The Rice Institute

Electrical Engineering 510

Thesis

THEORY OF WAVE SHAPING CIRCUITS

Submitted by
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The conventional theory of circuit analysis distinguishes between what are known as the transient and the steady state conditions of an electric circuit. In practice, however, the steady state condition has received more attention and the transient phenomena have been looked upon as somewhat of a nuisance. Arcing at switches, key clicks in transmitters, etc., are usually thought of as something to be avoided.

In recent years, however, more and more electronic circuits have utilized transient phenomena as the essence of performance. Television and radar are examples of equipments which employ such theory extensively.

These types of equipments consist largely of assemblies of simple component circuits, many of which have become standardized to a certain degree. The advantages of cataloging these standard components together with their respective nomenclature is appreciated when one realizes the neatness and functional simplicity of complex devices presented in block diagram form. For many purposes block diagrams are sufficient representation of complex devices. In cases where detailed information is to be presented, the introduction of block diagrams assist greatly in organizing and simplifying the analysis. The ultimate usefulness of block diagram representation will not be realized until complete standardization and utility is achieved. To this end, frequent use of block diagram representations wherever practicable should be encouraged.

It is observed, however, that associated with each component of a block representation is a small paragraph explaining the operation of the particular component. Thus, in general, there is one independent explanation for each standard component. Each of these independent explanations, however, seems to be built upon one underlying characteristic; namely, the transient behavior of the component. It seems expedient,
then, to accept this common factor and upon it attempt to construct a simple and rapid type of analysis which should lend itself to the explanation of most of the standard components.

In Part I of this paper such a type of analysis is derived and substantiated. In particular, the analysis is confined to the effects of typical waveforms applied to simple linear circuits. The application of this analysis is made to several standard components in Part II with the implication that most types of pulsed equipment which come in large boxes may be arranged into assemblies and sub-assemblies, each of which may be analysed by the methods outlined in Part I.

The analysis outlined in Part I is by no means new, for it employs for the most part such standard devices as Kirchhoff's Law, the mechanics of the exponential curve, the fact that energy is inherently a continuous function of time, etc. Furthermore, many of the methods are simply repetitions of the arguments commonly associated with typical components. The collection of this information and the molding of it into one body of knowledge forming the method of analysis, however, was done in the hope that it would be a contribution. The problem, then, is a study of transient behavior in standard component circuits.

It is to be noted that pure sine waves or simple combinations of pure sine waves are employed in circuits in which the steady state characteristics are of prominent importance. Correspondingly, transient considerations are required in circuits upon which are impressed odd shaped waveforms.

Transient behavior may be studied classically by the investigation of families of homogeneous differential equations. Such an investigation is accompanied by a certain amount of abstractness and tediousness. A heuristic method of solution is convenient for many circuits which are commonly used to produce certain desired transients. Such a method of
solution is often preferred by the engineer since it implies close association with the physical processes involved. Frequent reference to physical behavior throughout the analysis seems to yield successful results as may be appreciated when one considers typical arguments that usually accompany standard component circuits. In the formulation of the analysis, then, the heuristic atmosphere prevails in demonstrating the behavior of several simple linear circuits under the influence of typical waveforms designed to produce certain transient phenomena.

A second observation is apparent. The compilation of facts that form the analysis, Part I, together with the implied and stated applications of Part II may be useful as an instructional aid in elementary vacuum tube courses, since its content is closely associated with physical processes. It is a long established fact that a thorough grounding in the simple physical or tangible aspects of a study is conducive to a much more rapid transition into the complex as well as a more thorough appreciation of the complex. In Part II, vacuum tubes are used as switches, and the implied assumption is that linear amplifiers, modulators, etc. are more complex. It is felt that if a person were introduced to the study of vacuum tube circuits of the type set forth in Part II, having understood the methods of Part I, that such a person would have sufficient groundwork and physical concept to launch into the study of linear amplifiers, modulators, detectors, etc., with a sense of security and with much greater speed and comprehension.

The problem, then, is an investigation of the transient behavior of simple standard component circuits with the following objectives: (1) to compile and substantiate a type of rapid transient analysis applicable to simple standard component circuits with the implication that such an analysis may be used to investigate components or sub-assemblies constituting major units; and (2) to suggest this type of information as a groundwork upon which vacuum tube circuit theory may be built.
PART I
NOTES ON TYPICAL WAVEFORMS APPLIED TO SIMPLE LINEAR NETWORKS

The essential feature in the analysis of waveshaping circuits is a thorough understanding of the characteristics of RC and RL coupling circuits. It is proper, then, to consider them here.

THE RC CIRCUIT

Perhaps the most widely used component of all circuits is the RC circuit, consisting of a condenser in series with one or more resistors. Figure 1 (a) shows the simplest type of RC circuit. The wave forms associated with this circuit are shown in Figure 1 (b).

\[ i(t) = i_{\text{steady state}} + i_{\text{transient}} \]
\[ i_{\text{steady state}} = 0 \text{ by inspection.} \]
\[ i_{\text{transient}} = Ae^{-t/RC} \text{ by solution of } Ri + \frac{1}{C} \int idt = 0. \]
\[ i(t) = 0 + Ae^{-t/RC} \]

\[ i(t) = \left( \frac{E}{R} - \frac{Ec}{R} \right) e^{-t/RC} \text{ for } Ec(0) = E_0. \]  
(5)

\[ i(t) = \frac{E}{R} e^{-t/RC} \text{ for } Ec(0) = 0. \]  
(6)

\[ e_R(t) = i(t)R = E e^{-t/RC} \text{ for } Ec(0) = 0 \text{ as shown in Figure 2.} \]  
(7)

\[ e_c(t) = E - e_R(t) = E(1-e^{-t/RC}) \text{ for } Ec(0) = 0 \text{ as shown in Figure 2.} \]  
(8)
The quantity RC is known as the time constant, $T$, of the circuit. RC has the dimension time$^1$ and unit second or microsecond depending upon the choice of units for R and C. Most frequently encountered combinations are shown below:

<table>
<thead>
<tr>
<th>R</th>
<th>C</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>ohms</td>
<td>farads</td>
<td>seconds</td>
</tr>
<tr>
<td>megohms</td>
<td>microfarads</td>
<td>microseconds</td>
</tr>
<tr>
<td>ohms</td>
<td>microfarads</td>
<td>microseconds</td>
</tr>
<tr>
<td>megohms</td>
<td>micromicrofarads</td>
<td>microseconds</td>
</tr>
</tbody>
</table>

One time constant, $T$, then, is that value of $t$ which makes $e^{t/T} = 1$.

For the RC circuit of Figure 1 (a), $T = RC$, where $T$ is in microseconds if R is in ohms and C is in microfarads, etc.

For transient analysis it is important to understand the properties of the exponential curve. Consider Figure 1 (b). At the end of one time constant, the voltage across the resistor has decayed to approximately 37% of its initial value; at the end of two time constants, it has decayed to about 14% of its initial value, etc. Or from the other point of view, at the end of the first time constant, the voltage has changed by 63% of the difference between the initial voltage and the final voltage, etc.

To allow a more general treatment, consider any negative exponential curve $f(t/T) = e^{-t/T}$, $T$ greater than zero and finite, and $t$ greater than zero. The status of the change in $f(t/T)$ for integral values of $t/T$ is shown in Figure 2 and tabulated below:

<table>
<thead>
<tr>
<th>Number of Time Constants</th>
<th>Per Cent of (Initial Value - Final Value) which $f(t/T)$ has changed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1T</td>
<td>63.2</td>
</tr>
<tr>
<td>2T</td>
<td>86.5</td>
</tr>
<tr>
<td>3T</td>
<td>95.0</td>
</tr>
<tr>
<td>4T</td>
<td>98.2</td>
</tr>
<tr>
<td>5T</td>
<td>99.3</td>
</tr>
</tbody>
</table>

See Appendix I.
Of course \( f(t/T) \) never reaches its final value in finite time, but for most practical applications it may be assumed that \( f(t) = f(\infty) \), i.e., \( f(t/T) \) is equal to its final value after four time constants. So after four time constants the transient is expired.

It is also of interest to note that

\[
\left( \frac{d}{d(t/T)} e^{-t/T} \right)_t = -1
\]

i.e., the slope of the curve at the time \( t=0 \) is such that if maintained, it would cause the curve to complete its transition from initial value to final value in one time constant as shown by the dotted line in Figure 2.

In many circuits only a short portion of the exponential curve is used, and a convenient approximation is obtained by expanding \( f(t/T) \) in a Maclaurin series and neglecting terms in high powers of \( t \). Thus

\[
e^{-t/T} = 1 - t/T + t^2/2T^2 - t^3/6T^3 + \ldots
\]

The utility of such an approximation is, of course, governed by the number of terms than can be neglected. If all but the first two terms are neglected, an error of only 0.55\% is introduced provided \( t \) is smaller than or equal to 0.1 \( T \). With this assumption, linear variation as determined by the first derivative of \( e^{-t/T} \) at \( t = 0 \) is defined, i.e., \( f(t/T) \) is assumed to follow the dotted line in Figure 2 instead of the curve \( e^{-t/T} \).

See Appendix II
By inspection of Figure 2, the restriction that \( t \) is smaller than or equal to 0.1 \( T \) can be readily appreciated. This approximation indicates that the following relation holds:

\[
\text{Change in value} = \frac{t}{T} (\text{Final value} - \text{Initial value}) \quad \text{provided} \quad t \leq 0.1 \ T
\]  

In many cases it is useful to know how long it takes an exponential curve to go from one value to another. Consider Figure 3 and the following equations:

\[
f(t) = E e^{-t/T} \quad \text{(12)}
\]

\[
f(t_A) = E e^{-t_A/T} \quad \text{(13)}
\]

\[
f(t_B) = E e^{-t_B/T} \quad \text{(14)}
\]

To find:

\[
t = t_B - t_A
\]

\[
t_B = -T \log \left( \frac{f(t_B)}{E} \right) = -T \log f(t_B) + T \log E \quad \text{(15)}
\]

\[
t_A = -T \log \left( \frac{f(t_A)}{E} \right) = -T \log f(t_A) + T \log E \quad \text{(16)}
\]

\[
t = t_B - t_A = T \log f(t_A) - T \log f(t_B) \quad \text{(17)}
\]

\[
t = T \log \left( \frac{f(t_A)}{f(t_B)} \right) \quad \text{(18)}
\]
In order to make (18) more general, allow \( E \) in (12) to take on positive or negative values, and add the term \( g(t) \) to account for any initial conditions that might exist in the circuit. Then (12), (13), (14) become:

\[
\begin{align*}
    f &= E e^{-t/T} + g(t) \\
    f(t_A) &= E e^{-t_A/T} + g(t_A) \\
    f(t_B) &= E e^{-t_B/T} + g(t_B)
\end{align*}
\]  

(19)  

(20)  

(21)

In subtracting \( t_A \) from \( t_B \) as in (15), (16), and (17), the terms \( T \log E \) drop out and so allowing \( E \) to take on negative values offers no restriction. Equation (18) then becomes:

\[
\begin{align*}
    t &= T \log \frac{f(t_A) - g(t_A)}{f(t_B) - g(t_B)}
\end{align*}
\]  

(22)

The result, then, is perfectly general. If \( E \) is positive and \( g(t) = c \), a constant, the graph of Figure 3 holds. If \( E \) is positive and \( g(t) = c = 0 \) for all \( t \), then the general expression (22) reduces to (18). If \( E \) is negative and \( g(t) = -E \) for all \( t \), then the graph of Figure 4 holds, and it is seen that (22) applies also to a rising exponential.

