RICE UNIVERSITY

Efficiency and Productivity Analysis of Multidivisional Firms

by

Binlei Gong

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE

Doctor of Philosophy

APPROVED, THESIS COMMITTEE:

Dr. Robin C. Sickles, Chair
Reginald Henry Hargrove Professor of Economics and Statistics

Dr. Xin Tang
Associate Professor of Economics

Dr. Gustavo Grillon
Jesse H. Jones Professor of Finance

HOUSTON, TEXAS
MARCH 2016
Abstract

Efficiency and Productivity Analysis of Multidivisional Firms

by

Binlei Gong

Multidivisional firms are those who have footprints in multiple segments and hence using multiple technologies to convert inputs to outputs, which makes it difficult to estimate the resource allocations, aggregated production functions, and technical efficiencies of this type of companies. This dissertation aims to explore and reveal such unobserved information by several parametric and semiparametric stochastic frontier analyses and some other structural models. In the empirical study, this dissertation analyzes the productivity and efficiency for firms in the global oilfield market.
ACKNOWLEDGEMENTS

I would never have been able to finish my dissertation without the guidance of my committee members, comments from other scholars, help from friends, and support from my family.

I would like to express the deepest appreciation to my advisor and committee chair, Dr. Robin Sickles, for his excellent guidance, caring, patience, and providing me with an excellent atmosphere for doing research in the past four years. Dr. Robin Sickles is the person who leads me to the field of productivity and efficiency analysis. He let me work in the oilfield market to better observe and study this industry and encouraged me to integrate theory with practice.

I would like to thank my committee members, Dr. Xun Tang and Dr. Gustavo Grullon, who provided many insightful comments and suggestions from the perspective of economics and finance. Their wisdom, knowledge and commitment to the highest standards inspired and motivated me.

My sincere thanks also goes to Dr. Hülya Eraslan, Dr. Peter Hartley, Dr. Kenneth Medlock, Dr. Marc Santugini, and participants at presentations at the 25th Midwest Econometrics Group Meeting, 52nd Annual Meetings of the Missouri Valley Economic Association, 85th Annual Conference of the Southern Economic Association, and the 2016 Texas Camp Econometrics, for their feedbacks and questions.

I am grateful to Weatherford International for giving me the opportunity to closely observe firms in the oilfield market. I appreciate all the
helps and guidance from my supervisor Karen David-Green and my colleagues Suzanne Niemann and Bin Shao in the Investor Relations team. I would also like to thank Spears and Associates, Inc. for providing their high-quality data on the global oilfield market and James A. Baker III Institute for Public Policy for supporting my research on energy economics.

In addition, a thank you to two distinguished scholars who gave me suggestions on future studies. Dr. Steven Chu encouraged me to cover the entire petroleum industry and applies the model to multinational firms. Dr. Roger Myerson suggested me to introduce the local agency idea into the model.
Dedicated to my loving parents, Zhanguo Gong and Fengjuan Chen, for
giving birth to me at the first place and supporting me spiritually
throughout my life!

谨以此文献给我的父母，龚展国和陈风娟，感谢您们给了我生命，给了我无微不至的关怀！
4 Semiparametric Aggregated Production Function for Multidivisional Firms 78
4.1 Introduction ................................................. 78
4.2 Varying Coefficient Model ................................. 81
4.3 Semiparametric Model under Shape Constraints .......... 87
4.4 Empirical Study: The Productivity and Efficiency for Multidivisional Oilfield Firms ................................................. 90
4.5 Conclusion ..................................................... 96

Appendix A Appendices 98
A.1 Imputation Method ............................................. 98
A.2 Accuracy Test of the Estimation .............................. 116
A.3 OMR Data Introduction and Adjustment ..................... 126
A.5 Robustness Results Assuming T-L a Function ................. 131
## List of Figures

1. Structure of the Dissertation ............................................. 2
   1.1 Average efficiencies from different estimators in the two periods ... 34
2. The Estimated Cobb-Douglas Production Function ......................... 67
   2.2 Efficiency Level for BHI, HAL, SLB, WFT, and the Industry Average 70
3. Comparison of Average Efficiency Level in Various Methods ............. 76
4. Comparison of the Four Methods .......................................... 81
   4.2 Effect of Capital and Labor on Output in Various Methods ........... 91
   4.3 The Range of the Production Frontier in the “Varying Frontier” Method 92
   4.4 Estimated Production Frontiers in Various Methods .................... 93
   4.5 Technology Change in Various Methods ................................ 94
A.1 Efficiency Level for BHI, HAL, SLB, WFT, and the Industry Average (T-L Model) ...................................................... 132
List of Tables

1.1 Summary Statistics ........................................... 32
1.2 Stochastic Frontier Estimators ............................... 33
1.3 Calibration and Result of PMM ................................. 36
1.4 Calibration and Result of MXM ................................. 38

2.1 Oilfield Market Summary Statistics ......................... 64
2.2 Estimate of Cobb-Douglas Production Function ............... 65
2.3 Change in Share of Revenue by Segment for Multidivisional Firms .... 69
2.4 Efficiency Levels of the “Big Four” by Segment in 2014 ............ 71

3.1 Estimate of Transcendental Logarithmic Production Function ....... 77

4.1 Technical Efficiency Statistics ................................ 95
4.2 Technical Efficiency Class Interval ........................... 95
4.3 Efficiency Regression Result .................................. 96

A-1 OECD Data Summary Statistics ................................. 118
A-2 Accuracy Test Results .......................................... 123
A-3 Iteration Results by Different Initial Guess in 3 Segments Model ... 124
A-4 Iteration Results by Different Initial Guess in 9 Segments Model ... 125
A-5 Change in Share of Revenue by Segment for Multidivisional Firms (T-L Model) ........................................ 131
A-6 Estimate of Transcendental Logarithmic Production Function ....... 133
A-7 Efficiency Levels of the “Big Four” by Segment in 2014 (T-L Model) . 134
A-8 Technical Efficiency Statistics (T-L Model) ..................... 134
A-9  Technical Efficiency Class Interval (T-L Model) ................. 134
A-10 Efficiency Regression Result (T-L Model) ......................... 135
Introduction

Stochastic Frontier Analysis (SFA) is a method of economic modeling to measure the performance of firms that convert inputs to outputs. It has been widely used in Efficiency and Productivity Analysis to estimate production function and firm-level efficiency over the past three decades. However, efficiency defined by the Solow residual in SFA is a reduced-form concept and cannot be given a structural interpretation. The first chapter compares existing SFA with some structural models.

The classical SFA in the first chapter assumes a single production frontier, which is not realistic in some cases. For example, oilfield industry includes five segments; each has different players and different technologies. As a result, this industry has not only single division companies who only focus on one segment, but also multidivisional firms who have footprints in multiple segments. We may treat each segment as an industry and do stochastic frontier analysis for each segment, respectively. But often times the input allocation across divisions in multidivisional firms is unobserved, which prevent us to do SFA for each segment separately.

The next three chapters develop methods to estimate the production frontiers and technical efficiencies for these multidivisional firms: the second chapter opens the input “black box” to estimate the divisional level efficiencies using SFA for each
segment; the third chapter estimates the average segment efficiencies using a structural model while the fourth chapter estimates the aggregated production function for multidivisional firms using two semiparametric models, all without opening the input “black box”.

Therefore, this dissertation explores the productivity and efficiency analysis from two dimensions: 1) reduced form vs. structural model, and 2) single- vs. multi-segment assumption. The focus is on reduced-form SFA model with multi-segment concern, where two approaches (whether to open the input “black box”) are developed to evaluate the efficiencies of the multidivisional firms. Figure 1 provides the structure of this dissertation. More specifically, the summary of each chapter is as follows.

![Figure 1: Structure of the Dissertation]

The first chapter reviews and provides an extension of the benefits and drawbacks of the SFA paradigm, as well as other number index-based procedures that we will utilize in the analysis. This chapter then compares SFA with two structural models: the Pakes-McGuire Model (PMM) and Midrigan and Xu’s Model (MXM).
All three methods are used to estimate changes in firm-level efficiency in the British Isles before and after the 2007-2009 financial crisis under the canonical single production assumption. The empirical results indicate that overall productivity was not impacted to any substantial degree by the financial crisis, according to both SFA and the PMM. However, the productivity loss estimated by MXM due to financial friction from the recession was substantial. Since the production function for all the firms are assumed to be unique, this SFA model is called “Single Frontier” method.

The second chapter challenges the single frontier assumption when the targeted industry is multi-segment and has multidivisional firms. This chapter is concerned with specifying and estimating the productive characteristics of multidivisional firms at the divisional level. In order to accomplish this, the unobserved input allocations at the divisional level need to be estimated. By specifying a profit-maximizing neoclassical model and assuming undistorted input allocations but allowing for additive technical inefficiency shocks at the divisional level, the input allocations can be imputed. The accuracy of the imputed allocations is tested using data for OECD countries where the actual allocations are observed. This study then augments division-level information with the imputed inputs to estimate the segment (division)-specific productivity of firms that compete in the five segments of the global oilfield market, allowing for endogeneity of inputs. This model relaxes some key assumptions on which previous literature heavily relies. The empirical study finds evidence that those key assumptions are not always valid; thus, using them may lead to inaccurate results. To sum up, this chapter uses “Single Frontier” method on each segment and hence is called “Meta Frontier” method.

The third chapter estimates the average efficiency for each of the five segments in the global oilfield market using PMM structural model. In each segment, players (including unitary firms and the corresponding division of multidivisional firms) choose
investment, inputs, entry, and exit rationally to maximize their expected future total profit. This chapter calibrates the parameters to make the outcome statistics the same as the statistics in the data. In this model, we don’t need inputs and output data and hence don’t need to open the “black box” of inputs. The estimated average efficiency levels for the five segments are comparable with the ones in the second chapter.

The fourth chapter develops two semiparametric models to evaluate firm-level aggregated production frontiers and technical efficiencies without opening the input black box but allows for multi-segment concern. The second chapter estimates the segment-specific production frontiers that reflect the technologies utilized in each segment. However, the aggregated production function for a multidivisional firm depends on not only these technologies themselves, but also the frequencies of using them. This chapter uses revenue share by segment $\theta$ to capture such heterogeneity. Compared with the classic “Single Frontier” approach, the first semiparametric model, “Varying Frontier”, allows the variation in production function through replacing the constant technical parameters by some nonparametric function of $\theta$. The second semiparametric, however, further relaxes the formation of the production function to avoid rigid functional forms. This method is called “Shape-constrained Frontier” since the estimated production function must follow the economic theory to be monotone increasing and concave. This chapter ends by comparing the estimation results from different approaches.

In summary, all four chapters approach stochastic frontier analysis from different angles: the first chapter compares the empirical results of SFA with the competing structural models; the second chapter proposes a more realistic way to use the existing SFA in a multi-segment industry; the third chapter sets up a structural model in that multi-segment industry; and the last chapter creates two new estimators of SFA,
rather than changing the approach to use the existing SFA with a multiple-frontier concern. Essentially, the first chapter compares SFA and structural models under the traditional assumption of a single production frontier, while the next three chapters improve SFA and the structural models under a multi-segment assumption. The second chapter requires opening the input allocation but can derive both firm-level and divisional-level conditions. The third and fourth chapter doesn’t need to estimate input allocation. But the former can only describe segment average situations while the latter can only derive firm-level conditions.
Chapter 1

Productivity Loss during the 2007-2009 Financial Crisis in the British Isles

1.1 Introduction

The performance of a firm is usually measured by productivity, the ratio of a weighted average of outputs to a weighted average of inputs. Traditional productivity measurement includes labor productivity, capital productivity, and so on. However, these partial productivity measures may lead to a biased overall measure of productivity when firms use multiple inputs to produce outputs (Coelli et al. 2005). Total Factor Productivity (TFP) accounts for the effects on the total output that are not caused by inputs, which effectively evaluates overall productivity.

In the production function, TFP is the Solow residual. In a Cobb-Douglas production function, for example, the output \( Y \) is a function of total factor productivity \( A \), labor input \( L \), and capital input \( K \) with the form \( Y = AL^\alpha K^{(1-\alpha)} \). For a certain technology level, the upper bound of TFP is the efficiency level of TFP, which determines the production frontier. Many micro- and macro-level factors may create TFP loss, which keeps the actual production level below the frontier.
Stochastic Frontier Analysis (SFA) is a principal approach to estimate the overall frontier function and technical efficiency. The panel stochastic frontier analysis employed in this work below uses a normalization in which the firm with the highest TFP has a relative efficiency that is the highest of all the firms in the sample at that particular point in time (Schmidt and Sickles 1984). Although this reduced-form model can estimate the average efficiency level, and therefore predict the overall relative TFP loss for each unit, it cannot explain the sources of the inefficiency or TFP loss without a more detailed explanation of the inefficiency residual term.

Productivity defined by the Solow residual is a reduced-form concept and cannot be given a structural interpretation without a more formal structural model. This study introduces two structural models from the literature and compares their explicit restricted and reduced-form predictions with those from the unrestricted SFA model, using panel data from firms in the UK and Ireland.

The first structural model considered is the Pakes-McGuire model (PMM) that has been implemented to estimate it. This method computes the Markov Perfect Nash (MPN) Equilibria of an industry, where a firm’s profit is a function of its own level of efficiency and a vector specifying the efficiency level of all its competitors. This approach can estimate an average efficiency level by solving both the MPN equilibria and social planner’s problem.

The second structural model is Midrigan and Xu’s Model (MXM), which is able to decompose the residual and estimate the partial TFP loss due to specific factors (financial friction or financial misallocation, in this case). This approach sets up an economic structure and calculates the efficient allocation (with efficient TFP) to satisfy some equilibria. The factors of interest are introduced in the model along with some constraints, which can lead to a lower possible TFP and therefore cause TFP loss.
This research will analyze the effect of financial frictions on productivity in the British Isles before and after the 2007-2009 financial crisis. The overall TFP loss estimate using SFA and PMM, as well as the finance-caused TFP loss by MXM, will be compared. The ratio of partial TFP to overall TFP reveals the relative magnitude of the inefficiency as a result of the misallocation of capital.

Our findings indicate that the overall TFP was not impacted to any substantial degree by the financial crisis, according to both Stochastic Frontier Analysis and the Pakes-McGuire Model. However, the TFP loss due to financial friction estimated by Midrigan and Xu’s Model as a result of the recession was substantial.

The remainder of the chapter is structured as follows. Section 1.2 introduces the Stochastic Frontier Analysis. Section 1.3 describes the Pakes-McGuire Model, and Section 1.4 presents Midrigan and Xu’s Model. Section 1.5 provides data descriptions. Empirical results derived from the three approaches are presented and compared in Section 1.6. Section 1.7 concludes.

1.2 Stochastic Frontier Analysis

Aigner, Lovell, and Schmidt (1977) and Meeusen and Van den Broeck (1977) independently and simultaneously proposed the stochastic frontier production function model of the form

\[ \ln Y_i = x_i' \beta + \nu_i - u_i, \quad i = 1, \ldots, N, \]

which equals the deterministic frontier production function plus a symmetric random error variable, \( u_i \), to account for measurement errors and other sources of non-systematic statistic noise. \( Y_i \) is the output of firm \( i \), \( x_i \) is the vector of inputs typically in logarithms, and \( u_i \) is a non-negative random variable representing technical ineffi-
ciency (the distance to the frontier).

In most cases, \( \nu_i \) is assumed to follow a normal distribution that is independent of each \( u_i \). Both \( u_i \) and \( \nu_i \) are uncorrelated with independent variables \( x_i \). A variety of distributional assumptions are applied to \( u_i \). Aigner, Lovell, and Schmidt (1977) assumed \( u_i \) to be \( i.i.d. \) half-normal random variables and derived the Maximum Likelihood (ML) estimates. Stevenson (1980) introduced a normal truncated specification, while Greene (1990) considered the gamma specification.

The stochastic frontier literature in the early 1980s mainly consists of analyses for cross-sectional data. Schmidt and Sickles (1984) proposed three serious difficulties that the stochastic frontier model suffered at that time, including inconsistent firm-specific technical inefficiency estimations, strong assumptions about the distribution of technical inefficiency and statistical noise, and potentially incorrect assumptions that inefficiency is independent of the regressors. They provided a variety of estimators to solve these potential problems, given the panel data. The panel stochastic frontier model is

\[
\ln Y_{it} = \alpha + x_{it}' \beta + \nu_{it} - u_i = \alpha_i + x_{it}' \beta + \nu_{it}, \quad i = 1, \ldots, N, \ t = 1, \ldots, T. \tag{1.1}
\]

Although the random noise differs for each firm over time, the inefficiency term is assumed to be fixed for each individual in early panel data stochastic frontier analysis. If no time-invariant controlled variables are included, then the fixed-effects method (FIX) guarantees that the consistency does not hinge on a lack of correlation between the regressors and the individual effects. The random-effects method (RND) requires \( u_i \) to be uncorrelated with the regressors, but does allow time-invariant variables.

Cornwell, Schmidt, and Sickles (1990) introduced both a within estimator (CSSW) and a generalized least squares estimator (CSSG) that allow for time-variant individual effects by replacing the firm effect \( \alpha_{it} = \theta_{i1} + \theta_{i2} t + \theta_{i3} t^2 \). Sickles (2005) examined
various specifications of the time-variant firm effect $\alpha_{it}$ modeled in other research, including

$$\alpha_{it} = \gamma(t) \alpha_i = [1 + \exp(bt + ct^2)]^{-1} \alpha_i \quad (\text{Kumbhakar 1990}),$$

$$\alpha_{it} = \eta_{it} \alpha_i = \exp[-\eta(1 - T)] \alpha_i \quad (\text{Battese and Coelli 1992}),$$

$$\alpha_{it} = \theta_t \alpha_i \quad (\text{Lee and Schmidt 1993}),$$

and the very general

$$\alpha_{it} = c_{i1} g_{it} + c_{i2} g_{2t} + \ldots + c_{iL} g_{Lt} \quad (\text{Kneip 1994, Kneip, Sickles, and Song 2003}).$$

This study uses the Fixed Effects Estimator (FIX), Random Effect Estimator (RND), Kneip-Sickles-Song Estimator (KSS), and Battese-Coelli Estimator (BC). The first two are time-invariant estimators and the last two are time-varying effects estimators. This study also applies the Cobb-Douglas stochastic frontier function with a constant return to scale constraint, which restricts the sum of the coefficients of inputs to one.

$$\ln Y_{it} = \alpha_i + \beta_1 \ln K_{it} + \beta_2 \ln L_{it} + \delta t + \nu_{it} - u_{it} \quad \text{s.t. } \beta_1 + \beta_2 = 1$$

$$\Rightarrow \ln Y_{it} = \alpha_i + \beta_1 \ln K_{it} + (1 - \beta_1) \ln L_{it} + \delta t + \nu_{it} - u_{it} \quad (1.2)$$

$$\Rightarrow (\ln Y_{it} - \ln L_{it}) = \alpha_i + \beta_1 (\ln K_{it} - \ln L_{it}) + \delta t + \nu_{it} - u_{it}.$$

This study includes a linear time trend $\delta t$ for FIX, RND, and BC but not for KSS. $\alpha_i$ is constant over time for FIX and RND but time-variant for KSS and BC. $\nu_{it}$ is the noise, and $u_{it}$ is the nonnegative distance to the most-efficient level of TFP. The equation $\exp(\alpha_{it}) = \exp(\alpha_i - u_{it})$ provides the individual TFP level for firm $i$ at time $t$. Assuming that the largest individual TFP is the efficient level of TFP, the average level of TFP loss can be derived, which indicates the average level of efficiency in the economy. In practice, this chapter drops the top and bottom 5% of
the estimations to eliminate outliers.

\[
\text{TFP loss ratio} = 1 - \frac{\text{TFP}}{\text{TFP}^e} = 1 - \left( \frac{1}{T} \sum_{t=1}^{T} \left( \frac{1}{N} \sum_{i=1}^{N} \frac{\exp (\alpha_i - u_{it})}{\exp (\alpha_i)} \right) \right) = 1 - \left( \frac{1}{T \cdot N} \sum_{t=1}^{T} \sum_{i=1}^{N} \exp (-u_{it}) \right) = 1 - \text{Average TE}, \tag{1.3}
\]

where \(\text{TFP}^e\) is the efficient level of total factor productivity and \(\text{TFP}\) is the average level of total factor productivity. The ratio \(\frac{\text{TFP}}{\text{TFP}^e}\) equals the average technical efficiency (TE) that is typically used in stochastic analysis. This study can thus estimate the average technical efficiency using the stochastic frontier analysis and then derive TFP loss ratio.

### 1.3 Pakes-McGuire Model

This method computes the MPN equilibria (Maskin and Tirole 1988a, b) that are generated under the constraints of Ericson and Pakes (1992). This section employs the same notations and equations from the original work (p.7-36, Pakes, Gowrisankran, and McGuire, 1993).

#### 1.3.1 Model

The definition of “industry structure” is the range of efficiency levels among the various businesses in the model. A firm’s profits arise from the industry structure as well as to the individual level of operational efficiency. Over their operating life, the efficiency of businesses will change based on the stochastic environment of their operating expenditures. Decisions about investments, entries, and exits are made in order to achieve the highest level of future cash flow, based on their expected discounted value (EDV), according to the current information set. Each business
bases its decisions upon their prediction of the industry structure in the future. As a consequence, the true distribution of the future industry structure is determined, like a self-fulfilling prophecy. A MPN equilibrium is reached when the projected industry structure is the true distribution that results from the predictions. Thus, PMM calculates a subgame-perfect Nash equilibrium.

**Industry Structure:** \( W = \{0, ..., \bar{w}\} \) is the set of efficiency values for each firm, where 0 is zero efficiency and \( \bar{w} \) is the maximum level of efficiency. \( N \) is the maximum number of companies that can be simultaneously active in the industry. A state \([w, n]\) consists of a \( w \in W^* \), \( n \in N \), where \( W^* = \{(w_1, ..., w_N) | w_j \in W, w_1 \geq w_2 \geq ... \geq \bar{w}_N\} \). For any firm, \( w \) represents the economic environment, including the efficiency level of all firms, while \( n \) indicates which element of this vector is the efficiency of its own. \( W^* \) guarantees that the industry structure is represented as a weakly decreasing \( N \)-tuple to avoid having multiple \( w \) for the same industry structure.

**Investment:** A firm’s efficiency level for the next period is determined by a Markov process that depends on its current efficiency level, current investment, and exogenous factors. This model denotes \( x \) as the current investment, \( k \in W \) as the current efficiency level of a firm, and \( k' \in W \) as its efficiency level in the next period. Moreover, assume that \( \tau \) is the effect of firm-specific investment and \( v \) is the effect of other firm-invariant exogenous variables that are the same for all firms. Then the controlled Markov process for the evolution of \( k' \) is

\[
k' = k + \tau - v
\]

where \( p(\tau) = \begin{cases} 
(\alpha x) / (1 + \alpha x), & \text{if } \tau = 1 \\
1 / (1 + \alpha x), & \text{if } \tau = 0 
\end{cases} \) and \( p(v) = \begin{cases} 
\delta, & \text{if } v = 1 \\
1 - \delta, & \text{if } v = 0 
\end{cases} \)

**Maximum Level of Efficiency.** Eq. (1.4) ensures that efficiency can only im-
prove with investment and shows that the incremental efficiency in a period is bounded with probability one. Hence, there exists an upper bound of efficiency \( \overline{w} \). PMM computes \( \overline{w} \) as the maximum efficiency level that a monopolist would ever reach by starting with a very large efficiency level and computing the monopolist problem to see where the monopolist stops his or her investment.

**Exit and Entry:** Companies make the decision to exit if the future cash flow value drops below the stated scrap value of the business, which is denoted \( \phi \). The exiting business will only receive the current scrap value, not the current period profit. Business enters when the potential and expected future cash flow value is greater than the one-time cost of entry. The sunk cost of entry, \( X_E \), is a random variable uniformly distributed between \( X_{EL} \) (lowest) and \( X_{EH} \) (highest). The potential entrants know the draw of \( X_E \) once they decide to enter. If they enter, they will not receive profit for that period, and enter with efficiency level \( W_E \) or \( W_E - 1 \), depending on the value of \( \nu \).

### 1.3.2 The Algorithm

**Matrices:** Profit, \( \Pi \), is an iteration-invariant matrix that calculates the one-shot game profit for each possible industry structure. Each iteration begins with the investment matrix, \( X \), and the value function, \( V \), from the output of the last iteration.

**Iterative Procedure:** The algorithm calculates \( \Pi \) and iterates on \( X \) and \( V \) until the maximum of the element-by-element difference between successive iterations in these two matrices is below a specified tolerance level. Each time, the calculation is done separately for each of the industry structures, using only the old values of \( X \) and \( V \). Beginning with the most efficient company within the industry structure, its choice is updated using the most recent value of the iteration. The decision is renewed based on the value of investment, exit and entry. The value function is not
included. These figures are used to calculate the policies for the firm with the next-
highest efficiency. In turn, these updated choices are applied to the firm ranked third
in efficiency, and so on.

**Updating Exit and Entry**: By comparing the value function of an incumbent
competitor with the scrap value, a firm can predict if that incumbent competitor
exits. The PMM defines the strategy set so that a firm must exit if it perceives that a
competitor with a higher efficiency level than its own has exited. For any \([w, n]\), this
model defines \(w'\) as the industry structure that results after exit has been accounted
for, and \(m\) as the number of active firms in \(w'\). After the decision of exit, this model
iterates on whether there will be entry if \(m < N\). The value of future cash flows in
state \([w', n]\) is compared with the one-time sunk cost in this process.

**Updating Investment**: Each firm chooses an optimal investment policy based
on its perception of future competitors. The calculation is done separately for every
\([w, n] \in (W^*, N)\). The value function at the \(i^{th}\) iteration is

\[
V^i(w, n) = \max \left\{ \phi, \sup_{x \geq 0} \left[ \Pi(w', n) - cx + \frac{\beta \alpha x C_l(w' + e(n), n)}{1 + \alpha x} + \frac{\beta C_l(w', n)}{1 + \alpha x} \right] \right\},
\]

where

\[
C_l(w', n) = \lambda(w', n) \left\{ \sum_{\tau_1=0}^{1} \ldots \sum_{\tau_n=0}^{1} \sum_{\tau_N=0}^{1} \sum_{V=0}^{1} V^{i-1} [w' + W \_E e(n_e) + \tau - iv, n] \right. \\
Pr[\tau_1|x_1^{i-1}, v] \ldots Pr[\tau_n|x, v] \ldots Pr[\tau_N|x_N^{i-1}, v] \ p(v) \bigg) + [1 - \lambda(w', n)] \left\{ \sum_{\tau_1=0}^{1} \ldots \sum_{\tau_n=0}^{1} \ldots \right. \\
\left. \sum_{\tau_N=0}^{1} \sum_{V=0}^{1} V^{i-1} [w' + \tau - iv, n] \ Pr[\tau_1|x_1^{i-1}, v] \ldots Pr[\tau_n|x, v] \ldots Pr[\tau_N|x_N^{i-1}, v] \ p(v) \right\}
\]

In this value function, \(w'\) is the incumbent efficiency levels after updating for exit;
\(m(w')\) is the number of active competitors at \(w = w'\); \(c\) is the cost in dollars of a dollar
worth in investment (equals 1 if no tax); \( \lambda (w', n) \) is the probability of entry; \( e(j) \) is a vector, all of whose elements are zero except for the \( j^{th} \) element, which is one; \( \hat{i} \) is a vector, all of whose elements are one; \( \tau \) is the vector containing the random \( \tau \) of competitors; \( n_e \) is the position of the entrant for any industry structure; \( n_e = m(w') + 1 \) unless the permutation cycle has been reordered; \( w'_1, \ldots, w'_N \) are the elements of the vector \( w' \); and \( x_1, \ldots, x_N \) (except \( x_n \)) is the investment of the \( N - 1 \) competitors at \( w' \). A symbol (\( \_ \)) in a summation means to omit that element. \( Cl(\cdot) \) sums over the probability-weighted average of the possible states of future competitors, but not over the investing firm’s own future states. It also indicates the firm’s expected discounted value for each of the two possible realizations of the firm’s own investment process, \( \tau \).

