INVESTIGATION OF THE COMMON DEPTH POINT METHOD
BY MEANS OF SYNTHETIC SEISMOGRAMS

by

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A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
Master of Arts

Thesis Director's signature: 

HOUSTON, TEXAS
May 1968

3 1272 00481 2499
ABSTRACT

INVESTIGATION OF THE COMMON DEPTH POINT METHOD BY MEANS OF SYNTHETIC SEISMOGRAMS

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JOSE PONCE DE LEON

In seismic reflection shooting, some of the traces correspond to relatively large angles of incidence, especially when the method of common depth point is used. In an effort to observe the effect of the angles of incidence on traces with common depth point, approximations of synthetic seismograms at oblique incidence with and without multiples were constructed. The theory of constructing a synthetic seismogram is briefly explained, under the assumption that S waves do not exist.

After obtaining three approximations of synthetic seismograms for assumed shotpoint to-geophone distances of 400, 800, and 1200 feet respectively, the effects of initial suppression, filter, and automatic gain control were simulated. The three synthetic seismograms so obtained were staked together to obtain a single seismogram for various comparisons.

It is concluded that as the angles of incidence of the shot traces increase, so do the differences with
respect to a trace shot at normal incidence. From the comparison of the stacked traces it is proved that the common depth point stacking process leads to a better definition of the character of the reflections, in comparison with the conventional process.
ACKNOWLEDGEMENTS

I wish to express my deep appreciation to Ings. Santos Figueroa and Jesus Basurto, Exploration Managers of Petroleos Mexicanos, who greatly contributed to this program of study.

I also wish to thank Dr. J. C. DeBremaecker for his invaluable assistance and Drs. H. C. Clark and J. A. S. Adams for reading the manuscript and being on the thesis committee.

I am also grateful to Mrs. Martha L. Broussard by her assistance in the editing of the manuscript.
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I. INTRODUCTION

In the geophysical prospecting, reflections and transmissions are the meaningful quantities concerning waves, since the wave amplitudes, as they appear in a seismogram, are the resultant of a multitude of effects superimposed upon the direct reflection and refraction processes occurring at the various horizons.

Despite the fact that many areas have been covered by reflection seismic exploration in the search for "structure", there still may exist, in these same areas, unfound stratigraphic traps. These traps may be located by the application of new seismic field studies or by the reexamination, with modern techniques, of old seismic records.

One of these techniques, the common depth point (CDP) method, together with the synthetic seismogram and the electronic computer, can be very helpful in studying the effect of the angle of incidence on wide-angle reflection shooting. The present work is an attempt to study this method in a practical example. The procedure used to study the propagation of a plane wave at oblique incidence is based on the solution of the wave differential equation. The matrix method is used to systematize the analysis and to present the equations in a form suitable for computation. This work is concerned only with plane compressional waves and horizontal plane interfaces.
In a general form it is possible to write the steps followed in this work as follows:

1. Obtain a velocity log of a well in the area of interest.
2. Obtain velocities for each of the layers of equal travel time.
3. Compute the depths to each layer.
4. Compute angles of incidence, given a shotpoint-geophone distance, and the depth to each successive layer.
5. Compute the reflection and transmission coefficients in terms of the angle of incidence.
6. Compute the impulsional synthetic seismograms with and without multiples for each of the three different shotpoint-geophone distances.
7. Apply simulations of initial suppression, filter, and AGC to the impulsional synthetic seismograms obtained.
8. Add the synthetic seismograms with multiples to obtain a single seismogram. Do the same for the synthetic seismograms without multiples.
9. Compare the synthetic seismograms so obtained.
II. REMARKS CONCERNING THE CDP METHOD

The CDP method was designed mainly for the improvement of the signal-to-noise ratio and as a tool for attenuation of multiple reflections or combinations of multiple reflections that interfere with the true reflections.

The attenuation of multiple reflection is possible when velocity increases with depth. This is due to the fact that the NMO (normal moveout) is proportional to the surface distance between shotpoint and geophone, and inversely proportional to the square of average velocity. Therefore the multiple reflections will have greater NMO than the primary reflections arriving at the same time. If there is no velocity contrast between their paths, primary and multiple reflections cannot be distinguished on the basis of their moveout.

The application of the CDP technique is simply the selection of a number of shotpoints and geophone locations so that they have a common depth point with a number of different offset distances. The signals for all paths to the common depth point, after static and dynamic corrections, are added together to give one single record. On this record the primary reflections have been preserved by addition in phase.

Two types of CDP configurations are normally used. The split configuration is satisfactory for signal-to-noise ratio improvement. Since the longitudinal waves are the fastest, and are the ones of interest in reflection prospecting, it is important to separate the reflections
from the undesirable slower waves which in some cases interfere with the reflections. Thus in the split CDP spread the longitudinal waves travel along short surface distances and therefore the normal moveout is small. This allows an enhancement of data by improving the signal-to-noise ratio. Figure 1(a), shows the split CDP configuration. The off-end configuration is satisfactory for multiple reflections attenuation. This is because the greater surface distances produce big moveouts on the primary reflections and hence the multiple reflections will have bigger moveout so that the contrast between the primary and multiple reflections is very clear. Figure 1(b) shows the off-end CDP configuration. A spread of one mile total length is normally used. The spacing between geophone groups is 220 feet.

The effectiveness of the CDP method increases in proportion to the number of paths for a common reflection point on the subsurface. That is, a six-fold coverage is more effective than a three-fold coverage.

Besides proper geometry, the determination of the correct velocity which must be utilized to remove the normal moveout prior to stacking, is of critical importance. All long spread CDP records contain accurate velocity data which can be used to determine the correct function for normal moveout corrections prior to stacking. However the primary reflections are distorted by reverberations such that a deconvolution process must be applied before the primary reflections can be correctly identified and
(a) Split CDP Spread

(b) Off-end CDP Spread

Figure 1. CDP Configurations
used for the velocity determinations. Another important point in regard to the CDP method is that most of the traces are shot with large angles of incidence. In an effort to observe the effect of these angles of incidence on traces with common depth point, three approximations of synthetic seismograms at oblique incidence with and without multiples were constructed in the present work. They could then be compared with a synthetic seismogram at normal incidence.

The basic steps followed, in order to obtain such approximations of synthetic seismograms, are explained below.

First, we obtained from a velocity log the velocities for each of the layers of equal travel time, from which we can obtain the thickness of each of the layers and subsequently the depth corresponding to each layer.

Second, we computed the angles of incidence, assuming a shotpoint geophone distance of 400, 800, and 1200 feet respectively, for each synthetic seismogram, found by the following expression:

\[ \theta_i = \tan^{-1} \frac{x_j}{2z_k} \]

where:

- \( \theta_i \) = angle of incidence
- \( x_j \) = shotpoint geophone distance
- \( z_k \) = depth to bottom of layer \( K \)

We assumed that the rays travelled along straight lines.
We now were able to compute the reflection and transmission coefficients in terms of the angle of incidence as it is explained later in the section on reflection and transmission coefficients.

After we had obtained the reflection and transmission coefficients we calculated the approximation of impulsional synthetic seismograms. The three synthetic seismograms so obtained were then added together, since they all have common depth points, in the CDP method.

The rest of the work consisted of a comparison of the synthetic records with themselves and with the ones resulting from normal incidence. A reduction of multiples is observed in the stacked seismogram.
III. THE TECHNIQUE OF THE SYNTHETIC SEISMOGRAM

The source of energy for seismic exploration methods is usually close to an impulse. That is, in standard seismic methods, the energy from the normal charge size may be considered to be a single energy unit with a maximum amplitude of one. However, the reflection signal appearing on the seismic record is quite different from the unit impulse. The seismic recording represents the total effect of the entire recording system including the geophones, galvanometers, amplifiers, and the filtering effects of the earth.

