THE RICE INSTITUTE

EXPERIMENTS ON A ROTATING SUPERCONDUCTOR
AT AND ABOVE THE TRANSITION TEMPERATURE

by

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INTRODUCTION

When certain metals are cooled to within a few degrees of absolute zero, the resistivity of these metals drops to an immeasurably small value. The resistivity value depends on the presence of impurities, crystal structure of the metal, and mechanical strains. This phenomenon was first observed by Kamerlingh Onnes in 1911\textsuperscript{1}) and was called superconductivity. He found that a current induced in a lead ring at 4.2\textdegree{} K. did not decrease by as much as one part in forty thousand per hour and that the transition temperature at which the metal became a superconductor was a function of the exterior magnetic field. That is, by making the magnetic field large enough, one is able to make the superconductor return to a normal conductor.

If the metal had infinite conductivity, $\sigma$, below the critical temperature, the electric field within the superconductor would be zero. This follows from Ohm's Law, $\bar{E} = \frac{\bar{F}}{\sigma}$. From the law of induction

$$\text{curl } \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

and $\bar{E}=0$, one can conclude that $\bar{B}$ is constant with respect to time. This means that the magnetic induction vector, $\bar{B}$, will retain the value it had on passing through the transition temperature, and further changes in the magnetic field will not penetrate the superconductor. The magnetic flux is said to be frozen-in and, therefore, the state of the superconductor would depend on its previous history. If the
specimen passes through the transition temperature in a zero magnetic field and then is brought into a magnetic field, $H$, the $B$ inside of the superconductor will still be zero. But, if the specimen is cooled through the transition in the magnetic field, the $B$ inside the superconductor will equal $\mathcal{M}_0 \mathcal{H}$. 

The theory of the perfect conductor seemed to be confirmed with the persistent current experiments. However, in 1933 Meissner and Ochsenfeld announced that they had cooled a mono-crystal of tin in a magnetic field and found that the magnetic flux was pushed out of the specimen so that $B$ was equal to zero inside the tin. This is now called the "Meissner effect." One good reason this fact was not observed before 1933 is that even for the best obtainable spectroscopically pure metal, there exists a small amount of flux frozen-in when passing through the transition temperature in a magnetic field. This amount seems to depend on the quantity of impurities, on the crystal structure, and on the geometrical shape of the specimen, but more information is needed before anything quantitatively can be said on this.

In 1934 H. London and F. London developed their electrodynamical theory of superconductivity. They assumed the equation $\nabla \times (\lambda \mathbf{j}_s) = -\mathbf{B}$ where $\lambda$ is a constant depending on the superconductor, $\mathbf{j}_s$ is the density of the supercurrent, and $\mathbf{B}$ is the magnetic field. This equation relates the supercurrent and the magnetic induction and states that these supercurrents are determined by the magnetic field in which the superconductor finds itself. Using the assumed equation, Maxwell's equations,
and the acceleration equation for the motion of free electrons in an electric field, they were able to derive that $\vec{B}$ equals zero in a superconductor. The London theory (this theory is in Section VB) states that the magnetic field penetrates into a thin surface layer of the superconductor; however, this magnetic field penetration induces surface supercurrents which keep the value of $\vec{B}$ in the interior equal to zero. A picture of the principal distribution of the magnetic field about the superconductor and the surface currents, $j$, which are shown as lines of longitude, is shown in Figure 1.

\[ \text{Figure 1} \]

LINES OF $\vec{B}$ ABOUT A SUPERCONDUCTOR HAVING SURFACE CURRENTS, $j$. 
Infinite conductivity cannot describe the Meissner effect, for in the case of infinite conductivity this would indicate that $\mathbf{B}$ is a constant which is not necessarily equal to zero. But the Meissner effect for a simply connected domain can be explained by having $\mathbf{M} = 0$, i.e., perfect diamagnetism. An experiment which clearly demonstrates these statements has been published by Dr. W. V. Houston and Mr. Nils Muench. Their experiment consisted of a tin sphere suspended from a torsion fibre set in rotational oscillations in a magnetic field.

Above the transition temperature, damping followed the expected eddy current theory. But this theory does not hold below the transition temperature where the conductivity becomes large. The eddy current theory would predict a decrease in the period of the oscillations since there is a large restoring force when the conductivity becomes large, and this change in the period was not observed. In other words, their results supported the London theory of superconductivity which says that the supercurrents transmit force only to the surface of the superconductor and not as the normal current transmits a force to the lattice.

The present experiment is a continuation of the thesis work done at the Rice Institute Low Temperature Laboratory by Dr. W. F. Love and Mr. P. B. Alers on rotating superconductors. They observed that a rotating conductor on passing through the transition temperature showed a perfect Meissner effect. This thesis is an account of a rotating spherical conductor passing through the transition temperature in the presence of a horizontal magnetic field, i.e., a quantitative investigation
of this phenomenon, a perfect Meissner effect. Above the transition temperature eddy currents are induced in the sphere in an amount depending upon the conductivity and the speed of the rotation of the sphere. The purpose of this thesis is to show the similarities between the eddy currents and the supercurrents at the transition temperature.
II

EXPERIMENTAL APPARATUS

A. The Cryostat

The experimental apparatus (see Figure 2) was originally designed for fast speeds of rotation but has now been adapted for slow speeds. At the top of the apparatus is a brass plate upon which a stuffing box is mounted at the center for the drive rod. This drive rod extends only four inches below the brass plate. It is coupled with a textolite rod three-eighths of an inch in diameter, which extends downward to the textolite rotor housing. The pulley wheel on the drive rod is driven by a cord from a motor some twenty feet away. At this distance the magnetic field from the motor is negligible at the detector element when compared to the magnetic field of the eddy currents induced on the sphere. On the brass plate at the top of the apparatus there is a copper T-joint which is used for pumping on the system, in measuring the pressure of the helium vapor, and for the transfer of the liquid helium. Another opening in the brass plate is used to bring out the coaxial cable from the detector.