Equation (22) may be converted into more compact form by using the steady state value of \( f(t) \) as a reference line from which to make measurements in place of the line \( f = 0 \) axis. The general expression (22) then becomes:

\[
\begin{align*}
    t &= T \log \frac{E_A}{E_B}
\end{align*}
\]  

(23)

where \( E_A \) is the difference between the voltage at the beginning of the time interval and the voltage which the exponential is approaching; \( E_B \) is the difference between the voltage at the end of the time interval and the voltage which the exponential is approaching.

If \( E_A \) and \( t \) are known, (23) may be written to solve for \( E_B \):

\[
\begin{align*}
    E_B &= E_A e^{-t/T}
\end{align*}
\]  

(24)

As an example of the use of (23), consider the circuit of Figure 5.
For $t$ smaller than $t_1$ the condenser is discharged and the switch is open.

At some time $t_1$ the switch is closed driving the grid to -100 volts due to the fact that the potential across the condenser cannot change instantaneously. ($\frac{1}{2}CE^2 = \text{energy}$, and energy is inherently a continuous function of time.)

If the cut off of the tube is -15 volts, find how long the tube remains cut off.

![Figure 5(a)](image)

**FIGURE 5(a)**

![Figure 5(b)](image)

**FIGURE 5(b)**

\[ T = RC = 0.001 \text{ sec} = 1000 \mu \text{sec}. \]

Applying (23):

\[ E_A = 100 \text{ v}. \]

\[ E_B = 15 \text{ v}. \]

\[ t = T \log \frac{E_A}{E_B} = 1000 \log \frac{100}{15} = 1397 \mu \text{sec}. \]

Similarly, the circuit of Figure 6(a) may be analysed to give Figure 6(b).

\[ R_1 \text{ may be considered as the internal resistance of the source}. \]

![Figure 6(a)](image)

**FIGURE 6(a)**

Given $e_c$ is zero for $t$ smaller than zero.

Switch closed at $t = 0$

![Figure 6(b)](image)

**FIGURE 6(b)**
Since current must flow into the condenser a finite amount of time before the voltage across it can change, \( e_c \) will be zero just after the switch is closed. This fact aids a great deal in the heuristic analysis of a circuit, for hence the condenser may be considered a short circuit just after the switch is closed. A second thought on this characteristic of a condenser might be mentioned: the Fourier expansion of the current at the instant of closing the switch contains only terms of infinitely large frequencies. The condenser offers no impedance at these frequencies and hence no potential difference is developed. At \( t = 0 \), then, the circuit may be considered as simply two resistors in series with appropriate potentials developed across each as shown in Figure 6(b). The values of \( e_c \) and \( e_R \) vary as shown, and the voltage \( e_{(R+C)} = E - e_R \) has all the properties previously discussed.

These facts can be generalized for RC circuits containing one condenser to assist in rapid heuristic analysis. When any discontinuous change takes place in an RC circuit containing but one condenser, the voltage between any two points in the circuit may be plotted as follows: determine by inspection the voltage between the points in question just after the change takes place and the voltage between the points after steady state conditions have been reached; connect these points with an exponential curve having a time constant equal to the product of the capacitance of the condenser and the resistance which would be measured by placing an ohmmeter across the terminals of the condenser with all sources of emf short circuited (Thevenin generator).

In applying the above procedure, recall that the voltage across a condenser cannot change discontinuously, provided current is limited to finite values. Therefore, to determine the conditions in the circuit
just after the change takes place, the condenser may be replaced by a short circuit or by a battery having an emf equal to the voltage which was across the condenser just before the change takes place. Also recall that in the d.c. steady state condition, condensers are open circuits.

Such observations as are outlines above clearly distinguish this type of transient analysis from anything encountered in conventional alternating current theory.

The effect resulting from any discontinuous change in driving potential, then, may be determined readily by (1) observing the voltage across the condenser before the change takes place, (2) visualizing an equivalent circuit for the condition just after the change takes place, (3) visualizing an equivalent circuit for the steady state condition, and (4) calculating the effective time constant for the circuit.

An example employing this procedure follows. Consider the circuit of Figure 7(a). Given that the switch has been closed a long time. Find the potential from point A to ground after the switch is opened. (1) By inspection, the voltage at point A and the voltage across the condenser are both 50 volts prior to the opening of the switch. (2) Figure 7(b) is the equivalent circuit just after the switch is opened. Note that the condenser has been replaced by a battery of 50 volts potential with polarity similar to that of the condenser prior to the change. The potential of point A, then, jumps discontinuously from +50 volts to +90 volts at the instant of the change. (3) Figure 7(c) is the equivalent steady state circuit. For this condition the potential of point A is obviously +250 volts. (4) The transition from +90 volts to +250 volts is exponential, and the time constant for this curve is clearly $(10,000 + 40,000) \text{ohms} \times 0.01 \text{ufds} = 500 \text{u secs}$. The behavior of the potential of point A is then completely determined and the plot of voltage of point A vs time may be made. See Figure 7(d).
FIGURE 7
Assume, now, that the switch is closed again. Observing the four steps used above: (1) The charge on the condenser is 250 volts. (2) The two branches not containing the condenser may be replaced by a Thevenin generator. The resulting equivalent circuit for the instant of closing of the switch, then, is as shown in Figure 7 (e). The voltage A evidently jumps discontinuously from +250 volts to +139 volts. (3) Under steady state conditions, the voltage at A is again +50 volts. (4) The potential drops exponentially from +139 to +50 volts. The effective resistance in series with the condenser is 18,000 ohms, and so the time constant of the exponential variation is 180 usec. The curve is shown in Figure 7 (d).

Frequently a transient is interrupted by another sudden change in the circuit before equilibrium is reached. In this case, it is necessary to calculate the voltage in the circuit just before the second change takes place. Suppose, for example, that the switch in the circuit just considered is closed again 40 usec. after it is opened. The comments made in connection with Figures 7 (a), (b), and (c) obtain; in particular, when the switch is opened, the voltage at point A jumps discontinuously to +90 volts, and then it begins its exponential rise to +250 volts at a rate determined by $T_1 = 500 \text{ usec}$. With the present change in data, it is necessary to calculate the voltage at A 40 usec. after the beginning of this rise. Since 40 usec. is less than 0.1 $T_1$, equation (11) holds, and an error of less than one half of one per cent is introduced.  

Change in voltage at $A = \frac{40}{500} (250 - 90) = 13 \text{ volts}$.

The potential of point A, then, rises practically linearly from +90 volts to $90 + 13 = 103 \text{ volts}$.

So the condition of the circuit just before the second change takes place is such that the voltage at point A is 103 volts. Before visualizing the circuit just after the second change is made, it is required to find

$^3$ See Appendix II
the voltage on the condenser at the time of the change. Applying equation (11) again:

\[
\text{Change in condenser voltage} = \frac{40}{500} (250 - 50) = 16 \text{ volts.}
\]

At the instant the switch is closed, then, the condenser voltage is \(50 + 16 = 66\) volts, and in the corresponding equivalent circuit a battery of 66 volts may be used to replace the condenser. See Figure 7 (f). From this equivalent circuit it is seen that the potential at A drops discontinuously from +103 to +57 volts at the time the switch is closed. Continuing, it is clear that the voltage at A proceeds from +57 volts to +50 volts along an exponential curve of time constant 180 usec.

\[\text{FIGURE 7 (f)}\]

\[\text{FIGURE 7 (g)}\]

The waveform resulting from the above problem is shown in Figure 7 (g). Such a trapezoidal waveform of voltage is required in order to produce a linear sweep in magnetic deflection cathode ray tube circuits.

It is often required to design a circuit that will give linear change in voltage with time throughout some time interval \(dt\). This requirement can always be fulfilled with excellent practical accuracy by the use of an RC circuit as outlined in the foregoing pages. While it is true that the procedure outlined here admits but one condenser in the circuit, it is true that additional condensers of capacities much larger than the capacity of the condenser for which the transient behavior is calculated, may be replaced by batteries for the purpose of design since their time constants are much larger than the time constant which is principle to the transient desired.
In practical wave shaping circuits the above always is true; namely, that additional capacitors are made so large that their transient effects may be neglected. Hence the foregoing theory is extended to the general case, provided, of course, that the restrictions set forth concerning length of time interval and size of capacitors are observed.
**THE RL CIRCUIT**

The simple RL circuit may be treated in a manner similar to that of the RC circuit.

\[
i(t) = \frac{E}{R} \left(1 - e^{-\frac{Rt}{L}}\right) + i_0 e^{-\frac{Rt}{L}}
\]

for \(i(0) = i_0\) \hspace{1cm} (25)

\[
i(t) = \frac{E}{R} \left(1 - e^{-\frac{Rt}{L}}\right)
\]

for \(i(0) = 0\) \hspace{1cm} (26)

\[
e_R(t) = i(t)R = E(1 - e^{-\frac{Rt}{L}})
\]

for \(i(0) = 0\) \hspace{1cm} (27)

\[
e_L(t) = E - e_R(t) = E e^{-\frac{Rt}{L}}
\]

for \(i(0) = 0\) \hspace{1cm} (28)

Here the quantity \(L/R\) is the time constant and has the dimension time \(^4\) and the unit second or microsecond depending upon the choice of units for \(L\) and \(R\). Most frequently encountered combinations are shown below.

<table>
<thead>
<tr>
<th>(L)</th>
<th>(R)</th>
<th>(T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>henrys</td>
<td>ohms</td>
<td>seconds</td>
</tr>
<tr>
<td>millihenrys</td>
<td>kilohms</td>
<td>microseconds</td>
</tr>
<tr>
<td>microhenrys</td>
<td>ohms</td>
<td>microseconds</td>
</tr>
</tbody>
</table>

Note that in this simple series circuit, the voltage across the resistor is the same as the voltage across the condenser of the RC circuit, and the voltage across the inductor is the same as the voltage across the resistor of the RC circuit. Just as the voltage cannot change discontinuously across a condenser, so the current cannot change instantaneously in an inductor.

\(\text{(Li}^2/2 = \text{energy, a continuous function of time.)}\)

\(^4\) See Appendix I
The same comments concerning exponential curves, then, apply to the case of the RL circuit. In particular Figure 2 and equations (9), (11), (23), and (24) hold. Similarly a procedure for rapid analysis of the response of an RL circuit to discontinuous changes in voltage may be made: (1) Determine the current in the inductor just before the change takes place. (2) Visualize an equivalent circuit for the condition just after the change takes place remembering that the current in the inductor cannot change discontinuously. (3) Visualize an equivalent circuit for the steady state condition. (4) Calculate the effective time constant of the circuit.

One example should suffice. The switch in the circuit of Figure 9 (a) is initially open so that the current in the inductor is zero. When the switch is closed the current in the inductor remains zero (open circuit) for the first small increment of time, yielding the equivalent circuit of Figure 9 (b). For the steady state condition, the inductor may be replaced by a short circuit.

