AGU PUBLICATIONS

Geophysical Research Letters

RESEARCH LETTER

10.1002/2015GL065621

Key Points:
- Parameterized thermal history models assume quasi-equilibrium dynamical evolution
- Some plate tectonic models preclude quasi-equilibrium dynamical evolution
- Multiple equilibria also require nonequilibrium approaches

Correspondence to:
W. B. Moore,
william.moore@hamptonu.edu

Citation:
Moore, W. B., and A. Lenardic (2015), The efficiency of plate tectonics and nonequilibrium dynamical evolution of planetary mantles, *Geophys. Res. Lett.*, *42*, 9255–9260, doi:10.1002/2015GL065621.

Received 30 JUL 2015
Accepted 4 OCT 2015
Accepted article online 9 OCT 2015
Published online 11 NOV 2015

The efficiency of plate tectonics and nonequilibrium dynamical evolution of planetary mantles

W. B. Moore[1,2] and A. Lenardic[3]

[1]Department of Atmospheric and Planetary Sciences, Hampton University, Hampton, Virginia, USA, [2]National Institute of Aerospace, Hampton, Virginia, USA, [3]Department of Earth Science, Rice University, Houston, Texas, USA

Abstract Consideration of the structure of dynamical equilibria in terrestrial planets using simplified descriptions of the relevant heat transport processes (rigid-lid convection, plate tectonics, and heat pipe volcanism) reveals that if the efficiency of plate tectonic heat transport decreases at higher mantle temperature, then it cannot govern quasi-equilibrium dynamical evolution, and the system is always evolving away from the plate tectonic regime. A planet on which plate tectonics is less efficient at higher temperature stays in heat pipe mode longer, spends less time undergoing plate tectonics, and has a low and ever-decreasing Urey number during this phase. These conclusions are based solely on the structure of the equilibria in a system with less efficient plate tectonics in the past and are independent of the mechanisms leading to this behavior. Commonly used quasi-equilibrium approaches to planetary thermal evolution are likely not valid for planets in which heat transport becomes less efficient at higher temperature.

1. Introduction

Planets lose internal heat and cool over time through heat transport processes that include volcanism, convection and conduction. Despite substantial progress in simulating the process of planetary mantle convection with mobile plates, the primary problem for which these simulations were developed remains unsolved—how has the Earth's internal temperature evolved with time since its formation? The answer to this question depends critically on quantitative estimates of the efficiency of heat transport over time, and despite the progress alluded to above, there remains disagreement even on the sign of the trend of heat transport efficiency with temperature in the plate tectonic regime. This disagreement has implications not only for the reconstruction of Earth's thermal history for comparison with geological and petrological data but also for the method by which thermal histories are calculated for the Earth or any other planet that may have gone through a plate tectonic phase.

The thermal evolution of a planet depends on the balance between heat production and heat loss over time. The relationship typically assumed between planetary heat transport and internal temperature [*Schubert et al.*, 2001] is a power law relating the dimensionless heat flux (Nusselt number, *Nu*) to the Rayleigh number *Ra*, a measure of convective vigor:

$$Nu \sim Ra^{\beta}, \tag{1}$$

where β is a constant characteristic of a given heat transport regime. Dimensionally, this relationship is

$$F = C\frac{k\Delta T}{D}\left(\frac{\rho g \alpha \Delta T D^3}{\eta\kappa}\right)^{\beta}, \tag{2}$$

where k is thermal conductivity, ΔT is the difference between the interior and surface temperature, D is the depth of the mantle, ρ is its density, g is the acceleration of gravity, α is the thermal expansivity, η is the dynamic viscosity, and κ is the thermal diffusivity ($= k/\rho c_p$). More complex formulations accounting for the pressure, temperature, and stress dependence of viscosity [*Solomatov*, 1995; *Reese et al.*, 1998] include terms depending on the rheological parameters but preserve the form of (2). Although this appears to show that surface heat flux F depends on $\Delta T^{\beta+1}$, the temperature dependence of $1/\eta$ is strongly superlinear for mantle materials, and thus F decreases with temperature for all but a very small $\beta < 0$. The constant C depends on a number of factors including geometry and has proven very difficult to predict theoretically. In practice, C is evaluated based on numerical simulations of the complete equations in relevant geometries that have been run to steady or statistically steady state, or it can be tuned to match present Earth conditions (with uncertainty due to incomplete knowledge of those conditions).