Three thin-wall rods about fifteen inches long connect the brass plate and the cylindrical textolite tube three inches in diameter. Most of the wall of the textolite tube was removed to reduce the mass of the gear. This reduction of mass is very important because the less
mass there is to cool, the larger is the amount of liquid helium remaining after the necessary cooling to $4.2^\circ \text{K}$, which increases the time of the experiment. There is a circular aluminum plate at the bottom of the textolite tube which supports the rotor housing. At the top of the rotor housing there is another circular aluminum plate which maintains the alignment of the rotor housing.

The magnetic detection element shown in Figures 2 and 3 is placed inside a lucite tube, and the bottom of the tube is rigidly attached to the outer textolite tube in such a way that the bottom of the detector is on the same level as the center of the sphere in the rotor housing. The top of the lucite holder is attached so that the vertical alignment of the detector can be made.

The apparatus is placed inside a Dewar flask with the bottom side of the brass plate and the top of the Dewar sealed together by a collar and an O-ring set into the top of the Dewar. This collar is connected to the brass plate by four brass screws, and by uniform tightening of the screws, the O-ring is compressed which gives a vacuum-tight fit. The bottom half of the Dewar is a vacuum jacket which can easily be evacuated. In preparing for an experiment, an exchange gas of air for heat transfer purposes is placed within the jacket, and an overpressure of pure gaseous helium is placed within the Dewar itself. Exceptional care must be taken so that no air is allowed to enter the interior system. The oxygen from the air would liquefy, and since liquid oxygen is paramagnetic, this causes the magnetic detector to be inoperative. The Dewar is then placed in a liquid nitrogen bath for three hours.
FIGURE 3 - ROTOR AND LINES OF $\bar{B}$ ABOUT A ROTATING SPHERE

$\bar{B} = 10^{-5}$ W/M²
which cools the experimental gear to 80° K. Just before the transfer of liquid helium, the vacuum jacket is evacuated by an oil diffusion pump to a pressure of less than $4 \times 10^{-5}$ mm. of mercury and is sealed off by means of a stopcock. Upon the transfer of liquid helium, the air remaining in the vacuum jacket solidifies causing a further reduction in pressure. The Dewar is then returned to the liquid nitrogen bath which also reduces the amount of heat conduction into the liquid helium.
B. Magnetic Field Detection Equipment

The changes in the magnetic field were measured by a device known as the Magnetic Airborne Detector (AN ASQ-3) which was provided by the Office of Naval Research. Although certain alterations have been made on the Magnetic Airborne Detector, the operation of the detector is approximately the same as the original design. Figure 4 is a block diagram of the detector. The output of a 1,000-cycle oscillator is carefully filtered so that there does not exist any other frequency components, especially the 2,000-cycle component. After passing through the filter, the 1,000-cycle output then is sent through a resonant series circuit consisting of a capacitor and the detector element. This resonant series circuit is shunted by a 200-ohm resistor. Since this is a resonant circuit, the current in the detector element is determined by the applied voltage (13 volts) and the resistance of the detector element (130 ohms). The detector element is the only part of the above circuit that is cooled to 4.2° K.

If the detector element is in a magnetic field, a 2,000-cycle component is generated and is taken off to be fed through a filter which allows only the 2,000-cycle component to pass. Then the 2,000-cycle component is fed into a resistance-coupled amplifier with four stages. At the third stage two leads are taken off and connected to the oscilloscope. The load on the fourth stage consists of a 5,000-ohms resistor, and an Ayrton shunt and a galvanometer connected in parallel with a
FIGURE 4 - BLOCK DIAGRAM OF MAGNETIC DETECTION EQUIPMENT
resistance box and a 1.5-volt dry cell battery. The dry cell battery is used to buck out the current that exists when the magnetic field from the Helmholtz coils has been set so as to have a minimum deflection on the oscilloscope. A change in the magnetic field appears on the galvanometer as an unbalance. Field changes of $10^{-14}$ gauss gave deflection of three centimeters on the galvanometer scale with one-tenth sensitivity. The sensitivity reported for the equipment is better than ten microvolts per ten micro gauss at the output of the 2,000-cycle filter. The Appendix gives some details of the detector's operation.
G. Method of Measurement

This experiment can be divided into two parts. The first part of the experiment consisted of finding the dependency of eddy currents induced on the tin sphere with respect to the rotational speed of the rotor. Above the transition temperature the sphere was rotated at varying speeds, and the deflections on the galvanometer and the speeds of rotation were recorded. After each determination of speed versus amplitude, the rotor was then stopped and a check was made of the zero position of the galvanometer. At slow speeds the rotational speed was determined by the use of a stop watch. At faster rotational speeds where the revolutions could not be counted accurately, a stroboscope was used to measure the speed of rotation. The rotational speed was varied between 0.3 and 5.0 revolutions per second.

During the second part of the experiment, the specimen was rotated at a certain speed and passed through the transition temperature. The deflection on the galvanometer was observed during the transition. If, on passing through the transition temperature, a magnetic flux was frozen-in, the deflection on the galvanometer would oscillate with a frequency determined by the rotational speed of the rotor. That is, the sphere was not rotating at a speed fast enough to set up eddy currents large enough to make $\bar{B}$ approximately equal to zero in the interior of the sphere, and upon passing through the transition temperature part of the magnetic field that remained in the sphere was frozen-in. But on the other hand, if no flux was frozen-in, the galvanometer would retain
a steady deflection even if the rotation of the rotor was stopped. If
the rotor was then turned by hand, the galvanometer would retain its
steady deflection. This would indicate a perfect Meissner effect.

Most of the vertical component of the earth's magnetic field
was bucked out by external Helmholtz coils. The amount of current flowing
in the Helmholtz coils was varied until the 2,000-cycle component gener¬
at ed by the detector was at a minimum as seen on the oscilloscope.