An impulsional synthetic seismogram represents the response of the subsurface to a unit impulse, neglecting the effect of the recording equipment.

If the impulse response, or the transfer function of the whole system, excluding the earth, is known, the convolution of an impulsional synthetic seismogram with this impulse response will approximate a real seismogram.

Importance, Construction, and Use of Synthetic Seismograms

The limitations of the reflection seismograph and other field techniques often make the analysis of field data very difficult. If direct reference to a sonic log or other types of well logs is not sufficient to resolve
the interpretation, then a practical method is the construction of synthetic seismograms (SS) that approximately illustrate the effects of the basic parameters in the seismic process. A SS is valuable prior to the field program, at the shooting period, and during the interpretation. The practical use of SS stems from the fact that it is possible to study the changes produced in the SS by changes in the characteristics of the earth. The comparison of SS with field records allows better correlation between recorded reflections and their causes in the earth.

Several methods of constructing SS have been developed such as the mechanical analog method in which the values from the velocity log are used to construct a model whose acoustic impedance varies along its length in a manner analogous to the variation of acoustic impedance with depth in the ground. This method has not been much used in practice.

Another method is ascribed to Peterson (Peterson et al., 1955) in which the values of the reflection coefficients are approximated so that they are proportional to the amplitude changes of the velocity log if velocity is plotted on a logarithmic scale. This is the easiest way to produce the required sequence of reflected pulses. This method has the disadvantage in that multiple reflections are not included.

The digital computer method, in which the values from the velocity log are obtained at equal intervals of time, is a third method. This is the one used in the present work.
In this method a suitable program in a digital computer will yield the reflection coefficients at each of the assumed interfaces, and will also give the corresponding multiple reflections and the changes in amplitude due to transmission losses at the interfaces. This method allows the construction of synthetic seismograms either with or without multiples.

The synthetic seismogram is first computed for an impulsive source. This yields the impulsional synthetic seismogram (ISS), which is then convolved with the system response giving the desired SS.

If we neglect the distorting effect of the propagation and of the equipment, the series of reflection coefficients placed at instants of time separated by intervals equal to twice the travel time in the elementary layers into which the earth has been divided, represents the response of the subsurface to a unit impulse. Thus, the seismogram so obtained is called an impulsional synthetic seismogram without multiples.

By introducing the effects of multiple reflections, transmission coefficients, and the gain of the instruments the SS obtained constitutes, in general, the ideal theoretical response of the subsurface which field recording should obtain. However, this technique of calculating the SS does not give the most exact reflected signal possible because of the assumptions of plane wave source, of perfect elasticity of the layered media, and of the plane interfaces of the layers.

There is a direct relationship between the sequence, time, and the form of the reflected waves and the lithologic
well log. In other words the individual peaks and troughs on the seismogram correspond to individual high or low velocity zones in the lithologic section. Wide peaks or troughs represent thick beds while narrow peaks or troughs represent thin beds (Peterson et al., 1955). Each reflection has the same waveform as the time-invariant shot pulse, but each has its own characteristic amplitude, polarity, and time delay. Since the shot pulse is the input, the impulse response of the earth, which can be considered as a filter, is the set of reflection coefficients spaced properly in time.

The synthetic seismogram as a function of time \([S(t)]\) is given by

\[
S(t) = \gamma(t) * f(t) * e(t)
\]

in which \(\gamma(t)\) is the reflectivity function, \(f(t)\) is the shot pulse, and \(e(t)\) is the filter external to the earth.

For the case in which all multiples are included, the velocity function is sampled from the velocity log to give an "N" layered earth, and all reflections, primary and multiples, are obtained by solving the wave equation, taking into account the reflection and transmission coefficients at each interface. The basic relationship between the field and synthetic seismograms can be expressed using the transforms of the time functions.

If the technique is perfect, the SS should be identical to the corresponding noise-free field seismogram.
The simplest of the uses of the SS is the identification of the geological horizons giving rise to observed reflections. Multiple reflections can be identified by comparing SS which respectively include and exclude multiples.

Seismic Wave Propagation in Layered Media

In order to review the wave propagation across boundaries, we will examine a horizontal plane interface and a longitudinal (p) wave propagation along the vertical (y) axis. If a plane harmonic wave of angular frequency $\omega$ and incident amplitude $A_i$ travels downward in medium 1, then the incident disturbance, is the real part of (Lindsay, 1960):

$$g_i = A_i e^{i(\omega \zeta - k_1 y)}$$

where $\omega = 2\pi f$

$$k_1 = \frac{\omega}{C_1}$$

$\zeta$ = time variable

$A_i$ = real value of the amplitude coefficient of the incident plane harmonic wave.

The reflected wave in medium 1 can be represented by:

$$q_r = \frac{A}{r} e^{i(\omega \zeta + k_1 y)}$$

(2)
where $A_r =$ real value of the amplitude coefficient of
the reflected plane harmonic wave.

The transmitted wave into medium 2 can be represented by:

$$g_t = A_t e^{i(w_C-k_2 y)} \tag{3}$$

Where $A_t =$ real value of the amplitude coefficient
of the transmitted plane harmonic wave

$k_2 = \frac{w}{c_2}$

$c_2 =$ P wave velocity in medium 2

The incident, reflected, and transmitted waves, all have
the same frequency $f$, that is, a harmonic wave of given
frequency will retain this frequency regardless of the
medium through which it travels. The relationship between
$g_i, g_r,$ and $g_t$ depends on the boundary conditions. In
order to obtain $A_r$ and $A_t$ in terms of $A_i$ we need only
two boundary conditions, i.e. the stresses and displace¬
ments should be continuous across each interface between
layers.

The continuity of stresses yields:

$$A_i - A_r = K A_t \tag{4}$$

and

the continuity of displacements yields:

$$A_i + A_r = A_t \tag{5}$$
where $K$ is a function of the acoustic impedance of the medium. The acoustic impedance for a plane wave is defined as the product of the density by the wave velocity.

Solving Equations (4) and (5) for $A_r$ and $A_t$ in terms of $A_i$, we obtain:

$$A_r = rA_i$$

$$A_t = tA_i$$

where $r$ is defined as the amplitude reflection coefficient and $t$ is defined as the amplitude transmission coefficient resulting from an incident plane wave traveling downward in medium 1 to the interface.

Similarly, we define the coefficients $r'$ and $t'$ as the amplitude reflection and amplitude transmission coefficients respectively, resulting from an incident plane wave traveling upward in medium 2 to the interface.

The Velocity Log

The velocities used in the present work were sampled with constant time intervals of 2 milliseconds. The sonic
log was obtained in a well located in the Altamira
Municipality of the State of Tamaulipas, in Northern
Mexico.

The total number of layers which were sampled was
287 for a two-way vertical time of 1.152 seconds. The
velocities were plotted using an IBM-7040 digital computer.
The program in Fortran IV is shown in Appendix I. The
basic steps followed are:

1. - Read the number N of layers of equal travel
time, and the plotting symbols.
2. - Read the velocities of all of the N layers.
3. - Compute the two-way vertical time.
4. - Change the scale for the velocities.
5. - Test the maximum and minimum positions in the
plot.
6. - Write a plotting symbol in desired position, as
well as the layer number, the vertical time,
and the corresponding velocity.

7. - End.

The plot of the velocities is shown in Figure
2.

Reflection and Transmission Coefficients

Reflection and refraction of essentially planar
waves at interfaces between layers are of basic importance,
and it is interesting to know how they may change and what
makes them change.
The reflection and transmission coefficients for the general case of oblique incidence can be very closely approximated by computing the direct-reflected and the direct-transmitted portion for transition layers, and letting the thickness approach zero (Bortfeld, 1961). These approximations might be adequate for all cases occurring in reflection seismic prospecting.

