THE RICE INSTITUTE

THE ELASTIC SCATTERING OF PROTONS BY Fe$^{54}$

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I. INTRODUCTION

A large amount of experimental work has been done on excitation functions for charged-particle-induced reactions among the light nuclei (Z ≤ 20) in order to locate states of the compound nucleus. In addition, information regarding spins, parities, and resonance widths of the states has been sought.

It is of interest to perform similar measurements of nuclei of medium weight, but the density of states is larger than in the light nuclei for comparable excitation energies so that the resolution of the individual levels is difficult. However, special circumstances may alleviate the problem, e.g., if the target nucleus has zero angular momentum, compound states having only a limited range of spins are populated due to the effect of penetrability. The density of observable levels is then reduced, and the states are readily resolved. That this circumstance occurs was shown in the case of proton inelastic scattering from Fe$^{54}$ and Ti$^{46}$. The situation in these experiments was rendered even more favorable by the condition put on the scattering that it leave the target nucleus in its first excited state.

It was considered desirable to acquire more information about the states of the compound nucleus in this region of excitation (6 to 10 Mev). The study of the states by the elastic scattering of protons seemed to be the next logical step to attempt, and Fe$^{54}$ was chosen as the subject nucleus since much information was available about the compound states formed by this nucleus.
In considering elastic scattering from medium weight nuclei several questions arose:

1. Was it experimentally feasible to do elastic proton scattering with simple apparatus such as a crystal spectrometer?

2. If it were possible to detect the elastically scattered protons; then (a) would the yield associated with possible compound nucleus resonances be significant relative to the Rutherford background, and (b) if the resonant variations in yield were sufficient, would the spacings of the states generally be such that an analysis of line shapes could be made, yielding information about the spins of the states?

3. Would the states excited by inelastic scattering be observable in the elastic scattering, and if they were, would the spin assignments given by Prosser et al.\textsuperscript{1} be confirmed?

As will be shown in the succeeding sections an affirmative answer can tentatively be given these questions, although much more work needs to be done before the answers are complete.

It is of interest to give preliminary considerations to these questions. In regard to the first point, it could be anticipated that targets of the order of 10 Kev in thickness would be needed. This implies that a self-supporting foil would be required onto which the film of iron could be deposited. In order that the protons elastically scattered from this foil could be resolved from those scattered from Fe, the elements used in this foil needed
to have mass values greatly different from that of Fe. This enables advantage to be taken of the fact that the energy of the recoil protons is dependent on the mass of the scatterer. The choice of a supporting foil therefore seemed to lie between elements of low \( Z \) and those of high \( Z \). For purposes of comparison the separation in energy of the elastically scattered protons from C\(_{12}\) and Fe\(^{54}\) is given in Fig. 1 for scattering at 90\(^\circ\), 125\(^\circ\), and 150\(^\circ\), and for Fe\(^{54}\) and W for the angle where the separation is best, i.e., 150\(^\circ\). Since a crystal spectrometer with 5% resolution will not discriminate between protons due to the two elements Fe and W and it will discriminate between the protons due to Fe and C, C\(_{12}\) was chosen to be the self-supporting foil.

As to question 2(b), an estimate of the spacing of the states to be expected can be made from the data of Prosser et al. By assuming that Prosser et al. detected all the observable states in Co\(^{55}\) (180) in this energy interval of 1.4 Mev an upper limit to the average separation of the states can be made. This spacing is approximately 8 Kev. On the other hand, a rough estimate of a lower limit can be made using the factor \( 1/(2J+1) \) which is predicted by various theories of level densities\(^3\) to be proportional to the average spacing of states of different \( J \) where \( J \) is the spin of the state. Considering all the 180 states to be \( l = 2 \) states (of 69 resonances analyzed by Prosser et al. 67 were \( l = 2 \) states) the average spacing is approximately 4 Kev.
Questions 2(a) and 3 can not be answered in advance. In anticipation that resonances could be seen, a scattering matrix formulation for the cross sections for isolated resonances^4, which was hopefully believed applicable to this problem, was coded for an IBM 650 Computer.
II. EXPERIMENTAL METHODS AND PROCEDURE

A. Scattering Chamber and Spectrometer

A small scattering chamber using thin evaporated targets and scintillation spectrometers was designed to detect the scattered protons.