![Figure 9](a)

![Figure 9](b)

![Figure 9](c)

![Figure 9](d)
The voltage at point A is initially at +300 volts, drops instantaneously to +200 volts and then rises exponentially to its initial value of +300 volts. The time constant of the curve is evidently the inductance divided by the parallel combination of 5K with 10K: \( T_1 = \frac{50 \text{ mH}}{3.3 \text{ k}} = 15 \text{ usec} \). After about 4\( T_1 \) the steady state condition prevails and a current of 30 ma flows in the inductor.

If, now, the switch is opened, the 30 ma continues to flow in the inductor for an instant and the equivalent circuit of Figure 9 (c) obtains. A current flow of 30 ma in the inductor in the direction shown implies a voltage drop of 150 volts across the 5K resistor with the end more remote from the battery positive with respect to the end nearer the battery. Thus, when the switch is opened, the voltage at point A jumps instantly from +300 volts to \( 300 + 150 = 450 \) volts, and then decays exponentially back to 300 volts with a time constant of \( L/R = 50 \text{ mH/5K} = 10 \) usec. The behavior of the voltage at point A is shown in Figure 9 (d).
COUPLING CIRCUITS

Among the many commonly used types of coupling circuits, the circuits of Figure 10 are often employed. A voltage waveform is applied to the two elements in series and the voltage output is taken across one of the elements. It is convenient to classify RC and RL circuits with respect to the length of their time constants as compared with the period of the applied voltage. The following classifications are to be considered: circuits in which the time constant is short, comparable, and long with respect to the period of the applied wave.

Coupling Circuits with Short Time Constants

When the time constant of a RC or RL coupling circuit is short compared with the period of the applied wave, so that each transient response may be completed (4T) before the succeeding change is applied, the output taken across an appropriate element very nearly approximates the first time derivative of the input waveform. The term differentiating circuit is often applied to such RC and RL circuits.

In the above, the output is taken across the resistor in the RC circuit and across the inductor in the RL circuit. Recall from equations (7) and (28) that these voltages are identical if \( L/R = RC \). Observe, also, that the behavior of the output voltages of these circuits is the same as that discussed in connection with Figure 9. When a discontinuous, periodic waveform is applied to a differentiating circuit, then, the output consists of a synchronized sequence of discontinuities, each followed by an exponential
decay back to zero. The amplitude of the discontinuity in the output is equal to the amount by which the input changes, positive if the change is upward and negative if the change is downward.

Figure 11 shows output waveforms for typical input variations to either of the two circuits of Figure 10.

![Input and Output Waveforms](image)

**FIGURE 11**

Frequently limitations in the input circuit make it impossible to produce absolutely instantaneous changes in the waveform to be applied to the differentiating circuit. In practice, with proper choice of time constant in the driving circuit, it is possible to produce a waveform that is linear with a very steep slope even though the slope may not be infinite as desired. It is of interest, then, to investigate the response of the differentiating circuit when the input voltage variation is of the form \( e_o + kt \).

Consider the circuit of Figure 10 (a). Given that \( e_{input} = e_o + kt \). Determine \( e_{output} = e_R(t) \):

**Homogeneous Solution:** \( iR + \frac{1}{C} \int i dt = 0 \). So \( i = Ae^{-t/RC} \) by inspection. (29)

**Particular Solution:** \( iR + \frac{1}{C} \int i dt = e_o + kt \). (30)

or \( \frac{di}{dt} + \frac{i}{RC} = k/R \) So \( i = kC \) by inspection. (31)
Total Solution: \[ i = A e^{-t/RC} + kC \] (32)

\[ e_{out}(t) = e_R(t) = i(t)R = A e^{-t/RC} + kRC \] (33)

If at \( t = 0 \), \( e_R(t) = e_R(0) \), then \( AR = e_R(0) - kRC \).

Substituting:

\[ e_R(t) = \left( e_R(0) - kRC \right) e^{-t/RC} + kRC \] (34)

Likewise, the RL circuit of Figure 10 (b) may be solved.

Homogeneous Solution: \[ iR + L \frac{di}{dt} = 0. \] So \( i = A e^{-Rt/L} \), (35)

by inspection.

Particular Solution: \[ iR + L \frac{di}{dt} = e_0 + kt. \] So \( i = \frac{e_0}{R} - \frac{kL}{R^2} + \frac{k}{R} t \), (36)

by assuming solution of form \( i = a + bt \).

Total Solution: \[ i = \frac{e_0}{R} - \frac{kL}{R^2} + \frac{k}{R} t + A e^{-Rt/L} \] (37)

\[ e_{out}(t) = e_L(t) = e_0 + kt - i(t)R \]

\[ = e_0 + kt - e_0 - kt + \frac{kL}{R} = A e^{-Rt/L} \]

\[ = \frac{kL}{R} - A e^{-Rt/L} \] (38)

\[ -AR = e_L(0) - \frac{kL}{R} \] (39)

Substituting:

\[ e_L(t) = \frac{kL}{R} + \left( e_L(0) - \frac{kL}{R} \right) e^{-Rt/L} \] (40)

Both equations (34) and (40) have the equivalent

\[ e_{out} = \left( e_{out} \right)_0 - kT \right) e^{-t/T} + kT. \] (41)

The result of this expression may be illustrated as shown in Figure 12.

![Figure 12 (a)](image1)

![Figure 12 (b)](image2)
If \( t \) is less than or equal to 0.1T, then \( e^{-t/T} = 1 - t/T \), practically.

Furthermore, if \( (e_{\text{out}})_0 = 0 \), then equation (41) takes the form

\[
e_{\text{out}} = (0 - kT) (1 - t/T) + kT
= k t
\]

But (42) is precisely the variable term of the equation for the applied wave.

So if the initial output voltage is zero, for a short period of time the output voltage rises linearly with the applied voltage. At the end of about 0.1T, for example, the output voltage is nearly equal to the change in input voltage. Thus, if the time in which the voltage change takes place is very small (if \( kt \) has a very steep slope) compared with the time constant of the circuit, then the steeply sloping waveform may be considered as a discontinuous jump, as assumed previously, without introducing any practical error. This approximation to a discontinuous jump may easily be achieved in practice; i.e., it is possible to make the applied voltage change so quickly that the time constant of the differentiating circuit is long compared to the time of change but short compared with the period of the applied wave.

The criterion for the usefulness of this approximation may be stated more explicitly. Suppose a square wave is applied to a differentiating circuit. Then certainly the output waveform will be a sequence of jumps followed by exponentials as discussed previously. Now suppose that the sides of the square wave are not truly vertical; i.e., that \( \frac{d}{dt} (e_{\text{in}}) \neq \infty \), but rather equal to some finite value \( k \). Then this distorted square wave may be replaced by a theoretical square wave as stated above if, and only if, the magnitude of \( k \) is such that \( e_{\text{in}} \) reaches its final value (for a square wave, the horizontal portion) within some time \( t \) less than or equal to 0.1T. Both the slope and the magnitude of the change in input voltage, then, determine whether the wave may be considered as discontinuous or not.
In the event that the foregoing approximation does not apply, if the time of change is not short compared with the time constant, it is necessary to calculate the output voltage with the aid of equation (41).

When \( k \) is finite, the magnitude of the discontinuity in \( e_{\text{out}} \) is equal to the change in \( e_{\text{in}} \), as stated before. From (41) it can be seen that as \( k \) becomes smaller, the peak magnitude of \( e_{\text{out}} \) also becomes smaller, \( T \) fixed. Likewise, for a fixed \( k \), finite, the output peak amplitude becomes smaller as \( T \) is decreased. So for a given input waveform, reducing the time constant allows the circuit to be restored to its original condition more quickly, so that the cyclic frequency of the input wave may be increased. But the advantages gained by reducing the time constant are obtained at the expense of reduced amplitude. Figures 13 (a) and (b) illustrate the effect of varying \( k \) and \( T \). In all cases, of course, \( T \) is small compared with the period of the applied voltage. (\( T \) less than one-fourth the time between successive changes in input.)

To sketch the waveform \( e_{\text{out}} \) for a sloping input wave such that the time of change is greater than 0.1\( T \), equation (41) is used in which the factor \( e^{-t/T} \) is not replaced by \( (1 - t/T) \). From (41) it is seen that whenever an instantaneous change takes place in the voltage applied to a differentiating circuit, the same change appears in the output voltage, independent of the output voltage prior to the change. When a slow, linear change takes place in the applied voltage, the output voltage goes exponentially from its initial value to the value \( kT \) in about four time constants. It then retains this final value, \( kT \), as long as the input voltage continues to change at the same rate, \( k \). These facts were displayed in Figure 12.

Application of equation (41) to periodic waveforms result in curves as shown in Figures 13 (c) and (d). Graphs of \( \frac{d}{dt} (e_{\text{in}}) \) are given for comparison.
FIGURE 13
The action of the differentiating circuit on a sine wave is shown. To say that the time constant is small in the case of a sinusoidal input is equivalent to saying that the capacitive reactance is large compared with the resistance in the RC circuit, or that the resistance is large compared with the inductive reactance in the RL circuit. In either case, the output voltage is sinusoidal, of reduced amplitude and nearly $90^\circ$ out of phase with the input.

Coupling Circuits with Time Constants Comparable with Period of Applied Wave

Consider the circuit of Figure 14.

Assume the condenser to be discharged initially. On the first half cycle, the condenser charges to 63% of the applied voltage, 63 volts, since the time taken to traverse the first half cycle is equal to the time constant of the circuit. On the second half cycle, the voltage drops to 37% of 63 volts, 23 volts. At the completion of the third half cycle, the voltage across the condenser is 63% of $(100 - 23)$ volts + the initial 23 volts = 71
volts. After the fourth half cycle, 37% of 71 volts = 26 volts. Next, 
63%(100 - 26) + 26 = 72 volts, etc., until a steady condition is reached 
as shown in Figure 14 (a).

Because of the similarity of the output waveform of this circuit to 
the integral of the input, such circuits are often referred to as integrating 
circuits. Note that the average value of the output voltage is equal to 
the average value of the applied wave. Obviously this is always true for 
such a circuit.

The integrating circuit differs from the differentiating circuit in 
that the output of the former is taken across the condenser and the time 
constant of the circuit is of the same order of magnitude as the period of 
the applied wave.

Similarly, in an RL circuit of time constant comparable to one period 
of the input wave, the integrating effect is obtained by taking the output 
across the resistive element.

**Coupling Circuits with Long Time Constants**

Perhaps the most common of all RC circuits is the long time constant 
coupling circuit used in resistance coupled amplifiers. This type of 
circuit is shown in Figure 15. Analogous RL coupling circuits exist, but 
are not as popular as coupling links. For their analysis, the time constant, 
RC in the following discussion, should be replaced by $\frac{L}{R}$, and the output 
taken across the inductor.

![Input and Output Diagrams](image)
The output waveform of the long time constant circuit is seen to be a rather faithful reproduction of the input wave except that the average value is zero. For this reason, the condenser in such a circuit is sometimes called a blocking condenser since it blocks out the d.c. component of the input wave.

![Waveform Diagram](image)

FIGURE 16

A cycle by cycle analysis of the output voltage of Figure 16 reveals that after an initial transient during which the condenser accumulates the average value of the applied voltage, a stable condition is reached as shown in Figure 16 (c). It is clear that as the time constant is increased, the slope on the plateaus of the output wave becomes smaller and smaller, yielding a more perfect reproduction of the input wave. For example, if the time constant of the above circuit is increased by a factor of ten, the error in the horizontal portion of the wave is changed from \((51.25 - 48.75)\) volts to \((50.1 - 49.9)\) volts. In any event, the average value of the output voltage is zero.

