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STRATEGIC CAPACITY INVESTMENTS IN AN IMPERFECTLY COMPETITIVE WORLD NATURAL GAS MARKET

BY

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Abstract
In this paper, we develop a model for the world natural gas market where buyers and sellers are connected by a trading network. However, this paper extends Cigerli (2013) by relaxing the assumption of fixed supply capacities and allowing for natural gas producers to invest in their supply capacities. We assume a two period model with no uncertainty and show that there is a unique Cournot-Nash equilibrium and the open-loop Cournot-Nash equilibrium and closed-loop Cournot-Nash equilibrium investments coincide. We apply this model to a network formed by using BP's Statistical Review of World Energy 2010 major trade flows. Later, we change model parameters exogenously to analyze various policy scenarios. We find that producers respond to changes in market conditions by investing in their supply capacities instead of displacing their resources from other markets.

1 Introduction

This paper extends Cigerli (2013), which solved for the constrained Cournot equilibrium with n producers, each having a fixed supply capacity, and m consumers connected through a bipartite network. More specifically, in this paper we relax the assumption that producers have fixed supply capacities and instead allow them to invest in their supply capacities.

Models of capacity expansion in oligopolistic markets have tended to be applied most to electricity markets. This is because the perfect competition assumption is a strong one when it comes to restructured electricity markets. Even though there are several studies[1] looking at the operations of oligopolistic electricity markets, the literature on strategic investments in these markets is relatively new. Electricity market models dealing with both investments and operations start with Murphy and Smeers (2005), who considered three models of investment in generation capacity. The first model assumed perfect competition. The second model extended the Cournot model to include investments in new generation capacities, where capacity is simultaneously built and sold in long-term contracts (open-loop Nash equilibrium). The third model separated the investment and sales decision, assuming investment decisions are made in the first stage and sales in the second stage (closed-loop Nash equilibrium). Murphy and Smeers (2005) considered a simple electricity system where all demand and supply is concentrated at a single node and there are two generators behaving strategically. The most of the subsequent studies looked at the strategic investment problem in a duopolistic market. For instance, Ehrenmann and Smeers (2006) developed a two stage capacity expansion game under the assumption of duopoly. Their model was similar to Murphy and Smeers (2005), but unlike them, Ehrenmann and Smeers (2006) assumed no uncertainty. Genc and Zacoor (2010) extended the Murphy and Smeers (2005) two stage model to a dynamic duopoly with capacity investments under demand uncertainty. Genc and Zacoor (2010) characterized all the open-loop and closed-loop Nash equilibria of this game.

Ventosa et al. (2002) extended the capacity expansion problem from a duopolistic electricity market to an oligopolistic electricity market. However, they retained the assumption of a single demand node. They presented two approaches. In the first approach, firms choose their output and generating capacity under the assumption of Cournot competition. In the second approach, a “leader firm” chooses its capacity in the first stage, as in the Stackelberg game, and then in a second stage all the firms compete in quantity and capacity as in the Cournot game.

Our model adds to the strategic capacity investment literature by allowing for Cournot competition in a networked market with multiple demand nodes and multiple suppliers. However, our model makes a simplifying assumption that the network graph is fixed. A future extension of this paper would look for an equilibrium in a dynamic network graph with demand uncertainties over multiple periods. This is a difficult problem. There is even a conceptual issue in the problem with multiple periods. Hartley and Kyle (1989)

[1] Among others, see Wei and Smeers (1999), Daxhelet and Smeers (2001).

show that there can be multiple equilibria depending on what new investors conjecture about future investor behavior.

We modify Cigerli (2013) by allowing producers to invest in their supply capacities before making their production decisions. We show that this game can also be represented as a potential game and the open-loop Cournot-Nash[2] equilibrium and the closed-loop Cournot-Nash equilibrium of this potential game coincide. We apply this model to a world natural gas network formed by using BP's Statistical Review of World Energy, 2010. We then consider various changes to the basic model in a number of scenarios. We focus on how to look for strategic investment decisions.

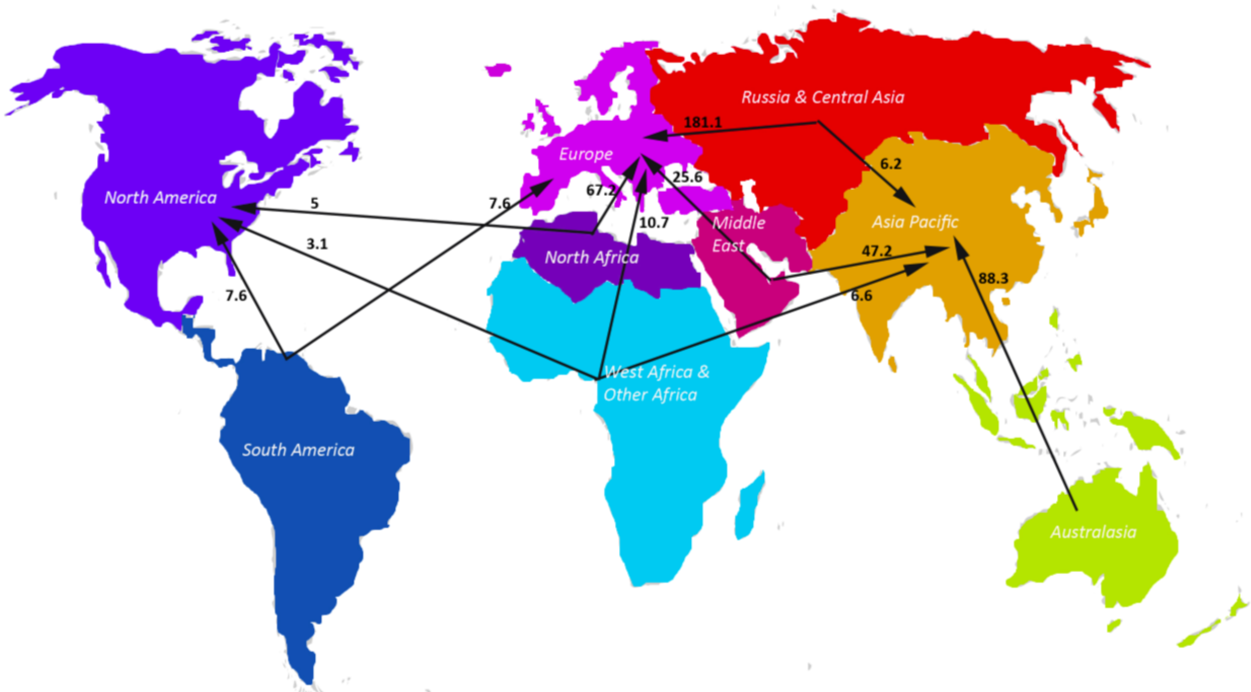
Section 2 introduces the world natural gas trade in 2009. In Section 3, we define our open-loop Cournot-Nash game model and solve for its unique Cournot-Nash equilibrium. Section 4 is devoted to analyzing different policy scenarios. The paper concludes in Section 5. In Section 6, we introduce a dynamic game which would be a future extension of this paper. In the appendix, we introduce the closed-loop Cournot-Nash game and also calibrate the model parameters.

