text{ Nm/deg}$
Figure 4.16: (a) Percent error in stiffness rendering and (b) virtual damping versus desired stiffness.

that the impedance control law using the compliant conduit force sensor successfully displays a linear, spring-damper impedance to the user.

For the full range of stiffness values, the percent error in virtual stiffness and the values of virtual damping are displayed in Fig. 4.16. In Fig. 4.16(a), there is a consistent bias in the virtual stiffness of approximately 2.5% above desired stiffness. Overall, this is very close to the desired stiffness values; however, it may indicate that the calibrations of the compliant conduit force sensor and load torque cell do not fully match and the true stiffness error is significantly lower.

The damping coefficients shown in Fig. 4.16(b) shows a linearly decreasing virtual damping as desired stiffness increases. When the desired stiffness is exactly equal
to the stiffness of the transmission, the motor does not need to move to provide the desired virtual stiffness. Since the additional compliance of the sensor is placed near the load interface, only the conduit compliance and not the damping or friction will be displayed to the user in this case. Therefore, for values below transmission stiffness, the damping will be positive while, above the stiffness, the damping will be negative. We can expect the point of zero virtual damping to correspond to the stiffness of the transmission. From Fig. 4.16(b) this is about 6 Nm/deg. With the stiffness of the compliant sensor, this gives a stiffness of the cable-conduit transmission of 6328 N/cm.

4.3.4 Motor Side Sensor Placement

This series of experiments testing force and impedance control were redone with the compliant conduit force sensor located on the motor-side of the conduit. The motor area is the intended location of any force sensors in NASA JSC’s soft robotic exoskeleton, so it is important to understand the limitations of control that exist when the force sensing is moved across the conduit.

The set of frequency, mean, and amplitude values used in Section 4.3.3 are repeated for motor side force tracking. To analyze the accuracy of the force tracking alone, only the motor side of the transmission was considered. Fig. 4.17 shows the compliant conduit force sensor tracking a sinusoidal force on the motor side for a subset of the frequency, mean, and amplitude combinations. The non-linear effects seen in Fig. 4.12 and Fig. 4.13 are not present in motor side force tracking. The RMSE of the force is shown in Fig. 4.18. Comparing these results to Fig. 4.14, the motor tracking error more closely matches the expected behavior in a linear system. The error increases with increasing frequency and amplitude. The error is not flat in mean force, though this same trend is seen in the load results for the same values.
Figure 4.17: Sinusoidal motor side force tracking for frequency (a) 0.1 Hz, (b) 0.5 Hz and mean and amplitude of (i) 90 N and 20 N, (ii) 90 N and 5 N, (iii) 30 N and 20 N, (iv) 30 N and 5 N.
Figure 4.18: Root Mean Square Error (RMSE) for motor side force tracking using the compliant conduit force sensor varying amplitude, mean, and frequency.

While the motor side force control is shielded from the non-linear “quantization” effect seen in Fig. 4.12 and this results in lower RMSE error of force tracking, in impedance control, the friction has a large effect. Fig. 4.19 shows the results of load torque versus load displacement for three stiffness values. Unlike the linear trends seen on the load side, the stiffness envelopes here resemble the Capstan relationship. The distribution of the damping confirms this. As in Fig. 4.15, a plot of the torque error versus load velocity is also provided in Fig. 4.19 for each value of desired stiffness. However, this does not result in a linear damping coefficient. Instead torque error
Figure 4.19: Rendered stiffness and damping with compliant sensor placed on the motor side for desired stiffness (a) $k = 0.05$ Nm/deg, (b) $k = 0.40$ Nm/deg, (c) $k = 0.75$ Nm/deg
Figure 4.20: Percent error of rendered stiffness versus desired stiffness. At each desired stiffness, points are plotted for both the pull and release stiffness values displays tendencies of both coulomb and viscous friction. The jump in torque error at zero velocity corresponds to Coulomb friction. Additionally, this is the friction present in the Capstan equation, as the magnitude of the jump depends on the load torque and, consequently, on the torque error.