A little thought relative to the practical uses of long time constant coupling circuits to pass discontinuous waveforms immediately brings to mind the possibility of variation in the driving source impedance throughout the cycle. Such a condition would add complications to the problem. If, for example, the input waveform to a long time constant coupling circuit, were caused by a biased diode, the time constant of the coupling circuit would change alternately from one half cycle to the next. Such conditions frequently occur in practice; namely, that the effective time constant of a coupling circuit for each first half cycle is different from
that for each second half cycle. Fortunately, such changes usually effect only the resistive component of the impedance and are likewise discontinuous with the voltage. (In fact, the discontinuities in impedance produce the discontinuities in voltage.)

Consider Figure 17. The switches are connected mechanically so that the time constant of the circuit is $R_1C$ when the input voltage is $E_1$, and $R_2C$ when the input voltage is $E_2$. $E_1$ or $E_2$, of course, may be considered positive, negative, or zero, and likewise, $R_1$ or $R_2$ may be zero if generalization is desired.

![Figure 17](image)

If either, or both, $R_1C$ or $R_2C$ is long compared with the switching interval, the average condenser voltage will reach some constant steady state value $E_c$ between $E_1$ and $E_2$. Furthermore, this steady state value will be reached when the change in charge on the condenser while the input voltage is $E_1$ is exactly equal to the change in charge on the condenser while the input voltage is $E_2$.

In practice, while one of the time constants may be influenced by the driving source, the other is purely arbitrary. The parameters for the above conditions, then, may nearly always be found.

The above conditions, which will now be assumed, may be stated as follows:

$$\text{average steady state condenser voltage} = E_c = \text{a constant}$$

and

$$\Delta q_1 = \Delta q_2$$ (43)
The steady state condition may be calculated immediately. Figure 17(b) is the equivalent circuit of Figure 17a when $E_1$ and $R_1$ are in the circuit, and Figure 17(c) for the condition that $E_2$ and $R_2$ are in the circuit. In each case the condenser is replaced by an equivalent constant potential $E_c$.

Assume that $E_1$ exceeds $E_2$, with the result that $E_c$ is less than $E_1$ but greater than $E_2$. If the time duration of $E_1$ as input voltage in a given cycle is denoted by $P_1$, etc, then $\delta q_1 = i_1 P_1$, and similarly for $\delta q_2$. Thus:

$$\delta q_1 = (E_1 - E_c)\frac{P_1}{R_1}$$

(44)

and

$$\delta q_2 = -(E_2 - E_c)\frac{P_2}{R_2}$$

(45)

Equating (44) and (45) as per (43)

$$(E_1 - E_c)\frac{P_1}{R_1} = -(E_2 - E_c)\frac{P_2}{R_2}$$

(46)

So

$$E_c = \frac{E_1 P_1 R_2 - E_2 P_2 R_1}{P_1 R_2 + P_2 R_1}$$

(47)

But in equation (46), $E_1 - E_c = E_{out1}$, the output voltage across the resistor when $R_1$ is in the circuit. Similarly $E_2 - E_c = E_{out2}$, the output under the alternate condition.

$$E_1 - E_c = E_{out1}$$

(48)

$$E_2 - E_c = E_{out2}$$

(49)

Making these substitutions in (46) and dividing thru by $C$, the following form is obtained:

$$E_{out1}\frac{P_1}{T_1} = - E_{out2}\frac{P_2}{T_2}$$

(50)

where $T_1$ and $T_2$ are $R_1C$ and $R_2C$, respectively. Recall that (50) holds only provided at least one $T$ is much greater than at least one $P$. Equation (50) is of the form:

$$\int_{0}^{P_1} E_{out1} \frac{1}{T_1} dt = \int_{0}^{P_2} E_{out2} \frac{1}{T_2} dt$$

(51)
It has been shown that for $T_1$, $T_2$ large enough, $E_{\text{out}1}$, $E_{\text{out}2}$ are practically independent of time. But (51) suggests graphical consideration of areas:

![Diagram showing areas $E_{\text{out}1}$ and $E_{\text{out}2}$ with shading to represent areas $A$ and $A_2$.]

So

$$A_1/T_1 = A_2/T_2 \quad \text{or} \quad A_1/A_2 = T_1/T_2 = R_1/R_2$$

(52)

So when the time constant does not change, the average value of output voltage is necessarily zero. When the time constant changes throughout a cycle, the average output voltage is in general not zero, but favors the polarity that provides the greatest product $(E - E_0)P$. 
PART II

APPLICATION OF THE METHODS OF PART I TO STANDARD WAVE SHAPING CIRCUITS

Recent developments in electronic equipment have made it necessary to employ many special types of non-sinusoidal voltage generators. In Part II several standard wave shaping circuits and generators of discontinuous waveforms are discussed. The analysis of these circuits differs from that of ordinary vacuum tubes in amplifiers and oscillators in two respects. First, since the wave forms are not sinusoidal, the concepts of reactance and impedance are meaningless and hence do not aid in the analysis. The methods of Part I are used instead. Second, grid variations in vacuum tube circuits are purposely made so great in most cases that linear amplification is not attained. Vacuum tubes are used merely as switches.

CLAMPING CIRCUITS

Consider the circuit of Figure 19 with the input voltage as shown. Assume that the resistance of a diode when conducting is 1000 ohms. (The average value of the conducting resistance of a 6H6 is about 1000 ohms.)

![Diagram](image_url)
The solution for the steady state condition for this circuit is shown in Figure 19 (e). It is observed that the circuit has two time constants. One with the diode conducting, one with the diode not conducting. Since one of the time constants is long (50,000 usec.) in comparison with the period of the applied wave (300 usec.), the procedure associated with equations (43) to (52) of Part I is justified.

By (47)
\[
E_0 = \frac{E_1P_1R_2 + E_2P_2R_1}{P_1R_2 + P_2R_1} = \frac{200 \times 100 \times 10^6 + 0 \times 200 \times 10^3}{100 \times 10^6 + 200 \times 10^3} = \frac{20,000 \times 10^6}{100.2 \times 10^6} = 200 \text{ volts, very nearly.}
\]

If an equivalent circuit were desired, the condenser could be replaced by a battery of 200 volts throughout the whole cycle.

By (48)
\[
E_{out_1} = E_1 - E_0 = 200 - 200 = 0 \text{ volts.}
\]

By (49)
\[
E_{out_2} = E_2 - E_0 = 0 - 200 = -200 \text{ volts.}
\]

These results are graphed in Figure 19 (c). The wave is clamped so that the positive excursions are always zero. Notice that the fact that \( E_0 \) is not exactly 200 volts and the fact that the plateaus of the output waveform are not perfectly horizontal are the two features that enable the circuit to function as it does. For practical results, of course, these two errors are so slight that they may be neglected.

If the diode of Figure 19 (b) were reversed, \( R_1 \) and \( R_2 \) would exchange values in the above equation for \( E_0 \), resulting in a new value for \( E_0 \) which would be very nearly zero (actually 0.1 volts). The new values of \( E_{out_1} \) and \( E_{out_2} \) would be very nearly 200 - 0 = 200 volts and 0 - 0 = 0 volts, respectively. The wave would be clamped so that the negative excursions would always be zero.
Further generalization can be made. If the parallel combination of the diode and the resistance, Figure 19 (b), were maintained at a constant potential $E$ above ground, then the equation (48) would become

$$E_{out1} = E_1 - E_0 + E = 200 - 200 + E = E$$

and (49) would be

$$E_{out2} = E_2 - E_0 + E = 0 - 200 + E = E - 200$$

That is, the output wave would be clamped in such a manner so that the positive excursions would always be $E$. Similarly, if the diode were reversed under these conditions, the output wave would be clamped such that the negative excursions would always be $E$ volts above ground. These statements are valid, of course, for any $E$, positive, negative, or zero.

The circuit of Figure 19 (b), then, is such a circuit in which the constant potential $E$ is zero. Figure 20 illustrates the content of the above argument for a square wave input. In each case the product $RC$ is made very much greater than the period of the applied wave to guarantee that at least one time constant is large with respect to the switching interval, justifying the use of the above method.

A circuit of this nature that fixes one extreme of the output voltage is variously called a clamping circuit, d.c. restorer circuit, d.c. reinsertion circuit, and perhaps others.

A clamping circuit, then, consists of a long time constant $RC$ coupling circuit with some device to vary the time constant of the circuit at predetermined intervals. Practically, this device is something that amounts to a diode switch as indicated in Figure 20. Furthermore, it is clear that if the cathode voltage of the diode is fixed, the positive extremes of the output wave are clamped at the cathode voltage. If the plate voltage of the diode is fixed, the negative extremes of the output wave are clamped at the plate voltage. An analogous solution holds for the long time constant RL circuit simply by taking the output across the inductor.
Figure 20
The usefulness of a clamping circuit is realized in circuits where one of the peak excursions (positive or negative) of the input wave to a device is required to be equal to or not greater than some prescribed value. Consider the circuit of Figure 21 in which the first two elements of a triode are used as a diode. If the product RC is made ten times, or preferably more, larger than the period of the applied wave, then clamping action will occur in the grid-cathode circuit such that the grid is never driven positive with respect to the cathode as shown in Figure 21 (c). This is, of course, nothing more than the conventional grid leak bias.

![Diagram of a diode and RC coupling circuit with input and grid voltage](image)

**FIGURE 21**

It will be recalled that if any periodic waveform is applied to a long time constant RC coupling circuit in which the time constant remains fixed throughout the cycle, the output waveform is nearly identical with that of the input, but the reference axis is changed such that the average value of the output voltage is zero. Now if the input waveform is changed slightly, the peak values of the output voltage will seek new levels so that the average output voltage remains zero. Such fluctuations in peak output voltage are often undesirable. The use of clamping networks in such circuits is indicated.
In order to emphasize the restrictions placed on a clamping circuit by assuming at least one time constant much longer than a period of the applied wave, and to illustrate the foregoing remarks, consider the proposed horizontal positioning control circuit for an electrostatic deflection cathode ray tube as shown in Figure 22 (a).

Assume that the sweep generator is driven so that at the end of each sweep, the spot remains at the right side of the screen until the next driving pulse is received by the sweep generator. Assume, further, that the output of the sweep generator is independent of sweep frequency or sweep duration, so that the voltage $A$ is a constant. Then $e_{\text{max}} - e_{\text{min}} = A$, a constant, also.

With no clamping action, diode not in the circuit, a given sweep frequency and sweep duration would yield an output averaging about the
zero axis, and the initial and terminal points of the travel of the spot on the screen would be determined by $e_{\text{min}}$ and $e_{\text{max}}$. For the input of Figure 22 (b), the output of (e) would appear, and the sweep would be fully contained on the screen, say. If, now, with the diode still out of the circuit, the sweep frequency would be decreased to some value associated with Figure 22 (d), the output would become as in (e), moving both the initial and terminal points of the travel of the spot on the screen to the left, causing the sweep to be off center. This condition would be undesirable.

By inserting the diode, the value $e_{\text{max}}$ is clamped at the voltage $E$ which may be varied by the horizontal positioning control. By clamping $e_{\text{max}}$ at $E$ by methods discussed previously, the value $e_{\text{min}}$ is fixed at the voltage $E - A$, so that the initial and terminal spots on the screen associated with these voltages are likewise fixed and independent of sweep frequency and sweep duration. Thus having centered the picture for one value of sweep frequency, it remains centered for all values of frequency and sweep duration.