However, in the present work, the assumption that all of the SV conversions are neglected, allows us to use the reflection and transmission coefficients for the case of liquid-liquid interface. To obtain such coefficients we consider the wave equation for a plane wave and the corresponding boundary conditions, i.e. continuity of vertical particle velocity and continuity of pressure, and their solution will lead us to determine the ratio of the amplitude of the reflected and refracted waves to the incident wave, reflection, and transmission coefficients respectively. The expressions for these coefficients (Officer, 1958) are:

\[
\begin{align*}
\rho_2 & \quad \sqrt{\frac{c_1^2 - \sin^2 \theta_j}{c_2^2 - \sin^2 \theta_j}} \\
\rho_1 & \quad 1 - \sin^2 \theta_j \\
\end{align*}
\]

\[
R = \frac{\rho_2}{\rho_1} - \frac{\sqrt{\frac{c_1^2 - \sin^2 \theta_j}{c_2^2 - \sin^2 \theta_j}}}{1 - \sin^2 \theta_j} \\
\]

\[
T = \frac{\rho_2}{\rho_1} + \frac{\sqrt{\frac{c_1^2 - \sin^2 \theta_j}{c_2^2 - \sin^2 \theta_j}}}{1 - \sin^2 \theta_j} \\
\]

--- (8)
where \( \rho_1 \) and \( \rho_2 \) are densities in medium 1 and 2 respectively, \( C_1 \) and \( C_2 \) are the P wave velocities in medium 1 and 2 respectively, and 
\[ \theta_1 \] is the angle of incidence.

In terms of the angles of incidence and refraction, the reflection and transmission coefficients can be expressed by:

\[
\begin{align*}
R &= \frac{\rho_2 \cot \theta_1 - \rho_1 \cot \theta_2}{\rho_2 \cot \theta_1 + \rho_1 \cot \theta_2} \quad \text{--------(10)}
\end{align*}
\]

\[
\begin{align*}
T &= \frac{2 \rho_1 \cot \theta_1}{\rho_2 \cot \theta_1 + \rho_1 \cot \theta_2} \quad \text{--------(11)}
\end{align*}
\]

where \( \theta_2 \) is the angle of refraction, given by:
\[ \theta_2 = \sin^{-1} \left( \frac{C_2}{C_1} \sin \theta_1 \right) \]

From Equations (8) and (9) we can see that the reflection and transmission coefficients depend on values of velocity and density on both sides of the boundary, and on the angle of incidence. This latter has a considerable effect on the reflection coefficient, since for angles of incidence less than the critical angle (given by \( \theta_c = \sin^{-1} \frac{C_1}{C_2} \); \( C_2 > C_1 \)) the reflection coefficient for longitudinal
plane waves varies continuously with increasing angle of incidence (Muskat and Meres, 1940 a). Beyond this angle of incidence, the radical $\sqrt{\frac{C_1^2}{C_2^2} - \sin^2 \theta_1}$ becomes imaginary. Thus the expression for the reflection coefficient becomes, for incidence more grazing than the critical angle:

$$r = \frac{\rho_2 + i \sqrt{\frac{\sin^2 \theta_1 - \frac{C_1^2}{C_2^2}}{1 - \sin^2 \theta_1}}}{\rho_1} \frac{\rho_2 - i \sqrt{\frac{\sin^2 \theta_1 - \frac{C_1^2}{C_2^2}}{1 - \sin^2 \theta_1}}}{\rho_1}$$

Or in another form

$$r = e^{i \epsilon}$$

where $\epsilon$ is real and given by the following expression

$$\epsilon = 2 \tan^{-1} \frac{\rho_1 \sqrt{\frac{\sin^2 \theta_1 - \frac{C_1^2}{C_2^2}}{\rho_2}}}{\rho_2 \sqrt{1 - \sin^2 \theta_1}}$$

This means that at angles of incidence greater than the critical angle, the amplitude of the reflected wave remains unity, but there is a phase shift of $\epsilon$ between the
reflected and incident waves. The phase shift varies from 0 degrees at the critical angle to 180 degrees at grazing incidence. The expression for the transmission coefficient becomes:

\[ t = \frac{2 \, e^{i\varepsilon/2}}{\rho_2 \left( \frac{\sin^2 \theta_1 - \frac{c_1^2}{c_2^2}}{\rho_1^2 + \frac{1}{2} - \sin^2 \theta_1} \right)^{1/2}} \]  

\[ \text{(13)} \]

It follows that the amplitude of the reflected wave is equal to that of the incident wave with a phase change of \( \varepsilon \), and no disturbance is transmitted in the interior of the second medium. However, if \( c_1^2/c_2^2 < \sin^2 \theta_1 \) the formulas represent a disturbance in the second medium which propagates along the interface and the amplitude decreases exponentially away from the surface into the lower medium. This wave exists only as a consequence of reflection at angles of incidence more grazing than critical.

Figure 3 shows how phase change varies as a function of the angle of incidence (Ewing et al., 1957) for the case of \( c_2 > c_1 \) and \( \rho_2 > \rho_1 \).
Figure 3. Phase Change $2\varepsilon$ for various angles of incidence
(Ewing et al., 1957)

The critical angle is determined solely by the ratio $C_1/C_2$ and occurs only for the case $C_2 > C_1$. For such a contrast there will always be a critical angle beyond which there is total reflection with a phase shift of the reflected wave with respect to the incident wave. For $C_2 < C_1$ total reflection occurs only at grazing incidence.
In practice, the fact that the reflection coefficient may change as a function of the angle of incidence is useful to improve the quality of reflections on a seismogram by choosing a suitable shotpoint geophone distance.

Experiments show that reflected waves stand out on the records for comparatively short distances from the shotpoint. The maximum distance, $d_m$, at which the reflected waves can be recognized is, generally, that at which headwaves are first observed.

The absence of reflected waves at distances greater than $d_m$ is explained by the geometric spreading of these waves.

Program to Compute the Reflection Coefficients

In order to compute the reflection coefficients for the three different distances, a program in Fortran IV has been written. It is as follows:

1.- Read the number $N$ of layers of equal travel time.
2.- Read the velocities of the $N$ layers.
3.- Read the three shotpoint-geophone distances $\chi$ for which the reflection coefficients are being computed.
4.- Compute the depth $Z$ to the bottom of each of the $N$ layers.
5.- Compute the three angles of incidence for the three distances $\chi$ and for each depth $Z$.
6.- Compare two adjacent velocities. If both are equal, then the reflection coefficient is zero.
7.- Compute the sine and cosine of the angles of refraction.

8.- Compute the three reflection coefficients for each interface.

9.- Compute the two-way vertical time and the real time or reflection time.

10.- Print a list of results including layer number, the depth, the three reflection coefficients corresponding to the three given distances, the angles of incidence, and the two-way vertical and reflection times.

11.- Punch the values of the reflection coefficients, that will be used in the programs to compute the synthetic seismograms.

12.- End.

The Fortran IV program is shown in Appendix II. The approximate running time for 287 layers is 120 sec.
The Synthetic Seismogram Without Multiples

The principles in construction of a synthetic seismogram are based on the fact that a seismic impulse traveling downward, at an interface between two layers of different acoustic impedances, produces both a reflection and a refraction, as is depicted in Figure 4 for the case of small angles of incidence.

\[ C_1 \]
\[ A_i \]
\[ \theta_1 \]
\[ A_r = rA_i \]
\[ C_2 \]
\[ \theta_2 \]
\[ A_t = (1-r^2)^{1/2} A_i \]

Figure 4. Reflection and Refraction of a Seismic Pulse

For the case of N layers, the approximation of a synthetic seismogram at oblique incidence, considering only primary reflections, will yield the amplitude of a plane wave reflected from an interface \( N \), given by the expression

\[ A_N = A_i r \prod_{n=1}^{N-1} (1-r_n^2) \]

\[ \text{(14)} \]
Since all of the reflection coefficients are less than one, equation (14) corresponds to a function in which the amplitudes decrease proportionally with time. Figure 5 shows how such a equation is obtained.

\[
A_N = (1-r_1^2)(1-r_2^2)\ldots(1-r_{n-1}^2)r_nA_i
\]

Figure 5. Amplitude of a plane wave reflected from Interface N.