2 World natural gas market

Taking account of the strategic interaction between suppliers adds to the complexity of our model. To simplify, we therefore aggregate producers and consumers into a small number of regions and equilibrium trade flows as shown in the world map in Figure 1.

[2]According to Fudenberg and Levine (1988), in the open-loop, players cannot observe the play of their opponents. In the closed-loop equilibrium, all past play is common knowledge at the beginning of each stage. Following their definition, in this paper we assume that in the open-loop producers do not know their competitiors' decisions in supply capacity investments and their current supply decisions, while in the closed-loop equilibrium they do know about the past plays of their competitiors, i.e, supply capacity investments, but they do not know about their competitors' current supply decisions.

Figure 1: Aggregated representation of producers and consumers and natural gas trade movements in 2009 (in Bcm)



Since each producer is connected to its domestic market, the number of producers and consumers is identical and in our case equals nine. In addition, six of the nine producers are exporters, and three of the nine consumers are importers. Producers and consumers are ordered[3] as Europe,[4] North America,[5] Asia Pacific,[6] South America,[7] West Africa,[8]

[3]They are labeled according this order. Producers: Europe labeled as 1, North America labeled as 2, Asia Pacific labeled as 3, South America labeled as 4, West Africa labeled as 5, North Africa labeled as 6, Russia labeled as 7, Middle East labeled as 8, Australasia labeled as 9. Consumers are in the same order as producers and labeled the same.

[4]Europe includes Austria, Belarus, Belgium, Bosnia, Bulgaria, Croatia, Czech Republic, Estonia, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, Macedonia, Moldova, Netherlands, Poland, Portugal, Romania, Serbia, Slovakia, Slovenia, Spain, Sweden, Switzerland, Turkey, United Kingdom and Ukraine.

[5]North America includes Mexico, U.S. and Canada.

[6]Asia Pacific includes Bangladesh, China, india, Japan, Myanmar, Pakistan, South Korea, Taiwan, Thailand and Vietnam.

[7]South America includes Argentina, Bolivia, Brazil, Colombia, Peru, Trinidad and Tobago, Venezuela.

[8]West Africa includes Angola, Equatorial Guniea, Mozambique and Nigeria.

North Africa,[9] Russia,[10] Middle East[11] and Australasia.[12]

According to the BP's Statistical Review of World Energy 2010, in 2009, North America's total natural gas consumption was 828 billion cubic meters (Bcm) and total production was 812.95 Bcm. In 2009, North America imported 42 percent of its natural gas from Trinidad and Tobago and 29 percent from Egypt.

In 2009, Europe's total natural gas consumption was 580.3 Bcm and total production was 288.1 Bcm. The production-to-consumption ratio for Europe was 0.49; thus, more than 50 percent of the natural gas consumed in Europe in 2009 was imported. Russia was the largest supplier of natural gas to Europe, with a 62 percent share of imports. The Middle East's share in European natural gas imports was 8.8 percent and North Africa's share was 23.3 percent.

In 2009, Asia Pacific's total natural gas consumption was 394.4 Bcm and its total production was 246.1 Bcm. The production-to-consumption ratio for the Asia Pacific was 0.62. Australasia supplied 59.5 percent of Asia Pacific's natural gas imports, making it Asia Pacific's largest supplier. The Middle East accounted for 31.8 percent of Asia Pacific's natural gas imports. Russia exported 6.2 Bcm of natural gas to the Asia Pacific in 2009, which was 3.7 percent of total imports. Before 2009, Russia had no natural gas exports to the Asia Pacific.

According to the BP's Statistical Review of World Energy in 2010, the U.S. Henry Hub natural gas price was 3.89 USD per million British thermal units (MMBtu). However, according to the OECD data on natural gas import costs, the U.S. LNG import cost was 4.52 USD per MMBtu. Due to our single price assumption for each region, the North American price in this model is 150 million USD per Bcm, which is approximately 4.18 USD per MMbtu.[13]

For the natural gas price in the Asia Pacific, we use LNG Japan price data reported by the BP's Statistical Review of World Energy in 2010, which is 9.06 USD per MMBtu. For natural gas price in the European market, we use the average of German import price, LNG and pipeline import prices for the European Union members provided by the OECD, which is 8.4 USD per MMbtu.

In our model, natural gas prices in the European and the Asia Pacific markets are close to each other and higher than the North American price. However, according to Figure 5 in Medlock (2012) the prices of natural gas at the U.S. Henry Hub, the UK National Balancing Point, the Platts Japan/Korea Marker were close before the Fukushima incident. We need to consider the historical natural gas price trends among these markets in our future research.

[9]North Africa includes Algeria, Egypt and Libya.

[10]Russia includes Armenia, Azerbaijan, Kazakhstan, Kyrgyzstan, Tajikistan, Turkmenistan, Uzbekistan and Russia.

[11]Middle East includes Iran, Israel, Kuwait, Lebanon, Oman, Saudi Arabia, Syria, Qatar, U.A.E., Yemen.

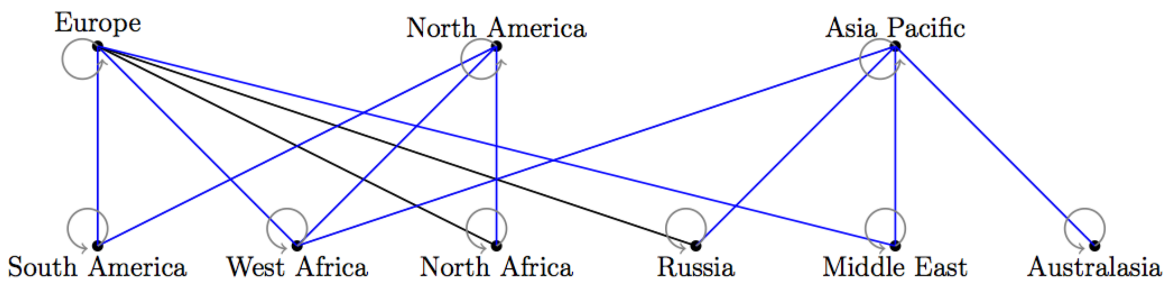
[12]Australasia includes Australia, Brunei, Indonesia, Malaysia, New Zealand, Philippines and Singapore.

