ANGULAR DISTRIBUTIONS OF PROTONS AND DEUTERONS IN THE REACTIONS \(^{40}\text{Ar}(d,p)\,^{41}\text{Ar}\) AND \(^{40}\text{Ar}(d,d)\,^{40}\text{Ar}\)

BY

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I. INTRODUCTION

The deuteron stripping reaction has become a valuable tool in nuclear spectroscopy, because of the fact that the angular distribution of the emitted particle is sensitive to the angular momentum ($\ell$ value) with which the neutron or proton is captured. By determining this $\ell$ value, one obtains information about the total angular momentum of the residual nucleus, since only a very limited number of total angular momenta can be formed for any given $\ell$ value. Although it has usually been possible to identify the orbital angular momenta of the captured particles using the Butler expression for the stripping cross section, the assumptions made in deriving this expression were unsatisfactory. More rigorous expressions have been developed, such as the one given by Tobocman and Kalos. The present experiment was undertaken with the ultimate purpose of obtaining data by which the newer theory could be tested.

We have studied the angular distributions of the elastically scattered deuterons, and of three proton groups produced in the bombardment of $^{40}$A with 4 Mev deuterons. The three proton groups correspond to transitions to the ground state, the first excited state, and the fifth excited state of $^{41}$A. Energy level diagrams of $^{40}$A, $^{41}$A and the compound nucleus $^{42}$K are given in Figure 1.

Pollard studied the energy levels of the proton groups from this reaction in 1949, using a proportional counter, at a bombarding energy

1 S. T. Butler, Proc. Roy. Soc. (London) A208, 559 (1951); cf. also
ENERGY LEVELS

STUDIED TO 8.5 MEV

ENDT AND BRAAMS R.M.P. 294,683

Figure 1
of 3.4 Mev; and in 1952 Gibson and Thomas\textsuperscript{3} took angular distributions of the elastically scattered deuterons and of the first three proton groups, using photographic emulsions, at a bombarding energy of 7.8 Mev.

In 1955, Burrows and Green\textsuperscript{4} obtained angular distributions for the first eight proton groups, using magnetic analysis, at a bombarding energy of 8.5 Mev.

The conclusion of Burrow's work is that the neutrons captured to form the ground state of $^{41}A$ are captured with $\mathcal{L}=3$; and that the other seven proton groups correspond to neutrons captured with $\mathcal{L}=1$. The $\mathcal{L}=3$ conclusion agrees with the Mayer Shell Theory prediction that the ground state of $^{41}A$ should be an $f_{7/2}^{-}$ state. However, the agreement of Burrow's angular distribution for the ground state group with the $\mathcal{L}=3$ Butler curve is not much better than its agreement with the $\mathcal{L}=2$ curve, and he attributes the poor fit to compound nucleus formation.

Pollard worked at a deuteron energy of 3.4 Mev, and observed that the most intense proton groups were those corresponding to transitions to the third, fifth, sixth and eighth excited state of $^{41}A$ (excitations of 1.39 Mev, 2.46 Mev, 2.79 Mev, and 3.36 Mev), the intensities of the higher excited states being higher; whereas Burrows worked at a deuteron energy of 8.5 Mev and found that the most intense proton group of all was the one corresponding to transitions to the third excited state.

state of $A^{41}$, at 1.39 Mev. Thus, higher energy deuterons seem to lead preferentially to lower levels in $A^{41}$.

It has been generally noticed in d-p reactions that there tend to be states in the final nucleus at excitations over 1 Mev which are reached much more often than the ground state. Schiffer$^5$ observed such intense states in the neighborhood of 2.3 Mev excitation of the residual nucleus for targets of Cr, Mn and Fe, at a bombarding energy of 3.6 Mev.

The deuteron energy used in the present experiment was below the Coulomb barrier; and since the Butler expression for the d-p cross section can be derived from the Born approximation, which assumes that the bombarding energy is large compared to the interaction energy, the experimental proton angular distributions might be expected to deviate considerably from the Butler curves. However, it has been shown that as far as the position of the peak of the angular distribution is concerned, the nuclear distortion of the incoming deuteron waves and outgoing proton waves largely cancels the Coulomb effect$.^1$ Nevertheless, it is of interest to compare low energy experimental results with more rigorous d-p cross section expressions, such as the one given by Tobocman and Kalos, and this was the main motive for the present experiment.

The angular distribution of the elastically scattered deuterons is also of interest, and is a necessary measurement for comparison with the theory of Tobocman and Kalos. The angular distribution of the

deuterons has not been studied previously except by Gibson. His experiment, with a deuteron energy of 7.8 Mev, gave a ratio to Rutherford scattering greater than one at the backward angles.

