Spin anisotropy due to spin-orbit coupling in optimally hole-doped Ba$_{0.67}$K$_{0.33}$Fe$_2$As$_2$

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We use polarized inelastic neutron scattering to study the temperature and energy dependence of spin space anisotropies in the optimally-hole-doped iron pnictide Ba$_{0.67}$K$_{0.33}$Fe$_2$As$_2$ ($T_c$ = 38 K). In the superconducting state, while the high-energy part of the magnetic spectrum is nearly isotropic, the low-energy part displays a pronounced anisotropy, manifested by a c-axis polarized resonance. We also observe that the spin anisotropy in superconducting Ba$_{0.67}$K$_{0.33}$Fe$_2$As$_2$ extends to higher energies compared with electron-doped BaFe$_2$As$_2$ ($TM$ = Co, Ni) and isovalent-doped BaFe$_2$As$_1.6$_P$_{0.6}$, suggesting a connection between $T_c$ and the energy scale of the spin anisotropy. In the normal state, the low-energy spin anisotropy for hole- and electron-doped iron pnictides near optimal superconductivity onset at temperatures similar to the temperatures at which the elastoresistance deviates from Curie–Weiss behavior, pointing to a possible connection between the two phenomena. Our results highlight the relevance of the spin-orbit coupling to the superconductivity of the iron pnictides.

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I. INTRODUCTION

The parent compounds of iron pnictide superconductors, such as LaFeAsO and BaFe$_2$As$_2$, form stripe antiferromagnetic (AF) order at $T_N$ below a tetragonal-to-orthorhombic structural transition temperature $T_S$ [inset in Fig. 1(b)] [1–4]. Superconductivity can be induced by partially replacing Ba by K in BaFe$_2$As$_2$ to form hole-doped Ba$_{1-x}$K$_x$Fe$_2$As$_2$ [5–9] or by partially replacing Fe by $TM$ ($TM$ = Co, Ni) to form electron-doped BaFe$_{2−x}$TM$_x$As$_2$ [10–13]. Importantly, the resulting phase diagrams exhibit significant asymmetry between electron- and hole-doping [Figs. 1(a) and 1(b)] [3,4]. For instance, while near optimal doping the stripe AF order becomes incommensurate for electron-doped BaFe$_{2−x}$TM$_x$As$_2$ [12,13] [see arrow in Fig. 1(b)], a double-Q tetragonal magnetic structure with ordered moments along the c-axis is observed in hole-doped Ba$_{1−x}$K$_x$Fe$_2$As$_2$ [see region of the phase diagram near the arrow and inset in Fig. 1(a)] [6–9].

Nevertheless, upon entering the superconducting state, a magnetic resonance mode appears in the magnetic spectrum in both cases at the AF wave vector (Q$_{AF}$) [14–17]. Furthermore, by measuring the splitting of the electronic states at high-symmetry points in reciprocal space [18], angle-resolved photoemission spectroscopy (ARPES) measurements find that spin-orbit coupling (SOC) is present in both electron- and hole-doped iron pnictides with a similar energy scale $\sim$10 meV [19]. Also common to both optimally-electron-doped BaFe$_{2−x}$TM$_x$As$_2$ [20,21] and -hole-doped Ba$_{1−x}$K$_x$Fe$_2$As$_2$ [22,23] is the presence of electronic nematic fluctuations, as revealed by the elastoresistance, i.e., the rate of change of the resistivity anisotropy with respect to applied in-plane uniaxial strain [Fig. 1(c)] [24]. The elastoresistance diverges with a Curie–Weiss form for both classes of materials as well as for isovalent-doped BaFe$_2$As$_1.6$P$_{0.6}$ [25]. Deviation from the Curie–Weiss behavior is seen in both optimally-electron- and -hole-doped BaFe$_2$As$_2$ at low temperatures, while no deviation is seen in BaFe$_2$As$_1.6$P$_{0.6}$ down to $T_c$ [Fig. 1(d)] [25].