One important restriction has been overlooked. All of the above is true provided at least one of the time constants of the circuit is long compared to the period of the applied wave. Of the two time constants, the longest is $RC$. As the sweep frequency approaches zero, the period of the wave approaches infinity, and, of course, the product $RC$ cannot be greater than this period for all values of frequency. As the sweep frequency is decreased, the centering adjustments outlined above become less and less effective until some value is reached when they are completely negligible. For this reason, laboratory oscillographs whose sweep frequencies go as low as one or two cycles per second do not employ the above type of centering circuit.
CLIPPING CIRCUITS

The purpose of a clipping circuit is to change a voltage waveform by removing all parts of the wave above or below some fixed value. The use of diodes in clipping circuits is common. Consider the circuits of Figure 23.

In all cases the resistor is very large compared with the conducting resistance of the diode. A sufficiently accurate analysis can then be made by assuming that the diode is practically a short circuit when conducting and an open circuit when not conducting.

In the first circuit of Figure 23, the diode conducts whenever the input voltage is positive. The output voltage is then practically zero. Thus, the positive portions of the wave are clipped. When the input voltage is negative, the diode is an open circuit and the output is identical with the input. The second circuit is simply a half wave rectifier clipping the negative peaks since the diode does not conduct when the input voltage is negative. The third and fourth circuits are like the first and second, respectively, except that the diodes are reversed.

That value of voltage which the input must exceed in order that clipping action is demonstrated may be made arbitrary by placing a fixed bias of desired value on the diode. Such circuits are shown in Figure 24.

Clipping may also be accomplished by the use of triodes. A simple method is to bias the triode so that plate current flows only during a portion of the cycle. If the peak-to-peak voltage of the input wave is greater than the distance from zero to cut-off, then a long time constant RC circuit may be used to couple the input to the tube; that is, grid leak biasing may be used to clamp the positive excursions of the input at zero. During the interval of each cycle while the input voltage is below cut-off, the tube does not conduct and hence a clipping action is effected. These facts are illustrated in Figure 25.
A second way in which triodes may be used as clippers is shown in Figure 26. The resistor $R$ is large compared with the effective grid-cathode resistance of the tube. (An average value of effective grid-cathode resistance with grid current flowing for a random choice of receiving tubes is about 1000 ohms.) So when the input voltage is positive, the grid conducts and effectively shorts out the input voltage in the same manner as was observed in the discussion of diodes. The grid voltage, then, can never go appreciably positive, and therefore, the positive half cycles of the input wave are clipped. This phenomenon is reflected in the output waveform. And, incidentally, any portion of the input wave which extends below the cut-off value will also be clipped. These results are sketched in Figure 26 (b).

It is clear that a circuit similar to the circuit at the bottom of Figure 24 may be used to make square waves out of sine waves by repeated applications of clipping and amplifying. In the circuit of Figure 26, the clipping action is effected in the grid-cathode circuit and the amplification is produced in the grid-plate circuit, evidently more economical than using diode tubes for clipping and conventional amplifiers for amplifying. The output of Figure 26, however, is not symmetrical since the entire upper half of the sine wave is removed and only a part of the lower half is clipped. This difficulty can be overcome by biasing the circuit so that the axis of the input sine wave is half way between zero and cut-off. In this case, equal parts of the negative and positive half cycles are removed.

If the circuit of Figure 26 were biased as stated above, the output waveform would be similar to a square wave except that the leading and trailing edges of the wave would be amplified segments from the center portion of a sine wave. By connecting this output to a second such triode circuit, the resulting waveform would be more nearly square, and each successive stage added would improve the character of the wave.
The input waveform of Figure 27 is biased so that when applied to the circuit of Figure 26, symmetrical clipping is effected. For convenience, the grid volts = 0 level is taken as the reference voltage level. With this notation, the horizontal axis of symmetry of the sine wave becomes the -E level, and the cut-off line becomes the -2E level. The peak amplitude of the applied sine wave can always be expressed as a constant times the voltage E. Thus, the peak amplitude is BE, where B is a constant and E is one-half the voltage between cut-off and the point where grid current begins to flow. Evidently B must be greater than unity if clipping action is assumed.

The effective grid input waveform for one stage, then, consists of a sinusoidal rise from -2E to 0, a plateau at 0, a sinusoidal decrease from 0 to -2E, etc. When amplified in the grid-plate circuit, the output is of a form indicated in solid black in Figure 28. This waveform, however, is simply a portion of a mythical sine wave of amplitude ABE, where A is the
gain of the stage assumed linear in the region $-2E$ to 0. The rise time, $dt$, is of some importance since by it the degree of perfection of the squareness of the waveform is indicated. For a perfect square wave, the rise time is zero; that is, the slope of the leading and trailing edges is infinite.

The rise time of the waveform at the output of the first stage is $dt_1$ as indicated under Figure 27. If, now, the output of the first stage, Figure 28, were applied to a second stage identical with the first, the resulting rise time would become $dt_2$ as indicated under Figure 28.

\[ \text{FIGURE 28} \]

\[
\begin{align*}
\text{ABE} \sin \omega t_2 - E &= -2E \quad (58) \\
\text{ABE} \sin \omega t'_2 - E &= 0 \quad (60) \\
dt_2 &= t'_2 - t_2 = \frac{2}{\omega} \sin^{-1} \frac{1}{AB} \quad (62)
\end{align*}
\]

For $n$ successive stages of clipping and amplifying in circuits identical with the above, the resulting rise time, $dt_n$, is given by

\[ dt_n = \frac{2}{\omega} \sin^{-1} \frac{1}{AnB} \quad (63) \]

provided stray and interelectrode capacitances may be neglected. Evidently the peak voltage of the initial sine wave input, the gain of each stage,
and the number of stages affect the rise time for a given sine wave frequency. For a number of stages in cascade, the ultimate limit in decreasing the rise time is set by the rapidity with which the voltage on the interelectrode capacities can be changed. Perfectly square waves, then, actually can never be obtained since the interelectrode capacities are never zero. The rise time may be reduced to a very small value though by the proper choice of tube and with a neat circuit arrangement.

The rise time, $dt$, of an approximate square wave is quite important in the following two respects. (1) If the rise time of an approximated square wave to be applied to a given circuit is such that $dt$ is less than about $0.1T$, where $T$ is the time constant of the circuit, then for that particular application, the approximated square wave may be regarded as perfectly square. See equation (42), Part I. (2) The rise time of a wave to be amplified has a direct bearing on the required frequency range of the amplifier. The rate at which the amplifier output rises, as the result of an approximately square wave input, is inversely proportional to the upper frequency that can be amplified by the amplifier without serious loss in gain. That is, the greater the band width of the amplifier, the more abruptly will the output rise, and hence, the more closely will the applied pulse be reproduced.

In the matter of obtaining square waves from sine waves by cascading circuits similar to that of Figure 26, little has been said about the method of coupling between stages or the manner in which the required bias is obtained. Consider Figure 29.

![Diagram](image)

FIGURE 29

In Figure 29, the long time constant RC network is used to couple stages. So the product $R_1C$ must be much greater than a period of the applied wave. Further, in order to obtain a clipping action, $R_2$ must be much larger than the grid-cathode conducting resistance. In order to bias the signal at a point midway between zero grid volts and cut-off, the value of $R_2$ becomes critical. The problem, then, is to determine $R_2$ such that the desired signal bias obtains.

The problem presents a situation similar to the one solved in connection with Figure 17 of Part I, namely, a circuit in which the time constant is alternately switched between two values, one of which ($R_1C$) is long compared with the period of the applied wave. The assumptions of equation (43) then hold; namely, the average steady state voltage on the condenser will be constant throughout the cycle and the change in charge on the condenser during each half cycle will be equal and opposite.

In the waveform of Figure 30, the desired method of biasing is assumed; i.e., the grid is biased at a value midway between cut-off and the point where grid current flows. The amplitude of the applied sine wave is taken as $BE$, as before, and for succeeding stages the amplitude is simply altered by a factor representing the voltage gain of the preceding stages. The horizontal axis is taken as the $\omega t$ axis, $\omega$ constant, and the initial point is so chosen that the associated phase angle is zero. The grid-cathode resistance is $r_g$. $A_1$, $A_2$, and $A_3$ are areas as shown. Evidently $A_1 = A_3$ for the desired bias assumed.
When \( e \) is positive, \( R_1 \) and \( (R_2 + r_g) \) are in parallel and

\[
e = \frac{i}{R_1} \frac{R_1 (R_2 + r_g)}{R_1 + R_2 + r_g}
\]

where \( i \) is the condenser current (Figure 29).

When \( e \) is negative, the \( (R_2 + r_g) \) branch is open and

\[
e = i R_1
\]
\[
A_1 + A_2 + A_3 = \left( \frac{1}{\omega t_1} \right) \int_{\pi - \omega t_1}^{\pi + \omega t_1} \text{d}t = \frac{R_1(R_2 + r_g)}{R_1 + R_2 + r_g} \int_{\pi - \omega t_1}^{\pi + \omega t_1} \text{d}t.
\]

But by equating change in charge as per equation (43) of Part I,

\[
\int_{\omega t_1}^{2\pi + \omega t_1} \text{d}t = \int_{\pi - \omega t_1}^{\pi + \omega t_1} \text{d}t.
\]

So the integrals in the numerator and denominator of (69) cancel for \(\omega\) constant, and (69) becomes

\[
\frac{A_1}{A_2 + A_3} = \frac{R_2 + r_g}{R_1 + R_2 + r_g}.
\]

Solving for \(R_2\), recalling that \(A_1 = A_3\),

\[
R_2 = \frac{A_1}{A_2} R_1 - r_g.
\]

But by (66), \(A_2 = 2E_B\). Therefore

\[
R_2 = \frac{A_1 R_1}{2E_B} - r_g.
\]

From (64), \(A_1 = 2BE \cos \omega t_1 + 2E_B t_1 - E_B\), where

\[
\omega t_1 = \sin^{-1} \frac{1}{B},
\]

so that

\[
A_1 = 2E \left( (B^2 - 1)^{1/2} + \sin^{-1} \frac{1}{B} - \frac{\pi}{2} \right).
\]
Substituting this value for $A_1$ into equation (73), the expression for $R_2$ becomes

$$R_2 = \frac{R_1}{\pi} \left( \left( B^2 - 1 \right)^{1/2} + \sin^{-1} \frac{1}{B} \right) - \frac{R_1}{2} - r_g. \quad (76)$$

For $B$ greater than about 10, $(B^2 - 1)^{1/2} = B$, and $\sin^{-1} \frac{1}{B} = \frac{1}{B}$ and may be neglected. Under these conditions

$$R_2 = R_1 \left( \frac{B}{\pi} - \frac{1}{2} \right) - r_g. \quad (77)$$

Signal biasing, then, lends itself to circuits similar to those of Figure 29. Recall that (77) holds provided $BE$ is greater than about 10E, $R_2$ is much greater than $r_g$, and that $R_1$ is such that $R_1C$ is much greater than the period of the applied wave. Equation (76) holds for any $B$.

