Topological Protection from Random Rashba Spin-Orbit Backscattering: Ballistic Transport in a Helical Luttinger Liquid

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The combination of Rashba spin-orbit coupling and potential disorder induces a random current operator for the edge states of a 2D topological insulator. We prove that charge transport through such an edge is ballistic at any temperature, with or without Luttinger liquid interactions. The solution exploits a mapping to a spin 1/2 in a time-dependent field that preserves the projection along one randomly undulating component (integrable dynamics). Our result is exact and rules out random Rashba backscattering as a source of temperature-dependent transport, absent integrability-breaking terms.

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The edge states of a quantum spin Hall (QSH) insulator realize a time-reversal symmetric helical Luttinger liquid (HLL): Two counterpropagating modes possess opposite spins and hence form a Kramers doublet [1–7]. Owing to the nontrivial bulk $\mathbb{Z}_2$ topology, the HLL provides an alternative realization for a quantum wire with strong spin-orbit coupling and an odd number of channels [8], distinct from (e.g.) carbon nanotubes [9–14]. In an ideal QSH insulator, the S_z component of the electron spin is conserved. Elastic backscattering of HLL edge carriers off of impurities is prohibited by the combination of spin U(1) and time-reversal symmetries; only forward-scattering potential disorder is allowed. The combination of pure potential disorder and Luttinger liquid interactions bosonizes [15,16] to a trivial free theory, leading to the prediction that edge electrons exhibit ballistic transport at any temperature [5]. These conclusions obtain in a *fixed realization* of the disorder, a robust version of topological protection that also applies to the surface states of 3D topological superconductors [17–19].

Axial spin symmetry in topological insulators is, however, *not* typically robust [1,2,20]. Rashba spin-orbit coupling (RSOC) arises whenever inversion symmetry is broken, as in HgTe/CdTe [21] and InAs/GaSb [22,23] heterostructures. The helical edge states then exhibit a twisted spin texture [20,24]. Neglecting the gapped bulk, electron annihilation operators at the edge can be expanded as

$$c_\uparrow(x) \simeq e^{ik_F x}R(x) - i\zeta e^{-ik_F x}\partial_x L(x),$$
$$c_\downarrow(x) \simeq e^{-ik_F x}L(x) - i\zeta^* e^{ik_F x}\partial_x R(x), \qquad (1)$$

where $R(x)$ and $L(x)$ destroy right- and left-moving edge mode electrons near the Fermi points $\pm k_F$. The parameter ζ encodes the strength of the RSOC. In the model of Refs. [20,24] $\zeta = 2k_F/k_0^2$, where k_0 sets the scale for rotation of the spin axis. In Eq. (1), we choose the quantization axis to coincide with the Kramers pair at $k = \pm k_F$. To lowest order in ζ, the electron density operator is given by

$$\rho = c_\uparrow^\dagger c_\uparrow + c_\downarrow^\dagger c_\downarrow \simeq R^\dagger R + L^\dagger L$$
$$- \{i\zeta e^{-2ik_F x}[R^\dagger \partial_x L - (\partial_x R^\dagger)L] + \text{H.c.}\}, \qquad (2)$$

where H.c. denotes the Hermitian conjugate. In a spinless Luttinger liquid, the time reversal operation $\mathcal{T}$ exchanges $R \leftrightarrow L$ ($\mathcal{T}^2 = +1$). The term on the second line of Eq. (2) is odd under this, and cannot contribute to ρ in the spinless case. This term is even under time reversal in the HLL, which sends $R \to L$ and $L \to -R$ ($\mathcal{T}^2 = -1$).

Two key attributes of HLLs with RSOC follow from Eq. (2). First, scalar potential disorder that couples to $\rho(x)$ generically induces a random backscattering component to the Dirac current operator, in the low-energy effective field theory of the HLL edge. Second, the screened Coulomb interaction $\rho^2(x)$ induces the usual Luttinger liquid interaction, as well as a one-particle umklapp interaction term. This interaction is irrelevant in the RG sense, due to an extra derivative.

