RICE UNIVERSITY

Design and Calibration of the
Suprathermal Ion Detector Experiment (SIDE)

by

Robert L. Shane

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

Thesis Director's signature:

Houston, Texas

May, 1969
ABSTRACT

Design and Calibration of the Suprathermal Ion Detector Experiment (SIDE)

Robert L. Shane

The SIDE is one of the experiments for the Apollo Lunar Surface Experiments Package (ALSEP). It is designed to measure mass, energy per unit charge, and flux of positive ions at the surface of the moon.

The SIDE contains two analyzers. One is for low energy ions; it consists of $E \times B$ velocity analyzer and a cylindrical curved plate energy analyzer aligned with a channeltron electron multiplier. The other is for high energy ions; it consists of a curved plate analyzer only. The combination of energy and velocity data provides both energy and mass analysis of the low energy ions. The high energy ions are energy analyzed only. Each element of the analyzers is discussed separately according to ion trajectories, passbands, and design characteristics.

The calibration procedures are described, sample curves given, and results presented.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Abstract</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table of Contents</td>
<td></td>
</tr>
<tr>
<td>I. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>II. Design Criteria</td>
<td>6</td>
</tr>
<tr>
<td>III. Instrument Description</td>
<td>7</td>
</tr>
<tr>
<td>A. Low Energy Analyzer</td>
<td>7</td>
</tr>
<tr>
<td>B. High Energy Analyzer</td>
<td>9</td>
</tr>
<tr>
<td>C. Operation</td>
<td>10</td>
</tr>
<tr>
<td>IV. Instrument Design Details</td>
<td>10</td>
</tr>
<tr>
<td>A. Velocity Filter</td>
<td>10</td>
</tr>
<tr>
<td>B. Curved Plate Analyzer (CPA)</td>
<td>14</td>
</tr>
<tr>
<td>V. Instrument Calibration</td>
<td>21</td>
</tr>
<tr>
<td>A. Low Energy Calibration</td>
<td>21</td>
</tr>
<tr>
<td>B. High Energy Calibration</td>
<td>25</td>
</tr>
<tr>
<td>C. Geometric Factor</td>
<td>26</td>
</tr>
<tr>
<td>Appendix</td>
<td></td>
</tr>
<tr>
<td>A. Equations of Motion for Particles in Crossed E and B fields</td>
<td>27</td>
</tr>
<tr>
<td>B. First Order Passband for the Velocity Filter</td>
<td>33</td>
</tr>
<tr>
<td>C. Potential, Electric Field, and Force Equations for the Curved Plate Analyzer</td>
<td>34</td>
</tr>
</tbody>
</table>
Appendix

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>D. Focusing Angle for a Curved Plate Analyzer</td>
<td>37</td>
</tr>
<tr>
<td>Figure Captions</td>
<td>41</td>
</tr>
<tr>
<td>Figures</td>
<td>44</td>
</tr>
<tr>
<td>Tables</td>
<td>62</td>
</tr>
<tr>
<td>Acknowledgments</td>
<td>70</td>
</tr>
<tr>
<td>References</td>
<td>71</td>
</tr>
</tbody>
</table>
I. INTRODUCTION

The Suprathermal Ion Detector Experiment, hereafter called SIDE, is designed to measure charged particle fluxes at the lunar surface. It consists of two analyzers to determine ionic constituents: a low energy analyzer for the lunar surface atmosphere and a high energy analyzer for the incident solar wind. Analysis of the SIDE data will yield flux of positive ions, energy spectra, lunar surface potential, and mass of ions observed by the low energy analyzer.

Many diverse models have been developed to predict the lunar atmosphere from assumptions concerning atmospheric sources and losses. The SIDE data will determine which model best describes the dominant mechanisms.

A few of the lunar atmosphere models are briefly described below; the sources and losses that each considers are given in Table I.

Edwards & Borst calculated the production of Xe and Kr from radiogenic sources such as (1) spontaneous fission of $^{238}U$, (2) thermal neutron fission of $^{235}U$ by cosmic ray bombardment, (3) decay of $^{129}I$, and (4) primeval gases. This results in a predominantly Xe and Kr atmosphere.
Singer\textsuperscript{2} points out that Edwards and Borst considered thermal losses only and disregarded losses due to a positive surface potential of +20 volts caused by photoionization. He concludes that Xe and Kr are relatively unimportant and that there is a permanent density of 80 hydrogen atoms per cubic centimeter from solar wind accretion.

Opik\textsuperscript{3}, assuming the solar wind is entirely protons, also obtains an atmosphere of H. However, he gets larger concentrations of H\textsubscript{2}O and CO\textsubscript{2}. The sputtering of the lunar surface by the solar wind causes the release of O which in turn combines with the H to form H\textsubscript{2}O. The remaining protons combine to give H\textsubscript{2}. The permafrost layers act as additional sources of H\textsubscript{2}O. The H\textsubscript{2}O as well as CO\textsubscript{2} is released by meteorite bombardment.

Watson et al\textsuperscript{4} maintain that the amount of volatile in the atmosphere is governed by the temperature of the solid phase. They obtain a lunar atmosphere consisting mainly of H\textsubscript{2}O vapor, the amount of which is controlled by the evaporation rate of the permafrost layers.

Nikada & Mihalev\textsuperscript{5}, assuming the solar wind composition is similar to the solar corona, add Ne and Ar to the lunar atmosphere. They consider the solar O and N
permanently bound to the moon because these elements are so chemically active.

Hinton and Taeusch and Vestine consider that the radioactive decay of K is an additional source of A.

Vestine and Michel estimate sources due to vulcanism result in H2O, SO2, CO2, and NH3.

Herring and Licht also obtain negligible Xe and Kr, but get SO2, CO2, and H2O due to residual volcanic activity based on terrestrial emission data.

Bernstein et al propose a null or negative surface potential and get an atmosphere that has all of the above gases except for Xe, Kr, H2O, and volcanic gases which they do not consider.

The above atmospheric models were based on the assumption of impact of the solar wind on the lunar surface, i.e., no bow shock exists; most evidence seems to support this assumption. Dolginov et al reported, from Luna 2 data, a magnetic field of ± 50 μ and from Explorer 35 data Ness et al reported a field of less than 16 μ at the moon's surface. However, Behannon estimates it to be 4 μ from Explorer 35 data while the moon crosses the neutral sheet and the field reverses. By comparing simultaneous data from Explorer 35 and 33,
Taylor et al. found that the moon had little effect on the interplanetary field. They say, "The magnetic effects of the moon on the interplanetary field can be summarized by the statement that the moon behaves like a poorly conducting nonmagnetic body with a negligible permanent magnetic field."

Data on the lunar ionosphere is obtained from occultation by the moon of radio sources. The angle of refraction of the radio waves as they traverse the lunar atmosphere is related to the index of refraction of the ionosphere. The refraction index is related to the electron number density and the wave frequency. By observing the occultation of the Crab Nebula and assuming ionization of one per thousand of the lunar atmosphere particles, Elsmore calculates an upper limit of $10^3$ electrons/cm$^3$ or $10^6$ particles/cm$^3$ for the lunar atmosphere. On the other hand, Hazard et al. calculate a limit of $10^2$ electrons/cm$^3$ from their observations of the occultation of Radio Source 3C 273.

Gringauz et al. provide a direct particle measurement indicating a lunar ionosphere. Lunik III found an increase in electron density above ambient beginning at about 10 moon radii. Also, from Luna 10, an
electron density of 80/cm$^3$ is obtained.

Weil and Barasch$^{23}$ theoretically calculate an electron density increasing with altitude above the lunar surface with a peak of 350 electrons/cm$^3$ at 0.6 lunar radii. This model is supported in part by the Stanford occultation measurements of Pioneer VII$^{24}$. SIDE data should delineate the predominant atmospheric sources. For example, if the masses are mainly atomic, the solar wind is the dominant source, but if they are molecular, vulcanism is dominant (Michel$^8$).

The particle temperature, derived from SIDE data, will show whether there is a bow shock or not: thermal particles, those that have been accommodated to the lunar surface temperature of 400$^0$K, imply a bow shock; fast, energetic particles imply incident solar wind particles and, therefore, no bow shock.

The SIDE mass identification determines whether the solar wind composition is similar to that of the solar corona or simply protons and alpha particles.

The SIDE obtains data on the lunar potential by stepping the potential of SIDE with respect to that of the moon's surface.
II. DESIGN CRITERIA

The instrumental design is based upon predictions by the models on lunar conditions and on minimal size and weight requirements. Mass analysis of the entire range is not practical because of space limitations. Therefore, the instrument is divided into two parts: a high energy analyzer only and a low energy analyzer which also includes mass analysis.

