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A Report on an Investigation of

ELECTRONIC BALANCING IN TWO-WAY AMPLIFIERS

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The nature of the following report might indicate that the investigation was largely theoretical, since most of the discussion is theoretical in character. As a matter of fact, much of the investigation involved a linear analysis of various circuits, which superficial investigation showed to be of some promise. At times these circuits were evolved systematically from the desired characteristics. At other times a purely exploratory approach was taken, in which any possibility suggested by the imagination was examined. Both approaches seemed to have a place, for both contributed to the small measure of progress that was made.

The lack of numerical and experimental data should not be considered indicative of a lack of laboratory work, however. Because the entire investigation involved a search for satisfactory circuits, and because such circuits were never discovered, the amount of data, other than diagrams, is necessarily meager. I felt that it would have been a waste of effort to take and record complete data on circuits which preliminary tests or mathematical analysis showed to be inadequate for the problem at hand. More often than not, when laboratory tests showed that a proposed circuit was inadequate, the reason was discovered from analysis rather than from experimental evidence. Thus, it is only natural that the report should be of a theoretical nature.

Although, as far as I am concerned, the work involved many original elements, I make no claim to originality without a more emphatic claim of indebtedness to the unnumbered contributors to the body of knowledge, tools, and techniques without which I would be helpless.

In making these acknowledgments— which must necessarily be incomplete— I wish to express a deeply felt sense of gratitude to the
institution which has provided the opportunity for study and experiment. Especially do I wish to thank Professor James S. Waters, through whose efforts and interest this opportunity to complete the study was made possible. It is with particular satisfaction that I record this statement of appreciation of one whom I regard highly as friend, counsellor, and teacher.

P.F.P.
May, 1948
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PART I. SUMMARY.

The usual two-way, single channel amplifiers, commonly known as two-way repeaters, depend for their operation on a network which matches accurately the impedance of the line or other load on the amplifiers. In many potential applications the design of this network is difficult, if not impossible; and even where this approach is feasible, satisfactory results are usually expensive.

The purpose of this investigation was to study the possibility of achieving balance automatically by electronic means. It was intended that this should be accomplished with standard tubes and components, and that the resulting equipment should be practical from an economic and technical standpoint.

Most of the investigation hinged around the possible use of a "phase splitter" circuit, which performed satisfactorily for constant resistance loads, but which failed to operate satisfactorily for reactive loads, or for changing loads. Five principal approaches were made to the problem of adapting this circuit to compensate electronically for its weakness. A sixth approach to the problem involved the relatively simple principle of balancing the output against the input of a constant gain amplifier. Under the conditions demanded by the system, however, no circuit was discovered which would perform adequately.

It was anticipated that inverse feedback would be a major tool in the solution. It is usually emphasized that inverse feedback improves circuits by making their gain independent of tube characteristics and of load impedance. However, the changes in input and output impedance of circuits accompanying the institution of feedback proved to be of
such a nature and magnitude that the necessary improvement was impossible in the circuits tested. While no direct proof was evolved to show that the problem, as considered, is without possibility of satisfactory solution, there is evidence that the general nature of the solution as conceived may be completely inadequate. In all probability, a completely fresh approach to the problem is necessary, if there is to be any satisfactory solution. What may constitute a satisfactory approach is not apparent to the writer at present, although there are other possibilities for two-way communication on a single channel. One which suggests itself is the possibility of switching or reversing the amplifier electronically at a supersonic rate, thus allowing simultaneous conversations. This, however, is not the sort of solution desired, for it limits the repeater to a rather narrow band of audible frequencies. For certain applications this solution might be adequate, if practical circuits were evolved. But such a solution is not a solution of the problem proposed for this investigation, and hence was not studied formally.
PART II. REVIEW OF THE SUBJECT.

Published material on two-way amplification is limited very largely to descriptions of bridge type systems utilizing the "hybrid-coil" or "bridge-transformer" as it is called. The principal application is in the Bell Laboratories Type 21 (two-way, one amplifier) and Type 22 (two-way, two amplifier) systems. Other devices described in the literature prove to be variations of these.

A. L. Albert has given a clear development of the basic principles involved in the use of the "hybrid-coil." Figure 1 illustrates the basic bridge structure of the hybrid-coil system. If $Z_w$ and $Z_e$ are identical impedances, and if the hybrid-coil is symmetrical, signals fed to its primary appear across $Z_e$ plus $Z_w$, but do not appear across $Z_i$. On the other hand, a part of any signal appearing across $Z_e$ or $Z_w$ alone would appear across $Z_i$. If $Z_w$ represents the input impedance of a transmission line, $Z_e$ that of a balancing network, and $Z_i$ the input impedance of an amplifier, an effective separation of line and local signals is achieved. The line signals are fed to the amplifier, as desired. The local signals are fed to the line, but do not enter the amplifier, provided the balance between $Z_w$ and $Z_e$ is perfect.

Figures 2 and 3 illustrate the application of this basic circuit to the problem of two-way amplification. Figure 2 is a simplification of the type 22 system, and figure 3 is a similar simplification of the type 21 system. The latter suffers considerable limitation, principally that of amplified echo, which is not characteristic of the type 22 system. The type 22 system has found wide application in two-way voice frequency systems in which careful matching of line impedances is possible.
Figure 1. "Bridge Transformer"

\[ Z_w \parallel Z_i \parallel Z_o \]

Figure 2. Simplified Type "22" Repeater.

Figure 3. Simplified Type "21" Repeater.
There are three definite limitations to the hybrid coil systems.

(1). Voltage gain is limited by the degree of balance obtainable. This, of course, is an inherent limitation of all systems depending upon balance for signal separation. Usually the voltage gain in practical systems is limited to values of the order of fifty. Although tremendously high gains are not necessarily desirable, improvement would reduce the number of repeater stations required by a transmission system.

(2). The use of a transformer limits the range of frequencies which may be amplified. This is not a serious limitation for voice frequency telephone systems, in which the frequency range is often limited purposefully. But such a limitation definitely rules out the use of hybrid-coil systems for high-fidelity or wide band applications, unless, of course, the carrier principle is used.

(3). Balance is limited by the accuracy of the matching network. The problem of balancing is a difficult one, and from a purely economic point of view it rules out the use of hybrid-coil systems for many private telephone and intercommunication applications. A further limitation exists in that hybrid-coil systems in no way provide for sudden changes in the impedance of the line such as take place on many private lines. For instance, if telephones are connected to the line in parallel, the impedance presented to the repeater may change radically with each additional unit that is connected to the line by the simple act of lifting the receiver.
PART III. OBJECTIVES OF THE INVESTIGATION.

Most of the private intercommunication systems using amplification are forced to use "push-to-talk" arrangements because their characteristics do not allow the use of currently available two-way amplifiers. The present investigation is concerned with the possibility of developing a two-way amplifier which overcomes some of the limitations inherent in the hybrid-coil systems, and which will remove the necessity of push-to-talk arrangements. The requirements of two common types of systems are in mind. One is the small private telephone system which does not justify elaborate balancing equipment. The other is the type of "intercom" system which uses a dynamic speaker as both receiver and microphone. In such a system, there is no separation of received and transmitted signal at the ends of the line. The nature of the electrical impedance of the dynamic speaker unit is such as to make line balancing networks impractical, if not impossible.