A recent experiment [25] has raised concerns that trivial edge states can masquerade as HLLs, while previous experiments [21–23,26] have not shown the anticipated ballistic transport at the lowest temperatures (T) for sufficiently long edges. A crucial theoretical task is to identify and understand mechanisms that might weaken topological protection and suppress the conductance at finite and zero T [3,20,27–31]. Although irrelevant, the one-particle umklapp interaction can be the dominant source of inelastic backscattering for $k_B T$ much less than the bulk gap in an isolated HLL, leading to T-dependent corrections to the edge conductance [20,27–29]. Phonon scattering [32], Kondo impurities [33–36], or charge puddles [37,38] can also give T-dependent corrections to transport. In this Letter we ignore these known mechanisms and focus upon the random Dirac current operator in a disordered HLL with RSOC.

Quenched disorder that couples to the backscattering kinetic operator on the second line of Eq. (2) has been termed "random RSOC" in previous studies [30,31]. Unlike backscattering (random mass) disorder in a spinless Luttinger liquid, short-range correlated random RSOC is irrelevant in the RG sense for an edge Luttinger parameter $K > 1/2$ [31]. Both the random RSOC and one-particle umklapp interaction can be simultaneously irrelevant, and map to similar operators in bosonization [28,30,39]. This suggests that both can be treated with perturbation theory, using bosonization to incorporate Luttinger liquid effects. Within this framework, random RSOC is predicted to give a T-dependent correction to transport that vanishes at $T = 0$ for $K > 1/2$ [31]. Moreover, it has been argued that for $K < 1/2$ the random RSOC can induce Anderson localization [30]. We show here that these conclusions are incorrect and miss important physics.

In this Letter we prove that charge transport is perfectly ballistic with Landauer conductance $G = e^2/h$ per edge, for a HLL with random RSOC at $T \geq 0$, with or without Luttinger interactions. We first solve the noninteracting problem exactly by transfer matrix, which is unitary up to a certain factor. This unitary matrix is equivalent to the evolution operator of a spin-1/2 magnetic moment in a random, two-component time-dependent magnetic field. The dynamics are integrable, since the evolution preserves the spin projection along one randomly undulating component of the field [40], and this translates into the absence of backscattering [2] for an edge connected to ideal leads. With Luttinger interactions, we map the problem onto one with a homogeneous (inhomogeneous) current operator (density-density interaction). The transformed theory is equivalent to a free Luttinger liquid, but with inhomogeneous Luttinger and charge velocity parameters. We corroborate these results with a numerical treatment of the edge wave functions and level statistics. Finally, we compare to a disordered, particle-hole symmetric spinless quantum wire [8], which also evades Anderson localization in 1D.

Model with random RSOC.—In terms of the two-component Dirac spinor $\Psi(x) \equiv [\,R(x) \quad L(x)\,]^{\mathsf{T}}$, the Hamiltonian of a noninteracting edge incorporating random RSOC [28,30–32,41–43] can be written as

$$H_0 = \int dx \Psi^\dagger \hat{h} \Psi(x), \tag{3a}$$

$$\hat{h} = \hat{j}(x)(-i\partial_x) - \frac{i}{2}\partial_x \hat{j}(x) + V(x). \tag{3b}$$

In Eq. (3b) the electric current operator reads

$$\hat{j}(x) = \boldsymbol{\gamma}(x) \cdot \hat{\boldsymbol{\sigma}}, \tag{4}$$

where $\hat{\boldsymbol{\sigma}} = (\hat{\sigma}^1, \hat{\sigma}^2, \hat{\sigma}^3)$ are the Pauli matrices and $\boldsymbol{\gamma}(x) = [\,\xi_1(x), \xi_2(x), \mathsf{v}_F(x)\,]$ encodes the random RSOC backscattering strengths $\xi_{1,2}(x)$ and the (possibly inhomogeneous) Fermi velocity $\mathsf{v}_F(x)$; $V(x)$ denotes the forward-scattering scalar potential. All scattering strengths are real functions. Equation (3) is invariant under time reversal $\mathcal{T}$, defined by $\Psi(x) \to i\hat{\sigma}^2\Psi(x)$ and $i \to -i$. The term in Eq. (3b) involving $\partial_x \hat{j}(x)$ is required by Hermiticity [cf. Eq. (2)].