To compute thermal histories, the heat flux scaling above is used to evolve the average mantle temperature T_m through an energy balance relationship:

$$C_m \frac{\mathrm{d}T_m}{\mathrm{d}t} = \langle H \rangle - \langle F \rangle \,, \tag{3}$$

where C_m is the heat capacity of the mantle, and $\langle H \rangle$ and $\langle F \rangle$ are the globally integrated heat production and surface heat flux, respectively. A dynamical equilibrium will exist when the right-hand side is zero and heat production balances heat loss, either strictly or in a time-averaged sense.

2. Heat Transport Regimes

There are four known regimes of heat transport in terrestrial planets with solid surfaces (a magma ocean phase may also occur prior to formation of a solid, stable crust). In small and/or cold bodies convective stresses are insufficient to drive flow, and heat transport occurs by conduction, which implies a linear relationship between internal temperature and heat flow. When convective stresses are too weak to break the cold and stiff lithosphere, convection proceeds in a limited portion of the mantle below the rigid lid and equation (2) applies. Our current understanding of rigid-lid convection is well developed [*Solomatov and Moresi*, 2000], and heat flow in this regime clearly shows a positive trend with increasing temperature.

At higher temperatures there may be a transition between rigid-lid convection as observed on the other terrestrial planets and plate tectonics as observed on Earth. This transition takes place in the Tectono-Convective Transition Window (TCTW) [*Weller and Lenardic*, 2012] and depends on whether the planet is warming or cooling, making metastability or bistability possible in the system [*Sleep*, 2000; *Tackley*, 2000; *Weller and Lenardic*, 2012]. In the plate tectonic regime, convective stresses overcome the strength of the lithosphere, allowing the cold material near the surface to participate in the flow. This increases the effective temperature gradient (relative to rigid-lid convection) and should result in an increase in heat transport at the transition [*Solomatov*, 1995; *Sleep*, 2000]. Since plate breaking is a local process, there will not necessarily be a discontinuous jump in heat flow but a gradual increase as the length of subduction zones increases. Once fully developed, the efficiency of plate tectonics as a heat transport process depends on the rate at which material is mixed downward from the surface. This rate depends on the buoyancy forces created by cooling at the surface, which should be greater in the past, but it is also very sensitive to the stiffness of the surface materials and the geometry of downwellings.

Both buoyancy and stiffness of surface materials may be affected by the melting processes that give rise to oceanic lithosphere and crust. Of particular interest has been the role of dehydration and stiffening of the mantle by melt extraction [*Ito et al.*, 1999]. These processes have been argued to result in a plate tectonic regime with negative β [*Korenaga*, 2003, 2008]. This negative β behavior has been invoked to explain petrological data that suggest a period in which the Earth cooled very little or possibly even warmed prior to 3 Gyr ago [*Herzberg et al.*, 2010; *Korenaga*, 2013].

Regardless of the trend of heat flow in the plate tectonic regime, at progressively higher interior temperatures, volcanism becomes more important as a heat transport process. The transition to the heat pipe regime is defined by the point at which volcanism dominates the heat transport. We note that for most terrestrial planets, heat pipes seem to transition directly to rigid-lid convection, while they remain currently active on Jupiter's moon Io [*O'Reilly and Davies*, 1981]. The predictions of the heat pipe model provide a good explanation for many features of Earth's most ancient geologic record [*Moore and Webb*, 2013]. In addition to transporting heat, volcanism is an important mass transport process, causing the surface temperature to be advected downward as old, cool flows are buried by newer flows. In the heat pipe regime, this profoundly affects the structure of the lithosphere, producing a thick, cold lid [*O'Reilly and Davies*, 1981; *Breuer and Moore*, 2007; *Moore and Webb*, 2013]. By removing heat and thus buoyancy from the active boundary layer beneath the lid, heat pipes also reduce the stresses on the lid, suppressing plate tectonics [*Moore and Webb*, 2013]. Heat pipe volcanism increases very strongly with internal temperature, due to the highly nonlinear dependence of melt extraction rates on melt fraction [*Faul*, 1997; *Wark and Watson*, 1998].