[13]This price reflects the natural gas price in Canada, U.S. and Mexico.

2.1 Schematic representation of the world natural gas trade

The world natural gas network formed using these statistics[14] is shown below.

Figure 2: Schematic representation



3 Model

3.1 Notation[15]

There are m markets[16] $d_1, ..., d_m$ and n firms[17] $f_1, ..., f_n$. They are embedded in a network that links markets with firms, and firms can supply only to the markets to which they are connected. This network will be represented as a set, $g = \langle D \cup F, L \rangle$, of *nodes* formed by markets $D = d_1, ..., d_m$, and firms $F = f_1, ..., f_n$ and a set of *links* L, each link joining a market with a firm. A link from d_i to f_j will be denoted as (i, j). We say that a market d_i is *linked* to a firm f_j if f_j supplies natural gas to market d_i, using the link joining the two. We will use $(i, j) \in g$ meaning that d_i and f_j are connected in g.

A graph is *connected* if there exists a path connecting any two nodes of the graph while ignoring direction of physical flows. This concept is important because in a connected graph any change affecting one node will impact all other nodes.

$N_g(d_i)$ will denote the set of firms linked with d_i in $g = \langle D \cup F, L \rangle$. More formally:

$$N_g(d_i) = \{f_j \in F \text{ such that } (i, j) \in g\} \tag{1}$$

and similarly $N_g(f_j)$ stands for the set of markets linked with f_j.

[14]The blue lines indicate that the natural gas is transported via LNG and the black lines indicate that the natural gas is transported via pipeline. Half of the natural gas exports from North Africa to Europe are carried via LNG and half of them are carried via pipeline. Each producer is connected to its domestic market, which is indicated by gray circle.

[15]We use the conventions set forth in Ilkilic (2010).

[16]We use terms "market", "consumer" and "buyer" interchangeably.

[17]We use terms "firm", "producer" and "seller" interchangeably.

3.2 Open-loop Cournot-Nash game[18]

In this section, we introduce the open-loop Cournot-Nash game, where capacity investment and production decisions are made simultaneously. In the appendix, we introduce the closed-loop Cournot-Nash game[19] show that in a two stage game with no uncertainty, its equilibrium coincides with the equilibrium of the open-loop Cournot-Nash game.

We assume that markets have linear inverse demand functions. Given a market d_i and a flow vector Q_g the price, p_i, at d_i is

$$p_i(Q_g) = \alpha_i - \beta_i h_i, \tag{2}$$

where α_i and β_i are positive constants and h_i is natural gas consumption in market d_i:

$$h_i = \sum_{f_j \in N_g(d_i)} q_{ij} \tag{3}$$

We assume that the natural gas producer has zero costs of production in the short run up to its production capacity, $\bar{S}_j = \bar{S}_j^0 + k_j$,[20] and the marginal cost of capacity investment is constant and positive, θ_j.[21] Therefore, the cost of expanding production capacity by k_j is equal to $\theta_j k_j$.

We also assume that cost of exporting natural gas is proportional to the export volume. Therefore, for firm f_j the short-run total cost of exporting is

$$T_j(Q_g) = \sum_{d_i \in N_g(f_j) \backslash d_j} \tau_{ij} q_{ij}, \tag{4}$$

where τ_{ij} is the marginal cost of exporting natural gas to market i.

Firm j's total supply is denoted as s_j:

$$s_j = \sum_{d_i \in N_g(f_j)} q_{ij}, \tag{5}$$

where $s_j \leq \bar{S}_j = \bar{S}_j^0 + k_j$.

Given a graph Q_g and a supply capacity of $\bar{S}_j$, firm j maximizes its profit by choosing q_{ij} and k_j. Then, the potential function of this game is:

[18] We use the same notation as in Cigerli (2013).

[19] An assumption that the supply capacity investment is not productive instantly, meaning that there is a lag between a producer's capacity investment and the production, would be equivalent to solving the closed-loop Cournot-Nash equilibrium. For further details, see (A.1).

[20] $\bar{S}_j^0$ is the starting capacity at the beginning of period 0 and k_j is the capacity expansion in that period.

[21] In the calibration, we approximate θ_j by using the inverse of the reserve to production ratio, $\left(\frac{R}{P}\right)^{-1}$.

$$
\begin{aligned}
P^\star(Q_g) = & \sum_{d_i \in N_g(f_j)} \alpha_i \left(\sum_{f_j \in N_g(d_i)} q_{ij} \right) - \sum_{d_i \in N_g(f_j)} \beta_i \left(\sum_{f_j \in N_g(d_i)} q_{ij}^2 \right) \\
& - \sum_{d_i \in N_g(f_j)} \beta_i \left(\sum_{1 \le j < k \le n} q_{ij} q_{ik} \right) - \sum_{d_i \in N_g(f_j) \setminus d_j} \sum_{f_j \in N_g(d_i)} \tau_{ij} q_{ij} - \sum_{f_j} \theta_j k_j \qquad (6)
\end{aligned}
$$

subject to

$$\bar{S}_j^0 + k_j \geq \sum_{d_i \in N_g(f_j)} q_{ij} \quad \text{for all } \; j \in F \tag{7}$$

and

$$q_{ij} \geq 0 \quad \text{for all } \; (i,j) \in g \tag{8}$$

and

$$k_j \geq 0 \quad \text{for all } \; j \in F. \tag{9}$$

It can be verified that for every link from firm j to market i, that is q_{ij}, and for every link that is not from firm j to market i, that is q_{-ij},

$$\pi_j(q_{ij}, q_{-ij}) - \pi_j(x_{ij}, q_{-ij}) = P^\star(q_{ij}, q_{-ij}) - P^\star(x_{ij}, q_{-ij}) \tag{10}$$

and for every firm j's capacity investment, that is k_j, and for every firm's, that is not firm j, capacity investment, that is k_{-j}, $P^\star(Q_g)$[22] satisfies

$$\pi_j(k_j, k_{-j}) - \pi_j(t_j, k_{-j}) = P^\star(k_j, k_j) - P^\star(t_j, k_{-j}) \tag{11}$$

A function $P^\star$ satisfying (10) and (11) is called a potential function, which requires

$$\frac{\partial \pi_j}{\partial q_{ij}} = \frac{\partial P^\star}{\partial q_{ij}} \quad \text{for all } \; (i,j) \in g \tag{12}$$

and

$$\frac{\partial \pi_j}{\partial k_j} = \frac{\partial P^\star}{\partial k_j} \quad \text{for all } \; (j) \in g \tag{13}$$

Under no uncertainty, choosing capacity investment and production amounts to the same thing as choosing capacity investment in the first stage and choosing production in the second stage.[23]

[22] Q_g is the vector of quantities in graph g.
[23] See Section (A.1) for an analysis of the two stage capacity investment and optimal production game.