Two general approaches to the problem are visualized. First, to investigate the possibility of developing a system in which the amplifiers are switched by voice signal or other automatic means. This would remove the necessity of balancing systems, since only one amplifier channel would be operative at any given time. Other problems, however, present themselves.

The approach chosen as the purpose of this investigation is: to investigate the possibility of procuring electronically a balance that is relatively independent of load impedance. It is intended that this goal be achieved, if possible, with a system that uses standard tubes and components, and which requires a minimum of critical adjustment and maintenance.
PART IV. ANALYSIS OF SYSTEM REQUIREMENTS.

The first investigations of the possibility of obtaining electronic balance were made without an analysis of the requirements of the entire system. It was understood that the system requirements would be severe, and the investigations were directed toward discovering how much could be accomplished. Subsequent investigation has shown the requirements to be even more severe than expected. This is especially true in regard to reflected voltage, as shown below.

A block diagram of the system, which involves two amplifiers, is shown in figure 4. \( A_1 \) and \( A_2 \) represent the two amplifiers which may be of standard design. \( S_1 \) and \( S_2 \) represent the balancing output systems to couple the amplifiers and line together. Since it was expected that balancing would be done electronically, these include power amplification stages.

To analyze the behavior of the system, it has been convenient to define certain transmission coefficients. It should be emphasized that for steady-state analysis these will in general be complex quantities. The numerical subscripts merely distinguish between the same type of coefficient for the two channels of amplification. The definitions below are for these coefficients involving the subscript one; exactly similar definitions hold for the same coefficient with subscript two.

The factor \( A_1 \) is the voltage amplification of the amplifier so designated, and is the complex ratio of the output voltage to the input voltage of that amplifier when excited with sinusoidal voltage.

For the system \( S_1 \) there are three distinct transmission or amplification coefficients. These, in general, are complex, and the \( k \)'s may be greater or less than unity, according to the circuit involved.
**Figure 4. The General Two-Way System.**

**Figure 5. Reflected Voltage and Input Impedance.**
B_{i} may be called an unbalance factor. It is a measure of the portion of the amplified voltage e' which is fed back into the input of amplifier number two (see figure 4). If the balance were perfect, this factor would be zero. The factor $k_{01}$ is a measure of the amplification of signal voltage from $e^0$ to $e_0$. The factor $k_{11}$ is the ratio of the voltage fed in from the line to amplifier two to the line voltage. The nature of this input factor is indicated for $S_2$ in figure 4.

Consider the behavior of the system when a signal $e_5$ is fed into $S_2$ on the left and transmitted to the line at the right as $e_0$. These voltages are assumed to be sinusoidal. A study of figure 4 shows the following relationships to hold:

\begin{align*}
(1) & \quad e_3 = k_{e1} e' \\
(2) & \quad e' = A_1 (k_{i2} e_3 + B_2 e'') \\
(3) & \quad e'' = A_2 B_1 e' \\
\end{align*}

Substituting (3) in (2) and rearranging; then substituting for $e'$ in (1), we get the relationship:

\begin{equation}
A = \frac{e_3}{e_3} = \frac{A_1 k_{e1} k_{i2}}{(1 - A_1 A_2 B_1 B_2)}
\end{equation}

where $A$ is the overall amplification from left to right. Equation (4) is thus an expression for the effective voltage gain of the system. It is important to note the presence of the unbalance factors in the denominator in the role of the feedback factor of feedback circuits. Indeed, if the balance is not perfect, the system involves amplified feedback, which may be positive or negative. The feeding back of the unbalance voltage is the root of the problem.

Assuming for the moment that all the factors in the denominator are real and positive, it is apparent that the product $A_1 A_2 B_1 B_2$ must not
be allowed to approach unity. If this happens, the amplification of the system becomes very large, and according to the well-known theory of stability in feedback systems the system may become unstable. Unless careful design of the system prevents this factor (when complex) from being equal to or greater than unity as it becomes real, the system will be unstable and will oscillate or "sing". This can be prevented if each AB product is less than unity in magnitude. To see what this means, suppose that $B_1 = B_2$ (although this is not necessary), and that the voltage amplifications $A_1$ and $A_2$ are 100. Then the unbalance factor must be less than 0.01 or 1%. This is a stringent requirement.

A more serious situation exists in regard to the voltage "reflected" through the feedback arrangement. The amplified unbalance voltage ($e'$ in figure 4) is "thrown back" to the input terminals. In the steady state, this appears as a change in impedance from that which exists without such feedback. If the time delay is considerable with respect to the wave being amplified, this feedback signal may appear as an echo. The situation is analogous to that on a transmission line with reflected signals from the receiving end.

The reflected voltage may be expressed:

\[(5)\quad e_r = k_{oZ} e'' = k_{oZ} k_{ij} A_{ij} \frac{A_1 B_1 e_s}{(1 - A_1 A_2 B_1 B_2)} = R e_s\]

Figure 5 shows an equivalent circuit for expressing the effect on steady state impedance. The factor $R$ is that defined in (5) above. If $e_r$ is absent, the input impedance is $Z_1$. However, with $e_r$ present, the ratio of input voltage to input current is derived in the following expressions:

\[(6)\quad i_s = \frac{e_s - e_r}{Z_i} = \frac{e_s (1 - R)}{Z_i}\]
Expression (7) shows that the factor $H$ should be kept different from unity to prevent instability and to allow impedance match. It also indicates that if the unbalance factors vary, the input and output impedances of the system will vary. This requires that the coefficients $B$ be kept small.

Using the figures of the example above, we see from expression (5) that the factors $B$ should be less than 0.01 to keep the product $A_1A_2B_1B_2$ and the product $A_1B_1A_2$ sufficiently small. The situation is relieved somewhat by the fact that in most circuits tested the $K_0K_1$ product is somewhat less than unity. The requirement of 0.01% balance is very exacting and difficult of achievement. Since voltage gains of the order of 100 are desirable, the goal for the balancing system must be of the order of a small fraction of 1%. In addition, good control of phase shift should be incorporated in the amplifier system. Moreover, the balance must be maintained with considerable change in load impedance.

These requirements proved too stringent for any system investigated, and resulted finally in abandoning this approach to the problem. An analysis of typical circuits investigated is contained in the following section.
PART V. APPROACHES TO THE PROBLEM.

Most of the investigation hinged around an attempt to develop a circuit to perform the function indicated by the systems $S_1$ and $S_2$ of figure 4. It was recognized that if this were done, attention probably would have to be given to the phase shift and frequency response characteristics of the amplifiers $A_1$ and $A_2$, but except possibly for the input circuit of these, it was assumed that they could utilize standard circuit arrangements with slight modifications.

The first basic circuit that was investigated is outlined in figure 6. An informal test of this circuit, before organized investigation of the problem was begun, indicated that the ability of the circuit to maintain balance with a dynamic load is limited. The reason is apparent from a simple description of its operation.