The chamber, shown in Fig. 2, is cylindrical in shape, 5 inches in diameter and 2-1/4 inches deep. The beam from the Van de Graaff enters the chamber through the pumping tube and passes through the target to be collected in an insulated Faraday cup. The target is located in the center of the chamber and can be rotated to any desired angle. There are three scintillation spectrometers mounted on the chamber for detecting protons scattered at 90°, 125°, and 150°, the angles being measured from the forward direction of the beam.

A spectrometer consists of a Dumont 6291 photomultiplier tube, a thin (50 mil) cesium iodide (thallium activated) crystal in contact with the photocathode of the tube, and collimating slits. The crystal is within the evacuated target chamber and is covered by a .02 mil thick nickel foil in order to increase the light gathering power of the optical system. The vacuum seal between the spectrometer and the chamber is made by two O-rings, one is compressed between the lucite spacer and the facing of the hole in the side of the chamber, the other is compressed between the opposite side of the spacer and the face of the photomultiplier tube as shown in Fig. 2. The collimating system consists of a
short tube with provision for disks with different size holes to be inserted at the ends, thus changing the effective solid angle.

A calibration of the counter efficiency was preformed by scattering protons off a foil of .02 mil Ni at 1.67 Mev and comparing with the Rutherford cross section. The effective solid angle was found to be \(5.2 \times 10^{-4}\) steradians, in agreement with the measured geometrical solid angle.

B. Preparation of Targets

In the gamma ray work targets were made by evaporating Fe\(_2\)O\(_3\) (Fe\(^{54}\) enriched) from a tungsten boat onto a 5 mil Au foil under vacuum. In this process some tungsten also evaporated, as revealed in the magnetic analysis of protons elastically scattered from one of these targets. The composition of this target was 57% Fe, 35% O, and 8% W. Although the W contamination was small, the larger Rutherford scattering cross section of W as compared to Fe could cause enough protons to be scattered from the W to obscure the peak due to the scattered protons from Fe.

Similarly, it was believed that any other of the higher Z materials capable of withstanding the high temperature necessary (about 1600° C) for the evaporation would themselves evaporate with the iron in excessive amounts. Hence attention was given to evaporation from carbon boats as a possible substitute.

Experience in evaporating Fe\(_2\)O\(_3\) from W boats had shown that
some 10 mg of material were necessary in order to obtain a target of 2 kev thick. To improve the efficiency of utilization of the iron isotopes, it was decided to attempt reducing the Fe$_2$O$_3$ before evaporation. Also some method was needed to prevent loss of material from the boat during the reduction of the Fe$_2$O$_3$.

The carbon boats were made of graphite. Graphite outgasses easier and more thoroughly than more solid carbon. The boats were 1/4 inch by 5/16 inch by 1-3/4 inches. A hole 1/8 inch by 3/16 inch and 1/4 inch deep was drilled into the center of the boat. The Fe$_2$O$_3$ was placed in this hole. This hole enabled this part of the boat to reach a very high temperature.

Carbon is customarily used in reducing Fe$_2$O$_3$ to iron. The method used to reduce the Fe$_2$O$_3$ was to put 3 mg of powered carbon into the boat with 2 mg of Fe$_2$O$_3$ and to cover the boat with a lid of carbon to contain the mixture while the heated boat was outgassing and the heat of fusion attendant to the reduction of the Fe$_2$O$_3$ was released. The boat was brought to white heat long enough to reduce the Fe$_2$O$_3$ and evaporate some iron, and then allowed to cool so that it would not become a getter for more gases when the evaporator was opened. The evaporator was opened, the target backing of a thin

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*The Fe$_2$O$_3$ used in this experiment was enriched to 96.66% in Fe$^{54}$. This enriched Fe$_2$O$_3$ was supplied by the Union Carbide Nuclear Company, a division of Union Carbide and Carbon Company.
self-supporting carbon foil put into position above the boat, and the carbon lid on the boat removed. The iron deposited on the lid was scraped off back into the hole in the boat. The evaporator was returned to approximately $10^{-6}$ mm Hg vacuum and the evaporation of the iron performed.

The self-supporting carbon foils used as target backing were made on 1/2 inch by 1 inch strips of tantulum heated to incandescence in methyl iodide gas at a pressure of 3 inches of Hg. Carbon alone was deposited on the filament and it was removed in the form of foils approximately 300 micrograms per square centimeter thick. The thickness of the foils was somewhat increased by carbon evaporated from the boat during the deposition of the iron target material.