Transfer-matrix solution.—The single-particle Schrödinger equation takes the form

$$\hat{h}\psi(x) = \varepsilon\psi(x), \tag{5}$$

where $\psi(x)$ is the two-component wave function with eigenenergy ε. We define the current norm $\|\hat{j}(x)\|$ and the normalized current operator $\hat{J}(x)$ as

$$\|\hat{j}(x)\| \equiv |\boldsymbol{\gamma}(x)|, \qquad \hat{J}(x) \equiv \hat{j}(x)/\|\hat{j}(x)\|, \tag{6}$$

where $\|\hat{j}(x)\|$ is the local random speed. In terms of the rescaled wave function $\varphi(x) \equiv \sqrt{\|\hat{j}(x)\|}\psi(x)$, the Schrödinger equation transforms to

$$(-i\partial_x)\varphi(x) = \hat{\mathcal{H}}_\varepsilon(x)\varphi(x), \tag{7}$$

where $\hat{\mathcal{H}}_\varepsilon(x)$ is composed of two components,

$$\hat{\mathcal{H}}_\varepsilon(x) = \hat{\mathcal{O}}_\varepsilon(x) + \hat{\Theta}(x), \tag{8a}$$

$$\hat{\mathcal{O}}_\varepsilon(x) = [\varepsilon - V(x)]\frac{\hat{J}(x)}{\|\hat{j}(x)\|}, \qquad \hat{\Theta}(x) = \frac{i}{2}\hat{J}\partial_x\hat{J}(x). \tag{8b}$$

$\hat{\mathcal{O}}_\varepsilon(x)$, $\hat{\Theta}(x)$, and $\hat{\mathcal{H}}_\varepsilon(x)$ are all Hermitian operators.

The transfer-matrix solution for the single-particle wave functions is

$$\psi_{\varepsilon,a}(x) = \left[1/\sqrt{\|\hat{j}(x)\|}\right]\hat{\mathsf{T}}_\varepsilon(x, -\infty)|a\rangle, \tag{9a}$$

where $|a = 1\rangle \equiv (\,1 \quad 0\,)^{\mathsf{T}}$ and $|a = 2\rangle \equiv (\,0 \quad 1\,)^{\mathsf{T}}$ label the degenerate Kramers pair, and the *unitary* transfer matrix generated by the Hermitian operator (8) reads

$$\hat{\mathsf{T}}_\varepsilon(x, x') \equiv \mathcal{P}\exp\left[i\int_{x'}^{x} dy \hat{\mathcal{H}}_\varepsilon(y)\right]. \tag{9b}$$

Here "$\mathcal{P}$" denotes path ordering. The normalization constant of the wave function (9a) is fixed in order to recover the RSOC-free physics, e.g., the density profile given by the U(1) axial anomaly in $(1+1)$ dimensions [see Eq. (12)]. Using the Heisenberg equation of motion for the transfer matrix $(-i\partial_x)\hat{\mathsf{T}}_\varepsilon(x, x') = \hat{\mathcal{H}}_\varepsilon(x)\hat{\mathsf{T}}_\varepsilon(x, x')$, one can prove the nontrivial relation

$$\hat{J}(x)\hat{\mathsf{T}}_\varepsilon(x, x') = \hat{\mathsf{T}}_\varepsilon(x, x')\hat{J}(x'), \tag{10}$$

which implies the *integrability* of the transfer matrix, as discussed below.

From the solution (9) we obtain the following conclusions: (i) The single-particle wave functions are extended and uniformly inhomogeneous (not rarely peaked or multifractal), with a probability density determined by the local random speed [see also Fig. 2(a)],

$$|\psi_{\varepsilon,a}(x)|^2 \equiv \psi^\dagger_{\varepsilon,a}(x)\psi_{\varepsilon,a}(x) = \|\hat{j}(x)\|^{-1}. \quad (11)$$