A signal $e'$ is impressed on the grids of $T_1$ and $T_2$ of figure 6, resulting in a signal of the same polarity at the cathodes of these tubes. If $Z_{k2}$ is of the proper value, the two voltages are exactly equal, and the net signal applied to the input of $A_2$ is zero. If the balance is not perfect, then the difference of these two cathode voltages, represented as $E_0'$, appears at the input of the amplifier $A_2$. On the other hand, if a signal $e_{g}'$ is impressed across the cathode of $T_1$ from the line, this entire voltage is applied to the grid of the input tube of $A_2$. Except for cathode degeneration, which has the effect of lowering the effective mutual conductance of the input tube, the entire line signal $e_{g}'$ is amplified by $A_2$.

Difficulty arises when the total impedance in the cathode circuit of $T_1$ changes. This change is due almost entirely to change in load impedance. Under these circumstances, $Z_{k2}$ and $e_0$ are not equal, and the unbalance voltage becomes appreciable. Without modification, then, this
Figure 6. A possible balancing circuit.

Figure 7. The "phase-splitter" balancing circuit.
circuit is unsuitable for the problem as described above.

A much simplified circuit which accomplishes the same results is shown in figure 7. This system has the advantage of requiring only one tube to do the task accomplished by two tubes in the above system. Furthermore, it allows the input connections to $A_2$ to be standard, single-ended. The circuit is essentially a phase splitting device, with one side loaded. It will be referred to in the following discussion as a "phase-splitter". The load in the plate circuit (figure 7) is a dummy load. This means that part of the output energy is lost. It will be recalled that this was the case in the hybrid-coil systems described earlier. It is also true of the circuit of figure 6. Ordinarily this is no serious handicap, since the amounts of power involved are very small—of the order of one watt or less.

The operation of the circuit with resistance loads may be explained by considering the application of a small instantaneous signal potential to the grid. It is assumed that the potential of point B—namely the B supply voltage—does not change, and hence for purpose of analysis of signal voltages point B may be considered at ground potential. Also, in this and other analyses, linearity of tube characteristics is assumed. Let us suppose that $e_1$ goes positive. Then $e_{k1}$, which is the same as $e_o$, is positive, while $e_p$ becomes negative. If the ratio of $R_1$ to $R_2$ is the same as the ratio of the magnitudes of $e_p$ and $e_{k1}$, then no voltage change appears at the tap to be fed into $A_2$. On the other hand, if $e_1$ is zero and a signal $e_0'$ is fed in from the line, the signal will be transmitted through to $A_2$ as follows. Suppose $e_0'$ is positive. The cathode is made positive with respect to the grid, which decreases the plate current and allows $e_p$ to go positive. Both ends of the voltage divider, $R_1$ plus $R_2$ are driven positive, and the voltage at the tap is therefore positive.
Similar arguments hold for negative signal pulses. If the system is linear, simultaneous signals applied to grid and cathode result in linear superposition of the two effects. When load and dummy impedances are general impedances including reactive elements, the operation is complicated by phase shift or time delay considerations.

In this circuit, as in the previous one, difficulty arises if the impedance in the cathode circuit changes, for this changes the ratio of $a_k$ to $e_p$, which results in unbalance at the voltage divider tap. Much of the investigation herein reported was concerned with trying to find by analysis and laboratory experiment a way to minimize or compensate electronically for this effect with changing load.

The possibility of plate loading was considered also. In this case the problem of unbalance remains unchanged. In addition, the circuit is not so efficient in transmitting signals from the line to $A_2$, since the effects of cathode injection are missing. This is true especially if the plate resistance of the tube is high.

Simple tests indicated that relative phase shift between the dummy and actual load voltages is one of the most serious factors in causing unbalance. This matter is discussed in Appendix A, where it is demonstrated that no balancing of the magnitudes of the voltages is sufficient if phase shift is present.

The attempts to modify the phase splitter circuit may grouped under five basic approaches. The first two named below were straightforward modifications of the system described above. After a number of schemes were tried without marked success, a study was made of the basic phase splitting circuit to investigate the possibility of using other component voltages for obtaining balancing action. An investigation for the case
of pentodes with high plate resistance is reported in detail in
Appendix B. This investigation suggested the fourth approach. The
fifth approach to the problem did not involve the use of the phase
splitter, necessarily. After this approach had yielded some circuits
with interesting properties, the possibility was suggested that approach
number six below might bear some results.

Briefly, the attempts were to:

1. Compensate for unbalance electronically.
2. Put balancing action in separate tubes.
3. Balance the voltage from cathode to grid against the voltage
   from ground to plate, in pentodes.
4. Balance the voltage from grid to ground against the voltage
   from cathode to plate, in pentodes.
5. Develop an amplifier whose gain is independent of load
   impedance, and balance input against output.

6. Develop a two-way amplifier with constant input impedance,
   to be used as a load for the phase splitter. This is closely
   allied with 2, above, but involves a completely different idea
   in the two-way amplifier.

The following sections contain descriptions of some of the basic circuit
ideas investigated. In several cases, more detailed accounts of circuits
and their analyses may be found in the Appendices.

1. Electronic compensation for unbalance.

   The first investigation involved an attempt to find a means by which
   unbalance voltage might be compensated for. Considerable effort was
   expended in seeking satisfactory solutions, but very few circuits seemed
   even plausible. Balancing unwanted voltage posed no serious difficulties,
except that the same scheme for balancing unwanted voltage also balanced out voltage fed through from the cathode circuit as signal coming in from the line.

This is illustrated in one of the more plausible appearing schemes, shown in Figure 8. If it is assumed that plate resistance is high, as in the case of pentodes, the following relations may be written. Letting

\[ R_K = k R_P \]

(1) \[ \zeta p_1 = (\zeta_5 - \zeta P R_K) G_{m1} = \frac{\zeta_5 G_{m1}}{1 + R_K G_{m1}} \]

(2) \[ \zeta p_2 = \zeta p_1 R_K G_{m2} = \frac{\zeta_5 G_{m1} G_{m2} R_K}{1 + R_K G_{m1}} \]

(3) \[ \zeta p_3 = - (\zeta p_1 + \zeta p_2 + \zeta p_3) R_P G_{m3} \]

\[ = - (\zeta p_1 + \zeta p_2) \left( \frac{R_P G_{m3}}{1 + R_P G_{m3}} \right) \]

(4) \[ \zeta p = -(\zeta p_1 + \zeta p_2 + \zeta p_3) R_P \]

\[ = - \zeta_5 \left[ \frac{G_{m1} (1 + G_{m2} R_K)}{1 + G_{m1} R_K} \right] R_P \]

(5) \[ \zeta K = \zeta p_1 R_K \]

The unbalance voltage is proportional to the sum \( (\zeta K + \zeta p) \). A rearrangement of the above relationships yields the following:

(6) \[ \frac{\zeta p + \zeta K}{\zeta K} = \left( 1 - \frac{1}{k} \left( \frac{1 + k G_{m2} R_P}{1 + G_{m3} R_P} \right) \right) \]

This equation shows that if the products \( G_{m2} R_P \) and \( G_{m3} R_P \) are much greater than unity, the unbalance becomes increasingly small, even with considerable change in \( k \). The amount by which \( k \) differs from unity is a measure of unbalance. For the usual design values, the above products are of the order of unity. If the value one is substituted in expression (6) above,
Figure 8. Typical circuit for compensation electronically.