This model denotes \( x^*[w', n] \) as the investment level that solves Eq. (1.5). To calculate it, this model first derives the optimal level of investment, \( x[w', n] \), given that this investment is nonzero and that the firm does not exit. The actual level of investment, therefore, is either this number or zero, where zero is the solution if the optimal investment still leads to an exit decision or if \( x[w', n] \) is negative. Let \( D_x \) denote the derivative with respect to \( x \). The first-stage investment can solve

\[
 x \left[ w', n \right] = \text{argsolv}_x \left\{ c = \beta \left[ D_x \left( \frac{\alpha x}{1 + \alpha x} \right) \left[ Cl \left( w' + e(n), n \right) - Cl \left( w', n \right) \right] \right] \right\}. \quad (1.6)
\]

It is worth to notice that

\[
 D_x \left\{ \frac{1}{1 + \alpha x} \right\} = \frac{\alpha}{(1 + \alpha x)^2} = \alpha [1 - p(x)]^2, \quad \text{where} \quad p(x) = \frac{\alpha x}{1 + \alpha x}. \]

So, if \( v_1 = Cl \left( w' + e(n), n \right) \) and \( v_2 = Cl \left( w', n \right) \), the investment can be rewritten as

\[
 x = \text{argsolv}_x \left\{ c = \beta \alpha [1 - p(x)]^2 (v_1 - v_2) \right\} \Rightarrow p(x) = 1 - \sqrt{\frac{1}{\beta \alpha (v_1 - v_2)}}.
\]
Taking the inverse of \( p(x) \), it can be seen that:

\[
x[w', n] = \frac{p(x)}{\alpha - \alpha p(x)}.
\]

It is straightforward to derive the optimal value function by plugging the optimal investment into Eq. (1.5) and computing

\[
V^i(w, n) = \max \left\{ \phi, \Pi(w', n) - cx[w', n] + \frac{\beta \alpha x[w', n] Cl(w' + e(n), n)}{1 + \alpha x[w', n]} + \frac{\beta Cl(w', n)}{1 + \alpha x[w', n]} \right\}.
\]

If \( V^i(w, n) = \phi \), then this model sets \( x = 0 \) with a probability of one. Hence, the actual investment is determined

\[
x^i[w', n] = I \{ V^i(w, n) > \phi \} x[w', n],
\]

where \( I\{\cdot\} \) is the indicator function that takes the value of one if the condition is true, and the value of zero otherwise.

**Calculating the Probability of Entry:** After the exit decision is made, the value of entry is simply the value of an incumbent realizing that there would be no entry and was constrained not to have profits or invest in the current period. The expected discounted value of entering is

\[
V^e(w') = \beta Cl[w' + WEe[m(w') + 1], m(w') + 1; \lambda = 0].
\]

A firm would like to enter if and only if \( V^e > X^E \), which is the random entry cost. Since the random cost is uniformly distributed between \( X^EL \) and \( X^EH \), the probability of entry by an incumbent whose competitors are specified by \( w' \) and
\[ m(w') < N \] is:

\[
\lambda(w', n) = \min \left\{ \max \left[ \frac{V^e(w') - X_{EL}}{X_{EH} - X_{EL}}, 0 \right], 1 \right\}.
\]

**Updating N**: This model starts with the one-firm issue and solve for its value function and optimal policies. Then it proceeds to the two-firm problem, using the fixed values that is solved for in the one-firm problem as the starting values for \( X \) and \( V \):

\[
V^0[(w_1, w_2), 1] = V^\infty(w_1), \forall w_1, w_2 \in W,
\]

\[
V^0[(w_1, w_2), 2] = V^\infty(w_2), \forall w_1, w_2 \in W,
\]

where \( V^\infty(\cdot) \) is the fixed point for the one-firm problem. Analogously, for the \( N \)-firm problem with \( N > 2 \), the starting values are

\[
V^0[w, n] = \begin{cases} 
V^\infty[(w_1, \ldots, w_{N-1}), n], & \text{if } n < N \\
V^\infty[(w_1, \ldots, w_{N-2}, w_N), n-1], & \text{if } n = N 
\end{cases}
\]

The elements of \( X \) are updated in the same way as \( V \). This process is then repeated until \((w', n) = 0\) for all \((w', n)\) with \( m(w') \geq N - 1 \).

**1.3.3 Profit Function**

The one-shot profit function that is utilized in this model is a homogenous-products Nash in quantities (Cournot) markets where differences in efficiency among companies are reflected by differences in marginal costs. Let producers’ different but constant marginal costs, \( \theta(w_n) \), be determined by multiplying a firm’s specific efficiency index and a common factor of price index. Accordingly, if \( s_\nu \) and \( s_\tau \) are the logarithms of...
the firm's efficiency index and of the factor price index, respectively, then \( w_n \equiv s\tau - s\nu \)
and \( \theta (w_n) = \gamma \exp(-w_n) \).

Let \( q_n \) be firm \( n \)'s output, \( Q = \sum q_n \), \( f \) be the fixed cost of production, and \( D \)
be the vertical intercept of the demand curve. The profits are given by

\[
\pi_n = p(Q)q_n - \theta(w_n)q_n - f = (D - Q - \theta(w_n))q_n - f
\]

The unique Nash equilibrium for this problem has quantities and price as

\[
q^*_{w_n} = \max \{ 0, p^* - \theta_i \} \quad \text{and} \quad p^* = \frac{1}{n^* + 1} \left[ D + \sum_{j=1}^{n^*} \theta(w_j) \right].
\]

where \( n^* \) is the number of firms with positive \( q^* \). Finally, the profit of the current
period is

\[
\pi(w, n) = \max \left\{ -f, \left[ p^*(w, n) - \theta(w_n) \right]^2 - f \right\} = \max \left\{ -f, \left[ \frac{1}{n^* + 1} \left( D + \sum_{j=1}^{N} \theta(w_j) - \theta(w_n) \right) \right]^2 - f \right\}
\]

1.3.4 Social Planner’s Problem

We are interested in finding out how the social planner will respond when the tech-
ology is exactly the same as that faced by the MPN competitors. The result enables
us to compare the average efficiency level (and TFP loss ratio) to the MPN case.

In the homogenous-products Nash in quantities profits model, the social planner
will set \( p = mc \) for the firm with the lowest marginal cost (with the highest efficiency
level), and produce until supply equals demand. The methodology used to solve the
social planner’s issues is similar to the one used in the case of MPN competitors. The
social planner’s role is to maximize the stochastic profit function or the projected value of social surplus, i.e. the producer plus consumer surplus. The value function is then constructed using the value function, as defined in Eq. (1.5).

Since the social planner controls the entire economy, any industry structure results in only one state, not in N states. Moreover, we do not need to form perceptions about entry and exit or the behavior of cohorts.

1.4 Midrigan and Xu’s Model

This study applies the benchmark model introduced by Midrigan and Xu (2014), following its set-up, decision rules, definition of equilibrium, TFP function, and first-best allocation of the economy. This model uses the same notations and equations from Midrigan and Xu’s Model (p.425-435 Midrigan and Xu, 2014).

1.4.1 Set up

The economy has a measure \( N_t \) of producers and a measure one of workers. The labor productivity and producer’s population grow at constant rates. Producers operate either in a traditional sector that uses only labor and an unproductive technology, or in a modern sector that uses capital and labor and a more productive technology. This study focuses on the financial misallocation in the modern sector. A one-time sunk entry cost is required for producers in the traditional sector who want to enter into the modern sector. Moreover, one-period uncontingent security and equity claims to producers’ profits are the only two kinds of financial instruments in the model.

**Traditional Sector Producers:** A certain amount, \((\gamma - 1) N_t\), of new producers enter the economy at the end of period \( t \), but only in the traditional sector. Producers in this sector face decreasing returns on technology \((\eta < 1)\) that produces output \( Y_t \).
using labor $L_t$ as the only factor of production:

$$Y_t = \exp(z + \epsilon_t)^{1-\eta} L_t^\eta. \quad (1.7)$$

This study assumes that entrants draw the permanent productivity component $z$ from some distribution $G(z)$, whose mean is normalized to unity. $\epsilon_t$ is a transitory productivity component that evolves over time according to a finite-state Markov process of $E = (e_1, \ldots, e_T)$ with transition probabilities $f_{i,j} = \Pr(e_{t+1} = e_j|e_t = e_i)$. Entrants draw their initial productivity component $e_i$ from the stationary distribution associated with $f$, which is denoted with $\overline{f_i}$.

All producers in the traditional sector aim to maximize their lifetime utility, which is $E_0 \sum_{t=0}^{\infty} \beta^t \log (C_t)$. However, the budget constraints they face depend on whether remaining in the traditional sector or switching to the modern sector.

On the one hand, the budget constraint for those who stay in the traditional sector is

$$C_t = Y_t - WL_t - (1 + r) D_t + D_{t+1}, \quad (1.8)$$

where $D_t$ denote the producer’s debt position, which is non-positive since these producers are not allowed to borrow. All entering producers have no wealth, i.e. the initial $D$ is equal to zero. Moreover, $W$ and $r$ are the equilibrium wage and interest rate.

On the other hand, traditional sector producers who enter the modern sector requires an investment equal to $\exp(z)\kappa$ units of output, which is proportional to the permanent productivity component. Besides internal funds, both of the two financial instruments including one-period risk-free debt and equity claims to future profits are potential channels to finance the physical capital, $K_{t+1}$, and intangible capital,
exp(z)κ. In terms of debt, the borrowing constraint is

$$D_{t+1} \leq \theta (K_{t+1} + \exp(z)\kappa), \quad (1.9)$$

where $\theta \in [0, 1]$ governs the strength of financial frictions in the economy, which requires the debt below a fraction of its capital stock. For the equity claims, this study denotes $P_t$ as the price of the claim to the entire stream of profits, where profits are defined as $\Pi^m_t = Y_t - WL_t - (r + \delta)K_t$ and $\delta$ is the capital depreciation rate. This model assumes that producers can only issue claims to a fraction, $\theta \chi$, of their future profits, where $\chi \in [0, 1]$. $\theta$ is characterizing the degree of financial development of the economy since it decides the producer’s ability to both borrow and issue equity. The budget constraint of a producer that enters the modern sector is therefore

$$C_t + K_{t+1} + \exp(z)\kappa = Y_t - WL_t - (1 + r)D_t + D_{t+1} + \theta \chi P_t. \quad (1.10)$$

**Modern Sector Producers:** The production function for the producers in the modern sector is

$$Y_t = \exp(z + e_t + \phi)^{1-\eta} (L_t^\alpha K_t^{1-\alpha})^\eta,$$

where $\phi \geq 0$ determines the relative productivity of this sector, $\alpha$ controls the share of labor in production, and $K_t$ is the amount of capital used in the previous period.

Producers in the modern sector can save and borrow at the risk-free rate, $r$, subject to the constraint (1.9). Their budget constraint is

$$C_t + K_{t+1} - (1 - \delta)K_t = Y_t - WL_t - (1 + r)D_t - \theta \chi \Pi^m_t + D_{t+1} \quad (1.11)$$
This model assumes, as is standard in the investment literature, that output at date \( t + 1 \) is produced with capital held in period \( t \). The choice of how much to invest at the end of period \( t \) is, however, measurable with respect to \( e_{t+1} \). This assumption of timing explains why the expected return of stock equals the risk-free return.

**Workers:** A unit measure of workers is available in the economy, each of whom supplies \( \gamma' \nu_t \) efficiency units of labor, where \( \nu_t \) is the worker’s idiosyncratic efficiency that evolves over time according to a finite-state Markov process. These workers have the same log preferences (utility function) as producers do. However, their budget constraint is

\[
c + a_{t+1} + \int P_t^i \omega_{t+1}^i di = W \gamma' \nu_t + (1 + r) a_t + \int (P_t^i + \Pi_{t}^{m,i}) \omega_t^i di,
\]

where \( a_t \) denote a worker’s holdings of risk-free assets and \( \omega_t^i \) denote the number of shares he or she owns of producer \( i \). The total asset holding, \( a_{t+1} + \int P_t^i \omega_t^i di \), are non-negative because this model assumes workers cannot borrow.

Once again, there is no aggregate risk in this economy since our assumption of the timing. As a result, the lack of arbitrage implies that the return on the risk-free security is equal to the expected return on equity claims:

\[
(1 + r) = \frac{E_t \left[ P_{t+1}^i + \Pi_{t+1}^{m,i} \right]}{P_t^i}.
\]

### 1.4.2 Recursive Formulation and Decision Rules

**Modern Sector Producers:** The risk-free assumption on capital implies that producer profits are solely a function of its net worth, which is denoted as \( A = K - D \). Moreover, profits, output, and the optimal choice of capital and labor are all homogeneous of degree one in \((A, \exp(z))\) so this model can rescale all variables by \( \exp(z) \)
including the rescaled net worth \( a = A/\exp(z) \). Given the new notation, the Bellman equation is

\[
V^m(a, e_i) = \max_{a', c} \log(c) + \beta \sum_m f_{i,j} V^m(a', e_j).
\]

Similarly, the budget constraint in Eq. (1.11) can be rewritten as

\[
c + a' = (1 - \theta \chi) \pi^m(\alpha, e) + (1 + r) a,
\]

where

\[
\pi^m(\alpha, e) = \max_{k,l} \exp(e + \phi)^{1-\eta} (l^\alpha k^{1-\alpha})^\eta - Wl - (r + \delta) k.
\]

Furthermore, the borrowing constraint in Eq. (1.9) reduces to

\[
k \leq \frac{1}{1 - \theta} a + \frac{\theta}{1 - \theta} \kappa.
\]

This model characterizes the producer’s net worth accumulation decision of the producer by

\[
\frac{1}{c(a, e_i)} = \beta \sum f_{i,j} \left[ (1 + r) + \frac{1}{1 - \theta} \mu(a', e_j) \right] \frac{1}{c(a', e_j)},
\]

where \( \mu(a, e) \) is the multiplier on the borrowing constraint (1.15). The producer’s return to savings increases with the expectation that the borrowing constraint will bind in future periods. Therefore, the producers have the incentive to accumulate net worth.

Accordingly, the decisions of the optimal level of capital and labor simplifies to

\[
\frac{\alpha n y(a, e)}{l(a, e)} = W
\]
and

$$(1 - \alpha) \frac{\eta y(a, e)}{k(a, e)} = r + \delta + \mu(a, e).$$ (1.18)

Dispersion of the net worth and productivity of businesses due to borrowing constraints causes dispersion in the marginal product of capital of individual producers. In turn, this causes TFP reductions due to misallocation. In the rescaled formulation of the problem, it is worth noting that the producer’s permanent productivity component, $z$, has no independent effect on allocations.

**Traditional Sector Producers:** The next thing to consider is the problem of producers in the traditional sector. Since capital is not an input for these producers, their net worth is $a = -d$. This model also denotes $x$ as their savings. The Bellman equation for such producers is

$$V^{\tau}(a, e) = \max_{a', c} \left\{ \log(c) + \beta \max \left\{ \sum_{j} f_{i,j} V^{\tau}(a', e_j), \sum_{m} f_{i,j} V^{m}(a', e_j) \right\}, \right.$$ subject to

$$c + x = \pi^{\tau}(e) + (1 + r) a,$$ (1.19)

where

$$\pi^{\tau}(e) = \max_{l} \exp(e)^{1-\eta l} - Wl.$$ 

In each period, the producer’s decision of whether to stay in the traditional sector or switch to the modern sector depends on the relative value of these two options. This decision also determines the evolution of its net worth. A producer who remains in the traditional sector simply inherits its past savings, $a' = x$, while a producer that
enters the modern sector has

\[ a' = x - \kappa + \theta \chi p(a', e_i), \quad (1.20) \]

where \( p(a', e_i) \) is the rescaled price of the equity claim to that satisfies

\[ p(a, e_i) = \frac{1}{1 + r} \sum_j f_{i,j} [p(a', e_j) + \pi^m(a', e_j)]. \quad (1.21) \]

The producers in the modern sector may have negative net worth since they can borrow against the intangible capital. Besides the collateral constraint in Eq. (1.15), the natural borrowing constraint,

\[ a > a_{\min} = -\left(1 - \theta \chi \right) \pi^m(a_{\min}, e_1) \frac{1}{r}, \quad (1.22) \]

which guarantees the producer’s solvency even under the worst possible sequence of productivity shocks. This constraint may be more stringent than the collateral constraint and motivate producers to accumulate enough savings before entering the modern sector even in the absence of a collateral constraint.

1.4.3 Equilibrium

This model denotes \( n_i^m(a, e) \) as the measure of modern-sector producers and \( n_i^r(a, e) \) as the measure of traditional sector producers. The population of producers in these two sectors sum to \( N_t = \gamma^t: \int_{A \times E} dn_i^m(a, e) + \int_{A \times E} dn_i^r(a, e) = N_t. \)

On the one hand, the number of producers in the modern sector evolves according
to

\[ n_{t+1}^m (A, e) = \int \sum_i f_{i,j} I_{\{a^m(a,e_i) \in A\}} d n_i^m (a, e_i) + \int \sum_i f_{i,j} I_{\{\xi(a,e_i) \in A\}} d n_i^r (a, e_i), \]

(1.23)

where \( \xi(a, e) \) is an indicator for whether a producer in the traditional sector switches, \( A = [a, \bar{a}] \) is the compact set of values that a producer’s net worth can take and \( A \) is a family of its subsets, \( a^m(.) \) is the amount of net worth for a producer in the modern sector, and \( a^r,s(.) \) is the savings decision of a producer who switches.

On the other hand, the measure of producers in the traditional sector is

\[ n_{t+1}^r (A, e) = \int \sum_i f_{i,j} I_{\{\xi(a,e_i) = 0, \ a^r(a,e_i) \in A\}} d n_i^r (a, e_i) + (\gamma - 1) N_t I_{\{0 \in A\}} f_j, \]

(1.24)

where \( f_j \) is the stationary distribution of the transitory productivity and \( a^r(.) \) is the net worth of a producer that stays in the traditional sector.

A balanced growth equilibrium must satisfy the following five conditions.

(I) the labor market clearing condition:

\[ \int_{A \times E} l^r (e) d n_i^r (a, e) + \int_{A \times E} l^m (a, e) d n_i^m (a, e) = L_t = \gamma^t, \]

(II) the asset market clearing condition:

\[ A_{t+1}^m + \sum_{i=m,r} \int_{A \times E} a_{t+1}^i (a, e) d n_i^i (a, e) = \int_{A \times E} k_{t+1}^m (a, e) d n_i^m (a, e), \]

(1.25)

or

\[ C_t + K_{t+1} - (1 - \delta) K_t + X_t = Y_t, \]

(1.26)

(III) producer and worker optimization,
(IV) the no-arbitrage condition in Eq. (1.21),
(V) the laws of motion for the measures in Eqs. (1.23) and (1.24).

All variables that have indicated with time subscripts grow at a constant rate $\gamma$ while others are consistent along a balanced growth path. Solving the balanced growth equilibrium is equal to solving the stationary system where all the time-variant variables are rescaled by $\gamma^t$.

1.4.4 Efficient Allocations

The value of TFP in the economy is reduced by financial frictions, which occur in two ways: either by affecting a business’ entry into the modern sector or by causing losses in the modern sector due to misallocation. The strength of these two paths is defined using two separate computations. The first computation determines the level of TFP losses in the modern sector as a result of capital misallocation among the producers in the group. The equilibrium of the model is taken as the stationary level. This calculation is similar to the one that was stated by Hsieh and Klenow (2009). The second computation determines the consumption, output, and TFP, in addition to measuring solutions for the problems of a planner where there are no limitations on the allocation of capital and labor across units of production. The broader question in this calculation is identifying the level of consumption in the economy and how it is limited by financial frictions that develop along the way in both intensive and extensive margins.

**TFP Losses from Misallocation in the Modern Sector:** Let $i$ index producers, $M$ be the set of all producers in the modern sector. Moreover, let $L$ and $K$ be the total amount of labor and capital used in that sector, respectively. Integrating the decision rules (1.17) and (1.18) across producers, the total amount of
output produced by the modern sector is

\[ Y = \exp(\phi)^{1-\eta} \left( \frac{\int_{i \in M} \exp(e_i) \left( r + \delta + \mu_i \right)^{(1-\alpha)\eta} di}{\left( \int_{i \in M} \exp(e_i) \left( r + \delta + \mu_i \right)^{(1-\alpha)\eta} di \right)^{(1-\alpha)\eta}} \right)^{1-\alpha \eta} \left( L_t^{\alpha} K_t^{1-\alpha} \right)^{\eta}. \] (1.27)

This expression shows that TFP of the modern sector is determined by the exogenous productivity gap, \( \phi \), and an endogenous component that depends on the measure of producers, their efficiency, and the extent to which they are bind.

To calculate the efficient level of TFP given a measure of \( M \) producers, this model allocates capital and labor across these producers so that marginal product of capital and labor are the same across producers in order to maximize total output in the modern sector. Accordingly, the efficient level of output is given by

\[ Y^e = \exp(\phi)^{1-\eta} \left( \int_{i \in M} \exp(e_i) di \right)^{1-\eta} \left( L^\alpha K^{1-\alpha} \right)^{\eta}. \] (1.28)

Comparing Eqs. (1.27) and (1.28) and using the fact that the shadow cost of capital, \( r + \delta + \mu \), is proportional to its average product, as in Eq. (1.18), the TFP losses from misallocation are

\[ \text{TFP losses} = \log \left( \int_{i \in M} \exp(e_i) \right)^{1-\eta} \left( \int_{i \in M} \exp(e_i) \left( \frac{y_i}{k_i} \right)^{(1-\alpha)\eta} \left( 1-\eta \right)^{(1-\alpha)\eta} \right)^{1-\alpha \eta} \left( \int_{i \in M} \exp(e_i) \left( \frac{y_i}{k_i} \right)^{(1-\alpha)\eta} \left( 1-\eta \right)^{(1-\alpha)\eta} \right)^{(1-\alpha)\eta}. \] (1.29)

To clarify Eq. (1.29), suppose that the logarithm of \( y_i/k_i \) and \( e_i \) are jointly normally distributed. Eq. (1.29) then reduces to

\[ \text{TFP losses} = \frac{1}{2} \left( \frac{1-\alpha \eta}{1-\eta} \right) \left( 1-\alpha \right) \eta \text{var} \left( \log \left( \frac{y_i}{k_i} \right) \right). \] (1.30)
so that the TFP losses are proportional to the variance of the average product of capital. In other words, higher variability in the average product of capital across producers generates more TFP losses.

**Efficient (First-Best) Allocations:** To calculate the efficient allocation, this model must also derive the optimal number of producers across the two sectors. This can be done by solving the social planner’s problem that is only constrained by the aggregate resource constraint in Eq. (1.26) and by the production technologies that we have assumed. Accordingly, this study chooses the amount of capital, $K$, the number of producers in the two sectors, $n^\tau_i$ and $n^m_i$, and the allocation of labor across those sectors, $L^\tau$ and $L^m$, to maximize

$$
\left( \sum_i \exp (e_i) n^\tau_i \right)^{1-\eta} (L^\tau)^{\eta} + \left( \sum_i \exp (e_i + \phi) n^m_i \right)^{1-\eta} ((L^m)^{\alpha}(K)^{1-\alpha})^{\eta} - \left( \delta + \frac{\gamma}{\beta} - 1 \right) K - \left( \gamma - 1 \right) \kappa \sum_i n^m_i
$$

subject to the restrictions on the measurements implied by Markov transition probabilities, $f_{i,j}$, and to the labor constraint, $L^\tau + L^m = 1$.

**1.4.5 Summary**

In short, the model has three kinds of players: workers, traditional producers, and modern producers. Traditional producers use only labor and unproductive technology, and cannot borrow money. Modern producers, on the other hand, use capital, labor, and more productive technology; they can also borrow. Traditional producers can become modern producers, but to do so they must incur a sunk entry fee, and they are allowed to borrow and issue claims to part of the future profit during that period of transformation. The amount that a producer can borrow is subject to collateral
constraints. Workers face uninsurable idiosyncratic labor income risk and have access to financial markets. There are two types of financial instruments available: a one-period non-contingent security and equity claims to producers’ profits.

These three kinds of players all try to maximize their lifetime utility. The equilibrium requires (I) a labor market clearing condition, (II) an asset market clearing condition, (III) producer and worker optimization, (IV) the no-arbitrage condition, and (V) the laws of motion. This study can use Eq. (1.3) to calculate the TFP loss due to financial misallocation. On the one hand, the actual TFP level, $TFP$, in the equilibrium can be derived under this setup. On the other hand, the efficient level of TFP, $TFP^e$, is the solution to the planner’s problem that is not restricted in any way concerning the allocation of labor and capital across firms.

1.5 Data

The data are derived from Fame dataset\(^1\) and cover the years 2005-2012. Fame contains comprehensive information, including balance sheets and profit and loss accounts for approximately half a million companies in the UK and Ireland. Each firm’s balance sheet provides total asset, liabilities, and shareholder funds information, while the profit and loss account provides operating profits, depreciation, amortization, impairment, remuneration, directors’ remuneration, and the number of employees. This information allows us to construct a more accurate measurement of the output ($Y$) and inputs ($L$ and $K$).

The labor expenditure is the sum of remuneration and directors’ remuneration. Remuneration includes wages and salaries, social security cost, pension costs, and other staff costs, while directors’ remuneration includes director’s fees, pension contributions, and other compensations. This study defines labor as the number of

\(^1\) http://www.bvdfinance.com/en-gb/our-products/company-information/national-products/fame
employees.

Capital expenditure is defined as the sum of depreciation, amortization, and impairment. This study uses the term “capital employed” to derive capital. Capital employed is the total assets less current liabilities. It is the value of the assets that contribute to a company’s ability to generate revenues (liquidity), including both sunk cost (not used in production) and capital (used in production). This study assumes the same ratio between sunk cost and capital across firms. It also adjusts all of the capital expenditures by the average depreciation rate in the UK (8%) to predict the total capital volume. The total sunk cost for all of the firms is the total capital employed minus the total capital. This method provides the average ratio between sunk cost and capital, which can be applied to estimate the capital in each firm.

The output is measured by value added. One way of calculating value added is to identify the net revenue of expenditure on intermediate inputs, while the other is to sum profit, depreciation cost and labor cost. Since Fame does not have intermediate input information, this study uses the second method. This research defines value added as the sum of operating profit, labor expenditure, and capital expenditure. Labor expenditure is the return, while the “cash” profit (operating profit plus capital expenditure) is a return to capital.

All series are real. This study deflates value added, labor cost, and capital cost using the Producer Price Index (PPI) and Consumer Price Index (CPI): PPI and CPI of Ireland for firms in the Republic of Ireland and the PPI and CPI of the UK for firms in England, Scotland, Wales, North Ireland, and British Crown dependencies.

This research dismisses observations that are likely the outcome of measurement error, such as observations with negative or missing figures of value added, expenditure of labor, and constructed capital series. This study also drops the top 0.5% and bottom 0.5% of observations of output-to-capital ratio, output per
capita, change in output, change in labor, and change in capital. This leaves us with $17,873 \times 8 = 142,984$ firm-year observations over an eight-year period from 2005 to 2012.

Table 1.1 shows the statistics of the Fame data. The average output is £33 million before the crisis and £38 million after the crisis. The average capital increases from £58 million before the crisis to £74 million after the crisis, while its expenditure increases from £5 million to £6 million. The average number of employees also increases from 671 to 737 between those two periods, while the labor cost increases from £17 million to £19 million. These statistics show that the output, capital input, and labor input increase by 14%, 28%, and 10%, respectively. Therefore, firms in the British Isles are actually using more capital input in their portfolios after the financial crisis. The output per capita is higher, while the output per capital is lower. The debt-to-output ratio decreases from 2.33 to 2.15, but the equity-to-output increases from 1.2 to 1.43 after the financial slowdown. These two ratios imply a significant change in the debt/equity structure in firms after the recession. In fact, equity occupies a much larger proportion of the total assets after the slowdown.

Table 1.1: Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>S.D.</td>
</tr>
<tr>
<td>Output ($Y$) (£ million)</td>
<td>33.04</td>
<td>446.3</td>
</tr>
<tr>
<td>Capital ($K$) (£ million)</td>
<td>58.00</td>
<td>971.3</td>
</tr>
<tr>
<td>Labor ($L$)</td>
<td>671.4</td>
<td>6914.2</td>
</tr>
<tr>
<td>Capital Expenditure ($\delta K$)</td>
<td>4.94</td>
<td>161.1</td>
</tr>
<tr>
<td>Labor Expenditure ($W_L$)</td>
<td>17.47</td>
<td>139.2</td>
</tr>
<tr>
<td>Debt to Output</td>
<td>2.33</td>
<td>40.64</td>
</tr>
<tr>
<td>Equity to Output</td>
<td>1.20</td>
<td>2.11</td>
</tr>
</tbody>
</table>

1.6 Quantitative Analysis

1.6.1 Stochastic Frontier Analysis Based Overall TFP Loss

Table 1.2 provides an estimation result of the Stochastic Frontier Analysis by different models (FIX, RND, KSS and BC) for both the pre-crisis period (2005-2007) and the post-crisis period (2008-2012). The first two rows in Table 1.2 give the estimated coefficient and standard error of the simple time trend, respectively. All estimators show that the technology change increases TFP in the first period, but decreases TFP after the financial crisis. The second two rows are the coefficient estimation and standard error of capital input, respectively.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FIX</td>
<td>RND</td>
</tr>
<tr>
<td>Time Trend</td>
<td>δ̂</td>
<td></td>
</tr>
<tr>
<td>S.E.</td>
<td>0.001</td>
<td>0.012</td>
</tr>
<tr>
<td>Capital</td>
<td>β̂₁</td>
<td></td>
</tr>
<tr>
<td>S.E.</td>
<td>0.004</td>
<td>0.003</td>
</tr>
<tr>
<td>Average TE</td>
<td>0.341</td>
<td>0.365</td>
</tr>
<tr>
<td>TFP Loss Ratio</td>
<td>0.659</td>
<td>0.635</td>
</tr>
</tbody>
</table>

The estimations in Table 1.2 help to predict the average technical efficiency. Table 1.2 also shows the TFP loss by different models for the pre- and post-crisis periods, which is one minus the average technical efficiency. The estimated TFP losses of the four models are robust and the average is 64.5% (equal to an average 35.5% technical efficiency) before 2008. These TFP losses by different models are all dropped after the recession and decrease by an average 1.3%, to 63.2% (equal to 36.8% average efficiency). These estimations imply that the average efficiency level increased after
Figure 1.1: Average efficiencies from different estimators in the two periods

The crisis. Figure 1.1 provides the average efficiencies from different estimators in the two periods. The KSS estimator indicates that the average efficiency was at low level in 2007 and 2008 but rebounded very soon above the pre-recession level.