Ordinarily, $R_1$ is of the order of one megohm, and $r_g$ of one kilohm, so that except for the restriction on signal amplitude (77) may be used conveniently to determine $R_2$. For such installations, the term $-r_g$ is negligible and (77) becomes simply

$$R_2 = R_1 \left( \frac{B}{\pi} - \frac{1}{2} \right). \quad (78)$$
NARROWING AND DELAYING CIRCUITS

The effects of narrowing and delaying circuits upon an input wave are illustrated in Figure 31. Such circuits find their usefulness in equipment in which predetermined sequences of events are required to exist within each cycle of an input wave. The output waveforms from narrowing or delaying circuits may be passed through differentiating circuits in order to form accurately timed discontinuous voltage peaks of short duration for use as synchronizing or trigger signals for further components.

A narrowing device may be assembled easily from the basic circuits already considered. The block diagram of Figure 32 is such an assembly. The square wave input is applied to a differentiating circuit, a long time constant RC circuit as described in Part I. The average output of the differentiating circuit is maintained at a level below the cut-off value of the triode tube to which it is applied, but such that positive peaks of the output of the differentiating circuit cause the tube to
FIGURE 32
conduct. The output of the triode, then, consists of a sequence of sharp fluctuations below the B-battery level, and the leading edges are synchronized with the leading edges of the input square wave. Although the manner of biasing may differ, the clipping action of the triode is the same as that discussed in connection with Figure 25. The output wave of the first triode is applied to a circuit identical with that of Figure 25, in which the first two elements of the tube are used to clamp the grid voltage at zero. The portions of the grid wave below cut-off are clipped and reasonably square pulses appear on the plate. Replacing the units of the block diagram with circuits previously considered, the schematic circuit of Figure 32 is obtained. Waveforms are recorded. Note that the width of the output wave may be varied at will by altering the time constant of the differentiating circuit or by changing $E_{cc}$ slightly. The rise time of the output waveform is zero, while the fall time is slightly greater than zero. If the latter is objectionable, the waveform may be passed through a diode clipper and amplified.

A delaying circuit may likewise be assembled from the basic networks already considered. Consider Figure 33. The delaying circuit is identical with the narrowing circuit except that the differentiating circuit in the input is replaced by an integrating circuit, an RC coupling network with a time constant comparable with the period of the applied wave. Since the output of an integrating circuit is taken across the condenser, it is necessary to insert a high resistance d.c. path in the grid circuit. The function associated with the second triode is identical with that of the corresponding tube in the narrowing circuit. The delay time can be controlled by altering the time constant of the integrating circuit of Figure 33 or by changing $E_{cc}$ slightly. Again, clipping may be required to reduce the rise and fall time. Delaying action may also be obtained by the use of multivibrators or by use of the phantastron circuit.
7. In Figure 33 note that the variation of delay time with the time constant of the integrating circuit or with $E_{ac}$ is not linear. In radar work it is convenient to provide a "step" or "spot" which may be moved across the time base of the scope by means of rotating a calibrated knob. When an operator lines up this step with a target, the range of the target may be read from the calibrated knob. The step is usually formed by differentiating the leading edge of a delayed wave and by applying the pulse so formed to the vertical deflection plates of the scope. To obtain a linear movement of the step across the screen, the time constant of the integrating circuit or the voltage of the battery must be varied exponentially. Exponentially wound potentiometers are not standard equipment, and furthermore, the calibrated knob would have to be corrected each time a replacement tube would be installed due to slight variations in cut-off potential. For this reason a more stable delay circuit with linear variation of time delay with some parameter was sought. The phantastron circuit is such a device. See footnote 6.
COUNTING CIRCUIT

Occasionally it is required to apply a pulse to a device after each n pulses of some reference wave. Such a division can be accomplished by means of the circuit of Figure 34.

The condenser \( C_2 \) is ten to twenty times as large as \( C_1 \). Assume that both condensers are initially discharged. The first pulse of the input wave causes the series diode to conduct putting \( C_1 \) and \( C_2 \) in series. As a result, a voltage equal to \( \frac{EC_1}{(C_1 + C_2)} \) appears on \( C_2 \). At the end of the pulse, the plate of the shunt diode is positive with respect to its cathode due to the voltage on \( C_1 \). Thus \( C_1 \) is dischaged. \( C_2 \), however, retains its charge since the input of the triggered circuit is required to be of such a nature that its impedance is very high until triggered. During the next pulse, the series diode again conducts and an additional voltage

\[
E \frac{C_1}{C_1 + C_2} \left( 1 - \frac{C_1}{C_1 + C_2} \right)
\]

appears on \( C_2 \). The condenser \( C_1 \) is again discharged through the shunt diode at the end of the pulse. On each pulse, the voltage across \( C_2 \) is increased by an amount equal to \( \frac{C_1}{(C_1 + C_2)} \) times the difference between \( E \) and the voltage across \( C_2 \) just prior to the pulse under consideration.
After n pulses, the voltage on $C_2$ is given by

$$E_{c2} = E_k \left( 1 + (1 - k) + (1 - k)^2 + \cdots + (1 - k)^{n-1} \right), \quad (79)$$

where \( k = \frac{C_1}{C_1 + C_2} \).

If $C_2$ were not discharged, its voltage would approach the peak voltage of the applied wave in a series of ever decreasing incremental steps such that a line passing through the mid points of the steps of the output would be an exponential curve from 0 to $E$ with a time constant dependent upon the output impedance of the driving source and the conducting resistance of the diodes for fixed values of $C_1$ and $C_2$.

The triggered circuit exhibits a very high impedance until $E_{c2}$ reaches some preset value at which time the input impedance of the triggered circuit drops nearly to zero. Such a triggered circuit could well be a thyatron tube. When $E_{c2}$ reaches the firing potential of the thyatron, the impedance drops and $C_2$ is discharged through the triggered circuit. The impedance then rises abruptly and the whole sequence is initiated again. Most stable operation can be obtained in practice by designing the triggered circuit to discharge $C_2$ at some potential within the first ten to fifteen steps of the output wave.
The output voltage of a sweep circuit for use as a time base with electrostatic cathode ray tubes most frequently takes the form of a saw tooth wave. Conventional circuits to generate such waveforms are shown in Figures 35 and 36.

The circuit of Figure 35 (a) is a common type of continuous sweep generator. The thyratron is normally not conducting, during which time the condenser C charges exponentially toward 280 volts through the resistance network. (R33 and R34 are small compared with R32.) When the voltage across C reaches the firing potential of the tube, which may be varied by the sync input, the tube conducts, and the condenser C is discharged very rapidly to the extinction potential of the tube with a short time constant determined by the conducting resistance of the thyratron and the value of C. The condenser again starts to charge through the resistances R32, R33, R34 and the cycle is repeated periodically. The output of the circuit is a familiar saw tooth waveform as indicated in Figure 35 (b). The portion of the exponential curve of voltage across C while the tube is not conducting that lies between the extinction and firing potentials is nearly linear with time and when amplified is applied to the horizontal deflection plates of a cathode ray tube to form a linear time base. Synchronization may be obtained by varying the firing potential as mentioned above. Frequency may be controlled by varying the time constant in the charge circuit. For this purpose it is convenient to replace C by a selection of condensers of arbitrary size that may be switched in or out at will.

The circuit of Figure 36 (a) is more appropriate when an intermittent sweep is required. The input voltage, which controls the frequency of the

FIGURE 37
sweep, should have steeply sloping sides and negative excursions of the duration required by the desired sweep duration time as may be appreciated from Figure 36 (b). The clamping circuit in the input maintains the positive excursions of the input wave at zero volts. With a wave as shown, the grid is normally at zero so that the plate voltage is low. When the tube is cut off for a short time, the condenser charges exponentially toward Ebb. This nearly linear rise in plate voltage is used as the sweep. Note that the length of the negative plateaus of the input voltage have a definite bearing upon the duration time of the sweep as well as upon the linearity.

In each of the above circuits, the degree of linearity is determined by the length of the exponential curve used. Much has been written on methods of improving the linearity of sweep voltages. 9

The length of time base which can be accommodated on a cathode ray tube of given diameter can be increased by using a circular sweep as the circumference rather than the diameter of the tube is used. Such a time base can be produced by applying sine waves of equal amplitude and frequency but differing in phase by 90° to the vertical and horizontal deflection plates. The resulting pattern is the well known Lissajous figure for the stated conditions, namely, a circle.

The circuit of Figure 37 may be used. The impedance of the resistor R and the reactor C are approximately equal. When the test signal is zero, the voltages applied to the X and Y axis amplifiers, then, are of about equal magnitude and are out of phase by 90°. Equalization of amplitudes may be accomplished by adjusting the X and Y axis amplifiers of the scope to make the pattern a perfect circle. The radius of the circle is proportional to the amplitude of the sine wave applied to the

9. See, for example, M. I. T. Radar School Staff, op. cit.
deflecting plates, which in turn may be varied by changing the gain of the tubes. The radius of the circle, then, may be varied by changing the screen voltage (the gm) on the two tubes of Figure 37. But the screen voltage may be controlled simply by applying an amplified version of the test signal to the screen circuit. As a result, the instantaneous radius of the circle changes as the signal voltage changes so that the signal waveform appears as a radial excursion upon the circular time base.

The determination of frequency ratios by means of conventional Lissajous patterns becomes quite inconvenient for ratios greater than about 5:1. By the use of a circular time base such ratios may be determined accurately and conveniently as large as about 282:1. If two sine wave frequencies are to be compared, F and f, respectively, with f greater than F, the wave of frequency F may be applied to the Sweep Sine Wave Input terminals of Figure 37, and the wave of frequency f may be applied to the Test Signal terminals. The resulting pattern, then, is a circular time base rotating at F cycles per second with sine wave radial excursions of frequency f. The Du Mont 208-B Oscillograph has a five inch scope. If the sweep is adjusted for a four inch diameter circle, and if it is agreed that sine waves may be easily counted if each cycle takes up 0.2 inch of the circumference, then the maximum ratio of f/F that may be observed conveniently is simply \( \frac{4\pi''}{0.2''} = 63 \). It is possible to increase the value of this ratio if one is willing to exert a little eye strain. By using a 4.5 inch diameter sweep circle and by counting peripheral sine waves spaced as closely as 0.05 inch, the maximum ratio becomes \( \frac{f}{F} = \frac{4.5\pi''}{0.05''} = 282 \). Thus, for \( F = 60 \) c.p.s., the maximum unknown frequency could be as great as \( f = 282F = 17,000 \) c.p.s. It is clear that the value F may be the output of a frequency doubler in which case the ratio would be even greater.
If a saw tooth wave were applied to the Test Signal terminals of Figure 37, the pattern on the screen would be a spiral provided the frequency of the saw tooth wave were comparable with the frequency of the sweep sine wave. In fact, if the two frequencies were equal, the spiral would consist of one turn; if the saw tooth frequency were half the sweep sine wave frequency, a spiral of two turns would result, etc.