G. Resolution of the Spectrometer

The resolution of the 150° scintillation spectrometer is given in Fig. 3 as a function of the energy of the scattered proton. The resolution at the lower energies shows the expected decrease, since there is less light output from the scintillation crystal.

Fig. 4 gives the pulse height distribution of the peaks due to the elastically scattered protons from C and Fe. The size of the C peak is reduced by a factor of 100 on the figure. Since the carbon foil is much thicker than the film of Fe, i.e., 72 Kev
Figure 3

Resolution of 150°

Spectrometer

Resolution (°)

Energy of scattered protons

8 6 4 2

2.0 3.0 4.0 5.0

FIG. 3
as compared to 2.5 kev at 1.67 mev, the carbon peak is much broader and has a high energy tail which lies under the peak due to the Fe. The thickness of the carbon foil in micrograms per square centimeter was computed from the cross section for elastic scattering at 3.8 mev. We wish to decrease the contribution of this high energy tail as much as possible. One method that can be used to accomplish this is to put stopping foils in the path of the scattered beam in order to slow down the less energetic particles more than the faster particles.

This method was used in each spectrometer by placing a 15.6 mg/cm² Ni foil against each crystal. Fig. 5 gives the separation of the Fe and C peaks with this foil in place. Fig. 6 and Fig. 7 give the experimental change in the separation of the peaks for 150° and 90°, as shown by the increased width of the valley between the tail of the C peak and the start of the Fe peak. The insertion of this foil does not decrease the intensity of the scattered beam.

D. Measurements

The proton beam from the 6.0 Mev electrostatic generator of the Rice Institute was used in this experiment. The energy of the proton beam was determined by measuring the field strength of the 90° analyzing magnet associated with the accelerator with a proton magnetometer. The beam energy regulation signal was taken from a
ENERGY SEPARATION BETWEEN
F AND C PEAKS

\[ \Delta E \] (MEV)

\[ 150^\circ \quad 150^\circ \]

\[ 125^\circ \quad 125^\circ \]

\[ 90^\circ \quad 90^\circ \]

---

WITH 15.6 MG/CM² NI FOIL

W/O NI FOIL

FIG. 5

\[ E_P \] (MEV)
PULSE HEIGHT DISTRIBUTION
150° COUNTER
3.9 MEV

ARBU. UNITS

Fig. 6

CHANNEL
43 45 41 39 37 35 33

W/O NI FOIL
WITH 15.6 MG/CM² NI FOIL
PULSE HEIGHT DISTRIBUTION
90° COUNTER
3.9 MEV

ARBITRARY UNITS

WITH 15.6 MG/CMP²
NI FOIL
W/O NI FOIL

CHANNEL

FIG. 7
pair of slits 1 mm apart located 4 m from the exit of the magnet. The beam was also defined by two pairs of slits at the entrance and exit of the magnet, with separations, respectively, of about 1.5 mm and 0.8 mm. These dimensions are compatible with an energy resolution of about 0.03% and a reproducibility of about 0.1%.

Energy steps of approximately 5 KeV were used in taking the complete excitation curve at 150° with a target which was 18 KeV thick at 1.67 MeV. Steps of approximately 1 KeV were used in taking the combined 90° and 150° data with a target, inclined at 45° to the beam, effectively 3.9 KeV thick at 1.67 MeV.

The two peaks corresponding to the scattered protons from Fe and C were observed by using an Atomic 20 Channel Differential Pulse Height Analyzer. A single channel analyzer, in parallel with the 20 channel pulse sorter, was set so that it counted all pulses that comprised the iron peak as viewed in the twenty channels. The single channel analyzer was adjusted by using the precision pulser associated with the twenty channel analyzer.

The gain of the amplifier was continually adjusted so that the valley between the carbon and iron peaks remained in one channel of the twenty channel analyzer.

This method of using a single channel analyzer eliminated reading and recording the counts in each channel of the twenty channel analyzer, which took approximately 1 minute, and waiting for the twenty channel analyzer to recycle, which took approximately
20 seconds. The single channel analyzer could be read with a glance and recycled in 1 second.

