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There is Time for Calculation in Speed Chess and Calculation Accuracy Increases with Expertise

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Abstract

The recognition-action theory of chess skill holds that expertise in chess is due primarily to the ability to recognize familiar patterns of pieces. Despite its widespread acclaim, empirical evidence for this theory is indirect. One source of indirect evidence is that there is a high correlation between speed chess and standard chess. Assuming that there is little-to-no time for calculation in speed chess, this high correlation implies that calculation is not the primary factor in standard chess. Two experiments were conducted analyzing 200 games of speed chess. In Experiment 1, we examined the distributions of move times and the key finding was players often spent considerable time on a few moves. Moreover, stronger players were more likely than weaker players to do so. Experiment 2 examined skill differences in calculation by examining poor moves. The stronger players made proportionally few blunders (moves that a two-ply search would have revealed to be errors). Overall, the poor moves made by the weaker players would have required a less extensive search to be revealed as poor moves than the poor moves made by the stronger players. Apparently, the stronger players are searching deeper and more accurately. These results are difficult to reconcile with the view that speed chess does not allow players time to calculate extensively and call into question the assertion that the high correlation between speed chess and standard chess supports recognition-action theory.

Keywords: chess, expertise, recognition-action theory, depth of search

Chess expertise has been actively researched since the seminal work of de Groot (1946). The most cited aspects of de Groot's research are his finding that memory for chess positions was strongly related to chess skill whereas the other variables he investigated such as the number of moves players looked ahead (depth of search) were not. Based on these findings, Chase and Simon (1973) proposed what is now known as the "recognition-action theory." According to this theory, expert chess players have very detailed information about chess positions stored in long-term memory and this information allows them to relate the position in the current game to previous games. The familiar perceptual structures in a position allow the masters to "come up with good moves almost instantaneously, seemingly by instinct and intuition" (Chase & Simon, 1973, p. 269). That is, the recognition-action mechanism allows recognition of patterns of pieces on the board, and these familiar patterns suggest possible moves. Chase and Simon (1973) explained the recognition-action mechanism by stating that "each familiar pattern serves as the *condition* part of a production. When this condition is satisfied by recognition of the pattern, the resulting *action* is to evoke a move associated with this pattern and to bring the move into short-term memory for consideration" (p. 269). As a result, players can often immediately "see" the best move. A key tenet of recognition-action theory is that stronger players "see" better moves because they are able to access more chess patterns. According to Chase and Simon (1973), "the *most important processes* underlying chess mastery are these immediate visual-perceptual processes rather than the subsequent logical-deductive thinking processes" (p. 215, emphasis added).

Despite its intuitive appeal and widespread acclaim, empirical evidence for recognition-action theory has been indirect. As noted, its initial development was based in large part on negative evidence: the failure to find a relationship between playing strength and variables such as how many moves a player looks ahead. More recently, a second type of indirect evidence has been taken as support for recognition-action theory. Specifically, it has been claimed that the relatively high quality of moves played by Kasparov in a simultaneous exhibition (Gobet & Simon, 1996) and the high correlation between players' results in speed chess and standard chess (Burns, 2004) are strongly supportive of recognition-action theory. This argument is based on the assumption that simultaneous exhibitions and speed chess do not provide sufficient time for calculation and therefore move choice in these situations is based predominantly on fast automatic processes. As Gobet and Simon (1996) stated, “In view of the slight extent to which the lack of time for search lowered the quality of Kasparov’s play in the simultaneous matches, we conclude that memory and access to memory through the recognition of clues is the *predominant* basis for his skill” (p. 54, emphasis added). Similarly, Burns (2004) concluded that strength of the correlation between speed and standard chess “strongly suggests that variance in the effectiveness of fast processes (such as recognition) accounts for most of the variance in chess skill” (p. 446).

Burns also computed what he called the Equalization Factor (EF) which represents the degree to which the stronger of two players does not do as much better against the weaker player in speed chess as would be expected from their regular chess rating. Interestingly, he found negative correlations between chess rating and the EF. Burns’ interpretation is as follows:

> “If the proportion of skill due to slow processes is greatest at low skill levels, when two weak players compete against each other in blitz, a relatively large amount of what determines their normal ratings may have been removed. In effect, blitz equalizes their skill level and thus disadvantages the stronger player. However, at

> higher levels of skill, where fast processes play a greater role, equalization should decrease" (p. 443).