Figure 9. Balancing action in auxiliary tube.
the expression takes on the value $0.5(1 - 1/k)$. This is exactly one-half the value of the expression if the balancing tubes are not present. This may be seen from (6) by setting the mutual conductances equal to zero. Exactly the same result could be derived independently. Experience and the analysis of system requirements show that this is not sufficient compensation to meet the expected demands on the circuit.

It is possible to add amplification to this basic system. Added amplification would have the effect of increasing the effective $G_m$ of tubes 2 and 3 by the amount of the voltage amplification. The balance would be improved thereby. But a consideration of the behavior of the system when a signal is injected at the cathode shows the limitation of such a procedure (other than the complication of the circuit). For this situation, $e_3$ is zero, and $e_2$ becomes the negative of $e_k$. Under these conditions, manipulation of the equations written above yields the expression:

$$(7) \quad e_P = e_k \left( \frac{G_m1 - G_m2}{G_m3} \right)$$

This shows that if $G_m2$ and $G_m3$ become very much larger than $G_m1$, $e_P$ approaches the negative of $e_k$. This is exactly the condition of balancing out the desired signal voltage as well as the undesired unbalance voltage. Thus, the system is unsuitable for the task imposed upon it. Too much compensation balances out the desired voltage. Partial compensation not only fails to correct for unwanted unbalance, but also requires greater gain in $A_1$ or $A_2$ to make up for the decrease in line voltage fed through. Any improvement is at least partially nullified.

The limitations of the circuit described above were encountered in every plausible circuit examined. No way of correcting or circumventing these difficulties was discovered, and the approach was abandoned.
2. **Balancing action in auxiliary tube.**

Two considerations indicated the desirability of investigating this matter. First, by putting the balancing action in a small tube, an economy of audio power is possible. Since the input impedance of $A_1$ and $A_2$ (figure 4) may be made high, only a voltage balance is necessary, and this may well be done in a small tube using small currents. Second, it is conceivable that perhaps if the balancing action were in a separate tube, the balance might be isolated from the effect of load impedance changes.

Figure 9 illustrates a simple application of the first idea. $T_1$ is the power output tube, and $T_2$ is a small auxiliary tube in which the balancing voltage is developed in the resistor $R_{p2}$. The principle of operation is identical with that of the single tube phase splitter. Unfortunately, the circuit offers no improvement in the matter of unbalance with changing load.

A study was made of the possibility of making the balancing action in the auxiliary tube independent of the load. No circuit was discovered in which this is accomplished. To do so would require that both the signal from the cathode and that applied to the grid of the power tube be applied to the auxiliary tube. Since the ratio of these changes with change in load impedance, no combination of these voltages is satisfactory.

3. **Balancing of voltage from cathode to grid against the voltage from ground to plate in pentodes.**

The circuit of figure 10 serves to illustrate the possibilities and limitations of this approach. Briefly, the operation is as follows. Tube $T_2$ and its associated circuit is a linear, low-gain amplifier.
**Figure 10.** Circuit for balancing cathode to grid voltage vs. ground to plate voltage.

**Figure 11.** Impedance in transformer secondary (Fig. 10)
Cathode degeneration is used to keep the gain low and the circuit linear.
It also has the effect of making the output resistance higher than it
would be without feedback. This proves to be important. If \( e_{o1} \) were
proportional to \( e_g \), operation with an input signal \( e_b \) would be as follows.
The voltage \( e_{p2} \) is proportional to the voltage \( e_{o1} \) and hence to \( e_g \). With
proper transformer polarity, \( e_{p2} \) may be balanced against \( e_g \).

The difficulty arises in providing for feed through from the cathode
circuit. If the grid is allowed to float, injection of voltage at the
cathode results in negligible change in plate current, and hence negli¬
gible voltage is supplied to \( R_1 \) plus \( R_2 \). Because of the unilateral
nature of the \( T_2 \) stage, no signal is fed through to the bottom of that
voltage divider. Hence it is necessary to tie the control grid of \( T_1 \) to
ground through \( R_g \), as shown.

For feed through from the cathode circuit, \( R_g \) should be made small.
This results in the condition shown schematically in figure 11. The
impedance presented to the \( T_2 \) stage is essentially \( R_g \) in series with \( Z_L \).
Both of these are small. A change in the value of \( Z_L \) with change in
load is in effect a change in load on the \( T_2 \) stage. This results in a
change in amplification which allows an unbalance voltage \( e_b \) to appear.
Lowering the output impedance of \( T_2 \) minimizes the change in amplification
with change in load. However, this is no real solution, since lowering
\( Z_{o1} \) has the same effect on feedthrough from the cathode circuit as in¬
creasing \( R_g \).

The net result is that it proved impossible to select design values
which gave satisfactory balancing action, and at the same time gave
adequate feed through from the cathode. Essentially the same circuit
as that shown in figure 10 was tested in the laboratory. These tests
showed that phase shift in the transformer was sufficient to prevent satisfactory balance. Also, the assumed linearity of the tubes was sufficiently inadequate to have appreciable effect. This, as was expected, placed limitations on the balance that could be achieved.

4. Balancing of voltage from ground to grid against the voltage from cathode to plate in pentodes.

In Appendix B an examination is made of various possible balancing voltages. Reference to the table in the Appendix shows that voltage $e_{13}$ (ground to grid) was equal to $e_{2a}$ (cathode to plate), when the plate resistor is adjusted to make the voltage from ground to plate equal to the voltage from cathode to grid. This corresponds to unity for the value of $k$ in the table of Appendix B. A straightforward attempt to use these voltages is considered in the Appendix, with the result that no voltage could be fed through from the cathode circuit.

Figure 12 shows a modification of the above circuit to overcome, in part, that difficulty. The amplifier $T_2$ serves the purpose of reversing the phase of the voltage $e_{2a}$ and relating it to ground potential. This is expressed in the relationship on the figure:

$$(8) \quad e = -a_1 e_{2a}$$

The signal $e_{1B}$ is transmitted by the stage $C$ (which may be a cathode follower circuit), so that we may write:

$$(9) \quad e_{1B} = a_2 e_{1B}$$

Adjustment of $a_1$ in the circuit of $T_2$ achieves balance so that no signal is fed to the transformer. If a signal $e_{12}$ is injected into the cathode circuit, a portion of it appears across $R_g$ as $e_{2B}$. This portion will be balanced out by the change in $e_{1A}$. The remaining portion, which appears
Figure 12. Voltage 1-3 vs. 2-A.

Figure 13. Unity gain circuit.
across $R_o$, will be passed effectively by $T_2$. Since the signal cannot be fed through $C$ in this direction, none of the signal appears at $B$, except for a fraction across $R_1$ by voltage divider action. If $R_1$ is small, only a small fraction is thus fed back through $C$. Thus, a portion of the cathode signal is fed through to the transformer.