In addition to observing the $90^\circ$ and $150^\circ$ proton excitation curves, the $1.41$ Mev gamma ray from the first excited state in Fe$^{54}$ was observed simultaneously, thus providing a correlation with the gamma ray excitation curve taken by Prosser et al. as given in Fig. 8.

The gamma ray was detected by placing a 3 inch by 3 inch NaI (thallium activated) crystal mounted on a 6363 photomultiplier tube beneath the chamber approximately 1 inch from the excited area of the target.

The energies of the resonances as seen by Prosser et al. were accurate to $\pm 0.2\%$ and the energies between adjacent resonances were accurate to within $\pm 1$ Kev. The gamma ray data taken in this experiment agreed very well with Prosser's in that the energies between adjacent resonances agreed to within $\pm 1$ Kev.

The beam current was measured initially with a standard integrating circuit. The use of a mechanical register, possible changes in calibration, leakage currents, and incomplete electron suppression were the basis for an assigned error in the accuracy of the beam current measurement of $\pm 4\%$.

Upon observing the complete excitation curve it was noticed that there were many small fluctuations in the cross section. The accuracy of the integration and the crude steps in energy
denied the identification of these fluctuations as resonances. In order to eliminate this ambiguity in succeeding work, a more accurate integrator was used along with finer steps. By using this integrator an error of only ± 0.6% was incurred.
FIG. 8 THE YIELD OF THE 1.41 MeV GAMMA RAY FROM Fe$^{54}$ AS A FUNCTION OF THE ENERGY OF THE INCIDENT PROTONS
III. RESULTS

Since earlier work on the elastic scattering from $^{56\text{Fe}}$ showed that resonances could be seen, a crude survey of the $^{54\text{Fe}}$ excitation curve was undertaken at $150^\circ$ in order to establish the strength of the resonances with respect to Rutherford cross section. This survey was made with an 18 kev thick target, measured at 1.67 Mev, and is shown in Fig. 9.

There were 13 large resonances resolved in the interval 3.0 to 4.8 Mev. Nineteen others were fairly prominent in this region. In addition to these there were many small fluctuations which could not be considered resonances because such crude steps were taken. From this excitation curve line shapes characteristic of states studied in light nuclei emerged. Also at high energies there was clustering of states. This effect was also found in the inelastic work with $^{54\text{Fe}}$ and $^{46\text{Ti}}$.

Because time did not permit the complete excitation curve to be run at high resolution, selected regions of the excitation curve were observed in 1 kev steps at $90^\circ$ and $150^\circ$ simultaneously with a target 3.9 kev thick at 1.67 Mev. The angle $90^\circ$ was chosen since the theoretical line shapes here were complementary with the line shapes at $150^\circ$. This angle helps pin down the range of spin values. The selected regions correspond to the regions in which one or more strong resonances were seen in the crude excitation curve. The results of the measurements are shown in Fig. 11 through Fig. 17 where the ratio of the cross section to the effective
background cross section is plotted for each energy interval.

The non-resonant cross section for Fe$^{54}$ (p,p), assuming that the only phase shift that entered into the calculations was the hard sphere phase shift, was computed using a program for phase shift analysis coded by Mr. P. D. Miller at the Rice Institute. This cross section corresponds to hard sphere scattering which includes Rutherford scattering. The formula used in this computation is given in Appendix I. A comparison between Rutherford scattering and this cross section for each of the angles 150° and 90° is given in Fig. 17 and Fig. 18 respectively.
COMPLETE EXCITATION CURVE AT 150°
18 KEV THICK TARGET
1.67 MEV

FIG. 9

E_p (MEV)

ARB. UNITS

HARD SPHERE SCATTERING
RUTHERFORD
90°

RUTHERFORD CROSS SECTION
AND
HARD SPHERE CROSS SECTION

\[ \frac{D\sigma}{D\Omega} \text{ (MB/STER.)} \]

RUTH.
HD. SP.
$\frac{D\sigma}{D\Omega}$

(MB/STER)

150°

RUTHERFORD CROSS SECTION

AND

HARD SPHERE CROSS SECTION

FIG. 18

PROTON ENERGY (MEV)
IV. ANALYSIS AND DISCUSSION OF RESULTS

The gamma rays seen at the same time the $90^\circ$ and $150^\circ$ elastically scattered protons were observed were compared to the gamma ray data taken by Prosser et al. This gave a correspondence of the energy scales. The energy scale given is that found by Prosser et al. The range of excitation in the compound nucleus, Co$^{55}$, was 8.16 to 10.19 Mev.