This interpretation is consistent with recognition-action theory in that it attributes differences among strong players to fast processes. However, it is not entirely consistent in that it implies that weaker players differ from stronger players in the quality of their slow processes.

Burns acknowledged that other hypotheses about the relationship between blitz performance and skill level are possible. One alternative is that the slow process of calculation is important for both skill levels, but in time pressure the weaker players are more prone to game-changing blunders than are the stronger players, and the rate of these blunders is not as strongly related to their rating as other aspects of their play. Thus, although Burns' finding of a negative correlation between chess rating and EF is a fascinating one, its support for recognition-action theory is indirect and more research would be required to interpret it unambiguously.

Interpretations of research on chess play under strict time limits are complicated by the possibility that time-consuming calculations take place on a small proportion of moves even when total time is limited. The assumption that move choice under strict time limits is devoid of slow processes would be undermined if it could be shown that players take a disproportionately long time on a small proportion of moves, thus allowing the opportunity for slow and detailed calculations in these key positions. Without this assumption that there are essentially no slow processes in speed chess, the argument by Burns (2004) that essentially all of the correlation between speed chess and standard chess is attributable to the presumed correlation between pattern recognition ability and expertise would be weakened: If players are doing slow and detailed calculating in some key positions, then their ability to do so accurately would almost certainly correlate with their ability to calculate accurately in standard chess. Thus, some or perhaps most of the correlation between speed chess and standard chess could be explained by calculating skill. Similarly, simultaneous exhibitions do not limit the time that can be taken on an individual move. It is not unlikely that Kasparov took the time he needed when faced with a complex position.

Experiment 1

This experiment investigated distribution of move times. Recognition-action theory claims that stronger players do not calculate more deeply than do weaker players and, therefore, this theory does not make specific predictions regarding differences in expertise in the distribution of move times. If, contrary to recognition-action theory, stronger players do more in-depth calculations than weaker players, then the times they take in key positions may be even more extreme than the times taken by weaker players. This may occur even though stronger players calculate more quickly than weaker players (Campitelli & Gobet, 2004).

Method

Subjects

The subjects were selected from those playing publicly on the website of the Internet Chess Club (ICC; http://www.chessclub.com). This is a club with various strengths of players in different time controls, including many of the best players in the world. The players receive a stable Elo rating only after having played a sufficient number of games in a specific time control. ICC has invested a significant amount of effort into the detection of frauds: computer fraud and having stronger players use others account. Moreover, the Elo rating (Elo, 1978) provides a standard and widely used quantitative scale for measuring the skill level of chess players. For the purpose of this study, players were selected so that their Elo ratings would to fall into two ranges: from 2300 to 2399 (stronger players) and from 1600 to 1699 (weaker players). The selection of the players was based on two criteria other than their ratings: (1) players within the Elo rating ranges must have played a game of speed chess for which the total time for each player was 5 minutes during data collection days and (2) players must have played with the opponents in the same Elo range. If a player met these criteria, we chose the player's most recent game for which the player and an opponent both were in the same Elo range. Based on this protocol, a total of 200 players (100 games) was selected. The stronger players in this present study had Elo ratings ranging from 2300 to 2397 (*n* = 100; *M* = 2,349; *SD* = 31); the weaker players had Elo ratings ranging from 1600 to 1699 (*n* = 100; *M* = 1,652; *SD* = 28).

Procedure

Time distributions. Times for opening moves and time-pressure moves were not included in the analysis. Opening moves were defined as moves 1 to 15 which often contain many "book moves" that are played from memory. This criterion was chosen to be conservative so that virtually all memorized moves would be eliminated. Time pressure moves were defined as moves made when players had less than 30 seconds left. When the players do not have much time left, they must play especially quickly. The weaker players made 133 time pressure moves and the stronger players made 421 time pressure moves. All other move times were included.

Results and Discussion

Figures 1 and 2 show the distributions of move times for the weaker players and for the stronger players, respectively. It is clear that both distributions are very positively skewed and that although the majority of moves are made quickly, a non-trivial number of moves are made after considerable time and presumably considerable thought.