The circuit was not tested in the laboratory or analyzed further, since experience with other similar circuits had indicated reason for unbalance. For one thing, in order to get an appreciable fraction of the signal across $R_o$, it is necessary that $R_g$ be small with respect to $R_o$. This means that with varying load on the cathode circuit of $T_1$ the load and hence the amplification of $C$ changes. Hence, the ratio of $e_{13}$ to $e_{18}$ is not constant. This means, of course, that unbalance will occur with change in load impedance. It is evident also, that the circuit is somewhat complicated, which is a decided disadvantage.

5. **Balancing of input and output of constant gain amplifier.**

This method failed because of one important property of inverse feedback. It is usually assumed that inverse feedback improves frequency response of amplifiers by making the voltage gain somewhat independent of the magnitude of the load impedance. This is done, however, by lowering the effective impedance of the vacuum tube circuit, so that changes in load impedance have comparatively small regulation effects which appear as changes in voltage gain. If the impedance of the load is made equal to the new lowered impedance of the tube, the advantage disappears. With this new value of load impedance, percentage changes produce the same percentage changes in gain as occurred before applying feedback. This apparent from Thevenin's Theorem. Appendix D
contains a detailed consideration of this fact.

Before the above limitations of feedback were recognized, a great deal of laboratory work was done on the circuits described below. Although they did not serve the purpose intended, they exhibit some characteristics that make them of possible interest in other applications.

Figure 13 shows a much simplified version of the first important circuit studied. The tube $T_3$ tries to operate as a high gain pentode amplifier have a voltage gain of approximately $G_{m3}R_p$. This tube drives the cathode follower output stage $T_1$ which is coupled to the line. The output voltage is fed back to the grid of tube $T_2$, which also tries to act as a high gain amplifier with a gain $G_{m2}R_p$. Because of the inverse phase relationship, the signal plate currents tend to cancel out. The effect is amplified inverse feedback, which yields a gain of approximately unity. For high $G_{m2}R_p$ in tubes 2 and 3, the output tends to follow the input very closely. Since this is the case, proper adjustment of the ratio $R_1$ to $R_2$ results in balance at point 5. On the other hand, injection of a signal from the line at points 1-2 is not balanced. If $R_1$ is small compared to $R_1$ and $R_2$, then very little of this signal appears at point 5. The principal effect of the small signal that does appear is to modify the impedance at the cathode circuit.

Other than the general difficulty of impedance matching effects, this circuit has some serious disadvantages. It is desirable to have $R_p$ and the $G_m$ of tubes 2 and 3 large. However, $R_p$ must carry the dc plate current of both tubes. Moreover, tubes with large $G_m$ draw comparatively large plate currents, so that definite limitations were placed on design. Several schemes were tried, to overcome this limitation. The idea of using a pentode as a dynamic load resistor was abandoned because of the
necessity of using fixed voltages on the screen grid. It was also
taken to use a supplementary amplifier between $R_p$ and the grid of
$T_1$ (with due consideration for phase relationships). Such circuits
as were tested proved to be unstable. Had they not been unstable, the
result would have been tremendously low output impedance at the cathode
of $T_1$.

A second difficulty with this circuit is that the grids must be
capable of handling the entire output voltage swing, which must be
comparatively large if appreciable power is to be supplied to the load.
Laboratory experience showed that with the designs available, grid swing
must be limited to the order of one-half volt to prevent excessive second
harmonic distortion. This distortion could not be balanced out by
adjustment of the $R_1$ to $R_2$ ratio.

The circuit above was analyzed partially, but this analysis was
not completed because the circuit of figure 14 (shown in a much simplified
form) was discovered. This circuit has similar characteristics to those
of the circuit considered above, with considerable improvement at the
points of disadvantage of the latter.

This circuit differs from the circuit of figure 13 in that feedback
reduces the net grid swing of a single tube, rather than the net signal
current in a single resistor. The resultant behavior at the terminals
is almost identical. Besides the obvious advantage of reducing the number
of tubes, this circuit overcomes two of the major operating limitations
of the previous circuit. For one thing, $R_p$ carries the dc current of
only one tube, allowing larger resistance values for the same plate
supply voltage. Perhaps even more important is the fact that the grid
swing of $T_2$ is only a small fraction of the signal and output voltages,
Figure 14. Unity gain circuit.

Figure 15. Phase splitter loaded with two-way stage.
which reduces very significantly the difficulties due to non-linear distortion.

It was hoped that this circuit would provide the solution to the problem by providing a constant gain with varying load. Preliminary experiments showed this to be the case for loads which were of the order required to match to a cathode follower circuit. Good balance between output and input was obtained. But when an attempt was made to inject a signal at the cathode (point 2 in figure 14), it was found that the circuit loaded down the oscillator far more than was anticipated. The behavior was as if the impedance were extremely low. An analysis of the impedance showed this to be the case (see Appendix C). For the tubes and circuit values being used, the output impedance at the cathode proved to be of the order of 2 to 3 ohms. Furthermore, it was shown in the analysis that when the load impedance $Z_L$ was made of this order of magnitude, percentage changes in load impedance were as effective in producing percentage changes in gain as if there were impedance match with no feedback. This characteristic seems to be common to all circuits, and is in keeping with Thévenin's Theorem.

Although the circuit of figure 14 did not prove to be of direct importance in the solution of the problem under investigation—except to point out emphatically the limitations of feedback as a tool— it is believed that the circuit may well be of some importance in other applications. For one thing, it could lend itself readily to applications involving driving single-ended power amplifiers in which considerable grid currents flow for a part of each cycle. Whereas ordinary transformers reduce output impedance at the expense of voltage output, this circuit does not lose driving voltage. Actually, it is a power amplifier with
unity voltage gain (for $R_1$ equal to $R_2$). It is the same sort of device as the cathode follower, except that it achieves values of impedance not ordinarily obtainable with a cathode follower. Like the latter, however, it suffers definite limitations in the amount of power which it can put out.

Even in circuits where transformer output is desirable or necessary, the circuit seems to have some advantages for low power work. Extremely low output impedances may be obtained with nominal turn ratios on the transformers. This simplifies transformer design, somewhat, especially in the matter of maintaining low frequency response by keeping the inductance of the windings reasonably high. The feedback tie can come from the secondary of the transformer as well as from the cathode, which for many applications would reduce some of the undesirable effects of the transformer.

6. Use of a two-way amplifier with constant input impedance as the load for the phase splitter.

Development of the constant gain amplifier described in the preceding section suggested the final approach attempted. It may be noted in figure 14 that the feedback tie allows a portion of a signal fed in either at the plate of $T_2$ or the cathode of $T_1$ to appear at the grid circuit. Since the input to a vacuum tube circuit operating class A is very high, it was thought that such a two-way amplifier might be used as a cathode load for the phase splitter. With the load of high and relatively constant impedance, the phase splitter could balance nicely.

Several possible circuit combinations, including that of figure 14 were studied. However, certain difficulties were common to all of these. Figure 16 will help to illustrate some of these.
Two difficulties present themselves. As shown in Appendix C, the input impedance to $T_2$ is not constant but is given by the expression:

$$ R_i = (R_1 + R_2) \frac{e_{o3}}{e_{o3} - e_{o1}}. \tag{10} $$

Since the ratio of $e_{o3}$ to $e_{o1}$ (due to the slight difference in notation of Figure 15 from that in the Appendix) is not constant, the input impedance which serves to load the phase splitter varies with the load present to the unity gain amplifier.