In practice it is desirable that both the field record and the synthetic seismogram have an average amplitude approximately uniform along the whole length. In the field record this is done by the AGC (Automatic Gain Control). In the synthetic seismogram a simulation of this AGC should be made. There exist some important
differences between the synthetic seismogram and the field record. This is due to the reflections between layers, which are propagated up to the first layer, and the multiple reflections of the surface as is shown in Figure 6.

For this reason the synthetic seismogram without multiples is considered, above all, as a correlation tool. The synthetic seismogram with multiples is a better means of analysis.

Figure 6. Some possible multiple reflections of the surface.
The synthetic seismogram calculated by the electronic computer is plotted as a time-series. The program to compute the approximation of synthetic seismogram at oblique incidence without multiples is shown in Appendix III of this work.

The Simulation of the Filter

The simulation of analog filters is based on the frequency-to-time transformation (S. Smith, pers. comm., 1965)

\[ w = -\frac{2i}{\Delta t} \frac{1-Z}{1+Z} \]

where \( Z = e^{-i\omega \Delta t} \)

If \( F(w) \) is a desired filter characteristic, then for any filter that has an analog realization \( F(w) = R(Z) \) where \( R(Z) \) will be a rational function

\[ R(Z) = \frac{\sum_{n=0}^{N} a_n Z^n}{\sum_{n=0}^{M} b_n Z^n} \]

The corresponding linear operator is:

\[ \sum_{n=0}^{M} b_n O_{k-n} = \sum_{n=0}^{N} a_n I_{k-n} \]

where \( I \) is the input to the filter and \( O \) is the output.
Thus we can write an expression for the filter output as:

$$O_k = \frac{1}{b_0} \left[ \sum_{n=0}^{N} a_n I_{k-n} - \sum_{n=1}^{M} b_n O_{k-n} \right] \hspace{1cm} (15)$$

This represents an equivalent frequency response to other filter operations and their minimum requirements on the length of filter coefficients reduces the computer time. For the present work two low pass filters with cut-off frequencies of 40 and 60 cps were computed, both with an attenuation ratio of 6 db/oct.

From the equation for a single pole low pass filter

$$R(Z) = \frac{1 + Z}{(1 + \frac{2}{f \Delta t}) + Z (1 - \frac{2}{f \Delta t})} = \sum_{n=0}^{N} a_n Z^n$$

$$= \sum_{n=0}^{M} b_n Z^n$$

where $f$ is the cut frequency and $\Delta t$ is the sampling time rate, we can compute the coefficients $a_n$ and $b_n$ in order to substitute them into equation (15).

Then for the case of the 40 cps filter we obtain:

$$O_k = \frac{1}{13.5} \left( I_k + I_{k-1} + 11.5 O_{k-1} \right) \hspace{1cm} (15a)$$

Similarly for the case of the 60 cps filter we obtain:

$$O_k = \frac{1}{9.33} \left( I_k + I_{k-1} + 7.33 O_{k-1} \right). \hspace{1cm} (15b)$$
Both filters were used in the computation of the synthetic seismograms. However, the synthetic seismograms computed with the 60 cps filter were chosen for the various comparisons.

The AGC Simulation

In the present work an AGC medium with a decay velocity of 80 db/sec was assumed. The following expression gives the relationship between the input and the output of the simulated AGC.

\[
O(i) = \frac{I(i)}{\left( \sum_{j=0}^{5} O^2(i-j) \right)^{1/2}}
\]

where \(I\) is the input and \(O\) is the output of the AGC.

The Initial Suppression Simulation

For the simulation of the initial suppression it was assumed that the decay velocity was 120 db/sec, from which the coefficients affecting the amplitudes were calculated. These coefficients were neglected after the 84th amplitude. For the first 84 amplitudes, each amplitude was multiplied by its respective factor of suppression.
Program to Compute a Synthetic Seismogram
Without Multiples

In order to obtain the amplitudes at constant intervals of time of 2 milliseconds, corresponding to a synthetic seismogram without multiples, a program in Fortran IV has been written. A brief description of the main steps followed, is given below.

1. - Read the shotpoint geophone distance and the number N of layers of equal travel time.
2. - Read the reflection coefficients for the N interfaces. These values of the reflection coefficients have been obtained from the output of the respective program, which gave those values on punched cards.
3. - Compute the amplitudes for the first ten layers, since it is assumed that the dynamite charge is placed at interface eleven.
4. - Compute the amplitudes for the rest of the layers.
5. - Introduce the simulation of the effect caused by the initial suppression when it is attenuating.
6. - Read the factors which affect the values of the first 84 amplitudes. The effect of the initial suppression on the rest of amplitudes is neglected.
7. - Multiply the amplitudes affected for their respective factor of suppression.
8.- Introduce the simulation of the filter.
9.- Introduce the simulation of the AGC.
10.- Compute the vertical travel time.
11.- Print a list of results including the number of the interface, the vertical travel time and the values of the amplitudes.
12.- Punch the values of the amplitudes on cards that will be used in the programs to plot the synthetic seismogram without multiples and the stack.
13.- End.

This program is shown in Appendix III. The approximate run time is 100 seconds.

Program to Plot a Synthetic Seismogram
With or Without Multiples

A program in Fortran IV was used to plot the amplitudes of a synthetic seismogram with or without multiples, and with the simulated effects of the initial suppression, filter, and AGC. Such a program has been done according to the following steps:

1.- Read the number N of layers and the plotting symbols.
2.- Read the amplitudes of the complete synthetic seismogram corresponding to a given shotpoint - geophone distance. The values of the amplitudes have been obtained from the output of the respective program which yield those values in punched cards.
3. - Compute the two-way vertical time.
4. - Normalize and round the values of the amplitude.
5. - Test for the maximum and minimum position in the plot.
6. - Print a plotting symbol in desired position, the two-way vertical time, and the normalized values of the amplitudes.
7. - End.

Appendix IV shows this program.

The Synthetic Seismogram With Multiples

In order to obtain a synthetic seismogram with multiples, the displacements are determined, step by step at each succeeding instant, at all of the interfaces reached by the waves at these times.

Assuming that the vertical travel time of an elastic wave through each layer is constant and that the source of the downgoing pulse is at times $\zeta = 0$, by using the Z-transform we can arrive at the equations which relate the wave coefficients in terms of the reflection and transmission coefficients.

Figure 7 depicts the wave coefficients, that is the displacement amplitudes of the downgoing waves as well as the upgoing waves both at the top and at the bottom of the layers.
Figure 7. Wave Coefficients.

The notations used in Figure 7 to denote the wave coefficients are as follows:

\[ b_{k,k+2n-1} \] is the displacement amplitude of the downgoing wave at the top of layer K occurring at time \( k+2n-1 \) where \( k = 0, 1, 2, \ldots \) and \( n = 0, 1, 2, \ldots \) is a suitable time index, \( b'_{k,k+2n} \) is the displacement amplitude of the downgoing wave at the bottom of the layer K occurring at time \( k+2n \), \( s_{k,k+2n-1} \) is the displacement amplitude of the upgoing wave at the top of layer K occurring at time \( k+2n-1 \), where \( k = 0, 1, 2, \ldots \) and \( n = 1, 2, 3, \ldots \), \( s'_{k,k+2n} \) is the displacement
amplitude of the upgoing wave at the bottom of layer $K$ occurring at time $k+2n$, where $n = 0, 2, 2, \ldots$. Let us now recall the concept of the $Z$-transform. This is a specialized Laplace transform with which we can transform time-series into the frequency domain, and work with the resultant function as a frequency function.