Aside from the fact that there are competing channels, pronounced deviations from Rutherford scattering are to be expected at smaller energies and smaller angles in the case of deuteron scattering than in the case of proton scattering, for example, on account of the large size of the deuteron. Even at deuteron energies below the Coulomb barrier, the proton may come close enough to the nucleus that the neutron is affected by the nuclear force in such a way that the deuteron does not break up, but does deviate from Rutherford behavior.
II. EXPERIMENTAL PROCEDURE

The deuteron source used in this experiment was the 6 Mev Rice Institute Van de Graaff accelerator. The beam from the accelerator passes through a 90° analyzing magnet, and the bombarding energy is determined by using the phenomenon of nuclear magnetic resonance. In this method, the magnetic moment of a hydrogen probe, or a lithium probe, is exposed to the analyzer field, and to an alternating field produced by an oscillator coil. When the oscillator frequency is equal to the Larmor frequency of the magnetic moment of the sample, energy is absorbed from the coil. This frequency may be determined using an oscilloscope and a frequency meter. The bombarding energies corresponding to certain Larmor frequencies have been determined through analysis of elastically scattered protons in a 180° absolute magnetic spectrometer, and thus the bombarding energy for any beam can be determined, since it varies in a known way with the Larmor frequency, the particle charge and the particle mass.

An independent check on the bombarding energy is provided by the magnet current, since the energies have been determined for certain magnet currents from other methods, so that for any given beam, the energy for a given magnet current may be read from a graph.

In this way, the deuteron bombarding energy was determined to be 4.03 Mev.

\[ \text{[\ldots]} \]

\textit{\footnote{Kendall F. Famularo, Ph.D. Thesis, The Rice Institute, 1952.}}
Linde argon, at pressures ranging around .8 cm Hg, was the target gas. The target chamber was the large volume gas target scattering chamber described in the Master's Thesis of John L. Russell, modified by the addition of a second particle detector, described in the Master's Thesis of R. R. Henry. Both detectors are scintillation counters using thallium activated CsI crystals.

The particle beam does not pass through a foil in entering this chamber. The separation of the accelerator high vacuum from the pressure of the target gas in the chamber is accomplished instead by means of a differentially pumped tube containing many 2mm holes through which the beam passes. A 1.5mm hole at each end serves to define the beam.

In obtaining the cross section for a nuclear reaction, one must
determine the following quantities:

(1) The number of particles received in a known small solid angle by the detector.

(2) The corresponding number of incident beam particles.

(3) The gas pressure.

(4) The angle of the emergent particles with respect to the incident beam.

(5) The gas temperature.

(6) The identity of the observed particles.

These points are illustrated in Figure 2. Each detector has two slits as shown. The effective solid angle $\Delta \Omega$ is determined by

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Figure 2

\[ \Delta \Omega = \frac{A}{R^2} \]

\[ \frac{T \sin \theta}{R} = \frac{W}{S} \]

SLIT SYSTEM
measuring the distance $R$ from the rear slit to the point where the detector axis intersects the beam (which is the center of the chamber), and the area $A$ of the rear slit. $R$ was determined by rotating the detector through exactly $180^\circ$, and measuring the diameter of the circle on which the rear slit moved, using a meter stick fixed above a diameter of this circle and a plumb bob whose tip was made to touch the slit edge in each of the two positions. This distance $S$ between the front and rear slits was determined in the same manner. The target length $T$, seen by the detector, can be seen from Figure 2 to be $T = \frac{WR}{S\sin\theta}$. Now, by the definition of the differential cross section, the number of particles $N$ received in the detector when $N_0$ incident beam particles have passed through the target length $T$ is given by

$$N = N_0 \sigma n T \Delta \Omega,$$

where $\sigma$ is the differential cross section in the laboratory system, and $n$ is the number of atoms per cm$^3$. $nT$ is the number of atoms per cm$^2$ in the target length seen by the beam, and $\sigma$ is the effective area per atom per unit solid angle $\Delta \Omega$. Since the solid angle is

$$\Delta \Omega = \frac{A}{R^2},$$

the geometrical factor $G$ is defined by

$$T \Delta \Omega = \frac{WR}{S\sin\theta} \frac{A}{R^2} = \frac{WA}{RS\sin\theta} = \frac{G}{\sin\theta},$$