In addition to its impact on the electronic spectrum [19,26], SOC also converts crystalline anisotropies into anisotropies in spin space, as seen from nuclear magnetic resonance studies [27]. The spin anisotropy resulting from SOC plays an essential role for the double-Q magnetic phase [6–9], in which the ordered moments align along the c axis [28]. If SOC was absent, the spin excitations in the paramagnetic tetragonal state of the iron pnictides would be isotropic in spin space [Fig. 1(e)]. However, due to the presence of a sizable SOC, an anisotropy is developed in the spin excitations, which can be quantitatively determined by neutron polarization analysis [29]. In the antiferromagnetically ordered phases of the parent compounds BaFe$_2$As$_2$ and NaFeAs [30,31], where the ordered moments point parallel to the orthorhombic a axis [inset in Fig. 1(b)], spin waves exhibit significant anisotropy, with c-axis polarized spin waves occurring at lower energy compared with b-axis polarized spin waves [32–34]. To elucidate the relevance of SOC to superconductivity, it is instructive to compare the behavior of the spin anisotropy in hole-doped and electron-doped BaFe$_2$As$_2$, since the maximum values of $T_c$ are quite different in these two cases—$T_c \approx 38$ K for optimally-hole-doped Ba$_{0.67}$K$_{0.33}$Fe$_2$As$_2$ and $T_c \approx 25$ K for optimally-electron-doped BaFe$_{1.86}$Co$_{0.14}$As$_2$. Previous analysis of the electron-doped case revealed that the spin anisotropy persists in the paramagnetic tetragonal phase for doping levels up to or slightly beyond optimal...
In this paper, we present polarized neutron scattering studies of spin excitations in optimally hole-doped BaFe$_2$As$_2$, where spin anisotropy at Q$_{MF} = (1,0,1)$ was found below $E \approx 7$ meV and up to $\sim 70$ K [36,37]. We associate the onset of normal-state spin anisotropy with the nematic susceptibility deviating from Curie–Weiss behavior measured via elastoresistance [see vertical arrows in Fig. 1(d)] [25], indicating an important role of spin excitations in transport properties of iron pnictides.

Upon entering the superconducting state, we find that, while at high energies ($E \geq 14$ meV), the spectrum is nearly isotropic as found in previous work [38], at low energies the resonance mode is strongly anisotropic, being dominated by a $c$-axis polarized component. We attribute this behavior to the fact that the superconducting state is close to the double-$Q$ magnetic phase, in which the magnetic moments point out of plane [7,8]. Indeed, by adding a spin-anisotropic term that favors $c$-axis spin orientation in a simple two-band theoretical model, we find that the resonance mode in the $c$-axis polarized channel has in general a lower energy than in other channels, and that this energy difference increases as the magnetically ordered state is approached. Our analysis also reveals an interesting correlation between the energy scale of the spin anisotropy in the superconducting state and $T_c$ [35–40,42], suggesting that SOC is an integral part of the superconductivity of iron pnictides.

II. EXPERIMENTAL RESULTS

Polarized inelastic neutron-scattering measurements were carried out by using the IN22 triple-axis spectrometer at Institut Laue–Langewin in Grenoble, France. We studied Ba$_{0.67}$K$_{0.33}$Fe$_2$As$_2$ single crystals ($a = b \approx 5.56$ Å, $c = 13.29$ Å) co-aligned in the $[H,0,L]$ scattering plane used in previous works [38,47]. We use the orthorhombic notation suitable for AF-ordered iron pnictides even though Ba$_{0.67}$K$_{0.33}$Fe$_2$As$_2$ has a tetragonal structure and is paramagnetic at all temperatures [15,38]. Thus, the momentum transfer is $Q = H a^* + K b^* + L c^*$, with $a^* = \frac{2\pi}{a} a$, $b^* = \frac{2\pi}{b} b$, and $c^* = \frac{2\pi}{c} c$, where $H$, $K$, and $L$ are Miller indices. In this notation, magnetic order in BaFe$_2$As$_2$ occurs at $Q_{AF} = (1,0,0)$ with $L = 1,3,5,\ldots$ [Fig. 1(e)]. Three neutron spin-flip (SF) cross sections $\sigma^\alpha_\parallel$, $\sigma^\alpha_\perp$, and $\sigma^\alpha_{SF}$ were measured, with the usual convention $x \parallel Q$, $y \perp Q$ and in the scattering plane, and $z$ perpendicular to the scattering plane. Magnetic neutron scattering directly measures the magnetic scattering function $S^\alpha(Q,E)$, which is proportional to the imaginary part of the dynamic susceptibility through the Bose factor, $S^\alpha(Q,E) \propto [1 - \exp(-\frac{E}{k_B T})^{-1}]\Im \chi^\alpha(Q,E)$ [48]. Following earlier works [34,36,38,40,41], we denote the diagonal components of the magnetic scattering function $S^\alpha_\parallel = M_\alpha$, $M_{\parallel}$ and $M_{\perp}$ can be obtained from measured SF cross sections through $\sigma^\alpha_\parallel = \sigma^\alpha_\perp = M_{\parallel}$ and $\sigma^\alpha_\parallel - \sigma^\alpha_\perp = M_{\perp}$, and $M_{\perp}$ are related to $M_{\parallel} = M_{100}$, $M_{\parallel} = M_{100}$, and $M_{\perp} = M_{001}$ through $M_{\parallel} = \sin^2 \theta M_{\parallel} + \cos^2 \theta M_{\perp}$ and $M_{\perp} = M_{\parallel}$ [34,36,38,40,41], with $\theta$ being the angle between Q and $a^*/a$. Anisotropy between $M_{\parallel}$ and $M_{\perp}$ at $Q_{AF}$ is allowed in the paramagnetic tetragonal state of iron pnictides and does not break four-fold rotation symmetry of the lattice because $Q_{AF}$ is at an edge of the Brillouin zone of the unfolded tetragonal (1-Fe) unit cell [46], as depicted in Fig. 1(f). Another manifestation of the lack of four-fold rotational symmetry for magnetic excitations.