Mariner 2 data indicate solar wind velocities ranging from 300 to 800 km/sec corresponding to fluxes from $10^7 - 10^9 \text{ cm}^2/\text{sec}$. The upper end of the high energy range is, therefore, 3500 ev. The lower end is 10 ev which slightly overlaps the high end of the low energy range.

The low energy range starts at 0.2 ev, which is slightly above lunar thermal energies, and extends to 48.6 ev, which is slightly higher than the low end of the high energy range.

The geometric factor is the constant which converts fluxes to counting rates. This constant is inherent in the physical construction of the instrument and is determined by the detection area, the acceptance angle, and the instrument efficiency. The SIDE geometric factor
is $10^{-6}$ cm$^2$.

The mass range includes the mass of all predicted lunar atmospheric elements; it extends from 1 amu (for hydrogen) through 130 amu (for xenon).

In order to obtain data on the lunar surface potential, the instrument potential is stepped ± 20 volts with respect to a screen grid which will be in contact with the lunar surface. The ± 20 volts corresponds to the highest estimate of the lunar potential.

The instrument's size is 12 by 16.3 by 5.35 inches and its weight is 21 pounds. Its operating temperature range for the electronics is from -30°C to 70°C. The instrument lifetime is one year.

III. INSTRUMENT DESCRIPTION

A. Low Energy Analyzer

This analyzer consists essentially of four components: (1) velocity filter, (2) low energy electrostatic curved plate analyzer (LECPA), (3) ion flight tunnel, and (4) electron multiplier (Figure 1). The velocity filter data is combined with the data from the LECPA to obtain particle mass in the low energy range.
The filter is a Wein velocity filter; it consists of a double C permanent magnet made of alnico. It weighs approximately 62.5 grams and has a field strength of $850 \pm 10$ gauss (Figure 2). This configuration provides minimal weight and size and ease of mounting. There are two parallel electric field plates, between the magnetic pole pieces, with a balanced potential on each, i.e., equal voltages of opposite polarity. The plate voltages are set according to the masses and energies to be observed by equating the forces on the particles.

$$eE = Bev \quad (1)$$

$E$ is the electric field, $B$ the magnetic field, $e$ the particle charge, and $v$ the particle velocity.

$$v = \frac{E}{B} \quad (2)$$

But the voltage $V$ on the plates of a parallel plate capacitor of separation $d$ is

$$V = Ed \quad (3)$$

$$V = Bvd \quad (4)$$

Substituting for $v$ in terms of energy $w$ and mass $m$ gives

$$v = \sqrt{\frac{2w}{m}} B d \quad (5)$$
These voltages range from 0.12 to 28 volts in 120 steps - 20 steps for each of the six energy levels (see Table 1).

The curved plate analyzer is basically a section of a cylindrical capacitor, i.e., two concentric cylindrically curved plates with a stepped balanced voltage on them (Figure 3). This design reduces background counts due to ultraviolet radiation and provides greater efficiency than a retarding potential analyzer configuration. The dimensions were determined from voltage and size considerations and geometric factor calculations. The voltage is 0.1 to 24.3 volts as required by the energy range being investigated (0.2 - 48.6 ev in multiples of three).

The ion flight tunnel (Figure 1) has entrance, exit, and baffeling slits for beam definition. All surfaces are gold plated and platinum blacked to minimize ultraviolet scattering.

The electron multiplier is a Bendix Channeltron - a solid state counting device which is operated at -3500 volts in order to post accelerate the ions and give them sufficient energy to produce secondary electrons.

B. High Energy Analyzer

The high energy analyzer consists of three components: (1) high energy electrostatic curved plate analyzer (HEPCA),
(2) ion flight tunnel, and (3) electron multiplier. These components are similar to those described for the low energy analyzer. The HECPA voltages are 2.5 to 825 volts in 20 steps (Table 3, Figures 3 and 4).

C. Operation

The SIDE operates as follows (see Figure 1). Low energy ions enter the entrance aperture and pass through the flight tunnel into the velocity filter which passes ions having a narrow range of velocities. These selected ions then pass into the curved plate analyzer which passes ions having only a narrow range of energies per unit charge. The passed ions are counted by the channeltron and have a known velocity and energy from which their masses may be determined. The high energy particles enter the entrance apertures and pass into the curved plate analyzer which measures their energy spectrum.

IV. INSTRUMENT DESIGN DETAILS

A. Velocity Filter

Ion focusing is determined from the equations for the Wein velocity filter (used in SIDE) derived by Herzog18.
The lateral displacement $y''$ of a particle which has already passed through the crossed field region is given by

$$
y'' = a \left[ -\frac{\alpha_m}{\alpha} \sin \frac{\rho}{\alpha_m} - \frac{\alpha_m}{\alpha} \beta \left( 1 - \cos \frac{\rho}{\alpha_m} \right) + \frac{\rho'}{\alpha} \cos \frac{\rho}{\alpha_m} \right]$$

$$+ X'' \left[ -\alpha' \cos \frac{\rho}{\alpha_m} - \rho \sin \frac{\rho}{\alpha_m} - \frac{\rho'}{\alpha_m} \sin \frac{\rho}{\alpha_m} \right]$$

(5)

The symbols are defined as follows (see Figure 5).

- $\alpha_m = \frac{m \gamma}{B e}$ is the radius of curvature of a particle traveling in a magnetic field only.
- $\alpha$ is the radius of curvature of ions with mass $m$ and velocity $v_0$.
- $L$ is the length of the field region.
- $\alpha'$ is the incident angle of particles entering the field region.
- $\beta = \frac{v}{v_0} - 1$ is the velocity percentage passband.
- $X''$ is the $X$-coordinate after leaving the field.
- $Y' = b - \alpha' \rho'$ is the $Y$-coordinate of the point where particles enter the field and $b$ is the $Y$-coordinate of the source.
- $l'$ is the object to field distance.
- $l''$ is the image to field distance.

$$q = \frac{a_m \cot \frac{\rho}{\alpha_m}}{\sin \frac{\rho}{\alpha_m}}$$

$$f = \frac{a_m}{\sin \frac{\rho}{\alpha_m}}$$
The point of direction focusing, i.e., where particles entering the field at small angles to the normal are re-focused (focal point), is obtained by setting $\alpha'$ equal to zero to make $\gamma''$ independent of $\alpha'$. This yields

$$f^2 = (l' - g)(l'' - g)$$  \hspace{1cm} (7)

The equations of motion in terms of time as a parameter, derived in Appendix A, are

$$x = a t + \frac{v}{\omega} (1 - \cos \omega t) + \frac{v}{\omega} (x_0 - a) \sin \omega t$$  \hspace{1cm} (8)

$$y = y_0 + \frac{a - x_0}{\omega} (1 - \cos \omega t) + \frac{v}{\omega} (\sin \omega t)$$  \hspace{1cm} (9)

where $\omega = \frac{q \Phi}{m}$ is the cyclotron frequency for a particle charge $q$ and particle mass $m$.

$v_0 = \frac{E}{B}$ = electric field/magnetic field is the straight through velocity.

The initial conditions are $\theta$ in the $x$ direction, $E$ in the $y$ direction, and at $t=0$, $y=y_0$, $y'=y_{0}$, $y''=y_{0}$, $x=x_0 = 0$, $\dot{x} = x_0$.

A velocity passband $\beta$ can be approximated as shown in Appendix B.

$$\beta = \frac{y_s v_0}{\frac{v^2}{2} + 2 y_s v_0}$$  \hspace{1cm} (10)

where $y_s$ is the off axis displacement at exit from field.

$L$ is the length of velocity filter.
\( \nu \) is the straight through velocity.

\( \omega \) is the cyclotron frequency.

It is apparent that the passbands become larger as the magnetic field decreases so that overlapping of passbands occurs for masses with atomic weight near 18. Thus, there is a minimum critical field above which overlapping does not occur. This field is found by numerically solving for the passbands and the particle trajectories for different values of \( B \). The resulting lower bound of \( B \) is 1,000 gauss.

Since the ions travel in cycloids, it is possible that masses as light as hydrogen might execute more than one cycle within the field region and pass through the velocity filter with a velocity other than the straight through velocity \( \nu \). If a well defined passband exists for particles executing one half cycloid or less, the critical energy for any mass that just completes one half cycloid as a function of filter velocity may be obtained\(^{19} \).

If the field region is of length \( L \), the particle velocity \( \nu \), and \( T \) the transit time for one half cycloid, then

\[
L = \nu T
\]  

(11)

The transit time is equal to the angle displacement \( \phi \)
divided by the angular velocity \( \omega = \frac{v}{L} \)

\[
T = \frac{\theta}{\omega} = \frac{\pi}{\omega} \tag{12}
\]

\[
L = \frac{\nu_0 \pi}{\omega} \tag{13}
\]

\[
L = \frac{\nu_0 m \pi}{e \phi} \tag{14}
\]

The energies of interest for any given mass are then

\[
E_{\text{energy}} \geq \frac{1}{2m} \left( \frac{e B L}{\pi} \right)^2 \tag{15}
\]

These critical energies are listed in Table 4 for \( L = 1 \text{cm} \) and \( B_{\text{eff}} = 785 \text{ gauss} \). \( B_{\text{eff}} \) is the experimental effective magnetic field.