Since these results follow the Capstan equation, the data is separated into pull phase and release phase, and linear fits with stiffness values are found for each. These
are more accurate values for the stiffness displayed to the user, compared to what could be found fitting a line to full data set. The errors in these rendered stiffness values are displayed in Fig 4.20. The error is on the order of ten times what is seen in Section 4.3.3 when a load side sensor is used. Additionally, due to the two stiffness values, the rendered stiffness changes by up to 50% of the desired stiffness when switching from pull to release. These stiffness values can also be used to estimate the
friction coefficient. The resulting friction estimates are shown in Fig. 4.21. Though the friction estimates of either phase stay moderately close to each other when the stiffness is low, they begin to separate as the stiffness increases. In addition, the friction coefficients and mean friction decrease with increasing stiffness. However, comparing the calculated friction coefficients to that found under static loading, there is significantly more friction present. This may be related to the viscous damping present, which may be inflating the estimated Capstan relationship at lower stiffness values. This would explain the decreasing friction, as the viscous friction effect may decrease with increasing stiffness as seen when the sensor is located on the load-side.

4.4 Summary and Conclusions

Testing was successfully completed on the compliant conduit force sensor, designed in Chapter 2, and cable conduit transmission using a Bowden cable testbed designed to closely match the soft robotic exoskeleton design. Conduit compression was confirmed to match near-side conduit tension, while the relationship between near- and far-side tension was matched to the model proposed in Chapter 3. This model was unaffected by the location or value of the added compliance, but was affected by conduit bend angle in the expected manner. Open loop frequency response for the tension transfer function was found and matched using the lumped parameter Bowden cable simulation, giving an estimate of the linear properties of the transmission. The full use of the compliant sensor was confirmed when force and impedance control performance were analyzed for placement of the sensor on both load and motor side. For load placement, the effects of Coulomb friction cause a stepped response and, therefore, an increased RMSE during sinusoidal force tracking; neither of these effects are seen when the sensor is placed motor-side and used to control motor force.
Impedance control tells the opposite story, where the Capstan relationship severely limits impedance rendering on the load side using motor side measurement. Load-side sensor placement for impedance control results in a linear spring-damper impedance, eliminating the non-linear friction effects as displayed to the user. The advantages of load-side spring placement during impedance rendering far outweigh the increased error of force tracking. While the compliant conduit force sensor has demonstrated successful use for both force and impedance control, the desired force sensor placement for the soft robotic exoskeleton is on the motor-side of the conduit. In the future, strategies will need to be developed to increase the accuracy of the impedance rendering using this sensing strategy.
Chapter 5

Conclusion

The use of robotic exoskeletons to aid in motor recovery has been shown successful in post-neurological injury treatment. However, previous devices have been limited by their weight and size. Soft robotic exoskeletons offer an alternative for rehabilitation that allows natural movements with a limited number of rigid structures and offers the possibility of portability for in home use or as an assistive rather than therapeutic device. To successfully build and implement these soft exoskeleton devices, the actuation strategies must be fully analyzed. The most popular of these for soft robotic exoskeletons are DC motors with cable-conduit transmissions, which have non-linear stiffness and Coulomb friction.

Using the philosophy of series elastic actuation, a compliant force sensor was designed to be placed in the cable-conduit transmission as an alternative to previous sensing and control strategies for Bowden cable systems. The compliant conduit force sensor design follows biologically inspired requirements and utilizes non-contact displacement sensing to fully meet the expected force, stiffness, and sensor placement needs.

A full understanding of the actuation and transmission is also necessary for understanding and overcoming the non-linear dynamics that are present. Using simple lumped parameter models and Coulomb friction, simulations were carried out to test theories of the transmission dynamics and confirm experimental results. However, it is the derived Capstan equation based model that easily allowed parameters of the
defining dynamics, the non-linear friction, to be found.

The sensor and cable-conduit modeling are brought together in an experimental Bowden cable testbed. First used to verify the sensor and model, further testing of the cable-conduit transmission provided insight into the force and impedance control capabilities of these transmissions. With placement on the load, or user, side of the cable-conduit transmission, the non-linear friction was eliminated in impedance control, but could be seen in a quantization of forces that could be achieved accurately during force control. Motor-side placement of the sensor displays accurate control of the cable tension on the motor side of the conduit, yet suffers in terms of impedance control. The rendered stiffness was shown to be limited by the Capstan equation and the non-linear damping remains as well. The results show what challenges will need to be overcome and what approaches may exist to make accurate impedance rendering using motor-side sensing a reality.
Bibliography


