THESIS

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THE PHOTOELASTIC INVESTIGATION OF STRESSES IN A WEDGE UNDER COMPRESSION LOADING.

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The object of this investigation is to determine the nature of the stresses occurring in a wedge when a load is applied to the apex. This problem is of importance in the design of cutters for rock cutting bits. The cutters of such bits have wedge shaped teeth around the periphery of a cylinder or cone. As the cutter rolls over the rock, each tooth in turn acts as a pivot for the cutter and is loaded first on one side, and then on the other. The spacing of the teeth is such that the individual tooth cannot be greatly tilted before the next tooth touches the rock and assumed its share of the load. Consequently, the stresses involved in the loading of the tooth as a cantilever will not be considered.

This problem was attacked by use of the method of photoelasticity. The history and theory of this method will not be given here, since this phase of the work has been thoroughly dealt with by numerous writers, notably Filon and Coker. In all of the writings consulted many of the details of practical procedure were omitted. In this paper, a more detailed account will be given, in the hope that it will be of service to persons undertaking for the first time work in this branch of applied mechanics.

The equipment used consisted of an optical bench, Nicol's prisms, quarter wave plates, lenses, camera, lights, loading frames, and bakelite models. The bench was built of a 3" structural H-Beam ten feet long, mounted on supports. The different elements of the optical train were assembled
on the bed, as shown diagramatically in Figure 1.

\[ L \] is the light source, \[ M \] is a condensing lens for converging the light from \( L \), \( P \) is the polarizing Nicol, \( Q \) are quarter wave plates, \( M \) and \( Q \) are large lenses of about 5 foot focal length, \( F \) is the loading frame, \( A \) is the analyzing Nicol and \( C \) the camera. - Two setups were used, one for employing white light and one for monochromatic light, the use of which will be explained later.

The white light used was an ordinary automobile headlight bulb in a suitable lantern. - The monochromatic light source was a mercury arc lamp supplied with a filter which transmitted a monochromatic light of about 5461 Å wave length. - This light produces the green line of the spectrum. - The two large lenses are arranged so that the light passing between them is a parallel beam. - This is necessary to prevent distortion of the images. - The Nicols were located at the focii of the lenses in order to get a maximum of light through them. - Two loading frames were used, one for calibration, and one for the tests.
The most essential factor in the successful use of photoelasticity is the preparation of the models. The material in this work was Bakelite of .309 inches thickness. Bakelite is about 5 times as sensitive to the stress optical effect as celluloid; but unfortunately, due to the high pressure used in its fabrication, it contains obstinate internal cooling strains. These strains must be removed by annealing. The presence of these strains can be easily detected by examining the models between crossed Nicols. When the strains have been removed, the whole model will appear uniformly opaque. It was found that the strains could be completely eliminated by heating in an oven to a temperature of between $85^\circ$ C. and $90^\circ$ C. for about one hour, and slowly cooling. Somewhat lower temperatures can be used, but in any case, the success of the process depends upon the retardation of the cooling. The time required for the models to reach room temperature should be at least six hours, or if possible even longer. The annealing of the models should be done after all machining and polishing have been completed, since these processes are likely to induce strains in the material. The edges of the models can be given a final trueing after the annealing by carefully grinding on emery paper.

The polishing of the Bakelite models was accomplished by grinding the surfaces on emery paper over a piece of plate glass. Four grades of emery paper were used: Nos. 1, 0, 00, and 0000. The models were first rubbed on paper No. 1, until scratches appeared in only one direction. These scratches were removed by rubbing at $90^\circ$ to them on paper No. 0.
This process was repeated on No. 00 and No. 0000 papers. The models were then buffed to give them a light polish, thus rendering them very transparent.