(ii) The density profile is defined via $n(x) \equiv \lim_{\eta\to 0}\sum_a \int d\varepsilon f(\varepsilon)\psi^\dagger_{\varepsilon,a}[x-(\eta/2)]\psi_{\varepsilon,a}[x+(\eta/2)]$, where $f(\varepsilon) = 1/(e^{\varepsilon/T}+1)$ is the Fermi-Dirac distribution. We recover the U(1) axial anomaly in $(1+1)$ dimensions [15,16] renormalized by the local speed,

$$n(x) = -V(x)/[\pi\|\hat{j}(x)\|]. \quad (12)$$

The absence of additional terms due to the random $\|\hat{j}(x)\|$ suggests that density-density interactions will not induce quantum (Altshuler-Aronov) corrections to transport [19,44], as we confirm below. (iii) The Kubo formula for the dc conductivity can be calculated via $\sigma_0 = (e^2/2hL)\int_{-\infty}^{\infty} d\varepsilon[-df(\varepsilon)/d\varepsilon]\int dx dx' F_\varepsilon(x,x')$, where $L\to\infty$ is the system size and $F_\varepsilon(x,x') = \mathrm{Tr}[\hat{J}(x)\hat{\mathsf{T}}_\varepsilon(x,x')\hat{J}(x')\hat{\mathsf{T}}^\dagger_\varepsilon(x,x')]$. Equation (10) implies that $F_\varepsilon(x,x') = 2$, independent of x, x'. Then, the Kubo formula suggests a temperature-independent, universal Landauer conductance

$$G_0 = \sigma_0/L = e^2/h. \quad (13)$$

The direct calculation of G_0 for a HLL with random RSOC connected to ideal leads confirms this result, as we now explain.

Integrable dynamics of a spin 1/2 in a random, but correlated magnetic field.—The purely ballistic transport in Eq. (13) can be interpreted in terms of the instantaneous eigenstates of a spin 1/2 evolving in a time-dependent magnetic field, since the integration of the transfer matrix [Eq. (9b)] between ideal leads is described by a corresponding spin rotation. We introduce the Hamiltonian

$$\hat{H}(t) = \sum_{\alpha=1}^{2}\hat{H}_\alpha(t), \qquad \hat{H}_\alpha(t) = \mathbf{B}_\alpha(t)\cdot\hat{\boldsymbol{\sigma}}, \quad (14a)$$

with the magnetic fields $\mathbf{B}_1(t)\perp\mathbf{B}_2(t)$ defined by

$$\mathbf{B}_1(t) = B_1(t)\mathbf{n}(t), \qquad \mathbf{B}_2(t) = \frac{1}{2}\mathbf{n}(t)\times\partial_t\mathbf{n}(t), \quad (14b)$$

where $B_1(t)$ and $\mathbf{n}(t)$ denote the magnitude and the direction of $\mathbf{B}_1(t)$, respectively. The connection between the spin model (14) and the edge model in Eq. (8) becomes manifest if we choose $\mathbf{B}_1(t) = \frac{1}{2}\mathrm{Tr}[\hat{\mathcal{O}}_\varepsilon(t)\hat{\boldsymbol{\sigma}}]$. The spin Hamiltonian $\hat{H}(t)$ precisely takes the form of Eq. (8) with time t replaced by the spatial coordinate x.

The time evolution of the spin is determined by the unitary operator $\hat{U}(t) = \mathcal{T}\exp[-i\int_0^t dt'\hat{H}(t')]$, where "$\mathcal{T}$" denotes time ordering. One can show the following for a differentiable but otherwise arbitrary field $\mathbf{B}_1(t)$: Starting from an eigenstate $\varphi(0)$ of $\hat{H}_1(0)$, $\varphi(t) = \hat{U}(t)\varphi(0)$ remains an instantaneous eigenstate of $\hat{H}_1(t)$. This statement is equivalent to the relation $\hat{U}^\dagger(t)\mathbf{n}(t)\cdot\hat{\boldsymbol{\sigma}}\,\hat{U}(t) = \mathbf{n}(0)\cdot\hat{\boldsymbol{\sigma}}$, which is Eq. (10) in the spin language.

In particular, setting $\mathbf{B}_1(0) = \mathbf{B}_1(T) = B\hat{z}$ and the initial state $\varphi(0) = |\uparrow\rangle$ (aligned along z), we obtain $\varphi(T) = \varphi(0)$ for any smooth $\mathbf{B}_1(t)$ in $t\in(0,T)$; i.e., there is no net rotation. Similarly, for an edge with random RSOC connected to ideal leads at $x = \pm L/2$, the current operator in Eq. (4) satisfies $\hat{j}(\pm L/2) = \mathsf{v}_F\hat{\sigma}^3$ (v_F is the uniform Fermi velocity in the leads). The transfer matrix in Eq. (9b) is therefore reflectionless, and this holds for all eigenenergies ε [Eq. (8)]. Since the true eigenstate ψ differs from φ only by a local Jacobian factor $\|\hat{j}\|^{1/2}$, identical on either side of the leads $\|\hat{j}\|^{1/2} = \sqrt{\mathsf{v}_F}$, the transmission coefficient is exactly unity so that backscattering is prohibited. We conclude that topological protection here corresponds to a special, integrable two-level system that depends upon an arbitrary random field, and which preserves the projection along this field. This is an explicit example of how perfect transmission is achieved for noninteracting edges in the presence of RSOC [2].