It may be concluded from Table 4 that the only cycloiding problem will be hydrogen and helium at 0.2 ev and hydrogen at 0.6 ev.

B. Curved Plate Analyzer (CPA)

The curved plate analyzer (Figure 3) is a device which allows ions with only one particular energy to pass on a circular path. The critical dimensions and parameters are: the radii of curvature, the analyzer constant, the ion focusing angle, the plate separation and width, and the energy passband.

The radii of curvature are determined by the geometric factor, the solid angles subtended by the slits (see Figure 6), and the lengths of the CPA. The geometric
factor is related to the incident unidirectional flux and the counting rate by

\[ GF = c \]  \hspace{1cm} (16)

where

- \( c \) is the counts per second,
- \( G \) the CPA geometric factor, and
- \( F \) the unidirectional flux in particles /cm\(^2\)/sec/ster. The geometric factor is proportional to the product of the area \( A \) of the entrance slit and the solid angle \( \varpi \).

\[ G_{\text{CPA}} = A \varpi \varepsilon \]  \hspace{1cm} (17)

in which \( \varepsilon \) is the detection efficiency. From this the solid angle may be calculated.

\[ \varpi = \frac{G}{A \varepsilon} \]  \hspace{1cm} (18)

The linear length \( \lambda \) of the CPA is derived from the solid angle as defined in Figure 7.

\[ \lambda = \frac{A}{A^2} \]  \hspace{1cm} (19)

\[ \lambda = (\frac{A}{A})^{1/2} \]  \hspace{1cm} (20)

\[ \lambda = (\frac{\lambda}{\lambda^{1/2}}^{1/2}) \]  \hspace{1cm} (21)

Since \( \lambda = r_o \theta = r_o (2\pi^{1/2}) \) \hspace{2cm} \( r_o)(2.22 \text{ radians}) \)

\[ r_o = \frac{\lambda}{2.22} \]  \hspace{1cm} (22)

where \( r_o \) is the radius of curvature of the CPA.

The inner and outer plate radii \( r_i \) and \( r_o \), respectively, are \( r_i = \frac{\Delta r}{2} \) and \( r_o + \frac{\Delta r}{2} \), where \( \Delta r \) is the plate separation.
The analyzer constant is derived from the voltage \( \nu \) impressed upon the plates of the CPA. The potential \( \phi \) of the CPA (Appendix C) is

\[
\phi = \frac{\nu}{2 \log\left(\frac{r_1}{r_2}\right)} \left[ \log\left(\frac{r}{r_2}\right) \right]
\]

(23)

where \( r_1 \) and \( r_2 \) are the radii of curvature and \( r \) the radial coordinate. The electric field \( E \) is the gradient of the potential; it is expressed as

\[
E = -\frac{\nu}{\log\left(\frac{r_1}{r_2}\right)} \frac{1}{r}
\]

(24)

and the force acting on a singly charged ion is

\[
F = -\frac{\nu}{\log\left(\frac{r_1}{r_2}\right)} \frac{\phi}{r}
\]

(25)

Therefore, the equation for the equilibrium orbit is

\[
\frac{\sigma \nu^2}{r} = \frac{\nu}{\log\left(\frac{r_1}{r_2}\right)} \frac{1}{r}
\]

(26)

This gives the energy \( w \) as

\[
\frac{1}{2} m \nu^2 = w = \frac{\nu}{2 \log\left(\frac{r_1}{r_2}\right)}
\]

(27)

Thus,

\[
w = \nu A
\]

(28)

where \( A = \frac{1}{2 \log\left(\frac{r_1}{r_2}\right)} \) is the analyzer constant.

The focusing angle of the CPA may be determined by several methods. Only two methods will be discussed: one is given below and the other in Appendix D.

The equations of motion for particles in \( E \) and \( \phi \) fields (Appendix D) are:

\[
\ddot{r} - r \dot{\phi}^2 = \frac{\kappa}{r} - \lambda r \dot{\phi}
\]

(30)
where \( r \) and \( \theta \) are polar coordinates, \( \kappa = \frac{\nu}{2y(r_n r)} \frac{c}{c_n} \), \( \lambda = \frac{\kappa c}{r_n} \), and \( n_1 \) and \( n_2 \) are radii of curvature of the plates.

Substituting equation (31) into equation (30), and considering orbits at small angles to circular orbits yields the equation for simple harmonic motion

\[
\ddot{x} + \frac{\lambda^2}{2} x = 0
\]  

(32)

where \( x \) is a small radial displacement from a circular orbit. The solution to the above equation is

\[
x = x_0 \sin \left( \frac{\lambda t}{\sqrt{2} \lambda} \right)
\]  

(33)

Focusing of divergent rays occurs for \( x = 0 \) and the focusing angle \( \phi_e \) is

\[
\phi_e = \frac{\pi}{\sqrt{2} \lambda} = 127^\circ 17'
\]  

(34)

This solution is valid for the CPA because it is independent of \( \lambda = \frac{\kappa c}{r_n} \) and therefore applies when \( \phi = 0 \).

The plate separation and width calculation procedure is illustrated in Figure 7.

The plate separation \( \Delta r \) is calculated from the slit width and the angle \( \theta \) defined in Figure 7; it is

\[
\theta \approx \tan \theta = \frac{\Delta x}{l}
\]  

(35)
in which $\Delta x$ is the distance from the plate to the slit edge. The plate separation is

$$\Delta r = 2(\Delta x) + \text{slit width}$$

(36)

The minimum plate width $w_{\text{min}}$ is calculated by the same method where the slit width is now the slit length, the angle $\theta$ is now the angle $\varphi$, and $\Delta x$ is now $\Delta y$.

$$w_{\text{min}} = 2 \Delta y + \text{slit length}$$

(37)

The energy passband may be obtained by solving the lateral displacement equation (D-4) for $\varphi$ and using the dimensions from Figures 19.

$$\beta = \frac{y^* - \cos \varphi \left[ y^* - y' \right]}{\alpha_e \left( 1 - \cos \varphi \right) + \frac{\Delta x}{\sqrt{\alpha_e}} \left( \frac{\Delta x}{\sqrt{\alpha_e}} + \frac{\alpha_e}{\Delta x} \right)}$$

(38)

Since $\varphi_e = \frac{\pi}{2}$, all the sine terms are zero and the cosine terms are -1. Then

$$\beta = \frac{y^* - \left[ y^* - y' \right]}{\alpha_e}$$

(39)

$\alpha_e$ is radius of curvature for ions traveling in radial electric fields.

Numerical values of the dimensions and parameters for LECPA and HECPA are similarly calculated except that those for LECPA are calculated for thermal particle fluxes (corresponding to a lunar surface temperature of $400^\circ\text{K}$, 18
a velocity of \(10^5\) cm/sec, and a number density of \(10^3/\text{cm}^3\); whereas, those for HEGPA are calculated for solar wind fluxes. Hence, only the calculations for HEGPA are given. The results for both HEGPA and LECPA are listed in Table 5.

For HEGPA the CPA length is numerically determined from the incident flux \(F\), which is fixed by the number density \(n\), the velocity \(v\), and the solid angle \(\phi\). Assuming that the density is \(10/\text{cm}^3\) and the velocity is \(4 \times 10^7\) cm/sec\(^17\), the particle flux is

\[
F = \frac{nv}{4\pi} = \frac{1}{4\pi} \times 10^8 \quad \text{particles/cm}^2/\text{sec/ster} \quad (40)
\]

Taking \(10^2\) as an acceptable count rate, the CPA geometric factor can be calculated from equation (16).

\[
G_{\text{CPA}} = \frac{C}{F} = \pi \times 10^{-6} \text{ cm}^2/\text{ster} \quad (41)
\]

For a channeltron efficiency of one percent and a slit area of \(0.16\) cm\(^2\), the solid angle is calculated from equation (18)

\[
\mathcal{N} = 1.96 \times 10^{-3} \quad \text{ster} \quad (42)
\]

The length of the HEGPA is then determined from equation (20)

\[
\mathcal{L} = 9.02 \text{ cm} \quad (43)
\]

and the radius of curvature from equation (22)
The analyzer constant calculated from equation (29) is
\[ A = 3.97 \frac{e \nu}{\nu_0 f} \]  

The ion focusing angle as derived above is 2.22 radians or $127^\circ 17'$. 

From equations (36 and 37), the plate separation is 0.50 cm and the plate width is 2.00 cm. To simplify fabrication, all plate widths were set at $\pi$ cm and all plate separations at 0.5 cm: both values are larger than the calculated minimums.