1.6.2 Pakes-McGuire Model Based on Overall TFP Loss

Pakes, Gowrisankaran, and McGuire (1993) defined several constants in their algorithm. The same parameters as the original paper, except for three in the profit function, are utilized. This study posits that the profit function changed before and after the financial crisis. PMM provides the outputs, including the efficiency level of all the active firms in every period, the average lifespan of firms, the average investment and profit in one period, and so on. To this end, this study pins down the three parameters in the profit function \((D, f, \text{ and } \gamma)\) by requiring that the model provide similar statistics with *Fame* data, including the average lifespan of firms and the investment-to-profit ratio. Then, the average efficiency level can be estimated.
All of these statistics are computed by simulating the industry from an economy starting with only one firm whose efficiency level is 11. This study also uses the value function, investment, and entry/exit decisions to evaluate the optimal policies and update the industry structure. We have separate, but similar, programs to evaluate the statistics for the MPN equilibrium and for the social planner’s problem before and after the crisis, respectively. The industry is simulated 10,000 times and the average efficiency level is the mean of all the active firms in those 10,000 periods.

Based on Fame data, the average lifespan of firms is 17.4 years before and 16.1 years after the crisis. Goodridge, Haskel, and Wallis (2012) provide the total annual investment in the UK from 2005 to 2011. The annual Gross Value Added (GVA) of the UK is available in the Regional Gross Value Added report from the Office for National Statistics (ONS). These two datasets provide the investment-to-value added ratio. Fame data provides the value added-to-profit ratio. Therefore, the average investment-to-profit ratio can be derived, which calculates to 0.634 before and 0.525 after the recession in 2008.

Table 1.3 presents the parameters used in the PMM, as well as the outputs. The average efficiency level before and after the crisis is 6.86 and 7.03, respectively. Since the highest efficiency level witnessed is 13 in both periods, this study predicts a 47.2% and 45.9% TFP loss before and after the crisis, respectively. Therefore, the overall TFP was not impacted to any substantial degree by the financial crisis.

1.6.3 Midrigan and Xu’s Model Based TFP Loss

This model groups the parameters into two categories. The first category includes preference and technical parameters that are difficult to identify using our data. This study assigns these parameter values that are already common in existing work. The second category includes parameters that determine the process for entrepreneurial
Table 1.3: Calibration and Result of PMM

<table>
<thead>
<tr>
<th></th>
<th>Pre-Financial Crisis</th>
<th>Post-Financial Crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data MPNE SP</td>
<td>Data MPNE SP</td>
</tr>
<tr>
<td>A. Used to calibrate model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average lifespan of firms</td>
<td>17.4 17.7 16.1 16.3</td>
<td></td>
</tr>
<tr>
<td>Average investment-to-profit ratio</td>
<td>0.634 0.644 0.525 0.538</td>
<td></td>
</tr>
<tr>
<td>B. Assigned parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant used in investment fn.</td>
<td>$\alpha$</td>
<td>3</td>
</tr>
<tr>
<td>Cost for a GBP investment</td>
<td>$c$</td>
<td>1</td>
</tr>
<tr>
<td>Maximum Number of firms</td>
<td>$N$</td>
<td>3</td>
</tr>
<tr>
<td>Highest efficiency level attainable</td>
<td>$\hat{w}$</td>
<td>19</td>
</tr>
<tr>
<td>Efficiency level for entrants</td>
<td>$W_E$</td>
<td>9</td>
</tr>
<tr>
<td>Sunk cost of entry</td>
<td>$X_E$</td>
<td>0.2</td>
</tr>
<tr>
<td>Lowest sunk entry cost</td>
<td>$X_EL$</td>
<td>0.15</td>
</tr>
<tr>
<td>Highest sunk entry cost</td>
<td>$X_EH$</td>
<td>0.25</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.92</td>
</tr>
<tr>
<td>Prob. of outside alternative rising</td>
<td>$\delta$</td>
<td>0.7</td>
</tr>
<tr>
<td>Scrap value at exit</td>
<td>$\phi$</td>
<td>0.1</td>
</tr>
<tr>
<td>C. Calibrated parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical intercept of demand</td>
<td>$D$</td>
<td>4</td>
</tr>
<tr>
<td>Fixed cost of production</td>
<td>$f$</td>
<td>0.2</td>
</tr>
<tr>
<td>Capital-to-cost parameter</td>
<td>$\gamma$</td>
<td>1</td>
</tr>
<tr>
<td>D. Result</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average efficiency</td>
<td>6.86 8.71 7.03 8.9</td>
<td></td>
</tr>
<tr>
<td>Max Efficiency level Appeared</td>
<td>13 – 13 –</td>
<td></td>
</tr>
</tbody>
</table>
productivity, as well as the size of the financing frictions. This study finalizes these parameters by requiring that the model accounts for the salient features of the Fame data. Table 1.4 summarizes the parameter values that we used in our experiments, as well as the results.

For the pre-crisis period, all of the statistics in Part A under “Data” of Table 1.4 is the average level using Fame information from 2005 to 2007 except autocorrelation and intangible investment-to-output. Autocorrelation is calculated using the entire dataset from 2005 to 2012. Intangible investment information is the average level for the UK during the stated period. This study assigned the parameters in Part B for the “Benchmark” to make the environment as close as possible to the data. Then, the parameters in Part C are calibrated to make the outcome statistics in Part A of the benchmark close to the statistics in “Data”. At the same time, the calibrated parameters in Part C of the benchmark also guarantee all the conditions of equilibrium. As a result, the TFP loss due to financial misallocation is 0.7% before the recession.

After the financial crisis, a significant decrease in debt is witnessed in the Fame data. The average Debt-to-Equity ratio decreases from 2.239 to 1.852. If we shrink $\theta$, the constraint of a firm’s debt, by the same proportion, then $\theta$ will decrease from 0.59 to 0.49. However, since not all the firms borrow as much as they can, the constraint decrease should be larger than the decrease in the actual debt ratio. Therefore, this study sets $\theta$ to $\sim$ 0.45 after the crisis. This research finds that the TFP loss due to financial friction increases from 0.7% to 1.8%.

Another method for estimating the TFP loss due to misallocation is the use of Eqs. (1.29) and (1.30). Using the variance of the average product of capital derived from Fame, the TFP losses (in Eq. (1.30)) are 0.626 and 0.893 for the pre- and post-crisis period, respectively. Therefore, this study can derive the TFP loss due
Table 1.4: Calibration and Result of MXM

<table>
<thead>
<tr>
<th>A. Used to calibrate model</th>
<th>Pre-Financial Crisis</th>
<th>Post-Financial Crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Benchmark</td>
</tr>
<tr>
<td>S.D. output growth</td>
<td>0.27</td>
<td>0.30</td>
</tr>
<tr>
<td>S.D. output</td>
<td>1.39</td>
<td>1.28</td>
</tr>
<tr>
<td>1-year autocorrelation</td>
<td>0.98</td>
<td>0.97</td>
</tr>
<tr>
<td>3-year autocorrelation</td>
<td>0.96</td>
<td>0.94</td>
</tr>
<tr>
<td>5-year autocorrelation</td>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td>Intangibles investment-to-output, %</td>
<td>12.0</td>
<td>11.1</td>
</tr>
<tr>
<td>Output growth rate, %</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Debt-to-output</td>
<td>2.3</td>
<td>2.6</td>
</tr>
<tr>
<td>Equity-to-output</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>var (log (y_i/k_i))</td>
<td>1.0227</td>
<td>0.9885</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Assigned parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>labor elasticity</td>
<td>α</td>
</tr>
<tr>
<td>span of control</td>
<td>η</td>
</tr>
<tr>
<td>capital depreciation</td>
<td>δ</td>
</tr>
<tr>
<td>discount factor</td>
<td>β(1 + μ)^{-1}</td>
</tr>
<tr>
<td>growth rate</td>
<td>γ</td>
</tr>
<tr>
<td>persistence unit worker state</td>
<td>λ_1</td>
</tr>
<tr>
<td>persistence zero worker state</td>
<td>λ_0</td>
</tr>
<tr>
<td>relative efficiency in modern sector</td>
<td>(1 + η) φ</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C. Calibrated parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>collateral constraint</td>
<td>θ</td>
</tr>
<tr>
<td>equity issuance constraint</td>
<td>χ</td>
</tr>
<tr>
<td>standard deviation transitory shocks</td>
<td>σ_ε</td>
</tr>
<tr>
<td>persistence transitory shocks</td>
<td>ρ</td>
</tr>
<tr>
<td>cost of entering modern sector</td>
<td>κ</td>
</tr>
<tr>
<td>wage</td>
<td>W</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D. Result</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss misallocation, %</td>
<td>46.5</td>
</tr>
</tbody>
</table>
to financial friction via Eq. (1.3), which yields as 46.5% before and 59.1% after the recession.

This method suggests a much larger TFP loss than the first approach. The major difference between the two approaches for estimating TFP loss due to financial friction is also seen in Midrigan and Xu’s finance and misallocation paper. They found that the TFP loss due to financial friction in Korea was 0.3% for the benchmark model, but 16.2% for the data model. A fairly large TFP loss was also found using Data from China and Colombia, as well as the findings of Hsieh and Klenow’s paper in 2009. Midrigan and Xu emphasized that Eq. (1.29) may overstate the TFP losses from misallocation due to financial frictions, since differences in average capital production may reflect differences in technologies, technological barriers to capital reallocation, or other inefficiencies, such as taxes or markups. Nevertheless, they found Eq. (1.29) to be useful, as it provides an upper bound for the losses from capital misallocation. However, both methods in the structural model suggest significantly increased TFP loss due to financial misallocation after the financial crisis.

1.7 Conclusion

This chapter estimates the overall TFP loss, as well as the loss specifically caused by financial friction in the British Isles both before and after the recession. The Stochastic Frontier Analysis estimates an average 64.5% and 63.2% overall TFP loss before and after the crisis, respectively. The overall TFP loss predicted by the Pakes-McGuire Model are 47.2% and 45.9% before and after the crisis, respectively. Therefore, the TFP loss derived by the reduced-form method and structural model are very close. Both of these estimates show that the TFP loss does not change significantly after the financial crisis. Actually, the loss even decreased a little bit after the recession.

However, the FTP loss due to financial friction estimated by Midrigan and Xu’s
Model more than doubled after the recession, which shows the negative effect of the financial restriction on productivity. Consequently, the ratio of finance-caused TFP loss increases significantly as a part of the overall TFP loss.

This study provides an example on how to use both the reduced-form and structural model to estimate overall productivity performance, as well as the proportion of loss caused by different factors. We can split the productivity loss for more insightful analysis, and we can also transform our results from TFP loss to an efficiency analysis.
2.1 Introduction

In productivity and efficiency analysis, input and output data are usually needed to estimate the production function. However, much of the data, especially the inputs, is incomplete or inaccessible to researchers. One example is seen when estimating productivity and efficiency in an industry. More and more integrated companies at present have a multidivisional form (MDF or M-form) and have footprints in multiple segments. They report total output and input data, and possibly division-level output as well, but not division-level inputs. Such information is not sufficient to reflect the company’s resources invested in the targeted segment and to estimate the segment-specific production function. In this case, the input allocations across divisions/segments for each firm need to be collected or imputed.

Such a problem also exists in modeling productivity by country/region. Many models of world productivity have been proposed in the literature, but most assume
a unique production function for the entire economy. However, different sectors or industries within an economy may utilize different technologies in the real world. Some industries are more labor-intensive, while others are more capital-intensive. Many economists overlook this economic structural heterogeneity while others choose to ignore such issues due to the lack of industry-level data.

These two examples illustrate two challenges that arise as a result of unobservable input allocations, or the “black box” within multidivisional organizations. For each multidivisional organization, the productivity levels measured by standard production methods do not provide the difference in performance across divisions. For each segment, the production function cannot be estimated, since the input allocated in that specific segment is unknown for those multidivisional firms. One of the goals in this study is to impute the input allocations at the division level.

In recent years, the setup of multidivisional firms and multiproduct firms in economics and finance studies is very similar, as most of the conglomerates adopt the M-form to produce multiple products, where each division presents one product line that produces one output. Many studies (McEwin 2011; Teece 1998) call such a business organization a multidivisional multiproduct firm. Section 2.1 introduces multidivisional firms and multiproduct firms, as well as standard assumptions and conventions adopted from the literature so that appropriate methods could be applied to multiproduct firms.

It is worth noting the relationship among the three words that this research frequently uses: product, division, and segment. Suppose a firm has two divisions - one producing oil and the other producing natural gas; it is said that this is a multidivisional multiproduct firm. Such a company has an oil division that generates the oil product in the oil segment, as well as a natural gas division that generates the natural gas product in the natural gas segment. Therefore, the words product, division,
and segment are one-to-one matching. For one product, the matching division is the department or branch of the firm that generates this product, while the matching segment refers to the market for that product.

De Loecker et al. (2015) point out the unobserved input allocations of multi-product firms. According to their setup, these multiproduct firms can be treated as multidivisional multiproduct firms. They exploit data on single-product firms to estimate the production function for each product line, assuming that the production function is the same for single- and multiproduct firms that generate the same product. The production functions estimated from single-product firms allow them to back out allocations of inputs across products within a multiproduct firm. There are several drawbacks to their paper:

Firstly, and most importantly, the validity of the key assumption in their work that production techniques for single- and multiproduct firms are the same for one product line is questionable. De Loecker et al. (2015) point out that this assumption is already implicitly employed in all previous work that pools data across single- and multiproduct firms (e.g., Olley and Pakes (1996) and Levinsohn and Petrin (2003)). However, they neither test this assumption nor give any empirical evidence.

Secondly, there may not be enough observations if there are only a few single-product firms, but many more multidivisional firms in a product line. For example, two of the five segments in the empirical study of the global oilfield market are dominated by multidivisional firms. There are only seven and eight single-division firms in the completion and production segments, respectively. Another example concerns world productivity analysis, where almost all countries are multi-product/division “firms” since they all have more than one industry.

Lastly, De Loecker et al. (2015) assume that the share of a firm’s inputs allocated to a given product line is constant, and thus independent of the input type. For
example, if a product line uses 30% of the firm’s labor, it also uses 30% of the firm’s capital. However, this assumption (assumption 4 in their paper) contradicts the product-specific production function assumption (assumption 1 in their paper). For a multiproduct firm, some product lines are relatively labor-intensive and need a higher labor share in the input portfolio, while other product lines are relatively capital-intensive and need a higher capital share in the input portfolio.

This study makes three central contributions that address the shortcomings of the previous literature discussed above. Firstly, this chapter’s method can test the key assumption that previous studies rely on, i.e., single- and multi-product/division firms have the same production technique for the same product/segment. Secondly, this research can estimate the production function and impute input allocations simultaneously in the absence of the key assumption and the constant share constraint of the input portfolio. Thirdly, this is the first productivity and efficiency study of the global oilfield market, or the oil and gas service industry, that addresses these problems. The empirical study shows that the key assumption and the input portfolio constraints are not always valid. Moreover, the division-level analysis provides valuable information for firms’ mergers and acquisitions (M&A) decisions.

This study develops a model to explore resource allocation across divisions within a multidivisional firm when division-level outputs, total inputs, and segment average input prices are available. The undistorted input allocations must simultaneously satisfy two systems of equations including segment-specific stochastic production functions and the maximization of the mathematical expectation of profit for each multidivisional firm.

An iterative method is used to jointly solve these two systems and impute undistorted input allocations. Then, stochastic frontier analysis is used to estimate the segment-specific production frontiers. Since there is one frontier for each segment,
this method is called the “Meta-Frontier” approach, while the standard stochastic frontier analysis assuming similar production frontier across segments is called the “Single-Frontier” approach.

This study tests the accuracy of the estimation using panel data for twenty-two Organization for Economic Co-operation and Development (OECD) countries from 2000 to 2006 where the actual input allocations are observed. The imputed undistorted inputs allocations are closer to the actual levels than the three competing estimations. Moreover, the iteration converges to the same point using different initial guesses.

This “Meta-Frontier” approach is used to explore the unobservable input allocations of multidivisional firms in the global oilfield market, which is composed of five segments. The results show that the production functions are different across segments, but all demonstrate constant returns to scale or very close to constant returns to scale. Moreover, the single- and multidivisional firms in the capital equipment segment have different production functions, which provides evidence that the key assumption is not always valid. On average, the multidivisional firms are as efficient as single-divisional firms when firm size is controlled. Multidivisional firms may have very different efficiency levels across segments and they prefer to invest in more efficient segments. This “Meta-Frontier” approach can estimate division level and firm-level efficiency as well as average efficiency for each segment.

The remainder of the chapter is structured as follows. Section 2 introduces the methodology of the “Meta-Frontier” approach. Section 3 presents an empirical application on the global oilfield market. Section 4 consists of the conclusions drawn.
2.2 The Model

2.2.1 Multidivisional Firms and Multiproduct Firms

A multidivisional form is an organizational structure that separates a company into individual divisions based on location or products. A multidivisional (form) firm is essentially divided into semiautonomous divisions that have their own unitary structures, and division managers are responsible for their own production and for maximizing profit division-wide. There is a central office that develops overall strategies to maximize overall profit company-wide. In other words, the headquarters are in charge of overall strategic decisions, while division managers are allowed to make their own operational choices.

Multidivisional firms, or conglomerates, have been popular in the United States since the 1960s and currently account for over 60% of the book assets and market equity of S&P 500 firms (Duchin, Goldberg, and Sosyura 2015). This type of firm is mostly studied in corporate finance and management research. The main focus includes a comparison between the U-form (unitary form) and M-form (multidivisional form), the principal-agent problem between headquarters and division managers, transfer pricing and tax minimization.

The M-form provides many advantages. Firstly, firms can be more productive by diversifying under the assumption of neoclassical diminishing returns within the industry (Maksimovic and Phillips 2002). Secondly, a large company has higher brand value and spillover effects that can be shared across divisions. Thirdly, division managers can more efficiently handle the day-to-day operations of their own divisions. Fourthly, even if some units of the firm fail, the other divisions can still be productive and profitable, which guarantees a more versatile, less risky, and more enduring organization. Fifthly, diversified firms can allocate capital better than market with
superior inside information (Williamson 1975). Lastly, the headquarters can expand highly productive divisions and abandon low producing divisions more effectively than the market (Stein 1997). To sum up, the M-form combines the advantages of a distinct brand and economies of scale for a large conglomerate, while maintaining the operational flexibility of a small firm.

The M-form also has some disadvantages. The headquarters of multidivisional firms may not be able to manage the various different businesses as effectively as single-division firms. In addition to coordination problems, conglomerates suffer from agency problems, wherein division managers may report incorrect information to maximize the profit division-wide, rather than firm-wide. Bureaucracy and a lack of managerial focus are also frequent problems. Moreover, a lack of transparency, such as the black box of input allocations across divisions that this study discusses, also discourages investors. As a result, the stock market may value a diversified group of businesses and assets at less than the sum of its parts, which is referred to as the “conglomerate discount.” Many scholars (Lang and Stulz 1994; Berger and Ofek 1995; Maksimovic and Phillips 2002) find that multidivisional firms have lower productivity levels than single-division firms.

It is important to clarify the definition and terminology in regard to multiproduct and multidivisional firms. Multiproduct firms are firms that produce multiple goods and face the problem of how to allocate inputs in each product line. Multidivisional firms are those that have multiple semi-autonomous divisions and face the problem of how to allocate inputs in each division. On the one hand, multiproduct firms take either a unitary form (e.g., a small farm that produces vegetables and fruits) or a multidivisional form (e.g., General Electric operates through multiple divisions: power & water, oil & gas, aviation, healthcare, transportation, capital, and energy management). On the other hand, a multidivisional firm can be adopted by either
single-product firms (e.g., an electricity company owning several power plants) or multiproduct firms (e.g., the GE case mentioned above).

Since the multidivisional form became popular in the 1960s, more and more studies focus on these organizational structures, especially for multiproduct firms. Williamson (1975) theorizes the information-processing advantages that a multiproduct firm can achieve by deploying a multidivisional form. Teece (1981) concludes that large multiproduct firms that adopted a multidivisional form always perform better than those choosing a unitary form. Many studies (Teece 1998; McEwin 2011) call these multidivisional multiproduct firms. Although more and more large firms are both multidivisional and multiproduct, some differences between the two were revealed in previous studies.

On the one hand, more multidivisional firm literature can be found in finance and management journals. These studies often discuss the principal-agent problem (Ross 1973; Hölmstrom 1979, 1982; Grossman and Hart 1983), since divisional managers are responsible for their own profit maximization, which may conflict with firm-level profit maximization. On the other hand, more multiproduct firm papers are being written by economists. These research studies center on product choices: how to concentrate on the most productive goods and drop the least productive goods in global markets (Baldwin, Caves, and Gu 2005; Bernard, Redding, and Schott 2011) and within national markets (Broda and Weinstein 2010; Bernard, Redding, and Schott 2010).

In recent years, both multidivisional and multiproduct studies began to consider the division-specific or product-specific production functions, rather than a unique production function at the firm-level. Moreover, as principal-agent problems have been solved, more recent researches (Maksimovic and Phillips 2002) study multidivisional firms in the absence of an agency problem. As a result of these two assumptions,
the difference between multidivisional firms and multiproduct firms are minimized, which reflects the fact that most conglomerates presently have both multiproduct and multidivisional characteristics. These firms set each product line as a division and let division managers make operational decisions. This study adopts these assumptions in order to better study conglomerates in the real world.

2.2.2 General Model

Suppose companies can have footprints in one or multiple segments in a “$M$ Inputs – $N$ Outputs/Segments – $T$ periods” economy. This subsection presents production functions to estimate productivity and efficiency, where stochastic frontier analysis\(^1\) is used as the production method. Therefore, the classic model, where one production frontier applies to all the firms, is denoted as “Single-Frontier” approach, while the new model, where segment-specific production frontiers exist, is denoted as “Meta-Frontier” approach.

2.2.2.1 Single-Frontier Analysis

The canonical stochastic frontier analysis for panel data, or what this study calls the “Single-Frontier” estimator, runs the regression for individual (country, region, company, etc.) $i$ at time $t$:

$$Y_{it} = f(X_{it}; \beta_0) \exp(\tau Z) \exp(\nu_{it}) \exp(-u_{it})$$  

where $Y_{it}$ is the aggregated output of individual $i$ at time $t$; $X_{it} = (X_{i1t}, X_{i2t}, ..., X_{iMt})$ vectors the $M$ types of inputs; $f(X_{it}; \beta_0) \cdot \exp(\tau Z)$ is the production frontier over time, where $f(X_{it}; \beta_0)$ is the time-invariant part of the production function, $\beta_0 =$

\(^1\) Stochastic frontier analysis (SFA) is a method of economic modeling widely used in productivity and efficiency analysis. See Kumbhakar and Lovell (2003) and Coelli et al. (2005) for more information about this method.
\((\beta_01, \beta_02, \ldots, \beta_{0M})\) is a vector of technical parameters to be estimated. \(Z\) vectors a group of year dummy variables, controls the production frontier change over time and \(\tau\) vectors the coefficients of the year dummy variables; \(\exp(\nu_{it})\) is the stochastic component that describes random shocks affecting the production process, where \(\nu_{it}\) is assumed to be normally distributed with a mean of zero and a standard deviation of \(\sigma_{\nu}\); and \(TE_{it} = \exp(-\mu_{it})\) denotes the time-variant technical efficiency defined as the ratio of observed output to maximum feasible output. \(TE_{it} = 1\) or \(\mu_{it} = 0\) shows that the \(i\)-th individual allocates at the production frontier and obtains the maximum feasible output at time \(t\), while \(TE_{it} < 1\) or \(\nu_{it} > 0\) provides a measure of the shortfall of the observed output from the maximum feasible output. This study uses the error components specification with time-varying efficiencies (Battese and Coelli 1992), where \(\mu_{it} = \exp(-\eta(t - T)) \times \mu_i\). To sum up, the input coefficients \(\beta_0\) of the production function in Eq. (2.1) are time-invariant, while the firm-specific intercept term is shifted by a common time varying component. This study will test if the coefficients \(\beta_0\) of the production function also change over time in the empirical analysis.

### 2.2.2.2 Meta-Frontier Analysis

As has been discussed, Eq. (2.1) assumes a unique production function across segments and can only derive firm-level efficiency without segment-specific production function concern. The “Meta-Frontier” analysis, however, assumes production frontiers are segment-specific, as different segment requires different technologies. In a “\(M\) Inputs – \(N\) Products/Segments – \(T\) periods” economy, each equation in the system of \(N\) equations (2.2) describes the production techniques for the corresponding
segment.

\[
\begin{cases}
Y_{i1t} = f_1(X_{i1t}; \beta_1) \exp(\tau_1 Z_1) \exp(\nu_{i1t}) \exp(-u_{i1t}) \\
\vdots \\
Y_{iNt} = f_N(X_{iNt}; \beta_N) \exp(\tau_N Z_N) \exp(\nu_{iNt}) \exp(-u_{iNt})
\end{cases}
\]  
(2.2)

where \( Y_{ijt} \) represents the observed scalar output and \( X_{ijt} \) vectors the unobserved inputs of individual \( i \) in segment \( j \) at time \( t \), respectively. \( f_j(X_{ijt}; \beta_j) \) is the heterogeneous production frontier for segment \( j \), where \( \beta_j = (\beta_{j1}, \beta_{j2}, \ldots) \) is a vector of segment-specific technical parameters; \( Z_j = (Z_{j1}, Z_{j2}, \ldots, Z_{jT}) \) vectors a group of year dummy variables to control the production frontier change across time and \( \tau_j = (\tau_{j1}, \tau_{j2}, \ldots, \tau_{jT}) \) vectors the coefficients of the year dummy variables; and \( u_{ijt} = \exp(-\eta(t-T)) \ast \mu_{ij} \) is the time-variant efficiency indicator. \( \nu_{ijt} \) is the noise that is considered normally distributed with a mean of zero and a standard deviation of \( \sigma_{\nu_j} \). The data pooled into the \( j \)th equation in Eq. (2.2) depends on the validity of the key assumption. If assuming that single-division firms and multidivisional firms have the same production techniques, all the players in segment \( j \) are included in the regression. Otherwise, the single-division firms are removed from the regression.

Using stochastic frontier analysis at divisional level, the “Meta-Frontier” approach can predict division-level efficiency for multidivisional firms (\( \hat{TE}_{ijt} = e^{-\mu_{ijt}} \)) and firm-level efficiency for single-division firms. The firm-level efficiency for the multidivisional firm \( i \) at time \( t \), \( \hat{TE}_{it} \), is the weighted average of its efficiency in each division.

\[
\hat{TE}_{it} = \sum_j \left[ \frac{R_{ijt}}{R_{it}} \hat{TE}_{ijt} \right],
\]

where \( R_{it} \) is the firm-level revenue for firm \( i \) at time \( t \) and \( R_{ijt} \) is the division-level revenue for firm \( i \) in division \( j \) at time \( t \).
Compared with “Single-Frontier”, “Meta-Frontier” approach considers the heterogeneity in production frontiers across segments and can derive divisional level efficiency. However, the unobserved input allocations at divisional level need to be imputed. The next three subsections present the assumptions, the methodology, and the accuracy test of the new imputation approach, respectively.

2.2.3 Assumptions

Before building the model to impute input allocations across divisions, some assumptions that apply in this study must be introduced.

Assumption 1. The multidivisional firms are profit-maximizing at firm-level and have undistorted input allocations in the absence of agency problems.

Each semiautonomous division has its own unitary structures and managers are responsible for their own business, which may be contrary to the goal of the entire company. This principal-agent problem in multidivisional firms has been studied by many economists since Ross (1973) and Hölstrom (1979). Hölstrom (1982) and Grossman and Hart (1983) solve the moral hazard of the division managers by designing a compensation scheme so that the semiautonomous divisions would not deviate from the equilibrium. Since the principal-agent problem was well studied in earlier years, Maksimovic and Phillips (2002) analyzes optimal firm size, growth, and resource allocation by using a profit-maximizing neoclassical model in the absence of an agency problem. This study follows their ideas by assuming no limit on the inputs one company can employ, indicating that the firm-level profit maximization problem is the first best and divisions do not have to compete for inputs. Then, division managers are willing to truthfully report their types and the profit-maximizing headquarters therefore knows the profit function for each division. The empirical results in Maksimovic and Phillips (2002) show that the majority of conglomerates’ actual
resource allocation is generally consistent with the undistorted/optimal allocation of resources across divisions to maximize profit. Therefore, the undistorted input allocations for profit-maximizing multidivisional firms are imputed to proxy the actual input allocations.