The number of moves, maximum time, the percentage of outside values, and the percentage of far-out values were computed for each player[1]. Table 1 shows the minima, maxima, means, and standard deviations of these measures for the two skill levels. The maximum time was log-transformed to reduce skew and therefore the significance tests represent tests of geometric means rather than arithmetic means. To adjust for possible violations of homogeneity of variance, the Welch correction was used in all *t* tests. The mean of the maximum times taken by the stronger players (*Geometric Mean* = 29.24) was 43% longer than the mean for the weaker players (*Geometric Mean* = 20.40), $t(160.2) = 4.53$, $p < 0.001$. The ratio of these geometric means is 1.43 (95% CI: 1.23 to 1.68).

The mean percentages of extreme times were also considerably larger for the stronger than for the weaker players. Figures 3 and 4 show back-to-back stem and leaf plots of these percentages of outside and far-out values, respectively, for the weaker and the stronger players. These reveal that the stronger players made more and higher percentages of outside and far-out values than did the weaker players.

The distributions for the percentages of outside and far-out values were extremely skewed and log transformations were not possible because some players had no outside or far-out values. Therefore, we computed significance tests on differences between the mean percentages of outside values and mean percentages of far-out values using randomization tests and bootstrap tests to calculate confidence intervals. Both the randomization and bootstrap tests were conducted using the statkey software provided by the Lock family (http://www.lock5stat.com/statkey/index.html). For each of these calculations, we used 10,000 samples. The mean percentage of outside values for the stronger players ($M = 4.19$) was 1.35 higher (which is 48% higher) than the mean percentage of outside values for the weaker players ($M = 2.84$), $p = 0.011$ (95% CI on difference: 0.20 to 2.50). For the far-out values, the mean for the stronger players of 2.92 was 1.56 higher (which is more than 100% higher) than the mean of 1.36 for the weaker players, $p = 0.002$ (95% CI on difference: 0.53 to 2.58).

We also analyzed the proportions of games in which players took considerable time on one or more moves. Although subjective, we chose to define times that consisted of 10% or more of the total allotted time (10% of 5 minutes = 30 seconds) as “considerable time.” The stronger players took 30 seconds or longer on one or more moves in 0.42 of their games compared to 0.22 of the games for the weaker players, $\chi^2(1, N = 200) = 9.19$, $p = 0.002$ (95% CI on difference: 0.07 to 0.32)[2]. The mean number of moves taken by the stronger players was 54% higher than the mean for the weaker players, $t(168.5) = 6.13$, $p < 0.001$ (95% CI on difference: 6.37 to 12.41). Thus, even with more moves and therefore less mean time available per move, the stronger players more frequently took a long time on some moves than did the weaker players.

These results support the view that stronger players more frequently take considerable time on some moves than do weaker players. A possible reason for this difference is that for difficult positions, stronger players are more able to benefit more from extra time than are weaker players. Consistent with this possibility are the findings of Moxley, Ericsson, Charness and Krampe (2012) who asked chess players at two skill levels to find the best moves in positions differing in difficulty both immediately after presentation and after five minutes of study. The finding most relevant to the present discussion is that in the most difficult position, there was strong evidence that experts benefitted from the extra time whereas there was minimal evidence that the non-experts were able to do so.

A likely explanation of this experiment’s move-time results is that in certain key positions, stronger players improve their move choice by engaging in more in-depth calculation. It is not clear how this result could be reconciled with recognition-action theory without making *ad hoc* assumptions.

Experiment 2

Burns (2004) noted that his results are consistent with the thesis that search does not improve much as a function of skill level. In Experiment 2 we address the question of skill differences in search by analyzing moves on which players choose very poor moves.

It is generally difficult to assess how far a player looked ahead on a given move from only the game score. However, the analysis of poor moves provides a way, albeit an indirect one, to estimate search depth. By analyzing the position in which a poor move is made, one can determine how many plies the player would have had to look ahead in order to have seen that the move was an error. Therefore, the number of plies required to see that a move is a poor move can be interpreted as one greater than the maximum depth of search on that move. Here we define "required ply value" as the number of moves necessary to look ahead to see that a move is a serious error.

As applied to skill differences, if stronger players search deeper than weaker players, then their poor moves will tend to have higher ply values than poor moves made by the weaker players. If the weaker players and the stronger players are looking ahead to the same degree, there is no reason to think that the poor moves made by the stronger players would have higher ply values than the poor moves made by the weaker players.