**FIG. 1.** The electronic phase diagrams of (a) hole- and (b) electron-doped BaFe$_2$As$_2$. While parent compounds of iron pnictides have stripe AF order [inset in panel (b)] [2], the tetragonal double-$Q$ $C_4$ AF order is found in hole-doped BaFe$_2$As$_2$ near optimal superconductivity [inset in panel (a)] [6–9]. $T_S$, $T_N$, $T_c$, and $T_r$ mark the tetragonal-to-orthorhombic structural transition, the paramagnetic-to-AF transition, the superconducting transition, and the transition into the $C_4$ magnetic phase. The phase diagrams in panels (a) and (b) are adapted from Refs. [43] and [4]. (c) Resistivity anisotropy $(\rho_a - \rho_b)/(\rho_a + \rho_b)$ of BaFe$_{1.90}$Ni$_{0.06}$As$_2$ and Ba$_{0.67}$K$_{0.33}$Fe$_2$As$_2$ under uniaxial pressure of $P = 15$ MPa measured by using a mechanical clamp that can vary applied pressure in situ [44,45]. (d) Elastoresistance [2] for optimally doped BaFe$_{1.91}$Ni$_{0.09}$As$_2$, Ba$_{0.67}$K$_{0.33}$Fe$_2$As$_2$, and BaFe$_{2}$As$_2$$_{1-\delta}$, adapted from Ref. [25]. The solid lines are Curie–Weiss fits to the data and the dashed lines represent deviations from the Curie–Weiss form. Vertical arrows mark the temperature at which such deviations begin. (e) Schematic of isotropic spin excitations (left) and anisotropic spin excitations (right), with the sizes of arrows centered at $Q_{AF}$ representing the intensities of spin excitations polarized along different directions. (f) In-plane spin anisotropy discussed in this work (represented by red and blue arrows of different sizes) preserves four-fold rotational symmetry of the tetragonal unit cell because $Q_{AF}$ is at an edge of the Brillouin zone of the unfolded tetragonal (1-Fe) unit cell [46], depicted by the shaded gray area.
By using the measured cross sections in panels (a)–(d), we find that \( \sigma^x \) with \( L \) for (a) \( L = 0 \), (b) \( L = 1 \), (c) \( L = 2 \), and (d) \( L = 3 \) measured at 2 K. By using the measured cross sections in panels (a)-(d), \( M_x \), \( M_y \), and \( M_z \) are obtained for (e) even and (f) odd \( L \). The solid lines are to guide the eye.

at \( Q_{AF} \) is the anisotropic in-plane correlation lengths seen in the paramagnetic tetragonal states of BaFe\(_2\)As\(_2\) [49] and CaFe\(_2\)As\(_2\) [50]. By obtaining \( M_x \) and \( M_z \) at two equivalent wave vectors with different \( \theta \), it is then possible to obtain \( M_x \), \( M_y \), and \( M_z \) [33,34,36,40].

Figure 2 summarizes constant-\( Q \) scans at 2 K and \( Q = (1,0,L) \) with \( L = 0, 1, 2, \) and 3. From Figs. 2(a)–2(d), it is clear that \( \sigma^x > \sigma^y > \sigma^z \) below \( E \approx 14 \) meV, meaning that spin anisotropy exists below this energy while excitations above this energy are isotropic as shown in previous work [38]. Although magnetic order is fully suppressed in Ba\(_{0.67}\)K\(_{0.33}\)Fe\(_2\)As\(_2\), the spin gap \( E_s \) in the superconducting state displays strong \( L \) dependence, with \( E_s \approx 0.75 \) meV for odd \( L \) and \( E_s \approx 5 \) meV for even \( L \) [15,38]. From Figs. 2(b) and 2(d), we observe that the small gap for odd \( L \) is due to \( M_x \), with \( \sigma^x \approx \sigma^y > \sigma^z \) for \( E \lesssim 5 \) meV. Magnetic excitations are gapped in the same energy range for even \( L \), as can be seen in Figs. 2(a) and 2(c), with \( \sigma^x \approx \sigma^y > \sigma^z \). \( M_x \), \( M_y \), and \( M_z \) for even and odd \( L \) are shown in Figs. 2(e) and 2(f), respectively. While \( M_y \) is weakly \( L \) dependent, \( M_x \) clearly displays different behavior for even and odd \( L \). Because, in the energy range \( 5 \lesssim E \lesssim 10 \) meV, \( M_x \) dominates and is dispersive along \( L \), we uniquely identify it with the the anisotropic resonance that disperses along \( L \) which was previously observed in the same sample [38].