These experiments were performed with a tin sphere, one and
one-half inches in diameter, and a tantalum cylinder, one inch in diameter
and one and one-half inches long. The transition temperature for tin
is 3.69° K. so in order to pass through the transition temperature of
tin, it was necessary to pump on the liquid helium until the absolute
pressure of the helium vapor was 441 mm. of mercury. Since the transi¬
tion temperature of tantalum is 4.38° K., tantalum when immersed in liquid
helium at a pressure of 760 mm. of mercury is in the superconducting
state. By not allowing the helium vapor to escape to the air, an over¬
pressure was built up. At 890 mm. of mercury of pressure the tantalum
returned to a normal conductor. With an overpressure on the helium
greater than 890 mm. of mercury, the tantalum was rotated as a normal
conductor, and by reducing the pressure to the atmospheric pressure,
the tantalum passed through its transition temperature while rotating.
The experiments on the metals, tin and tantalum, were performed with the same apparatus. The only difference was a change in the rotor housing since the tin is in a spherical shape and the tantalum is in a cylindrical shape.

A. Results on Tantalum

Tantalum has the property of freezing in most of the magnetic flux that exists in the material when passing through the transition temperature. Figure 6 shows the frozen-in moment as a function of the angle of the rotor with respect to the position of the detector element. This large frozen-in moment normally occurs in superconducting tantalum. In this graph the tantalum was not rotating when it passed through the transition temperature in a horizontal magnetic field. Such a large frozen-in moment existed that the less sensitive oscilloscope was used to give an indication of its amount. This was necessary because there is less amplification of the 2,000-cycle at the oscilloscope. The position, zero of the rotor, was chosen arbitrarily, and a rotation of the drive rod gave a new position of the axis of the frozen-in moment with respect to the detector element, which remained fixed in space.

The next part of the experiments on tantalum consisted of rotating the tantalum when passing through the transition temperature.
FIGURE 6
FROZEN-IN MOMENT
OF TANTALUM

SCOPE READING (cm.)

ANGLE OF ROTOR (degrees)
FIGURE 7
MEISSNER EFFECT OF TANTALUM

SCOPE READING (cm.)

0  0.5  1  1.5  2

ANGLE OF ROTOR (degrees)

0  60  120  180  240  300  360  0
The plot of the magnetic flux through the detector element as a function of the angle of the rotor with respect to the detector is shown in Figure 7. There was a steady deflection with a small variation superimposed. The speed of rotation was 0.35 revolutions per second during the transition.

The data for Figure 6 and Figure 7 are shown below:

<table>
<thead>
<tr>
<th>Position of Rotor (In Deg.)</th>
<th>Amplitude of Signal on Scope (In cm.)</th>
<th>Position of Rotor (In Deg.)</th>
<th>Amplitude of Signal on Scope (In cm.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>0</td>
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</tr>
<tr>
<td>30</td>
<td>0.3</td>
<td>30</td>
<td>1.8</td>
</tr>
<tr>
<td>60</td>
<td>0.7</td>
<td>60</td>
<td>1.7</td>
</tr>
<tr>
<td>90</td>
<td>1.5</td>
<td>90</td>
<td>1.65</td>
</tr>
<tr>
<td>120</td>
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<td>120</td>
<td>1.6</td>
</tr>
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<td>150</td>
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<td>150</td>
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</tr>
<tr>
<td>180</td>
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<td>180</td>
<td>1.7</td>
</tr>
<tr>
<td>210</td>
<td>3.6</td>
<td>210</td>
<td>1.65</td>
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<td>3.2</td>
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<td>1.55</td>
</tr>
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<td>270</td>
<td>2.6</td>
<td>270</td>
<td>1.6</td>
</tr>
<tr>
<td>300</td>
<td>1.75</td>
<td>300</td>
<td>1.7</td>
</tr>
<tr>
<td>330</td>
<td>0.9</td>
<td>330</td>
<td>1.8</td>
</tr>
<tr>
<td>360</td>
<td>0.45</td>
<td>360</td>
<td>1.9</td>
</tr>
</tbody>
</table>
B. Results on Tin

The tin sphere is a better behaved superconductor than tantalum in that tin contains a smaller amount of frozen-in flux. Figures 8 and 9 are the plot of the galvanometer reading as a function of the rotational speed. Before going into the discussion of this graph, it would be best to explain what the magnetic detector element measures. The detector element will measure only a vertical component of the magnetic induction, $\vec{B}$. By looking at Figure 3 the lines of $\vec{B}$ are seen to be bent around the rotating specimen. If only a small amount of eddy currents is induced on the surface of the sphere, most of the lines of $\vec{B}$ continue straight on through the detector element and pass through the sphere. As the eddy currents increase, the lines of $\vec{B}$ are forced over the sphere. This gives a vertical component of $\vec{B}$ in the detector element which appears as an unbalance on the galvanometer scale. The graphs show an exponential rise with a gradual levelling off. The curves are still slightly rising with increases of the rotational speed.

With the sphere rotating at 4.6 revolutions per second, it was observed that the amplitude of the flux being kicked out by the eddy currents increased as the sphere was cooled. The deflection on the galvanometer increased from 45 cm. to 51 cm. as the temperature decreased. The galvanometer deflection reached the maximum value, 51 cm., above the transition and remained there while passing through the transition temperature.
FIGURE 8
EDDY CURRENTS IN TIN AT 4.2°K.
The data for Figure 8 and Figure 9 are shown below:

<table>
<thead>
<tr>
<th>Galvanometer Deflection (Cm.)</th>
<th>Rotational Speed (Rev/sec)</th>
<th>Galvanometer Deflection (Cm.)</th>
<th>Rotational Speed (Rev/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>.55</td>
<td>11</td>
<td>.32</td>
</tr>
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<td>17</td>
<td>.80</td>
<td>19</td>
<td>.41</td>
</tr>
<tr>
<td>21.3</td>
<td>1.15</td>
<td>25</td>
<td>.47</td>
</tr>
<tr>
<td>26</td>
<td>1.80</td>
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<td>.56</td>
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</tr>
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<td>43</td>
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</tr>
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<td>45</td>
<td>2.0</td>
</tr>
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<td>47</td>
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</tr>
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<td></td>
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<tr>
<td></td>
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<td>50.5</td>
<td>2.55</td>
</tr>
</tbody>
</table>
IV

DISCUSSION OF RESULTS

The Meissner effect for tantalum is practically non-existent, and Figure 6 shows the large frozen-in moment. This is what might be expected since pure tantalum in bulk quantity is made in small flakes randomly orientated. This was deduced from ultrasonic observation at The Rice Institute by Dr. W. C. Overton, Jr.\(^6\) from the fact that the sound waves were scattered so badly on passing through tantalum.