Method

Procedures

Poor moves. In order to identify poor moves, the moves were analyzed using the game-analysis facility of Fritz 13 chess playing program (ChessBase, Hambury, Germany). This program has long been one of the world's strongest commercially available computer chess programs. The criteria used to identify poor moves were based on Chabris and Hearst's (2003) protocols for defining a "true blunder." Here we refer to these moves as "poor moves" rather than blunders because they are not necessarily stupid, ignorant, or careless mistakes (Merriam-Webster Online Dictionary). It is possible that a poor move could be made because the position was complex and difficult. For example, if a move required a player to look 10-ply ahead in order to see that it is a losing move, this move is better thought of as a poor move rather as a blunder.

An objective evaluation of each move was obtained from Fritz 13. When the threshold is set to 10 ply, this program analyzes every move with a 10-ply exhaustive search. This means that this program looks five moves ahead for each player in each position to choose the best move.

The first criterion in determining that a move is a poor move is that it must be least 1.5 pawns worse than the program's choice for the best move. Moves that meet this first criterion are called "candidate poor moves." The 1.5-pawn criterion was chosen since this size is generally considered theoretically large enough to be sufficient to win (see, e.g., Hartmann, 1989). The second criterions is that it must not be the case that either of the players had an advantage of 3.0 or more before and after the candidate poor move. The logic is that if a player still has a clearly winning or losing position after a move, this move has no practical consequences.

Determining Required Ply Values. A move identified as a poor move is, by definition, at least 1.5 pawns worse than the program's choice for the best move based on a 10-ply search. Once a poor move is identified, the program is used to analyze the move based on a 9-ply search. If the move played is still evaluated as at least 1.5 pawns worse than the program's choice, then the program is used to do an 8-ply search. The depth of search is reduced by one until the program no longer identifies the move as a poor move. The ply value is then defined as the lowest depth of search for which the move is determined to be an error. For example, if the program identifies a move as a poor move with a 6-ply search but not with a 5-ply search, then this move has a required ply value of 6 since if the player would have accurately searched 6 plies ahead, the poor move would have been avoided. We define a blunder as an error with a ply value of one or two.

Results and Discussion

Figure 5 contains a back-to-back dot plot of the required ply values. An inspection of this figure reveals that the weaker players made more poor moves in general and more blunders (1-ply or 2-ply poor moves) in particular.

Significance tests and confidence intervals for percentages of poor moves were computed using randomization and bootstrap tests. The percentage of poor moves was computed for each player and the mean percentage of poor moves was higher for the weaker players ($M = 9.19$) than it was for the stronger players ($M = 4.82$), $p < 0.001$ (95% CI on difference: 1.8 to 7.4).

To avoid non-independence of values in the computation of the inferential statistics for required ply values, if a player made more than one poor moves in a game, one of the poor moves was randomly selected for analysis. Table 2 shows the proportion of players making each of the n-ply required value for poor moves as a function of skill level. Note that if a player made, for example, both a 3-ply and a 5-ply error, only one of these errors was randomly selected to be included in Table 2. The proportion of weaker players (0.38) making at least one blunder was higher than the proportion of stronger players (0.23), $\chi^2 (1, N = 200) = 5.31$, $p = 0.021$ (95% CI on difference: 0.02 to 0.27).

In order to assess skill differences in depth of search when poor moves were made, the required ply values of the two groups were compared. The required ply values were log-transformed to reduce skew which means significance tests on means pertain to geometric rather than arithmetic means. The mean required ply value for poor moves was higher for the stronger players (*Geometric Mean* = 3.76) than it was for the weaker players (*Geometric Mean* = 2.67), $t(106.4) = 3.74$, $p < 0.001$. The ratio of these geometric means is 1.41 (95% CI on difference: 1.18 to 1.69). Thus, even when stronger players made poor moves, they, on average, searched deeper than did the weaker players.

In sum, the stronger players made fewer blunders and their poor moves had higher required ply values than those of the weaker players. This strongly suggests that their accuracy of search was greater than that of the weaker players. These findings are consistent with the view that stronger players calculate more and search deeper than weaker players even under time pressure.