Two types of interference bands result from the stressing of Bakelite in a beam of polarized light. These are isoclinics and monochromatics. In the theory of elasticity, these bands are respectively, the lines along which the directions of the principal stress make constant angles with a given direction, and the lines along which the difference between the principal stresses is constant. The parametric equations of these lines are:

\[ \tan 2 \alpha = \frac{2\overline{xy}}{\overline{x}y - \overline{y}x} \quad (1) \]

for the isoclinic, and

\[ (\overline{x} - \overline{y})^2 + 4\overline{xy}^2 = R^2 \quad (2) \]

where \( \alpha \) and \( R \) are the parameters and the other quantities are stresses; \( \overline{x} \) being the stress in the \( x \) direction across a surface perpendicular to \( x \); \( \overline{y} \) the stress in the \( y \) direction across a surface perpendicular to \( y \); and \( \overline{xy} \) a stress in the \( y \) direction across a surface perpendicular to \( x \).

These two equations completely determine the stress conditions in a two-dimensional problem. The actual solution of these equations involves an integration process too complicated for engineering purposes. Coker has devised a graphical integration method (which will be explained by a specific problem) for handling these equations. Equation (2) can be written in a more convenient form, as follows:

\[ (F - q) = K \quad (3) \]
where $P$ and $Q$ are the principal stresses, and $K$ is the parameter which contains the wave length of the light, the thickness of the material of the model, the relative retardation of the light rays due to the different refractive indices of the two oppositely polarized rays, and the stress optical coefficient.

Since the shear in a stressed material is equal to $\frac{1}{4}(P - Q)$ it follows that the isochromatic lines are also the lines of constant shear.

Before discussing the use of the isoclinic and isochromatic bands, it is of interest to consider their optical significance. - The isochromatic bands are so called because they are lines along which the wave length of the light is constant. - If a stressed model is viewed in a beam of white polarized light, it appears covered with varicolored bands, or fringes, arranged in the order of the spectrum. - Each color follows along the directions of the isochromatic bands. - If the light used is monochromatic these bands follow the same pattern as the colored bands, but are merely black and white. - The black bands representing complete interference of the refracted light, and the white bands complete reinforcement.

When the principal axes of stress are parallel to the axes of the polarizer and the analyzer, all light colors are extinguished and a black band appears. - This band is an isoclinic, and as before stated, follows the direction of the principal stress. - From what has been said it will be seen that to form the isoclinic lines, plain polarized light must be used. - For this purpose the quarter wave plates shown in Fig. 1 are removed, and the axes of the Nicols set at $90^\circ$ to each other. - White
light is used because the isoclinics, which are black, are easily distinguishable from the isochromatics, which are colored. To completely map the isoclinics, the Nicols are rotated and lines drawn for different angles of the Nicols. In the maps of isoclinics made for the wedge, the Nicols were rotated at 5° intervals.

Since the isoclinics are dependent upon the angle of the polarizer and analyzer, they can be completely eliminated if circularly polarized light is used. For this purpose the quarter wave plates shown in Fig. 1 are used. The axes of these plates must be placed at 45° to the axes of the Nicols. The Polarizer and analyzer must be both right-handed, or both left-handed circular polarizers. Since the quarter wave plate is a true quarter wave for only one wave length of light, monochromatic light must be used. With this apparatus only the isochromatic bands will appear, and these can be easily photographed.

We now turn to the use of these bands for calculations. There are several methods for determining the stress optical coefficient and relative retardation in order to evaluate $K$ in equation (3). By far, the simplest method is by means of the calibration beam. This method consists of eliminating one of the principal stresses by means of a beam in pure bending. Figure (2) is a sketch of the beam used.
In equation (3) let \( P \) be the horizontal principal stress and \( q \) the vertical principal stress. For the loading condition shown, \( q \) will be zero in the middle portion of the bar. Then we have only the \( P \) stresses to evaluate.

The calibration beam was cut from the same slab of Bakelite as the other models. The cross section of the beam was 1.158" x .309". This gives a section modulus, \( \frac{I}{C} = .0691 \).

Several loads were hung on the ends of the beam, starting with light weights and increasing the load in order to watch the formation of the fringes. Four of these pictures are shown in Fig. 3.