To gain further insight into the spin anisotropy of Ba\(_{0.7}\)K\(_{0.3}\)Fe\(_2\)As\(_2\), we carried out temperature scans at \( Q_{AF} = (1,0,1) \) for \( E = 3 \) meV and \( E = 9 \) meV, as shown in Figs. 3(a) and 3(b). At \( E = 3 \) meV, the spin anisotropy with \( \sigma^x > \sigma^y \) persists up to \( \sim 100 \) K. Although below \( T_c \), the magnetic signal is suppressed in all three SF cross sections, the normal-state anisotropy persists [Fig. 3(a)]. At \( E = 9 \) meV, the spin anisotropy disappears above \( T_c \), suggesting that the main contribution to the spin anisotropy in the superconducting state arises from the anisotropic resonance mode [38]. In Figs. 3(c) and 3(d), \( \sigma^y - \sigma^x \propto M_y \) and \( \sigma^y - \sigma^x \propto M_z \) are shown. At \( E = 3 \) meV, \( M_y > M_z \), while a clear resonance mode with an order-parameter-like temperature dependence is seen in \( M_y \), \( M_z \) remains constant across \( T_c \). The temperature onset of spin anisotropy is more clearly seen in Figs. 3(e) and 3(f), which plots \( \sigma^y - \sigma^x \propto M_y \) for \( E = 3 \) meV and \( E = 9 \) meV, respectively.

To obtain the temperature dependence of \( M_x \), \( M_y \), and \( M_z \), we measured \( \sigma^x \), \( \sigma^y \), and \( \sigma^z \) at \( Q_{AF} = (1,0,3) \) for \( E = 3 \) meV and \( E = 9 \) meV [44]. Combining the temperature dependence for \( L = 1 \) and \( L = 3 \), \( M_x \), \( M_y \), and \( M_z \) are obtained for odd \( L \) as shown in Figs. 4(a) and 4(b). At \( E = 3 \) meV, \( M_y > M_z \) within the probed temperature range, and all three channels decrease in intensity below \( T_c \). At \( E = 9 \) meV, \( M_x \) and \( M_y \) display a weak temperature...
Without SOC, the energy of the resonance mode is the same for all polarizations, being close to 2Δ far from the putative magnetic quantum phase transition inside the superconducting dome \([U \ll U_c^*]\) in Fig. 4(c), but vanishing as the transition is approached \([U \to U_c^*]\) in Fig. 4(c). SOC, however, promotes a spin anisotropy term that makes the magnetic moments point along the \(c\) axis for hole-doped compounds \([28]\). As a result, the energy of the resonance mode polarized along the \(c\) axis is suppressed much faster as the magnetic transition is approached, yielding \(\omega_c < \omega_{ab}\) [Fig. 4(c)]. This behavior is in qualitative agreement with our experimental results, with the resonance seen in \(M_4\) indeed at lower energies. It should also be noted that our model does not capture the broadening of the resonance, which is rather pronounced in the experimental data. Our simple model has two additional consequences: first, as the system is overdoped and moves farther from the magnetically ordered state, the resonance mode should become more isotropic. While spin anisotropy persists in slightly overdoped BaFe\(_{1.5}\)Fe\(_2\)As\(_2\) \((T_c = 36 \text{ K})\) \([39]\), how it evolves in K-well-overdoped samples remains to be seen. Furthermore, because in electron-doped compounds the moments point along the \(a\) direction, the resonance is expected to be polarized along the \(a\) axis. Although this is the case in electron-doped NaFe\(_{0.985}\)Co\(_0.015\)As \([40]\), the sample studied had long-range AF order. For electron-doped Ba(Fe\(_{0.94}\)Co\(_{0.06}\))\(_2\)As\(_2\) \([37]\), the anisotropic resonance was argued to be also polarized along \(c\) axis, based on the assumption \(M_x = M_y\) and the observation \(M_x > M_y = 0\) for the anisotropic resonance. As we have shown here and in previous work \([36]\), even in the tetragonal state \(M_x\) and \(M_y\) are not necessarily the same and it is unclear whether there is also significant resonance spectral weight polarized along the \(a\) axis in previous work \([37]\). Spin anisotropy of spin excitations has also been detected in the superconducting states of LiFeAs \([54]\) and FeSe\(_{0.5}\)Te\(_{0.5}\) \([55]\), consistent with significant spin-orbit coupling detected by ARPES \([19,26]\) in these systems.