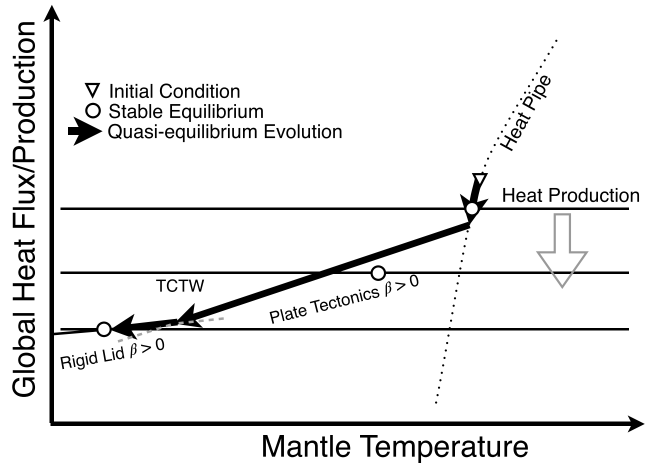

Figure 1. A schematic diagram of planetary heat flow and heat production in the three heat transport regimes versus temperature. Heat production by radioactive decay is shown as solid lines with the outlined arrow indicating the evolution with time. Heat transport in the rigid lid ($\beta > 0$, solid, dashed), plate tectonics ($\beta > 0$, solid, dashed), and heat pipe (dotted) regimes are shown. Metastable branches are shown in the TCTW (grey dashed). Stable equilibria are indicated by the circles, and initial conditions by the triangles. Quasi-equilibrium evolution in this system follows the black arrows.

3. Equilibria in Planetary Thermal Evolution

Figures 1 and 2 illustrate the evolving structure of dynamical equilibria in planetary thermal evolution for positive and negative β, respectively. For clarity of illustration, constant β in each regime has been assumed, but the conclusions we reach do not depend on this, only on the presence of some domain of negative β behavior. In each diagram, the integrated heat production rate $\langle H \rangle$ is independent of temperature (horizontal lines) but gradually decaying with time as indicated by the outlined arrow. Dynamic equilibria are found where the heat production equals the heat transport and are indicated by circles. The stability of these equilibria are determined by the relative slopes of the heat production and heat transport curves. When the slope of the heat transport is greater than the slope of the heat production, then slight perturbations from an equilibrium cause the system to return to the equilibrium, making it a stable point [*Tozer*, 1965]. Conversely, when the slope of the heat transport is less than the slope of the heat production (either more negative or simply less positive), small perturbations grow, driving the system away from the unstable equilibrium.

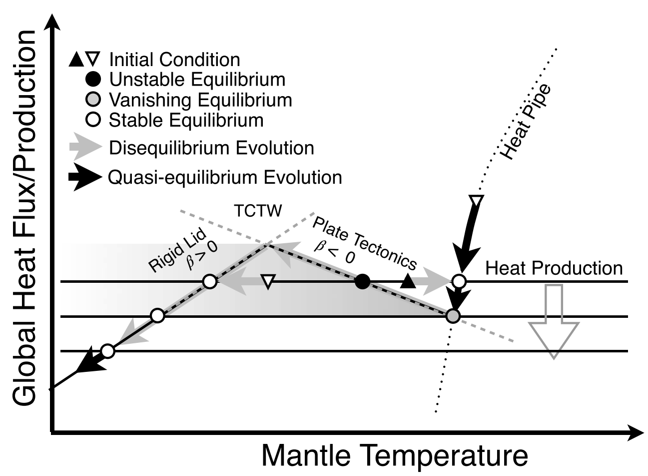

Figure 2. As in Figure 1, but for negative β plate tectonics. All equilibria in the negative β plate tectonic regime are unstable, and the grey circle indicates the vanishing of the two high-temperature equilibria. Disequilibrium and quasi-equilibrium evolution trajectories from various initial states (triangles) are shown by the gray and black arrows, respectively. The shading indicates the region of equilibrium states that cannot be reached from states on the hot side of the plate tectonic regime.

Figure 1 shows the behavior (black arrows) of a system cooling from a hot initial temperature (inverted triangle) in heat pipe mode, transitioning to a positive β plate tectonic mode, and finally passing through the TCTW (with the potential for metastable evolution along the grey dashed path) before transitioning to the rigid-lid mode. Throughout this evolution, it is possible (though not guaranteed) for the system to chase the evolving stable equilibrium (white circles) in a quasi-equilibrium fashion. Heat transport in such evolving systems generally exceeds the equilibrium value, leading to a Urey number (ratio of heat production to heat transport) somewhat less than unity and gradual cooling. A Urey number less than unity reflects the adjustment timescales of the convecting system, which tends to lag behind the ever-decreasing heat production rate. Adjustment timescales vary in the different heat transport regimes, generally becoming longer at lower temperatures.