The passbands as calculated from equation (39) are 11.4\% for LECPA and 6.1\% for HECPA.

Collimation of the incoming ion beam is achieved by placing the entrance slit 6 cm from the HECPA. This gives a new length including the flight path of 15.00 cm for HECPA and 15.50 cm for LECPA (Figure 7). The instrument geometric factors become $1.14 \times 10^{-6}$ cm$^2$-ster for HECPA and $1.06 \times 10^{-6}$ cm$^2$-ster for LECPA.

The look angles $\theta_i$ and $\theta_f$ of the instrument are the maximum angles from the normal to the slits for which particles can reach, from the entrance slits, the
HECPA or the velocity filter. They are defined from their tangents. These tangents are determined by the slit dimensions and the distance from the entrance aperture to the HECPA or to the velocity filter.

\[
\begin{align*}
\theta_y &= \theta^o \\
\theta_y &= \varphi^o
\end{align*}
\]

where the \( y \)-axis is parallel to the entrance slit width, the \( y \)-axis is parallel to its length, and the \( x \)-axis is parallel to the normal to the slit.

V. INSTRUMENT CALIBRATION

A. Low Energy Calibration

The SIDE is placed in a vacuum chamber and bombarded with ions of various mass numbers, which depend on atmospheric constituents and known gases leaked into the system. The ions are given energies corresponding to those listed in Table 2. The data is printed out by a Franklin printer.

The curved plate analyzer passband of the LECPA can be determined by plotting the counts in each side frame (SF) versus energies within a narrow range centered about the six being investigated. A SF is each detection interval. Each voltage change corresponds to a new SF and to a different energy or mass sample. The width at half
maximum of the envelope of these curves is the passband. Figure 8 shows a sample curve indicating a passband of about 10%, which compares quite well with the 11.4% theoretically predicted.

Absolute mass calibration is dependent upon the sensitivity of the instrument for any given gas. Two factors must be known: the number of ions of a certain mass in the beam incident on the instrument and the fraction of counts due to ions of that mass in each SF. These factors are determined in the following way. The counts in any SF are made by ions having many different masses; each ion is present in proportion to a constant which depends only on the SF. The count rate can be written as

$$c_s = \sum_{i=1}^{n} F_i \alpha_i$$  \hspace{1cm} (47)

where $F_i = \frac{I_i}{A}$ is the unidirectional flux of the $i^{th}$ mass, $c_s$ is the count rate of the $s^{th}$ SF, $A$ is the entrance slit area, and $I_i$ is the current of the $i^{th}$ mass. By leaking the gases anticipated in the lunar atmosphere into the chamber and taking data points at several pressures, 20 simultaneous equations can be obtained for any given SF.
These simultaneous equations can be solved by computer for the coefficients $a_{ii}$ to $a_{20}$. By repeating the process for each of the 20 SF and the six energies, all the $a_{ij}$ coefficients are obtained and the following matrix equation is set up:

$$ [c_i] = [a_{is}] [F_i] $$

Inversion then gives the matrix equation used to obtain the unidirectional flux from the observed counting rates.

$$ [F_i] = [a_{is}]^{-1} [c_i] $$

where

$$ [a_{is}]^{-1} = \frac{\text{cofactor } a_{is}}{\text{det } (a_{is})} $$

The calibration system is illustrated in Figure 9 and a block diagram of the complete calibration instrumentation is given in Figure 10. Directly under the ion gun is an electronic beam switcher (EBS) which is a CPA with one plate made of screen. The SIDE is located directly under the EBS and the ion gun; a quadrapole
mass analyzer is oriented with the output of the EBS. In operation, when the plates of the EBS are grounded, the beam enters the SIDE and the data is printed on a Franklin printer and recorded on magnetic tape by the SDS 910. When the proper voltage is placed on the EBS, the beam is bent into the quadrapole and the resulting spectrum is plotted on a X-Y plotter. Thus $F_i$ can be calculated and $\xi_i$ is observed; the combination gives sufficient data to compute $a_i$.

Some masses have been identified by leaking known gases into the system and plotting counts versus SF. Figures 11-15 are typical results; the peaks correspond to the calculated mass locations in Table 2 using the effective magnetic field strength mentioned earlier. This effective field was determined by using an ion beam known to be predominantly Na$^+$ produced by an ion gun using a Li source and consequently producing only one large peak located at SF 9. Using the known energy, mass, and voltages the effective magnetic field was calculated. This field value then was used to derive the mass locations in Table 2.

The mass resolution given by $\frac{m}{2m}$ is about 2 for the SIDE.

Angular calibration is needed to get the instrument...
response to fluxes entering the detector off axis. The instrument was tipped relative to the beam direction (the x-axis) for rotations of $\pm 5^\circ$ around the z-axis and $\pm 15^\circ$ around the y-axis. The y-axis is parallel to the entrance slit length and the z-axis is parallel to the slit width. Figures 16 and 17 are plots of counts versus angle for these calibrations. The curves are not centered about zero degrees due to a misalignment between the ion gun and the SIDE. The extent of the curves is limited by the mechanical tilting fixture. However, by extrapolating the curves it can be seen that the counts drop by 50% for angles of incidence of $\pm 2.5^\circ$ on the y-axis and $\pm 4^\circ$ on the z-axis.

B. High Energy Calibration

The high energy calibration is considerably more simple than that for low energy because there is no mass analysis. The data shows that the HECPA has a pass-band of approximately 10%. The angular calibration yields a 50% peak reduction for angles of $2^\circ$ in z direction and $4^\circ$ in the y direction. A typical graph is displayed in Figure 18.
C. Geometric Factor

Comparing the counts observed during calibration to the number of incident particles gives the experimental geometric factor of the instrument. It is $1.9 \times 10^{-6}$ cm$^2$-ster. However, for the purposes of data reduction, the efficiency of the instrument has been calibrated and programmed into a data reduction tape assuming an unidirectional beam. The remaining factor needed is the solid angle $\Omega$. This angle can be calculated by integrating over the angles obtained for 50% decrease in count rate.

$$\Omega = \int \frac{dA}{x^2}$$

$$\Omega = \iiint \sin \theta \, d\theta \, d\phi$$

$$\Omega \approx 10^{-3} \text{ ster}$$
Appendix A

Derivation of the Equations of Motion for Particles in Crossed E and B Fields

The straight through velocity may be obtained from the Lorentz force equation:

$$\vec{F} = q (\vec{v} \times \vec{B} + \vec{E})$$

where the straight through velocity occurs when the force is zero.

$$v_0 = \frac{E}{B}$$

Combining the Lorentz equation for a magnetic field with Newton's third law allows the definition of angular frequency $\omega$.

$$\vec{F} = q \vec{v} \times \vec{B} = \frac{mv_t}{r}$$

$$q B = \frac{mv_t}{r}$$

$$\nu = r \omega$$

$$\omega = \frac{q B}{m}$$

where the magnetic field is in the z direction and the electric field is in the y direction.
The force components are

\[ \begin{align*}
    m \ddot{x} &= g \dot{y} \beta, \\
    m \ddot{y} &= g E - g \dot{x} \beta, \\
    m \ddot{z} &= 0
\end{align*} \tag{A-1} \]

To solve for the \( y \) motion differentiate the \( \ddot{y} \)-equation and combine with the \( \ddot{x} \)-equation.

\[ \begin{align*}
    \dddot{y} + \frac{g^2 \beta^4}{m^2} \dot{y} &= 0 \\
    \omega^2 + \frac{g^2 \beta^4}{m^2} \omega &= 0 \\
    \omega \left( \omega^2 + \frac{g^2 \beta^4}{m^2} \right) &= 0 \\
    y &= a_1 + b_1 e^{i \omega t} + c_1 e^{-i \omega t} \\
    y &= a + b \cos \omega t + c \sin \omega t \tag{A-2}
\end{align*} \]

Where \( \omega = \frac{g \beta}{m} \). The initial conditions specify the constants; they are \( t=0, \ y=y_0, \ \dot{y}=\dot{y}_0, \ \ddot{y}=\ddot{y}_0 \).

\[ \begin{align*}
    \dddot{y} &= -b \omega \sin \omega t + c \omega \cos \omega t \\
    \dddot{y}_0 &= c \omega, \quad c = \frac{\dddot{y}_0}{\omega} - \frac{\dddot{y}_0 m}{g \beta} \\
    \dddot{y} &= -b \omega^2 \cos \omega t - c \omega^2 \sin \omega t \\
    \dddot{y}_0 &= -b \omega^2, \quad b = -\frac{\dddot{y}_0}{\omega^2} \\
    b &= -\dddot{y}_0 \left( \frac{m}{g} \right)^2
\end{align*} \]
but from equation (A-1)

\[ \ddot{y}_0 = \frac{g}{m} E - \frac{E}{m} \dot{x}_0 \]

from equation (A-2)

\[ y_0 = a_1 + b \quad , \quad a_1 = y_0 + \dot{y}_0 \left( \frac{m}{E} \right)^2 \]