From the Fourier transfer function

$$\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$$

a complex variable $Z$ is substituted for $e^{-i\omega}$, and substituting summation for integration, we arrive at

$$\sum_{t} f(t)Z^t$$

So now we can represent a time-series by using the amplitudes of the time points as coefficients of $Z$, and the time value itself as the exponent of $Z$. We then have a polynomial with respect to the complex variable $Z$, which is in the frequency domain. In other words the $n$th order polynomial in $Z$, $F(Z) = a_0 + a_1Z + a_2Z^2 + \ldots + a_nZ^n$, is called the $Z$-transform of the $n$th order delay filter. The constants $a_0, a_1, a_2, \ldots a_n$, are the weighting coefficients of the delay filter.

Going back to the wave coefficients we can apply the $Z$-transform to them in order to have the information contained either at the top or at the bottom of a given layer. Thus the $Z$-transform of the downgoing waves at the top of
layer $K$ is given by

$$B_k(Z) = b_{k,k-1}Z^{k-1} + b_{k,k+1}Z^{k+1} + b_{k,k+3}Z^{k+3} + \cdots.$$  

Similarly the Z-transform of the downgoing waves at the bottom of layer $K$ is

$$B'(k)(Z) = b'_{k,k}Z^k + b'_{k,k+2}Z^{k+2} + b'_{k,k+4}Z^{k+4} + \cdots$$

These two Z-transforms are related by a Z-transform $H_k(Z)$. Assuming the absorption-free case, the effect of layer $K$ is equivalent to introducing a time delay of one unit from the top to the bottom of the layer. Since $Z$ represents a pure delay of one time unit (Robinson and Treitel, 1964) we conclude that $H_k(Z) = Z$ and

$$B'_k(Z) = Z B_k(Z)$$

Therefore, the term $b_{k,k+3}Z^{k+3}$ for example, is associated with a time delay of $k+3$ from the instant that the pulse has excited the system.

As before, the Z-transform of the upgoing waves at the top of layer $K$ is

$$S_k(Z) = S_{k,k+1}Z^{k+1} + S_{k,k+3}Z^{k+3} + S_{k,k+5}Z^{k+5} + \cdots$$

whereas the Z-transform of the upgoing waves at the bottom of layer $I$ is

$$S'_k(Z) = S'_{k,k}Z^k + S'_{k,k+2}Z^{k+2} + S'_{k,k+4}Z^{k+4} + \cdots$$

Again, the last two Z-transforms are related by a Z-transform characteristic of layer $K$, i.e. $S_k(Z) = H_k(Z)S'_k(Z)$, therefore $S_k(Z) = Z S'_k(Z)$.  

We now are able to arrive at the equations which relate the wave coefficients in terms of the reflection and transmission coefficients.

Figure 8. depicts the general case at the interface K at time instant \( \zeta \).

\[
\begin{array}{ccc}
\zeta - 1 & \zeta & \zeta + 1 \\
K - 1 & & \\
& b_{k, \zeta - 1} & \rightarrow & S_{k, \zeta + 1} \\
K & \rightarrow & b_{k, \zeta} & \rightarrow & S_{k, \zeta} \\
& S_{k+1, \zeta} & \rightarrow & b_{k+1, \zeta} \\
K + 1 & \rightarrow & S'_{k+1, \zeta} & \rightarrow & b'_{k+1, \zeta + 1} \\
K + 1 & & \\
\end{array}
\]

Figure 8. Wave Coefficients for the General Case.
The coefficient $S'_{k,\zeta}$ is the result of both the reflection of $b'_{k,\zeta}$ and the transmission of $S_{k+1,\zeta}$. Thus we can write the equation

$$S'_{k,\zeta} = r'b'_{k,\zeta} + t'S_{k+1,\zeta} \quad \ldots \ldots \quad (16)$$

On the other hand the coefficient $b_{k+1,\zeta}$ is the result of both the reflection of $S_{k+1,\zeta}$ and the transmission of $b'_{k,\zeta}$. Thus, we can write a second equation

$$b_{k+1} = r'S_{k+1,\zeta} + t'b'_{k,\zeta} \quad \ldots \ldots \quad (17)$$

By combining equations (16) and (17) we obtain

$$tS'_{k,\zeta} = r'b_{k+1,\zeta} + (tt'-rr')S_{k+1,\zeta} \quad \ldots \ldots \quad (18)$$

Equations (17) and (18) define the situation at the interface $k$ and at time $\zeta$.

By multiplying these equations by $Z^\zeta$ and taking summations over $\zeta = k, k+2, k+4, \ldots$, we can rewrite them as $Z$-transforms:

$$t \sum_{\zeta} S'_{k,\zeta} Z^\zeta = r \sum_{\zeta} b_{k+1,\zeta} Z^\zeta + (tt'-rr') \sum_{\zeta} S_{k+1,\zeta} Z^\zeta \quad \ldots \ldots \quad (19)$$

$$t \sum_{\zeta} b'_{k,\zeta} Z^\zeta = \sum_{\zeta} b_{k+1,\zeta} Z^\zeta - r' \sum_{\zeta} S_{k+1,\zeta} Z^\zeta \quad \ldots \ldots \quad (20)$$

or

$$tS'_k(Z) = rB_{k+1}(Z) + (tt'-rr')S_{k+1}(Z) \quad \ldots \ldots \quad (21)$$

$$tB'_k(Z) = B_{k+1}(Z) - r'S_{k+1}(Z) \quad \ldots \ldots \quad (22)$$
The reflection and transmission coefficients for the downgoing and upgoing waves are related by:

\[ r = -r' \]

and \[ tt' - rr' = 1 \]

Therefore equations (21) and (22) become:

\[ t S_k'(Z) = rB_{k+1}(Z) + S_{k+1}(Z) \quad \text{--------(23)} \]

\[ t B_k'(Z) = B_{k+1}(Z) + r S_{k+1}(Z) \quad \text{--------(24)} \]

Using now the fact that \( B_k'(Z) = Z B_k(Z) \) and \( S_k(Z) = Z S_k'(Z) \) we obtain:

\[ t S_k(Z) = Zr B_{k+1}(Z) + Z S_{k+1}(Z) \quad \text{--------(25)} \]

\[ t B_k(Z) = \frac{1}{Z} B_{k+1}(Z) + \frac{r}{Z} S_{k+1}(Z) \quad \text{--------(26)} \]

Equations (25) and (26) can be expressed in matrix form (Thomson, 1950) as follows:

\[
\begin{bmatrix}
S_k(Z) \\
B_k(Z)
\end{bmatrix} =
\begin{bmatrix}
\frac{Zr}{t} & \frac{Z}{t} \\
\frac{1}{Zt} & \frac{r}{Zt}
\end{bmatrix}
\begin{bmatrix}
B_{k+1}(Z) \\
S_{k+1}(Z)
\end{bmatrix}
\quad \text{---------(27)}
\]

Since the reflection and transmission coefficients \( r \) and \( t \) respectively, correspond to the interface \( K \), i.e. between the layers \( K \) and \( K+1 \), we can define the matrix \( R_k \) as:
\[ R_k = \begin{bmatrix} \frac{Z}{t_k} & \frac{Z r_k}{t_k} \\ \frac{r_k}{Z t_k} & \frac{1}{Z t_k} \end{bmatrix} \]  \hspace{1cm} (28)

where \( k = 1, 2, 3, \ldots \).