**FIG. 3**

- \( L = 24.18 \) lb. on Each End
- \( L = 30.78 \) lb. on Each End
- \( L = 35.18 \) lb. on Each End
- \( L = 41.78 \) lb. on Each End
The central dark band in the pictures is the neutral axis of the beam. It will be noted that this band does not coincide with the geometrical center line of the beam, but is displaced slightly to one side, the tension side. This shows that there was an initial compressive stress in the beam. This can be compensated for in the following manner. The stress in the outer fibers can be computed from the dimensions in Fig. 2:

\[ S = \frac{Ma}{I} = \frac{1.719L}{0.069} = 24.86L \]  

(4)

Where \( L \) is the load on each end of the beam.

Let \( P \) be the stress in the outer fibers, \( P_e \) the initial stress, \( b \) half the depth of the beam, and \( e \) the distance between the neutral axis and the center line.

Referring to Figure 4, we have:

\[ P_e = \frac{e}{b} P \]  

(5)

Then, the stress on the compression side of the beam is equal to \( P + P_e \), and on the tension side is \( P - P_e \).

In order to minimize errors in scaling, the photographs of the beams were enlarged to twice the size, and the calibration made from these enlargements. Figure 4 is drawn to scale from the picture of the beam with 18 Kg. on each end. The horizontal lines are the dark bands of the photographs. It was found that the value of \( P_e \) was the same for all loads, and that all the pictures gave practically the same value for each stress.
fringe. - In plotting the calibration curve shown on page No. 24, only the average value obtained from the 15 Kg. load and 18 Kg. load were used. - The computations for the latter load are given below. - The weight of the supports for the test weights was 2.187 Lb. Then,

\[ L = 18 \times 2.2 = 41.8 \text{ Lb} \]

\[ S = 24.86 \times 41.8 \]

\[ = 1040 \text{ Lb/in}^2 \]

\[ b = .760 \text{ inches } e = .055 \text{ inches} \]

\[ P_e = 1040 \times \frac{.055}{.760} = 75.3 \text{ Lb/in}^2 \]

\[ P_t = 965 \text{ Lb/in}^2 \text{ in tension}, \]

\[ P_c = 1115 \text{ Lb/in}^2 \text{ in compression}. \]

These are values at the outer fibers and are laid off along AB and CD in Fig. 4. - The other points on the curve were obtained by scaling the lengths of the lines intercepted between AC and BD. - The fringes obtained from the pictures extend only to the fourth order, but fifth and sixth order fringes can be evaluated with reasonable accuracy by extending the curve beyond the last observed point.

In making the photoelastic tests, five kinds of wedges were used. - Three small wedges of 90°, 60° and 45° included angles were used for determining the effect of inclined loading. These wedges had nearly sharp points, about 1/64" being rounded off to prevent upsetting of the point. - Two large wedges of 60° included angle were used to show the effect of truncating the wedge by means of a radius at the point and by means of a flat
point. The method of loading the wedge is illustrated in Fig. 5.

The weight on the tongue $T$ was three times as great as $W$ due to the mechanical advantage of the lever $L$. The bearing plate $B$ was marked with $5^\circ$ graduations by means of which the inclination of the model $M$ could be read. Pictures were made for symmetrically load conditions and various angles of inclination. Many different combinations of weights and angles were photographed, both for the small and the large wedges. The ones given on the following pages were selected from a total of about 80 which were taken. Those not included in this paper were discarded because the fringe patterns showed internal strains; because of similarity to those shown; or because they had no particular bearing on the problem.
FIG. 6
60° Wedge
0° Inclination
12.81 Lb. Load

FIG. 7
60° Wedge
5° Inclination
12.81 Lb. Load

FIG. 8
60° Wedge
10° Inclination
12.81 Lb. Load

FIG. 9
60° Wedge
15° Inclination
12.81 Lb. Load

FIG. 10
60° Wedge
0° Inclination
23.81 Lb. Load

FIG. 11
60° Wedge
5° Inclination
23.81 Lb. Load
FIG. 12
15° Wedge
0 Inclination
12.81 Lb. Load