**Assumption 2.** *Spillover effects on input prices are division-invariant functions of firm size measured by input costs.* This assumption can be divided into the following three subassumptions: (i) Input costs can measure firm size, (ii) Firm size can affect input prices in each division, and (iii) The effect of firm size on input prices is similar across divisions.

Input costs can measure firm size. One of the advantages of the multidivisional form is its size effect due to knowledge spillover and economy of scale. Shalit and Sankar (1977) list the five most commonly used measures of firm size, which consist of total dollar annual sales, total assets, number of employees, equity stock, and market value of the firm. This study uses labor and capital cost as the measurement of firm size so that both the number of employees and total assets are taken into account.

Firm size can affect input prices in each division. Sönmez (2013) presents a literature survey on firm survival and concludes that the survival probability of a firm increases with firm size. On the one hand, Burdett and Mortensen (1998) and Coles (2001) discuss how firms trade off higher wages against a lower quit rate when they expect a higher likelihood of survival. They also find that in equilibrium, large firms with many employees pay high wages, while small firms with few employees pay low wages. On the other hand, many studies have found that larger firms pay less for capital (Botosan 1997; Petersen and Rajan 1994; Poshakwale and Courtis 2005; Gebhardt, Lee, and Swaminathan 2001; Reverte 2012), since large firms have a higher survival probability and lower risk. As a result, larger firms are more capital-intensive than smaller ones due to factor prices differing by firm size (Söderbom and Teal 2004).
Based on these studies, the price of capital and the price of labor are affected by firm size.

The effect of firm size on input prices is similar across divisions. Duchin, Goldberg, and Sosyura (2015) point out that the spillovers (within-firm peer effects) in compensation and capital expenditure are equally strong across related and unrelated divisions within a firm. If the effect is not equal across segments, less-benefiting divisional managers will lobby and negotiate with the headquarters. Levin (2002) presents a model where the actions of the firm toward a group of employees affect the expectations of everyone else, which implies the importance of wage equity across divisions. Therefore, multidivisional firms would pay the same premium/discount based on the segment average compensation for employees to maintain a similar quit rate across divisions. The capital prices across divisions within the same multidivisional firm reflect the same company risk above the segment-specific systematic risk. Therefore, the capital price has the same premium/discount (relates to the risk of the M-firm) based on segment average price (relates to systematic risk of the segment). This study assumes an equal effect of firm size across divisions. That is, the premium/discount on input prices due to spillover effects is equal across divisions.

**Assumption 3.** *Capital is chosen prior to the realization of the production, while other inputs are chosen at the time production takes place.*

This assumption specifies the timing of input choices, which is consistent with classic productivity frameworks. In the framework of Olley and Pakes (1996), the labor used in time $t$ is chosen when the productivity shock at time $t$ is observed, while the capital used in time $t$ is chosen at time $t-1$ because $K_t = (1 - \delta_{t-1}) K_{t-1} + I_{t-1}$, where $K_t$, $\delta_t$, and $I_t$ are capital, depreciation rate and investment, respectively, at time $t$. Ackerberg, Caves, and Frazer (2006) describe those inputs that are chosen at the time production takes place as “perfectly variable” inputs. In Levinsohn and Petrin
(2003), labor and intermediate inputs are both “perfectly variable” inputs, while capital is not. This study allows inputs other than capital to be “perfectly variable”. The timing to make input decisions decides the information that is available to expect profit. Moreover, this study assumes that firms predict the productivity in the next period equal to a log-normal distribution conditional on the realized values in the current period. The same rule applies to prices and inputs.

2.2.4 Imputation Algorithm

Notationally let:

- \( R_{ijt} \) be the division-level revenue for firm \( i \) in division \( j \) at time \( t \);
- \( I_{it} \) be the information set for firm \( i \) at time \( t \);
- \( E[R_{ijt}|I_{it-1}] \) be the expected revenue for firm \( i \) in division \( j \) at time \( t \) given the information set at time \( t - 1 \);
- \( X_{ijt}^k \) be unobserved \( k \)-th input of firm \( i \) in division \( j \) at time \( t \) for all \( k < M \), and the last input, \( X_{ijt}^M \), be the division-level capital input;
- \( p_{jt} \) be the average price of the output in segment \( j \) at time \( t \);
- \( c_{jt}^k \) be the average price of the \( k \)-th input in segment \( j \) at time \( t \);
- \( r_{jj'}^k \) be the ratio of average price for the \( k \)-th input between segments \( j \) and \( j' \) at time \( t \);
- \( W(i,t) \) be a subset of \( W = (1, 2, ..., N) \), indicating the segments where firm \( i \) at time \( t \) has a footprint;
- \( \beta_j = (\beta_{j1}, \beta_{j2}, ...) \) be a vector of technical parameters in segment \( j \)'s production function.
• \( \nu_{ijt} \sim N(0, \sigma_{\nu j}^2) \) be the white noise in segment \( j \)'s production function;

• \( \tau_{jt} \) be the coefficient of the time \( t \)'s dummy variable in segment \( j \)'s production function;

• \( A_{ijt} = \exp(\alpha_j + \tau_{jt} - \mu_{ijt}) \) be the (total-factor) productivity in segment \( j \)'s production function;

• \( \delta_{jkl} \) be the coefficient of the intersection between the \( k \)-th input and \( l \)-th input in segment \( j \)'s production function if the function takes Transcendental Logarithmic form;

• \( \log(x_{ijt})|\log(x_{ijt}) \sim N(\log(x_{ijt}) + \mu_{xj}, \sigma_{xj}^2) \) for all \( x = p, A, X^k \), which is the conditional log-normal distribution mentioned in Assumption 3.

Theorem 1. Under Assumption 1 - 3, the undistorted input allocations in multidivisional firms satisfy: (i)

\[
\begin{align*}
\frac{\partial E[R_{ijt}|I_{it}]}{\partial X_{ijt}^k} &= \frac{c_j^k}{c_j^{t'}} = r_{j_jt}^k, \forall i, t, \forall k < M, \forall j, j' \in W(i, t) \\
\frac{\partial E[R_{ijt}|I_{i(t-1)}}{\partial X_{ijt}^k} &= \frac{c_j^{M-1}}{c_j^{t'-1}} = r_{j_j't-1}^M, \forall i, t, \forall j, j' \in W(i, t) \\
\sum_{j \in W(i,t)} X_{ijt}^k &= X_{it}^k, \forall i, k, t
\end{align*}
\]

(ii) When the production function has a Cobb-Douglas form, (i) derives

\[
\begin{align*}
X_{ijt}^k &= \frac{\beta_{jk} R_{ijt}}{\sum_{j' \in W(i,t)} \beta_{j'k} R_{ijt} r_{j_j't}^k \exp\left(\nu_{ijt} - \nu_{ij't} + 0.5\sigma_{\nu j'}^2 - 0.5\sigma_{\nu j}^2\right) X_{it}^k}, \forall k < M \\
X_{ijt}^M &= \frac{\beta_{jM} R_{ijt}}{\sum_{j' \in W(i,t)} \beta_{j'M} R_{ijt} r_{j_j't}^M (\gamma_{j'/\gamma_j}) (z_{ij't}/z_{ijt}) \exp\left(\nu_{ijt} - \nu_{ij't} + 0.5\sigma_{\nu j'}^2 - 0.5\sigma_{\nu j}^2\right) X_{it}^M}
\end{align*}
\]
where
\[ z_{ijt} = \frac{p_{jt-1} A_{ijt-1}}{p_{jt} A_{ijt}} \prod_{k=1}^{M-1} \left( \frac{X_{ijt-1}^k}{X_{ijt}^k} \right)^{\beta_{jk}} \]
\[ \gamma_j = \exp \left( \mu_{pj} + \mu_{Aj} + \sum_{k=1}^{M-1} \beta_{jk} \mu X_{kij} + \frac{\sigma_{pj}^2 + \sigma_{Aj}^2 + \sum_{k=1}^{M-1} \beta_{jk}^2 \sigma_{Xk}^2}{2} \right) \]

(iii) When the production function has a Transcendental Logarithmic form, (i) derives

\[
\left\{
\begin{array}{l}
\frac{\partial E[R_{ijt} | I_{it}]}{\partial X_{ijt}^k} = (\beta_{jk} + \sum_{l=1}^M \delta_{jkl} \ln X_{ijt}^l) R_{ijt} X_{ijt}^k \exp \left( \nu_{ijt} + 0.5 \sigma_{vij}^2 \right) \\
\frac{\partial E[R_{ij't} | I_{it-1}]}{\partial X_{ij't}^k} = z_{ijt}(\beta_{j'k} + \sum_{l=1}^M \delta_{j'kl} \ln X_{ij't}^l) R_{ij't} X_{ij't}^k \exp \left( \nu_{ij't} + 0.5 \sigma_{vij'}^2 \right) \gamma_j \\
\frac{\partial E[R_{ij't} | I_{it-1}]}{\partial X_{ij't}^k} = z_{ij't}(\beta_{j'k} + \sum_{l=1}^M \delta_{j'kl} \ln X_{ij't}^l) R_{ij't} X_{ij't}^k \exp \left( \nu_{ij't} + 0.5 \sigma_{vij'}^2 \right) \gamma_j' \\
\sum_{j \in W_{i,t}} X_{ijt}^k = X_{it}^k, \quad \forall i, k, t
\end{array}
\right.
\]

Applying Eqs. (2.3) - (2.5) are derived from the first order condition of profit maximization problem and can be solved jointly with the segment-specific production functions using an iterative method. Finally, the iterative method can impute undistorted input allocations, which can be applied in the “Meta-Frontier” approach (through Eq. (2.2)) to estimate the efficiency at divisional level.

### 2.2.5 Accuracy Test of the Imputation

This study tests whether the new imputation method can derive accurate estimations of input allocations using a panel data for OECD countries where actual input allocations are observed. The result shows that the new imputation method can always
derive input allocations closer to the actual allocations than the other three competing imputation methods. Moreover, the iteration converges to the same point using different initial guesses. See details of the accuracy study in Appendix A.2.

2.2.6 Endogeneity Problem

Olley and Pakes (1996) point out that the restricting of an industry involved significant changes in input quantities, which generates a simultaneity problem when estimating production functions. Endogeneity arises because input choices are determined by some factors that are unobserved to the econometrician, but observed by the firm (Ackerberg, Caves, and Frazer 2006), such as the firm’s beliefs about its productivity and efficiency. This problem is more pronounced for inputs that adjust rapidly (Marschak and Andrews 1944), such as the oilfield market where the decisions of the companies depend heavily on exploration and production (E&P) spending from the oil and gas firms (affected by oil price) and the business cycles. Many companies divest capital and cut headcount aggressively when the oil price goes down. Therefore, the inputs may be endogenous in the production function, which leads to biased OLS estimates.

One of the solutions to an endogeneity problem is a fixed-effects estimation, which requires the assumption that the efficiency terms are constant across time. Therefore, this method cannot derive time-variant efficiency. Another set of two-step techniques, advocated by Olley and Pakes (1996), is to use observed investment to “control” for unobserved productivity shocks (efficiency). Levinsohn and Petrin (2003) extend the idea by using intermediate inputs instead of investment to solve the simultaneity issue. They claim the benefit is strictly data-driven, since many datasets have significant amounts of observations with zero investment, which makes investment an invalid proxy. However, as Ackerberg, Caves, and Frazer (2006) note, both of the models
suffer from the collinearity problems in the first step, such that the coefficients of the exogenous inputs cannot be identified. Moreover, intermediate data cannot be found for firms in the oilfield market.

The third approach, which is also the most widely used method to solve the endogeneity problem, is instrumental variables (IV) estimation. Amsler, Prokhorov, and Schmidt (2015) review Two-Stage Least Square (2SLS) and applied a Corrected 2SLS (C2SLS) to solve the endogeneity problem in stochastic frontier analysis when the production is linear, such as Cobb-Douglas (C-D) production function. For C2SLS, the first step is to estimate the model by 2SLS and derive the residuals using the instruments. In the second step, these 2SLS residuals are decomposed using the maximum likelihood method, just as in classic stochastic frontier analysis. A somewhat similar two-step procedure is built by Guan et al. (2009). For a Transcendental Logarithmic (T-L) production function, however, Amsler, Prokhorov, and Schmidt (2015) suggest that the control function method is more efficient than the C2SLS method. Moreover, they explain how to reduce the number of instrument variables needed, while still getting consistent estimators in the control function method.²

This study uses the C2SLS method for the linear C-D production function and the control function method for the nonlinear T-L production; both are recommended in Amsler, Prokhorov, and Schmidt (2015). The control function method can also test the exogeneity of the inputs using t-tests for the significance of the reduced form residuals. For the selection of instrument variables, Levinsohn and Petrin (2003) mention that the potential instruments include input prices and lagged values of input use. Lagged values of inputs are valid instruments if the lag time is long enough to break

² For example, suppose two inputs, labor and capital, are both endogenous. At least five instruments are needed since all of the two inputs, their square terms, and their intersection are endogenous in the T-L production function. However, under some additional assumptions, consistent estimators can be obtained using only two control functions, not five. This point has been made by some economists, including Blundell and Powell (2004), Terza, Basu, and Rathouz (2008) and Wooldridge (2010). See detailed discussion in Amsler, Prokhorov, and Schmidt (2015)
the dependence between the input choices and the serially correlated shock. Blundell and Bond (2000) and Guan et al. (2009) both emphasize the input levels lagged at least two periods can be valid instruments. This study chooses input price and lag two input quantities\(^3\) to be IVs.

### 2.3 Empirical Study of the Oilfield Market

This section imputes the undistorted input allocations and does “Meta-Frontier” analysis for multidivisional firms in the global oilfield market, where five segments exist.

#### 2.3.1 The Oilfield Market

Most literature concerning stochastic frontier analysis assumes a unique production frontier for all the firms in an industry or an economy. This default assumption is not realistic in some industries with different segments, as each has different players and technologies. The oilfield service industry, which is studied in this section, is a great example of such an industry.

The oilfield market, or oil and gas exploration and service, is a complex process that involves specialized technology at each step of the oil and gas supply chain. Most oil and gas companies, even vertically integrated giants like Chevron and Exxon Mobil, choose to rent or buy part of the necessary equipment from oilfield services firms. Companies in the oilfield market provide the infrastructure, equipment, intellectual property, and services needed by the international oil and gas industry to explore for and extract crude oil and natural gas, and then transport it from the earth to the refinery, and eventually to the consumer. Therefore, this industry has many diverse

\(^3\) The estimation results using lag two and lag three input quantities are robust in the empirical study.
product lines.

Firms in this industry have a total market capitalization of over $4 trillion USD, generating total revenues over $400 billion USD in 2014. The significant development of new technologies, such as hydraulic fracturing and offshore drilling, has resulted in a 10% compound annual growth rate (CAGR) over the past decade. As conventional oil and gas resources are now being exhausted, oil and gas companies are currently paying more attention to unconventional oil and gas, offshore production, and aging reservoirs to maintain a steady supply. The higher exploration and production (E&P) spending of the oil and gas companies translates to higher revenues for the oilfield service companies, which guarantee the dramatic growth and bright future of the oilfield market.

The Oilfield Market Report (OMR) by Spears divides the oilfield industry into five segments: 1) exploration, 2) drilling, 3) completion, 4) production, and 5) capital equipment, downhole tools and offshore services (capital equipment, hereafter). OMR reports segment-level revenue for the 114 public firms in the field: 68 firms are single-division and 56 firms are multidivisional (28 firms do business in two segments, 10 firms are active in three segments, seven firms have footprints in four segments, and only one firm covers all five segments). There are four diversified oilfield firms (the “Big Four”): Baker Hughes,\(^4\) Halliburton, Schlumberger and Weatherford.

### 2.3.2 Oilfield Data

This study applies the present model in the oilfield market using deflated revenue as the output, number of employees as the first input, and capital as the second input. This study assumes different production functions for the five segments and builds a

\(^4\) Halliburton agreed to acquire Baker Hughes in November, 2014 and the merger is still being processed in 2015. Since the two firms were separate entities from 1997-2014, this study still regards them as two individual firms.
“2 Inputs – 5 Products/Segments – 18 Periods” industry. Division-level revenue data from 1997 to 2014 for each of the 114 public firms are collected from the three waves of the OMR (2000, 2011, and 2015) dataset. Appendix A.3 introduces this report, the method employed to combine the three waves of data, and the detailed segmentation of the oilfield market. The Bureau of Labor Statistics publishes the Producer Price Index (PPI) by North American Industry Classification System (NAICS) division. The average output price index for each of the five segments, as well as the overall oilfield market, can thus be calculated. The output price indices deflate the revenue, which can be regarded as output.

Data on the annual overall revenue, the number of employees, and total capital for the 114 public firms in during same period is collected from Thomson ONE, Bloomberg, and FactSet. The total capital is the accounting capital, which is the sum of equity and long-term debt. This study adjusts the capital data following the approach of Berlemann and Wesselhöft (2014), which uses a unified perpetual inventory method (PIM). Appendix A.4 explains the data-generating process. Since the labor and capital data are the year-end values, the values at periods $t$ and $t+1$ are averaged to get the average value at period $t+1$. The overall revenue of a firm is not always equal to the total revenue in the oilfield market as reported by the OMR. In some cases, the former may be larger because the company has some business outside the oilfield market. On the other hand, the former could be smaller, as the OMR adds the acquired firm’s revenue to the mother firm’s revenue even in the years before acquisition. The input proportionality assumption suggested by Foster, Haltiwanger, and Syverson (2008) is used to adjust the labor and capital used in the oilfield market.

Besides the input and output quantity data, this study also collects input and output prices data. The capital price is the sum of the depreciation rate and interest rate. The depreciation rate is calculated using the depreciation and capital data from
Thomson ONE, Bloomberg, and FactSet. Using the firm-level beta\(^5\), the risk-free rate, and the expected market return from the same database, the firm-level interest rate can be estimated with a capital asset pricing model (CAPM). For labor price, many international firms have compensation cost information, but North American firms have no such information published. The Bureau of Labor Statistics has average compensation per employee figures for each NAICS division in the Labor Productivity and Cost (LPC) Database. The labor prices of those firms without wage information are set equal to the corresponding NAICS division average. This study calculates the average labor price and capital price in each segment.

Table 2.1 summarizes the input and output in the oilfield market and each of the five segments. Firm-level labor and capital are observed, but segment-level labor and capital are not. The average revenues in drilling and completion are significantly higher than those of other segments. The exploration and drilling segments have a high average wage while the capital equipment segment has a low average wage. Moreover, the capital price in exploration is relatively high when compared to the other segments.

\[2.3.3 \quad \text{Empirical Results}\]

This study imputes the input allocations assuming that the production function takes C-D and T-L forms, respectively. The stopping criterion \(c=1\times10^{-6}\) was attained after eight iterations in the C-D case and after eleven iterations in the T-L case. This subsection reports the results for the C-D case and Appendix A.5 provides the corresponding results for the T-L case. Overall, the findings are consistent in the C-D case and T-L case.

\(^5\) In finance, the beta of an investment or a company is a measure of the risk arising from exposure to general market movements as opposed to idiosyncratic factors. The market portfolio of all investable assets has a beta of unity.
Table 2.1: Oilfield Market Summary Statistics

<table>
<thead>
<tr>
<th>Unit</th>
<th>Value Added $1 \cdot 10^9</th>
<th>Output Price Index</th>
<th>Labor Price of $1,000</th>
<th>Capital Price of $1 \cdot 10^9</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oilfield Industry</td>
<td>1.7</td>
<td>2.0</td>
<td>6.1</td>
<td>76.0</td>
<td>1.4</td>
</tr>
<tr>
<td>I) Exploration</td>
<td>0.6</td>
<td>1.7</td>
<td>–</td>
<td>80.1</td>
<td>–</td>
</tr>
<tr>
<td>II) Drilling</td>
<td>1.4</td>
<td>2.4</td>
<td>–</td>
<td>79.8</td>
<td>–</td>
</tr>
<tr>
<td>III) Completion</td>
<td>1.2</td>
<td>2.1</td>
<td>–</td>
<td>75.3</td>
<td>–</td>
</tr>
<tr>
<td>IV) Production</td>
<td>0.5</td>
<td>2.1</td>
<td>–</td>
<td>76.1</td>
<td>–</td>
</tr>
<tr>
<td>V) Capital Equip</td>
<td>0.8</td>
<td>1.9</td>
<td>–</td>
<td>68.3</td>
<td>–</td>
</tr>
</tbody>
</table>

2.3.3.1 Estimations of Production Functions

Table 2.2 presents the estimation results of the “Single-Frontier” approach in Eq. (2.1) and “Meta-Frontier” approach in Eq. (2.2) on the oilfield market data.

The first two columns of Table 2.2 report both the MLE estimation (assuming inputs are exogenous) and the C2SLS estimation (assuming inputs may not be exogenous) of the total oilfield industry’s production frontier, respectively. The method discussed by Amsler et al. (2015) is followed to test the exogeneity of labor and capital. The capital is assumed to be exogenous, while labor is allowed to be endogenous, which is consistent with the literature (Levinsohn and Petrin, 2003; Olley and Pakes, 1996). Griliches and Mairesse (1998) remark that fixed effect estimators have frequently led researchers to find point estimates for the capital coefficient that are very low and often not significantly different from zero. This problem appears to have been addressed as the coefficient on capital in the C2SLS model (0.23 with a t-value of 19.2) is both statistically and economically more significant than the estimation without considering endogeneity (0.13 with a t-value of 8.6). Moreover, the sum of
Table 2.2: Estimate of Cobb-Douglas Production Function

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Entire Industry</td>
<td>Exploration Industry</td>
<td>Drilling Industry</td>
<td>Completion Industry</td>
<td>Production Industry</td>
<td>Capital Equipment</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(w/o IV)</td>
<td>(w/ IV)</td>
<td>(w/ IV)</td>
<td>(w/ IV)</td>
<td>(w/ IV)</td>
<td>(w/ IV)</td>
<td>(w/ IV)</td>
<td>(w/ IV)</td>
</tr>
<tr>
<td>lnL</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Both</td>
<td>.579***</td>
<td>.726***</td>
<td>.647***</td>
<td>.804***</td>
<td>.944***</td>
<td>.519***</td>
<td>.664***</td>
<td>.916***</td>
</tr>
<tr>
<td></td>
<td>(.016)</td>
<td>(.013)</td>
<td>(.037)</td>
<td>(.022)</td>
<td>(.025)</td>
<td>(.027)</td>
<td>(.032)</td>
<td>(.032)</td>
</tr>
<tr>
<td>lnK</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Both</td>
<td>.134***</td>
<td>.226***</td>
<td>.283***</td>
<td>.185***</td>
<td>.072***</td>
<td>.309***</td>
<td>.187***</td>
<td>.173***</td>
</tr>
<tr>
<td></td>
<td>(.016)</td>
<td>(.011)</td>
<td>(.03)</td>
<td>(.019)</td>
<td>(.024)</td>
<td>(.023)</td>
<td>(.027)</td>
<td>(.036)</td>
</tr>
<tr>
<td>time dummy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Both</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Intercept</td>
<td>1.62***</td>
<td>-1.28***</td>
<td>-.721***</td>
<td>-1.7***</td>
<td>-2.24***</td>
<td>-.137</td>
<td>-.672***</td>
<td>-2.17***</td>
</tr>
<tr>
<td></td>
<td>(.170)</td>
<td>(.095)</td>
<td>(.269)</td>
<td>(.136)</td>
<td>(.164)</td>
<td>(.192)</td>
<td>(.228)</td>
<td>(.222)</td>
</tr>
<tr>
<td># of firms</td>
<td>114</td>
<td>113</td>
<td>18</td>
<td>53</td>
<td>44</td>
<td>28</td>
<td>22</td>
<td>22</td>
</tr>
<tr>
<td># of obs.</td>
<td>1525</td>
<td>1298</td>
<td>218</td>
<td>606</td>
<td>493</td>
<td>325</td>
<td>257</td>
<td>285</td>
</tr>
<tr>
<td>Wald test</td>
<td>353</td>
<td>31</td>
<td>6.4</td>
<td>.83</td>
<td>1.4</td>
<td>85.3</td>
<td>48.9</td>
<td>16.4</td>
</tr>
</tbody>
</table>

Notes: Significant at: *10, **5 and ***1 percent; Standard error in parentheses.

the output elasticities of the labor and capital factor inputs is closer to unity (0.96 vs. 0.71) in the C2SLS estimation than for the MLE estimation, indicating a more constant returns to scale (CRS) technology when endogeneity is considered.

This study first uses the “Meta-Frontier” approach to impute input allocations at the divisional level and estimate segment-specific production functions, assuming that both single- and multidivisional firms have the same production techniques within a segment. Then, the validity of this assumption is tested for each regression and it is found that the production techniques for single- and multidivisional firms are significantly different in the capital equipment segment, which indicates that the key assumption previous studies rely heavily on is not always valid. Therefore, the
observation from single-division firms cannot solely be used to estimate the production function that multidivisional firms also follow.

Since the production techniques are different for single- and multidivisional firms in the capital equipment segment, this study drops the observations of single-division firms in that segment from the production function and reruns the iterations to reach the undistorted input allocations. Columns 3 - 6 of Table 2.2 list the estimated production frontiers for each of the first four segments, and Columns 7 and 8 show the estimated production frontiers in the capital equipment segment for single- and multidivisional firms, respectively.

The production frontiers are different across segments. Compared with the drilling, completion, and capital equipment segments, the exploration and production segments are relatively more capital-intensive. This finding implies that the constant share in the input portfolio across segments assumed in De Loecker et al. (2015) is not a valid assumption.

The Wald test in Table 2.2 shows the returns to scale: the exploration, drilling, and completion are constant returns to scale (CRS), the production segment and single-division firms in the capital equipment segment are decreasing returns to scale (DRS), and multidivisional firms in the capital equipment segment are increasing returns to scale (IRS).

Table 2.2 also presents the average efficiency for the oilfield market and each segment. The top and bottom 5% of the estimated efficiencies are dropped to eliminate possible outliers. After controlling the endogeneity problem, the average efficiency for the oilfield market as a whole is 0.541. For segment-level average efficiency, the exploration segment (0.719) and the multidivisional firms in the capital equipment segment (0.673) are the highest, followed by the production (0.659) and drilling (0.633) segments, while the completion segment (0.606) and the single division firms in the
capital equipment segment (0.610) are the lowest.

This study tests if the coefficients of inputs in the production frontier are time-invariant by adding the intersections between each year dummy variable and each input into the production equations. All the coefficients of the intersections are insignificantly different from zero, which supports \( f(X_t; \beta_0) \) in Eq. (2.1) is time-invariant as is modeled. Results for each year are not reported here and are available on request.

The estimators in Table 2.2 make it possible to draw 3D images of the production frontiers. Figure 2.1 plots the production frontiers for the oilfield market. The left is the MLE estimation and the right is the C2SLS estimation. These 3D images visualize the production-labor-capital relation. It is obvious that the elasticity of capital in the C2SLS estimation is larger than that in the MLE estimation.

![Figure 2.1: The Estimated Cobb-Douglas Production Function](image)
2.3.3.2 Investment Decision and Efficiency

In the oilfield market, mergers, acquisitions, investment and divestment are common ways to change the share of revenue by segment for multidivisional firms. These companies take such strategic actions frequently in response to market volatility and to compete with peers. Compared with firms only generating revenue in one segment, multidivisional firms can improve their overall efficiencies not only by improving division-level efficiency, but also by transferring more resources and business from less efficient segments to more efficient ones. Suppose the revenues of a firm in segments I and II were half-and-half, while its efficiency in the two segments was 0.1 and 0.7, respectively. After a decade, this firm improved efficiency by 0.1 in each segment. During the same period, this firm divested in segment I and invested in segment II, so that the share of revenue by segment changed from 1:1 to 1:3 in segments I and II. Therefore, the aggregate efficiency for this firm changed from 0.4 to 0.65 in ten years. The improved efficiency in each segment contributes to increase of 0.1, while the change in business portfolio contributes to increase of 0.15 in overall efficiency. Table 2.3 checks if the multidivisional firms in the oilfield market invested more in higher efficient divisions than in lower efficient divisions. For each multidivisional firm, the change is calculated in the share of revenue by segment between the first year and last year in the dataset. Two-divisional firms, on average, transfer 7.5% of their business from the low-efficiency division to the high-efficiency division. This transfer from the low- to high-efficiency division is not that significant for firms that are present in more than two segments. Overall, multidivisional firms transferred about 4% of their business from low- to high-efficiency divisions.
Table 2.3: Change in Share of Revenue by Segment for Multidivisional Firms

<table>
<thead>
<tr>
<th>Segments Entered</th>
<th>Firm #</th>
<th>Lowest Efficient Division Share Change</th>
<th>Highest Efficient Division Share Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>28</td>
<td>-7.5%</td>
<td>7.5%</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>-5.6%</td>
<td>1.1%</td>
</tr>
<tr>
<td>4 &amp; 5</td>
<td>8</td>
<td>-0.4%</td>
<td>0.9%</td>
</tr>
<tr>
<td>Total</td>
<td>46</td>
<td>-3.2%</td>
<td>4.3%</td>
</tr>
</tbody>
</table>

2.3.3.3 “Big Four” Comparison and the HAL/BHI Merger

Thus far, this section presents the average efficiency at the industry and segment levels. This subsection compares the firm-level and division-level efficiency of the “Big Four”, namely Schlumberger (SLB), Halliburton (HAL), Baker Hughes (BHI), and Weatherford (WFT).