$$\mathcal{L} = \sum_{d_i \in N_g(f_j)} \alpha_i \left(\sum_{f_j \in N_g(d_i)} q_{ij} \right) - \sum_{d_i \in N_g(f_j)} \beta_i \left(\sum_{f_j \in N_g(d_i)} q_{ij}^2 \right) - \sum_{d_i \in N_g(f_j)} \beta_i \left(\sum_{1 \le j < k \le n} q_{ij} q_{ik} \right)$$
$$- \sum_{d_i \in N_g(f_j) \setminus d_j} \sum_{f_j \in N_g(d_i)} \tau_{ij} q_{ij} + \sum_{f_j \in N_g} \lambda_j \left(\bar{S}_j + k_j - \sum_{d_i \in N_g(f_j)} q_{ij} \right) + \sum_{f_j \in N_g} \mu_j k_j$$
$$+ \sum_{d_i \in N_g(f_j)} \sum_{f_j \in N_g(d_i)} \iota_{ij} q_{ij} \tag{14}$$

There exists $\lambda_j^\star$, $\mu_j^\star$ and $\iota_{ij}^\star$ such that $q_{ij}^\star$, $\lambda_j^\star$, $\mu_j^\star$ and $\iota_{ij}^\star$ that satisfy the following Kuhn-Tucker optimality conditions:

$$\frac{\partial \mathcal{L}}{\partial q_{ij}} = \alpha_i - 2\beta_i q_{ij} - \beta_i \left(\sum_{f_k \in N_g(d_i) \setminus \{f_j\}} q_{ik} \right) - \lambda_j - \tau_{ij} - \iota_{ij} = 0 \tag{15a}$$
$$\frac{\partial \mathcal{L}}{\partial k_j} = -\theta_j + \lambda_j + \mu_j = 0 \tag{15b}$$
$$\frac{\partial \mathcal{L}}{\partial \lambda_j} = \bar{S}_j^0 + k_j - \sum_{d_i \in N_g(f_j)} q_{ij} \ge 0 \tag{15c}$$
$$\frac{\partial \mathcal{L}}{\partial \mu_j} = k_j \ge 0 \tag{15d}$$
$$\frac{\partial \mathcal{L}}{\partial \iota_{ij}} = q_{ij} \ge 0 \tag{15e}$$
$$\lambda_j \frac{\partial \mathcal{L}}{\partial \lambda_j} = \lambda_j \left(\bar{S}_j^0 + k_{ij} - \sum_{d_i \in N_g(f_j)} q_{ij} \right) = 0 \tag{15f}$$
$$\mu_j \frac{\partial \mathcal{L}}{\partial \mu_j} = \mu_j k_j = 0 \tag{15g}$$
$$\iota_{ij} \frac{\partial \mathcal{L}}{\partial \iota_{ij}} = \iota_{ij} q_{ij} = 0 \tag{15h}$$

Therefore, equilibrium trade flow from firm j to market i is[24]

$$q_{ij}^\star = \begin{cases} \dfrac{\alpha_i - \tau_{ij} - \lambda_j - \beta_i \left(\sum_{f_k \in N_g(d_i) \setminus \{f_j\}} q_{ik} \right)}{2\beta_i} & \text{if } \frac{\partial \pi_j}{\partial q_{ij}}\Big|_{Q_g} \ge 0 \\ 0 & \text{if } \frac{\partial \pi_j}{\partial q_{ij}}\Big|_{Q_g} < 0 \end{cases} \tag{16}$$

[24]Note that, for $i = j$, $\tau_{ij} = 0$.

and if $k_j > 0$ then $\mu_j = 0 \implies \theta_j = \lambda_j \implies \dfrac{\partial C(k_j)}{\partial k_j} = \lambda_j^{\star}$.

Theorem 1: The Cournot game has a unique Nash equilibrium.

Proof: Proof of Theorem 1 in Cigerli (2013) applies to the proof this theorem. Note that the constraints of this game are linear functions of the new choice variables that we introduced in this paper, k_j.

Proposition 1: When there is no uncertainty, the open-loop Cournot-Nash equilibrium and the closed-loop Nash equilibrium investments coincide.

Proof: See Section (A.1) in the appendix.

4 Scenario analysis

In this section, we analyze the same scenarios[25] as in Cigerli (2013) by using the same methodology.[26]

4.1 Scenario I: Increased competition between Russia and the Middle East

In this scenario, we consider bringing Iraqi gas to the European market via a pipeline through Turkey, "Nabucco" pipeline. We incorporate this scenario into our model by using the RWGTM's cost estimate[27] for pipeline from Iraq to Istanbul, Istanbul to Bulgaria and Bulgaria to Austria. We get the marginal cost of exporting to Europe by taking the weighted average[28] of marginal costs of exporting natural gas to Europe via pipeline and via LNG[29], which decreases to 237.97 million USD. With this reduction, the Middle East increases supply to Europe from 25.6 Bcm to 56.36 Bcm by expanding its supply capacity by 30.68 Bcm which is 7.3 percent of its supply capacity in 2009. Unlike the fixed capacity scenario, the Middle East does not simply reallocate its resources.[30] When Nabucco is built there will be more competition in the European market for all producers that are connected to it: Europe, South America, West Africa, North Africa and Russia. They will decrease their supply to Europe to avoid further decline in the equilibrium natural gas price in Europe. For instance, South America's equilibrium supply to Europe decreases from 7.6 Bcm to 2.47 Bcm. Similarly, Russia's equilibrium supply decreases from 181.1 Bcm to 175.97 Bcm.

Under this scenario, equilibrium total supply to Europe increases from 580.3 Bcm to 585.43 Bcm, which decreases the equilibrium price in Europe from 300 million USD per Bcm to 294 million USD per Bcm. Since, producers do not shift their resources between

[25]We do not consider the scenarios with an exogenous change in supply capacity.

[26]Equilibrium trade flows and supply capacity investments are provided in Table (2) and Table (3).

[27]We consider tariffs paid to transit countries plus the operating and maintenance costs.

[28]This scenario assumes that 20 percent of natural gas is carried via pipeline and 80 percent is carried via LNG.

[29]The cost of exporting natural gas via LNG is calibrated in the previous section.

[30]This result changes as the marginal cost of expanding supply capacity changes.

markets, neither the consumption nor the price changes in North America and the Asia Pacific.

Under this scenario, profits of all producers that are connected to Europe (except the profits of the Middle East) decline. This is due to a 6 million USD per Bcm decline in the European price. Profits of the Middle East are 0.75 billion USD higher than its profits with fixed supply capacity. With capacity investments, the Middle East is able to increase supply to Europe without shifting supply from the Asia Pacific and domestic markets.