Equation (27) becomes:
\[ \begin{bmatrix} S_k(Z) \\ B_k(Z) \end{bmatrix} = R_k \begin{bmatrix} S'_{k+1}(Z) \\ B'_{k+1}(Z) \end{bmatrix} \hspace{1cm} (29) \]

For the interface \( k = 0 \) we use the equations:
\[ t_0 S'_0(Z) = r_0 B_1(Z) + S_1(Z) \hspace{1cm} (30) \]
\[ t_0 B'_0(Z) = B_1(Z) + r_0 S_1(Z) \hspace{1cm} (31) \]

or in matrix form:
\[ \begin{bmatrix} S'_0(Z) \\ B'_0(Z) \end{bmatrix} = P_0 \begin{bmatrix} S_1(Z) \\ B_1(Z) \end{bmatrix} \hspace{1cm} (32) \]

\[ P_0 = \begin{bmatrix} 1 & r_0 \\ t_0 & t_0 \\ t_0 & 1 \\ t_0 & t_0 \end{bmatrix} \]

Finally, we can write an expression of Z-transforms of the upgoing and downgoing waves for the case of N
layers.

\[
\begin{bmatrix}
S'_0(Z) \\
B'_0(Z)
\end{bmatrix} = P_0 \begin{bmatrix} R_1 & R_2 & R_3 & \cdots & R_k & \cdots & R_{N-1} \\
S_N(Z) & B_N(Z)
\end{bmatrix} \cdots (33)
\]

Equation (33) is the desired result expressing the particle displacement in terms of the reflection and transmission coefficients of the individual layers and the travel time across the layers.

Forming the product of matrices \( R_1 \ R_2 \ \cdots \ R_k \ \cdots \ R_{N-1} \) we can define a matrix \( A \) such that

\[
A = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix} = R_1 \ R_2 \ \cdots \ R_k \ \cdots \ R_{N-1}
\]

The elements of the product matrix \( A \) can then be expressed as a polynomial in \( Z \) and equation (33) becomes

\[
\begin{bmatrix}
S'_0(Z) \\
B'_0(Z)
\end{bmatrix} = P_0 \begin{bmatrix} S_N(Z) & B_N(Z)
\end{bmatrix} \cdots \cdots \cdots (34)
\]

in which the product of the three matrices of the right hand side, is a polynomial in \( Z \) itself. By taking the inverse transform of the polynomial we obtain the time function by the substitution of \( Z = e^{-i\omega} \).

Therefore, the approximation of an impulsional synthetic seismogram at oblique incidence, with multiple reflections may be expressed as a time function, as follows:
\( S(t) = a_0 \delta(t) + a_1 \delta(t-2\zeta) + \ldots + a_n \delta(t-2n\zeta) \)

for the case that the stress pulse is a unit impulse located just above the first layer, at time \( \zeta = 0 \). The coefficients \( a_0, a_1, \ldots, a_n \) determine the magnitude of the unit impulses separated in time, given by twice the travel time across each of the layers, received by a geophone located also just above the first layer. These coefficients \( a_0, a_1, \ldots, a_n \) are determined by the characteristic impedances of the layers. That is, the coefficients are functions of the reflection and transmission coefficients.

Similarly, for the case that the stress pulse is not located just above the first layer, i.e. a buried source, the corresponding synthetic seismogram may be expressed as

\( S(t) = a_0 \delta(t-\zeta) + a_1 \delta(t-3\zeta) + a_2 \delta(t-5\zeta) + \ldots + a_n \delta(t-(2N+1)\zeta), \)

in which the series of unit impulses are received by the geophone at equal intervals of time, given by twice the travel time across each of the layers but the first unit impulse is received by the geophone at time \( \zeta \) after the instant that the stress pulse excites the system.

As before the coefficients \( a_0, a_1, a_2, \ldots, a_n \) are functions of the reflection and transmission coefficients, and they determine the amplitude of the unit impulses.
Program to Compute a Synthetic Seismogram
With Multiples

The computation of the synthetic seismograms with multiples has been carried out by using a program in Fortran IV, prepared according to the following steps:

1. - Read the shotpoint-geophone distance and the number N of layers of equal travel time.
2. - Read the reflection coefficients for the N interfaces. These values have been obtained from the output of the program to compute the reflection coefficients.
3. - Read the constant NSS. If it is zero, an impulsional synthetic seismogram will be computed. If it is one, the complete synthetic seismogram including the simulations of the filter, AGC, and initial suppression will be computed. Read also the interface number NK at which the source is assumed, the interface number NS at which the geophone is assumed, and the number of impulses or samples desired for which the synthetic seismogram is being computed.
4. - Compute the product of the matrices in order to arrive at the impulsional synthetic seismogram.
5. - Compute the vertical travel time.
6. - Print a list of results in case that only the impulsional synthetic seismogram is desired. Otherwise continue with the program.

7. - Introduce the simulation of the initial suppression. Read the factors which affect the values of the first 84 amplitudes. The effect of the initial suppression on the succeeding amplitudes is neglected.

8. - Introduce the simulation of the filter. Two cases result from the 40 cps and 60 cps filters imposed.

9. - Introduce the simulation of the AGC.

10. - Print a list of results including the number of the interface, the vertical travel time, and the values of the amplitudes.

11. - Punch the values of the amplitudes corresponding to a selected filter. These punched cards will be used in the programs to plot the synthetic seismogram, and the stack of 3 synthetic seismograms.

12. - End.

Appendix V lists this program. The approximate running time to process 480 data points is 3.5 minutes.

Program for Stacking Three Synthetic Seismograms With or Without Multiples

Similar to the program used to plot a synthetic seismogram with or without multiples, is this program to
plot a stack of three synthetic seismograms, prepared in Fortran IV, by the following scheme:

1.- Read the number N of layers of equal travel time and plotting symbols.

2.- Read amplitudes of the complete synthetic seismograms corresponding to each of the three shotpoint-geophone distances. These values of amplitudes have been obtained from the respective programs which yield as their outputs such values in punched cards.

3.- Compute the two-way vertical travel time.

4.- Change scale for the values of the amplitudes.

5.- Compute the algebraic sum of the three final amplitudes.

6.- Normalize and round the values of the sums.

7.- Test for the maximum and minimum positions in the plot.

8.- Print the vertical time, the normalized values of the stack, and the plotting symbol in desired position.

9.- End

Appendix VI shows this program. The approximate running time is 70 sec for a total of 480 data points.
IV. ANALYSIS AND COMPARISON OF THE SYNTHETIC SEISMOGRAMS OBTAINED

The analysis and comparison of the synthetic seismograms obtained should be carried out on elements that can readily be compared, such as the character, frequency, and shape of the signal picked out. The comparison, according to the nature of the problem, has to have a certain degree of freedom. In other words, it should be, above all, used to explain the differences between the traces to be compared.

Comparison Between Synthetic Seismograms at Normal and Oblique Incidence Without Multiples

Figure 9 shows one synthetic seismogram at oblique incidence, without multiples, corresponding to a shotpoint-geophone distance of 400 feet, and one synthetic seismogram at normal incidence also without multiples (Palafox, 1968). The great differences in shape and amplitude of the signals between 0.504 and 0.556 sec and between 0.688 and 0.740 sec are due to an introduction of fake values of velocities at 0.552 and 0.724 sec in the computation of the synthetic seismogram at oblique incidence. Besides these, no other great differences are noticed. Only slight differences in amplitude in the first 0.360 sec of the traces are probably the effect of the angle of incidence, though small in general, because of the short offset, in the curve
A few other not very important differences are observed in the deepest part of the traces.

The Synthetic Seismogram at Normal and Oblique Incidence With Multiples

Curve (a) of Figure 10 corresponds to a synthetic seismogram at normal incidence with multiples (Palafox, 1968) whereas curve (b) corresponds to a synthetic seismogram at oblique incidence with multiples, for an offset of 400 feet. It can be seen that at small depths the differences are probably produced by the effect of the angles of incidence, although these are not great because of the short shotpoint geophone distance. As the depth increases some other divergences can be observed; among these are small waves around 0.756, 0.948, 1.096, 1.128 and 1.172 sec in the trace at oblique incidence. The number of the multiples increases with depth. Thus the comparison of the traces discloses the location and importance of the more outstanding deep multiples. However both synthetic traces with multiples are very helpful in the search for true deep reflections. By comparing a field seismogram with a synthetic one, the energy arrivals seen only in the field seismogram are very probably true reflections.
Comparison Between Two Synthetic Seismograms at Oblique Incidence, With Different Shot Offsets

The interesting point in this comparison is to remark that the traces obtained for different offset distances are not entirely similar one to another, but they present differences along the seismogram. Curves (a) and (b) of Figure 11 represent two synthetic seismograms at oblique incidence with multiples, corresponding to a shotpoint-geophone distance of 400 and 1200 feet respectively.