Luttinger interactions.—We consider the model in Eq. (3) for an edge spanning $|x|\le L/2$, connected to ideal leads. The Hamiltonian incorporating Luttinger interactions is given by

$$H = H_0 + \int dx U(x)[\Psi^\dagger\Psi(x)]^2, \quad (15)$$

where $U(x) = U\theta(L/2-|x|)$, and $\theta(x)$ is the unit step function. Inspired by the transfer-matrix solution (9a), we introduce the rotated fermion field

$$\Phi(x) \equiv \sqrt{\|\hat{j}(x)\|/\mathsf{v}_F}\quad \hat{\mathsf{T}}_0^\dagger(x,-L/2)\Psi(x). \quad (16)$$

The operators $\Phi(x)$ and $\Phi^\dagger(x')$ satisfy rescaled canonical anticommutation relations since the rotation in Eq. (16) is *nonunitary*. However, one is free to perform this transformation in a path integral formalism, because Eq. (16) is a linear change of variables, up to a disorder-dependent Jacobian that cancels between numerator and denominator for a correlation function. Note that $\Phi(x) = \Psi(x)$ for $|x| > L/2$. Exploiting Eq. (10), Eq. (15) reduces to

$$H = \int dx\{\Phi^\dagger\mathsf{v}_F\hat{\sigma}^3(-i\partial_x)\Phi(x) + \tilde{U}(x)[\Phi^\dagger\Phi(x)]^2\}, \quad (17)$$

where $\tilde{U}(x) \equiv \mathsf{v}_F^2 U(x)/\|\hat{j}(x)\|^2$. This transformed theory with a homogeneous (inhomogeneous) kinetic term (Luttinger interaction) is equivalent to a free boson theory [15,16]. The conductance in a Landauer setup for the $\Phi(x)$ fermions is therefore given by Eq. (13), *independent* of

$\tilde{U}(x)$ [45–47]. Bosonizing Eq. (17) in a fixed realization of disorder gives the Luttinger parameter $K(x)$ and charge velocity $\mathsf{v}_c(x)$ [39],

$$K(x) = 1/\sqrt{[1+\chi(x)][1-\chi(x)+2\tilde{U}(x)/\pi\mathsf{v}_F]},$$
$$\mathsf{v}_c(x) = \mathsf{v}_F\sqrt{[1-\chi(x)+2\tilde{U}(x)/\pi\mathsf{v}_F]/[1+\chi(x)]}, \qquad (18)$$

where $\chi(x) \equiv \mathsf{v}_F/\|\hat{j}(x)\| - 1$. As usual, while the mapping to a free boson parametrized by $K(x)$ and $\mathsf{v}_c(x)$ is exact, we expect that the explicit formulas in Eq. (18) are correct only to linear order in the perturbations χ and $\tilde{U}$ [15]. In the noninteracting case $\tilde{U}=0$, we have $K(x) \simeq 1 + \boldsymbol{O}(\chi)^2$ and $1/\mathsf{v}_c(x) \simeq 1/\|\hat{j}(x)\| + \boldsymbol{O}(\chi)^2$.