where

$$G = \frac{WA}{RS},$$

so that

$$\sigma = \frac{N}{N_0} \frac{\sin\theta}{n G}.$$

The slit dimensions have been measured using a traveling microscope; and the $G$ factor of each counter is known to three significant figures.
Before each use of this chamber, it is necessary to go through an alignment procedure, to make certain that the beam passes through the axis of rotation of the counters. Otherwise, the distance $R$ and hence the $G$ factor would vary with the rotation of the counter. This was accomplished as follows: The Faraday cup, which may be seen in the photograph (Figure 3), opposite the nozzle of the beam inlet tube, was removed. A strong light was shined down the beam inlet tube, and a telescope was focused on this light, through the Faraday cup mount. A vertical straight edge mounted on one of the counter housings was made to intercept each edge of the light beam, and the counter position corresponding to the center of the beam was read from an angle scale which is located below the chamber. Then the counter was rotated through a half circle until the straight edge again intersected the beam, and the angle reading, corresponding to the beam center, was again determined. If the two readings were found to differ by exactly $180^\circ$, the alignment was true. Otherwise, the beam inlet tube was adjusted to correct the error.

The number of incident beam particles corresponding to a given number of counts is determined by measuring the total amount of charge received in the Faraday cup in the same time interval. The current integrator which was used to measure this charge was built by Robert Perry and Edwin Kashy, using essentially the same circuitry as that devised by R. R. Henry, and described by Phillip Miller in his Ph.D. Thesis. It works on the following principle:

\[ \text{Number of particles} = \frac{\text{Charge in Faraday cup}}{\text{Current sensitivity}} \]

\[ \text{Current integrator by R. R. Henry, described by Phillip Miller in his Ph.D. Thesis.} \]

\[ \text{Phillip D. Miller, Ph.D. Thesis, The Rice Institute, 1958.} \]
A known capacitor is charged to a known voltage, and the beam current is used to discharge this capacitor. A D.C. amplifier, which monitors the voltage on the capacitor, may be set to throw a relay and stop the counting when the voltage falls to a certain value. From the initial voltage, the final voltage, and the capacity, the charge is determined. This current integrator has several scales, and the scales used were calibrated in connection with the experiment. In this calibration, an external voltage, provided by a wet cell and a potentiometer, was applied to the capacitor through an external resistance of 1 Megohm and in the same direction as the original voltage. When the counting circuit was closed, the capacitor voltage decayed exponentially toward the external voltage instead of toward zero. Using the charge scale with the smallest capacity (to get the smallest RC time), the amplifier gain was set so that the integrator cut off after a time of about 16 times RC (RC was 1/4 second) so that the capacitor voltage was very nearly equal to the external voltage. This cutoff voltage was the same for all scales since it depended only on the amplifier gain setting. The external voltage was immediately removed and measured with a type K potentiometer. Then the capacitor charge was allowed to decay through an accurate external resistance, with no external voltage. The time required for the voltage to reach the known cutoff value was determined by means of a 60 cycle scaler gated by one of the integrator relays. The initial voltage was determined by balancing against a standard voltage. From these quantities, the capacitance C was easily determined, and the actual charge values, given on the
basis of zero cutoff voltage, were as follows:

<table>
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<th>Rated Charge</th>
<th>Actual Charge</th>
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<tr>
<td>2.5 micro coulombs</td>
<td>2.501 m. c.</td>
</tr>
<tr>
<td>10 m. c.</td>
<td>9.872 m. c.</td>
</tr>
<tr>
<td>50 m. c.</td>
<td>49.430 m. c.</td>
</tr>
<tr>
<td>100 m. c.</td>
<td>99.290 m. c.</td>
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During the experiment, the same cutoff voltage was used as in the calibration.

The Faraday cup, which collects the beam charge, is described in its latest form in Russell's Thesis. This system must be at a high vacuum to prevent gas ions from conducting charge away from the cup. For this reason, the Faraday cup is isolated from the chamber by a thin aluminum foil, and pumped continuously with a diffusion pump. Also, it is equipped with a liquid air trap. Pressures as low as $2 \times 10^{-5}$ mm Hg may be regularly obtained in this manner. Since beam particles passing through the foil tend to knock electrons into the Faraday cup, thus neutralizing part of the beam charge, the cup is provided with an electrostatic suppressor ring at a potential of about -200 volts, and a small permanent magnet which deflects the electrons without appreciable deflecting the heavier beam particles. These measures also prevent the beam particles from knocking electrons out of the Faraday cup.

The chamber pressure was read with an oil manometer filled with butyl phthalate, the specific gravity of which is known to be 1.04578. A cathetometer, accurate to .1 mm, was used to read the manometer, and

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10 John L. Russell, Ph.D. Thesis, The Rice Institute, 1959,
the manometer was backed by a diffusion pump. The argon was admitted to the chamber through a reducer and a needle valve. A mechanical pump and a diffusion pump pumped at two ports in the beam inlet tube, so that, by adjusting the needle valve, an equilibrium pressure could be reached.