General Discussion

The findings in Experiment 1 that players take considerable time on select moves and that stronger players do this more often than weaker players are difficult to reconcile with the view that speed chess does not allow players more than a trivial amount of time to look ahead. The possibility that players are engaging in detailed calculation on at least some moves provides an alternative interpretation of the high correlation between speed chess and standard chess found by Burns (2004). Specifically, it may be that calculation is involved in both speed chess and standard chess and that the better one can calculate in a relatively short period of time (say, 20 seconds), the better one can calculate in a longer period of time. Thus, the correlation between speed chess and standard chess could be mediated largely by calculating ability.

Experiment 2 provides two types of indirect evidence that stronger players search deeper and calculate more accurately than weaker players even when time is severely limited: (1) stronger players made proportionally few blunders than weaker players and (2) although both weaker and stronger players made their share of poor moves, the poor moves made by the stronger players tended to be higher-ply poor moves.

We think it is important to consider skill differences in the accuracy of search as well as in the depth of search. The results of Experiment 2 suggest that stronger players search more accurately than weaker players because it is unlikely that blunders made by the weaker players were due to depth of search rather than accuracy of search. Specifically, there can be little doubt that players with ratings over 1600 conduct at least a 2-ply search. We suspect that the classic results showing no difference between experts and non-experts would have been different if accuracy of search rather than depth of search had been considered. The disassociation of depth and accuracy of search is evident in the protocols of players in de Groot's classic study. The strongest player studied by de Groot was probably one-time world champion Alexander Alekhine. In Alekhine's analysis of the variations stemming from the correct first move, his maximum depth of search was only five plies (including the correct move itself). Despite his relatively shallow search, Alekhine correctly decided on the move based on positional considerations. A much lesser player (E1) did a 10-ply search of this variation and was therefore credited with conducting the greatest depth of search of any player studied. Unfortunately, his third move (ply 5) was not a legal move and therefore his subsequent analyses were meaningless. Clearly one would get a misleading impression of the relative tactical skills and quality of the slow processes of these two players if one relied solely on depth of search. This illustrates one of the risks associated with asserting that skill differences are primarily due to differences in the recognition of familiar patterns and the actions associated with them based on the failure to find differences in depth of search.

It should be recognized that there are ways other than those specified by recognition-action theory to find a good move quickly. One is to apply well known chess principles. For example, a bishop on the same color as many of its own pawns is known as a “bad bishop” because the pawns restrict the bishop’s mobility. A move preventing a pawn that restricts the opponent’s bishop from moving could be found very quickly by a player who understood this concept. Gobet (1997) made a similar point in discussing his template theory, although we think this and other chess principles are more abstract than the recognition of familiar patterns posited by recognition-action theory or even the templates in Gobet’s template theory.

Kasparov's performance in the simultaneous exhibition against international masters does not provide unambiguous support for recognition-action theory. Although time was limited overall to keep the exhibition from taking an extremely long time, there was no limit on the time that Kasparov could take on any one move. There likely was enough time for Kasparov to do a considerable amount of calculation in the most tactically complicated positions. Ironically, there is a limit on the time the participants in a simultaneous exhibition can take since they are required to move as soon as the player giving the exhibition arrives at their tables, although some exhibitions allow players one or two opportunities to pass.

Despite its wide acclaim and frequent citation, the evidence for recognition-action theory is not without its interpretational problems. The present results call into question the interpretation that the high correlation between speed chess and standard chess supports recognition-action theory and the claim that the recognition-action mechanism is the predominant basis of the skill. They also suggest that differences in calculation accuracy contribute in an important way to differences in playing strength.

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Footnotes

[1] The outside values are the values from 1.5 interquartile ranges to 3.0 interquartile ranges above the 75th percentile of the time distribution; the far-out values are values more than 3.0 interquartile ranges above the 75th percentile.

[2] This and other confidence intervals for differences between two independent proportions were computed using the procedure recommended by Newcombe (1998).

Table 1

Descriptive Statistics as a Function of Player Strength

	Weaker Players				**Stronger Players**			
	M	*SD*	*Min*	*Max*	*M*	*SD*	*Min*	*Max*
Number of Moves	17.49	8.62	4	39	26.88	11.76	4	55
Maximum Time	23.93	13.51	3	73	32.46	16.03	10	97
Percentage of Outside Values	2.84	3.65	0	11.76	4.19	4.23	0	17.65
Percentage of Far-Out Values	1.36	3.01	0	14.29	2.92	3.92	0	16.67

Note. Data from 87 weaker players and 93 stronger players are included in this table. Data from games in which fewer than 5 moves were played are not included. Times are in seconds.