Noninteracting energy level statistics, comparison to Dyson.—Single particle energy levels in random systems typically exhibit Poissonian or Wigner-Dyson energy level statistics, associated to localized or ergodic wave functions [48]. Since the helical edge with random RSOC is solved by an integrable transfer matrix, we do not expect Wigner-Dyson statistics. Using the Heisenberg equation of motion for $\Phi(t,x)$ in Eqs. (16) and (17) with $\tilde{U}=0$ gives the static Schrödinger equation $\|\hat{j}(x)\|\hat{J}(-L/2)(-i\partial_x)\Phi(x) = \varepsilon\Phi(x)$. On a periodic ring with circumference L, the eigenenergies $\{\varepsilon_n\}$ with $n \in \mathbb{Z}$ (doubly degenerate) can be obtained via the Bohr-Sommerfeld quantization, which leads to $\varepsilon_n = \pm 2\pi n \Delta_L$. Here, the level spacing is $\Delta_L^{-1} = \oint_L dx \|\hat{j}(x)\|^{-1}$. For a fixed realization of disorder, the energy levels are equally spaced, as for a clean system. We confirm this by numerically diagonalizing the original Hamiltonian (3b) in momentum space [39,49]. The result is the sharp delta-function-like level-spacing distribution shown in Fig. 1.

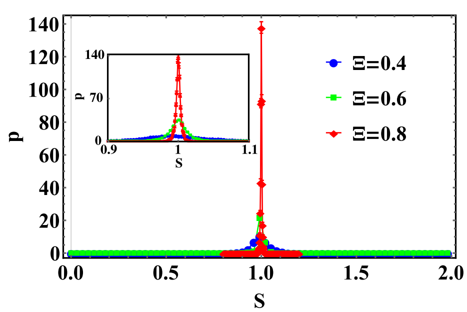

FIG. 1. δ-type level spacing distribution $p(s)$ for the noninteracting helical edge model, obtained by numerical diagonalization of the Hamiltonian Eq. (3b) in momentum space. We sample every other level to account for the Kramers degeneracy. Here $s = |\varepsilon_n - \varepsilon_{n-2}|/\Delta$ is the normalized level spacing near energy ε_n, while Δ is the average of $|\varepsilon_n - \varepsilon_{n-2}|$ over the chosen set of levels. The parameter Ξ is the disorder correlation length in units of the inverse ultraviolet momentum cutoff [39,49]. Inset: The broadening of the δ-type distribution with different values of Ξ [39].

We conclude that the helical edge with a random backscattering kinetic operator (induced by RSOC) possesses extended states, shows perfect ballistic transport, and exhibits clean level statistics. It is interesting to contrast these results for Eq. (3) to a nontopological 1D system that incorporates backscattering, but also possesses extended states. This is the random mass ("Dyson," class BDI) Dirac model [8,50,51], which has the Hamiltonian

$$\hat{h}_{\text{Dyson}} = \mathsf{v}_F\hat{\sigma}^3(-i\partial_x) + m(x)\hat{\sigma}^2.$$

This is particle-hole symmetric in every realization of disorder, and arises as the continuum limit of a 1D lattice model perturbed with weakly random nearest-neighbor hopping. In sharp contrast to the HLL with random RSOC, however, the extended states exist only near zero energy (the localization length diverges at $\varepsilon = 0$). Moreover, an extended Dyson state is quasilocalized, consisting of a *few* isolated peaks with stretched exponential tails, separated by large distances [50]. Because of this rarefied structure the *typical* Landauer conductance decays as $G_{\text{typ}} \sim (e^2/h)\exp(-2\sqrt{2DL/\pi})$, where D is the variance of the random mass and L is the length [51]. Position-space profiles of random-RSOC edge state and Dyson wave functions are shown in Fig. 2. For the helical edge state (a), the wave function is ergodic, with only a modulated profile [Eq. (11)].

Discussion.—Although the combination of random RSOC and Luttinger interactions does not affect transport, in QSH materials the low-temperature conductance will be affected by other scattering mechanisms. Finite-temperature corrections, likely to be power law in temperature, may arise