If β is negative, on the other hand, we see a very different evolution as illustrated in Figure 2. Since the slope of the plate tectonic heat transport is negative, all equilibria in this regime are unstable. A cooling system therefore does not transition to plate tectonics until the stable equilibrium along the heat pipe branch vanishes (grey circle), at which point the only remaining equilibrium state is at low temperature in the rigid-lid ($\beta > 0$) regime. The system therefore evolves toward this state in a nonequilibrium manner (grey arrow), becoming increasingly out of dynamical equilibrium with a Urey number that decreases ever more quickly as the mantle cools and plate tectonics moves faster and faster. Upon passing through the TCTW (or into a positive β domain), the system may begin to follow the stable equilibrium, and the Urey number may rebound somewhat.

Any negative β domain implies a region of unstable equilibria that prevents evolution through it into the cooler stable states that overlap it in heat flux (shaded region in Figure 2). This dynamical shadow will exist for any initial condition on the hot side of the unstable equilibrium, eliminating the possibility for quasi-equilibrium behavior in the range of overlap. This is a generic result for dynamical systems with multiple equilibria: pairs of stable points are separated by an unstable one, and the transitions between stable branches will be discontinuous and history dependent [*Thom*, 1977; *Arnold*, 1986]. A purely energetic formulation of mantle convection exhibits precisely this behavior [*Crowley and O'Connell*, 2012], and the transition dynamics and history dependence has been demonstrated in full numerical solutions [*Lenardic and Crowley*, 2012]. In an evolving system, the shadow extends beyond the overlap region, since it will take time to cool from the vanishing equilibrium toward the remaining stable equilibrium on the positive β branch, during which time the heat production will have further decayed.

Note that in all cases the thermal evolution is monotonic. The unstable nature of the plate tectonic equilibrium for $\beta < 0$ prevents evolution into this regime until after the mantle has cooled below all possible equilibrium states. The only way to generate increasing mantle temperatures is to somehow end up on the plate tectonic branch on the hot side of the unstable equilibrium (black triangle in Figure 2) which drives the system in a nonequilibrium manner toward the heat pipe regime (at an ever-accelerating rate). This requires an initial condition out of the heat pipe regime that should be the immediate successor to the magma ocean phase of Earth's history [*Moore and Webb*, 2013]. Arguments that the early Earth warmed due to an inefficient stagnant-lid regime prior to transitioning to plate tectonics [*Korenaga*, 2013] imply equivalently nonequilibrium behavior.

4. Discussion

The different thermal evolution trajectories implied by different signs of β have significant implications for planetary evolution. First, when plate tectonics is more efficient at higher temperature (Figure 1), the transition out of heat pipe occurs at higher heat flow (and hence earlier) than when it is less efficient at high temperature (Figure 2), given the same rigid-lid behavior. Second, when a planet finds itself as far out of dynamical equilibrium as it does in the plate tectonic regime in the negative β case, it cools very rapidly, pushing it toward the TCTW at a rate that increases with time, since the heat sources are decaying all the while. Thus, negative β plate tectonics should be short-lived compared to the positive β phase and observed to accelerate rapidly prior to a sudden transition to rigid lid.

More generally, integrating the thermal history of a planet via (3) requires the specification of the heat transport in terms of the mantle temperature. All parameterizations of heat loss versus convective vigor used for thermal history modeling are strictly valid only for steady state or statistically steady state. It was noted from the earliest days of parameterized thermal history modeling that using steady state scalings for a transient problem could limit the validity of results [*Davies*, 1980]. The argument for the utility of quasi-equilibrium models has traditionally been made in terms of adjustment timescales [*Sharpe and Peltier*, 1979; *Davies*, 1980; *Christensen*, 1985a]. If the timescale over which the convecting mantle adjusts to changes in the heat sources that drive convection is very fast relative to the timescale over which heat sources decay, then the mantle system can be treated as moving from one local quasi–steady state to another as heat sources decay with time.

For cases with positive β, this argument has been tested and the quasi-equilibrium approach validated by comparing parameterized thermal history models to numerical models that solve the full equations for thermal convection and allow for decaying heat sources [*Daly*, 1980; *Christensen*, 1985b]. The assumption behind the basic argument can be recast to say that the system tracks a moving equilibrium as it evolves and that the convective balance of that equilibrium determines the present heat flux to a reasonable approximation. This is equivalent to saying that the Urey number does not depart too far from unity, but the neighborhood of validity remains untested.