Figure 2.2 shows the efficiency of the four diversified oilfield services companies at the firm level over time. Baker Hughes had the highest efficiency level, followed by Schlumberger and Halliburton, which are very close to the industry average efficiency, while Weatherford is below industry average in terms of firm-level efficiency.\(^6\)

Looking at the divisional level, Table 2.4 provides detailed efficiency levels by division for the four giants in 2014. The efficiency levels within the companies across

\(^6\) Although not listed in this study, the estimated efficiencies for these four companies are much higher compared with the industry average when overlooking the endogeneity problem. This is because the MLE method predicts a much lower returns to scale (see Column 1 in Table 2.2) and hence overestimates the efficiency of big companies. For the same reason, the MLE approach predicts that Schlumberger and Halliburton are more efficient than relatively small Baker Hughes.
Figure 2.2: Efficiency Level for BHI, HAL, SLB, WFT, and the Industry Average

segments were very different. Schlumberger, the biggest company in the oilfield industry, had footprints in all five segments. This firm has very high efficiency in the exploration segment and no significant weakness in terms of division-level efficiency. Halliburton, the second largest firm in the field, has the highest efficiency in the exploration segment, but the lowest efficiency in the production segment among the “Big Four”. Baker Hughes, which does business in only three segments, had the highest efficiency in the production segment. Compared with its peers, Weatherford is the smallest diversified oilfield company and is not as efficient as the other three companies in most segments.

In the fourth quarter of 2014, Halliburton (number two in the field) spent $35 billion USD to acquire Baker Hughes (number three in the field). The comparison of efficiency at the divisional level explains the incentive of this merger. Comparing the four segments that both Schlumberger and Halliburton entered,\(^7\) Halliburton is

\(^7\) The capital equipment segment that only Schlumberger entered can be ignored in this comparison because it only generates 0.5% of Schlumberger’s total revenue.
slightly more efficient than Schlumberger in the exploration (0.95 vs. 0.90), drilling (0.70 vs. 0.68), and completion (0.62 vs. 0.61) segments, but significantly less efficient in the production (0.61 vs. 0.73) segment. Baker Hughes, however, is very efficient in the production segment (0.91). Moreover, the efficiencies in Baker Hughes’ drilling and completion segments are also slightly higher than that of Halliburton and Schlumberger. Therefore, Halliburton can improve the efficiency in its production segment dramatically, while maintaining the efficiency in other segments after the merger. The new company can be as competitive as Schlumberger in every major segment.

Table 2.4: Efficiency Levels of the “Big Four” by Segment in 2014

<table>
<thead>
<tr>
<th>Segments</th>
<th>Schlumberger</th>
<th>Halliburton</th>
<th>Baker Hughes</th>
<th>Weatherford</th>
<th>Oilfield Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exploration</td>
<td>0.90</td>
<td>0.95</td>
<td>–</td>
<td>–</td>
<td>0.71</td>
</tr>
<tr>
<td>Drilling</td>
<td>0.68</td>
<td>0.70</td>
<td>0.72</td>
<td>0.53</td>
<td>0.61</td>
</tr>
<tr>
<td>Completion</td>
<td>0.61</td>
<td>0.62</td>
<td>0.64</td>
<td>0.55</td>
<td>0.61</td>
</tr>
<tr>
<td>Production</td>
<td>0.73</td>
<td>0.61</td>
<td>0.91</td>
<td>0.71</td>
<td>0.63</td>
</tr>
<tr>
<td>Capital Eqpt</td>
<td>0.63</td>
<td>–</td>
<td>–</td>
<td>0.50</td>
<td>0.61</td>
</tr>
</tbody>
</table>

2.4 Conclusion

This chapter aims to estimate the distribution of inputs across segments within a firm since such information is necessary for division-level productivity and efficiency analysis but in many cases remains unobserved. Solving a system of segment-specific production functions and a system of firm-level profit maximization problem simultaneously, this chapter builds a model to impute the undistorted input allocations. This approach can test if some assumptions that previous studies rely on are valid.
and derive a result even in the absence of those assumptions.

The accuracy of the undistorted input allocations is tested using panel data from OECD countries where the actual division-level inputs are available. The undistorted estimation outperforms the three competing imputation methods. In the empirical study, the input allocations for all the multidivisional firms in the global oilfield market are estimated, which make it possible to perform reduced-form stochastic frontier analysis for each of the five segments. This “Meta-Frontier” approach predicts division-level and firm-level efficiency as well as the average efficiency for each of the five segments. Evidence is also found that the assumptions used by previous works are not always valid.
3.1 Introduction

This chapter develops a structural model for each of the five segments in the oilfield market where investments, inputs, entry, and exit are endogenous variables rationally chosen by firms to maximize their expected future total benefit. This model is used to estimate the average efficiency level for each segment without opening the “black box” of input. The result can be compared with the “Meta-Frontier” estimation in the second chapter.

Pakes-McGuire Model (PMM) is developed in Pakes, Gowrisankran, and McGuire (1993) and Pakes and McGuire (1994), which is a structural model that is widely used to study oligopoly markets (Borkovsky, Doraszelski, and Kryukov 2010). Tan (2006) applies PMM to study the market structure of U.S. Cigarette Market where four largest firms (Philip Morris, R.J.Reynold, Brown & Williamson, and Lorillard) dominate the industry. This dynamic oligopoly model is also applied in other industries such as hospital industry (Gowrisankaran and Town 1997), local cement industry
(Ryan 2012), pharmaceutical industry (Filson 2012), and banking industry (Cor-bae and D’Erasmo 2014). Many studies (Lebanon 1975; Barreau 2002; Yuan and Zhang 2008) point out that the oilfield market is characterized by the presence of an oligopoly which is composed of several major firms (Schlumberger, Halliburton, Baker Hughes, and Weatherford) and other much smaller oil and gas service businesses. The high concentration ratio in market share also supports that the oilfield market is an oligopoly market\textsuperscript{1}.

Therefore, PMM is applicable for the oilfield market and is utilized for each of the five segments respectively. This model does not use division-level input and output data. Instead, the average lifespan of firms and the average investment-to-profit ratio of each segment are needed. For each segment, firms’ decisions upon entry and exit, as well as investment, are simulated. This method computes the MPN equilibria (Maskin and Tirole 1988a, 1988b) that are generated under the constraints of Ericson and Pakes (1992). The algorithm from the original work (p.7-36, Pakes, Gowrisankran, and McGuire, 1993) is employed, which is introduced in the first chapter.

### 3.2 Results

The three parameters in the profit function ($D$, $f$, and $\gamma$) are pinned down by requiring that the model provides similar statistics with the real-world data, including the average lifespan of firms and the investment-to-profit ratio. Both of these statistics are computed by simulating the industry from an economy starting with only one firm whose efficiency level is 11. This study also uses the value function, investment, and entry/exit decisions to evaluate the optimal policies and update the industry structure. This study uses separate, but similar, programs to evaluate the statistics

\textsuperscript{1} The number of companies that have over 5% market share is 5, 5, 5, 6, and 10 in the exploration, drilling, completion, production, and capital equipment segment, respectively.
for the MPN equilibrium for the five segments; each is simulated 10,000 times and the average efficiency level is the mean of all the active firms in those 10,000 periods. The assigned parameters are similar to the one in Pakes, Gowrisankaran, and McGuire (1993).

Due to limited data, the average lifespan of firms is estimated based on the firms deactivated in the period 2011 - 2013. The companies who exit the market can be found by comparing the 2011, 2012, 2013, and 2014 versions of OMR. This research cannot track the deactivated firms before 2011, due to a lack of consecutive annual OMR before 2011. The life length and segments that these firms entered are then collected to estimate the average lifespan of firms for each of the five segments, respectively.

Annual investment and profit data can be downloaded from Thomson ONE. For each segment, the average investment-to-profit ratio is the mean of investment over profit for all the single-division firms in that segment from 1997 to 2014.

Figure 3.1 compares the average efficiency level of various methods for each of the five segments. The “Meta-Frontier” estimators are derived from the C2SLS “Meta-Frontier” approach, while the PMM estimators are derived from the structural model. Table 3.1 presents all the parameters used in the PMM, as well as the outputs. Both methods support that PMM-estimated segment average efficiencies are lower than those estimated by the C2SLS “Meta-Frontier” approach. However, all the results support that the exploration segment is on average the most efficient, followed by the drilling, production, and capital equipment segments, while the completion segment is on average the least efficient.
Figure 3.1: Comparison of Average Efficiency Level in Various Methods
Table 3.1: Estimate of Transcendental Logarithmic Production Function

<table>
<thead>
<tr>
<th></th>
<th>Exploration</th>
<th>Drilling</th>
<th>Completion</th>
<th>Production</th>
<th>Capital Equipment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data MPNE</td>
<td>Data MPNE</td>
<td>Data MPNE</td>
<td>Data MPNE</td>
<td>Data MPNE</td>
</tr>
<tr>
<td>Average lifespan of firms</td>
<td>32.0</td>
<td>31.3</td>
<td>22.6</td>
<td>22.4</td>
<td>24.9</td>
</tr>
<tr>
<td>Average investment-to-profit ratio</td>
<td>0.53</td>
<td>0.52</td>
<td>0.61</td>
<td>0.61</td>
<td>0.55</td>
</tr>
<tr>
<td>Constant used in investment fn.</td>
<td>$\alpha$</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost for a GBP investment</td>
<td>$c$</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum Number of firms</td>
<td>$N$</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Highest efficiency level attainable</td>
<td>$\hat{w}$</td>
<td>19</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Efficiency level for entrants</td>
<td>$W_E$</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sunk cost of entry</td>
<td>$X_E$</td>
<td>0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lowest sunk entry cost</td>
<td>$X_{EL}$</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Highest sunk entry cost</td>
<td>$X_{EH}$</td>
<td>0.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.92</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prob. of outside alternative rising</td>
<td>$\delta$</td>
<td>0.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scrap value at exit</td>
<td>$\phi$</td>
<td>0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical intercept of demand</td>
<td>$D$</td>
<td>7.4</td>
<td>5</td>
<td>6.37</td>
<td>9.86</td>
</tr>
<tr>
<td>Fixed cost of production</td>
<td>$f$</td>
<td>2.1</td>
<td>0.69</td>
<td>1.3</td>
<td>4.34</td>
</tr>
<tr>
<td>Capital-to-cost parameter</td>
<td>$\gamma$</td>
<td>2</td>
<td>0.3</td>
<td>6</td>
<td>2.1</td>
</tr>
<tr>
<td>Average efficiency</td>
<td></td>
<td>7.889</td>
<td>6.274</td>
<td>8.494</td>
<td>8.22</td>
</tr>
<tr>
<td>Max Efficiency level Appeared</td>
<td></td>
<td>13</td>
<td>11</td>
<td>16</td>
<td>15</td>
</tr>
<tr>
<td>Average efficiency Index</td>
<td></td>
<td>0.607</td>
<td>0.570</td>
<td>0.531</td>
<td>0.548</td>
</tr>
</tbody>
</table>
4.1 Introduction

One industry may have multiple segments. For example, the global oilfield market has five segments including exploration and production. Since the technologies utilized in exploration and production are different, the production function is segment-specific. The second chapter imputes the resource allocation and further estimates the segment-specific production functions. Then the productivity and efficiency analysis is launched for each segment, respectively. This “Meta Frontier” method derives division-level efficiencies and then aggregates to firm-level efficiency but fails to predict the aggregated production function. For a multidivisional firm who has footprints in multiple segments, different production technologies are used to convert inputs to outputs. Therefore, the aggregated production function for this firm is not equal to any of the segment-specific production functions, but a combination of them. This chapter attempts to estimate the aggregated production function for multidivisional firms.
Since multidivisional firms use different production technologies, some weight index is needed to derive the aggregated production function. The revenue share by division, $\theta_{it}$, is an eligible weight to capture the heterogeneity in technologies since it indexes the proportion of business using each of the segment-specific techniques. In other words, $\theta_{it}$ measures the frequency of using every segment-specific technology in a multidivisional firm. Imagine a “$M$ inputs – $N$ Products/segments – $T$ periods” industry, $\theta_{it} = (\theta_{i1t}, \theta_{i2t}, \ldots, \theta_{imt})$ where $\theta_{ijt} = R_{ijt}/\left(\sum_{j=1}^{N} R_{ijt}\right)$ $\forall j = 1, 2, \ldots, N$ and $R_{ijt}$ is the revenue for firm $i$ in segment/division $j$ at time $t$.

As a standard single frontier analysis, the first chapter ignores the heterogeneity in production technologies across segments. The second chapter imputes input allocation and estimates segment-specific production functions rather than the aggregated production function for each multidivisional firm. Since the spillover effects of using multiple technologies simultaneously are unknown, this chapter introduces two semiparametric methods to estimate the aggregated production function when firms may have different technologies in their portfolio and use them at different frequencies.

The first method is a varying coefficient model where every technical parameter of a production function (e.g. Cobb-Douglas or Translog) is a nonparametric function of some threshold variables ($\theta_{it}$ in this case) rather than a constant. This model allows changes in production frontier in response to changes in business portfolio $\theta_{it}$ and hence be called “Varying Frontier”.

In the second model, the formation of the production function is relaxed to avoid rigid functional forms (e.g. Cobb-Douglas or Translog) by semiparametric methods. In economic theory, production functions are monotone increasing and concave with respect to inputs. A semiparametric production function with monotonicity and concavity restrictions is introduced and called “Shape-constrained Frontier”.

Compared with the “Meta Frontier” method in the second chapter, both the
“Varying Frontier” and “Shape-constrained Frontier” methods estimate the production frontier and efficiency for each firm without opening the input black box but still considers heterogeneity across segments/divisions. This chapter ends by comparing the results from “Single Frontier”, “Varying Frontier” and “Shape-constrained Frontier” approaches.

Suppose a population of firms uses one input to generate one output in segments $A$ and $B$. Figure 4.1 compares all the four approaches: 1) the upper left figure shows the idea of the classic “Single Frontier” method. The x- and y-axis present the firm-level input and output, respectively. The vertical distance of a firm’s allocation to the frontier decides the firm-level inefficiency. That is, all firms compete with each other directly; 2) the bottom left figure shows the “Meta Frontier” approach, which in this case has two frontiers (one for segment $A$ and one for segment $B$). The x- and y-axis present the segment-level input and output. For a firm that only enters one segment, its efficiency is measured by the distance to the frontier of that segment. For a firm that has footprint in both segments, it receives two segment-level efficiency scores, which can be aggregated to a firm-level efficiency by some weighted average techniques. In this model, a firm’s division only competes with producers in the same segment; 3) the upper right figure shows the production frontier that shifts according to $\theta$, the share of revenue from segment $B$ within a firm. This figure reflects the “Varying Frontier” which estimates the overall production frontier for multidivisional firms based on their utilization of various technologies; and 4) the bottom right figure shows the “Shape-constrained Frontier” where the frontier can be any monotone increasing and concave function in absence of the formation assumption (e.g. Cobb-Douglas or Translog).

The remainder of the chapter is structured as follows. Section 2 introduces the methodology of varying coefficient stochastic frontier model (“Meta Frontier” ap-
Section 3 builds a semiparametric model with shape constraint ("Shape-constrained Frontier" approach) to relax the priori assumption of the functional form of the production function. Section 4 presents an empirical application on the global oilfield market. Section 5 consists of the conclusions drawn.

4.2 **Varying Coefficient Model**

This section develops a partial linear semiparametric varying coefficient stochastic frontier analysis ("Varying Frontier") to estimate the aggregated production function for multidivisional firms and further predict firm-level efficiency without opening the
input black box but still considers different technologies across divisions.

4.2.1 General Model

Imagine a “$M$ inputs – $N$ Products/segments – $T$ periods” industry; the traditional stochastic frontier analysis under a unique production function assumption is

$$Y_{it} = f (X_{it}; \beta_0) \exp(\tau Z) \exp(\nu_{it}) \exp(-u_{it})$$

which is introduced in Eq. (2.1) and $\beta_0 = (\beta_{01}, \beta_{02}, \ldots)$ is a vector of segment-invariant technical parameters of different inputs.

In this chapter, the revenue share by segment $\theta_{it}$ is introduced to capture the heterogeneity in total revenue and production technologies. This chapter uses $\theta_{it}$ as a weight index and adds into the aggregated production function:

$$Y_{it} = f (X_{it}; \beta_0, \theta_{it}) \exp(\tau Z) \exp(\nu_{it}) \exp(-u_{it})$$

The effect of the portfolio of the business $\theta_{it}$ can be either dependent or independent with the rest of the production function. If it is independent (i.e., $f (X_{it}; \beta_0, \theta_{it}) = f (X_{it}; \beta_0) \cdot m(\theta_{it})$), then a transfer to the traditional multiproduct stochastic frontier analysis is possible, where one product is a function of all inputs and all other products. Adams, Berger, and Sickles (1999) and Liu (2014) use such a canonical regression to check the efficiency of the banking industry with multiple outputs and inputs, where the former and latter papers model $f (X_{it}; \beta_0)$ nonparametrically and parametrically, respectively. This study assumes a Cobb-Douglas form and set a production function where $\theta_{it}$ is independent with $f (X_{it}; \beta_0)$

$$\ln (Y_{it}) = r (\theta_{it}) + \sum_{k=1}^{M} \beta_k (\ln X_{it}^k) + \tau Z + \nu_{it} - u_{it},$$
where \( r(\theta_{it}) \) is a nonparametric functions of \( \theta_{it} \). Similar to the “Single Frontier” equation mentioned in chapter 2 (Eq. (A-16)), this function in Eq. (4.3) also has a unique production function \( f(X_{it}; \beta_0) \). However, the intercept \( r(\theta_{it}) \) is a nonparametric functions of \( \theta_{it} \) rather than a constant \( \alpha \). In this chapter, “Single Frontier” refers to the one in Eq. (4.3).

This section focuses on the other situation, where \( \theta_{it} \) can directly affect the production function though their effects on the technical parameters. This model is inspired by the smooth/varying coefficient model (see Hastie and Tibshirani (1993)) and therefore called the varying production frontier, where the coefficients are nonparametric functions of some “threshold” variables (\( \theta_{it} \) in this case).

\[
Y_{it} = f \left( X_{it}; \beta_0' = r(\theta_{it}) \right) \exp(\tau Z) \exp(\nu_{it}) \exp(-u_{it}). \tag{4.4}
\]

Eq. (4.4) allows the change in aggregated production function when revenue share \( \theta \) changes. For example, if a multidivisional firm has major business in segment A and minor business in segment B, then the aggregated production function of this firm is likely to be closer to production function in segment A, as this company uses production technology from this segment more frequently. Since using multiple technologies jointly can lead to nonlinear spillover effects caused by shared R&D investment, joint inputs, and so on, we cannot simply take the weighted average of the segment-specific production functions. Hence, a nonparametric function \( r(\cdot) \) is used to control the nonlinear combination of technologies.

4.2.2 Semi-Varying Coefficient Stochastic Frontier Analysis

Productivity and efficiency analysis is dominated by two approaches: the parametric stochastic frontier analysis (SFA) and the nonparametric deterministic data envelop-
ment analysis (DEA). Each method has its own strengths and drawbacks: stochastic frontier analysis is suitable for noisy data, but requires the priori assumption of an explicit functional form; data envelopment analysis does not require specified functional form, but does not allow for statistical noise since no stochastic component is included. In recent years, many new semiparametric and nonparametric stochastic frontier techniques have been applied to narrow the gap between SFA and DEA. Such development results in new methods to better model the aggregated production function for multidivisional firms who use multiple production technologies.

Fan, Li, and Weersink (1996) propose a semiparametric method that allows for statistical noise and does not need to specify the functional form of the production frontier. Their approach, known as semiparametric frontier analysis, has the form

\[ y = f(x) + \epsilon = f(x) + \mu + \nu - u \] (4.5)

where \( f(x) \) is a semi- or nonparametric production function. Similar to parametric stochastic frontier analysis, \( u \) is a non-negative technical inefficiency term and \( \nu \) is a statistical noise term. \( \mu \) is a constant that guarantees the expected value of \( \epsilon \) equals zero. Therefore, \( \epsilon = \mu + \nu - u \) is the disturbance term with a zero mean.

In practice, the semiparametric model is solved in two steps: in the first step, the semi- or nonparametric regression \( y = f(x) + \epsilon \) is run to retrieve the residuals \( \hat{\epsilon} \); in the second step, the residual is decomposed as \( \hat{\epsilon} = \mu + \nu - u \) using normal stochastic frontier analysis where \( \hat{\epsilon} \) is the dependent variable and a constant is the only independent variable. Henningsen and Kumbhakar (2009) adopt this approach in their applied study on Polish farms, where they use logarithmic output and input quantities for three reasons: 1) the elasticities are easier to interpret; 2) the observations are more equally distributed when using constant bandwidths; and 3) the usual specification of the production function is easier to adopt.
As Henningsen and Kumbhakar (2009) point out, the nonavailability of software used to prevent applied studies to widely use this approach. This restriction has disappeared in recent years. Take R as an example, the “np” package (Hayfield and Racine (2008)), the “gam” package (Hastie and Tibshirani (1990)), or the “gamlss” package (Stasinopoulos and Rigby (2007)) can be used in the first step and the “frontier” package (Coelli, Henningsen, and Henningsen (2012)) can be used in the second step.

This section uses the varying coefficient model (VCM) for the production function $f(x)$ in Eq. (4.5). Hastie and Tibshirani (1993) first introduce VCM in the form

$$Y = X_1 r_1 (\theta_1) + \ldots + X_p r_p (\theta_p) + \epsilon$$

where $\theta_1, \ldots, \theta_p$ change the coefficients of $X_1, \ldots, X_p$ through unspecified functions $r_1 (\cdot), \ldots, r_p (\cdot)$. The coefficients are nonparametric functions that are not constant, hence the name “varying/smooth coefficient model”. VCM is initially applied to model time-variant coefficient functions in censored data in survival analysis.

In production analysis, environmental factors can only affect the frontier neutrally if treated as independent variables ($X$). Some varying coefficient stochastic frontier analysis treats the environment factors as $\theta_i$ and allows their effect on the frontier to be non-neutral. Research and Development (R&D) is such an environmental factor that it is believed to affect the frontier directly. Other examples of such “threshold” variables include tax rate, firm size, firm age, etc. (Kumbhakar and Sun 2013).

Sun and Kumbhakar (2013) estimate stochastic production frontier in a Norwegian forest using a cross-section of 3,249 active forest owners. Zhang et al. (2012) develop a varying coefficient production function to study China’s high technology industry, where panel data spanning the period 2000-2007 is used. Both of these studies use R&D-varying coefficient production functions. However, they use an average
production function, not a stochastic frontier analysis.

This section generates a partial linear semi-varying coefficient stochastic frontier analysis to model the OMR panel data for the oilfield market introduced in the second chapter where the revenue distribution by segment $\theta$ can directly affect the technical parameters and the frontier has a Cobb-Douglas (C-D) form.

$$\ln Y_{it} = \alpha + r_1(\theta_{it}) \ln L_{it} + r_2(\theta_{it}) \ln K_{it} + \tau Z + \nu_{it} - u_{it} \quad (4.6)$$

where $Y_{it}$, $L_{it}$, and $K_{it}$ are the output, number of employees, and capital employed for firm $i$ at time $t$, respectively.

There are two nonparametric approaches to estimate the $r_1(\cdot)$ and $r_2(\cdot)$ in Eq. (4.6): the kernel-based method (Hu 2014; Fan and Huang 2005; Su and Ullah 2006; Fan and Li 2004; Sun, Carroll, and Li 2009) and the spline-based method (Hastie and Tibshirani 1993; Ahmad, Leelahanon, and Li 2005). Fan and Zhang (2008) think that kernel smoothing methods are more reasonable, as the varying coefficient model is a local linear model, while Kim (2013) argues that spline methods are more attractive for their flexibility to involve multiple smoothing parameters. However, both methods have some disadvantages: the former may suffer from the “curse of dimensionality” and the latter may encounter computational challenges, since the number of spline basis functions can be large.

Since there are five variables in $\theta_{it}$ that will cause a “curse of dimensionality”, this section selects the penalized B-spline approach to estimate the production function. It is assumed that the inefficiency term is time-invariant ($u_{it} = u_i$) so that the Least Square Dummy Variable (LSDV) can be used to derive a fixed effect estimator. Lu, Zhang, and Zhu (2008) present results on the strong consistency and asymptotic normality for penalized B-spline estimators of such a varying coefficient model.

This paper uses the two-step approach in Henningsen and Kumbhakar (2009) to
estimate Eq. (4.6): 1) a penalized B-spline method is used to derive consistent coefficients and predict the residuals \( \hat{\epsilon} \) in Eq. (4.5) in the first step; 2) then, a normal stochastic frontier analysis is used where \( \hat{\epsilon} \) is the dependent variable and a constant is the only independent variable. This paper also develops a varying coefficient stochastic frontier analysis where the production function has a Transcendental Logarithmic (T-L) form to check the robustness of the varying coefficient model.

### 4.3 Semiparametric Model under Shape Constraints

Compared with the semi-varying coefficient model in Section 4.2, the formation of the production function in this section is relaxed to avoid rigid functional forms by nonparametric methods. However, flexible parameterizations can cost fidelity to economic theory and lead to implausible prediction. This dilemma is tackled by semiparametric method subject to some restrictions suggested by economic theory. The stochastic semiparametric frontier approach allows for noise and do not impose a priori assumption on the functional form, but most importantly, it still follows economic theory. Different from the approach in Fan, Li, and Weersink (1996), the production function \( f(\cdot) \) is modeled semiparametrically rather than nonparametrically.

In economic theory, production functions are monotone increasing and concave with respect to inputs (Diewert and Wales 1987). The property of monotonicity guarantees that firms can always produce more with more inputs. And the property of concavity guarantees decreasing marginal products when input grows. A semiparametric function \( \xi(\cdot) \) with monotonicity and concavity restriction is inspired by
As is proposed by Ramsay (1998), the positive exponential functional embedded in the integral transformation guarantees non-negative first derivative $\xi'(x) = \exp(\cdot) \geq 0$ and hence achieves global monotonicity. On the other hand, we assume $g(x) = x^2$ in the second integral assures the second derivative $\xi''(x) = \xi'(x)[-g(h(w))] \leq 0$ since $\xi'(x) \geq 0$ and $g(h(w)) = [h(w)]^2 \geq 0$, which obtains concavity for $\xi(x)$. These two transformations change a constrained problem into an unconstrained one. Finally, only the function $h(w), w \in [0,1]$ needs to be modeled. This study opts to use spline method to model $h(w)$ nonparametrically. Specially, the truncated power series splines are

$$\Phi(x) = (1, x, \ldots, x^p, (x - k_1)^p, \ldots, (x - k_M)^p)^T,$$

where $0 < k_1 < \ldots < k_M < 1$ are a series of knots of the spline basis functions, $(x)_+ = \max(x,0)$, and $p$ is a positive integer. Then $h(x) = c^T \Phi(x)$ where $c$ vectors the coefficients with compatible dimension.

Using this semiparametric method $\xi(\cdot)$ with shape constraint, a new aggregated stochastic frontier production can be built:

$$Y_{it} = A_{\theta} \cdot \left[ \prod_{k=1}^{M} \xi_k(X_{it}^k) \right] \cdot \exp(\tau Z) \cdot \exp(\nu_{it}) \cdot \exp(-u_{it})$$

where $A_{\theta} = \exp[r(\theta_{it})]$ and $r(\theta_{it})$ is a nonparametric function of $\theta_{it}$. Other notations in Eq. (4.8) follows the ones in Eq. (4.1). $\xi_k(X_{it}^k)$ is a monotone increasing and concave

\footnote{An alternative shape constrained additive models is developed in Pya and Wood (2015). Readers are encouraged to use their R package “scam” to relax the formation of the production functions.}
function of the $k$th input.