Substituting and writing \( \frac{E}{g} \) as \( \frac{1}{w} \) and \( v_0 \) as \( \frac{E}{g} \) gives

\[ y = y_0 + \frac{v_0 - \dot{x}_0}{w} (1 - \cos \omega t) + \frac{\dot{v}_0}{w} \sin \omega t \]

(A-3)

The same method is used to solve the x-equation of motion.

\[ m \ddot{x} = g \dot{y} \]

\[ m \ddot{y} = g E - g \dot{x} \]

Differentiating and substituting gives

\[ \ddot{x} + \frac{E}{m} E \ddot{x} - \frac{E}{m} E E = 0 \]

Let

\[ \rho = \frac{d}{dt} \]

\[ \ddot{\rho} + \frac{E}{m} E \rho = \frac{E}{m} E \]

The particular solution is

\[ \rho_p = \frac{E}{g} \]
The homogeneous solution is obtained from the following equation:

\[ \ddot{x} + \frac{k^2}{m} x = 0 \]

\[ \omega^2 + \frac{k^2}{m} = 0 \]

\[ f_1 = c_1 e^{i \frac{\omega}{m} t} + c_2 e^{-i \frac{\omega}{m} t} \]

\[ \dot{x} = \frac{1}{\omega} - c_1 e^{i \frac{\omega}{m} t} + c_2 e^{-i \frac{\omega}{m} t} \]

Integration yields

\[ x = \frac{E}{\omega} t + A \cos \frac{\omega}{m} t + c \sin \frac{\omega}{m} t + c_f \]

The initial conditions determine the constants:

\[ t = 0, \quad x = x_0 = 0, \quad \dot{x} = \dot{x}_0, \quad \ddot{x} = \ddot{x}_0 \]

\[ \dot{x} = \frac{E}{\omega} - \frac{k^2}{m} A \sin \frac{\omega}{m} t + \frac{k^2}{m} C \cos \frac{\omega}{m} t \]

\[ \dot{x}_0 = \frac{E}{\omega} + C \frac{k^2}{m} \]

\[ c = \frac{m}{\frac{k^2}{\omega^2}} \]

\[ \ddot{x} = -A \frac{k^2}{m} \cos \frac{\omega}{m} t - C \frac{k^2}{m} \sin \frac{\omega}{m} t \quad (A-4) \]

from equation (A-1)

\[ \ddot{x} = \frac{k^2}{m} \gamma \]

Equating equation (A-1) with equation (A-4) at \( t=0 \)
gives
\[
\frac{\mathbf{m}}{\mathbf{A}} \mathbf{g} \hat{y}_0 = -\mathbf{A} \frac{\mathbf{g}^2}{\mathbf{A}^2} \mathbf{B}.
\]
\[
\mathbf{A} = -\frac{\mathbf{m}}{\mathbf{g}} \hat{y}_0
\]
\[
\hat{x}_0 = \mathbf{A} + \mathbf{c}_3 = 0
\]
\[
\mathbf{c}_3 = -\mathbf{A} = \frac{\mathbf{m}}{\mathbf{g}} \hat{y}_0
\]

Substitution yields the $\mathbf{x}$-equation.
\[
\mathbf{x} = \mathbf{v}_0 t + \frac{\mathbf{v}_0}{\omega} (1 - \cos \omega t) + \frac{1}{\omega} (\mathbf{v}_0 - \mathbf{v}_0) \sin \omega t
\]

To get an explicit solution for $y$ in terms of $x$, the time must be obtained from the $\mathbf{x}$-equation and substituted into the $y$-equation. When the velocity and mass of the particles are such that less than one cycloid is executed during passage through the selector, small angle approximations may be used
\[
\cos \Theta = 1 - \frac{\Theta^2}{2} , \quad \sin \Theta = \Theta
\]
\[
\mathbf{x} = \mathbf{v}_0 t + \frac{\mathbf{v}_0}{\omega} (1 - \cos \omega t) + \frac{1}{\omega} (\mathbf{v}_0 - \mathbf{v}_0) \sin \omega t
\]
define \[\Delta \mathbf{v} = \mathbf{v}_0 - \mathbf{v}_0\]
\[
\mathbf{x} = \mathbf{v}_0 t + \frac{\mathbf{v}_0^2}{\omega^2} w t^2 - \Delta \mathbf{v} t
\]
\[
\mathbf{x} = \frac{\mathbf{v}_0}{\omega^2} w t^2 + t (\mathbf{v}_0 - \Delta \mathbf{v})
\]
\[
\frac{\mathbf{v}_0}{\omega^2} w t^2 + 2 \frac{\mathbf{v}_0}{\omega} t - 2 x = 0
\]
\[
t = \frac{-2 \mathbf{v}_0 \pm \sqrt{4 \mathbf{v}_0^2 - 8 \mathbf{x} \omega \mathbf{v}_0}}{2 \omega \mathbf{v}_0}
\]

31
This may be substituted into the $y$-equation. However, a simple approach is to let $\cos \approx 1$. Then $t = \frac{x}{X_0}$ and $y$ becomes:

$$y = y_0 + \frac{\Delta v}{\omega} (1 - \cos \frac{\omega x}{X_0}) + \frac{v_0}{\omega} \sin \frac{\omega x}{X_0}$$
Appendix B

First Order Passband for the Velocity Filter

A passband equation may be derived for the small angle case. It is shown in Appendix A that the time for traveling the length of the selector $L$ approximates the straight through time. Using this condition and assuming that $y_0 = 0$ and $\dot{y}_0 = 0$ gives the desired result.

$$t = \frac{x}{x_0}$$

$$y_s = \Delta v \left( 1 - \cos \frac{\omega t}{x_0} \right)$$

$$y_s = \Delta v \left\{ 1 - \sum_{i=0}^\infty \frac{(-1)^i}{2^i} \left( \frac{\omega x_s}{x_0} \right)^i \right\}$$

$$y_s = \Delta v \frac{\omega}{2} \left( \frac{x_s}{x_0} \right)^2$$

$$\Delta v = v_0 - \dot{x}_0 \quad \dot{x}_0 = -\Delta v \quad x_0 \approx v_0 \quad -\Delta v \quad \Delta v$$

The second order terms in $(\Delta v)^2$ are dropped. Then,

$$y_s = \Delta v \frac{\omega}{2} \frac{x_s}{v_0^2 - 2 \gamma s \Delta v}$$

$$\Delta v = \frac{\gamma s \gamma_s}{\frac{\gamma_s}{x_0} + 2 \gamma s \Delta s}$$

$$\beta = \frac{\Delta v}{\gamma s} = \frac{\gamma s \gamma_s}{\frac{\gamma_s}{x_0} + 2 \gamma s \Delta s}$$

\[ (B-1) \]
Appendix C

The Potential, Electric Field, and Force Equations for the Curved Plate Analyzer

Laplace's equation for the potential is

$$\nabla^2 \phi = 0$$

which in cylindrical coordinates is

$$\nabla^2 \phi = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

When $\phi$ is a function of $r$ only,

$$\nabla^2 \phi = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \phi}{\partial r} \right) = 0$$

$$\frac{\partial \phi}{\partial r} = \text{constant} = C_1$$

$$d \phi = \frac{C_1}{r} \, dr$$

$$\phi = C_1 \log r + C_2$$

Letting $r_i$ and $r_a$ be the coordinates of the curved plates and applying the boundary conditions $\phi(r_i) = -\frac{V}{2}$ and $\phi(r_a) = \frac{V}{2}$ the constants of integration may be evaluated.
\[ \phi(r_2) = c_1 \log r_2 + c_2 = \frac{\nu}{2} \]

\[ \phi(r_1) = c_1 \log r_1 + c_2 = -\frac{\nu}{2} \]

(C-1)

Combining yields

\[ c_1 \log r_2 - c_1 \log r_1 = \nu \]

\[ c_1 \log \left(\frac{r_2}{r_1}\right) = \nu \]

\[ c_1 = \frac{\nu}{\log \left(\frac{r_2}{r_1}\right)} \]

Substituting in equation (C-1) gives

\[ c_2 = -\frac{\nu}{2} - c_1 \log r_1 \]

\[ = -\frac{\nu}{2} - \frac{\nu}{\log \left(\frac{r_2}{r_1}\right)} \log r_1 \]

\[ = -\frac{\nu}{2 \log \left(\frac{r_2}{r_1}\right)} \left[ \log \left(\frac{r_2}{r_1}\right) + 2 \log r_1 \right] \]

\[ = -\frac{\nu}{2 \log \left(\frac{r_2}{r_1}\right)} \left[ \log \left(\frac{r_2 r_1}{r_1^2}\right) \right] \]