A spiral time base extends the advantages of the circular time base since it produces a useful sweep of \( \frac{N}{F} \) seconds, where \( N \) is the number of turns in the spiral and \( F \) is the frequency in c.p.s. of the sine wave at the Sweep Sine Wave Input terminals. For \( N = 4 \) and \( F = 60 \), \( \frac{N}{F} = 0.0667 \) seconds, a very long time base. The saw tooth wave form can be mixed with the test signal in the test signal amplifier, and, as expected, the signal pattern consists of radial excursions from the spiral. Synchronization, of course, is critical to obtain a stationary pattern. The resulting spiral sweep is linear with respect to time in the number of radians it traverses per second, not in the linear distance it moves.
The Eccles-Jordan trigger circuit\textsuperscript{10}, often referred to as the flip-flop, has two stable states because of the direct current connections from the plate of each tube to the grid of the other (R2, R5 of Figure 38) which cause the conducting tube to bias the nonconducting tube negatively, and the nonconducting tube to bias the conducting tube positively. In the circuit of Figure 38 assume that \( E_{bb} \) is initially turned on with zero volts across each of the sync. terminals. Since it is exceptionally unlikely that both tubes conduct identically even though the circuits are symmetrical, it may be assumed that V3, say, conducts slightly more current than V4. Under these conditions the voltage developed across R1 is greater than that developed across R4 \((R1 > R4)\) resulting in a lower plate voltage on V3. A decrease in plate voltage on V3 reflects a decrease in grid bias on V4 through \( R_2 \), which, in turn, results in an increase in plate voltage on V4. The latter change is transmitted through the resistor R5 to increase the grid bias on V3 causing it to conduct more heavily. When V3 conducts more heavily, its plate voltage decreases more so causing the grid of V4 to go lower, causing the plate of V4 to increase, resulting in a further increase in the grid voltage of V3, etc., until V4 is completely cut off, halting the cause-effect sequence. The circuit is in a stable state with V3 conducting and V4 cut off.

Now if a negative pulse were applied to the Sync. 3 terminals of sufficient amplitude to cut off V3, or if a positive pulse were applied at Sync. 4 of sufficient amplitude to cause V4 to conduct, then the entire process would be repeated in reverse order until the second stable state would be reached; namely, V3 cut off and V4 conducting. The duration of the initiating pulse, of course, would need only to be momentary.

Furthermore, since, in the stable condition first discussed, a negative pulse at Sync. 4 would only drive V4 farther into the cut-off region and hence have no effect, and since a positive pulse at V3 which is already passing maximum current would have no effect, therefore, the terminals of Sync 3 and Sync 4 may be connected in parallel without altering the function of the circuit at all.

Each reversal in stability must be initiated by a sync. pulse of appropriate sign. Although the transition from one stable state to the other is a result of a series of incremental changes, the shift is completed in very little time.

The design of the circuit leads to a compromise between two opposing factors: an attempt to decrease the actual transition time in changing from the first stable state to the second, and an attempt to provide short time constant paths for the return to a quiescent condition so that the circuit may be prepared to accept the subsequent sync. pulse which would return the circuit to its initial stable state. Increasing the values of R1, R2, R4, and R5 would increase the gain of the circuit and hence decrease the transition time. Increasing the value of C1 and C2 would accelerate the transfer of the plate signal to the grid, likewise decreasing the transition time. But large values of the resistors and condensers just considered would make the time constant of the grid circuits large and thereby delay the return of the grids to a quiescent condition. A compromise must be effected. With proper parameters, circuits similar to Figure 38 can be made with a transition time of about one microsecond, and which can accept initiating pulses at the rate of one each five microseconds. 11

11. A. W. Burks, op. cit.
It is evident from the foregoing that the Eccles-Jordan trigger circuit executes one alteration (one half of a complete cycle) for each appropriate sync. pulse. It is possible, however, to bring about a complete cycle of operation for each single sync. pulse by revising the circuit so that only one stable condition exists in which the one tube is cut off due to the conduction of the other. Such a circuit resembles a resistance coupled amplifier with one tube normally cut off due to current flow in the other. See Figure 39 (a). Under normal conditions the voltage across R3 is zero so that V2 is conducting. R2 and R4 are of such values that the voltage drop across R4 is sufficient to maintain the grid of V1 below cut-off. When a positive pulse of sufficient amplitude to cause V1 to conduct is applied to the sync. terminals, the conducting tube current causes a voltage drop across R1 so that $e_{b1}$ decreases. Employing the methods of Part I, the voltage across C1 cannot change instantaneously so that when the potential at the upper part of C1 is reduced, the potential at the lower terminal is likewise reduced causing a reduction in $e_{o2}$. A reduction in $e_{o2}$ decreases the current flow in V2 causing a smaller voltage drop across R4 which increases the bias on V1, etc., until V2 is cut off and V1 is conducting. At this stage, V1 is fully self-biased by the voltage across R4, which is not sufficient to cause its own grid to be below cut-off. The above sequence of incremental changes takes place very rapidly. The voltage across C1, now, is still practically at its original high value of ($E_{bb}$ - the value of $e_{R4}$ with V2 conducting and V1 cut off), while the potential between points X and Z, $e_{p1}$, is considerably lower. That is, the voltage on C1 is high, and the voltage across R3 is negative, cutting off V2, so that their sum is $e_{p1}$. C1, then, discharges exponentially to a new low value of $e_{p1} - e_{R3}$ with a time constant of C1 ($R3 + r_p$), where $r_p$ is the plate resistance of V1. The affect of the parallel path ($R1 + R4$) is negligible. At some
FIGURE 39
time less than $4C_1 (R_3 + r_p)$, $e_{R3}$ rises to the cut-off level of $V_2$ causing $V_2$ to conduct, increasing $e_{R4}$, decreasing $e_{e1}$. Decreasing $e_{e1}$ causes less current to flow in $V_1$, reducing the voltage drop on $R_1$, increasing the potential at point $X$. The discharge curve of $C_1$ is terminated prior to its completion, and since the voltage between points $X$ and $Y$ cannot change instantaneously, the grid of $V_2$ is driven more positively causing the voltage $e_{e1}$ to decrease, etc., until $V_1$ is cut off and $V_2$ is conducting. The voltage on $C_1$ then rises exponentially to $E_{bb} - e_{R4}$ with the time constant

$$C_1 (\frac{R_1 + R_4 + \frac{R_2 r_g}{R_3 + r_g}}{R_3 + r_g})$$

where $r_g$ is the grid-cathode conducting resistance. Again, the affect of $R_3$ may be neglected since it is much greater than $r_g$. The time constant then becomes simply

$$C_1 (R_1 + R_4 + r_g).$$

At this point the cycle of events is complete, and the circuit remains quiescent until another sync. pulse is applied.

The circuit of Figure 39 is usually referred to as a one-shot multivibrator. The one-shot multivibrator differs from the Eccles-Jordan trigger circuit in that the former executes one complete cycle for each sync. impulse, while the latter, which has two stable states, completes only a half cycle for each sync. pulse.

Useful output waveforms of the one-shot multivibrator, of course, may be taken across arbitrary points. The plate voltage of $V_2$, $e_{b2}$, provides a nearly perfect rectangular wave, as seen in Figure 39 (b). The width of the pulse of $e_{b2}$ is determined by the $C_1$ discharge curve of $e_{e2}$ as shown. Slight variations in cut-off value from tube to tube which may be used at $V_2$ would result in lack of stability of pulse width due to the small angle ($A$ of Figure 40 (a)) at which the exponential curve intersects the cut-off level. The circuit may be altered so that
the discharge curve of Cl is such that $e_{O2}$ tends to rise exponentially toward $E_{bb}$ instead of toward zero, in which case the angle of intersection of the $e_{O2}$ curve with the cut-off level is greatly increased as shown in Figure 40 (b). Such a circuit would be much more stable with respect to pulse width than the one of Figure 39. This more stable circuit is often referred to as a degenerative positive-bias multivibrator, and is discussed in detail by Bertram.

It is possible to insert feedback paths in a one-shot multivibrator so that the circuit will maintain this abrupt type of oscillation independent of any external initiating pulse. Such circuits are called free-running multivibrators, and are classified in general according to the method of feedback coupling. If the coupling is from the plate of each tube to the grid of the other, for example, the multivibrator is said to be plate coupled. If one of the plate to grid paths of the plate coupled circuit is replaced by a cathode to cathode coupling, the circuit is called a cathode-coupled multivibrator, etc. These and others are thoroughly discussed in several publications. Methods of controlling and synchronizing free-running multivibrators also offer a field of study.

12. This change may be accomplished simply by removing R3 from point Z and reconnecting it at the positive side of $E_{bb}$.

M. I. T. Radar School Staff, op. cit.
F. E. Terman, op. cit.
In addition to the difference in cyclic performance of the multivibrator as compared with the Eccles-Jordan trigger circuit, it is noted that d.c. paths exist in the latter, while condensers are always present in the coupling paths of the former. It is by virtue of this difference, namely, that the discharging of the condenser in the coupling path starts the second half cycle, that the multivibrator completes a full cycle for each initiating pulse (whether external as in the case of the one-shot type, or internal as in the free-running type), while the Eccles-Jordan simply shifts from the one stable state to the other.

Realisation of this distinction suggests the possibility of making a one-shot circuit in which the initiating pulse is applied to the first half of an Eccles-Jordan which triggers the second half of a multivibrator, which in turn reflects the change upon the first half of the Eccles-Jordan, at which point the sequence is terminated due to the fact that the Eccles-Jordan is not capacitively coupled to the multivibrator. Such a device would complete one cycle for each sync. pulse applied to the Eccles-Jordan.

The circuit of Figure 41 consists of a half of an Eccles-Jordan trigger circuit and a half of a multivibrator. The assignment of names to the tubes, of course, depends upon with which tube the condenser C1 is associated. In any event, the plate of V1 is connected to the grid of V2 by a d.c. path, and the plate of V2 is coupled to the grid of V1 by an energy storing path as is required for whole-cycle operation.

At any time, the voltage across C1 plus the voltage across R1 is equal to $e_{b2}$, so that in any stable state the voltage across C1 is identically $e_{b2}$ leaving the voltage across R1, $e_{c1}$, zero. So in the stable state, the grid of V1 is zero, leaving the grid of V2 below cutoff. When a negative sync. pulse of sufficient amplitude to cut off V1 is applied to the circuit, $e_{b1}$ rises abruptly (assuming the sync. pulse
has a steep leading edge) to $E_{bb}$, driving the grid of V2 positive. The voltage $e_{c2}$ would go far into the positive region except that R4 limits the grid current so that $e_{c2}$ remains at some value slightly above zero. When V2 conducts, $e_{b2}$ drops, driving $e_{c1}$ far below cut-off since the voltage across the condenser cannot change abruptly. $C_1$ discharges with a time constant $R_1C_1$ (R3 in parallel with V2 evidently contribute a negligibly small amount), causing $e_{c1}$ to vary exponentially toward zero. At the point where $e_{c1}$ reaches cut-off, V1 begins to conduct, reducing $e_{b1}$ and $e_{c2}$ until V2 is cut off and V1 is conducting, the initial stable state. A particularly square pulse may be obtained in this manner.
Figure 41
APPENDIX I

DIMENSIONS OF CERTAIN ELECTRICAL QUANTITIES

Systems of units and dimensions are purely arbitrary. In establishing such systems for electrical quantities it is convenient to employ the conventionally established fundamental units and dimensions wherever possible, e.g., the units gram, centimeter, and second (gram, cm., sec.) and the dimensions mass, length, and time (M, L, T). For unique resolution of electrical quantities it is necessary to introduce a fourth fundamental unit with its associated dimension. Suitable as such a fourth fundamental unit is any one of the following: permittivity of free space, permeability of free space, an arbitrary unit of charge as the coulomb, an arbitrary unit of resistance, inductance, capacitance, temperature, and perhaps others.