Table 2

The proportion of players making each of the required ply value for poor moves as a function of skill level (100 players per group).

	Weaker Players	**Stronger Players**
1- or 2-ply	0.38	0.23
3-ply	0.09	0.08
4-ply	0.07	0.03
5-ply	0.04	0.07
6-ply	0.03	0.06
7-ply	0	0.03
8-ply	0.02	0.04
9-ply	0	0.03
10-ply	0	0.04

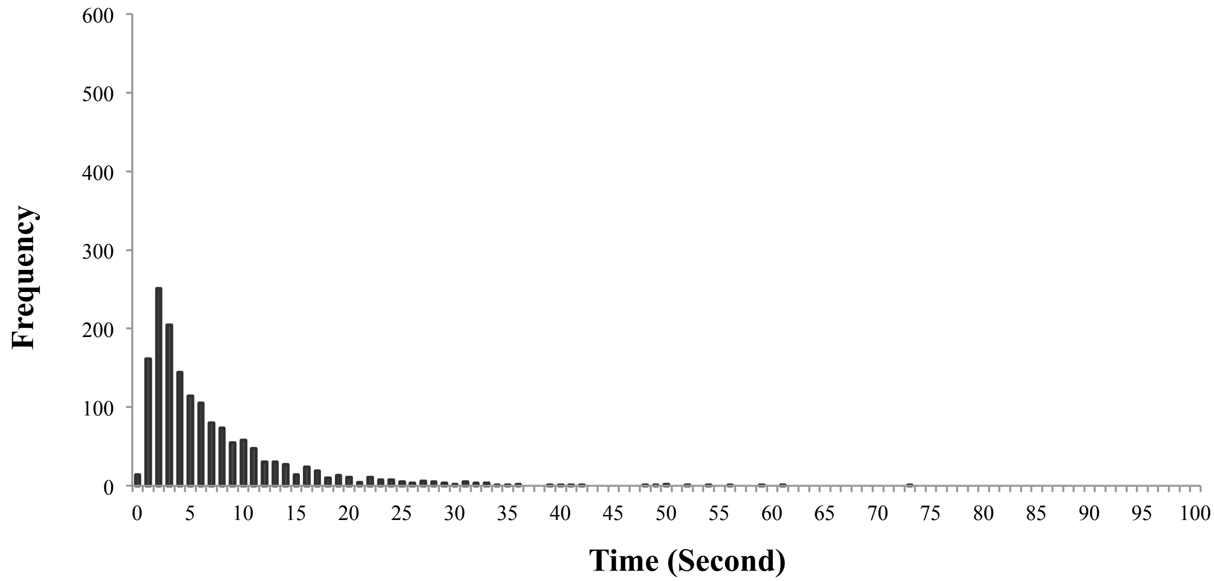

Figure 1. Time distribution of the moves from the weaker players.