A further limitation of the scheme is apparent when it is recognized that the signal originating in the unity gain amplifier and appearing as $e_{o2}$ is subjected to voltage divider action as it is transmitted to the phase splitter, $T_3$. The effective signal appearing at the cathode of $T_3$ is

$$ e_{o3} = e_{o1} \frac{R_{o3}}{(R_1 + R_2 + R_{o3})}. \tag{11} $$

This makes it desirable to keep $R_1$ and $R_2$ small. But equation (10) shows this involves loading down the cathode circuit of $T_3$, which makes percentage variations of $R_1$ of serious consequence.

Other possible circuits showed the same difficulties. The principle on which this approach fails is general in nature. In order that an amplifier operate bilaterally, there must be a feedback tie. The presence of this tie results in changes in output voltage reflecting to modify the input impedance. This has generally been recognized in regeneration theory as applied to oscillators but its importance for most amplifiers is so slight that it is not commonly accounted for except in the case of the cathode follower. In that case, it is a decided advantage.
PART VI. CONCLUSIONS.

The requirements of the proposed system proved to be too severe for any circuit arrangement studied. The principal problem centered in the requirement that the system should be capable of operating with a variable load impedance. No system was discovered which would compensate satisfactorily for such a change.

It was anticipated that inverse feedback would play a major role in the solution. However, certain effects of feedback which are usually considered secondary served to nullify the improvements. The usual treatments of inverse feedback, as found in the literature, emphasize the fact that they improve circuits by making them relatively independent of tube characteristics and of changes in load impedance. However, the changes in output and input impedance which accompany feedback proved to be of such a nature and magnitude as to prevent the necessary improvement. In other words, the presence of a feedback tie produced unwanted as well as desired effects, and the former were too severe to be neglected.

The present evidence does not warrant a positive statement that satisfactory solution of the problem is impossible. Yet there is evidence that the general nature of the solution as conceived herein may well violate fundamental principles. In all probability, any satisfactory solution of the problem, if it exists, must lie in a direction different than that explored in this investigation. What the nature of this solution may be is not apparent to the writer at this time.

There are other possibilities for simultaneous transmission in either direction on a single line besides the carrier and the balanced repeater methods. One possibility which suggests itself is that of switching or reversing the amplifier electronically at a supersonic rate, thus allowing simultaneous two-way transmission of signals of audible frequencies. Such
a system, if the problems could be solved practically and economically, would suffer some serious limitations. For one thing, transmission would be limited to a fairly narrow band of audio and lower supersonic frequencies. The system probably would not be suitable for high fidelity transmission. This would not be too serious a limitation for some applications, if the circuits were simple and dependable enough. However, this problem was not formally investigated, for it lies outside the scope of the problem set for this investigation.

Probably the most significant positive result of this investigation was the development of the unity gain amplifier of figure 14. The unusually low output impedance of this circuit suggest several possible uses, where a stiff source of relatively low power handling capacity is desired.

Also, the phase-splitter balancer circuit may find application in certain types of coupler circuits or anti-sidetone circuits in which the balance requirements are not as severe as those for repeater operation.
APPENDIX A. ANALYSIS OF THE PHASE-SPLITTER CIRCUIT.

The relationships listed below refer to the notation of figure A-1. In addition to that defined by the figure, the following notation will be used:

- \( Z_L' \) = the combined impedance of \( Z_L \) in parallel with \( R_p \).
- \( G_m \) = the transconductance or mutual conductance of the tube.
- \( r_p \) = the dynamic plate resistance of the tube.
- \( \mu \) = the amplification factor of the tube, taken to be \( \frac{G_m R_p}{r_p} \).

Making use of the usual linear approximations of electronic circuit analysis, we may set down the following relationships for the circuit of figure A-1:

1. \[ e_o = \lambda_p Z_L' \]
2. \[ \lambda_p = G_m e_3 - \frac{\lambda_p (R_p + Z_L')}{r_p} \]
3. \[ e_p = -\lambda_p R_p \]
4. \[ e_3 = e_s - e_o \]

By proper substitutions and rearrangements in equations (1) through (4) we obtain the relationship

5. \[ \frac{e_o}{e_3} = \frac{Z_L' G_m R_p}{R_p + r_p + (G_m R_p + 1) Z_L'} = \frac{Z_L' \mu}{R_p + r_p + \mu + 1) Z_L'} \]

This ratio is the \( k_o \) of figure 4. If \( R_p \) is zero, this expression reduces to the well known expression for the voltage gain of a cathode follower.

By inspection we see that the ratio

6. \[ \frac{e_o}{e_p} = -\frac{Z_L'}{R_p} \]

must hold, since the same current flows through each impedance. The
Figure A-1. Phase-splitter.

Figure A-2. Unbalance due to phase shift

Figure A-3.
situation is somewhat more complex in the case of pentodes, since the screen current flows in the cathode circuit but not in the plate circuit.

For balancing the two voltages, $e_o$ and $e_p$, it is necessary that they satisfy two conditions:

(a) Their ratio must be constant in magnitude.

(b) The ratio must be negative and real.

The relationship of these two voltages to each other and to the unbalance voltage, $e_b$, is shown in figure A-2a. We may express this relationship mathematically as follows:

$$e_b = e_o - \frac{e_o - e_p}{R_1 + R_2} R_2 = \frac{e_p R_2 + e_o R_1}{R_1 + R_2}$$

For perfect balance, it is necessary that the numerator of the last member in expression (7) should vanish. The vector diagram of figure A-2b shows that balance is not possible if $e_o$ and $-e_p$ are not in phase. Laboratory experience confirmed this and indicated that phase shift may be the principal offender in most cases of unbalance.

Since an approximation to impedance matching is necessary, it is desirable to know the effective output impedance of the circuit. This impedance may be derived analytically by holding $e_s$ at zero and applying a voltage at the cathode. We shall call this $e_o$ as before, although this now represents an impressed voltage. Thus, in figure A-1,

$$Z_o = \frac{e_o}{i_k}$$

In addition to (8) and expressions (1) to (4), we may write the following:

$$i_p = i_k + i_L$$

$$e_o = i_k R_k$$
Again by a process of substitution and rearrangement we obtain the desired result:

\[
Z_o = \frac{R_K(R_P + r_p)}{R_K(Gm r_p + 1) + R_P + r_p} = \frac{R_K(R_P + r_P)}{R_K(\mu + 1) + R_P + r_P}
\]

If \( r_P \) is large, as in the case of pentodes, (11) simplifies approximately to:

\[
Z_o = \frac{R_K}{R_K Gm + 1} \quad r_P \gg R_P, R_K
\]

If we allow \( R_K \) to become infinite, we obtain the output impedance of the tube alone, which is approximately the reciprocal of the mutual conductance, as in the case of the simple cathode follower.