When passing slowly through the transition temperature while in rotation, tantalum showed a Meissner effect with a small frozen-in moment superimposed. This can be observed in Figure 7 since upon rotation of the sphere, there were no large changes in the field within the detector element. The field distribution was similar to Figure 1 which shows field distribution about a perfect superconductor except that Figure 1 pertains to a sphere rather than a cylinder. The eddy currents induced on the cylinder were of such magnitude that they removed most of the magnetic flux in the interior of the tantalum. And upon passing through the transition temperature, the eddy currents changed over into supercurrents. It was found that the cylinder had to pass slowly through the transition temperature so that the residual flux could be removed from the interior.
At 4.2° K, the tin sphere did not reach a saturation speed; that is, a speed was not attained where further increases of rotational speed would not give a larger deflection. The curves in Figures 8 and 9 are still slightly rising with increases of the rotational speed. It was impossible to further increase the speed, since five revolutions per second is the maximum speed attainable with the present apparatus.

The data for Figures 8 and 9 showed that there existed a change in the sensitivity, as the only difference in the two runs was a change in the zero vertical magnetic field setting of the current in the Helmholtz coils and a change in the resistance in the bucking circuit. The difference in the vertical magnetic field on the detector element for the runs in Figures 8 and 9 was approximately $1.5 \times 10^{-3}$ gauss. This change in the zero field setting of the Helmholtz coils would not appreciably alter the large horizontal magnetic field (0.20 gauss). Therefore, the galvanometer deflection for a given rotational speed should be the same in the two runs. As can be seen in Figure 9, there existed a 47-cm. galvanometer deflection for 2.5 revolutions per second, while in Figure 8 there was a 29-cm. deflection.

This effect is negligible at room temperature, however, it does exist at liquid air temperatures but to a lesser degree than at liquid helium temperatures. Further work at liquid air temperatures is necessary before anything of a quantitative nature can be said on the cause of this variation in the sensitivity.
The increase in the galvanometer deflection as the tin sphere was cooled may be attributed to an increase in the conductivity, \( \sigma \). This is because the sphere was rotating at the same speed and because the eddy currents are a function of \( \sqrt{\sigma \omega} \). That is, an increase in the conductivity increases the strength of the magnetic moment of the eddy currents in the same manner as an increase of angular velocity.

The magnetic field distribution about a rotating sphere in a horizontal magnetic field (as shown in Figure 3) must satisfy the equation \( \nabla^2 \vec{B} = j \omega \mu \sigma \vec{B} \) where \( \omega \) is the angular velocity, \( \mu \) is the magnetic permeability, and \( \sigma \) is the electrical conductivity.

The eddy currents induced by the magnetic field produce a magnetic field which outside the sphere is equivalent to a magnetic dipole of strength \( \left( \frac{4\pi I}{M_o} \right) \frac{B D}{2} \) at the center of the sphere. \( B \) is the magnetic field in which the sphere is rotated, and \( D \) is given by
\[
D = -a^3 \left\{ 1 - \frac{3}{10a} \frac{\sinh \frac{10a}{\mu} - \sin \frac{10a}{\mu}}{\cosh \frac{10a}{\mu} - \cos \frac{10a}{\mu}} \right\} \\
-ja^3 \left\{ \frac{3}{10a} + \frac{3}{10a} \frac{\sinh \frac{10a}{\mu} + \sin \frac{10a}{\mu}}{\cosh \frac{10a}{\mu} - \cos \frac{10a}{\mu}} \right\}
\]
with \( \rho = \omega \mu \sigma \), and \( a \) is the radius of the sphere. This is derived in Section VA, Eddy Current Theory. When the conductivity, \( \sigma \), and the angular velocity, \( \omega \), are low, the imaginary component is the predominant term. The axis of the dipole will then be perpendicular to the applied magnetic field. If either the conductivity or angular velocity become very large, the imaginary component approaches zero, and the dipole axis then coincides with the direction of the magnetic field. At 4.2° K. for five revolutions per second and \( \sigma = 4 \times 10^{-6} \text{ mhos} \), the imaginary component meter...
is only one-tenth of the real component. The magnetic dipole axis is then almost parallel with the applied field.

The London theory requires that the magnetic field of a superconducting sphere must satisfy the equation

$$\nabla^2 \bar{B} - \frac{\mu}{\lambda} \bar{B} = 0$$

where $\lambda = \frac{m}{\gamma c^2}$, $\gamma$ is the number per unit volume, $m$ is the mass, and $e$ is the charge of the superconducting electrons. In Section VB it is shown that the magnetic field of the superconducting sphere is also the field of a magnetic dipole which has a strength given by

$$M = -\left(\frac{4\pi}{M_0}\right)\frac{a^3}{2} \left(1 - \frac{3}{\sqrt{4 \pi a}} \coth \frac{4a}{\sqrt{a}} - \frac{3}{a^2} \right)$$

Comparing this with the dipole set up by the eddy currents, one sees that they are equal in the limit of very fast speeds of rotation with high conductivity. At about 3.8° K. with the sphere rotating at 4.6 revolutions per second, the detector element could not observe any change in the magnetic field when the sphere became superconducting. That is, the field change at the detector element was less than $10^{-4}$ gauss. The skin depth, $\frac{1}{\sqrt{\mu c^2}}$, for the eddy currents is 0.6 mm. at five revolutions per second while the skin depth, $\frac{1}{\sqrt{\mu a}}$, of the supercurrents is $1.6 \times 10^{-5}$ mm.
A. Eddy Currents

A sphere rotating in a horizontal magnetic field will have eddy currents induced on the surface of the sphere, and this will cause a change in the distribution of the magnetic field about the sphere. How the magnetic field changes in the vicinity of the sphere can be determined by the electromagnetic theory.