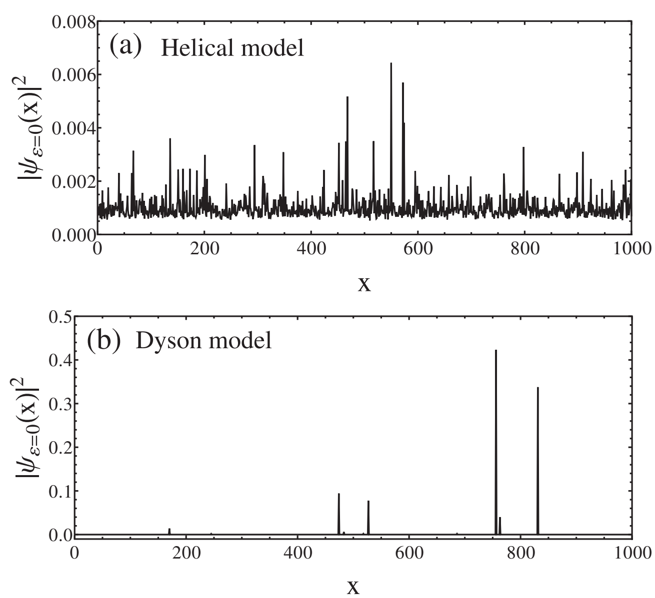

FIG. 2. Typical single-particle delocalized wave functions for the helical model (a) and the Dyson model (b) at energy $\varepsilon = 0$ (momentum space exact diagonalization [39]).

due to various irrelevant time-reversal symmetric mechanisms such as inelastic umklapp processes [3,20,27–29], phonon scattering [32], or Kondo impurities [33–36]. More recently, it has been suggested that the corrections due to scattering off charge puddles in the bulk [37,38] give rise to a much weaker temperature dependence and might dominate the low temperature transport in existing materials [38]. These studies have been performed in terms of the physical $\Psi(x)$ fermion [Eq. (3)]. We suggest that, as long as the random RSOC is present, it is necessary to carry out calculations in terms of the rotated $\Phi(x)$ fermions [Eq. (16)]. We leave this work to future study.