The counter angles, read from azimuth circles below the chamber, may be determined to within about 3 minutes of arc. The position of the angle zero is determined by shining a light down the beam inlet tube and taking a reading with the light spot on each edge of each slit. The mean value of the two readings for the front slit is determined, and similarly, for the back slit. The two mean values are made to coincide by adjustment of the slits, if necessary, and the final mean value is the zero.

The gas temperature was determined from a mercury thermometer in contact with the lid of the chamber. If the heating effect of the beam introduced any appreciable error in the target temperature, one would observe an error which would be a function of the beam current. This has not been observed and it is thus assumed that the error is very small.

In this experiment, there was no difficulty in distinguishing the elastically scattered deuterons from the various proton groups, because of the great difference in the cross sections. The proton groups were identified on the basis of their energies. The groups with the higher energies correspond to transitions to lower excited states in the final nucleus.
The overall RMS error in the functioning of the equipment has been estimated by Russell to be 3%, the largest single error being 2% for detection efficiency.

The total RMS error involved in calibrating the current integrator is estimated to be 1%, and the total RMS error involved in measuring the geometrical quantities is estimated to be 4.5%. The other errors associated with the equipment are much smaller.

The possibility of contamination of the target gas was considered. The chamber was evacuated, and a background spectrum was taken at 15°. The observed background did not agree with the structure observed for the $^{40}_1$ (d,p) $^{41}_1$ reaction.

Also, a spectrum was taken with air as the target, and the spectrum obtained with air did not agree with any feature of the argon spectrum.

The chamber was immediately refilled with argon, and a new spectrum was taken, in which the original result was obtained. Thus, these features were ascribed to argon.

A 37 mg/cm$^2$ aluminum foil was used to help separate the proton groups by means of the curvature of the range-energy relation. The minimum proton range was 75 mg/cm$^2$ of aluminum (at backward angles for the lowest energy proton group), and it was felt that any appreciably thicker foil would introduce a serious straggling effect.

The energy spectra were taken using a very thin (.2mm) thallium activated cesium iodide crystal; a thin crystal was necessary on account
of the high gamma ray background. A DuMont 6292 photomultiplier was used, and the counts were recorded with a 20 channel differential pulse height analyzer.

In the proton angular distribution, single data points as long as 20 minutes were taken, but the statistical error is nevertheless large.
III. EXPERIMENTAL RESULTS

The results are given in Figures 4 through 9. Figure 4 is the proton pulse height distribution obtained at a laboratory angle of $15^\circ$, in which the discernable groups are taken to be $P_0$, $P_1$ and $P_5$. The belief that the intense group is $P_5$ agrees almost exactly with the Q value and intensity relative to the ground state given by Pollard. The observed laboratory energy for this group differs somewhat from the energy corresponding to Burrows' Q value. For example, at $15^\circ$ in the laboratory, the laboratory energy corresponding to his Q value for $P_5$ is 5.29 Mev. The observed energy calculated on the basis of channel position, is 5.69 Mev. This is a difference of 7%. On the basis of Pollard's relative intensity figures, the intense group could conceivably be $P_3$, since he reports $P_3$ as having nearly the intensity of $P_5$. But, using Burrows' Q value for $P_3$, the laboratory energy at $15^\circ$ would be 6.48 Mev, which is further from the observed value than the energy corresponding to $P_5$. This, together with the fact that Pollard reported $P_5$ as being more intense than $P_3$, leads to the conclusion that this group is $P_5$. The energy observed for $P_1$ agrees fairly well with Burrows' Q value. At $15^\circ$ in the lab, the observed energy is 7.00 Mev and the energy corresponding to his Q value is 7.31 Mev. This is a difference of 4.3%. In calculating the observed energies, the ground state Q value was taken as 3.90 Mev, which is the value given by Gibson and agrees closely with Pollard's value of 3.84 Mev. But, the observed energies calculated on the basis of channel position are not very accurate.
It will be noticed that $P_1$ and $P_0$ are of about equal intensity, and this agrees with Pollard's work. As was pointed out before, Burrows' relative intensity figures do not apply at this energy (he worked at 8.5 Mev).

It will be noticed that $P_5$ is contaminated by a small group on the high energy side; this is taken to be $P_4$. The intensity ratio given for these two groups by Pollard is about 14:1.