FIG. 13
15° Wedge
0 Inclination
23.81 Lb. Load

FIG. 14
15° Wedge
5° Inclination
12.81 Lb. Load

FIG. 15
15° Wedge
10° Inclination
12.81 Lb. Load

FIG. 16
15° Wedge
12° Inclination
12.81 Lb. Load

FIG. 17
30° Wedge
0 Inclination
12.81 Lb. Load
FIG. 18
90° Wedge
0° Inclination
23.81 lb. Load

FIG. 19
90° Wedge
5° Inclination
12.81 lb. Load

FIG. 20
90° Wedge
10° Inclination
12.81 lb. Load

FIG. 21
90° Wedge
15° Inclination
12.81 lb. Load

FIG. 22
Round Point Wedge
0° Inclination
12.81 lb. Load

FIG. 23
Flat Point Wedge
0° Inclination
12.81 lb. Load
**FIG. 24**
Round Point Wedge
0° Inclination
23.81 Lb. Load

**FIG. 25**
Round Point Wedge
0° Inclination
34.81 Lb. Load

**FIG. 26**
Flat Point Wedge
0° Inclination
23.81 Lb. Load

**FIG. 27**
Round Point Wedge
10° Inclination
23.81 Lb. Load

**FIG. 28**
Flat Point Wedge
10° Inclination
23.81 Lb. Load
It is interesting to note the intricate pattern of stress fringes in the supporting block in some of the pictures, e.g., Figs. 12, 13 and 17. In these pictures, the block used was an unannealed piece of Bakelite. Some idea of the internal strains can be had by comparing these pictures with Figs. 6 and 8, in which a carefully annealed block was used.

As stated in the earlier pages, these isochromatic fringes furnish only a part of the data necessary for a solution of the problem. Each fringe in the pattern is a region of a certain difference in principal stress.

From the calibration curve, we see that the first fringe represents a $P - Q$ of 280 lb./in.$^2$, the second fringe a $P - Q$ of 580 lb./in.$^2$, and so on. These fringes, however, do not give any intimation as to the actual value of either $P$ or $Q$. It is the purpose of the isoclinics to establish these values.

The isoclinics for a symmetrically loaded wedge, and a wedge inclined at $15^\circ$ to the vertical were sketched by focusing the image of the wedge on a piece of tracing paper attached to a pane of glass and tracing the lines with a pencil. These lines were photographed, but they could not be distinguished from the isochromatic fringes on black and white. Color photography is necessary to separate the black isoclinics from the colored isochromatics.

Figures 29 and 30 show the isoclinic sketches. It was found that isoclinics for the inclined wedge were quite similar to those for the symmetrically loaded wedge, except that they were more crowded on one side, and on the opposite side a
dark area appeared. – This dark area can also be seen on many of the isochromatic pictures. – Its significance will be brought out in a discussion of the fringes at the conclusion of this paper.

FIG. 29

FIG. 30

FIG. 31

FIG. 32
From these isoclines, the lines of principal stress are next constructed. This is accomplished by drawing short lines at intervals across each isoclinic, as shown in Fig. 31. These short lines are parallel to the axis of one of the Nicols and represent the tangents to the one family of principal stress lines. There will be another set of lines parallel to the other Nicol representing tangents to the other family of principal stress lines. The principal stress lines shown in Fig. 32 are mapped by following the tangents from one isoclinic to the next. The result is a set of radial lines through the apex and a set of concentric circles with the apex of the wedge as a center.

Having the map of the directions of the principal stresses, and knowing the value of \( P - Q \) for any point, we are now prepared to make a complete analysis of the stress distribution in the wedge. The graphical method due to Coker will be employed.