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[1] C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 146802 (2005).
[2] C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 226801 (2005).
[3] C. Wu, B. A. Bernevig, and S.-C. Zhang, Phys. Rev. Lett. **96**, 106401 (2006).
[4] B. A. Bernevig and S.-C. Zhang, Phys. Rev. Lett. **96**, 106802 (2006).
[5] B. A. Bernevig, T. L. Hughes, and S.-C. Zhang, Science **314**, 1757 (2006).
[6] X.-L. Qi and S.-C. Zhang, Rev. Mod. Phys. **83**, 1057 (2011).
[7] M. Z. Hasan and C. L. Kane, Rev. Mod. Phys. **82**, 3045 (2010).
[8] F. Evers and A. D. Mirlin, Rev. Mod. Phys. **80**, 1355 (2008).
[9] H. Suzuura and T. Ando, Phys. Rev. Lett. **89**, 266603 (2002).
[10] T. Ando and H. Suzuura, J. Phys. Soc. Jpn. **71**, 2753 (2002).
[11] Y. Takane, J. Phys. Soc. Jpn. **73**, 9 (2004).
[12] H. Sakai and Y. Takane, J. Phys. Soc. Jpn. **74**, 1521 (2005).
[13] M. Caselle and U. Magnea, J. Stat. Mech. (2006) P01013.
[14] P. M. Ostrovsky, I. V. Gornyi, and A. D. Mirlin, Phys. Rev. Lett. **98**, 256801 (2007).
[15] A. O. Gogolin, A. A. Nersesyan, and A. M. Tsvelik, *Bosonization and Strongly Correlated Systems* (Cambridge University Press, Cambridge, England, 1998).
[16] J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena*, 4th ed. (Oxford University Press, New York, 2002).
[17] A. P. Schnyder, S. Ryu, A. Furusaki, and A. W. W. Ludwig, Phys. Rev. B **78**, 195125 (2008).
[18] M. S. Foster, H.-Y. Xie, and Y.-Z. Chou, Phys. Rev. B **89**, 155140 (2014).
[19] H.-Y. Xie, Y.-Z. Chou, and M. S. Foster, Phys. Rev. B **91**, 024203 (2015).
[20] T. L. Schmidt, S. Rachel, F. von Oppen, and L. I. Glazman, Phys. Rev. Lett. **108**, 156402 (2012).
[21] M. König, S. Wiedmann, C. Brüne, A. Roth, H. Buhmann, L. W. Molenkamp, X.-L. Qi, and S.-C. Zhang, Science **318**, 766 (2007).
[22] I. Knez, R.-R. Du, and G. Sullivan, Phys. Rev. Lett. **107**, 136603 (2011).
[23] I. Knez, C. T. Rettner, S.-H. Yang, S. S. P. Parkin, L. Du, R.-R. Du, and G. Sullivan, Phys. Rev. Lett. **112**, 026602 (2014).
[24] A. Rod, T. L. Schmidt, and S. Rachel, Phys. Rev. B **91**, 245112 (2015).
[25] F. Nichele, H. J. Suominen, M. Kjaergaard, C. M. Marcus, E. Sajadi, J. A. Folk, F. Qu, A. J. A. Beukman, F. K. de Vries, J. van Veen, S. Nadj-Perge, L. P. Kouwenhoven, B.-M. Nguyen, A. A. Kiselev, W. Yi, M. Sokolich, M. J. Manfra, E. M. Spanton, and K. A. Moler, arXiv:1511.01728.
[26] T. Li, P. Wang, H. Fu, L. Du, K. A. Schreiber, X. Mu, X. Liu, G. Sullivan, G. A. Csathy, X. Lin, and R.-R. Du, Phys. Rev. Lett. **115**, 136804 (2015).
[27] N. Lezmy, Y. Oreg, and M. Berkooz, Phys. Rev. B **85**, 235304 (2012).
[28] N. Kainaris, I. V. Gornyi, S. T. Carr, and A. D. Mirlin, Phys. Rev. B **90**, 075118 (2014).
[29] Y.-Z. Chou, A. Levchenko, and M. S. Foster, Phys. Rev. Lett. **115**, 186404 (2015).
[30] A. Ström, H. Johannesson, and G. I. Japaridze, Phys. Rev. Lett. **104**, 256804 (2010).
[31] F. Geissler, F. Crépin, and B. Trauzettel, Phys. Rev. B **89**, 235136 (2014).
[32] J. C. Budich, F. Dolcini, P. Recher, and B. Trauzettel, Phys. Rev. Lett. **108**, 086602 (2012).
[33] J. Maciejko, C. Liu, Y. Oreg, X.-L. Qi, C. Wu, and S.-C. Zhang, Phys. Rev. Lett. **102**, 256803 (2009).
[34] Y. Tanaka, A. Furusaki, and K. A. Matveev, Phys. Rev. Lett. **106**, 236402 (2011).
[35] B. L. Altshuler, I. L. Aleiner, and V. I. Yudson, Phys. Rev. Lett. **111**, 086401 (2013).
[36] O. M. Yevtushenko, A. Wugalter, V. I. Yudson, and B. L. Altshuler, Europhys. Lett. **112**, 57003 (2015).
[37] J. I. Väyrynen, M. Goldstein, and L. I. Glazman, Phys. Rev. Lett. **110**, 216402 (2013).
[38] J. I. Väyrynen, M. Goldstein, Y. Gefen, and L. I. Glazman, Phys. Rev. B **90**, 115309 (2014).
[39] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.116.086603 for the bosonization of the backscattering kinetic operator on the second line of Eq. (2) and the 1-particle umklapp interaction that obtains from $\rho^2(x)$, the derivation of Eq. (18), and details of the numerical scheme used to generate Figs. 1 and 2.
[40] E. Barnes, Phys. Rev. A **88**, 013818 (2013).
[41] E. Ya. Sherman, Phys. Rev. B **67**, 161303(R) (2003).
[42] L. E. Golub and E. L. Ivchenko, Phys. Rev. B **69**, 115333 (2004).
[43] D. G. Rothe, R. W. Reinthaler, C.-X. Liu, L. W. Molenkamp, S.-C. Zhang, and E. M. Hankiewicz, New J. Phys. **12**, 065012 (2010).

[44] B. L. Altshuler and A. G. Aronov, in *Electron-Electron Interactions in Disordered Systems*, edited by A. L. Efros and M. Pollak (North-Holland, Amsterdam, 1985).
[45] D. L. Maslov and M. Stone, Phys. Rev. B **52**, R5539(R) (1995).
[46] V. V. Ponomarenko, Phys. Rev. B **52**, R8666(R) (1995).
[47] I. Safi and H. J. Schulz, Phys. Rev. B **52**, R17040(R) (1995).
[48] For a recent review, see, e. g., V. E. Kravtsov, arXiv:0911.0639.
[49] Y.-Z. Chou and M. S. Foster, Phys. Rev. B **89**, 165136 (2014).
[50] L. Balents and M. P. A. Fisher, Phys. Rev. B **56**, 12970 (1997).
[51] H. Mathur, Phys. Rev. B **56**, 15794 (1997).