The overlap of the $P_1$ and $P_0$ peaks and the statistical error are the main sources of error in determining these cross sections. The statistical errors are about 7% for the ground state proton group, 6% for $P_1$, and 3% for $P_5$. For $P_0$ and $P_1$, the error caused by the overlap was estimated on the basis of the agreement between the cross sections calculated for different runs under the same conditions. Figures 5, 6 and 7 give the experimental angular distributions for the three proton groups, and the Butler curves are given in Figures 5 and 6. The laboratory cross sections calculated, using the formula discussed above, were converted into center of mass cross sections through the use of the conversion factor

$$\sigma_{\text{LAB}} = (1 - \chi^2) \left[ \chi \cos \theta - \left( 1 - \chi^2 \sin^2 \theta \right)^{\frac{1}{2}} \right]^{-2}$$

where

$$\chi = \left( \frac{M_1 M_3}{M_2 M_4} \right)^{\frac{1}{2}} \left[ 1 + \frac{M_1 + M_2}{M_2} \frac{Q}{E_1} \right]^{-\frac{1}{2}}$$

and where $\theta$ is the laboratory angle of the observed particle; $M_1$ is the mass of the incident particle; $M_2$ is mass of the target particle. $M_3$ is the mass of the observed particle, $M_4$ is the mass of the residual nucleus, $Q$ is the energy release, and $E_1$ is the laboratory bombarding energy.

Values of the conversion factor were taken from a table.

Tables for the Transformation of Angular Distribution Data from the Laboratory System to the Center of Mass System - Jerry B. Marion and Shell Development Company.
PROTON PULSE HEIGHT DISTRIBUTION

Figure 4
PROTON ANGULAR DISTRIBUTION, $P_5$

EXPERIMENTAL

L=1 BUTLER CURVE

Figure 5
PROTON ANGULAR DISTRIBUTION, $P$

- EXPERIMENTAL

- $L=1$ BUTLER CURVE

Figure 6
PROTON ANGULAR DISTRIBUTION, $P_0$

Figure 7
Figure 8 gives the angular distribution of the elastically scattered deuterons. The maximum statistical error was 2%, but the error estimated from non-repetition of results is about 9%. Figure 9 gives the ratio to Rutherford scattering for this distribution. This curve shows one minimum and maximum that is characteristic of the wave nature of the process. A plausibility argument for the large deviation from Rutherford scattering as far forward as $90^\circ$, is given below.
IV. DISCUSSION OF RESULTS

This section will first give an outline of the way in which the ordinary Butler expression was used to obtain the curves given in Figures 5 and 6. This is followed by a classical argument showing that the general nature of the d-p angular distribution is determined by the momentum and energy conservation laws. The classical argument leads to an illustration of the way in which the nuclear and Coulomb forces oppose one another in their effect on the d-p angular distribution.

At this point a simple argument is given to show that the effect of the nuclear force on the proton in the incoming deuteron is not by any means negligible in the present experiment. This is done in order to show that one cannot neglect nuclear distortion effects in comparison with Coulomb distortion. The section concludes with a plausibility argument concerning the large deviation from Rutherford scattering observed in the angular distribution of the elastically scattered deuterons.

(A) Butler Theory

Neglecting all neutron reduced widths, except one, the Butler expression for the d-p stripping center of mass cross section is

\[
\sigma = \frac{A}{[(kR)^2 + (\kappa R)^2]^2} \left[ j_\ell(kR) \frac{2}{\beta n} h_\ell^{(1)}(ikR) - \frac{2}{\beta n} j_\ell^{(1)}(kR) \right]_{n=R}^2.
\]

In this expression, A is angle independent; \( h_\ell^{(1)} \) is the spherical Hankel function of the first kind; \( j_\ell \) is the spherical Bessel function;
and
\[ K_N = \sqrt{\frac{2M_{NT}E_N}{h^2}}. \]

\( E_N \) is the magnitude of the binding energy of the neutron in the final nucleus, and \( M_{nt} \) is the reduced mass of the neutron-target nucleus system. Choosing the zero of energy as the energy of a neutron-proton-target nucleus system, the energy balance equation is
\[ E_d - \epsilon = E_p - E_N. \]

where \( E_d \) is the energy of relative motion of the deuteron-target system; \( \epsilon \) is the deuteron binding energy; and \( E_p \) is the energy of relative motion of the proton-residual nucleus system.