4.2 Scenario II: Decreased competition between Russia and the Middle East

In this scenario, Russia and the Middle East[31] collude to maximize their joint profits. Given the natural gas network we had in 2009, the joint profit of Russia and the Middle East[32] after collusion is

$$\begin{aligned}\Pi_{78}(Q_g) =&\alpha_1(q_{17}+q_{18}) - \beta_1(q_{11}+q_{14}+q_{15}+q_{16}+q_{17}+q_{18})(q_{17}+q_{18}) + \alpha_3(q_{37}+q_{38})\\ &- \beta_3(q_{33}+q_{35}+q_{37}+q_{38}+q_{39})(q_{37}+q_{38}) - \tau_{17}q_{17} - \tau_{18}q_{18} - \tau_{37}q_{37} - \tau_{38}q_{38}\\ &- \theta_7 k_7 - \theta_8 k_8 \end{aligned} \tag{17}$$

subject to

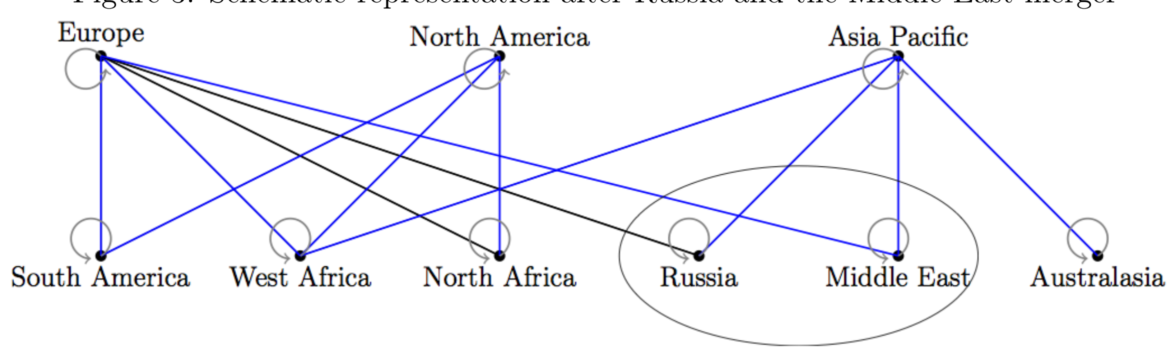
$$q_{17}+q_{37}+q_{77} \le \bar{S}_7 + k_7\,, \quad q_{18}+q_{38}+q_{88} \le \bar{S}_8 + k_8$$

and

$$q_{17},\, q_{37},\, q_{77},\, q_{18},\, q_{38},\, q_{88},\, k_7,\, k_8 \ge 0. \tag{18}$$

The graph of this new network is:

Figure 3: Schematic representation after Russia and the Middle East merger



After the merger, Russia and the Middle East reduce their combined output and their equilibrium supplies to each of the markets they share, namely Europe and the Asia

[31] We call this a "merger" between Russia and the Middle East.

[32] We label the merged Russia and Middle East supplier as 78.

Pacific. The new equilibrium outcome is that the links from Russia to the Asia Pacific and from the Middle East to Europe carry zero flows. This occurs because Russia has a lower marginal cost of exporting natural gas to Europe, while the Middle East has a lower marginal cost of exporting natural gas to the Asia Pacific.

The equilibrium supply of Russia and the Middle East to Europe is 185.38 Bcm after the merger. The pre-merger supply from Russia to Europe was 181.1 Bcm and from the Middle East to Europe was 25.6 Bcm. Similarly, the equilibrium supply of Russia and the Middle East to the Asia Pacific is 48.45 Bcm after the merger. The pre-merger supply from Russia to the Asia Pacific was 6.2 Bcm and from the Middle East to the Asia Pacific was 47.19 Bcm.

As a result of collusion, prices rise in both Europe and the Asia Pacific. In the new equilibrium, total supply to Europe decreases from 580.3 Bcm to 576 Bcm, which increases the equilibrium price from 300 million USD per Bcm to 304.45 million USD per Bcm. In the new equilibrium, total supply to the Asia Pacific decreases from 394.39 Bcm to 393.1 Bcm, which increases the equilibrium price in the Asia Pacific from 320 million USD per Bcm to 321.62 million USD per Bcm.

Neither the consumption nor the equilibrium price change in North America. This is because producers increase their supply to Europe and the Asia Pacific by expanding their supply capacity and not by shifting supplies from other markets.

In response to Russian and Middle Eastern collusion, other suppliers connected to Europe and the Asia Pacific invest in their supply capacities to increase their supply to Europe and the Asia Pacific. For instance, West Africa increases its supply capacity by 5.52 Bcm, which is approximately 18.13 percent of its supply capacity in the reference case. Its capacity investment is the highest of all other firms supplying Europe or the Asia Pacific. That is because it is the only producer that is connected to *both* Europe and the Asia Pacific. West Africa increases supply to Europe by 4.27 Bcm and to the Asia Pacific by 1.24 Bcm and expands its supply capacity by 5.52 Bcm.

The joint profit of Russia and the Middle East increases by 1.06 billion USD compared to total joint profits in 2009. However, their joint profit decreases by 1.12 billion USD compared to a scenario with a merger but holding supply capacities fixed. The impact of such collusion would be more dramatic on the equilibrium prices if producers were constrained by their supply capacities.

4.3 Scenario III: An increase in Asia Pacific's natural gas demand

According to the IEA's 2010 World Energy Outlook, China's demand is projected to grow faster than any other region, at an average of almost 6 percent per year 2008-2035. The IEA report projects that from 2008 to 2015 Asia's demand will grow from 341 Bcm to 497 Bcm a year.

The expected demand increase in the Asia Pacific is incorporated into our model by increasing the choke price in the Asia Pacific by 5 percent. In response, all producers that are connected the Asia Pacific increase supplies by expanding their supply capacities.

Hence, the total production in the Asia Pacific, West Africa, Russia, the Middle East and Australasia increase.

West Africa expands its supply capacity by 5.32 Bcm which corresponds to 17.4 percent of its supply capacity in 2009. On the other hand, Russia increases its supply capacity by 5.22 Bcm which is around 0.77 percent of its supply capacity in 2009.

With the increase in Asia Pacific's demand, equilibrium supply to the Asia Pacific increases from 394.34 Bcm to 421.07 Bcm and the equilibrium price increases from 320 million USD per Bcm to 326 million USD per Bcm. Neither the consumption nor the equilibrium price change in any other region.

The increase in the Asia Pacific price increases the profits of producers connected to the Asia Pacific. Moreover, the Asia Pacific makes more profit under this scenario than the scenario with fixed supply capacities. Under fixed supply capacities, the Asia Pacific is not able expand its supply capacity to respond to an increase in the demand for natural gas in its domestic market. On the other hand, all other dominant producers make more profits with the fixed supply capacities, as the equilibrium prices in all three importing markets were higher.