A simple change of the coordinate system allows the sphere to remain stationary, and the magnetic field, $\vec{B}$, to rotate around the sphere. In this new system of coordinates the magnetic field at a great distance is given by $\vec{B}$ equals $\vec{B}_0 e^{j\omega t}$; however, the real and the imaginary part of $\vec{B}$ must be used to give a rotating field.

Maxwell's equation, if one neglects displacement currents (the maximum $\omega$ for the rotating sphere is five revolutions per second), becomes

$$\text{curl } \vec{H} = \vec{J}$$

Since the sphere is a normal conductor having conductivity, $\sigma$, and permeability, $\mu$, the current density, $\vec{J}$, flowing will obey Ohm's Law

$$\vec{J} = \sigma \vec{E}$$
The magnetic vector potential is related to the magnetic induction vector, \( B \), by the equation

\[
\text{curl } \mathbf{A} = \mathbf{B}
\]

The electric field is given in terms of \( \mathbf{A} \) by

\[
\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \text{grad (scalar function)}
\]

\[
\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t}
\]

because in the static case the electric field, \( \mathbf{E} \), is zero. If the equation, \( \text{curl } \mathbf{A} = \mathbf{E} \), is put in terms of \( \mathbf{A} \), the equation becomes

\[
\text{curl } \text{curl } \mathbf{A} = \mu_0 \frac{\partial^2 \mathbf{A}}{\partial t^2}
\]

Because of the restriction on the electric field, \( \mathbf{E} \), the divergence of \( \mathbf{A} \) is zero. The equation to be solved then reduces to

\[
\nabla^2 \mathbf{A} = \mu_0 \frac{\partial^2 \mathbf{A}}{\partial t^2}
\]

Since this equation is a linear differential equation, use can be made of the principle of superposition. Find the solution of the above equation when the applied field is \( \mathbf{B}_2 = \mathbf{B}_0 e^{j\omega t} \) (only the real part), i.e., an alternating field in the \( Z \) direction. Then find another solution for \( \mathbf{B}_1 = \mathbf{B}_0 e^{j(\omega t + \frac{\pi}{2})} \) (only the real part), and the addition of these two solutions will give the solution for the field rotating about the sphere. First consider the problem when the applied field is \( \mathbf{B}_2 = \mathbf{B}_0 e^{j\omega t} \).

Choose the polar axis of a system of spherical coordinates to coincide
with the Z axis. This alternating magnetic field will be independent of \( \phi \) and will have no \( \phi \) component. For this case the solution is (Smythe, *Static and Dynamic Electricity*, Page 395)

\[
\vec{A} = \frac{-j}{2\pi} \sum_n \left[ A_n P_n'(\cos \theta) + B_n Q_n'(\cos \theta) \right] \left[ C_n I_{n+\frac{1}{2}}(j\mu r) + D_n I_{n+\frac{1}{2}}(j\mu r) \right] e^{j\omega t}
\]

where \( P_n^1 \) and \( Q_n^1 \) are Legendre functions of the first and second kind, \( I_{n+\frac{1}{2}} \) and \( I_{-(n+\frac{1}{2})} \) are modified Bessel functions, and \( \mu = \frac{\omega}{c} \).

To determine the arbitrary constants, \( A_n^1, B_n^1, C_n^1, \) and \( D_n^1 \), boundary conditions must be applied. At a large distance the magnetic vector potential, \( \vec{A} \), must approach the vector potential of the original field

\[
\vec{A} = \vec{\Phi} \frac{1}{2} B r \sin \theta = \vec{\Phi} \frac{1}{2} B r \left[ P_n^1(\cos \theta) \right]
\]

The curl of this \( \vec{A} \) gives only a Z component which is the applied field, \( B_2 = B_0 e^{-j\omega t} = \vec{B} \). Thus, \( n = 1 \), and since \( Q_1'(0) = \infty \), the \( B_1 \) is defined as zero. For the region outside the sphere where the conductivity, \( \sigma \), is equal to zero, the solution for \( \vec{A}_0 \) is given by the Legendre's function times \( \left[ E_r + F_r \right] \). This is the solution of \( \nabla^2 \vec{A} = 0 \), where \( \vec{A} \) is not a function of \( \phi \) and \( r > 1 \).

The magnetic vector potential outside of the sphere, \( \vec{A}_0 \), is given by

\[
 \vec{A}_0 = \vec{\Phi} \frac{1}{2} B \left( r + \frac{d}{r} \right) \sin \theta
\]
Where \( D \) is an arbitrary constant to be determined, the coefficient of \( r \) has to equal one so that at large distances \( \vec{A} \phi \) is equal to the vector potential of the original field, and \( a \) is the radius of the sphere.

Since \( \vec{A} \phi \) must be finite when \( r = 0 \), only the modified Bessel function, \( I_{3/2}[(y/\rho)^{1/2} r] \), can be used inside the sphere. This gives

\[
\vec{A} \phi \bigg|_{\alpha < r < a} = \vec{A} \phi \bigg|_{r = 0} + BC r^{-1/2} \left( I_{3/2}[(y/\rho)^{1/2} r] \right) \sin \theta
\]

Requiring that the boundary conditions at \( r = a \) equals a

\[
A \phi \bigg|_{r = a} = A \phi \bigg|_{r = 0}
\]

\[
\frac{\partial}{\partial r} \left[ r \sin \theta A \phi \bigg|_{r = a} \right] = \frac{2}{2 r} \left[ r \sin \theta A \phi \bigg|_{r = a} \right]
\]

be satisfied, gives

\[
C = \frac{3 a^{3/2} (y/\rho)^{-1/2}}{I_{3/2}[(y/\rho)^{1/2} a]} \quad D = -a^{3} \frac{I_{3/2}[(y/\rho)^{1/2} a]}{I_{1/2}[(y/\rho)^{1/2} a]}
\]

The following identities were used in obtaining \( C \) and \( D \)

\[
I_{n}'(\nu) = I_{n-1}(\nu) - \frac{2n}{\nu} I_{n}(\nu)
\]

\[
I_{n}'(\nu) = I_{n+1}(\nu) + \frac{2n}{\nu} I_{n}(\nu)
\]