The resulting equation for the potential is

\[ \phi = \frac{\nu}{\log \left(\frac{r_2}{r_1}\right)} \log r - \frac{\nu}{2 \log \left(\frac{r_2}{r_1}\right)} \log \left(\frac{r_2 r_1}{r_1^2}\right) \]

\[ = \frac{\nu}{2 \log \left(\frac{r_2}{r_1}\right)} \left[ 2 \log r - \log \left(\frac{r_1^2}{r_2}\right) \right] \]

\[ = \frac{\nu}{2 \log \left(\frac{r_2}{r_1}\right)} \left[ \log \left(\frac{r_2^2}{r_1^2}\right) \right] \]
The electric field is given by

\[ E = - \nabla \phi \]

\[ = - \frac{d \phi}{dr} \]

\[ E = - \frac{V}{\log (\nu / \lambda)} \frac{1}{r} \]

and the force acting on a singly charged positive ion is

\[ F = q E \]

\[ F = - \frac{V}{\log (\nu / \lambda)} \frac{1}{r} \]
Appendix D

Focusing Angle for a Curved Plate Analyzer

Method One

The equations of motion for particles in E and B fields are

\[ m \ddot{r} - m r \ddot{\phi}^2 = F_r(r) \]

\[ \frac{d}{dt} (m r^2 \dot{\phi}) = r e v_r B \]

The force equation is

\[ F_r = e \dot{E}_r + e \ddot{\phi} \times \vec{B} \]

\[ = e E_r - e r \dot{\phi} \vec{B} \]

Substitution gives

\[ \ddot{r} - r \ddot{\phi}^2 = \frac{\kappa}{r} - \frac{e e}{m} r \dot{\phi} \]

where

\[ \kappa = \frac{-v}{\log(n/h)} \frac{e}{m} \]

Define \( \lambda = \frac{d\phi}{dt} \) so that

\[ \ddot{r} - r \dot{\phi}^2 = \frac{\kappa}{r} - \lambda r \dot{\phi} \quad \text{(D-1)} \]

and

\[ \frac{1}{r} \frac{d}{dt} (r^2 \dot{\phi}) = \lambda \dot{r} \]

\[ \frac{d}{dt} (r^2 \dot{\phi}) = \lambda r \dot{r} \]

\[ = \frac{A}{2} \frac{d}{dt} r^2 \]

37
Integrating gives

\[ \dot{\phi} = \frac{\kappa}{r^2} + \frac{A}{r^2} \quad (D-2) \]

Substitute this equation into equation (D-1)

\[ \ddot{r} - \frac{\kappa}{r^2} \left( \frac{A}{r^2} \right) = \kappa r - \lambda r \left( A^2 + \frac{A}{r^2} \right) \]

\[ \ddot{r} - r^2 \dot{r}^2 - \frac{A}{r} + \frac{A^2}{r^2} = \frac{\kappa}{r^2} - \frac{A^2}{r^2} - \frac{A^2}{r} \quad (D-3) \]

\[ \ddot{r} = \frac{A^2}{r^2} + \frac{\kappa}{r^2} - \frac{A^2}{r} \]

For circular orbits \( \ddot{r} = 0 \), \( r = f \)

\[ \frac{g A^2}{f^2} - \frac{\kappa}{f} = \frac{A^2}{f^2} \]

For near circular orbits \( r = f + x \), \( \ddot{r} = 0 \), \( A = 0 \),

\[ A^2 = \frac{\kappa}{f^2} \quad , \quad x \ll f \]

Substituting into equation (D-3)

\[ \ddot{x} = \frac{A^2}{(f + x)^3} + \frac{\kappa}{f + x} - \frac{A^2}{f^2} \left( f^2 + x^2 \right) \]

\[ \ddot{x} = \frac{A^2}{f^2 \left( 1 + \frac{x}{f} \right)^3} + \frac{\kappa}{f \left( 1 + \frac{x}{f} \right)} - \frac{A^2 \left( f^2 + x^2 \right)}{f^3} \]

38
Since $\frac{K}{f} \ll 1$, the denominators can be expanded.

$$\ddot{x} = \frac{A^2}{f^2} \left(1 - \frac{2x}{f}\right) + \frac{A^2}{f} \left(1 - \frac{x}{f}\right) - \frac{A^2}{f^2} (f + x) \quad \text{(D-4)}$$

Equation (D-4) and the conditions for circular orbits give

$$\ddot{x} = \frac{2Kx}{f^2} - \lambda^2 x$$

Substituting $\kappa = \frac{2\lambda^2}{f}$ yields the equation for simple harmonic motion.

$$\ddot{x} + \frac{\lambda^2}{4} x = 0$$

Its solution is

$$x = x_0 \sin \frac{\lambda t}{\sqrt{2}}$$

The divergent rays will be reunited when $x = 0$ or when

$$\frac{\lambda t}{\sqrt{2}} = \pi$$

However, $\theta = \dot{\theta} t$, and for circular orbit conditions, equation (D-2) shows that $\dot{\theta} = \frac{A}{2}$. Substituting gives the desired angle

$$\theta_c = \frac{\lambda}{2} \frac{\sqrt{2} \pi}{\lambda} = \frac{\pi}{\sqrt{2}} = 127^\circ 17'$$

39
Method Two

This method takes the equations for mass spectrometers derived by Herzog and, putting in the conditions for a curved plate analyzer, immediately gives the proper equations. The lateral displacement of a particle after passing through the CPA is given as

\[
y'' = a_e \left[ -\frac{e \omega}{m} \sin \psi \, \phi_e + \frac{1}{2} \left( 1 - \cos \psi \, \phi_e \right) + \frac{y_1}{a_e} \cos \psi \, \phi_e \right] \\
+ x'' \left[ -\frac{e \omega \cos \psi \, \phi_e}{m} + \frac{1}{2} \psi \sin \psi \, \phi_e - \frac{y_1}{a_e} \psi \sin \psi \, \phi_e \right]
\]

(D-4)

where

\[
a_e = \frac{m_0 v_0^2 \lambda \log \left( \frac{\psi}{\nu_2} \right)}{e (\nu_1 - \nu_2)}
\]

Where the definitions of page 11 apply plus \( \phi_e \) is the exit angle of the particles, \( a_e \) is the radius of curvature for ions traveling in a radial electric field.

For the case where the entrance and exit slits are at the boundaries of the field, \( \nu' = \nu'' = 0 \) then

\[
\frac{1}{\sin^2 \psi \, \phi_e} = \frac{\cos^2 \psi \, \phi_e}{\sin^2 \psi \, \phi_e}
\]

or

\[
\cos \psi \, \phi_e = \pm 1
\]

There will be focusing when \( \phi_e = \frac{\nu_1}{\nu_2} \).
Figure Captions

Figure 1
Low energy flight tunnel A, velocity filter B, Curved Plate Analyzer C, and Channeltron D.

Figure 2
Velocity filter magnet with electric field plates between the pole pieces.

Figure 3
Dimensions and radii of curvature of LECPA and HECPA.

Figure 4
High energy flight tunnel A, curved plate analyzer B, and channeltron C.

Figure 5
Illustration of symbols used in Herzog's equations for a Wein velocity filter.

Figure 6
Illustration of the definition of a solid angle.
Figure 7
Diagram of a straightened out CPA used to calculate the plate separation or plate width ($\mu_{in}$, 4r) (depending on which way it is viewed), and the CPA length $L_{CPA}$ which defines the radius of curvature.

Figure 8
Plot of counts vs energy for various SF.

Figure 9
Calibration system showing SIDE, ion accelerator (gun), EBS, and quadrapole.

Figure 10
Block diagram of the entire calibration instrumentation.

Figures 11 - 15
SIDE mass calibration curves where each figure corresponds to one of the following gases being leaked into the vacuum chamber: $H_2$, $CO_2$, He, $N_2$, Xe.

Figures 16 - 17
Angular calibration curves for the y-axis and z-axis respectively.
Figure 18
High energy calibration curves.