The normal-state spin anisotropy at low energies persists to a temperature significantly higher than \(T_c\) \([70 \text{ K in BaFe}_{1.09}\)Ni\(_{0.06}\)As\(_2\) \([36]\) and \(\sim 100 \text{ K in BaFe}_{0.67}K_{0.33}\)Fe\(_2\)As\(_2\); Fig. 3(e)] for both electron- and hole-doped BaFe\(_2\)As\(_2\) near optimal doping. The temperature at which spin anisotropy onsets is similar to the temperature at which the nematic susceptibility deviates from Curie–Weiss behavior \([25]\), suggesting a common origin for both phenomena [Fig. 1(d)]. For optimally doped BaFe\(_2\)As\(_1.4\)P\(_{0.6}\), whose nematic susceptibility shows no deviation from the Curie–Weiss form [Fig. 1(d)] \([25]\), no spin anisotropy is observed right above \(T_c\) \([42]\). While disorder is likely to play an important role in explaining this deviation from Curie–Weiss behavior in the elastoresistance \([25]\), our results suggest that the spin anisotropy may also be important. Indeed, previous inelastic neutron scattering experiments revealed the intimate relationship between nematicity and magnetic fluctuations \([56–58]\). Theoretically, the nematic susceptibility increases with increasing magnetic fluctuations in all polarization channels \([24]\). However, once a spin anisotropy sets in, fluctuations related to the spin components perpendicular to the easy axis increase more slowly with decreasing temperature. As a result, the nematic susceptibility should also increase more slowly, which may contribute to the deviation from Curie–Weiss behavior observed experimentally.

FIG. 4. Temperature dependence of \(M_x\), \(M_y\), and \(M_z\) for (a) \(E = 3 \text{ meV}\) and (b) \(E = 9 \text{ meV}\). The solid lines are to guide the eye and dashed vertical lines mark \(T_c\). The insets in panels (a) and (b) show binned \(M_x\), \(M_y\), and \(M_z\) from panels (a) and (b) for \(T \leq 25 \text{ K}\) and \(T \geq 38 \text{ K}\). The insets show the same \(y\)-axis label as panels (a) and (b), (c) Difference between the energies of the \(ab\)-polarized and the \(c\)-polarized resonance modes obtained in our theoretical model. \(U\) is the electronic interaction that triggers long-range magnetic order with the moments pointing along the \(c\) direction, the resonance is expected to be polarized along the \(a\) axis. Although this is the case in electron-doped NaFe\(_{0.985}\)Co\(_0.015\)As \([40]\), the sample studied had long-range AF order. For electron-doped Ba(Fe\(_{0.94}\)Co\(_{0.06}\))\(_2\)As\(_2\) \([37]\), the anisotropic resonance was argued to be also polarized along \(c\) axis, based on the assumption \(M_x = M_y\) and the observation \(M_x > M_y = 0\) for the anisotropic resonance. As we have shown here and in previous work \([36]\), even in the tetragonal state \(M_x\) and \(M_y\) are not necessarily the same and it is unclear whether there is also significant resonance spectral weight polarized along the \(a\) axis in previous work \([37]\). Spin anisotropy of spin excitations has also been detected in the superconducting states of LiFeAs \([54]\) and FeSe\(_{0.5}\)Te\(_{0.5}\) \([55]\), consistent with significant spin-orbit coupling detected by ARPES \([19,26]\) in these systems.

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Finally, the maximum energies at which spin anisotropy is observed in the superconducting states of several doped BaFe$_2$As$_2$ compounds are plotted as a function of $T_c$ in Fig. 4(d). Note that the spin anisotropy of the resonance in the superconducting state is also present in BaFe$_2$As$_2$P$_{0.6}$, despite the absence of spin anisotropy in the normal state [42]. We note a clear positive correlation between the energy scale of the spin anisotropy and $T_c$, suggesting SOC to be an important ingredient for understanding superconductivity in iron pnictides.

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[42] Polarized neutron scattering results on BaFe$_2$As$_4$P$_{0.6}$. 