The field outside the sphere and how it changes with \( D \) can be obtained by using the equations

\[
I_{3/2}(\nu) = \frac{2}{\pi \nu} \left[ (1 + \frac{3}{\nu^2}) \sinh \nu - \frac{3}{\nu} \cosh \nu \right]
\]

\[
I_{1/2}(\nu) = \frac{1}{\pi \nu} \left[ \sinh(\nu) \right]
\]
to solve for $D$. From these equations it follows that

$$D = -a^3 \left\{ 1 - \frac{3}{2r^3} \left( \frac{\sin \theta \rho \phi}{\rho \phi} \right)^2 - \frac{3}{\rho \phi} \right\} \left( \frac{\sin \theta \phi \rho}{\sin \theta \rho \phi} \right)^2 \left( \frac{\cos \phi \rho \phi}{\sin \theta \phi \rho} \right)^2$$

By use of

$$\frac{1}{\rho} = \frac{1}{2} r^2$$

$$\sinh (x+jx) = \sinh x \cos x + j \cosh x \sin x$$

$$\cosh (x+jx) = \cosh x \cos x + j \sinh x \sin x$$

$D$ can be separated into its real and imaginary components.

$$D = -a^3 \left\{ 1 - \frac{3}{2r^3} \frac{\sin \theta \rho \phi - \sin \theta \phi \rho}{\cosh \theta \rho \phi - \cos \theta \phi \rho} \right\}$$

$$-j a^3 \left\{ -\frac{3}{r^2} + \frac{3}{2r^3} \frac{\sinh \theta \phi \rho + \sin \theta \phi \rho}{\cosh \theta \phi \rho - \cos \theta \phi \rho} \right\}$$

The magnetic field outside the sphere is determined by taking the curl of $A^\phi$. This gives

$$B_\theta = -B \left( 1 - \frac{D}{2r^3} \right) \sin \theta$$

$$B_r = B \left( 1 + \frac{D}{2r^3} \right) \cos \theta$$

The field distribution is similar to a variable magnetic dipole of strength $\frac{DBe^{j\omega t}}{2(4\pi)}$ along the polar axis with the applied field, $B$, also parallel to the polar axis. This is the solution for $B_z = B_0 e^{j\omega t}$.

The solution for $B_x = B_0 e^{j\omega t}$ can be greatly simplified by a rotation of the axes. In the new system of coordinates, the $X'$ axis coincides with the $X$ axis in the previous system, i.e., a rotation of $90^\circ$ about the $Y$ axis. This alternating magnetic field will be independent of $\phi$ and will not have a $\theta$ component. This gives the same
magnetic field distribution in the new coordinates as existed for the previous problem. The solution is

\[ B_{\theta'} = -\left(1 - \frac{\mathcal{D}}{\kappa^2} \right) \sin \theta, \quad B_0 e^{j(\omega t + \varphi)} \]
\[ B_{\phi'} = \left(1 + \frac{\mathcal{D}}{\kappa^2} \right) \cos \theta, \quad B_0 e^{j(\omega t + \varphi)} \]

As before, this is similar to a variable magnetic dipole of the same strength, \( \frac{\mathcal{D}}{\kappa^2} B_0 e^{j(\omega t + \varphi)} \), along the \( Z' \) axis; however, in the previous set of coordinates, this dipole is along the \( X \) axis. The total solution of the original problem is the addition of these two solutions. Since the strength of one magnetic dipole is \( 90^\circ \) out of phase with the strength of the other magnetic dipole, the resultant magnetic dipole will rotate with the angular velocity, \( \omega \), and is given by \( \frac{\mathcal{D}}{\kappa^2} B_0 e^{j\omega t} \) (real and imaginary part). Thus, it can be seen that the dipole axis is rotating and is out of phase with respect to the applied field by an amount determined by the real and imaginary components of \( \mathcal{D} \). Returning to the laboratory coordinates where the sphere is rotating, the dipole moment is then fixed in space for a given angular velocity, \( \omega \), and a conductivity, \( \sigma \).
B. Electrodynamics of a Superconducting Sphere

When a sphere becomes superconducting in a magnetic field, the magnetic field is pushed out of the metal. This is called the Meissner effect. The distribution of the magnetic field about the superconducting sphere can be obtained from Maxwell's equations and the London equation,

$$\text{curl} (\lambda \vec{J}) = -\vec{B}, \quad \lambda = \frac{m}{\hbar c^2}$$

where $\vec{J}$ is the current density, $m$ is the mass, $n$ is the number per unit volume, and $e$ is the electronic charge of the superconducting electrons. Neglecting the displacement currents and taking the curl of the above equation, gives

$$\text{curl} \text{curl} (\lambda \vec{J}) = -\text{curl} \vec{B}$$

$$\text{curl} \text{curl} (\vec{J}) = -\frac{\hbar}{\lambda} \vec{J}$$

The current density must satisfy this equation in the superconductor.

Consider a superconducting sphere of radius, $R$, in a magnetic field which has only a component along the polar axis. The field inside the sphere must be cancelled by an equal and opposite directed field. "Because of the symmetry of the sphere and the magnetic field having only a $Z$ component, there can only be a current parallel to the equator, such that, $\vec{J}_\theta = 0$, $\vec{J}_r = 0$, and $\vec{J}_\phi$ is of the form $\vec{J}_\phi = \hat{\phi} \rho \sin \phi \cos \phi$." This is taken from Dr. F. London's book, Superfluids. 7)

The current, $\vec{J} \phi$, must satisfy the equation

$$\text{curl} \text{curl} (\vec{J}_\phi) + \nabla^2 \vec{J}_\phi = 0$$
where \( \beta = \frac{1}{\mu \lambda} \).