According to a solution derived by Meisanger, which will not be given here, the differential expressions for the principal stresses are:

\[
dP = (P - Q) \cot \beta d\phi \tag{5}
\]

\[
dq = (P - Q) \tan \beta d\phi \tag{6}
\]

In these equations \( P - Q \) are the values obtained from the isochromatics; \( \beta \) is the angle which each isoclinic in turn makes with the principal stress line along which the integration proceeds; and \( d\phi \) is the increment of the angles of the isoclinic. In the problem under consideration the isoclinics were drawn at 5° intervals. Thus \( d\phi \) is 5° or 0.0873 radians.

The graphical integration is taken in steps of...
from one isoclinic to the next along each principal stress line. This integration is expressed by means of the following summation:

$$P - P_0 = \sum_{n=0}^{N} \left( P - Q \right)_{n+\frac{1}{2}} \left( \alpha_{n+1} - \alpha_n \right) \cot \frac{1}{2} \left( \beta_n + \beta_{n+1} \right)$$

and a similar expression for $Q - Q_0$ in which $\cot \frac{1}{2} \left( \beta_n + \beta_{n+1} \right)$ is replaced by $\tan \frac{1}{2} \left( \beta_n + \beta_{n+1} \right)$. Here $(\alpha_{n+1} - \alpha_n) = d\phi$, and $P_0$ is the stress at the boundary which in this case is equal to zero. $(P - Q)_{n+\frac{1}{2}}$ is the value of $P - Q$ at the midpoints between the isochromatics. $\frac{1}{2} \left( \beta_n + \beta_{n+1} \right)$ is the average of the angles of two adjacent isoclinics and gives the angle at the midpoint represented by $(P - Q)_{n+\frac{1}{2}}$. These midpoints are used because they give a closer approximation.

The values of the quantities given in equation (7) were tabulated for six pairs of principal stress lines in the neighborhood of the point of the wedge. The angles were measured by superposing the principal stress map upon the isoclinic sketch. A sample sheet of these data for the $P$ stresses (see Fig.32) are shown on page 19. It was discovered in tabulating these results that $\cot \beta$ is zero in most cases, and nearly zero in the others. The closer to the point the principal stress lines were taken, the more closely $\cot \beta$ approached zero.

Using this value in equation (7) we learn that along the $P$ lines shown in Fig.32 the principal stress is zero. From this we infer that all of the principal stresses lie along the radial $Q$ lines. Assuming this to be the general law, it is unnecessary to proceed further with the integration. The stress $P$ is uniquely determined by the order of the stress fringe at any point, and its numerical value can be taken directly from the calibration curve.
### SAMPLE DATA SHEET FOR INTEGRATING ALONG $Q$-LINES.