The analysis of spin $1/2$ particles on spin 0 nuclei is particularly simple to perform since there is only one way in which a resonance of a particular $J$ and parity can be formed. In the scattering matrix formulation, the scattering matrix is just a number. The formula for phase shift analysis can be constructed for spin $1/2$ on spin 0, whereas any formulism for higher spins or for nuclei not having spin 0 is very complicated.

A formulism for the elastic scattering of charged particles near a resonance level of a compound nucleus is given by J. M. Blatt and L. C. Bedienharn in terms of the scattering matrix formalism. As pointed out in the introduction, line shapes for $\ell = 0, 1, 2, 3,$ and 4 were computed on an IBM 650 Computer and are given in Fig. 19. The formula coded for the computer and a brief description of its derivation is given in Appendix I.

It must be emphasized that the theoretical curves are for isolated resonances only.

The analysis of the data consisted of comparing the calculated curves to the experimental shapes to see if there was any correspondence
in the shapes. The shapes allowed the assignment of \( l \) values to several of the states observed in the selected regions. For a number of states, data from the inelastic scattering provided an independent check on the assignments.

There are no contradictions between the shapes of the proton resonances and the assignments of the \( J \) values to these resonances from the angular distributions of the gamma rays. The proton data distinguished between \( J = 1/2^+ \) and \( 1/2^- \) states and gave a \( J \) assignment to two states which were detected by gamma rays which had an isotropic angular distribution.

In general, the states which were indicated by strong gamma rays gave only small variations in the proton yield.

There was possibly only one state that did not have a gamma ray associated with the proton resonance. This is the \( l = 0 \) state at 4.513 Mev.
THEORETICAL DIFFERENTIAL CROSS SECTIONS
ASSUMING ISOLATED RESONANCES

\[
\frac{d\sigma}{d\Omega} = \frac{J+\frac{1}{2}}{J+\frac{1}{2}}
\]

(RUHS/MERADIAN)

\( \theta = 90^\circ \)

\( \theta = 150^\circ \)

\( \theta = 180^\circ \)

\( \theta = \frac{3}{2} \)

\( \theta = \frac{3}{2} \)

\( \theta = \frac{3}{2} \)
V. ANALYSIS OF THE RESONANCES

I. 3.130 Mev Resonance  Fig. 10.

This resonance has a shape that is characteristic of $\ell = 2$ states. There is a gamma ray associated with this resonance, but data by Prosser et al. did not extend this low in energy. Since we do not have the J value of the state from the gamma ray work, we can only say that this is an $\ell = 2$ state.

II. 3.293 Mev Resonance  Fig. 11.

This corresponds to state 2 of the data of Prosser et al. See Fig. 8. The J value of this resonance was not given, but it was stated that the gamma ray had an isotropic angular distribution. This means that the state was either $1/2^+$ or $1/2^-$ unless there was some chance interference that gave an isotropic distribution. The shape of the proton resonance at 150° corresponds to $J = 1/2^+$, $1/2^-$, or $3/2^-$. The absence of a large dip and the presence of a small bump in the cross section at 90° indicates an odd $I$ state. While the gamma ray data shows this to be a $J = 1/2^-$ state, we can say that it is a $J = 1/2^-$ state.

III. 3.424 Mev Resonance  Fig. 12.

This is state 6 as numbered by Prosser et al. The proton shape corresponds to an $I = 2$ resonance. Since the gamma ray data gives a 5/2 assignment, we can say that this is a 5/2$^+$ state.
IV. 3.481 Mev Resonance  Fig. 12.

State 7 did not give an appreciable yield in either the 90° or 150° curve, but a state lying to the high energy side of 7, which showed up in the gamma ray data, gave a strong elastic yield. From the 150° curve it appears to be a $J = 1/2^+, 1/2^-$, or $3/2^-$ state. The data at 90° together with the choices at 150° shows that it is an $\ell = 1$ state.

V. 3.688 Mev Resonance  Fig. 13.

This resonance is between states 18 and 19. There appears a small bump in the gamma yield which could be associated with this state. The 150° data shows that we have a $1/2^+, 1/2^-$, of $3/2^-$ state. The 90° data with its symmetric dip together with the 150° choices shows that this is a $1/2^+$ state.