The field outside the sphere is then given by the superposition of the constant magnetic field, \( B_0 \), with the magnetic field of a dipole moment

\[
B_\text{ext} = \left( B_0 + \frac{2m}{r^3} \frac{\mu_0}{4\pi} \right) \cos \theta
\]

\[
B_\phi = -\left( B_0 - \frac{m}{r^3} \frac{\mu_0}{4\pi} \right) \sin \theta \quad r > R
\]

where \( M \) is the magnetic dipole moment of the sphere, and the \( \frac{\mu_0}{4\pi} \) enters because of the use of the rationalized NES system. The dipole axis coincides with the polar axis since there exists only a \( \phi \) component of the supercurrents.

To find the currents and the field inside the sphere, \( J_\phi \) is substituted in

\[
\text{curl} \text{ curl} J_\phi + \beta^2 J_\phi = 0
\]

and then the equation to be solved is

\[
f''(r) + \frac{2}{r} f'(r) - \left( \frac{2}{r^2} + \beta^2 \right) f(r) = 0
\]

which has the general solution

\[
f(r) = \frac{A}{\beta^2} \left\{ \sinh \beta r - \frac{\beta}{\beta^2} \cos \beta r \right\} + \frac{B}{\beta^2} \left\{ \cosh \beta r - \beta \sinh \beta r \right\}
\]

where \( A \) and \( D \) are integration constants. Because \( \frac{\cosh \beta r}{r^2} \) becomes \( \infty \) as \( r \to 0 \), \( D \) is defined as zero. The current becomes
The equation
\[ \text{curl} (\lambda \vec{\nabla} \phi) = -\vec{B} \]
determines the magnetic field inside the sphere.

\[ B_r^i = -\lambda (\text{curl} \vec{\nabla} \phi)_r = -\frac{\lambda}{r^3} \sin \phi \left[ \frac{\partial}{\partial \phi} (\sin \phi \partial \phi) \right] \]

\[ B_\theta^i = -\frac{2\lambda A}{r^3} \left[ \sinh (\beta r) - \beta r \cosh (\beta r) \right] \cos \phi \]

\[ B_\phi^i = -\frac{2\lambda A}{r^3} \left[ (1+\beta^2 r^2) \sinh (\beta r) - \beta r \cosh (\beta r) \right] \cos \phi \]

\[ B^\phi = 0 \]

The constants, \( M \) and \( A \), are determined by the boundary conditions at \( r = R \):

\[ B_r^i = B_\theta^i = B_\phi^i = B^\phi = 0 \]

\[ A = \frac{3RB_0}{2\lambda R^2 \sinh (\beta R)} = \frac{3RB_0}{2M R \sinh (\beta R)} \]

\[ M = -\frac{\lambda}{\mu_0} \frac{R^3 B_0}{2} \left[ 1 - \frac{3}{\beta R} \coth (\beta R) + \frac{3}{\beta^2 R^2} \right] \]
At a large distance from the sphere, the field from the supercurrents is equivalent to a magnetic dipole, M, of this strength.
VI

CONCLUSION

In conclusion, the following statements can be made regarding the experimentation with tin and tantalum rotating at the transition temperature.

1) Experimentally, the eddy currents in the tin sphere have the same space distribution as the superconducting Meissner currents within the sensitivity of the magnetic detection equipment.

2) Tantalum metal which has been cintered together to make the bulk specimen will perform like a normal superconductor if the magnetic field within the interior of the metal is largely removed by the rotation of the specimen while passing through the transition temperature.
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APPENDIX

DETAILS OF MAGNETIC DETECTOR OPERATION

A. The 2,000-Cycle Output Voltage

The detector element is a solenoid three centimeters long of fine copper wire singly wound on a lucite spool with approximately seventy-two turns per centimeter. The diameter of the lucite spool is about 0.8 centimeter. In the center of the lucite spool is a glass tube about one-tenth of an inch in diameter and on which a strip of molybdenum permalloy, one-thousandth of an inch thick, is placed. This permalloy-rolled core is the material that makes the detector work.

When the current flows in the solenoid, the permalloy is magnetized and reaches saturation with a small applied field. On the ring-shaped end of the permalloy there exists a large divergence of the magnetization, \( \nabla \cdot \mathbf{M} \). Since the divergence of \( \mathbf{B} \) is always zero, the divergence of \( \mathbf{M} \) is equal to the negative divergence of the magnetic field strength, \( \nabla \times \mathbf{H} \). The divergence of \( \mathbf{M} \) integrated over a thin flat volume including the end of the permalloy can be represented by a surface integral of \( \mathbf{M} \) over the end of the permalloy which is, by similar reasoning, equal to the surface integral of \( \mathbf{H} \) over the end. The field distribution of \( \mathbf{H} \) is similar to the electric field of a ring having a surface charge density, \( \mathbf{M} \). To calculate the value of \( \mathbf{H} \) at the center of the solenoid due only to the divergence of the magnetization, we make use of the above
analogy. The surface area of one end of the permalloy is given by

\[ S = \pi (r_2^2 - r_1^2) = \pi (r_2^2 - r_1^2)(r_2 + r_1) \approx 2\pi r_1^2 \Delta r \]

where the inner radius, \( r_1 \), of the core is one-twentieth of an inch and the difference in radii, \( \Delta r \), is one-thousandth of an inch. Therefore, the value of \( S \) is \( \frac{\pi}{9000} \). The electric field of a charged ring at a distance \( \frac{L}{2} \) from the center of the loop on its axis is

\[ E = \frac{N S}{4\pi \epsilon_0 (L/2 + \frac{D}{2})^2} = FN (4.4 \times 10^{-5}) \]

where \( \frac{L}{2} \) equals three-fourths of an inch, i.e., in the center of the solenoid axis. This magnetic field strength, \( B_H \), due to the divergence of \( H \) is in the opposite direction to the field, \( H \), from the solenoid. The effect of the other end adds to this field; therefore, the field, \( B_H \), at the center is \( -8.8 \times 10^{-5} \). With most materials this can be neglected, but permalloy has a value of \( B \) in the range of 10,000 to 30,000. Using a value, 10,000 for \( B \), it can be seen from the following equations that this field, \( B_H \), is large. Inside the permalloy we have

\[ \beta = \mu H_c = \mu_0 H_c + \mu H \]

where \( H_c \) is the field produced by the current in the solenoid.

\[ H = \left( \frac{\mu - 1}{\mu_0} \right) H_c \approx \mu H_c \quad (\mu_0 = 1 \text{ emu}) \]
Therefore, the field, $\bar{H}$, is approximately equal to $-0.9 \bar{H}_0$, and inside the core there exists a large demagnetizing factor due to the divergence of the magnetization on the ends. This keeps the $\bar{H}$ inside the core from changing appreciably; therefore, it can be said that after the saturation of the permalloy, $\bar{B}$ does not change.