Since plate loading was considered in some circuits, it was necessary to know the output impedance of the plate circuit. It is assumed that \( Z_k' \) is now merely \( R_p \) (figure A-1). Referring to figure A-3, we may write:

\[
Z_o' = \frac{E_p}{\xi_L'}
\]

\[
\xi_r + \xi_L' = \xi_P = -\frac{E_p}{R_P} + \xi_L'
\]

\[
\xi_P = -\xi_P R_K G_m + \frac{E_p - \xi_P R_K}{r_P}
\]

When proper combination is made, these yield:

\[
Z_o' = R_P \left[ \frac{r_P + R_K(Gm r_P + 1)}{R_K(Gm r_P + 1) + R_P + R_P} \right]
\]

By allowing \( R_P \) to become infinite in the above expressions we obtain the output impedance of the tube alone:

\[
Z_o' = r_P' = \left[ r_P + R_K(Gm r_P + 1) \right]
\]
APPENDIX B. EXAMINATION OF POSSIBILITIES FOR BALANCE VOLTAGES IN PENTODES.

In order to investigate possibilities for balance voltages, a numerical investigation was made, under certain assumptions. First, linearity was assumed. Second, it was assumed that plate current depends only on voltage from cathode to control grid—see expression (2) below. Third, it was assumed that screen current is proportional to control grid voltage.

Referring to figure B-1, we may write the following relations:

(1) \( e_{z3} = e_{13} - e_{12} \)
(2) \( e_{1A} = -k e_{z3} \)
(3) \( e_{zA} = e_{1A} - e_{12} \)
(4) \( e_{3A} = e_{1A} - e_{13} \)

If \( e_{13} \) is the independent variable, \( e_{12} \) depends upon the cathode load. The factor \( k \) in (2) above depends upon the mutual conductance of the tube and the value of the resistor, \( R_p \).

The procedure of the analysis is as follows. Assume that the plate resistor, \( R_p \), is adjusted to give balance for a resistance load in the cathode circuit equal to the output impedance of the tube. Then variations in \( e_{12} \) (such as would occur with change in cathode load) are assumed, and the resulting voltages are compared. The \( e \)'s are assumed to be sinusoidal. Some results are tabulated below, for comparison.
Figure B-1. Balance possibilities.

Figure B-2. Circuit balancing $e_{13}$ vs. $e_{2A}$
Table I. Balance voltage possibilities. Refer to figure B-1.

a. Input = $e_{13}$

<table>
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<tr>
<th></th>
<th>$e_{13}$</th>
<th>$e_{12}$</th>
<th>$e_{23}$</th>
<th>$e_{1A}$</th>
<th>$e_{2A}$</th>
<th>$e_{3A}$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+2</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>+2</td>
<td>+0.8</td>
<td>+1.2</td>
<td>-1.2</td>
<td>-2</td>
<td>-3.2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>+2</td>
<td>+1.2</td>
<td>+0.8</td>
<td>-0.8</td>
<td>-2</td>
<td>-2.6</td>
<td>1</td>
</tr>
<tr>
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<td>1-j0.2</td>
<td>-1+j0.2</td>
<td>-2</td>
<td>-3+j0.2</td>
<td>1</td>
</tr>
<tr>
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<td>+2</td>
<td>1-j0.2</td>
<td>1+j0.2</td>
<td>-1-j0.2</td>
<td>-2</td>
<td>-3-j0.2</td>
<td>1</td>
</tr>
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<td>-0.5-j0.2</td>
<td>-1.5</td>
<td>-2-j0.2</td>
<td>1</td>
</tr>
<tr>
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<td>1-j0.2</td>
<td>-2+j0.4</td>
<td>-3+j0.2</td>
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<td>1-j0.5</td>
<td>-2+j1.0</td>
<td>-2.5-j0.5</td>
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</table>

b. Input = $e_{12}$

<table>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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<td>0</td>
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<td>1</td>
</tr>
<tr>
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<td>+1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>--</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
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<td>+1</td>
<td>-0.5</td>
<td>+0.5</td>
<td>-0.5</td>
<td>--</td>
<td>1</td>
</tr>
</tbody>
</table>

Line 1 in the table above shows the assumed condition for balance of $e_{12}$ and $e_{1A}$, which is the condition upon which phase-splitter operation is based. The succeeding lines assume various conditions of unbalance as shown in the $e_{12}$ column. With $k$ equal to one, it is evident that $e_{23}$ and $e_{1A}$ are equal and opposite (actually assumed), and that $e_{13}$ and $e_{2A}$ were balanced. With $k$ other than unity, phase shift is not completely cared for in the latter, as shown in lines 7 and 8.

The last three lines, 9 through 11, show conditions for a signal input from points 1 and 2, which corresponds to a signal coming in from the line. If $e_{13}$ is zero, $e_{2A}$ remains zero, which complicates an attempt to balance $e_{13}$ against $e_{2A}$ directly. Before this was determined analytically,
a circuit balancing those voltages was tried experimentally in the laboratory. The results were as indicated above. No signal was fed through from the cathode.

Figure 8-2 shows the elements of this circuit. T2 is a unity gain amplifier stage employing cathode degeneration to make the circuit linear and independent of voltage from cathode to plate. An extension of this circuit is shown in figure 8 in the main body of the report.
APPENDIX C. ANALYSIS OF UNITY GAIN CIRCUIT.

The circuit of figure C-1 proved to have some interesting and potentially useful characteristics which make it worth studying in detail. A linear analysis is made below, with symbols as defined on figure C-1. Currents drawn by the divider $R_1$ plus $R_2$ are neglected. From an inspection of the circuit diagram, the following relations may be deduced:

1. $i_{p_1} = G_{m_1} e_{g_1}$

2. $e_{g_1} = e_s - (e_3 - e_{o_2}) \frac{R_2}{R_1 + R_2} = e_s k + e_{o_2} (1 - k)$
   where $k = \frac{R_1}{R_1 + R_2}$

3. $i_{p_2} = (e_{o_1} - e_{o_2}) G_{m_2} - \frac{e_{o_2}}{r_{p_2}} = e_{o_1} G_{m_2} - e_{o_2} (G_{m_2} + \frac{1}{r_{p_2}})$

4. $e_{o_2} = i_{p_2} Z_k'$

5. $e_{o_1} = -i_{p_1} Z_L'$

By a rearrangement of the above expressions, we may derive the following one for the voltage gain of the circuit:

6. $A_L = \frac{e_{o_2}}{e_S} = -\frac{k G_{m_1} G_{m_2} Z_k' Z_L' Z_k'}{[1 + Z_k' G_{m_2} (1 + (1 - k) Z_L' G_{m_1}) + \frac{Z_k'}{r_{p_2}}]}$

In the case of cathode loading, it is desirable to make $Z_L'$ equal to $R_0$ and as large as possible. Ordinarily, this value of $R_0$ is considerably larger than $Z_k'$. Considering the following rearrangement of (6)

6a. $A_L = \frac{k G_{m_1} G_{m_2}}{\left[ \frac{1}{Z_k' Z_L'} + G_{m_2} \left( \frac{1}{Z_L'} + (1 - k) G_{m_1} + \frac{1}{Z_L' r_{p_2}} \right) \right]}$

we see that these lead to the approximate relation for gain shown below.
Figure C-1. Unity Gain Circuit.
(6b) \[ A_2 = \frac{k}{1-k} = 1 \quad \text{for } k = 0.8 \]

From (6a) and (6b) it is apparent that changes in \( Z_k' \) have very little effect on the voltage gain. In practical circuits, \( C_m R_o \) may have values of the order of 200. \( R_p \) may have values in the range 50,000 to 100,000 ohms. These values render the gain very constant for considerable variations in \( Z_k' \).