Since \( E_p - E_d = Q \), the energy release of the reaction, we have that
\[ E_N = Q + \epsilon, \]

\[ K_N = \sqrt{\frac{2M_{NT}}{h^2}(Q + \epsilon)}. \]

The only angle dependent part of the Butler expression is the wave number \( K \), defined as the magnitude of
\[ K = \frac{M_T}{M_N + M_T} \overrightarrow{K_P} - \overrightarrow{K_d}, \]

where
\[ E_p = \frac{h^2K_p^2}{2M_{pr}}, \]
\[ E_d = \frac{h^2K_d^2}{2M_{pr}}, \]

\( M_{pr} \) is the proton-residual nucleus reduced mass. From the law of cosines,
\[ K = \left[ K_d^2 + 2 \frac{M_T}{M_N + M_T} K_d K_p \cos \theta_{cm} + \left( \frac{M_T}{M_N + M_T} K_p \right)^2 \right]^{\frac{1}{2}}, \]

where \( \theta_{cm} \) is the center of mass angle at which the proton emerges with
respect to the incident beam.

The parameter $R$ in the Butler expression is known as the interaction radius, and the Butler distribution is very sensitive to the value chosen.

For a particular value of $q$, the general expression for the cross section may be simplified through the use of relations between the functions and their derivatives.

Using the relation

$$\frac{2}{\pi n} \frac{\partial}{\partial n} j_{\ell}(kn) = \frac{k}{2\ell+1} \left[ j_{\ell-1}(kn) - (\ell+1) j_{\ell+1}(kn) \right],$$

and the recursion relation

$$j_{\ell+1}(kn) = \frac{2\ell+1}{kn} j_{\ell}(kn) - j_{\ell-1}(kn),$$

we get

$$\frac{2}{\pi n} \frac{\partial}{\partial n} j_{\ell}(kn) = k j_{\ell-1}(kn) - \frac{\ell+1}{n} j_{\ell}(kn).$$

This relation also holds for the Neumann function $n_{\ell}(kn)$, and since by definition

$$h_{\ell}^{(1)}(ik_{n}n) = j_{\ell}(ik_{n}n) + in_{\ell}(ik_{n}n),$$

the corresponding result for $h_{\ell}^{(1)}$ is

$$\frac{2}{\pi n} h_{\ell}^{(1)}(ik_{n}n) = ik_{n} h_{\ell-1}^{(1)}(ik_{n}n) - \frac{\ell+1}{n} h_{\ell}^{(1)}(ik_{n}n).$$

Using this relation, the $q=1$ Butler expression can be written

$$\sigma = B \left[ (kR)^{2} + k^{2} R \right]^{2} \left[ j_{1}(kR) - \frac{2R^{2}}{1 + k^{2} R} + kR j_{0}(kR) \right]^{2},$$

where $B$ is a constant with respect to angle.

Using the above results, Butler curves were calculated for the proton groups corresponding to the fifth and first excited states of $^{41}$A. In these calculations, a value $R = 5.47$ Fermis, or $(1.2 \sqrt[3]{41} + 1.37)$ Fermis was found to give reasonable results.

\footnote{L. I. Schiff, "Quantum Mechanics", (McGraw-Hill), page 78}
A frequently used value is
\[ R = (1.22 \frac{3}{A} + 1.7) F \]
but it gave results which were slightly worse. It will be noticed that both Butler curves have peaks near 25°. The results for the ground state proton group are not sufficiently accurate to compare with the theory.

(B) Conservation Laws

Considering that the Butler cross section may be derived from the Born approximation, it may seem surprising that the position of the experimental peak agrees fairly well with the Butler peak for \( J = 1 \) in the case of \( P_5 \) and \( P_1 \), since the bombarding energy was below the Coulomb barrier, where the Born approximation is not applicable.

This may be understood on the basis of a semi-classical argument. Since the target nucleus \( \text{A}^{40} \) has zero total angular momentum, it seems reasonable to view the residual nucleus \( \text{A}^{41} \) as consisting of an \( \text{A}^{40} \) core (whose spin is zero), and a neutron whose motion about the core determines the total angular momentum of the nucleus. Either of two orbital angular momenta \( \mathcal{J} \) could couple with the neutron spin to form a given total angular momentum \( J \); but in forming a nuclear level with a definite parity, one of these \( \mathcal{J} \) values is rejected because it has the wrong parity. A classical argument shows that the angular distribution of the protons emitted in a d-p reaction will be characteristic of the \( \mathcal{J} \) value with which the neutron is captured.

\[ \text{-----------------------------} \]
13 "Progress in Nuclear Physics" (Pergamon, 1953) Vol. 3
In order for the d-p process to lead to a level in A$^{41}$ with orbital angular momentum $L = \sqrt{L(L+1)}$, the linear momentum $P$ associated with the relative motion of the neutron - target nucleus system must satisfy $PR > \sqrt[3]{L(L+1)}$, where $R$ is the radius of the target nucleus.