4.4 Scenario IV: Increase in importers' natural gas demand

In this scenario, we consider an increase in demand for natural gas from all importing countries. According to IEA's Energy Outlook, global demand for natural is projected to increase by 50 percent to 5 trillion cubic meters in 2035.[33]

These demand increases are incorporated into our model by increasing the choke prices in Europe, North America and Asia Pacific by 2 percent. With a 2 percent increase in the choke prices, all producers invest in their supply capacities in order to increase supplies to importing regions. For instance, West Africa expands its supply capacity by 11.29 Bcm, which corresponds to 37 percent of its supply capacity in 2009. On the other hand, Russia expands its supply capacity by 4.51 Bcm, which corresponds to 0.67 percent of its supply capacity in 2009.

Under this scenario, total supplies to Europe, North America and the Asia Pacific increase. Due to the increase in demand, the equilibrium prices in the importing regions increase. For instance, total supply to Europe increases by 15.17 Bcm and the equilibrium price increases by 2.6 million USD per Bcm. Similarly, total supply to North America increases by 26.13 Bcm and the equilibrium price increases by 1.6 million USD per Bcm. The equilibrium price in the Asia Pacific increases by 2.8 million USD per Bcm due to the increase in total supply by 10.66 Bcm.

The profit of each producer increases as the equilibrium prices in the importing regions increase. All producers would make more profits if there were fixed supply capacities, or if they cooperated and did not expand their supply capacities. However, it is hard to maintain such a cooperative behavior as cheating is profitable.

[33]See http://www.iea.org/newsroomandevents/pressreleases/2012/november/name,33015,en.html

4.5 Scenario V: Russia to China pipeline

In this scenario, we assume that Western Siberia and China are connected through a pipeline. To incorporate this into our model, we use the RWGTM's cost estimates for pipeline routes from West Siberia to China. We assume that 30 percent of natural gas from Russia to the Asia Pacific is carried via pipeline and 70 percent is carried via LNG.

If 30 percent of natural gas is carried via pipeline, the marginal cost of exporting one Bcm of natural gas from Russia to the Asia Pacific decreases by 74.72 million USD per Bcm. With this reduction, Russia increases supply to the Asia Pacific from 6.19 Bcm to 53.5 Bcm. Russia meets this supply to the Asia Pacific by increasing its supply capacity by 47.3 Bcm. When a Russia-China pipeline is built there will be more competition in the Asia Pacific for all producers that are connected to it: the Asia Pacific, West Africa, the Middle East and Australasia. In response, they decrease their supply to the Asia Pacific. For instance, equilibrium supply from Australasia to the Asia Pacific decreases by 10.17 Bcm.

Under this scenario, neither the consumption nor the equilibrium prices change in Europe and North America. This is because Russia increases supply to the Asia Pacific without displacing supplies from other markets. However, total supply to the Asia Pacific increases by 10.17 Bcm, which decreases the equilibrium price by 13.2 million USD per Bcm. If there were no capacity expansions, Russia's equilibrium supply to the Asia Pacific would be 5.65 Bcm lower and hence the equilibrium prices would be 1.83 million USD per Bcm higher.

Under this scenario, profit of Russia increases by 3.66 billion USD, which is 1.53 billion USD more than its profits increase when supply capacities are fixed. All other producers connected to the Asia Pacific make less profits both because their market share decreases and also because the equilibrium price in Asia Pacific declines. On the other hand, producers that are not connected to the Asia Pacific make the same profits as in 2009. This is because equilibrium trade flows in these links do not change. Therefore, equilibrium prices remain unchanged. However, other producers prefer a scenario where the Russia-China pipeline is built and the supply capacities are fixed to this scenario. Under the fixed supply capacities, Russia displaces its supplies from Europe and its domestic market to the Asia Pacific which increases prices in the European market.

5 Conclusions

In this paper, we solved for the equilibrium strategic capacity investments and trade flows in a network model of the world natural gas market which consists of consumers, producers (which are represented as strategic Cournot players) and links connecting them. We assumed a two period model with no uncertainty and showed that this game has a unique Nash equilibrium. We also showed that the open-loop Cournot-Nash equilibrium and closed-loop Cournot-Nash equilibrium investments of this game coincide. Our paper contributes to the literature in strategic capacity investments by allowing for Cournot competition in a networked market with multiple demand nodes and multiple suppliers. In this paper, we assume that the strategic capacity investments are continuous. However, in reality economies of scale will make the capacity investments lumpy. A good extension of this paper would follow Hartley and Kyle (1989), where demand grows smoothly over time and the investment is the only cost which has a fixed size. In their paper, they show that there is an efficient investment path which is a function of the investment sequence and investment times. They also show the oligopolistic market can have multiple equilibria depending on what investors believe about future investment decisions. Similar problem can be applied to this network problem to solve for the strategic investment path with lumpiness.

We looked at the same scenarios as we looked at in our first paper and compared the results. We find that producers respond to changes in market conditions by investing in their supply capacities instead of displacing their resources from other markets.

A Appendix

A.1 Two stage capacity investment and production game (Closed-loop Cournot-Nash game)

Proof of Proposition 1: We consider a two stage game where in the first stage producers invest in their supply capacities and in the second stage they choose their production. We assume that there is no uncertainty.
In the second stage producers maximize their profit subject to their supply capacity constraints. Given a graph Q_g and a supply capacity of $\bar{S}_j$, firm j maximizes its profit by choosing q_{ij}.

$$\max_{q_{ij}} \left\{ \sum_{d_i \in N_g(f_j)} \alpha_i q_{ij} - \sum_{d_i \in N_g(f_j)} \beta_i q_{ij} h_i - \sum_{d_i \in N_g(f_j) \setminus d_j} \tau_{ij} q_{ij} \right\} \tag{19}$$

$$= \max_{q_{ij}} \left\{ \sum_{d_i \in N_g(f_j)} \alpha_i q_{ij} - \sum_{d_i \in N_g(f_j)} \beta_i q_{ij}^2 - \sum_{d_i \in N_g(f_j)} \beta_i q_{ij} \sum_{f_k \in N_g(d_i) \setminus f_j} q_{ik} - \sum_{d_i \in N_g(f_j) \setminus d_j} \tau_{ij} q_{ij} \right\}$$

subject to

$$\sum_{d_i \in N_g(f_j)} q_{ij} \leq \bar{S}_j = \bar{S}_j^0 + k_j \tag{20a}$$

$$q_{ij} \geq 0 \quad \text{for all} \ \ (i,j) \in g \tag{20b}$$

where $\bar{S}_j^0$ is the initial capacity at the beginning of period 0.