Figure 19
Illustration of symbols used in Herzog's equations for a CPA.
y_1, y_3, y_5, y_6, y_8 = 0.2 cm
y_2, y_9 = 0.3 cm
y_4 = 0.6 cm
y_7 = (r_o - r_1) = 0.5 cm
r_1 = 1.75 cm
r_o = 2.25 cm
x_1, x_4 = 5.0 cm
x_2, x_3 = 0.64 cm
x_5, x_6 = 1.0 cm
z_1 = 0.8 cm
L = 1.0 cm
x_7 = 0.848 cm
FIGURE 2

- ELECTRIC FIELD PLATES
- MAGNET

A = 1.00 cm
B = 3.810 cm
C = 0.600 cm

NOTE: ALL DIMENSIONS IN CENTIMETERS.
LECPA

Fj = 1.75 cm
r = 2.25 cm
rj = 3.75 cm
ro = 4.25 cm
ri = 4.00 cm

HECPA

APERTURE

0.8 X 0.2 cm

ri = 1.75 cm
ro = 2.25 cm
r = 2.00 cm

ri = 3.75 cm
ro = 4.25 cm
r = 4.00 cm

FIGURE 3
\( ds = \text{element of surface area} \)
\( dA = \text{area of sphere cut by cone formed at } O \)
\( d\Omega = \frac{dA}{r^2} = \text{solid angle subtended by } ds \text{ at } O \)
\( n = \text{unit normal to } ds \)
\( d\Omega = \frac{\bar{n} \cdot \bar{T}}{r^3} ds \)


**FIGURE 7**

ENTRANCE SLIT

STRAIGHTED OUT CPA

\[ \Delta x \]
\[ \frac{l_{CPA}}{2} \]

HE CPA

CPA = 9.02 cm
\( \lambda_{fl} = 6.00 \text{ cm} \)

LE CPA

CPA = 4.50 cm
\( \lambda_{fl} = 1.10 \text{ cm} \)
SN-5, H LEAKED, 48.6 ev

- • = SF4
- ▲ = SF10
- ■ = SF11
- ○ = SF19

COUNTS

10

4

3

2

10

55 60 65 70 75 VOLTS

FIGURE 8
FIGURE 10
SN-5, H₂ LEAKED, 48.2 ev

COUNTS

10^4

10^3

10^2

10

5 10 15 20

SIDE FRAME NUMBER

FIGURE 11
SN-5 CO$_2$ LEAKED, 48.6 ev

COUNTS

MASS (amu)

SIDE FRAME NUMBER

FIGURE 12
FIGURE 13

SN-5, HE LEAK, 48.6 ev

COUNTS

10^4

10^3

10^2

10

2.5 4 7 12 28 90 130

MASS (amu)

SIDE FRAME NUMBER
SN-5, N₂ LEAKED, 48.6 ev

**Figure 14**

- **Y-axis**: Counts
- **X-axis**: Side frame number
- **Mass (amu)**: 2.5, 4, 7, 12, 28, 90, 130

The graph shows a plot with a logarithmic scale on the y-axis, indicating the counts, and a linear scale on the x-axis, indicating the side frame number. The data points are spread across the mass range, with a notable peak at around 28 amu.
## Table 1
Source and Loss Mechanisms for Lunar Atmosphere Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Sources*</th>
<th>Losses**</th>
<th>Predominant Atm Gases</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Edwards &amp; Borst</td>
<td>1-5</td>
<td>1</td>
<td>Kr, Xe</td>
<td>1</td>
</tr>
<tr>
<td>Singer</td>
<td>1-6</td>
<td>1-3</td>
<td>H</td>
<td>2</td>
</tr>
<tr>
<td>Opik</td>
<td>5,6,8,9</td>
<td>1-5</td>
<td>H₂O, CO₂, N</td>
<td>3</td>
</tr>
<tr>
<td>Watson</td>
<td>9</td>
<td>1,3</td>
<td>H₂O</td>
<td>4</td>
</tr>
<tr>
<td>Nakada &amp; Michalov</td>
<td>5,7</td>
<td>6,3</td>
<td>H, Ne, A</td>
<td>5</td>
</tr>
<tr>
<td>Hinton &amp; Taeusch</td>
<td>5,7-10</td>
<td>1,2,3</td>
<td>Ne, A</td>
<td>6</td>
</tr>
<tr>
<td>Vestine</td>
<td>9,10,11</td>
<td>1,5</td>
<td>A, CO₂, SO₂, H₂O</td>
<td>7</td>
</tr>
<tr>
<td>Bernstein</td>
<td>1,2,4,5,7,10</td>
<td>1,3,6,2</td>
<td>A, Ne, H, Kr, Xe</td>
<td>8</td>
</tr>
<tr>
<td>Herring &amp; Licht</td>
<td>6,11,10</td>
<td>1,3,6</td>
<td>SO₂, CO₂, H₂O</td>
<td>9</td>
</tr>
</tbody>
</table>

*Sources
1. Spontaneous fission of U²³⁸
2. Cosmic rays - n + U²³⁸ - fission
3. Thermal fission O₁⁸ (,n) Ne²¹
4. I¹²⁹ - Xe decay
5. Primeval gases
6. Solar wind - H only
7. Solar wind - coronal composition
8. Meteoritic bombardment
9. Permafrost
10. Radioactive decay K - A
11. Vulcanism

**Losses**

1. Thermal
2. Lunar surface potential - photoionization and solar wind.
3. Solar wind - elastic scattering and charge exchange
4. Ionic
5. Permanent shadow areas
6. Chemical bonding to surface
### Table 2

Low Energy, Plate Voltage, Mass for each SF

<table>
<thead>
<tr>
<th>( \text{w (ev)} )</th>
<th>SF</th>
<th>( \text{V (volts)} )</th>
<th>MASS (amu), ( \text{Beff = 785 gauss} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>48.6</td>
<td>0</td>
<td>29.0</td>
<td>2.46</td>
</tr>
<tr>
<td>48.6</td>
<td>1</td>
<td>26.3</td>
<td>2.98</td>
</tr>
<tr>
<td>48.6</td>
<td>2</td>
<td>23.8</td>
<td>3.65</td>
</tr>
<tr>
<td>48.6</td>
<td>3</td>
<td>21.4</td>
<td>4.51</td>
</tr>
<tr>
<td>48.6</td>
<td>4</td>
<td>19.2</td>
<td>5.60</td>
</tr>
<tr>
<td>48.6</td>
<td>5</td>
<td>17.1</td>
<td>7.06</td>
</tr>
<tr>
<td>48.6</td>
<td>6</td>
<td>14.5</td>
<td>9.83</td>
</tr>
<tr>
<td>48.6</td>
<td>7</td>
<td>13.3</td>
<td>11.68</td>
</tr>
<tr>
<td>48.6</td>
<td>8</td>
<td>11.6</td>
<td>15.35</td>
</tr>
<tr>
<td>48.6</td>
<td>9</td>
<td>10.0</td>
<td>20.66</td>
</tr>
<tr>
<td>48.6</td>
<td>10</td>
<td>8.59</td>
<td>28.00</td>
</tr>
<tr>
<td>48.6</td>
<td>11</td>
<td>7.30</td>
<td>38.77</td>
</tr>
<tr>
<td>48.6</td>
<td>12</td>
<td>6.40</td>
<td>50.44</td>
</tr>
<tr>
<td>48.6</td>
<td>13</td>
<td>5.13</td>
<td>78.51</td>
</tr>
<tr>
<td>48.6</td>
<td>14</td>
<td>4.25</td>
<td>114.38</td>
</tr>
<tr>
<td>48.6</td>
<td>15</td>
<td>3.50</td>
<td>168.66</td>
</tr>
<tr>
<td>48.6</td>
<td>16</td>
<td>2.89</td>
<td>247.37</td>
</tr>
<tr>
<td>48.6</td>
<td>17</td>
<td>2.41</td>
<td>355.72</td>
</tr>
<tr>
<td>48.6</td>
<td>18</td>
<td>2.07</td>
<td>482.17</td>
</tr>
<tr>
<td>48.6</td>
<td>19</td>
<td>1.87</td>
<td>590.83</td>
</tr>
<tr>
<td>16.2</td>
<td>0</td>
<td>16.7</td>
<td>2.46</td>
</tr>
<tr>
<td>16.2</td>
<td>1</td>
<td>15.2</td>
<td>2.98</td>
</tr>
<tr>
<td>16.2</td>
<td>2</td>
<td>13.7</td>
<td>3.65</td>
</tr>
<tr>
<td>16.2</td>
<td>repeats</td>
<td>12.4</td>
<td>repeats</td>
</tr>
<tr>
<td>16.2</td>
<td>3</td>
<td>11.1</td>
<td></td>
</tr>
<tr>
<td>16.2</td>
<td>4</td>
<td>9.66</td>
<td></td>
</tr>
<tr>
<td>16.2</td>
<td>5</td>
<td>8.36</td>
<td></td>
</tr>
<tr>
<td>16.2</td>
<td>6</td>
<td>7.66</td>
<td></td>
</tr>
<tr>
<td>16.2</td>
<td>7</td>
<td>6.68</td>
<td></td>
</tr>
<tr>
<td>16.2</td>
<td>8</td>
<td>5.78</td>
<td></td>
</tr>
<tr>
<td>16.2</td>
<td>9</td>
<td>4.96</td>
<td></td>
</tr>
<tr>
<td>16.2</td>
<td>repeats</td>
<td>4.21</td>
<td></td>
</tr>
<tr>
<td>16.2</td>
<td>10</td>
<td>3.69</td>
<td></td>
</tr>
<tr>
<td>16.2</td>
<td>11</td>
<td>2.96</td>
<td></td>
</tr>
<tr>
<td>16.2</td>
<td>12</td>
<td>2.45</td>
<td></td>
</tr>
<tr>
<td>16.2</td>
<td>13</td>
<td>2.02</td>
<td></td>
</tr>
<tr>
<td>16.2</td>
<td>14</td>
<td>1.67</td>
<td></td>
</tr>
<tr>
<td>16.2</td>
<td>15</td>
<td>1.39</td>
<td></td>
</tr>
<tr>
<td>16.2</td>
<td>16</td>
<td>1.20</td>
<td></td>
</tr>
<tr>
<td>16.2</td>
<td>17</td>
<td>1.08</td>
<td></td>
</tr>
<tr>
<td>5.4</td>
<td>18</td>
<td>9.65</td>
<td></td>
</tr>
<tr>
<td>5.4</td>
<td>19</td>
<td>8.77</td>
<td></td>
</tr>
<tr>
<td>5.4</td>
<td>20</td>
<td>7.93</td>
<td></td>
</tr>
</tbody>
</table>