Suppose that one neglects the Coulomb field entirely, viewing the neutron as falling into a circular orbit, whose radius is the radius of A$^{40}$. We may easily see that for a given orbital angular momentum $L$, the direction of emission of the proton is uniquely determined (for a given bombarding energy and Q value).

In considering the consequences of the conservation laws, we may view the process as a two step reaction, in which the deuteron breaks up and then the neutron falls into its orbit. But, we may also view it as a single step process, in which the deuteron - target nucleus system goes over into the proton - residual nucleus system. From the first point of view, letting $\vec{P}_d$, $\vec{P}_p$ and $\vec{P}_n$ denote the laboratory momenta of the deuteron proton and neutron respectively, $\vec{P}_d = \vec{P}_p + \vec{P}_n$. If $\vec{v}_N$ is the laboratory velocity of the neutron, the center of mass of the neutron - target nucleus system moves with a laboratory velocity given by

$$\vec{v}_c = \frac{m_N}{m_N + m_T} \vec{v}_N$$

where $m_T$ is the mass of the target and $m_N$ is the mass of the neutron.
It is easily seen that the energy of relative motion of the neutron target system is
\[ E_c = \frac{1}{2} m_{NT} v_N^2, \]
where
\[ m_{NT} = \frac{m_N m_T}{m_N + m_T}. \]

The momentum associated with this relative motion is given by
\[ P = m_{NT} v_N. \]

In the classical case, \( P \) would satisfy the equation
\[ PR = L; \]
therefore,
\[ v_N = \frac{1}{m_{NT}} \frac{L}{R}. \]

So, the laboratory momentum of the neutron is of magnitude
\[ P_N = \frac{m_N}{m_{NT}} \frac{L}{R}. \]

Looking at the reaction from the other point of view,
\[ \overrightarrow{P}_D = \overrightarrow{P}_P + \overrightarrow{P}_R, \]
where \( \overrightarrow{P}_r \) is the laboratory momentum of the residual nucleus. Therefore,
\[ \overrightarrow{P}_R = \overrightarrow{P}_N, \]
which simply says that in capturing the neutron the nucleus is forced to recoil with the momentum the neutron had to begin with. Then, by the law of cosines,
\[ P_R^2 = P_P^2 - 2 P_D P_P \cos \theta_P + P_P^2. \]
where $\theta_p$ is the angle of $\vec{P}_p$ with respect to $\vec{P}_d$. Also, by the conservation of energy,

$$\frac{P_p^2}{2m_p} + \frac{P_{R}^2}{2m_{R}} = \frac{P_d^2}{2m_d} + Q,$$

where $Q$ is the energy release of the reaction. From the energy equation,

$$P_p = \sqrt{2m_p(E_d + Q - E_R)}$$

where

$$E_d = \frac{P_d^2}{2m_d}, \quad E_R = \frac{P_R^2}{2m_R}. $$

$E_d$ and $E_R$ are the laboratory energies of the deuteron and the residual nucleus.

This, together with the equations

$$P_R = P_N = \frac{m_N}{m_{NT}} \frac{L}{R},$$

and

$$\cos \theta_p = \frac{P_d^2 + P_p^2 - P_R^2}{2P_dP_p},$$

completely determines $\theta_p$ for a given set of the quantities $E_d$, $Q$, $L$ and $R$.

Thus, to obtain the angular deflection required by the conservation laws, neglecting Coulomb and nuclear force effects, we insert

$$L = \hbar \sqrt{l(l+1)},$$

in these expressions. In the actual wave mechanical process, the protons do not of course emerge all at the same angle, but the peak of the angular distribution should be near the classical angle, calculated taking all forces into account. If $l = 0$, $P_n = 0$ and $\vec{P}_p = \vec{P}_d$, so that the angular distribution has its peak at $\theta_p = 0$, neglecting
Coulomb and nuclear effects.

(C) Coulomb and Nuclear Effects

The size of the Coulomb effect may be easily estimated. We may approximate the average deflection by the deflection corresponding to deuterons with impact parameters equal to the sum of the target radius and the deuteron radius. This would be less than the deflection of elastically scattered protons with this energy and impact parameter, and greater than the corresponding deflection of elastically scattered deuterons. The laboratory angle of deflection $\Theta$ of such a deuteron is given by

$$\cot \Theta = \frac{m_d}{m_T} \cot \Theta + \cot \Theta,$$

where

$$\cot \Theta = \frac{2 \lambda}{b}.$$  

Here $\Theta$ is the center of mass angle of deflection, $\lambda$ is the impact parameter, $b$ is the collision diameter, and $V$ is the initial laboratory velocity of the deuteron.