This study uses the product of each input’s effect ($\prod_{k=1}^{M} \xi_k(X^k_{it})$) rather than the summation of them ($\sum_{k=1}^{M} \xi_k(X^k_{it})$) for the following reasons: (1) the product of positive monotone increasing and concave functions are still monotone increasing and function for each input; (2) the multiplication (instead of the summation) allows positive (rather than zero) cross-productivity effects. Many studies (Nicholson and Snyder 2011; Thompson 2011) point out positive cross-productivity effects are the most prevalent case. Take labor and capital as an example, the positive cross-productivity effects reflect workers would have higher marginal productivity if they have more capital. If summation is adopted, the effects of different inputs are additive and independent, which ignores the joint effects; (3) when the multiplication form is employed, a linear production function analogous to previous models is easy to be derived by taking log of production function. As a result, the advantage of logarithmic output described in Henningsen and Kumbhakar (2009) can be benefited.

The logarithm of equation Eq. (4.8) becomes

$$\ln (Y_{it}) = r(\theta_{it}) - \sum_{k=1}^{M} \ln[\xi_k(X^k_{it})] + \tau Z + \nu_{it} - u_{it},$$

(4.9)

where

$$\xi_k(X^k_{it}) = \int_0^{X^k_{it}} \exp \left[ \int_0^s - (c_k^T \Phi (w))^2 dw \right] ds.
$$

here $c_k^T$ in $\xi_k(X^k_{it})$ for all $k$ are the flexible tools for curve fitting and need to be estimated. The estimated shape of Eq. (4.9) can be compared with the one in Cobb-Douglas production function in Eq. (4.3).
4.4 Empirical Study: The Productivity and Efficiency for Multidivisional Oilfield Firms

This empirical study applies the described models into the public firms in the global oilfield market. Data description is given in the second chapter. This study estimates and compares the production frontiers and the firm-level efficiencies using the “Single Frontier” approach in Eq. (4.3), the “Varying Frontier” approach in Eq. (4.6), and the “Shape-constrained Frontier” approach in Eq. (4.8).

4.4.1 Production Frontiers

The two semiparametric models cannot derive the coefficients of the production function that are comparable with the ones in the “Single Frontier” model. Therefore, this study visualizes the respective effect of labor and capital on output under different approaches in Figure 4.2. The effects of labor and capital in all the three methods are robust. Compared with “Single Frontier” and “Varying Frontier” methods that assume Cobb-Douglas form, the “Shape-constrained Frontier” estimated effect of capital has a higher second derivative and hence less concave, which reflects a relatively slower declining of the marginal effect when capital grows. As a result, the “Shape-constrained Frontier” estimated output is relatively lower for smaller firms who use less capital compared with the ones derived by the other two methods. For the effect of labor, however, the “Shape-constrained Frontier” predicts an effect very similar to the one in the Cobb-Douglas form.

The “Varying Frontier” curves in Figure 4.2 describe the average effect of the varying coefficients. Figure 4.3 provides the range of such varying effect for firms with different business portfolios, which shows the variation of the aggregated production function under the “Varying Frontier” method. It is clear that the capital effect varies
a lot when technologies in different segments are utilized at different frequencies. But the labor productivities in different segments are very robust.

Figure 4.4 presents the production frontiers to show the “Output-Labor-Capital” relations graphically using 3D images and contour graphs. Overall, the comparisons among the three methods show that Cobb-Douglas is a valid assumption, as relaxing this form assumption can lead to robust results.

### 4.4.2 Technology Change

The coefficients of the year dummy variables $\tau$ indicate the production frontiers change by time, which is affected by technology, market, and other time-variant factors. A greater coefficient of a certain year dummy variable implies higher attainable productivity from inputs to outputs compared with the base year, which is 1999 in the study. The improvement in productivity could be results of new technology and equipment, or higher demand due to economic boom, or management innovation.
However, the major reason is always the economic performance. If the world economy is in expansion, higher demand of energy drives oil and gas companies to increase production and E&P Spending, which pushes the growth and innovation of the oilfield market. The larger cake spurs the competitors in the oilfield market to invest on new technology and equipment to increase market share and margins. Since it takes time to deploy the new technology and equipment, there will be a time lag between the economy performance and the productivity performance.

In Figure 4.5, the “Single Frontier”, “Varying Frontier”, and “Shape-constrained Frontier” estimators are robust and all support the idea above. The economy grew rapidly before the productivity dropped by about 10% in 2002 follows the Early 2000s recession. The 2003 is the beginning of the Golden Time for oilfield market, followed by a six-year straight increase until 2008. The accumulative growth in productivity during the period is about 30 percent. In 2009, however, the productivity was fallen by 15 percent sharply after the Global Financial Crisis since 2007. The market is
Figure 4.4: Estimated Production Frontiers in Various Methods
recovery in recent years: the productivity stopped falling down in 2010, followed by significant gains in the next two years and back to the 2008 peak level. In 2014, the productivity decreased again because of the oil price crash.

4.4.3 Technical Efficiency

This subsection presents the efficiency results as follows. First, the statistics of the estimated firm-level efficiencies are provided. Subsequently, the efficiency class intervals are displayed. Finally, the effects of firm size and being a multidivisional firm on efficiency are estimated. The result and comparison derived from a T-L production function is given in Appendix A.5.

Table 4.1 summarizes the distribution of the efficiency scores in the oilfield market. The average efficiency level of the industry is 0.42-0.52 in different methodologies. The estimation of the “Shape-constrained Frontier” is lowest, followed by the “Single Frontier”, while the “Varying Frontier” is highest.
Table 4.1: Technical Efficiency Statistics

<table>
<thead>
<tr>
<th></th>
<th>Single Frontier</th>
<th>Varying Frontier</th>
<th>Shape-constrained Frontier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.43</td>
<td>0.52</td>
<td>0.42</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.14</td>
<td>0.26</td>
<td>0.14</td>
</tr>
<tr>
<td>25% quantile</td>
<td>0.26</td>
<td>0.40</td>
<td>0.25</td>
</tr>
<tr>
<td>50% quantile</td>
<td>0.38</td>
<td>0.51</td>
<td>0.36</td>
</tr>
<tr>
<td>75% quantile</td>
<td>0.56</td>
<td>0.63</td>
<td>0.56</td>
</tr>
<tr>
<td>Maximum</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 4.2 presents the distribution of firms by technical efficiency scores, which is classified into four efficiency class intervals. For each of the efficiency class intervals and the overall average estimated technical efficiency scores, the lower and upper bounds of 95% Confidence Interval (CI) are obtained by using a bootstrap technique, called Efron’s nonparametric bias-corrected and accelerated (BCa) method, with 10,000 replications (Briggs, Mooney, and Wonderling 1999). The majority of oil and gas service companies fall into a category of 30 - 75% technical efficiency in all the three methods.

Table 4.2: Technical Efficiency Class Interval

<table>
<thead>
<tr>
<th>Efficiency</th>
<th>Single Frontier</th>
<th>Varying Frontier</th>
<th>Shape-constrained Frontier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>#obs.</td>
<td>mean (95% CI)</td>
<td>#obs.</td>
</tr>
<tr>
<td>&lt; = 0.3</td>
<td>27</td>
<td>.22 (.20-.23)</td>
<td>9</td>
</tr>
<tr>
<td>0.3 - 0.5</td>
<td>33</td>
<td>.39 (.37-.41)</td>
<td>34</td>
</tr>
<tr>
<td>0.5 - 0.75</td>
<td>19</td>
<td>.60 (.58-.64)</td>
<td>36</td>
</tr>
<tr>
<td>&gt; 0.75</td>
<td>10</td>
<td>.84 (.80-.91)</td>
<td>10</td>
</tr>
<tr>
<td>Total</td>
<td>89</td>
<td>.43 (.39-.48)</td>
<td>89</td>
</tr>
</tbody>
</table>
In order to explore the effect of company size and being a multidivisional firm on efficiency, this study regresses firm-level efficiency (\( \hat{T}_E_i \)) on the log of firm-level revenue (\( \ln R_i \)) and a dummy variable of multidivisional firm\(^2\) (\( MD_i \)). Table 4.3 reports the estimation results. All three regressions indicate that larger firms have advantages over smaller firms in terms of efficiency\(^3\). When firm size is controlled, multidivisional firms on average have neither an advantage nor a disadvantage in efficiency over single-division firms in the oilfield market.

Table 4.3: Efficiency Regression Result

<table>
<thead>
<tr>
<th></th>
<th>Single Frontier</th>
<th>Varying Frontier</th>
<th>Shape-constrained Frontier</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{T}_E_i )</td>
<td>.077***</td>
<td>.123***</td>
<td>.085***</td>
</tr>
<tr>
<td>( \ln R_i )</td>
<td>(.017)</td>
<td>(.011)</td>
<td>(.018)</td>
</tr>
<tr>
<td>( MD_i )</td>
<td>-.037</td>
<td>-.042</td>
<td>-.026</td>
</tr>
<tr>
<td></td>
<td>(.044)</td>
<td>(.025)</td>
<td>(.044)</td>
</tr>
<tr>
<td>Intercept</td>
<td>-.062</td>
<td>-.275***</td>
<td>-.129</td>
</tr>
<tr>
<td></td>
<td>(.114)</td>
<td>(.078)</td>
<td>(.118)</td>
</tr>
<tr>
<td>R^2</td>
<td>0.19</td>
<td>0.58</td>
<td>0.21</td>
</tr>
</tbody>
</table>

4.5 Conclusion

This chapter develops two semiparametric stochastic frontier analysis to control the heterogeneity in output allocation among multidivisional firms. These companies use multiple production technologies and their aggregated production functions depend

\(^2\) This study uses data and estimation from 2013 since this is the year with the most active firms in the dataset.

\(^3\) This result is consistent with the opinion of Schlumberger’s CEO Paal Kibsgaard, who said in the 4Q2014 Earnings Call that scale is essential to drive performance in the oilfield market. The industry-leading size and integration capabilities are key competitive advantages.
on the frequency of using each of the technologies. The heterogeneity in output allocation (frequency) is captured by revenue share across divisions $\theta$.

Compared with the classic “Single Frontier” approach, the first semiparametric model, “Varying Frontier”, allows the variation in production function through replacing the constant technical parameters by some nonparametric function of $\theta$. Therefore, the formation of the production function is still assumed (e.g. Cobb-Douglas or Translog). The second semiparametric, however, relaxes the formation of the production function to avoid rigid functional forms. This method is called “Shape-constrained Frontier” since the estimated production function must follow the economic theory to be monotone increasing and concave.

The empirical results of the global oilfield market show: 1) the labor productivity is robust under different methods and across segments; 2) the capital productivity is slightly different under different methods and across segments; 3) Cobb-Douglas form is a valid assumption as relaxing this functional form derive similar production function; and 4) the technology change of this industry and the technical efficiency of the companies are reported.
A.1 Imputation Method

This appendix presents the imputation method including how to derive Theorem 1 under Assumption 1 - 3 in Chapter 2. A simple example is introduced first to illustrate how to impute undistorted input allocations in multidivisional firms when they maximize profits. Then, the general model is given under a “M Inputs – N Products/Segments – T periods” economy. Finally, parametric production function specification is discussed.

A.1.1 A Simple Example of the Model

A.1.1.1 Setup

This model assumes that firms use two inputs: labor and capital. All firms can operate in a maximum of two segments following segment-specific production functions. Suppose that a total of six companies is observed: 1) firms A and B do business only in segment I; 2) firms C and D produce only in segment II; and 3) firms E and F have footprints in both segments. Therefore, firm E (F) is a multidivisional firm.
with division $E_1$ ($F_1$) in segment $I$ and division $E_2$ ($F_2$) in segment $II$. As a result, segment $I$ has four competitors (firms $A$, $B$, $E_1$, and $F_1$), and segment $II$ also has four competitors (firms $C$, $D$, $E_2$, and $F_2$).

A.1.1.2 First System of Equations: Segment-specific Production Functions

The first system of equations consists of two segment-specific production functions, one for each segment. The production functions are assumed to be stochastic and have a Cobb-Douglas form.

\[
\begin{align*}
\ln Y_{i1t} &= \alpha_{i1t} + \beta_{1L} \ln (L_{i1t}) + \beta_{1K} \ln (K_{i1t}) + \nu_{i1t}, \quad \forall i, t \\
\ln Y_{i2t} &= \alpha_{i2t} + \beta_{2L} \ln (L_{i2t}) + \beta_{2K} \ln (K_{i2t}) + \nu_{i2t}, \quad \forall i, t
\end{align*}
\]

(A-1)

where $Y_{ijt}$, $L_{ijt}$, and $K_{ijt}$ are production, labor, and capital, respectively, for firm $i$ in segment/division $j$ at time $t$; the first equation is the production function frontier of segment $I$ and the second equation is the production function frontier of segment $II$. $\beta_{mn}$ is the coefficient of input $n$ at segment $m$. $\nu$ is the noise that is considered normally distributed with a mean of zero and a standard deviation of $\sigma_\nu$. The productivity term $\alpha_{ijt} = \alpha_{jt} - \mu_{ijt}$ has different characteristics under different production methods. In the stochastic frontier analysis, the technical efficiency term $\mu_{ijt}$ has bounded support (positive) and the exponential of $-\mu_{ijt}$ indicates the efficiency level of firm $i$ in segment/division $j$ at time $t$ ($TE_{ijt} = e^{-\mu_{ijt}}$). In the Olley-Pakes framework, the technical efficiency term is a first-order Markov process with unbounded support. All the coefficients $\beta_{mn}$ can be estimated if the input allocations for firms $E$ and $F$ are given.

It is worth noting the observations in each of the two regressions in Eq. (A-1). If single-division and multidivisional firms are assumed to have the same produc-
tion technique within a segment, then the first equation includes $A$, $B$, $E_1$, and $F_1$ (i.e. $\forall i = A, B, E_1, F_1$), while the second equation includes $C$, $D$, $E_2$, and $F_2$ (i.e. $\forall j = C, D, E_2, F_2$). If single-division and multidivisional firms are assumed to have different production techniques within a segment, then the first equation includes only $E_1$ and $F_1$ (i.e. $\forall i = E_1, F_1$), while the second equation includes only $E_2$ and $F_2$ (i.e. $\forall j = E_2, F_2$). In other words, single-division firms are removed from the regression if the latter assumption is selected.

A.1.1.3 Second System of Equations: Profit Maximization

The second system of equations is the profit maximization problem for multidivisional firms $E$ and $F$. Under Assumption 1, multidivisional firms maximize the net-present-value of discounted future profits by maximizing firm-level profits period-by-period, because firms’ decisions don’t have any intertemporal consequences. Therefore, the division managers are willing to report their true profit functions in each period so that the headquarters can better predict firm-level profit function and value function. Under Assumption 3, this study follows the framework in Olley and Pakes (1996) so that the labor used in time $t$ is chosen when the productivity shock at time $t$ is observed, while the capital used in time $t$ is chosen at time $t-1$.

Since this study develops stochastic production functions, the profits are also stochastic, rather than deterministic. Therefore, profit-maximizing firms maximize the mathematical expectation of profit (Zellner, Kmenta, and Dreze 1966). In this model, firm $i$ first decides capital allocations to maximize the expected firm-level profit of time $t$ knowing the production function and price information of time $t-1$ and then decides labor allocations to maximize the expected firm-level profit of time.
t knowing the production function and price information of time \( t \).

\[
\begin{align*}
\max_{K_{ijt}} E[\pi_{it} | I_{it-1}] &= \max_{K_{ijt}} \sum_{j=1,2} \left[ p_{jt-1} A_{ijt-1} L_{ijt-1}^\beta j L_{ijt}^\beta j K_{ijt}^\beta j e^{\sigma^2_j/2} \gamma_j - c_{ijt-1}^L - c_{ijt-1}^K \right] \\
\max_{L_{ijt}} E[\pi_{it} | I_{it}] &= \max_{L_{ijt}} \sum_{j=1,2} \left[ p_{jt} A_{ijt} L_{ijt}^\beta j L_{ijt}^\beta j K_{ijt}^\beta j e^{\sigma^2_j/2} - c_{ijt}^L - c_{ijt}^K \right]
\end{align*}
\]

(A-2)

where

\[
\begin{align*}
c_{ijt}^L &= h_{ist} w_{jt} L_{ijt} \quad \text{and} \quad c_{ijt}^K = h_{ist} \rho_{jt} K_{ijt} \\
\gamma_j &= \exp \left( \mu_{p_{jt}} + \mu_{A_{jt}} + \beta_{jL} \mu_{L_{jt}} + \frac{\sigma_{p_{jt}}^2 + \sigma_{A_{jt}}^2 + \beta_{jL}^2 \sigma_{L_{jt}}^2}{2} \right) \\
\delta_j &= \exp \left( \mu_{\omega_{jt}} + \mu_{L_{jt}} + \frac{\sigma_{\omega_{jt}}^2 + \sigma_{L_{jt}}^2}{2} \right) \quad \text{and} \quad \varphi_j = \exp \left( \mu_{\rho_{jt}} + \sigma_{\rho_{jt}}^2/2 \right) \\
h_{ist} &= h_{ist} \left( w_{1t} L_{11t} + w_{2t} L_{22t} + \rho_{1t} K_{11t} + \rho_{2t} K_{22t} \right) \quad \forall s = 1, 2
\end{align*}
\]

\( \pi \) is the profit, and \( p \) is the output price; \( A_{ijt} \) equals \( \exp(\alpha_{ijt}) \) in Eq. (A-1) and indicates the productivity; \( p_{jt}, \omega_{jt} \) and \( \rho_{jt} \) are the segment average price of output, labor and capital in segment \( j \) at time \( t \), respectively; \( h_1 \) and \( h_2 \) are the spillover effect on labor and capital price that follows Assumption 2, respectively. Firms predict \( \log (x_{ijt}) | \log (x_{ijt-1}) \sim N \left( \log (x_{ijt-1}) + \mu_{x_{jt}}, \sigma_{x_{jt}}^2 \right) \), where \( x = p, A, L, \omega, \rho \) and all the \( \mu \) and \( \sigma \) are known. Therefore, \( \gamma_j, \delta_j, \varphi_j \), and \( \sigma_{x_{jt}}^2 \) are known by companies. They are constant across firms and time, but vary across segments. Eq. (A-2) applies to firm \( i = E, F \).

Setting the first order condition of Eq. (A-2) to zero, the result is:

\[
\begin{align*}
\beta_1 K_{11t-1} A_{11t-1} L_{11t-1}^\beta j L_{11t}^\beta j K_{11t}^\beta j e^{\sigma^2_j/2} \gamma_1 - \rho_{1t-1} c_{1t-1} = 0 \\
\beta_2 K_{22t-1} A_{22t-1} L_{22t-1}^\beta j L_{22t}^\beta j K_{22t}^\beta j e^{\sigma^2_j/2} \gamma_2 - \rho_{2t-1} c_{1t-1} = 0
\end{align*}
\]

(A-3)
and
\[
\begin{align*}
\beta_1 L p_{1t} A_{iit} L_{i1t}^{\beta_1 L - 1} & K_{i1t}^{\beta_1 K} \exp \left( \frac{\sigma_{\epsilon_1}^2}{2} \right) - w_{1t} c_{2t} = 0 \\
\beta_2 L p_{2t} A_{i2t} L_{i2t}^{\beta_2 L - 1} & K_{i2t}^{\beta_2 K} \exp \left( \frac{\sigma_{\epsilon_2}^2}{2} \right) - w_{2t} c_{2t} = 0
\end{align*}
\] (A-4)

where
\[
\begin{align*}
c_{1t} &= h_{i1t}' (w_{1t} L_{i1t} \delta_1 + w_{2t} L_{i2t} \delta_2) + h_{i2t} (\rho_{1t} K_{i1t} \varphi_1 + \rho_{2t} K_{i2t} \varphi_2) \\
c_{2t} &= h_{i1t}' (w_{1t} L_{i1t} + w_{2t} L_{i2t}) + h_{i1t} + h_{i2t} (\rho_{1t} K_{i1t} + \rho_{2t} K_{i2t})
\end{align*}
\]

This study first solves the “perfectly variable” input labor in Eq. (A-4). From Eq. (A-1), the following is known:

\[
R_{ijt} = p_{jt} A_{ijt} L_{ijt}^{\beta_{ijt} L} K_{ijt}^{\beta_{ijt} K} \exp (\nu_{ijt})
\]

\[\Leftrightarrow p_{jt} A_{ijt} L_{ijt}^{\beta_{ijt} L - 1} K_{ijt}^{\beta_{ijt} K} = R_{ijt} / [L_{ijt} \exp (\nu_{ijt})] \] (A-5)

\[\Leftrightarrow p_{jt} A_{ijt} L_{ijt}^{\beta_{ijt} L} K_{ijt}^{\beta_{ijt} K - 1} = R_{ijt} / [K_{ijt} \exp (\nu_{ijt})] \] (A-6)

Plug Eq. (A-5) into Eq. (A-4), and the result is

\[
\begin{align*}
\beta_1 L & \exp \left( \frac{\sigma_{\epsilon_1}^2}{2} \right) R_{11t} / [L_{11t} \exp (\nu_{11t})] = w_{1t} c_{2t} \\
\beta_2 L & \exp \left( \frac{\sigma_{\epsilon_2}^2}{2} \right) R_{22t} / [L_{22t} \exp (\nu_{22t})] = w_{2t} c_{2t}
\end{align*}
\]

Then, this study divides the first by the second equation. Adding the observed firm-
Plugging Eq. (A-6) into Eq. (A-8):

\[
\begin{aligned}
\begin{cases}
\beta_{1L} R_{it1} L_{it2} \exp (\nu_{it2} + 0.5 \sigma_{v2}^2) = w_{it} \\
\beta_{2L} R_{it2} L_{it1} \exp (\nu_{it1} + 0.5 \sigma_{v1}^2) = w_{it} \\
L_{it1} + L_{it2} = L_{it}
\end{cases}
\end{aligned}
\] (A-7)

In Eq. (A-7), segment actual revenues \( (R) \) and average segment labor price \( (\omega) \) are observed. The rest of the variables, including the production parameters \( (\beta) \) and the noises \( (\nu) \), can be estimated in Eq. (A-1).

After solving the labor allocations, this study solves the capital inputs. Eq. (A-3) can be written as

\[
\begin{aligned}
\begin{cases}
\beta_{1K} p_{it1} A_{it1} \left( \frac{L_{it1} - 1}{L_{it1}} \right)^{\beta_{1L}} p_{it} A_{it} L_{it1}^{\beta_{1L}} K_{it1}^{\beta_{1K} - 1} \left[ \exp \left( \frac{\sigma_{v1}^2}{2} \right) \right] \gamma_1 = \rho_{1t-1} c_{1t-1} \\
\beta_{2K} p_{it2} A_{it2} \left( \frac{L_{it2} - 1}{L_{it2}} \right)^{\beta_{2L}} p_{it2} A_{it2} L_{it2}^{\beta_{2L}} K_{it2}^{\beta_{2K} - 1} \left[ \exp \left( \frac{\sigma_{v2}^2}{2} \right) \right] \gamma_2 = \rho_{2t-1} c_{1t-1}
\end{cases}
\end{aligned}
\] (A-8)

Plugging Eq. (A-6) into Eq. (A-8):

\[
\begin{aligned}
\begin{cases}
\beta_{1K} z_{it1} \left[ R_{it1} / [ K_{it1} \exp (\nu_{it1})] \right] \exp \left( \frac{\sigma_{v1}^2}{2} \right) \gamma_1 = \rho_{1t-1} c_{1t-1} \\
\beta_{2K} z_{it2} \left[ R_{it2} / [ K_{it2} \exp (\nu_{it2})] \right] \exp \left( \frac{\sigma_{v2}^2}{2} \right) \gamma_2 = \rho_{2t-1} c_{1t-1}
\end{cases}
\end{aligned}
\]

where

\[
z_{ijt} = \frac{p_{jt-1} A_{ijt-1}}{p_{jt} A_{ijt}} \left( \frac{L_{ijt-1}}{L_{ijt}} \right)^{\beta_{ijt}}
\]
Similar to the transformation of labor, this study divides the first equation by the second. Adding the observed firm-level capital input equation, the result is

\[
\begin{align*}
\beta_1 K z_{i1t} R_{i1t} K_{i2t} \exp (\nu_{i2t} + 0.5 \sigma_{v1}^2) \gamma_1 &= \rho_{1t-1} \\
\beta_2 K z_{i2t} R_{i2t} K_{i1t} \exp (\nu_{i1t} + 0.5 \sigma_{v2}^2) \gamma_2 &= \rho_{2t-1} \\
K_{i1t} + K_{i2t} &= K_t
\end{align*}
\]

\[
\iff K_{i1t} = \frac{\beta_1 K R_{i1t} \rho_{2t-1}}{\beta_1 K R_{i1t} \rho_{2t-1} + \beta_2 K R_{i2t} \rho_{1t-1} (\gamma_2/\gamma_1) \exp (\nu_{i1t} - \nu_{i2t} + 0.5 \sigma_{v2}^2 + 0.5 \sigma_{v1}^2)} K_t \\
K_{i2t} = \frac{\beta_2 K R_{i2t} \rho_{1t-1}}{\beta_1 K R_{i1t} \rho_{2t-1} (\gamma_1/\gamma_2) \exp (\nu_{i2t} - \nu_{i1t} + 0.5 \sigma_{v1}^2 - 0.5 \sigma_{v2}^2) + \beta_2 K R_{i2t} \rho_{1t-1}} K_t
\]

In Eq. (A-9), segment actual revenues (\(R\)), average segment capital price (\(\rho\)), and average segment labor price (\(\omega\)) are observed. If Eq. (A-1) can be estimated, other variables, including the production parameters (\(\beta\)), the productivity shock (\(A\)), the noises (\(\nu\)) and their variances (\(\sigma_{\nu}^2\)), as well as \(z\) and \(\gamma\), are all known. Then the capital allocations in Eq. (A-9) can be solved.

Eqs. (A-7) and (A-9) consist of the solutions of the second system of equations, which can be combined into Eq. (A-10)

\[
\begin{align*}
L_{i1t} &= \frac{\beta_1 L R_{i1t} w_{2t}}{\beta_1 L R_{i1t} w_{2t} + \beta_2 L R_{i2t} w_{1t} \exp (\nu_{i1t} - \nu_{i2t} + 0.5 \sigma_{\nu1}^2 - 0.5 \sigma_{\nu2}^2)} L_{it} \\
L_{i2t} &= \frac{\beta_2 L R_{i2t} w_{1t}}{\beta_1 L R_{i1t} w_{2t} \exp (\nu_{i2t} - \nu_{i1t} + 0.5 \sigma_{\nu1}^2 - 0.5 \sigma_{\nu2}^2) + \beta_2 L R_{i2t} w_{1t}} L_{it} \\
K_{i1t} &= \frac{\beta_1 K R_{i1t} \rho_{2t-1}}{\beta_1 K R_{i1t} \rho_{2t-1} + \beta_2 K R_{i2t} \rho_{1t-1} (\gamma_2/\gamma_1) \exp (\nu_{i1t} - \nu_{i2t} + 0.5 \sigma_{\nu2}^2 - 0.5 \sigma_{\nu1}^2)} K_t \\
K_{i2t} &= \frac{\beta_2 K R_{i2t} \rho_{1t-1}}{\beta_1 K R_{i1t} \rho_{2t-1} (\gamma_1/\gamma_2) \exp (\nu_{i2t} - \nu_{i1t} + 0.5 \sigma_{\nu1}^2 - 0.5 \sigma_{\nu2}^2) + \beta_2 K R_{i2t} \rho_{1t-1}} K_t
\end{align*}
\]
where

\[
\begin{align*}
\gamma_j &= \exp \left( \mu_{pj} + \mu_{Aj} + \beta_{jL} \mu_{Lj} + \frac{\sigma_{pj}^2 + \sigma_{Aj}^2 + \beta_{jL}^2 \sigma_{Lj}^2}{2} \right) \\
\end{align*}
\]

This system says that for each input, the ratio of expected marginal revenue equals the input price ratio across divisions. In other words, the cost for an input to generate additional expected revenue is equal across divisions at the undistorted allocations; hence, this study calls it the "equal (expected) marginal revenue per cost" condition. If the first system of equations in Eq. (A-1) is estimated, the input allocations for both firms E and F can be imputed in the second system of equations in Eq. (A-10).

### A.1.1.4 Iterative Algorithm

These two systems of Eqs. (A-1) and (A-10) need to be jointly solved to derive division-level inputs and segment-specific production functions. This study implements an iterative method through the following procedures.

1. **Step 1:** For each input, preset a reasonable guess of the initial allocations. This study assumes input proportionality, which is recommended by Foster, Haltiwanger, and Syverson (2008) and utilized as the first guess of division-level inputs.\(^1\)

2. **Step 2:** Estimate the segment-specific production functions in Eq. (A-1) using the initial guess of division-level inputs in Step 1.

3. **Step 3:** Update the division-level input allocations in Eq. (A-10) using the estimated segment-specific production functions derived in Step 2.

4. **Step 4:** Repeat Steps 2 and 3 until a given stationary threshold of the change in

\(^1\) In practice, other methods will also be used to set the initial allocations to see if the iteration converges at the same undistorted allocations. The methods and results are discussed in Appendix B.
coefficients is achieved by successive iterations.