$$\mathcal{L} = \sum_{d_i \in N_g(f_j)} \alpha_i q_{ij} - \sum_{d_i \in N_g(f_j)} \beta_i q_{ij} h_i - \sum_{d_i \in N_g(f_j) \setminus d_j} \tau_{ij} q_{ij} + \lambda_j \left(\bar{S}_j - \sum_{d_i \in N_g(f_j)} q_{ij} \right)$$
$$+ \sum_{d_i \in N_g(f_j)} \iota_{ij} q_{ij} \tag{21}$$

Then there exists $\lambda_j^\star$ and $\iota_{ij}^\star$ such that $q_{ij}^\star$, $\lambda_j^\star$ and $\kappa_{ij}^\star$ satisfy the following Kuhn-Tucker optimality conditions

$$\frac{\partial \mathcal{L}}{\partial q_{ij}} = \alpha_i - \tau_{ij} - \beta_i \left(\sum_{f_k \in N_g(d_i) \setminus \{f_j\}} q_{ik} + 2q_{ij}^\star \right) - \lambda_j + \iota_{ij} = 0 \tag{22a}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_j} = \bar{S}_j - \sum_{d_i \in N_g(f_j)} q_{ij} \geq 0 \tag{22b}$$

$$\frac{\partial \mathcal{L}}{\partial \iota_{ij}} = q_{ij} \geq 0 \tag{22c}$$

$$\lambda_j \frac{\partial \mathcal{L}}{\partial \lambda_j} = \lambda_j \left(\bar{S}_j - \sum_{d_i \in N_g(f_j)} q_{ij} \right) = 0 \tag{22d}$$

$$\iota_{ij} \frac{\partial \mathcal{L}}{\partial \iota_{ij}} = \iota_{ij} q_{ij} = 0 \tag{22e}$$

We get the Cournot-Nash equilibrium[34] flow of $q_{ij}^\star$

$$q_{ij}^\star = \begin{cases} \dfrac{\alpha_i - \tau_{ij} - \lambda_j - \beta_i \left(\sum_{f_k \in N_g(d_i) \setminus \{f_j\}} q_{ik} \right)}{2\beta_i} & \text{if } \frac{\partial \pi_j}{\partial q_{ij}}\Big|_{Q_g} \geq 0 \\ 0 & \text{if } \frac{\partial \pi_j}{\partial q_{ij}}\Big|_{Q_g} < 0 \end{cases} \tag{23}$$

in the first stage, producer j chooses his optimal capacity investment

$$\max_{\bar{k}_j} \Pi_j^\star(q_{ij}^\star) - C(\bar{S}_j) \tag{24}$$

Proposition 2: Firm j's profit maximizing supply capacity is obtained by solving $C'(\bar{k}_j) - \lambda_j^\star = 0$.
Proof: Let

[34]Ilkilic (2010) shows that the unconstrained Cournot game in a bipartite graph has a unique Nash equilibrium.

$$V_j(\bar{S}_j, \bar{S}_{-j}) = \max_{q_{ij}} \Pi(q_{ij}) \tag{25}$$

subject to

$$\sum_{d_i \in N_g(f_j)} q_{ij} \leq \bar{S}_j \tag{26}$$

By the Kuhn-Tucker optimality conditions:

$$\lambda_j^\star \left(\bar{S}_j - \sum_{d_i \in N_g(f_j)} q_{ij}^\star \right) = 0 \tag{27}$$

Hence,

$$V_j(\bar{S}_j, \bar{S}_{-j}) = \Pi_j^\star(q_{ij}^\star) + \lambda_j^\star \left(\bar{S}_j - \sum_{d_i \in N_g(f_j)} q_{ij}^\star \right) \tag{28}$$

By the envelope theorem

$$\frac{\partial V_j(\bar{S}_j, \bar{S}_{-j})}{\partial \bar{k}_j} = \underbrace{\frac{\partial \Pi_j^\star(q_{ij}^\star)}{\partial \bar{k}_j}}_{=0} + \lambda_j^\star \tag{29}$$

Hence, $\max_{\bar{S}_j} \Pi_j^\star(q_{ij}^\star) - C(\bar{S}_j)$ is $\lambda_j^\star - C'(\bar{k}_j) = 0$ since $\bar{S}_j = \bar{S}_j^0 + k_j$.

A.2 Calibration

In order to quantitatively evaluate different policy scenarios, we first need to calibrate the theoretical model. To calibrate the model parameters, we use the production, consumption, price and trade flow data in 2009. The price data is obtained from international Energy Agency's (IEA) website and other country websites. The data on production, consumption, and trade flows are obtained from BP's Statistical Review of World Energy 2010.

For calibration, we use the first order conditions of our model. The first order conditions with respect to equilibrium flows are same as the ones in Cigerli (2013).[35]

Example: The South American producer labeled as 4, has the objective[36]

$$\max_{q_{14}, q_{24}, q_{44}, k_4} \Pi_4(Q_g) = \max_{q_{14}, q_{24}, q_{44}, k_4} \{p_1 q_{14} + p_2 q_{24} + p_4 q_{44} - \tau_{14} q_{14} - \tau_{24} q_{24} - \theta_4 k_4\} \tag{30}$$

[35]The reason is that we have the same network with same equilibrium trade flows, production, consumption and price.

[36]For the sake of identification of the problem, we assume that the cost of transporting natural gas to the domestic market is zero.

subject to

$$q_{14} + q_{24} + q_{44} \leq \bar{S}_4 + k_4 \quad \text{and} \quad q_{14}, q_{24}, q_{44}, k_4 \geq 0 \tag{31}$$

By considering the links that carry positive flows[37] in equilibrium, we get the first order conditions as:

$$q_{14}: \quad \alpha_1 - 2\beta_1 q_{14} - \beta_1(q_{11} + q_{15} + q_{16} + q_{17} + q_{18}) - \tau_{14} - \lambda_4 - \iota_{14} = 0 \tag{32a}$$

$$q_{24}: \quad \alpha_2 - 2\beta_2 q_{24} - \beta_2(q_{22} + q_{25} + q_{26}) - \tau_{24} - \lambda_4 - \iota_{24} = 0 \tag{32b}$$

$$q_{44}: \quad \alpha_4 - 2\beta_4 q_{44} - \lambda_4 - \iota_{44} = 0 \tag{32c}$$

$$k_4: \quad -\theta_4 + \lambda_4 + \mu_4 = 0 \tag{32d}$$

We assume an interior solution for the capacity constraint,[38] $q^{\star}_{14} + q^{\star}_{24} + q^{\star}_{44} < \bar{S}_4$, this implies $\lambda_4 = 0$. Therefore, the first order condition in (32d) $\implies \theta_4 = \mu_4$ meaning that Kuhn-Tucker condition (15g) is satisfied when $k_4 = 0$.[39] We approximate the marginal cost of expanding production capacity by the inverse of reserves[40] to production ratio. If we assume that countries have the same production technologies, then producers with a higher reserves to production ratio must have lower costs of supply capacity expansion. However, our numerical results are sensitive to the choice of this marginal cost parameter.[41]

We apply the same equilibrium condition to each producer from 1 to 9, and get twenty one equations. We also have 9 equations, 1 price equation for each market.[42]

[37]According to Ilkilic (2010), links that carry zero flows in equilibrium have no role in determining the equilibrium.