<table>
<thead>
<tr>
<th>Isoclinic</th>
<th>Angle $\phi$ (Degrees)</th>
<th>$(\alpha_n - \alpha_n)$ Degrees</th>
<th>Slope of Isoclinic $\phi + \beta$ Degrees</th>
<th>Angle $\beta$ Degrees</th>
<th>$\frac{1}{2}(\beta_n + \beta_{n+1})$ (Degrees)</th>
<th>Cot. $\frac{1}{2}(\beta_n + \beta_{n+1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>0</td>
<td>5</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>5</td>
<td>5</td>
<td>95</td>
<td>90</td>
<td>90</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>10</td>
<td>5</td>
<td>100</td>
<td>90</td>
<td>90</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda_4$</td>
<td>15</td>
<td>5</td>
<td>105</td>
<td>90</td>
<td>90</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda_5$</td>
<td>20</td>
<td>5</td>
<td>110</td>
<td>90</td>
<td>90</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda_6$</td>
<td>25</td>
<td>5</td>
<td>114</td>
<td>89</td>
<td>89$\frac{1}{2}$</td>
<td>.0087</td>
</tr>
<tr>
<td>$\lambda_7$</td>
<td>30</td>
<td>5</td>
<td>119</td>
<td>89</td>
<td>89$\frac{1}{2}$</td>
<td>.0175</td>
</tr>
<tr>
<td>$\lambda_8$</td>
<td>-30</td>
<td>5</td>
<td>61</td>
<td>91</td>
<td>91</td>
<td>-0175</td>
</tr>
<tr>
<td>$\lambda_9$</td>
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<td>5</td>
<td>66</td>
<td>91</td>
<td>90$\frac{1}{2}$</td>
<td>-0087</td>
</tr>
<tr>
<td>$\lambda_{10}$</td>
<td>-20</td>
<td>5</td>
<td>70</td>
<td>90</td>
<td>90</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda_{11}$</td>
<td>-15</td>
<td>5</td>
<td>75</td>
<td>90</td>
<td>90</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda_{12}$</td>
<td>-10</td>
<td>5</td>
<td>80</td>
<td>90</td>
<td>90</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda_{13}$</td>
<td>-5</td>
<td>5</td>
<td>85</td>
<td>90</td>
<td>90</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda_{14}$</td>
<td>0</td>
<td>5</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>0</td>
</tr>
</tbody>
</table>
Using these values, curves on the following pages were plotted from the fringe patterns for the various wedges and loadings. All the curves were drawn for the edge of the wedge in which the greater compressive stress was produced. Similar curves could have been drawn for interior regions, but the internal stresses can never exceed the boundary stresses, so these curves would have no importance. The ordinates on the graphs are the distances from the apex and the abscissas are ratios between the stress at a given point to the average stress at the top of the wedge. This ratio was used rather than the actual stresses in order that direct comparisons for any load could be readily made. The points that were plotted were the intersections of the fringes with the edge of the wedge.

Examination of the fringe patterns reveals that the fringes are circular arcs, except as they approach the edges. It will be noted, particularly in the cases of the wedges inclined to the line of the load, that the fringes break sharply as they approach the edges. This is an inherent fault of the models, and numerous trials to correct it were unsuccessful. The models were carefully annealed several times, and the edges were accurately finished before and after annealing. The optical train was checked several times to be certain of alignment and focus. It is possible that further annealing would correct this fault.

In plotting the curves and in making calculations the centers of the circles were located, and the arcs projected to the edge of the wedge. To facilitate measuring the intersections of the fringes with the edges, maps of the fringe patterns were traced from the photographs, and used instead of the pictures.
Several important properties of the isochromatic fringes were discovered from a study of the photographs. As before stated, all the fringes are circular arcs, but what is more interesting is that the centers of all the arcs lie on a straight line, and the circles pass through the apex.

Consider first the symmetrically loaded wedge. The locus of centers of the fringes is a vertical line through the apex. Starting with the upper fringe and moving to the point the radius of the arc decreases, and the location of the centers approaches the apex. The limiting point for the center is at the apex. As the order of the fringes increases, and the center of the arcs moves toward the point of contact, the number of degrees of the arc intercepted by the sides of the wedge increase. This causes a greater crowding of the fringes at the edges of the wedges and shows that the stress along the edge increases at a greater rate than along internal radii.

We now examine the wedges inclined to the line of load. For all of these wedges the circles also pass through the apex of the wedge, however, the lines of centers are inclined to the axes of the wedges, and the greater the inclination of the wedge, the greater is the angle between the line of centers and the axis of the wedge. This relation is shown in the table on next page. Figure 33 shows the construction for the line of centers.
A line drawn perpendicular to the line of centers of the fringes and through the apex will will be tangent to all the fringes at the apex. - For small angles of inclination this perpendicular line will lie outside of the wedge. - As the inclination of the wedge to the load is increased a point is reached where the tangent line falls within the wedge. - In the pictures where this condition exists, the tangent line is represented by a dark fringe radiating from the apex. - This band represents a region of tensile stress. - It is the first order tensile stress fringe, and is somewhat distorted and wide at the upper part due to the effect of the corner and the condition of the loading. - This fringe fades out towards the apex for moderate angles, and disappears as the compression fringes crowd together at the contact point.