The inductance, $L$, is small since $\bar{B}$ is small inside the core. This follows from the definition of the coefficient of self-inductance, $L$, of the solenoid, which is given by

$$ \oint \bar{B} \cdot d\bar{s} = \angle I $$

where $\bar{B}$ is the magnetic induction due to the current, $I$, in the solenoid, and the integral is over the surface of the solenoid. If the solenoid was filled with permalloy, the $L$ would be exceptionally large, and a larger supply of power would be needed to saturate the permalloy.

If the detector element is in a zero magnetic field, the electromotive force produced by the time variation of the flux within the solenoid will not have a 2,000-cycle component. But there will exist a 2,000-cycle component if the detector element is in magnetic field, $\bar{H}$, parallel to the axis of the solenoid. The external magnetic field, $\bar{H}$, which is being measured will affect the electromotive force produced only if the magnetic field has a component parallel to the axis of the solenoid.

Figures 5a and 5b are taken from the article, "Air-Borne Magnetometer for Search and Survey," and are shown to help explain how the Fourier analysis can represent the electromotive force produced. With a 1,000-cycle current in the solenoid, the $\bar{B}$ due to this current
is given by $\mathbf{M}_0 \cos \omega t$ where $H_0$ is the maximum value of the field produced by the current in the solenoid, and $\omega$ is equal to $2\pi$ times the frequency, 1,000. This assumes that $B$ is a linear function of $H$. The sinusoidal field, $\mathbf{M}_0 \cos \omega t$, is in addition to the field, $\mathbf{H}$, inside the detector element. When the field saturates the permalloy, the magnetic induction, $\mathbf{B}$, remains practically constant so that $\frac{d^2}{dt^2} B$ equals zero. The heavy line in Figure 5a represents the value of $\mathbf{B}$ within the solenoid as a function of the time.

In Figure 5b the electromotive force produced in the solenoid is drawn as a function of the time. One can think of the impressed magnetic induction, $\mathbf{B}$, as given by

$$\mathbf{M}_0 \mathbf{H} + \mathbf{M}_0 \cos \omega t = \mathbf{B}. $$

It should be remembered that this $\mathbf{M}$ is not the $\mathbf{M}$ of the permalloy which is very large (10,000-30,000), but is determined by the geometry of the permalloy core in the solenoid. Therefore, the electromotive force per turn produced by the variation of the magnetic induction, $\mathbf{B}$, is given by

$$e = -A \frac{d^2}{dt^2} = A \mathbf{M}_0 \omega \sin \omega t$$

The electromotive force produced when $\mathbf{B}$ is constant is zero, and the heavy curve in Figure 5b shows the electromotive force as a function of the time. Since this is an odd function, it can be represented in a series of sines in the interval, 0 to $\pi$. The coefficient, $a_2$, 

FIGURE 5
(a) VARIATION OF FLUX IN DETECTOR ELEMENT WITH TIME
(b) ELECTROMOTIVE FORCE PER TURN CORRESPONDING TO VARIATION OF FLUX IN FIGURE 5a
of the 2,000-cycle component is given by

$$E_2 = \frac{2\mu M A H_0}{\pi} \int \frac{\pi}{2} + \sin^{-1}\left(\frac{B_m + u H}{H_0}\right)$$

$$\frac{\pi}{2} - \sin^{-1}\left(\frac{B_m - u H}{H_0}\right)$$

$$E_2 = \frac{4\mu M A H_0}{3\pi} \left[ \left[ \left( \frac{B_m + u H}{H_0} \right)^2 \right]^{3/2} - \left[ \left( \frac{B_m - u H}{H_0} \right)^2 \right]^{3/2} \right]$$

The coefficient of the second harmonic is a function of the impressed field, $H_0$, from the current in the solenoid, the magnetic induction, $B_m$, of the saturated permalloy, and the magnetic field, $H$, which is being measured.


B. The Sensitivity

The sensitivity is given by the derivative of the output of the electromotive force, the coefficient, \( e_2 \), with respect to field, \( \bar{H} \).

This derivative is

\[
\frac{dC_2}{dH} = \frac{4\pi A u_0 m}{H} \left[ \left( \frac{N}{2\mu_0} \right) \left( 1 - \left( \frac{B_m}{\mu_0 H_0} \right)^2 \right)^{1/2} \left( \frac{B_m - \mu_0 H}{\mu_0 H_0} \right) \left( 1 - \frac{B_m - \mu_0 H}{\mu_0 H_0} \right)^{1/2} \right]
\]

For small values of the field, i.e., \( H \approx 0 \), the sensitivity is given by

\[
\frac{dC_2}{dH} = \frac{8\pi A u_0 m}{H} \left( \frac{B_m}{\mu_0 H_0} \right) \left( 1 - \left( \frac{B_m}{\mu_0 H_0} \right)^2 \right)^{1/2}
\]

The sensitivity is at its maximum value when \( \mu_0 H_0 = \sqrt{2} B_m \); therefore, the current in the solenoid necessary for best operation must give a field, \( \bar{H} \), which satisfies this equation. In the article, "Air-Borne Magnetometer for Search and Survey," it is reported that the measured sensitivity was experimentally checked and there was found to be a close agreement with the calculated sensitivity. The expansion of \( e_2 \) by a series gives

\[
e_2 \approx -\frac{24\pi A B_m H}{\mu_0 H_0^2} \left[ 1 - 2\left( \frac{B_m^2 + (H_0 H)^2}{(H_0 H)^2} \right) \right]
\]

For small fields of the order of \( 10^{-2} \) gauss, the coefficient, \( e_2 \), is proportional to the field, \( \bar{H} \).