Although the fluctuations are small, the 90° curve has a dip corresponding to state 19 ($5/2$). This is typical of $\ell = 2$ states.

VI. 3.783 Mev Resonance  Fig. 14.

This is state 24 which was found to be $J = 1/2$ by Prosser et al. since the gamma ray associated with this state was isotropic in angular distribution. The data at 150° shows that this state is $\ell = 0$ or 1. This with the dip at 90° gives a $J = 1/2^+$ assignment to this state.
VII. 4.066 Mev Resonance  Fig. 15.

This is state 43 (5/2). Although the fluctuations are small there is a dip and rise at 90° and a rise and dip at 150° characteristic of \( l = 2 \) states.

VIII. 4.077 Mev Resonance  Fig. 15.

This is state 44 (5/2). The same analysis as that given for 43 can be given here.

IX. 4.088 Mev Resonance  Fig. 15.

The gamma ray associated with state 45 (5/2) has an intensity of about twice that of any other gamma ray from the lower regions to 4.3 Mev. See Fig. 8. There is only a small bump at 90° corresponding to this state. At 150° there is a small rise and dip compatible with \( l = 2 \) states. The fluctuations are so small here that we can not say anything definite about this state.

X. 4.139 Mev Resonance  Fig. 15

This is resonance #47 (5/2). It is the only gamma-ray-analyzed resonance that shows up clearly in this cluster of states. This resonance had a directly measured total width of 5.29 Kev. This was the largest total width seen by Prosser et al. in their gamma ray work. At 150° there is clearly a large rise but the gamma ray corresponds to the sloping rise at 90° and to the sloping fall
at 150°. This indicates an \( \lambda = 2 \) state. Since the gamma ray analysis gave \( J = 5/2 \), we may assign \( 5/2^+ \) to this state.

XI. Fig. 15 in general.

It is obvious that there are more resonances here than those analyzed by Prosser et al. Besides the eight analyzed resonances there were 14 small gamma rays seen that did not have sufficient intensity to be analyzed by an angular distribution.

Some of the resonances show large yield in the 90° and 150° curves, e.g., between states 39 and 40, 42 and 43, and 45 and 46.

The breakdown of the weak gamma rays as shown on the figure is 4 between 39 and 40, none between 40 and 41 and between 41 and 42. There are 5 between 42 and 43, and none between 43 and 44 and between 44 and 45. There are 2 between 45 and 46 and 3 between 46 and 47.

So some of the states that showed up weakly in the inelastic scattering data showed up prominently in the elastic scattering data.

In particular there appears to be an \( \lambda = 2 \) state just after state 42 at 4.038 Mev.

XII. 4.513 Mev Resonance Fig. 16

This resonance does not have any apparent gamma ray associated with it. From the 150° data the resonance appears to be \( \lambda = 0 \) or 1. The 90° data with the 150° choices indicates a \( J = 1/2^+ \) state. The dip in the 90° data is so pronounced that we may give this state a
\( J = \frac{1}{2}^+ \) assignment even though there are many more states surrounding this state to which assignments can not be made.

A thorough analysis of this data would include complete excitation curves at several angles. The resonance shapes could then be matched with the theoretical shapes by taking into account the target thickness, varying the half width of the resonance, and obtaining an accurate evaluation of the background due to the high energy tail of the distribution due to the elastically scattered protons from \( \text{C}^{12} \). In the data presented in the figures there has been no attempt to subtract the background due to this effect. The Rutherford curve given in the figures represents a normalization to the general trend of the cross section from 1.8 to 2.9 Mev. This curve is given so that an idea of the amplitudes of the resonances may be determined and should not be considered as an indication of the correct cross section for the resonances. The curve for hard sphere scattering which includes Rutherford scattering is also given but again the value of the rough estimate of Rutherford cross section was used in order to place this curve in its approximate position.
VI. SUMMARY AND CONCLUSIONS

We are now in position to deal further with the questions posed in the introduction. Question 2 (a) and (b) asked whether it were possible to detect the elastically scattered protons and whether the states would be spaced far enough apart so that individual states could be analyzed. We have seen that some of the states are widely enough spaced so that their shapes can be analyzed, and the results corroborate the conclusions from the inelastic scattering data. The states which showed prominently in the inelastic scattering gave only weak fluctuations in the elastic cross section.