The existence of local-in-time equilibrium points is not sufficient to justify the assumption of quasi–steady state evolution that is at the core of all parameterized thermal history models, however. The necessary and sufficient condition is that the local equilibrium point be a stable one. If that is not the case, then the mantle will not evolve toward the local dynamical equilibrium and the use of scalings based on dynamical equilibrium will not be valid. Furthermore, it is impossible to use numerical quasi-steady solutions to quantify the scaling parameters (such as C or β), since such solutions do not exist and the system instead evolves away from the unstable equilibrium and out of the negative β regime.

Our discussion of the timescales and rates of these processes is necessarily vague, as the detailed time evolution cannot be quantified from equilibrium considerations alone and dynamical timescales will vary strongly with both mantle temperature and the regime of transport. Thus, the validity of quasi-steady approaches may degrade even within a single model evolution. Equilibrium heat flow scalings can lose validity not only for negative beta behavior but also during transitions between tectonic regimes. For example, when the mantle of a terrestrial planet transitions from a heat pipe to a plate tectonic mode, there will be an adjustment period where the internal mantle temperature structure will not approximate a dynamic equilibrium. One such transition would occur at the initiation of subduction, which would deposit cold sinking slab material to the base of the mantle. The strong associated temperature inversion within the mantle would be a transient feature that would decay in magnitude over time [e.g., *Langan and Sleep*, 1982; *Lenardic and Kaula*, 1994]. Once the internal mantle thermal structure adjusts, a subadiabatic thermal gradient can be maintained [*Bunge et al.*, 2001], and equilibrium scalings have been developed that account for a subadiabatic mantle geotherm [*Moore*, 2008]. During the adjustment, however, equilibrium scalings could significantly misrepresent heat transfer properties. The degree to which this could affect the results of thermal histories with a regime transition, even if the regimes on both sides of the transition are associated with a positive beta scaling, would depend on the adjustment timescale.

In conclusion, while a negative β plate tectonic phase (which may only apply to part of the plate tectonic regime) has been hypothesized [*Korenaga*, 2003], it will be short-lived and very much out of dynamical equilibrium with a rapidly declining Urey ratio less than one. Quasi-equilibrium approaches to integrating the thermal history of such planets are not likely to be valid, even in the absence of metastability or bistability in the system, and the results of such integrations are of questionable relevance for planetary evolution. This conclusion is independent of the mechanisms that lead to $\beta < 0$ and depends solely on the equilibrium structure of any system with a domain of negative β behavior. Reliable thermal histories of planets with negative β or multistable behavior can only be computed using approaches which do not rely on the assumption of evolution about a nearby, stable equilibrium.

Acknowledgments
Norm Sleep is thanked for a thought-provoking review. W.B.M. acknowledges funding from NSFEAR-1246983 and A.L. acknowledges funding from NSFEAR-0944156. This work is theoretical in nature and no specific data sources were used outside of works cited.

References

Arnold, V. I. (1986), *Catastrophe Theory*, 2nd ed., Springer, Berlin.

Breuer, D., and W. B. Moore (2007), Dynamics and thermal history of terrestrial planets, the Moon, and Io, in *Treatise on Geophysics*, vol. 10, edited by G. Schubert, pp. 299–348, Elsevier, Amsterdam.

Bunge, H.-P., Y. Ricard, and J. Matas (2001), Non-adiabaticity in mantle convection, *Geophys. Res. Lett.*, *28*, 879–882, doi:10.1029/2000GL011864.

Christensen, U. R. (1985a), Heat transport by variable viscosity convection, *Phys. Earth Planet. Inter.*, *37*, 183–205.

Christensen, U. R. (1985b), Thermal evolution models for the Earth, *J. Geophys. Res.*, *90*, 2995–3007, doi:10.1029/JB090iB04p02995.

Crowley, J. W., and R. J. O'Connell (2012), An analytic model of convection in a system with layered viscosity and plates, *Geophys. J. Int.*, *188*, 61–78, doi:10.1111/j.1365-246X.2011.05254.x.

Daly, S. F. (1980), Convection with decaying heat sources: Constant viscosity, *Geophys. J. R. Astron. Soc.*, *61*, 519–547.

Davies, G. F. (1980), Thermal histories of convective Earth models and constraints on radiogenic heat production in the Earth, *J. Geophys. Res.*, *85*, 2517–2530, doi:10.1029/JB085iB05p02517.