Finally, the estimated division-level inputs are the undistorted allocations of the unobserved inputs, and the estimated production functions present how inputs are converted into output in each segment.

After the input allocations are imputed from the iterations, this study tests if the key assumption that single-division and multidivisional firms use the same production techniques within a segment is correct. For each segment, this study creates a dummy variable of multidivisional firms and then regresses the output on inputs, the dummy, and the intersections between each input and the dummy variable. If any coefficient of the intersections and the dummy variable is statistically significant, then the key assumption that most previous works (Olley and Pakes 1996; Levinsohn and Petrin 2003; De Loecker et al. 2015) rely on is invalid. On the other hand, evidence is achieved to support this key assumption if all the coefficients of the intersections are insignificant.

This study recommends imputing input allocations assuming the key assumption is valid first so that both observations of single-division and multidivisional firms can be used in the first system of equations. This is especially important with a small sample size. If the test then shows the assumption to be invalid, the iterations can be rerun after removing the observations of single-division firms in the first system of equations.

The next two subsections generalize the model, allowing for more inputs, segments, and multidivisional firms, as well as different production functional forms. The algorithm is analogous to the simple model above and can derive Theorem 1. Those who are not interested in the details can skip the rest of Appendix A.
A.1.2 General Model

A.1.2.1 Setup

As an extension of the simplified model, the general model considers a generalized production function and analyzes a “M Inputs – N Products/Segments – T periods” economy. Moreover, firms can have footprints in one or multiple segments and enter or exit segments over time.

A.1.2.2 Single-Frontier Analysis

The canonical stochastic frontier analysis for panel data, or what this study calls the “Single-Frontier” estimator, runs the regression for individual (country, region, company, etc.) $i$ at time $t^2$:

$$Y_{it} = f(X_{it}; \beta_0) \exp(\tau Z) \exp(\nu_{it}) \exp(-u_{it}) \quad (A-11)$$

where $Y_{it}$ is the total output of individual $i$ at time $t$; $X_{it} = (X_{it1}, X_{it2}, ..., X_{itM})$ vectors the $M$ types of inputs; $f(X_{it}; \beta_0) \cdot \exp(\tau Z)$ is the production frontier over time, where $f(X_{it}; \beta_0)$ is the time-invariant part of the production function, $\beta_0 = (\beta_{01}, \beta_{02}, ..., \beta_{0M})$ is a vector of technical parameters to be estimated. $Z$ vectors a group of year dummy variables, controls the production frontier change across time and $\tau$ vectors the coefficients of time; $\exp(\nu_{it})$ is the stochastic component that describes random shocks affecting the production process, where $\nu_{it}$ is assumed to be normally distributed with a mean of zero and a standard deviation of $\sigma_\nu$; and $TE_{it} = \exp(-\mu_{it})$ denotes the time-variant technical efficiency defined as the ratio of observed output to maximum feasible output. $TE_{it} = 1$ or $\mu_{it} = 0$ shows that the $i$-th individual allocates at the production frontier and obtains the maximum feasible output.

---

$^2$ Eq. (A-11) is the same as Eq. (2.1)

$^3$ In the simple example for this study, this production function has a Cobb-Douglas form.
output at time \( t \), while \( TE_{it} < 1 \) or \( u_{it} > 0 \) provides a measure of the shortfall of the observed output from the maximum feasible output. This study uses the error components specification with time-varying efficiencies (Battese and Coelli 1992), where 
\[
\mu_{it} = \exp(-\eta(t - T)) \ast \mu_i.
\]
To sum up, the coefficients of the production function in Eq. (A-11) are time-invariant, while the firm-specific intercept term is shifted by a common time varying component. This study will test if the coefficients of the production function also change over time in the empirical analysis.

### A.1.2.3 Meta-Frontier Analysis

As has been discussed, Eq. (A-11) assumes a unique production function across segments and can only derive firm-level efficiency without segment-specific production function concern. The “Meta-Frontier” analysis in a “\( M \) Inputs – \( N \) Products/Segments – \( T \) periods” economy imputes the input allocations by solving two systems of equations simultaneously.

Each equation in the first system of \( N \) equations (A-12) describes the production techniques for the corresponding segment:

\[
\begin{align*}
Y_{i1t} &= f_1(X_{i1t}; \beta_1) \exp(\tau_1 Z_1) \exp(\nu_{i1t}) \exp(-u_{i1t}) \\
& \quad \vdots \\
Y_{iNt} &= f_N(X_{iNt}; \beta_N) \exp(\tau_N Z_N) \exp(\nu_{iNt}) \exp(-u_{iNt})
\end{align*}
\]

(Eq. A-12)

where \( Y_{ijt} \) represents the observed scalar output and \( X_{ijt} \) vectors the unobserved inputs of individual \( i \) in segment \( j \) at time \( t \), respectively. \( f_j(X_{ijt}; \beta_j) \) is the heterogeneous production frontier for segment \( j \), where \( \beta_j = (\beta_{j1}, \beta_{j2}, ...) \) is a vector of segment-specific technical parameters; \( Z_j = (Z_{j2}, Z_{j3}, ..., Z_{jT}) \) vectors a group of year dummy variables to control the production frontier change across time and \( \tau_j = \)

\footnote{Eq. (A-12) is the same as Eq. (2.2)}
(\tau_{j2}, \tau_{j3}, \ldots, \tau_{jT}) \) vectors the coefficients of the year dummy variables; and \( u_{ijt} = \exp(-\eta(t-T)) \cdot \mu_{ij} \) is the time-variant efficiency indicator. \( \nu_{ijt} \) is the noise that is considered normally distributed with a mean of zero and a standard deviation of \( \sigma_{\nu_{ij}} \). The data pooled into the \( j \)th equation in Eq. (A-12) depends on the validity of the key assumption. If assuming that single-division firms and multidivisional firms have the same production techniques, all the players in segment \( j \) are included in the regression. Otherwise, the single-division firms are removed from the regression.

The second system of equations solves the profit maximization problem of multidivisional firms. For firm \( i \) at time \( t \), the mathematical expectation of the profit function is

\[
\max_{X_{ijt}^M} \mathbb{E}[\pi_{it} | I_{it-1}] = \max_{X_{ijt}^M} \sum_{j \in W(i,t)} \left( \mathbb{E}[p_{jt} Y_{ijt} | I_{it-1}] - \mathbb{E}\left[ \sum_{k=1}^{M} h_{ikt} c_{jkt} X_{ijt}^k | I_{it-1} \right] \right)
\]

\[
\max_{X_{ijt}^k} \mathbb{E}[\pi_{it} | I_{it}] = \max_{X_{ijt}^k} \sum_{j \in W(i,t)} \left( \mathbb{E}[p_{jt} Y_{ijt} | I_{it}] - \mathbb{E}\left[ \sum_{k=1}^{M} h_{ikt} c_{jkt} X_{ijt}^k | I_{it} \right] \right), \forall k < M
\]

(A-13)

where

\[
h_{ikt} = h_{ikt} \left( \sum_{k=1}^{M} c_{jkt} X_{ijt}^k \right), \forall k = 1, 2, \ldots, M
\]

and

\[
Y_{ijt} = f_j(X_{ijt}; \beta_j) \exp(\tau_j Z_j) \exp(\nu_{ijt}) \exp(-u_{ijt}), \forall j \in W(i,t)
\]

(A-14)

where \( Y_{ijt} \) and \( X_{ijt}^k \) represent the observed scalar output and the unobserved \( k \)-th input of individual \( i \) in segment \( j \) at time \( t \), respectively. The last input, \( X_{ijt}^M \), is the division-level capital input. \( h_{ikt} \) is the spillover effects on the \( k \)-th input for firm \( i \) at time \( t \). \( c_{jkt} \) is the average price of the \( k \)-th input in segment \( j \) at time \( t \). Similar to the
simple example in Subsection A.1.1.3, the general model also assumes that firms predict $\log(x_{ijt}) | \log(x_{ijt}) \sim N(\log(x_{ijt}) + \mu_{xj}, \sigma^2_{xj})$ where $x = p, A, X^k, \omega, \rho$ and all the $\mu$ and $\sigma$ are known. $W(i, t)$ is a subset of $W = (1, 2, ..., N)$, indicating the segments in which individual $i$ at time $t$ has a footprint. When estimating region or country productivity, $W(i, t)$ is usually similar to $W$ since most countries have production in every sector or segment. A counterexample is Singapore which has a negligible agriculture industry. For company productivity analysis, $W(i, t)$ is smaller than $W$ in most cases, as only a few integrated companies do business in every segment. Take this study’s empirical study as an example; only one out of 114 firms does business in all five segments of the oilfield market.

Using the same strategy in Eq. (A-2), this study first solves all the “perfectly variable” inputs and then solves the capital input. By solving the first order condition of Eq. (A-13), the “equal (expected) marginal revenue per cost” condition can be derived. Adding the observed firm-level inputs, the first part of Theorem 1 (Eq. (2.3)) is derived

$$\begin{cases}
\frac{\partial E[R_{ijt} | I_{it}]}{\partial X^k_{ijt}} = \frac{c^k_{jt}}{c^k_{j't}} = r^k_{jj't}, \forall i, t, \forall k < M, \forall j, j' \in W(i, t) \\
\frac{\partial E[R_{ij't} | I_{t-1}]}{\partial X^M_{ij't}} = \frac{c^M_{j't-1}}{c^M_{jj't}} = r^M_{jj't-1}, \forall i, t, \forall j, j' \in W(i, t)
\end{cases}$$

(A-15)

where $c^k_{jt}$ is the average price of the $k$-th input in segment $j$ at time $t$; $r^k_{jj't}$ represents the ratio of average price for the $k$-th input between segments $j$ and $j'$ at time $t$.

Suppose $N(i, t)$ is the number of elements (i.e., the number of segments entered by firm $i$ at time $t$) in set $W(i, t)$, and the system of Eq. (A-15) has $\sum_{i,t} N(i, t) \cdot M$
equations with \( \sum_{i,t} N(i,t) \cdot M \) unknown division-level inputs (i.e. the black box of input allocations). If the first system of equations in Eq. (A-12) can be estimated, then system (A-15) is just-identified and the input allocations can be imputed.

This study looks for the undistorted allocations of segment-level inputs \((X_{ijt}^k)\) that jointly solves the system of Eq. (A-12) and the system of Eq. (A-15). In order to find the undistorted input allocations and the segment-specific production functions that satisfy Eq. (A-12) and Eq. (A-15) simultaneously, this study implements the same iterative method discussed in Subsection A.1.1.4.

### A.1.3 Parametric Production Function Specification

Since the stochastic frontier analysis is a parametric approach, the mathematical form for the production function needs to be specified. This study discusses the solution of the undistorted input allocations when the production function \(f_j(X_{ijt}; \beta_j)\) has the most widely used forms: the Cobb-Douglas and Transcendental Logarithmic forms.

#### A.1.3.1 Cobb-Douglas Production Function

Given a C-D form, the canonical model in Eq. (A-11) has the form:

\[
Y_{it} = \exp(\alpha) \prod_{k=1}^{2} (X_{it}^k)^{\beta_{0k}} \exp(\tau Z) \exp(\nu_{it}) \exp(-u_{it})
\]

\[
\Leftrightarrow \ln Y_{it} = \alpha + \sum_{k=1}^{2} \beta_{0k} \ln(X_{it}^k) + \tau Z + \nu_{it} - u_{it}
\]  

(A-16)
The segment-specific production function in Eq. (A-12) becomes:

\[
\begin{align*}
\ln Y_{i1t} &= \alpha_1 + \sum_{k=1}^{M} \beta_{1k} \ln(X_{i1t}^k) + \tau_1 Z_1 + \nu_{i1t} - u_{i1t} \\
\vdots & \hspace{2cm} \text{(A-17)} \\
\ln Y_{iNt} &= \alpha_N + \sum_{k=1}^{M} \beta_{Nk} \ln(X_{iNt}^k) + \tau_N Z_N + \nu_{iNt} - u_{iNt} 
\end{align*}
\]

The “equal marginal revenue per cost” constraint is then:

\[
\begin{align*}
\frac{\partial \mathbb{E}[R_{ijt}|I_{it}]}{\partial X_{ijt}^k} &= \beta_{jk} R_{ijt} X_{ijt}^k \exp \left( \nu_{ijt} + 0.5 \sigma_{v_{ijt}}^2 \right) = r_{jj't}^k, \forall i, t, \forall k < M, \forall j, j' \in W(i, t) \\
\frac{\partial \mathbb{E}[R_{ij't}|I_{it-1}]}{\partial X_{ij't}^k} &= \beta_{jk} z_{ij't} R_{ij't} X_{ij't}^M \exp \left( \nu_{ij't} + 0.5 \sigma_{v_{ij't}}^2 \right) \gamma_j = r_{jj't-1}^M, \forall i, t, \forall j, j' \in W(i, t) \\
\end{align*}
\]

where

\[
\begin{align*}
z_{ijt} &= \frac{p_{jt-1}}{p_{jt}} \frac{A_{ijt-1}}{A_{ijt}} \prod_{k=1,\ldots,M-1} \left( \frac{X_{ijt-1}^k}{X_{ijt}^k} \right)^{\beta_{jk}} \\
A_{ijt} &= \exp (\alpha_j + \tau_{jt} - u_{ijt})
\end{align*}
\]

Given that and the observed firm-level inputs, it is easy to solve the system of equations:

\[
\begin{align*}
X_{ijt}^k &= \frac{\beta_{jk} R_{ijt}}{\sum_{j' \in W(i,t)} \beta_{jk} R_{ij't} r_{jj't}^k \exp \left( \nu_{ij't} - \nu_{ij't} + 0.5 \sigma_{v_{ij't}}^2 - 0.5 \sigma_{v_{ijt}}^2 \right) X_{ij't}^k}, \forall k < M \\
X_{ijt}^M &= \frac{\beta_{jM} R_{ijt}}{\sum_{j' \in W(i,t)} \beta_{jM} R_{ij't} r_{jj't}^M \left( \gamma_{j'/j} \right) \left( z_{ij't} / z_{ijt} \right) \exp \left( \nu_{ij't} - \nu_{ij't} + 0.5 \sigma_{v_{ij't}}^2 - 0.5 \sigma_{v_{ijt}}^2 \right) X_{ij't}^M} \quad \text{(A-18)}
\end{align*}
\]
where
\[ \gamma_j = \exp \left( \mu_{pj} + \mu_{Aj} + \sum_{k=1}^{M-1} \beta_{jk} \mu_{X_{kj}} + \frac{\sigma_{pj}^2 + \sigma_{Aj}^2 + \sum_{k=1}^{M-1} \beta_{jk}^2 \sigma_{X_{kj}}^2}{2} \right) \]

Eq. (A-18) is the second part of Theorem 1 (Eq. (2.4)). The solution exists when all the parameters \( \beta_{jk} \) are positive. Finally, the iteration method is used to solve the undistorted input allocations and technical parameters jointly from Eqs. (A-17) and (A-18).

**A.1.3.2 Transcendental Logarithmic Production Function**

Given a T-L form, the production function in Eq. (A-11) becomes:

\[
\ln Y_{it} = \alpha + \sum_{k=1}^{M} \beta_{ok} \ln X_{it}^k + \frac{1}{2} \sum_{k=1}^{M} \sum_{l=1}^{M} \delta_{0kl} \ln X_{it}^k \ln X_{it}^l + \tau Z + \nu_{it} - u_{it}
\]

where
\[ \delta_{0kl} = \delta_{0lk}, \forall k, l \in (1, 2, \ldots, M) \]

Similarly, the segment-level production function in Eq. (A-12) becomes:

\[
\begin{align*}
\ln Y_{i1t} &= \alpha_1 + \sum_{k=1}^{M} \beta_{1k} \ln X_{i1t}^k + \frac{1}{2} \sum_{k=1}^{M} \sum_{l=1}^{M} \delta_{1kl} \ln X_{i1t}^k \ln X_{i1t}^l + \tau_1 Z_1 + \nu_{i1t} - u_{i1t} \\
& \vdots \\
\ln Y_{iNt} &= \alpha_N + \sum_{k=1}^{M} \beta_{Nk} \ln X_{iNt}^k + \frac{1}{2} \sum_{k=1}^{M} \sum_{l=1}^{M} \delta_{Nkl} \ln X_{iNt}^k \ln X_{iNt}^l + \tau_N Z_N + \nu_{iNt} - u_{iNt}
\end{align*}
\]

where
\[ \delta_{jkl} = \delta_{jlk}, \forall k, l \in (1, \ldots, M), \forall j = 1, \ldots, N \]
Interestingly, this ratio itself is a C-D function.

\[
\begin{align*}
\frac{\partial E[R_{ij'}|I_{it}]}{\partial X^k_{ij'}^{t}} &= (\beta_{jk} + \sum_{l=1}^{M} \delta_{jk} l \ln X^l_{ij'}) R_{ij'} X^k_{ij'}^{t} \exp \left( \nu_{ij't} + 0.5 \sigma_{ij'}^2 \right) = r^k_{ij't}, \forall k < M \\
\frac{\partial E[R_{ij'}|I_{it}]}{\partial X^k_{ij'}^{t}} &= \frac{\partial E[R_{ij}|I_{it}]}{\partial X^k_{ij'}^{t-1}} = z_{ij} \left( \beta_{jk} + \sum_{l=1}^{M} \delta_{jk} l \ln X^l_{ij} \right) R_{ij} X^k_{ij}^{t} \exp \left( \nu_{ij't} + 0.5 \sigma_{ij'}^2 \right) \gamma_j = r^M_{ij't-1} \\
\frac{\partial E[R_{ij'}|I_{it}]}{\partial X^k_{ij'}^{t-1}} &= \frac{\partial E[R_{ij}|I_{it}]}{\partial X^k_{ij'}^{t}} = z_{ij'} \left( \beta_{jk} + \sum_{l=1}^{M} \delta_{jk} l \ln X^l_{ij'} \right) R_{ij't} X^k_{ij'}^{t} \exp \left( \nu_{ij't} + 0.5 \sigma_{ij'}^2 \right) \gamma_{j'} = r^M_{ij't-1}
\end{align*}
\]

where

\[
z_{ij} = \frac{p_{jt-1}}{p_{jt}} A_{ij} - 1 \prod_{k=1,\ldots,M-1} \left( \frac{X^k_{ij}^{t-1}}{X^k_{ij}^{t}} \right)^{\beta_{jk}} \prod_{k,l=1,\ldots,M} \frac{\exp \left( \frac{1}{2} \delta_{ijkl} \ln X^k_{1l} \ln X^l_{1l} \right)}{\exp \left( \frac{1}{2} \delta_{ijkl} \ln X^k_{1l} \ln X^l_{1l} \right)}
\]

\[
A_{ij} = \exp \left( \alpha_j + \tau_j - u_{ij} \right) \quad \text{and} \quad k + l < 2M
\]

\[
\gamma_j = e^{\left( \mu_{j} + \sum_{k=1}^{M} \beta_{jk} \mu_{X^k_{ij}} + \sum_{k=1}^{M} \beta_{jk} \sigma_{ij}^2 + \sum_{k,l=1}^{M} \beta_{ijkl} \sigma_{ij}^2 \sigma_{kl}^2 + \sum_{k,l=1}^{M} \left( \beta_{jk} \mu_{X^k_{ij}} + \beta_{jl} \sigma_{ij}^2 \right) \right)}
\]

Compared with Eq. (A-17) in the C-D case, Eq. (A-19) in the T-L case adds the \( \sum_{l=1}^{M} \delta_{jk} l \ln X^l_{ij} \) part, which makes the latter case much more complicated to solve. Interestingly, this ratio itself is a C-D function.

In the T-L case, the system of equations in Eq. (A-15) becomes

\[
\begin{align*}
\frac{\partial E[R_{ij}|I_{it}]}{\partial X^k_{ij}^{t}} &= (\beta_{jk} + \sum_{l=1}^{M} \delta_{jk} l \ln X^l_{ij} \right) R_{ij} X^k_{ij}^{t} \exp \left( \nu_{ij't} + 0.5 \sigma_{ij'}^2 \right) = r^k_{ij't}, \forall k < M \\
\frac{\partial E[R_{ij'}|I_{it}]}{\partial X^k_{ij'}^{t}} &= \frac{\partial E[R_{ij}|I_{it}]}{\partial X^k_{ij'}^{t}} = z_{ij} \left( \beta_{jk} + \sum_{l=1}^{M} \delta_{jk} l \ln X^l_{ij} \right) R_{ij} X^k_{ij}^{t} \exp \left( \nu_{ij't} + 0.5 \sigma_{ij'}^2 \right) \gamma_j = r^M_{ij't-1} \\
\frac{\partial E[R_{ij}|I_{it}]}{\partial X^k_{ij'}^{t}} &= \frac{\partial E[R_{ij'}|I_{it}]}{\partial X^k_{ij'}^{t-1}} = z_{ij'} \left( \beta_{jk} + \sum_{l=1}^{M} \delta_{jk} l \ln X^l_{ij'} \right) R_{ij't} X^k_{ij'}^{t} \exp \left( \nu_{ij't} + 0.5 \sigma_{ij'}^2 \right) \gamma_{j'} = r^M_{ij't-1}
\end{align*}
\]

\[
\sum_{j \in W_{it}(t)} X^k_{ij} = X^k_{it}, \forall i, k, t
\]

(A-20)

Eq. (A-20) is the third part of Theorem 1 (Eq. (2.5)). This system of equations is just-identified, where the number of unknown variables equals the number of equa-
tions. However, whether a solution exists depends on the initial inputs, i.e., $\beta_{jk}, \delta_{jkl}, \ Y_{ijt}, X_{it}^{k}, r_{jijt}^{k}, M,$ and $N(i, t)$. The T-L production function is a flexible functional form and a generalization of the C-D production function (Allen and Hall 1997). Compared with the C-D form, the T-L form does not need the perfect substitution between inputs restriction or the linear input-output restriction (Klacek, Vošvrda, and Schlosser 2007). However, an easy solution cannot be obtained for the T-L form in Eq. (A-20) as was done for the C-D form solution in Eq. (A-18).
A.2 Accuracy Test of the Estimation

A.2.1 OECD Data

This appendix uses a panel data for OECD countries to test whether the imputation method estimate accurate input allocation. Many studies treat countries or regions as firms and use a production frontier approach to estimate the productivity and efficiency of the whole economy. Koop, Osiewalski, and Steel (1999) decompose output change into technical, efficiency and input changes for seventeen OECD countries by using stochastic frontier methods. Mastromarco and Ghosh (2009) use stochastic frontier analysis to derive the total factor productivity for fifty-seven developing countries. Koop, Osiewalski, and Steel (1995) measure the source of growth in four regions, consisting of East Asia, Africa, Latin America, and West. Other studies focus on cross-state or cross-province rather than cross-country productivity analysis such as the thirty provinces of China (Wei and Hao 2011) and the forty-eight contiguous US states (Puig-Junoy 2001). However, all of these studies utilize one production function for the entire economy without considering differences across segments. With regard to segment-specific production functions, the “Meta-Frontier” method can impute segment-level input allocations in each country.

This study uses panel data for twenty-two OECD countries\(^5\) from 2000 to 2006 where labor and capital are the inputs and the value-added is the output. The dataset is mainly collected from the Structural Analysis (STAN) Database, the Annual National Accounts (ANA) Database, and the Monthly Monetary and Financial Statistics (MEI) Database, all of which are in the OECD iLibrary\(^6\).

---

\(^5\) These twenty-two OECD countries are Australia, Austria, Belgium, the Czech Republic, Denmark, Estonia, Finland, France, Germany, Hungary, Iceland, Ireland, Italy, Japan, the Netherlands, New Zealand, Poland, the Slovak Republic, Spain, Sweden, the United Kingdom, and the United States.

\(^6\) http://www.oecd-ilibrary.org/
Based on the sector classification of OECD data, the total economy of a country can be divided into primary, secondary, and tertiary sectors and further classified into nine industries:

I) the primary sector is segmented into 1) agriculture, hunting, forestry and fishing, and 2) mining and quarrying;

II) the secondary sector is divided into three categories, consisting of 3) manufacturing, 4) electricity, gas and water supply, and 5) construction; and

III) the tertiary sector includes 6) wholesale, retail trade, restaurants and hotels, 7) transport, storage and communications, 8) finance, insurance, real estate and business services, and 9) community, social, and personal services.

Our dataset has segment-level output, inputs allocations, and price data for each country. The output is value added in US dollars by segment and country. For the labor input, total employment and compensation for employees are collected. The former is a total quantity and the latter is divided by the former, thus providing labor price information. For the capital input, net capital stock/net fixed assets, consumption of fixed assets, and interest rate are available. The consumption of fixed assets divided by the net capital stock/net fixed asset is the depreciation rate. The price of capital, or the user cost of capital, is the sum of the depreciation rate and the interest rate. Table A-1 provides summary statistics for the inputs and outputs by segment.

A 3 Segments model is set up for OECD countries, where the output is value added \( Y_{it} \), the two inputs are labor \( (X^1_{it} = L_{it}) \) and capital \( (X^2_{it} = K_{it}) \), and the three segments are the primary, secondary, and tertiary sectors. In order to check the robustness of the results, this study further classifies the economy using nine industries and imputes the undistorted input allocations for a 9 Segments model.
Table A-1: OECD Data Summary Statistics

<table>
<thead>
<tr>
<th>Unit</th>
<th>Value Added</th>
<th>Labor Price of Added</th>
<th>Capital Price of Added</th>
<th>Capital</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Economy</td>
<td>1.196</td>
<td>18.572</td>
<td>22.69</td>
<td>3.295</td>
<td>7.95</td>
</tr>
<tr>
<td>I) Primary Sector</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agriculture, Hunting, Forestry &amp; Fishing</td>
<td>0.018</td>
<td>0.668</td>
<td>10.58</td>
<td>0.064</td>
<td>9.78</td>
</tr>
<tr>
<td>Mining &amp; Quarrying</td>
<td>0.013</td>
<td>0.064</td>
<td>37.22</td>
<td>0.064</td>
<td>9.83</td>
</tr>
<tr>
<td>II) Secondary Sector</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.190</td>
<td>2.865</td>
<td>28.27</td>
<td>0.254</td>
<td>11.26</td>
</tr>
<tr>
<td>Electricity, Gas &amp; Water Supply</td>
<td>0.025</td>
<td>0.165</td>
<td>37.79</td>
<td>0.140</td>
<td>7.97</td>
</tr>
<tr>
<td>Construction</td>
<td>0.066</td>
<td>1.417</td>
<td>19.89</td>
<td>0.068</td>
<td>10.88</td>
</tr>
<tr>
<td>III) Tertiary Sector</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trade, Restaurants &amp; Hotels</td>
<td>0.195</td>
<td>3.948</td>
<td>15.64</td>
<td>0.182</td>
<td>10.02</td>
</tr>
<tr>
<td>Transport, Storage &amp; Communications</td>
<td>0.087</td>
<td>1.074</td>
<td>36.16</td>
<td>0.279</td>
<td>9.72</td>
</tr>
<tr>
<td>Finance, Insurance, Real Estate &amp; Business Svs</td>
<td>0.323</td>
<td>2.483</td>
<td>27.32</td>
<td>1.607</td>
<td>6.72</td>
</tr>
<tr>
<td>Community, Social &amp; Personal Services</td>
<td>0.284</td>
<td>5.890</td>
<td>23.73</td>
<td>0.728</td>
<td>7.98</td>
</tr>
</tbody>
</table>

Since all estimations of the technical parameters ($\beta_{jk}$) are positive, the system of equations in Eq. (A-12) has a solution for all the observations in the C-D case. When T-L production functions are assumed, however, less than 10% of the observations have solutions in Eq. (A-14) and can be refined. Therefore, only the accuracy of the estimation when production functions have C-D forms is tested.

### A.2.2 Testing Method

“Equal (expected) marginal revenue per cost” estimation. The undistorted allocations of segment-level inputs $\hat{X}_{ijt}^k$ for the OECD countries imputed from the “Meta-Frontier” approach are called the “equal (expected) marginal product per cost”
estimation, which satisfies

\[
\begin{align*}
\frac{\beta_{jk} R_{ijt} X_{ijt}^k \exp \left( \nu_{ijt} + 0.5 \sigma_j^2 \right)}{\beta_{j'k} R_{ij't} X_{ij't}^k \exp \left( \nu_{ij't} + 0.5 \sigma_{j'}^2 \right)} &= r_{j'j't}, \forall i, t, \forall k < M, \forall j, j' \in W(i, t) \\
\frac{z_{ijt} \beta_{jk} R_{ijt} X_{ijt}^M \exp \left( \nu_{ijt} + 0.5 \sigma_j^2 \right)}{z_{ij't} \beta_{j'k} R_{ij't} X_{ij't}^M \exp \left( \nu_{ij't} + 0.5 \sigma_{j'}^2 \right)} &= r_{j'j't-1}, \forall i, t, \forall j, j' \in W(i, t)
\end{align*}
\]

(A-21)

where

\[
z_{ijt} = \frac{p_{jt-1} A_{ijt-1}}{p_{jt} A_{ijt}} \prod_{k=1,\ldots,M-1} \left( \frac{X_{ijt-1}^k}{X_{ijt}^k} \right)^{\beta_{jk}}
\]

This section tests whether method 1 (Eq. (A-21)) imputes allocations that are closer to the actual allocations than those based on the other three methods (Eqs. (A-22) - (A-24)). The three alternative methods for generating the missing allocations are presented below.