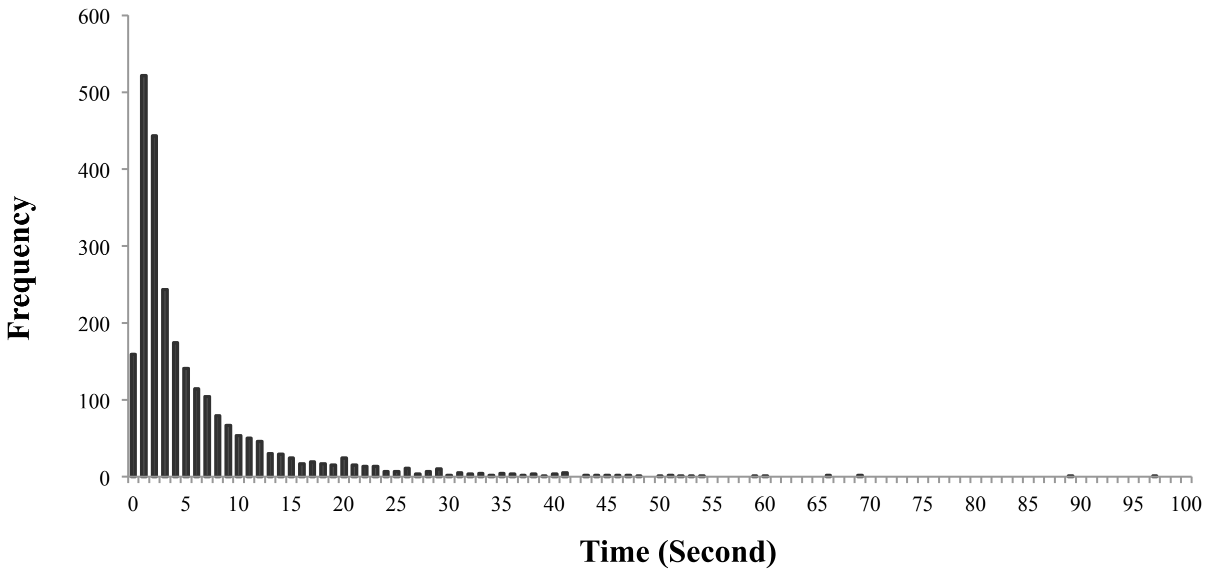

Figure 2. Time distribution of the moves from the stronger players.

Weaker Players (*n* = 87)	Stem	**Stronger Players (*n* = 93)**
	17	6
	16	7
	15	0
	14	
	13	
	12	9
8511	11	1158
50	10	0005
11	9	1
30	8	0338
7711	7	0
77333	6	13357789
6693333	5	366666999
8553220	4	2223333579 ← Arithmetic mean = 4.19
Arithmetic mean = 2.84 → 8332	3	23344688
8	2	147889
	1	9
0000000000000000000000000000000000000000000000000	0	000000000000000000000000000000000

Figure 3. The back-to-back stem and leaf plot of the percentages of outside values for the weaker and the stronger players.

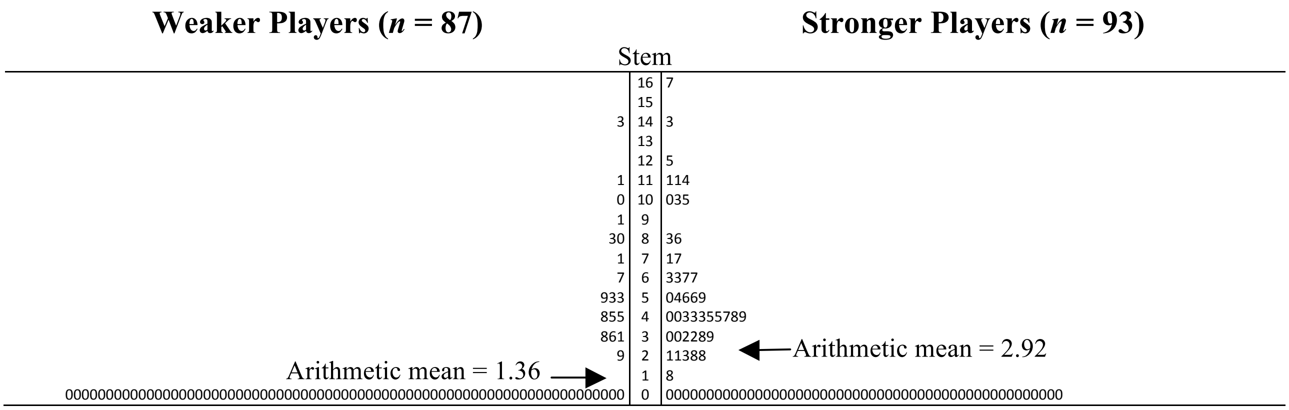

Figure 4. The back-to-back stem and leaf plot of the percentages of far-out values for the weaker and the stronger players.

Weaker Players		ply	Stronger Players	
N	60 50 40 30 20 10	ply	10 20 30 40 50 60	N
59	00000000000000000000000000000000000000000000000000000000000	1	00000000000000000000000000000000	32
7	0000000	2	000000000	9
10	0000000000	3	0000000000000000	16
13	0000000000000	4	00000	5
7	0000000	5	000000000000	12
5	00000	6	00000000	8
3	000	7	0000	4
6	000000	8	0000	4
0		9	00000	5
3	000	10	000000	6

Figure 5. The back-to-back dot plot of all required ply values for poor moves. The weaker players made 113 poor moves, whereas the stronger players made 101 poor moves.