It can be seen that as the offset distance increases, the angles of incidence increase and so do the differences between the traces.

In both traces, many of the characteristics seems to be common, many others however appear quite different. Let us look at, for example, the times of 0.260, 0.304, 0.828, 0.860, 0.980 and so on. These differences in shape and amplitude, mainly, do not need to be explained as they are very probably an effect of the differences in angles of incidence, since these are the only variable parameters in both traces. Again, in the deepest section of the seismograms, where the synthetic seismograms go beyond the velocity log (comprising only multiples), the location of the deep multiples is rather difficult to define by the comparison.
Comparison Between Two Synthetic Seismograms at Oblique Incidence, With and Without Multiples

This comparison indicates the influence of the multiple reflections. Figure 12 shows two synthetic seismograms at oblique incidence, both with the same shotpoint-geophone distance of 400 feet. Curve (a) is without multiples and curve (b) is with multiples.

The differences between the traces are quite obvious since curve (b) gives in addition to curve (a), changes in details due to the modifications produced by the effect of multiples.

The comparison is useful to define the multiple reflections, when the field seismogram is compared with a synthetic one.

The origin of the multiple reflections is rather difficult to determine, because each multiple is the result of the addition of a large number of small multiples which corresponding to different travel paths, all interfering at the same time.

A synthetic seismogram which includes only the first order multiples is generally not adequate to give an approximation of the actual trace with multiples.
Comparison Between a Stack of Three Synthetic Seismograms and a Synthetic Seismogram, at Oblique Incidence

Figure 13 shows two synthetic traces. One of them represents a single synthetic seismogram without multiples at oblique incidence, with a shotpoint-geophone distance of 400 feet. The other one is the result of stacking three synthetic seismograms at oblique incidence, without multiples, corresponding to offset distances of 400, 800, and 1200 feet respectively.

From this comparison we can only observe the effectiveness of the stacking process as an improvement, though slight, of the signal-to-noise ratio. It was pointed out that the effectiveness of the stacking process increases as the number of traces with common depth point stacked increases. Thus, a better signal-to-noise ratio would be obtained by stacking four or more traces with common depth point.

Comparison Between Two Stacks of Three Synthetic Seismograms at Oblique Incidence, With and Without Multiples

Figure 14 shows two stacks of three synthetic seismograms at oblique incidence with and without multiples, respectively.

This comparison is very similar to that of Figure 12 in which two synthetic seismograms, with and without
multiples are compared to observe the differences between them due to the influence of multiple reflections. In this case the two traces better define the events and therefore the location of multiple reflections is more precisely defined.

The improvement of the signal-to-noise ratio observable in both of the traces is expected after the stacking process of the three synthetic seismograms with common depth points.

Comparison Between a Stack of Three Synthetic Seismograms and Two Synthetic Seismograms, at Oblique Incidence

A final comparison between the seismograms obtained is shown in Figure 15 in which the bottom trace represents a stack of three synthetic seismograms at oblique incidence with offset distances of 400, 800, and 1200 feet, respectively. The two top traces represent two synthetic seismograms at oblique incidence, corresponding to 400 and 1200 feet offset distances, respectively.

At first glance there are not any great differences between the three traces, but with a more careful inspection, it can be seen that there is an improvement of the signal-to-noise ratio after stacking in the bottom trace.

Another interesting enhancement of data is observable in the last portion of the seismograms, in which the deep multiples appearing on the two top traces, are strongly reduced in the stacked trace.
We recall that the greater the number of traces with common depth point stacked, the greater reduction of multiples, as well as better signal-to-noise ratio. Thus, with a stack of six synthetic seismograms with common depth points, for instance, the improvements would be remarkable.
V. CONCLUSIONS

From a study of the several comparisons it can be concluded that when the traces are shot with small angles of incidence, no great differences are noticed with respect to a trace shot at normal incidence. However, as the angle of incidences increased its effect produces several differences such as small additional waves in some cases, small distortions in others, as well as the production of deep multiples and reduction of the amplitude of reflections.

On the other hand, the comparisons between stacked traces reveals the data enhancement of the CDP stacking process in comparison to the conventional process. This leads to a better definition of the character of the reflections and therefore to an improved interpretation by the more precise picking of the reflections.
VI. SELECTED REFERENCES


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Zagst, E. F., 1965, Horizontal stacking improves seismic data: Oil and Gas Jour. August 16, V. 63, p. 97-104.

APPENDIX I

Program to Plot Velocities
Obtained from Velocity Log
C PLOT OF THE VELOCITIES OBTAINED FROM THE VELOCITY LOG

REAL V

INTEGER J

DATA V/(5001 »

16 7 FORMAT(2X,80.I8)

C READ THE VELOCITIES V OF THE N LAYERS

READ(5,11) V, VI, I = 1, N

13 6 FORMAT(65.S12)

C READ PLOTTING SYMBOLS

READ(5,7) BLANK, STAR

16 7 FORMAT(2X)

DO 60 I = 1, N

20 T=I

C COMPUTE THE TWO WAY VERTICAL TRAVEL TIMES

TW(I)=T+0.004

21 60 CONTINUE

C CHANGE OF SCALE FOR THE VELOCITIES

VVI=V/325.0 + 0.5

22 6 FORMAT(5.4)

C PLOTTING THE VELOCITIES

WRITE(6,15)