A continued program of performing elastic and inelastic scattering using the medium weight nuclei should be undertaken. From this series, information about spacing, clustering, and range of \( \lambda \) values would be obtained.

From the work so far on Ni\(^{58}\), Ti\(^{46}\), and Fe\(^{54}\) we see that \( \lambda = 2 \) states are very abundant in the compound nucleus in the ranges observed. There also seems to be clustering of states both in the gamma ray work and in the elastic scattering work.

Since the shapes of the elastic resonances agree very well with the theoretical shapes, we can find information on the level spacings of the lower \( \lambda \) values. The inelastic scattering work appears to give most of the \( \lambda = 2 \) states; so a serious study of the level densities could be undertaken.
APPENDIX I

The expression for the differential cross section $d\sigma$ for scattering into the solid angle element $d\Omega$ at an angle $\theta$ to the incident beam is well known:

$$d\sigma = |f(\theta)|^2 d\Omega$$

where $f(\theta)$ is the scattering amplitude.

We can set $f(\theta)$ corresponding to the scattering of spinless charged particles by a nucleus of zero spin as:

$$f(\theta) = f_c(\theta) + f_{CH}(\theta) + f_{Rx}(\theta)$$

where $f_c(\theta)$ is the scattering amplitude for pure Rutherford scattering, $f_{CH}(\theta)$ is the difference between the scattering amplitude from a charged hard sphere and $f_c(\theta)$, and $f_{Rx}(\theta)$ is the nuclear resonance scattering amplitude, which we assume to be associated with a resonance of angular momentum $l$, of partial width $\Gamma_l$, total width $\Gamma$, and resonance energy $E_0$.

We introduce the notation

$$z = \frac{Z_a Z_x e^2}{2 M^2}$$

where $Z_a$ and $Z_x$ are the atomic numbers of the incident particle and the target nucleus, respectively, and $M$ is the reduced mass for the relative motion in the center-of-gravity system. We also define phase shifts $\phi_\ell$ and $\psi_\ell$ through

$$\phi_\ell \equiv \phi_\ell - \sigma_\ell \quad \exp(2i\phi_\ell) = \frac{G_\ell(R) - i F_\ell(R)}{G_\ell(R) + i F_\ell(R)}$$

$$\psi_\ell = \sigma_\ell - \sigma_0 \quad \exp(2i\psi_\ell) = \frac{(\ell + i\eta)(\ell - 1 + i\eta) \cdots (1 + i\eta)}{(\ell - i\eta)(\ell - 1 - i\eta) \cdots (1 - i\eta)}$$
In terms of this notation we have

\[
 f_c(\theta) = -2 \cos^2 \frac{\theta}{2} \exp \left[-2 \sqrt{\eta \ln \sin \frac{\theta}{2}}\right]
\]

\[
 f_{CH}(\theta) = i \pi \eta \sum_{k=0}^{\infty} (2k+1)^{1/2} \exp \left[2i \Psi_k\right] [1-\exp(2i \Psi_k)] Y_{1,0}^{\pm}(\theta)
\]

\[
 f_{R\lambda}(\theta) = i \pi \eta (2R+1)^{1/2} \exp \left[2i (\Psi_{\lambda}+\Phi_{\lambda})\right] \frac{\mu_{\lambda} \exp(2i\Theta) Y_{2,0}^{\pm}(\Theta)}{(E-E_0)^2-(\gamma_2^2)} \gamma_2
\]

The cross section for elastic scattering is then given by

\[
 d\sigma = \left| f_c(\theta) + f_{CH}(\theta) + f_{R\lambda}(\theta) \right|^2 d\Omega
\]

\[
 = \left[ |f_c|^2 + 2 R.P. (f_c^* f_{CH}) + |f_{CH}|^2 \right] d\Omega
\]

\[
 + \left[ 2 R.P. (f_c^* f_{R\lambda}) + 2 R.P. (f_{CH}^* f_{R\lambda}) \right] d\Omega
\]

\[
 + \left| f_{R\lambda} \right|^2 d\Omega
\]