If we consider the intense proton group $P_5$, a calculation of $\Theta_P$ from the conservation laws, neglecting the Coulomb and nuclear effects and using a nuclear radius $1.5 \hat{A}^{\frac{2}{3}} = 5.13$ Fermis gives the result (assuming $l=1$),

$$\Theta_P = 2.7^\circ.$$

The experimental peak is seen to be near $30^\circ$. But, an estimate of the Coulomb deflection indicates that it should be of the order of
30°, which would be in addition to the value already calculated for ΘP. That is, if there were Coulomb distortion only, the peak would be displaced to about 57°. This is an illustration of the fact that the nuclear distortion of the incoming deuteron waves, and the outgoing proton waves, acts to largely cancel the effect of the Coulomb field as far as the position of the proton peak is concerned, although both effects reduce the size of the cross section. It is just this fortuitous circumstance which explains the fact that the Butler expression has been fairly successful in cases where the Born approximation does not apply at all.

It is found that the simple one term Butler expression for the cross section, obtained by neglecting all neutron reduced widths except one, has zeros, whereas the experimental distribution does not. This can be understood partly on the basis of the Coulomb effect: The Coulomb deflection is a function of impact parameter, and since the observation in effect averages over the contributing impact parameters (or more precisely partial L values), the Coulomb effect tends to blur the spectrum, and prevent it from having zeros.

(D) The Nuclear Force Effect on the Proton

The Butler theory completely neglects the effect of the nuclear potential of the target on the proton, as well as the Coulomb effect. It can be argued that the large size of the deuteron makes it possible for the neutron to come within range of the nuclear force while the proton is largely unaffected. But, the weakness of this argument is
that it neglects the internal motion of the deuteron. A proton confined within a deuteron has a momentum uncertainty of the order of
\[ \Delta P = \frac{\hbar}{R} \]
where \( R \) is the radius of the deuteron, which is about 4.3 Fermis.

We may take the average magnitude of the velocity of the proton with respect to the center of mass of the deuteron as being at least of the order of
\[ V = \frac{\hbar}{m_p R} \]

We may then judge the validity of the above mentioned approximation on the basis of the number of passes the proton makes across the deuteron volume in a time
\[ \gamma = \frac{2\chi}{V} \]

where \( \chi \) is the impact parameter and \( V \) is the deuteron velocity. \( \gamma \) is the effective collision time, during which most of the interaction takes place. In the case of 4 Mev deuterons incident on \(^{40}\)A and with \( \chi \) taken as the sum of the nuclear and deuteron radii, the result is that the proton makes between one and two passes in the time \( \gamma \), which indicates that the approximation is not a very good one.

(E) The Elastically Scattered Deuterons

The pronounced deviation from Rutherford scattering for the elastically scattered deuterons even at forward angles may be made to seem plausible by the following argument. If the closest distance of approach to the center of the nucleus is calculated for deuterons scattered through 90° (by the Coulomb force only), it is found to be
6.97 Fermis, while the sum of the nuclear radius and the deuteron radius is about 8.75 Fermis, showing that there is a considerable overlap of the deuteron and nuclear wave functions, which should cause a considerable deviation from Rutherford scattering.
V. CONCLUSIONS

It is believed that the angular distribution for the ground state proton group is not accurate enough to yield a definite $\ell$ value assignment because of the overlap between $P_0$ and $P_1$, as well as the statistical error. Higher resolution of the scintillation counters is needed to improve these measurements.

The $P_1$ and $P_5$ angular distributions seem to be typical of $\ell = 1$ stripping curves, in agreement with Burrow's results. $P_5$ is the most intense group seen, at all angles. This agrees with Pollard's observation that $P_5$ was very intense. But, Pollard also reported that $P_3$ was nearly as intense as $P_5$, and $P_3$ was not detected in this experiment. These facts point to a strong resonant behavior of the yield curve.

In spite of the fact that $P_5$ is contaminated somewhat by $P_4$, it is believed that this angular distribution is more accurate than the distribution for $P_1$ or $P_0$.

The most accurate angular distribution is that for the elastic deuterons. The most peculiar feature of the ratio to Rutherford scattering for the elastic deuterons is that the curve deviates from a ratio of one almost at once, rather than remaining level, for some time, and then falling off. Data from an earlier experiment indicates the possibility of a resonance behavior in the deuteron scattering in this energy range. It is planned to examine this question more fully in the future.
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