**“Equal revenue per input” estimation.** If both the differences in price and production function across segments are ignored in Eq. (A-21), then the segment-level input allocations are in proportion to the actual revenue across segments:

\[
\frac{R_{ijt}}{R_{ij't}} \frac{X_{ijt}^k}{X_{ij't}^k} = 1, \forall k = 1, 2, \forall j, j' \in W(i, t)
\]

(A-22)

The input proportionality assumption is widely used, as it requires the least amount of information. For example, Foster, Haltiwanger, and Syverson (2008) allocate inputs based on products’ revenue shares. This study regards this estimation as the benchmark estimation since it is a reasonable first estimate; it is used in the first step of the iterative method. In business, financial analysts use revenue per employee and revenue per capital as efficiency ratios. This “Equal revenue per input” estimation assumes that these efficiency ratios are equal across segments within a company.

\footnote{\( \gamma_j \) is removed because empirical data shows that the difference between segments is negligible.}
“Equal (expected) marginal revenue per input” estimation. In order to test whether considering the difference in price across segments in Eq. (A-21) improves the accuracy of the estimation, this study also estimates the input allocations under the condition

\[
\begin{align*}
\beta_{jk} R_{ijt} X_{ijt}^k \exp (\nu_{ijt} + 0.5\sigma_j^2) & = 1, \forall i, t, \forall k < M, \forall j, j' \in W(i, t) \\
\beta_{j'k} R_{ijt} X_{ijt}^{j'} \exp (\nu_{ijt} + 0.5\sigma_{j'}^2) & = 1, \forall i, t, \forall j, j' \in W(i, t)
\end{align*}
\]

(A-23)

where the differences in market price of an input across divisions are ignored. This is called the “equal (expected) marginal revenue per input” approach.

“Equal revenue per cost” estimation. Similar to the benchmark estimation in Eq. (A-22), this method considers average revenue rather than expected marginal revenue. However, this approach also takes input price information into consideration.

\[
\frac{R_{ijt}}{R_{ij't}} = r_{ij't}^{k}, \forall k = 1, 2, \forall j, j' \in W(i, t)
\]

(A-24)

The estimation assumes that the average revenue per input is proportional to the price of the same input across segments. This approach guarantees that the average costs of the value-added are equal across segments and is therefore denoted as the “equal revenue per cost” estimation.

To sum up, only the undistorted estimation (“equal expected marginal revenue per cost”) considers both segment-specific input prices and production functions. The benchmark estimation “equal revenue per input” is the easiest to derive, as it requires no additional information. The “equal revenue per cost” estimation takes input price into account on the basis of the benchmark estimation and is likely to be close to the actual level. Both of these estimations ignore the segment-specific technical parameters, but are more computationally friendly than other methods, since
no segment-specific production regressions (stochastic frontier analysis in this case) and iterations are involved. The “equal marginal revenue per input” estimation, on the other hand, considers segment-specific production function but ignores the heterogeneity in input prices across segments. This estimation can therefore substitute the undistorted estimation if segment-level input prices are unobserved. Based on the amount of information used, this study predicts that the undistorted estimation is the most accurate and that the benchmark estimation is the least accurate. There is no measurable evidence about the relative accuracy between the “equal revenue per cost” and “equal marginal revenue per input” estimations. The advantage of the former is its lower computational burden, while the advantage of the latter is the fewer amounts of data (segment-level inputs) needed.

This study uses the mean square error (MSE) and the mean absolute error (MAE) methods for the accuracy test. Suppose the actual segment-level value of the $k$-th input is $X_{ijt}^k$, while the values for the undistorted estimation (“equal expected marginal revenue per cost”), “equal revenue per input” estimation, “equal revenue per cost” estimation, and “equal marginal revenue per input” estimation are $\hat{X}_{ijt}^k(1)$, $\hat{X}_{ijt}^k(2)$, $\hat{X}_{ijt}^k(3)$, and $\hat{X}_{ijt}^k(4)$, respectively. Then, the mean square error of the input allocations can be calculated using

$$MSE(p) = \frac{\sum_k \sum_i \sum_j \sum_t \left[ (\hat{X}_{ijt}^k(p) - X_{ijt}^k) / X_{ijt}^k \right]^2 \sum_k \sum_i \sum_j \sum_t \left[ I(X_{ijt}^k \neq 0) \right]}{\sum_k \sum_i \sum_j \sum_t \left[ I(X_{ijt}^k \neq 0) \right]}$$

Similarly, the mean absolute error of the input allocations can be calculated using

$$MAE(p) = \frac{\sum_k \sum_i \sum_j \sum_t \left| (\hat{X}_{ijt}^k(p) - X_{ijt}^k) / X_{ijt}^k \right| \sum_k \sum_i \sum_j \sum_t \left[ I(X_{ijt}^k \neq 0) \right]}{\sum_k \sum_i \sum_j \sum_t \left[ I(X_{ijt}^k \neq 0) \right]}$$

where $I(\cdot)$ is the indicator function that has a value of one if the argument is correct and a value of zero otherwise. The numerator of the MSE (MAE) is the sum of
the square error (absolute error) of the segment-level estimation of the inputs to the actual level. The denominator is the number of nonzero segment-level values for the inputs. Finally, this study calculates and compares MSE and MAE to check the accuracy of those estimations, where a lower error or deviation implies a more accurate estimation.

In the iterative method to estimate the undistorted allocations, the benchmark estimator in Eq. (A-22) is set as the initial guess. The other two competing methods in Eqs. (A-23) and (A-24) can also be alternative initial guesses of the iteration. This study checks if the iteration converges to the same allocations when given various initial allocations, which implies the robustness of the “Meta-Frontier” estimation.

**A.2.3 Test Results**

This study uses stopping criterion $c=1 \times 10^{-6}$ for the iteration process. When the mean error of the production parameters in two consecutive iterations is smaller than this criterion, it is believed that a stationary threshold has been achieved and that the estimated segment-level input locations satisfy the assumption that the ratio of expected marginal products equals the price ratio of an input across segments for each country.

Table A-2 presents the error results of the 3 Segments model and the 9 Segments model. The first model takes four iterations to pass the criterion and the second model takes thirty iterations to achieve a stationary condition.

The first four columns of Table A-2 show the MSE and MAE for the undistorted estimation and the other three estimations, respectively, for comparison. The fifth column shows the MSE and MAE ratios of the undistorted estimation and the benchmark estimation while the sixth column shows the MSE and MAE ratios of the undistorted estimation and the most accurate estimation among the three competing
Table A-2: Accuracy Test Results

<table>
<thead>
<tr>
<th>Model</th>
<th>Error Type</th>
<th>Error (1)</th>
<th>Error (2)</th>
<th>Error (3)</th>
<th>Error (4)</th>
<th>Error (1)/Error (2)</th>
<th>min_{p≠1} Error (p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 Segments MSE</td>
<td>0.160</td>
<td>0.985</td>
<td>0.377</td>
<td>0.989</td>
<td>16%</td>
<td>42%</td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>MAE</td>
<td>0.242</td>
<td>0.373</td>
<td>0.340</td>
<td>0.354</td>
<td>65%</td>
<td>71%</td>
</tr>
<tr>
<td>9 Segments MSE</td>
<td>1.532</td>
<td>7.341</td>
<td>2.327</td>
<td>7.617</td>
<td>21%</td>
<td>66%</td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>MAE</td>
<td>0.612</td>
<td>1.093</td>
<td>0.657</td>
<td>1.093</td>
<td>56%</td>
<td>93%</td>
</tr>
</tbody>
</table>

Notes: \( \text{error}(1) \) is the error of the undistorted “equal marginal revenue per cost” estimation; \( \text{error}(2) \) is the error of the “equal revenue per input” estimation; \( \text{error}(3) \) is the error of the “equal revenue per cost” estimation; and \( \text{error}(4) \) is the error of the “equal marginal revenue per input” estimation.

Compared with the benchmark “equal revenue per input” estimate, the undistorted estimation significantly decreases the mean square error and hence improves the accuracy. In the 3 Segments model, the MSE of the undistorted estimation is 16% of that of the benchmark estimation. A significant improvement in estimating the input allocations is also achieved in the 9 Segments model, where the MSE of the undistorted estimation is 21% of that of the benchmark estimation. Moreover, the MAE of the undistorted estimation is around 60% of that of the benchmark estimation in both the 3 Segments model and 9 Segments model, which also indicates that the undistorted estimation is more accurate than the benchmark input proportionality estimation used in Foster, Haltiwanger, and Syverson (2008).

The numbers in the last column are all smaller than one, which implies that the undistorted estimation is more accurate than all three competing estimations. Among the three estimations for comparison, the “equal revenue per cost” estimation that uses input price information is the most accurate one, while the other two are very close in their inaccuracy.
Moreover, this study uses all three competing methods (Eqs. (A-22) – (A-24)) to impute the input allocations and set each of these three allocations as the initial guess in the iteration, respectively. In Eq. (A-18), it can be found that the variation of input allocations only depends on the variation of the production functions, because all the revenue and prices in the equation are observed and fixed. Therefore, if the three initial allocations can derive similar production functions, these initial allocations can also impute similar undistorted input allocations.

Tables A-3 and A-4 present the estimated labor coefficient and capital coefficient in each segment for the 3 Segments model and the 9 Segments model, respectively. All the coefficients derived by the three different initial allocations are consistent, indicating that the iterative method converges to the same undistorted input allocations using those different initial guesses.

Table A-3: Iteration Results by Different Initial Guess in 3 Segments Model

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I) Primary Sector</td>
<td>Labor</td>
<td>0.467</td>
<td>0.467</td>
</tr>
<tr>
<td></td>
<td>Capital</td>
<td>0.385</td>
<td>0.385</td>
</tr>
<tr>
<td>II) Secondary Sector</td>
<td>Labor</td>
<td>0.701</td>
<td>0.702</td>
</tr>
<tr>
<td></td>
<td>Capital</td>
<td>0.287</td>
<td>0.286</td>
</tr>
<tr>
<td>III) Tertiary Sector</td>
<td>Labor</td>
<td>0.644</td>
<td>0.644</td>
</tr>
<tr>
<td></td>
<td>Capital</td>
<td>0.302</td>
<td>0.302</td>
</tr>
</tbody>
</table>

Notes: (1) is the estimation using “equal revenue per input” derived allocation as the initial guess; (2) is the estimation using “equal revenue per cost” derived allocation as the initial guess; and (3) is the estimation using “equal marginal revenue per input” derived allocation as the initial guess.
Table A-4: Iteration Results by Different Initial Guess in 9 Segments Model

<table>
<thead>
<tr>
<th>Segment</th>
<th>Labor</th>
<th>Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Agriculture, Hunting, Forestry &amp; Fishing</td>
<td>0.407</td>
<td>0.379</td>
</tr>
<tr>
<td></td>
<td>0.408</td>
<td>0.379</td>
</tr>
<tr>
<td></td>
<td>0.408</td>
<td>0.377</td>
</tr>
<tr>
<td>2) Mining &amp; Quarrying</td>
<td>0.483</td>
<td>0.343</td>
</tr>
<tr>
<td></td>
<td>0.483</td>
<td>0.343</td>
</tr>
<tr>
<td></td>
<td>0.483</td>
<td>0.343</td>
</tr>
<tr>
<td>3) Manufacturing</td>
<td>0.673</td>
<td>0.319</td>
</tr>
<tr>
<td></td>
<td>0.672</td>
<td>0.319</td>
</tr>
<tr>
<td></td>
<td>0.673</td>
<td>0.319</td>
</tr>
<tr>
<td>4) Electricity, Gas &amp; Water Supply</td>
<td>0.456</td>
<td>0.364</td>
</tr>
<tr>
<td></td>
<td>0.456</td>
<td>0.364</td>
</tr>
<tr>
<td></td>
<td>0.456</td>
<td>0.364</td>
</tr>
<tr>
<td>5) Construction</td>
<td>0.625</td>
<td>0.291</td>
</tr>
<tr>
<td></td>
<td>0.625</td>
<td>0.291</td>
</tr>
<tr>
<td></td>
<td>0.625</td>
<td>0.291</td>
</tr>
<tr>
<td>6) Trade, Restaurants &amp; Hotels</td>
<td>0.670</td>
<td>0.222</td>
</tr>
<tr>
<td></td>
<td>0.670</td>
<td>0.222</td>
</tr>
<tr>
<td></td>
<td>0.670</td>
<td>0.222</td>
</tr>
<tr>
<td>7) Transport, Storage &amp; Communications</td>
<td>0.633</td>
<td>0.203</td>
</tr>
<tr>
<td></td>
<td>0.633</td>
<td>0.203</td>
</tr>
<tr>
<td></td>
<td>0.633</td>
<td>0.203</td>
</tr>
<tr>
<td>8) Finance, Insurance, Real Estate &amp; Business Svs</td>
<td>0.602</td>
<td>0.311</td>
</tr>
<tr>
<td></td>
<td>0.602</td>
<td>0.311</td>
</tr>
<tr>
<td></td>
<td>0.602</td>
<td>0.311</td>
</tr>
<tr>
<td>9) Community, Social &amp; Personal Services</td>
<td>0.660</td>
<td>0.328</td>
</tr>
<tr>
<td></td>
<td>0.660</td>
<td>0.328</td>
</tr>
<tr>
<td></td>
<td>0.660</td>
<td>0.328</td>
</tr>
</tbody>
</table>

Notes: (1) is the estimation using “equal revenue per input” derived allocation as the initial guess; (2) is the estimation using “equal revenue per cost” derived allocation as the initial guess; and (3) is the estimation using “equal marginal revenue per input” derived allocation as the initial guess.
A.3 OMR Data Introduction and Adjustment

This study uses data from the Oilfield Market Report (OMR) by Spears & Associates in Chapters 2-4. This report details the global oilfield equipment and service markets associated with five macro-segments: exploration, drilling, completion, production, and capital equipment. Spears & Associates began tracking the oilfield market in 1996 and publish its OMR annually. Each year, the report not only releases new data for the current year, but also updates previously published data. Most numbers in the OMR are estimates developed by Spears through five sources: public company reports (about 100 firms), published information, interviews (about 2,000 discussions), trade shows, and site visits.

There are several advantages of using the OMR dataset. Firstly, this report brings estimations under the same criteria. Different firms have different segmentations, so direct use of their revenue declarations by product line from their financial reports is not wise. Secondly, this dataset is widely used by most firms and clients in the field. Thirdly, Spears has investigated the numbers through many sources to confirm its estimations in the past twenty years. Lastly, the OMR is updated each year, which alters any incorrect numbers according to the newest information.

In this study, three versions of the OMR (2000, 2011, and 2015) are used to collect firm-level data from 1997 to 2014, which is denoted as OMR1997–2014. OMR2000 includes firm-level revenue by segment from 1997 to 2000, OMR2011 includes firm-level revenue by segment from 1999 to 2011, and OMR2015 includes firm-level revenue by segment from 2005 to 2014. Since different waves of data have different market divisions, this study uses the market segmentation of OMR2015 and adjusts the other two datasets to acquire statistically comparable numbers.

The revision in OMR2000 consist of 1) the “Mud Logging” segment being renamed as the “Surface Data Logging” segment; 2) the “Field Processing Equipment” seg-
ment being removed from the market; 3) the “Offshore O&M Services/Contracting” segment being added to the “Offshore Contract Drilling” segment; and 4) the “Production Logging” segment being added to the “Wireline Logging” segment. Moreover, the “Casing & Cementation Products” segment in both OMR2000 and OMR2011 is added to the “Completion Equipment & Services” segment. Finally, the “Pressure Pumping Service” segment in both datasets is divided into the “Cementing” and “Hydraulic Fracturing” segments.

The OMR1997–2014 contains share and size analysis for 32 micro-market segments within the 5 macro-segments from approximately 600 companies working around the world. OMR1997–2014 gives detailed revenue by segment for 275 companies, 114 of which are public firms that publish complete financial information annually. The other 300 smaller companies have been added to “Others” in the report. The detailed segmentation is as follows:

I) Exploration segment includes 1) Geophysical Equipment & Services;


IV) Production segment includes 22) Artificial Lift, 23) Contract Compression Services, 24) Floating Production Services, 25) Specialty Chemicals, 26) Well Servicing; and
A.4 Estimating Capital Stocks Using Perpetual Inventory Method

The perpetual inventory method (PIM) is the most widely employed approach to estimate capital stocks in many statistical offices. Berlemann and Wesselhöft (2014) review the three PIM approaches used most frequently in the literature, consisting of the steady state approach, the disequilibrium approach, and the synthetic time series approach. After comparing the advantages and disadvantages of those three methods, they are able to combine them into a unified approach in order to prevent the drawbacks of the various methods. Their approach follows the procedure proposed by de la Fuente and Doménech (2006).

The PIM interprets a firm’s capital stock as an inventory of investments. The aggregate capital stock falls in the depreciation rate per period. Therefore, the capital stock in period $t$ is a weight sum of the history of the capital stock investment:

$$ K_t = \sum_{i=0}^{\infty} (1 - \delta)^i I_{t-(i+1)} $$

However, a complete time series of past investments from day one is not available for many companies. Thomson ONE, Bloomberg, and FactSet only cover the recent portion of investment history. Suppose the investment can only be tracked back to period $t_1$, then the current capital stock can be estimated by using

$$ K_t = (1 - \delta)^{t-t_0} K_{t_0} + \sum_{i=0}^{t-1} (1 - \delta)^i I_{t-(i+1)} $$  \hspace{1cm} (A-25)

Therefore, the information needed to calculate capital stock includes a time series of investment $I_{t-(i+1)}$, the rate of depreciate $\delta$, and the initial capital stock $K_{t_0}$. Firstly, de la Fuente and Doménech (2006) propose smoothing the time-series in-
vestment data since the economies are on their adjustment path towards equilibrium rather than staying in a steady state most of the time. Hence, this study smooths the observed capital expenditure (investment) using a regression $I_t = \alpha_i + \beta_1 t + \epsilon$ for each firm. Secondly, this study follows the lead of Kamps (2006) and uses time-varying depreciation schemes, which seems to be the most plausible variant. The time-variant smooth depreciation rate can be estimated as the fitted value of the regression $\delta_t = \alpha + \beta_2 t + \epsilon$. This study collects a given firm’s annual depreciation and total capital data to calculate the depreciation rate in accounting and use this information to run the regression. Finally, the initial capital stock at time $t_0$ can be calculated from the investment $I_{t_1}$, the long-term investment growth rate $g_I$, and the estimated depreciation rate $\delta$: $K_{t_0} \approx I_{t_1}/(g_I + \delta_{t_1})$, where the growth rate $g_I$ is $\beta_1$ and the investment $I_{t_1}$ is the fitted value in the same regression. Similar to the method used in Berlemann and Wesselhöft (2014), this study assumes all the years before $t_1$ without desegregated data have the same constant depreciation rate as year $t_1$. But for all the recent years that we have investment data, the depreciation rate is time variant. Therefore, Eq. (A-25) becomes:

$$K_t = \prod_{i=t_1}^{t} (1 - \delta_i) I_{t_1} / (g_I + \delta_{t_1}) + \sum_{i=0}^{t-1} \prod_{j=t-(i+1)}^{t-1} (1 - \delta_j) I_{t-(i+1)}$$

In our empirical study, $t$ is 2014 for most companies that are still active while $t_1$ presents the first year of investment data and varies across firms.
A.5 Robustness Results Assuming T-L a Function

In Chapters 2 and 4, the functional form of the production function is assumed to be Cobb-Douglas (C-D). This appendix gives the estimation results if Transcendental Logarithmic (T-L) is the formation of the production function. In the T-L model, no solution for the system of equations in Eq. (A-20) exists for about 16% of the observations. To sum up, the estimated results in the T-L case are consistent with the ones in the C-D case, supporting the robustness of our approach. However, C-D model guarantees all the input allocation can be updated as long as the technical parameters are positive.

Table A-5: Change in Share of Revenue by Segment for Multidivisional Firms (T-L Model)

<table>
<thead>
<tr>
<th>Segments</th>
<th>Firm</th>
<th>Lowest Efficient Division</th>
<th>Highest Efficient Division</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entered</td>
<td>#</td>
<td>Share Change</td>
<td>Share Change</td>
</tr>
<tr>
<td>2</td>
<td>28</td>
<td>-13.5%</td>
<td>13.5%</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>-4.9%</td>
<td>2.2%</td>
</tr>
<tr>
<td>4 &amp; 5</td>
<td>8</td>
<td>1.6%</td>
<td>-1.1%</td>
</tr>
<tr>
<td>Total</td>
<td>46</td>
<td>-8.9%</td>
<td>8.0%</td>
</tr>
</tbody>
</table>
Figure A.1: Efficiency Level for BHI, HAL, SLB, WFT, and the Industry Average (T-L Model)
Table A-6: Estimate of Transcendental Logarithmic Production Function

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industry</td>
<td>Entire Industry</td>
<td>(w/o IV)</td>
<td>Explor. Drilling</td>
<td>(w/ IV)</td>
<td>Expl. Drilling</td>
<td>(w/ IV)</td>
<td>Product. Equipment</td>
<td>(w/ IV)</td>
</tr>
<tr>
<td>lnL</td>
<td>.368*** (.065)</td>
<td>.074 (.063)</td>
<td>-.321*** (.120)</td>
<td>-1.04*** (.098)</td>
<td>-.107*** (.099)</td>
<td>-.698*** (.089)</td>
<td>-.891*** (.180)</td>
<td>-.990*** (.243)</td>
</tr>
<tr>
<td>lnK</td>
<td>.291*** (.04)</td>
<td>.524*** (.043)</td>
<td>.153 (.174)</td>
<td>1.04*** (.079)</td>
<td>1.32*** (.130)</td>
<td>3.14*** (.106)</td>
<td>2.06*** (.200)</td>
<td>.749*** (.135)</td>
</tr>
<tr>
<td>lnL * lnL</td>
<td>-.034*** (.007)</td>
<td>-.080*** (.006)</td>
<td>-.049** (.023)</td>
<td>-.173*** (.014)</td>
<td>-.229*** (.018)</td>
<td>-.092*** (.013)</td>
<td>-.154*** (.014)</td>
<td>-.117*** (.025)</td>
</tr>
<tr>
<td>lnK * lnK</td>
<td>-.013*** (.005)</td>
<td>-.020*** (.006)</td>
<td>.034 (.026)</td>
<td>-.030** (.013)</td>
<td>-.075*** (.025)</td>
<td>-.003 (.013)</td>
<td>.063* (.012)</td>
<td>.044*** (.013)</td>
</tr>
<tr>
<td>lnL * lnK</td>
<td>.045*** (.01)</td>
<td>.086*** (.01)</td>
<td>-.062 (.046)</td>
<td>.152*** (.025)</td>
<td>.273*** (.037)</td>
<td>.007 (.022)</td>
<td>.139*** (.052)</td>
<td>.014 (.031)</td>
</tr>
<tr>
<td>control lnK</td>
<td>– (.01)</td>
<td>.151*** (.044)</td>
<td>.513*** (.131)</td>
<td>.060 (.061)</td>
<td>.213** (.093)</td>
<td>.128 (.081)</td>
<td>-.228 (.147)</td>
<td>.045 (.095)</td>
</tr>
<tr>
<td>time dummy</td>
<td>yes (.202)</td>
<td>yes (.219)</td>
<td>yes (.523)</td>
<td>yes (.328)</td>
<td>yes (.284)</td>
<td>yes (.284)</td>
<td>yes (.762)</td>
<td>yes (.685)</td>
</tr>
<tr>
<td>Intercept</td>
<td>1.59*** (.114)</td>
<td>.080 (.113)</td>
<td>2.67*** (.18)</td>
<td>2.91*** (.53)</td>
<td>1.99*** (.44)</td>
<td>3.77*** (.28)</td>
<td>.242 (.22)</td>
<td>3.83*** (.22)</td>
</tr>
<tr>
<td># of firms</td>
<td>114 (.152)</td>
<td>113 (1298)</td>
<td>18 (218)</td>
<td>53 (606)</td>
<td>44 (493)</td>
<td>28 (325)</td>
<td>22 (285)</td>
<td>22 (285)</td>
</tr>
<tr>
<td># of obs.</td>
<td>1525 (.484)</td>
<td>1298 (.592)</td>
<td>18 (.659)</td>
<td>53 (.751)</td>
<td>44 (.490)</td>
<td>28 (.633)</td>
<td>22 (.492)</td>
<td>22 (.655)</td>
</tr>
<tr>
<td>Avg. Eff.</td>
<td>.484 (.484)</td>
<td>.592 (.592)</td>
<td>.659 (.659)</td>
<td>.751 (.751)</td>
<td>.490 (.490)</td>
<td>.633 (.633)</td>
<td>.492 (.492)</td>
<td>.655 (.655)</td>
</tr>
</tbody>
</table>

Notes: Significant at: *10, **5 and ***1 percent; Standard error in parentheses.
Table A-7: Efficiency Levels of the “Big Four” by Segment in 2014 (T-L Model)

<table>
<thead>
<tr>
<th>Segments</th>
<th>Schlumberger</th>
<th>Halliburton</th>
<th>Baker Hughes</th>
<th>Weatherford</th>
<th>Oilfield Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exploration</td>
<td>0.66</td>
<td>0.89</td>
<td>–</td>
<td>–</td>
<td>0.67</td>
</tr>
<tr>
<td>Drilling</td>
<td>0.74</td>
<td>0.73</td>
<td>0.84</td>
<td>0.66</td>
<td>0.72</td>
</tr>
<tr>
<td>Completion</td>
<td>0.57</td>
<td>0.46</td>
<td>0.62</td>
<td>0.42</td>
<td>0.50</td>
</tr>
<tr>
<td>Production</td>
<td>0.71</td>
<td>0.67</td>
<td>0.73</td>
<td>0.62</td>
<td>0.61</td>
</tr>
<tr>
<td>Capital Eqpt</td>
<td>0.46</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Table A-8: Technical Efficiency Statistics (T-L Model)

<table>
<thead>
<tr>
<th></th>
<th>Single Frontier</th>
<th>Varying Frontier</th>
<th>Shape-constrained Frontier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.48</td>
<td>0.52</td>
<td>0.42</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.16</td>
<td>0.25</td>
<td>0.14</td>
</tr>
<tr>
<td>25% quantile</td>
<td>0.33</td>
<td>0.40</td>
<td>0.25</td>
</tr>
<tr>
<td>50% quantile</td>
<td>0.43</td>
<td>0.50</td>
<td>0.36</td>
</tr>
<tr>
<td>75% quantile</td>
<td>0.61</td>
<td>0.61</td>
<td>0.56</td>
</tr>
<tr>
<td>Maximum</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table A-9: Technical Efficiency Class Interval (T-L Model)

<table>
<thead>
<tr>
<th>Efficiency</th>
<th>Single Frontier</th>
<th>Varying Frontier</th>
<th>Shape-constrained Frontier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>#obs.</td>
<td>mean</td>
<td>(95% CI)</td>
</tr>
<tr>
<td>&lt; = 0.3</td>
<td>18</td>
<td>.24</td>
<td>(.22-.25)</td>
</tr>
<tr>
<td>0.3 - 0.5</td>
<td>38</td>
<td>.39</td>
<td>(.37-.41)</td>
</tr>
<tr>
<td>0.5 - 0.75</td>
<td>20</td>
<td>.61</td>
<td>(.58-.64)</td>
</tr>
<tr>
<td>&gt; 0.75</td>
<td>13</td>
<td>.88</td>
<td>(.83-.93)</td>
</tr>
<tr>
<td>Total</td>
<td>89</td>
<td>.48</td>
<td>(.44-.53)</td>
</tr>
</tbody>
</table>
Table A-10: Efficiency Regression Result (T-L Model)

<table>
<thead>
<tr>
<th></th>
<th>Single Frontier</th>
<th>Varying Frontier</th>
<th>Shape-constrained Frontier</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\text{TE}}_i$</td>
<td>.095***</td>
<td>.121***</td>
<td>.085***</td>
</tr>
<tr>
<td></td>
<td>(.018)</td>
<td>(.012)</td>
<td>(.018)</td>
</tr>
<tr>
<td>$\ln R_i$</td>
<td>-.039</td>
<td>-.042</td>
<td>-.026</td>
</tr>
<tr>
<td></td>
<td>(.044)</td>
<td>(.027)</td>
<td>(.044)</td>
</tr>
<tr>
<td>$MD_i$</td>
<td>-.135</td>
<td>-.263***</td>
<td>-.129</td>
</tr>
<tr>
<td></td>
<td>(.121)</td>
<td>(.077)</td>
<td>(.118)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.24</td>
<td>0.56</td>
<td>0.21</td>
</tr>
</tbody>
</table>
References