It can be seen from the photographs that in all the wedges tested the tension band appears at an angle of inclination somewhere between 5° and 10°. - The included angle of the wedge does not have any great effect upon the inclination necessary to appearance of the band. - On the 90° wedge inclined at 10° to the load, the construction of the line of centers is such that its normal appears to fall slightly outside of the wedge, but the picture shows a tension fringe. - This is due to inaccuracy of
construction and to the fact that the edge of the wedge is relatively short and thus is more influenced by the straight part of the model. Although not evident from the pictures, the construction of the line of centers and their normals indicates that the tension fringes appear in the smaller angle wedge at slightly smaller angles of inclination. From this we infer that as the angle of inclination of the wedge increases the bending moment in the slender wedge will be greater.

Turning now to the curves, we note several general properties. All of the curves are roughly rectangular Hyperbolas having as asymptotes the ordinate in the neighborhood of zero stress ratio and the abscissa of nearly zero distance from the apex. As the curves slope downward they converge, so that close to the point of the wedges an increase in the angle of inclination does not greatly increase the stress ratio. At greater distances from the apex the increase of inclination causes larger changes in stress. It will be noted that for a given distance from the apex the stress ratio in the wedges of smaller included angle are more readily increased by increase of the inclination. Furthermore, in the slender wedges the regions of given stress move further up the edge, therefore these wedges should be strengthened by generous fillets at the bases, since the stress concentrations in sharp corners would be higher.

Although fringes close to the point of contact are obscured, due to yielding of the material, the curves indicate a stress near the apex of probably 70 or 80 times the average stress at the top.

The effect of truncating the wedge by means of a radius
45° INCLUDED ANGLE WEDGE
LOAD 12.8 LBS.

STRESS RATIO

DISTANCE FROM APEX: INCHES

0° ANGLE
6° ANGLE
12° ANGLE
90° INCLUDED ANGLE WEDGE
LOAD 12.8 LBS

DISTANCE FROM Apex, Inches

STRESS RATIO

15° ANGLE
10° ANGLE
5° ANGLE
0° ANGLE
○ - ROUND POINT WEDGE
□ - FLAT POINT WEDGE

LOADS 23.8 LBS

DISTANCE FROM APEX - INCHES

STRESS RATIO

10° ANGLE

0° ANGLE
or flat point is shown by curves and photographs of the larger wedge. These curves show a general relation between the two types of wedges. In order to have a better comparison with the sharp pointed wedges and to facilitate measurements, the distances used in the round pointed wedge were obtained by drawing a line through the point of contact parallel to the edges and determining the distances of points of intersection of the fringes with this line from the point of contact. For the flat pointed wedge the distances along a line through the center of the flat edge for symmetrical loading were used. For the inclined flat point wedge the distances were taken along the edge from the corner in contact.

The curves show that the stress ratio is approximately the same on all the curves near the contact point. However, the round pointed wedge has lower stresses in regions removed from the contact point than the flat pointed wedge has when inclined. The flat pointed wedge has the lowest stress ratio when symmetrically loaded. The high stresses produced at the corners when inclined would soon cause them to become rounded, and thus approach the round point wedge.

The loads used on the large wedge were much greater than on the small wedge, and consequently many more fringes were formed. The pictures and curves show that the stress at the contact point is probably 100 times greater than the average at the load. On the wedge with the greatest load fringes up to the 12th order can be counted by extrapolating the calibration curve. This would indicate a stress of 3500 lb./in². Beyond the 12th fringe, the material has exceeded its yield point and
is opaque. - At the contact point the stress is considerably above the ordinary ultimate tensile stress. - The material is able to withstand these incredible stresses since it is supported on all sides and is in effect under hydrostatic pressure.

A discussion of the stresses in the supporting block is not given here since the problem of a flat plate with a concentrated load has been dealt with by several writers. - It will merely be mentioned that at equal distances in a vertical line from the point of contact, the stress is the same in both the wedge and the block.

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