The reason for this constancy of gain is apparent when the output impedance at the cathode is evaluated. In addition to the relations (1) to (5) above, we note in figure C-1 that:

\[(7) \quad jL_2 = j\rho_Z - j\kappa \]

From these we derive the following expression for output impedance at the cathode:

\[(8) \quad R_o = -\left(\frac{e_{oZ}}{I_{LZ}}\right)_{r_z=0} = \frac{R_k}{1 + \frac{R_k}{\rho_Z} + R_k G_m Z_L} \]

If we use the following values of the parameters which may be considered representative: \( R_k = 1000 \) ohms; \( G_m Z_L' = 200 \); \( G_{m2} = 5000 \) microhms; \( \rho_Z = 5000 \) ohms; and \( k = 0.8 \); we find the output impedance to be approximately two ohms.

If we consider the tube alone, which may be done by allowing \( R_k \) to become infinite, we get the simplified expression:

\[(8a) \quad R_o' = \frac{1}{\frac{1}{\rho_Z} + G_m Z_L} \]

This yields approximately the same value of output impedance, since the former value was essentially the tube impedance (effective) in parallel with \( R_k \).
The expression for gain at the plate of tube 1 was found to be

\[ A_1 = \frac{E_{o1}}{E_{s1}} = -\frac{k Z' L' G_{m1}}{1 + (1-k) \frac{Z' L' Z_k' G_{m1} G_{m2}}{1 + Z_k' (G_{m2} + \frac{1}{r_{p2}})}} \]

For large \( Z_k' \), which is normal for plate loading, the expression reduces to the following approximation:

\[ (9a) \quad A_1 = -\frac{k Z' L' G_{m1}}{1 + (1-k) Z' L' G_{m1}} \]

which approaches a maximum of

\[ (9b) \quad A_1 = -\frac{k}{1-k} = -1 \quad \text{for} \quad k = 0.5 \]

A little study will show that the gain \( A_1 \) at the plate is not as independent of \( Z_k' \) as was the gain \( A_2 \) of \( Z_k' \). This shows up also, in the fact that the output impedance at the plate, \( R_{o1} \), is not as low as \( R_{o2} \).

The expression for output impedance at the plate was derived to be:

\[ (10) \quad R_{o1} = -\left( \frac{E_{o1}}{Z_{L1}} \right) e_{s1} = \frac{R_p \left[ 1 + Z_k' (G_{m2} + \frac{1}{r_{p2}}) \right]}{1 + Z_k' (G_{m2} + \frac{1}{r_{p2}}) + k G_{m1} R_p G_{m2} Z_k'} \]

If we allow \( R_p \) to become infinite, we get the output impedance of the tube alone. The expression simplifies to

\[ (10a) \quad R_{o1}' = \frac{1 + Z_k' (G_{m2} + \frac{1}{r_{p2}})}{k G_{m1} G_{m2} Z_k'} \]

Substituting the following representative values: \( R_k = Z_k' = 1000 \text{ ohms} \); \( r_{p2} = 10,000 \text{ ohms} \); \( k = 0.5 \); \( G_{m1} = G_{m2} = 5000 \text{ microhms} \), we get the result that \( R_{o1} \) is approximately 480 ohms. For normal purposes, then, plate loading has no advantage over a single cathode follower circuit.

Some possible applications were considered where this was not so; e.g., where bilateral transmission was desired.
As explained in the body of the report, it was desired to use this system as a two-way stage to serve as the load for a phase-splitter. In that connection the input impedance was of prime importance. Neglecting any possible grid current of tube 1, we have:

\[
(11) \quad R_i = \frac{E_s}{\lambda_5} = \frac{(R_1 + R_2)E_s}{E_s - E_0}
\]

For cathode loading of tube 2, the gain is relatively independent of load impedance, unless the load is made small enough to match approximately the output impedance of the tube. Since this condition was necessary for the purposes considered, the gain was not independent of load changes, as a study of equation (6) will show. Equation (11) shows that a change in voltage gain—i.e., a change in the ratio of \(e_{02}\) to \(e_3\)—results in a change in input impedance, \(R_i\). This is actually another example of the situation considered early in the report and symbolized in figure 5.
APPENDIX D. IMPEDANCE MATCHING AND VARIATION OF GAIN WITH LOAD.

The fact that inverse feedback changes the effective output impedance of a vacuum tube is well known. It is recognized that improvement of frequency response is a direct result of this, although the fact has not received sufficient emphasis in the literature. This may be appreciated by use of Thévenin's theorem and the usual equivalent circuit of an amplifier. One direct consequence is that if the load impedance is made equal to the effective internal impedance, the effect of impedance variation on voltage gain is exactly the same as if there were no feedback but impedance match. This may be made clear by simple examples.

Consider first the simple cathode follower circuit of Figure 2-1. We may write the well known expression for the output impedance as follows:

\[ R_o = \frac{1}{G_m + \frac{1}{r_p}} \]

Similarly the expression for voltage gain is:

\[ A = \frac{G_m R_k}{1 + R_k \left( G_m + \frac{1}{r_p} \right)} \]

Calculation shows that the results are not affected in principle if we neglect the term \( 1/r_p \), so that for our purposes we may write simply

\[ R_o = \frac{1}{G_m} \quad (1a) \quad A = \frac{G_m R_k}{1 + G_m R_k} \quad (2a) \]

If we let the cathode load \( R_k \) equal the output impedance \( R_o \), we find that the voltage gain is 0.5. Now if we let \( R_k \) equal to \( R_o (1+k) \), simple calculation shows the result to be:

\[ A = \frac{1 + k}{2 + k} \quad (2b) \]
**Figure D-1. Cathode Follower.**

**Figure D-2. Cathode Follower Equivalent Circuit.**

**Figure D-3. Triode Equivalent Circuit.**
This is exactly the result indicated by Thévenin's theorem, which says we may treat the system as a single source with impedance $R_0$ and generated voltage $e_s$. The value of the voltage is found by allowing $R_x$ to become infinite in expression (2), which makes the gain unity and the open circuit voltage equal to $e_s$. Simple calculation on the basis of the equivalent circuit, figure 5-2, shows that for the conditions assumed above identical results are obtained.

Moreover, if we compare the results with those for a simple triode amplifier whose equivalent circuit is given in 5-3, we find similar conditions. A percentage change in voltage gain with a given percentage in load impedance away from the condition of impedance match is identical in the two cases. This is obvious from a comparison of the equivalent circuits.

Calculations for other circuits show similar results. In the above discussion pure resistance loads were considered. Obviously, similar results would be obtained for general impedances containing reactive elements. Also, the discussion does not depend upon the cause of variation of load impedance. Usually, this is a variation of impedance with frequency, but other reasons for change are quite possible.
REFERENCES