[38]We make this assumption only when calibrating the parameters. This assumption is realistic especially in 2009, where due to the global recession, producers had excess supply capacities. When analyzing alternative scenarios we do not impose this assumption.

[39]This is because θ_4 is positive and constant.

[40]We obtain proved reserves data from BP's Statistical Review of World Energy, 2010.

[41]If the cost of expanding capacity is sufficiently high, the producer chooses to displace its resources rather than invest in capacity. The resulting outcome would then be the same as in Cigerli 2013.

[42]Natural gas import prices are usually different for each importer and this price may be different from the domestic producer's price. However, our model assumes that there is a single price of natural gas in each region, which is determined by the total supply of producers connected to that region.

Table 1: Network parameters

	Parameter	Value
Choke price in Europe	α_1	904.27
Choke price in North America	α_2	302.9
Choke price in Asia Pacific	α_3	832.83
Choke price in South America	α_4	260.02
Choke price in West Africa	α_5	220.01
Choke price in North Africa	α_6	199.97
Choke price in Russia	α_7	130.03
Choke price in Middle East	α_8	200.01
Choke price in Australasia	α_9	239.99
Slope of European inverse demand curve	β_1	1.041
Slope of North America's inverse demand curve	β_2	0.184
Slope of Asia Pacific's inverse demand curve	β_3	1.3003
Slope of South America's inverse demand curve	β_4	0.965
Slope of West Africa's inverse demand curve	β_5	10.912
Slope of North Africa's inverse demand curve	β_6	1.445
Slope of Russia's inverse demand curve	β_7	0.134
Slope of Middle East's inverse demand curve	β_8	0.2894
Slope of Australasian inverse demand curve	β_9	1.083
Marginal cost of exporting from South America to Europe	τ_{14}	292.08
Marginal cost of exporting from South America to North America	τ_{24}	148.59
Marginal cost of exporting from West Africa to Europe	τ_{15}	288.85
Marginal cost of exporting from West Africa to North America	τ_{25}	149.43
Marginal cost of exporting from West Africa to Asia Pacific	τ_{35}	311.41
Marginal cost of exporting from North Africa to Europe	τ_{16}	230.02
Marginal cost of exporting from North Africa to North America	τ_{26}	149.07
Marginal cost of exporting from Russia to Europe	τ_{17}	111.41
Marginal cost of exporting from Russia to Asia Pacific	τ_{37}	311.93
Marginal cost of exporting from Middle East to Europe	τ_{18}	273.34
Marginal cost of exporting from Middle East to Asia Pacific	τ_{38}	258.62
Marginal cost of exporting from Australasia to Asia Pacific	τ_{39}	205.18
Europe's marginal cost of supply capacity investment	θ_1	0.054
North America's marginal cost of supply capacity investment	θ_2	0.089
Asia Pacific's marginal cost of supply capacity investment	θ_3	0.038
South America's marginal cost of supply capacity investment	θ_4	0.019
West Africa's marginal cost of supply capacity investment	θ_5	0.006
North Africa's marginal cost of supply capacity investment	θ_6	0.017
Russia's marginal cost of supply capacity investment	θ_7	0.012
Middle East's marginal cost of supply capacity investment	θ_8	0.006
Australasia's marginal cost of supply capacity investment	θ_9	0.021

Table 2: Equilibrium trade flows (in Bcm)

Route	2009	Scenario I	Scenario II	Scenario III	Scenario IV	Scenario V
From Europe to Europe	288.10	282.98	292.33	288.10	290.55	288.10
From North America to North America	813.00	813.00	813.00	813.00	819.23	813.00
From Asia Pacific to Asia Pacific	246.10	246.10	247.32	251.42	248.21	235.93
From South America to Europe	7.60	2.47	11.86	7.60	10.08	7.60
From South America to North America	7.60	7.63	7.59	7.61	14.24	7.61
From South America to South America	134.70	134.70	134.69	134.70	134.69	134.70
From West Africa to Europe	10.70	5.57	14.98	10.70	13.19	10.70
From West Africa to North America	3.10	3.09	3.11	3.08	9.76	3.10
From West Africa to Asia Pacific	6.60	6.60	7.85	11.94	8.74	0.00
From West Africa to West Africa	10.08	10.08	10.08	10.08	10.08	10.08
From North Africa to Europe	67.20	62.07	71.47	67.20	69.68	67.20
From North Africa to North America	5.00	4.99	4.96	5.00	11.61	5.00
From North Africa to North Africa	69.20	69.20	69.19	69.20	69.19	69.20
From Russia to Europe	181.10	175.97	185.38	181.10	183.59	181.09
From Russia to Asia Pacific	6.20	6.20	0.00	11.54	8.34	53.48
From Russia to Russia	485.50	485.44	485.44	485.39	485.39	485.39
From Middle East to Europe	25.60	56.36	0.00	25.60	28.09	25.60
From Middle East to Asia Pacific	47.20	47.20	48.45	52.54	49.34	37.03
From Middle East to Middle East	345.60	345.53	345.54	345.53	345.53	345.54
From Australasia to Asia Pacific	88.30	88.30	89.53	93.63	90.43	78.13
From Australasia to Australasia	110.80	110.80	110.79	110.79	110.79	110.80

Table 3: Equilibrium supply capacity investments (in Bcm)

	2009	Scenario I	Scenario II	Scenario III	Scenario IV	Scenario V
Europe	288.10	0.00	4.23	0.00	2.45	0.00
North America	813.00	0.00	0.00	0.00	6.23	0.00
Asia Pacific	246.10	0.00	1.22	5.32	2.11	0.00
South America	149.90	0.00	4.25	0.00	9.11	0.00
West Africa	30.48	0.00	5.53	5.32	11.29	0.00
North Africa	141.40	0.00	4.22	0.00	9.09	0.00
Russia	672.80	0.00	0.00	5.23	4.52	47.17
Middle East	418.40	30.69	0.00	5.28	4.56	0.00
Australasia	199.10	0.00	1.23	5.33	2.12	0.00

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