The separation above is into three parts: potential scattering, interference between potential and resonance scattering, and pure resonance scattering. By expanding in terms of Clebsch-Gordon coefficients and combining some \( \gamma_{\lambda} \)'s and replacing \((2k+1)\) by \((2J+1)/(2I+1)(2 \lambda + 1)\) in the resonance terms we finally get the following formula for the elastic scattering of charged particles near an isolated resonance level of the compound nucleus:
\[ d\sigma_{\alpha\alpha} = \frac{\alpha^2}{(2I+1)(2I+1)} \sum_{L=0}^{\text{max}} R_L(\alpha,\alpha) \cdot P_L(\cos \theta) d\Omega \]

\[ + \frac{Z^2}{2} \csc^2 \theta \, d\Omega \]

\[ -2 \, \alpha^2 \sum_{L=0}^{\infty} (2L+1) \sin \phi_L \cos \left[ 2 \eta \sin \theta + \phi_L + \phi_L' \right] \]

\[ \times \csc^2 \theta \cdot P_L(\cos \theta) \, d\Omega \]

\[ + \frac{\alpha^2}{2} \sum_{L=0}^{\infty} \sum_{L' = 0}^{\infty} \sum_{L'' = 1}^{L+1} \left[ \frac{(2L+1)(2L'+1)(2L''+1)}{(2L'00L'0'(00L0))} \right]^2 \]

\[ \times \sin \phi_L \sin \phi_L' \cos \left[ (2 \Psi_L + \Psi_L') - (2 \Psi_L + \Psi_L') \right] \cdot P_L(\cos \theta) d\Omega \]

\[ + \frac{\alpha^2}{2} \frac{(2J_0+1)}{(2I+1)(2I+1)} \sum_{L=L_{\text{min}}}^{J_0+I+1} \frac{\Gamma_{\alpha L}}{\sqrt{[(E - E_0)^2 + (\gamma k)^2]}} \frac{1}{V_4} \]

\[ \times \csc^2 \theta \sin \left[ 2 \eta \sin \theta + 2 \psi_L + 2 \phi_L + \beta \right] \cdot P_L(\cos \theta) d\Omega \]

\[ - \frac{\alpha^2}{2} \frac{(2J_0+1)}{(2I+1)(2I+1)} \sum_{L=0}^{\infty} \sum_{L' = 0}^{J_0+I+1} \sum_{L'' = 1}^{L+1} \left[ \frac{(2L''+1)}{(2L'00L'0'(00L0))} \right]^2 \]

\[ \times \left[ (\eta L'001 \eta L'0'100 L0) \right]^2 \sin \phi_L' \cdot \frac{\Gamma_{\alpha L}}{\sqrt{[(E - E_0)^2 + (\gamma k)^2]}} V_4 \]

\[ \times \sin \left( \beta + 2 \Psi_L + 2 \phi_L' - 2 \Psi_L' - \phi_L' \right) \cdot P_L(\cos \theta) d\Omega \]
The interpretation of the various terms is as follows: the first term is pure resonance scattering. The following term is pure Rutherford scattering. The next two terms represent the correction due to the finite size of the nucleus. The next to last term is the interference between resonance scattering and pure Rutherford scattering, while the last term is the finite nuclear size correction to this interference term.

The part of this equation that is resonance dependent and Rutherford scattering was coded for an IBM 650 Computer. The parts that are non-resonant are terms 3 and 4, excepting Rutherford scattering.

In this program, \( \Gamma \) was considered to be equal to the total width \( \Gamma \). Setting the partial width equal to a value less than the total width would decrease the effects of the last two terms, i.e., the interference between resonance scattering and pure Rutherford scattering and its nuclear size correction.

The formula used in the calculation of the non-resonant cross section using phase shifts was

\[
d\sigma = |f_c|^2 d\Omega
\]

where

\[
f_c = -\frac{\alpha}{2} \csc^2 \frac{\theta}{2} e^{i\eta x_n c \text{csc} \frac{\theta}{2}}
\]

\[
+ \sum_{l=0}^{2N} e^{i\alpha_x (2l+1)} e^{i\delta_l} \sin \delta_l P_l (\cos \theta) 
\]

\[
\alpha_x = 2 \sum_{l=1}^{2N} \tan^{-1} \eta/l
\]

\[
\eta = \frac{\Gamma}{2}\left[\frac{\Gamma}{2} + \frac{1}{2}\right]
\]

\[
\delta_l = \delta_l - \Delta
\]
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