Three systems in common use are the cgs electrostatic system (esu), the cgs electromagnetic system (emu), and the rationalized mks system (mks). The esu system employs the following fundamental units: gram, centimeter, second, and permittivity of free space (gram, cm., sec., K_0) associated, respectively, with the following dimensions: mass, length, time, and permittivity (M, L, T, K). The emu system utilizes the same except that the unit of permittivity of free space with its dimension K is replaced by the unit of permeability of free space (u_0) with its dimension u. The mks system uses the following units: kilogram, meter, second, and coulomb (k., m., sec., q) with the dimensions mass, length, time, and charge (M, L, T, Q).

Discussions concerning choice of the size of the above units or whether certain units contain the pure numeric $4\pi$ as a factor are
given adequate attention in many texts. It is the intention here to give the dimensional equations for certain electrical quantities, in particular, to show that the combinations resistance x capacitance and inductance / resistance offer the proper dimensions to be considered as time constants of electrical circuits.

Table I shown the derivation of the dimensions for certain electrical quantities leading to the dimensional equations for capacitance, inductance, and resistance. Recall that all quantities in the esu system are resolved into the fundamental dimensions of that system: M, L, T, K; quantities in the emu system are resolved into the dimensions M, L, T, u; and those of the mks system are resolved into the dimensions M, L, T, Q.

The starting point for the esu and emu systems is the dimensional equation

\[ f = \frac{q1q2}{K r^2} = \frac{m_1m_2}{u r^2} = M L T^{-2}. \]

Subsequent formulas in the esu system are clearly derived. The crossover point for expressing electrostatic quantities in terms of electromagnetic dimensions lies in the equation

\[ h = \frac{2\pi}{r}. \]

Through the use of this formula, current and further electrical quantities may be expressed in terms of magnetic field strength, magnetic pole strength, etc. Dimensions of quantities in the mks system are easily obtained due to the fact that Q is a fundamental dimension in the system.

In Table II a summary of the results derived in Table I is given. Furthermore, the fact that resistance x capacitance and inductance / resistance exhibit the dimension T in all three systems is clearly shown.


16. For elaborate data on units and dimensions, see M. I. T. E. E. Staff, Applied Electronics, John Wiley & Sons, Appendix B.
## APPENDIX I

### TABLE I

**DERIVATION OF DIMENSIONAL EQUATIONS FOR CERTAIN ELECTRICAL QUANTITIES**

<table>
<thead>
<tr>
<th>ESU</th>
<th>EMU</th>
<th>MKS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FORCE</strong> = ( f = \frac{g \cdot b}{K \cdot r^2} ) = MLT^{-2}</td>
<td><strong>FORCE</strong> = ( f = \frac{m \cdot \nu}{\mu \cdot r^2} ) = MLT^{-2}</td>
<td><strong>CHARGE</strong> = ( q = \frac{Q}{Q} ) = ML^{-2}Q^{-1}</td>
</tr>
<tr>
<td><strong>CHARGE</strong> = ( q = \sqrt{K \cdot v^2} ) = ML^{-3}T^{-2}K^{-\frac{1}{2}}</td>
<td><strong>MAG. POLE STRENGTH</strong> = ( m = \sqrt{K \cdot v^2} ) = ML^{-3}T^{-2}K^{-\frac{1}{2}}</td>
<td><strong>CURRENT</strong> = ( i = \frac{Q}{Q} ) = ML^{-1}T^{-1}Q^{-1}</td>
</tr>
<tr>
<td><strong>CURRENT</strong> = ( i = \frac{g}{b} ) = ML^{-3}L^{-3}T^{-2}K^{-\frac{1}{2}}</td>
<td><strong>MAG. FIELD STRENGTH</strong> = ( H = \frac{M \cdot \nu}{\mu \cdot r^2} ) = ML^{-3}T^{-2}K^{-\frac{1}{2}}</td>
<td><strong>CURR.</strong> = ( i = \frac{Q}{Q} ) = ML^{-1}T^{-1}Q^{-1}</td>
</tr>
<tr>
<td><strong>POTENTIAL</strong> = ( V = \frac{\text{ENERGY}}{g} ) = ML^{-3}L^{-3}T^{-2}K^{-\frac{1}{2}}</td>
<td><strong>POTENTIAL</strong> = ( V = \frac{\text{ENERGY}}{\frac{Q}{Q}} ) = ML^{-3}L^{-3}T^{-2}K^{-\frac{1}{2}}</td>
<td><strong>POTENTIAL</strong> = ( V = \frac{\text{ENERGY}}{\frac{Q}{Q}} ) = ML^{-3}L^{-3}T^{-2}K^{-\frac{1}{2}}</td>
</tr>
<tr>
<td><strong>CAPACITY</strong> = ( \frac{g}{b} ) = ML^{-3}L^{-3}T^{-2}K^{-\frac{1}{2}}</td>
<td><strong>CAPACITY</strong> = ( \frac{\text{ENERGY}}{V^2} ) = ML^{-3}L^{-3}T^{-2}K^{-\frac{1}{2}}</td>
<td><strong>CAPACITY</strong> = ( \frac{\text{ENERGY}}{V^2} ) = ML^{-3}L^{-3}T^{-2}K^{-\frac{1}{2}}</td>
</tr>
<tr>
<td><strong>INDUCTANCE</strong> = ( \frac{\text{ENERGY}}{\frac{Q}{Q}} ) = ML^{-3}L^{-3}T^{-2}K^{-\frac{1}{2}}</td>
<td><strong>INDUCTANCE</strong> = ( \frac{\text{ENERGY}}{V^2} ) = ML^{-3}L^{-3}T^{-2}K^{-\frac{1}{2}}</td>
<td><strong>INDUCTANCE</strong> = ( \frac{\text{ENERGY}}{V^2} ) = ML^{-3}L^{-3}T^{-2}K^{-\frac{1}{2}}</td>
</tr>
<tr>
<td><strong>RESISTANCE</strong> = ( \frac{\text{POWER}}{V^2} ) = ML^{-3}L^{-3}T^{-2}K^{-\frac{1}{2}}</td>
<td><strong>RESISTANCE</strong> = ( \frac{\text{POWER}}{V^2} ) = ML^{-3}L^{-3}T^{-2}K^{-\frac{1}{2}}</td>
<td><strong>RESISTANCE</strong> = ( \frac{\text{POWER}}{V^2} ) = ML^{-3}L^{-3}T^{-2}K^{-\frac{1}{2}}</td>
</tr>
</tbody>
</table>
## APPENDIX I

### TABLE II

**DIMENSIONS OF CERTAIN ELECTRICAL QUANTITIES**

<table>
<thead>
<tr>
<th>QUANTITY</th>
<th>ESU</th>
<th>EMU</th>
<th>MKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>POTENTIAL</td>
<td>$M^{1/2}L^{1/2}T^{-1}K^{1/2}$</td>
<td>$M^{1/2}L^{3/2}T^{-3/2}Q^{1/2}$</td>
<td>$ML^2T^{-2}Q^{-1}$</td>
</tr>
<tr>
<td>CURRENT</td>
<td>$M^{1/2}L^{3/2}T^{-2}K^{1/2}$</td>
<td>$M^{1/2}L^{1/2}T^{-1}M^{-1/2}$</td>
<td>$T^{-1}Q$</td>
</tr>
<tr>
<td>CHARGE</td>
<td>$M^{1/2}L^{3/2}T^{-1}K^{1/2}$</td>
<td>$M^{1/2}L^{1/2}T^{-1}M^{-1/2}$</td>
<td>$Q$</td>
</tr>
<tr>
<td>CAPACITANCE</td>
<td>$LK$</td>
<td>$L^{-1}T^2M^{-1}$</td>
<td>$M^{-1}L^2T^2Q^2$</td>
</tr>
<tr>
<td>INDUCTANCE</td>
<td>$L^{-1}T^2K^{-1}$</td>
<td>$LM$</td>
<td>$ML^2Q^{-2}$</td>
</tr>
<tr>
<td>RESISTANCE</td>
<td>$L^{-1}TK^{-1}$</td>
<td>$LT^{-1}M$</td>
<td>$ML^2T^{-1}Q^{-2}$</td>
</tr>
<tr>
<td>RESISTANCE $\times$ CAPACITANCE</td>
<td>$T$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>INDUCTANCE $\div$ RESISTANCE</td>
<td>$T$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
</tbody>
</table>
By inspection it may be verified that only those combinations of 
electrical parameters that reduce to the form resistance $\times$ capacitance or 
inductance$/$resistance may qualify as time constants of electrical circuits.

From the results of Table II, familiar equations are seen to be 
dimensionally correct in each of the three systems.

By comparison of the dimensions of two identical quantities from the 
esu and emu systems, it may be observed that the expression $(uK)^{-1/2}$

must have the dimensions L T$^{-1}$, a velocity. From comparison of the units 
of any two identical quantities from the esu and emu systems (both are cgs 
systems), the pure numeric $3 \times 10^{10}$ is obtained. The velocity of light 
or of electromagnetic radiation is given by $v = c (uK)^{-1/2}$, where

$$c = \frac{\text{electromagnetic unit of charge}}{\text{electrostatic unit of charge}} = 3 \times 10^{10}, \text{ a numeric.}$$

Now $u = K = 1$ for a vacuum, by definition. The velocity of light in a 
vacuum, then, is $c (uK)^{-1/2}$, where $c = 3 \times 10^{10}$, a numeric, and

$$(uK)^{-1/2} = 1 \text{ with dimensions L T}^{-1}$$

and units cm/sec.

So the magnitude of the velocity in a vacuum is given by c, a dimensionless 
constant, and the dimensions are given by $(uK)^{-1/2}$.

One other item that might be called to attention is the fact that 
in the mks system, no one of the four fundamental dimensions is raised 
to a fractional power. This simplicity is due to the choice of the fourth 
fundamental dimension, Q, and is lauded as one of the advantages of the 
system.
APPENDIX II

APPROXIMATE FORMULA FOR $e^{-x}$

Given $f(x) = e^{-x}$ with $x$ not negative. Consider the Maclaurin expansion:

$$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \cdots .$$

If all but the first two terms are neglected, the following approximate formula is obtained:

$$e^{-x} = 1 - x .$$

The per cent error introduced in the approximation has a direct bearing on the utility of the formula.

$$\% \text{ error} = 100\% \frac{\text{error}}{\text{true value}} = 100\% \frac{e^{-x} - (1 - x)}{e^{-x}}$$

$$= 100\% \left( 1 + xe^x - e^x \right) .$$

<table>
<thead>
<tr>
<th>$x$</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.00</td>
</tr>
<tr>
<td>0.1</td>
<td>0.55</td>
</tr>
<tr>
<td>0.2</td>
<td>2.4</td>
</tr>
<tr>
<td>0.4</td>
<td>10.6</td>
</tr>
<tr>
<td>0.6</td>
<td>27.2</td>
</tr>
<tr>
<td>0.8</td>
<td>55</td>
</tr>
<tr>
<td>1.0</td>
<td>100</td>
</tr>
<tr>
<td>$\infty$</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

So for $x$ between 0 and 0.1, % error lies between 0 and 0.55%.

Put $x = t/T$, then for $t$ between 0 and 0.1T, % error lies between 0 and 0.55%.
BIBLIOGRAPHY