64
<table>
<thead>
<tr>
<th>(w_{(ev.)})</th>
<th>(SF)</th>
<th>(V) (volts)</th>
<th>MASS (amu), (B_{eff} = 785) gauss</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.4</td>
<td></td>
<td>4.83</td>
<td></td>
</tr>
<tr>
<td>5.4</td>
<td></td>
<td>4.42</td>
<td></td>
</tr>
<tr>
<td>5.4</td>
<td></td>
<td>2.86</td>
<td></td>
</tr>
<tr>
<td>5.4</td>
<td></td>
<td>2.43</td>
<td></td>
</tr>
<tr>
<td>5.4</td>
<td></td>
<td>2.13</td>
<td></td>
</tr>
<tr>
<td>5.4</td>
<td></td>
<td>1.71</td>
<td></td>
</tr>
<tr>
<td>5.4</td>
<td></td>
<td>1.42</td>
<td></td>
</tr>
<tr>
<td>5.4</td>
<td></td>
<td>1.17</td>
<td></td>
</tr>
<tr>
<td>5.4</td>
<td></td>
<td>.963</td>
<td></td>
</tr>
<tr>
<td>5.4</td>
<td></td>
<td>.805</td>
<td></td>
</tr>
<tr>
<td>5.4</td>
<td></td>
<td>.691</td>
<td></td>
</tr>
<tr>
<td>5.4</td>
<td></td>
<td>.624</td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td></td>
<td>5.57</td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td></td>
<td>5.06</td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td></td>
<td>4.58</td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td></td>
<td>4.12</td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td></td>
<td>3.69</td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td></td>
<td>3.29</td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td></td>
<td>2.79</td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td></td>
<td>2.55</td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td></td>
<td>2.23</td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td></td>
<td>1.93</td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td></td>
<td>1.65</td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td></td>
<td>1.40</td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td></td>
<td>1.23</td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td></td>
<td>.987</td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td></td>
<td>.817</td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td></td>
<td>.673</td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td></td>
<td>.556</td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td></td>
<td>.464</td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td></td>
<td>.399</td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td></td>
<td>.360</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td></td>
<td>2.22</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td></td>
<td>2.92</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td></td>
<td>2.64</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td></td>
<td>2.38</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td></td>
<td>2.13</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td></td>
<td>1.90</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td></td>
<td>1.61</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td></td>
<td>1.47</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td></td>
<td>1.29</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td></td>
<td>1.11</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td></td>
<td>.954</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td></td>
<td>.811</td>
<td></td>
</tr>
<tr>
<td>$w$ (ev)</td>
<td>$SF$</td>
<td>$V$ (volts)</td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
<td>------------</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>1.86</td>
<td>1.86</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>1.69</td>
<td>1.69</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>1.53</td>
<td>1.53</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>1.37</td>
<td>1.37</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>1.23</td>
<td>1.23</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>1.10</td>
<td>1.10</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.93</td>
<td>0.93</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.85</td>
<td>0.85</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.74</td>
<td>0.74</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.64</td>
<td>0.64</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.55</td>
<td>0.55</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.46</td>
<td>0.46</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.40</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.32</td>
<td>0.32</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.27</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.22</td>
<td>0.22</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.18</td>
<td>0.18</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.15</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.13</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.12</td>
<td>0.12</td>
<td></td>
</tr>
</tbody>
</table>

MASS (amu), $B_{eff} = 785$ gauss
Table 3
High Energy, Plate Voltage for each SF

<table>
<thead>
<tr>
<th>SF NUMBER</th>
<th>ENERGY (eV)</th>
<th>VOLTAGE (volts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3500</td>
<td>875.0</td>
</tr>
<tr>
<td>2</td>
<td>3250</td>
<td>812.5</td>
</tr>
<tr>
<td>3</td>
<td>3000</td>
<td>750.0</td>
</tr>
<tr>
<td>4</td>
<td>2750</td>
<td>687.5</td>
</tr>
<tr>
<td>5</td>
<td>2500</td>
<td>625.0</td>
</tr>
<tr>
<td>6</td>
<td>2250</td>
<td>562.5</td>
</tr>
<tr>
<td>7</td>
<td>2000</td>
<td>500.0</td>
</tr>
<tr>
<td>8</td>
<td>1750</td>
<td>437.5</td>
</tr>
<tr>
<td>9</td>
<td>1500</td>
<td>375.0</td>
</tr>
<tr>
<td>10</td>
<td>1250</td>
<td>312.5</td>
</tr>
<tr>
<td>11</td>
<td>1000</td>
<td>250.0</td>
</tr>
<tr>
<td>12</td>
<td>750</td>
<td>187.5</td>
</tr>
<tr>
<td>13</td>
<td>500</td>
<td>125.0</td>
</tr>
<tr>
<td>14</td>
<td>250</td>
<td>62.5</td>
</tr>
<tr>
<td>15</td>
<td>100</td>
<td>25.0</td>
</tr>
<tr>
<td>16</td>
<td>70</td>
<td>17.5</td>
</tr>
<tr>
<td>17</td>
<td>50</td>
<td>12.5</td>
</tr>
<tr>
<td>18</td>
<td>30</td>
<td>7.5</td>
</tr>
<tr>
<td>19</td>
<td>20</td>
<td>5.0</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
<td>2.5</td>
</tr>
</tbody>
</table>

67
Table 4

Critical energies for Cycloidal Motion

where $B_{eff} = 785$ gauss

<table>
<thead>
<tr>
<th>Mass (amu)</th>
<th>Energy (ve) *</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.990</td>
</tr>
<tr>
<td>2</td>
<td>1.50</td>
</tr>
<tr>
<td>4</td>
<td>0.748</td>
</tr>
<tr>
<td>12</td>
<td>0.249</td>
</tr>
<tr>
<td>q=2,28</td>
<td>0.214</td>
</tr>
<tr>
<td>16</td>
<td>0.187</td>
</tr>
<tr>
<td>18</td>
<td>0.166</td>
</tr>
<tr>
<td>20</td>
<td>0.150</td>
</tr>
<tr>
<td>28</td>
<td>0.107</td>
</tr>
<tr>
<td>32</td>
<td>0.094</td>
</tr>
<tr>
<td>40</td>
<td>0.074</td>
</tr>
<tr>
<td>64</td>
<td>0.047</td>
</tr>
<tr>
<td>83</td>
<td>0.036</td>
</tr>
<tr>
<td>131</td>
<td>0.023</td>
</tr>
</tbody>
</table>

* For energies smaller than the critical value, the corresponding mass cycloids.
Table 5

LECPA and HECPA

Dimensions and Parameters

LECPA

\[ r_0 = 2.00 \text{ cm} \]
\[ r_1 = 1.75 \text{ cm} \]
\[ r_2 = 2.25 \text{ cm} \]
\[ \Delta r = 0.50 \text{ cm} \]
\[ \omega = 3.14 \text{ cm} \]
\[ A = 1.99 \text{ joules/volt} \]

HECPA

\[ r_0 = 4.00 \text{ cm} \]
\[ r_1 = 3.75 \text{ cm} \]
\[ r_2 = 4.25 \text{ cm} \]
\[ \Delta r = 0.50 \text{ cm} \]
\[ \omega = 3.14 \text{ cm} \]
\[ A = 3.99 \text{ joules/volt} \]
ACKNOWLEDGMENTS

The guidance of my advisor, Dr. John W. Freeman, Jr., was invaluable throughout this project. In addition, many helpful discussions and comments were made by Dave Young, Carlos Warren, Dr. Hans Balsinger, and Dr. Kent Hills.

Many thanks are due my parents, Mr. and Mrs. L. E. Shane, for their assistance and moral support, and Mrs. Polly Tovar for her stenographic services.

The research reported here was supported by NASA Contract NAS 9-5911.
REFERENCES