C WRITE TITLES FOR PLOTTING BEGINNING IN A NEW PAGE


IC BLANK THE LINE

34 DO 30 L = 1, 80

35 LINE(L)=BLANK

36 30 CONTINUE

40 J=V(I)

C TEST FOR MAXIMUM POSITION FOR THE VELOCITIES

41 IF(J-80) 41, 42, 41

42 J=80

43 41 CONTINUE

45 J=1

C TEST FOR MINIMUM POSITION FOR THE VELOCITIES

46 IF(J-1) 41, 42, 42

47 J=1

48 41 CONTINUE

50 WRITE(6,16)


C WRITE A * IN DESIRED POSITION

55 LINE(J)=STAR

56 WRITE(6,17)


C WRITE A * IN DESIRED POSITION

65 LINE(J)=STAR

66 WRITE(6,17)


C WRITE A * IN DESIRED POSITION

75 CONTINUE

80 CONTINUE

85 STOP
APPENDIX II

Program to Compute the Reflection Coefficients
CONTINUE
APPENDIX III

Program to Compute a Synthetic Seismogram Without Multiples
C APPROXIMATION OF A SYNTHETIC SEISMOGRAM AT OBlique incidence
C WITHOUT MULTIPLES - NEGLECTING ALL S V CONVERSIONS
C WITH INITIAL SUPPRESSION, AGC AND FILTER SIMULATIONS
DIMENSION R(1500),I(1500),J(1500),U(1500),A(I),N(1500),JU(1500),NE(1500)
2I(1500),MT(1500),PL(1500),X(1500),Y(1500),Z(1500),NE(1500),NE(1500)
C 1500000, AG(15000), XP(15000), AM(15000), AM(15000), XP(15000), AM(15000)
C 4Ag(15000)
C READ THE NUMBER OF LAYERS OF EQUAL TRAVEL TIME
READ(5,301)NL
C READ THE REFLECTION COEFFICIENTS FOR ALL OF THE INTERFACES
FORMAT(10F7.4) 10 D 7 IC=1,9
C FIRST PART - IMPULSIONAL SYNTHETIC SEISMOGRAM
C FORMAT (2X.89HSYNTHETIC SEISMOGRAM WITHOUT MULTIPLES / )
READ(10,10)NL
C PRINT THE RESULTS
WRITE(*,10)JU=10,N.2
STOP
END
C C C C C
C READ THE FACTORS WHICH AFFECT THE AMPLITUDES
C BY INITIAL SUPPRESSION SIMULATION
READ(15,265) (WPI.l., l=1,8)
C FORM HORIZONTAL SYNTHETIC SEISMOGRAM WITHOUT MULTIPLES
WRITE(4,309)
C PRINT THE VALUES OF THE AMPLITUDES THAT WILL BE USED IN THE C PROGRAM TO PLOT THEM
WRITE(*,310)
STOP
END
C SIMULATION OF THE ATTENUATION CAUSED BY THE INITIAL SUPPRESSION
C READ THE FACTORS WHICH AFFECT THE AMPLITUDES
C SIMULATION OF THE attenuation caused by the initial suppression
C READ (5,265) (WPI.1., l=1,8)
C FORM HORIZONTAL SYNTHETIC SEISMOGRAM WITHOUT MULTIPLES
WRITE(4,309)
C PRINT THE VALUES OF THE AMPLITUDES THAT WILL BE USED IN THE C PROGRAM TO PLOT THEM
WRITE(*,310)
STOP
END
APPENDIX IV

Program to Plot a Synthetic Seismogram
With or Without Multiples
0 WRITEC
1 PLOT OF AN APPROXIMATION OF SYNTHETIC SEISMOGRAM
2 AT VARIOUS INCIDENCE, WITH OR WITHOUT MULTIPLES
3 NEGLECTING ALL SV CONVERSIONS
4 CONSIDERING STRAIGHT PATH TO EACH REFLECTOR
5 WITH INITIAL SUPPRESSION, FILTER, AND AGC SIMULATIONS
6 REAL LINE
7 INTEGRAL J
8 DIMENSION A(1000),T(1000),LINE(1000)
9 READ THE NUMBER N OF LAYERS EQUALLY SPACED IN TIME
10 FORMAT(15)
11 REAL LINE
12 INTEGRAL J
13 DIMENSION A(1000),T(1000),LINE(1000)
14 READS,Y(1:N),N
15 FORMAT(15)
16 REAL LINE
17 INTEGRAL J
18 DIMENSION A(1000),T(1000),LINE(1000)
19 READS,Y(1:N),N
20 FORMAT(15)
21 REAL LINE
22 INTEGRAL J
23 DIMENSION A(1000),T(1000),LINE(1000)
24 READS,Y(1:N),N
25 FORMAT(15)
26 REAL LINE
27 INTEGRAL J
28 DIMENSION A(1000),T(1000),LINE(1000)
29 READS,Y(1:N),N
30 FORMAT(15)
31 REAL LINE
32 INTEGRAL J
33 DIMENSION A(1000),T(1000),LINE(1000)
34 READS,Y(1:N),N
35 FORMAT(15)
36 REAL LINE
37 INTEGRAL J
38 DIMENSION A(1000),T(1000),LINE(1000)
39 READS,Y(1:N),N
40 FORMAT(15)
41 REAL LINE
42 INTEGRAL J
43 DIMENSION A(1000),T(1000),LINE(1000)
44 READS,Y(1:N),N
45 FORMAT(15)
46 REAL LINE
47 INTEGRAL J
48 DIMENSION A(1000),T(1000),LINE(1000)
49 READS,Y(1:N),N
50 FORMAT(15)
51 REAL LINE
52 INTEGRAL J
53 DIMENSION A(1000),T(1000),LINE(1000)
54 READS,Y(1:N),N
55 FORMAT(15)
56 REAL LINE
57 INTEGRAL J
58 DIMENSION A(1000),T(1000),LINE(1000)
59 READS,Y(1:N),N
60 FORMAT(15)
61 REAL LINE
62 INTEGRAL J
63 DIMENSION A(1000),T(1000),LINE(1000)
64 READS,Y(1:N),N
65 FORMAT(15)
66 REAL LINE
67 END
APPENDIX V

Program to Compute a Synthetic Seismogram With Multiples
C IMPULSIOUS SYNTHETIC SEISMOGRAM WITH MULTIPLES
325 105 IF(JK=JK-1) /*
C END OF THE FIRST PART OF THE PROGRAM
C SECOND PART OF THE PROGRAM
C CONSTRUCTION OF THE SYNTHETIC SEISMOGRAM WITH MULTIPLES
326 IF(JK+L=JK+1) /*
C CONTINUE
327 IF(JK+L<JK+1) /*
C CONTINUE
328 IF(JK+L>JK+1) /*
C CONTINUE
329 IF(JK+L=JK+2) /*
C CONTINUE
330 IF(JK+L>JK+2) /*
C CONTINUE
331 IF(JK+L=JK+3) /*
C CONTINUE
332 IF(JK+L>JK+3) /*
C CONTINUE
333 IF(JK+L=JK+4) /*
C CONTINUE
334 IF(JK+L>JK+4) /*
C CONTINUE
335 IF(JK+L=JK+5) /*
C CONTINUE
336 IF(JK+L>JK+5) /*
C CONTINUE
APPENDIX VI

Program to Plot a Stack of Three Synthetic Seismograms With or Without Multiples
STACK OF THREE APPROXIMATIONS OF SYNTHETIC SEISMOGRAMS
AT OBLIQUE INCIDENCE, WITH OR WITHOUT MULTIPLES
CONSIDERING STRAIGHT PATH TO EACH REFLECTOR
WITH INITIAL SUPPRESSION, FILTER, AND AGC SIMULATIONS

REAL LINE
INTEGER J
DIMENSION A1(1000), A2(1000), A3(1000), BLANK, DOT, STAR
READ THE NUMBER N OF LAYERS EQUALLY SPACED IN TIME
READ(5,9)
RE A0(5,9)
READI 5,5)
C READ PLOTTING SYMBOLS
READ(5,10)(All), I=1,N)
READ(5,10)(A2(I),I=1,N)
READI 5,10)(A3 I I),1=1,N)
FORMAT!
C READ AMPLITUDES A1, A2, A3
C A1, A2, AND A3 ARE THE AMPLITUDES OF THE COMPLETE SYNTHETIC
C SEISMOGRAMS WITH OR WITHOUT MULTIPLES
C CORRESPONDING TO THREE DIFFERENT SHOTPOINT-SEISMOGRAM DISTANCES
DO 20 1=1,N
CONTINUE
C ALGEBRAIC SUM OF THE THREE FINAL AMPLITUDES
CONTINUE
PLOTTING THE AMPLITUDES
WRITE TITLES FOR PLOTTING BEGUNING IN A NEW PAGE
WRITE(6,252)
25 FORMATT(1X,4SIVER,TIME STACK AMPLITUDES)
C CHANGE OF SCALE FOR THE AMPLITUDES
DO 30 I=1,N
CONTINUE
C WRITE A LINE OF DOTS THAT WILL BE THE AMPLITUDE AXIS
DO 35 J = 1,61
LINE J = DOT
CONTINUE
C WRITE A DOT IN LINE 431 TO PRODUCE THE TIME AXIS
LINE 431 = DOT
C INTRODUCING A CONSTANT TO ROUND VALUES OF THE AMPLITUDES
C TEST FOR THE MAXIMUM POSITION OF THE AMPLITUDES
C TEST FOR THE MINIMUM POSITION FOR THE AMPLITUDES
C WRITE A * IN DESIRED POSITION
C WRITE A BLANK IN SELECTED POSITION, WHICH MIGHT HAVE BEEN ON AXIS
C PUT DOT BACK IN AXIS LOCATION, IN CASE IT WAS BLANKED
STOP
END