Faul, U. H. (1997), Permeability of partially molten upper mantle rocks from experiments and percolation theory, *J. Geophys. Res.*, *102*, 10,299–10,311.

Herzberg, C., K. Condie, and J. Korenaga (2010), Thermal history of the Earth and its petrological expression, *Earth Planet. Sci. Lett.*, *292*, 79–88, doi:10.1016/j.epsl.2010.01.022.

Ito, G., Y. Shen, G. Hirth, and C. J. Wolfe (1999), Mantle flow, melting, and dehydration of the Iceland mantle plume, *Earth Planet. Sci. Lett.*, *165*, 81–96.

Korenaga, J. (2003), Energetics of mantle convection and the fate of fossil heat, *Geophys. Res. Lett.*, *30*(8), 1437, doi:10.1029/2003GL016982.

Korenaga, J. (2008), Urey ratio and the structure and evolution of Earth's mantle, *Rev. Geophys.*, *46*, RG2007, doi:10.1029/2007RG000241.

Korenaga, J. (2013), Initiation and evolution of plate tectonics on Earth: Theories and observations, *Annu. Revi. Earth. Planet. Sci.*, *41*, 117–151, doi:10.1146/annurev-earth-050212-124208.

Langan, R. T., and N. H. Sleep (1982), A kinematic thermal history of the Earth's mantle, *J. Geophys. Res.*, *87*, 9225–9235.

Lenardic, A., and J. W. Crowley (2012), On the notion of well-defined tectonic regimes for terrestrial planets in this solar system and others, *Astrophys. J.*, *755*, 132, doi:10.1088/0004-637X/755/2/132.

Lenardic, A., and W. M. Kaula (1994), Tectonic plates, D′′ thermal structure, and the nature of mantle plumes, *J. Geophys. Res.*, *99*, 15,697–15,708.

Moore, W. B. (2008), Heat transport in a convecting layer heated from within and below, *J. Geophys. Res.*, *113*, B11407, doi:10.1029/2006JB004778.

Moore, W. B., and A. A. G. Webb (2013), Heat-pipe Earth, *Nature*, *501*, 501–505, doi:10.1038/nature12473.

O'Reilly, T. C., and G. F. Davies (1981), Magma transport of heat on Io: A mechanism allowing a thick lithosphere, *Geophys. Res. Lett.*, *8*, 313–316.

Reese, C. C., V. S. Solomatov, and L. N. Moresi (1998), Heat transport efficiency for stagnant lid convection with dislocation viscosity: Application to Mars and Venus, *J. Geophys. Res.*, *103*, 13,643–13,657.

Schubert, G., D. L. Turcotte, and P. Olson (2001), *Mantle Convection in the Earth and Planets*, Cambridge Univ. Press, New York.

Sharpe, H. N., and W. R. Peltier (1979), A thermal history model for the Earth with parameterized convection, *Geophys. J. R. Astron. Soc.*, *59*, 171–203.

Sleep, N. H. (2000), Evolution of the mode of convection within terrestrial planets, *J. Geophys. Res.*, *105*, 17,563–17,578, doi:10.1029/2000JE001240.

Solomatov, V. S. (1995), Scaling of temperature- and stress-dependent viscosity convection, *Phys. Fluids*, *7*, 266–274.

Solomatov, V. S., and L. N. Moresi (2000), Scaling of time-dependent stagnant lid convection: Application to small-scale convection on Earth and other terrestrial planets, *J. Geophys. Res.*, *105*, 21,795–21,817.

Tackley, P. J. (2000), Self-consistent generation of tectonic plates in time-dependent, three-dimensional mantle convection simulations 2. Strain weakening and asthenosphere, *Geochem. Geophys. Geosyst.*, *1*(8), 1026, doi:10.1029/2000GC000043.

Thom, R. (1977), Catastrophe theory, *Nature*, *270*, 658, doi:10.1038/270658a0.

Tozer, D. C. (1965), Heat transfer and convection currents, *Philos. Trans. R. Soc. A*, *258*, 252–271.

Wark, D. A., and E. B. Watson (1998), Grain-scale permeabilities of texturally equilibrated, monomineralic rocks, *Earth Planet. Sci. Lett.*, *164*, 591–605.

Weller, M. B., and A. Lenardic (2012), Hysteresis in mantle convection: Plate tectonics systems, *Geophys. Res. Lett.*, *39*, L10202, doi:10.1029/2012GL051232.