Temperature Enhanced Alpha-Decay and the S-Process

by

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Calculation Procedure for Temperature Dependent  Alpha-Decay</td>
<td>9</td>
</tr>
<tr>
<td>References</td>
<td>32</td>
</tr>
<tr>
<td>Acknowledgments</td>
<td>33</td>
</tr>
</tbody>
</table>
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Abstract

In this paper temperature enhanced alpha-decay at high stellar temperatures is investigated in conjunction with s-process nucleosynthesis. Temperature dependent alpha-decay is assumed to be the result of more energetic, hence faster, decay from thermally excited nuclear states. An analytical formula for the temperature dependent half-life is derived as a function of the atomic number and weight, temperature, and the alpha-decay Q-value of the isotope. The effect of such alpha-decay upon s-process abundances is discussed and information about s-process neutron flux strengths is deduced based on the calculated alpha-decay half-lives of the isotopes Nd 144 and Sm 150. The neutron flux-temperature relationships are compared to specific constant temperature carbon burning models. The carbon burning phases of massive stars are considered to be possible sources of solar system s-process abundances. Finally, information regarding the time between s-process termination and final interstellar injection of s-process products is derived using the decay calculations.
Introduction

At high stellar temperatures alpha-decay may be strongly temperature dependent even though for laboratory conditions it is apparently insensitive to any thermal, as well as mechanical and chemical, effects. Isotopes which have measured half-lives in the laboratory of over a billion years may decay in seconds or less at temperatures of the order $10^9$ deg K ($T_9 = 1$). It is conceivable, therefore, that temperature enhanced alpha-decay might have an appreciable influence in determining the final isotopic abundances produced by high temperature nucleosynthesis.

Temperature dependent alpha-decay should be the result of the more energetic, hence faster, alpha-decay from thermally excited nuclei. The interaction of the thermal radiation field with atomic nuclei by

$$N + \gamma \rightarrow N^*$$

excites at a given instant only a small fraction of an ensemble of isotopic nuclei for temperatures up to the order of $T_9 \approx 2$, but the strong energy dependence of the alpha-decay process may allow the faster decay from the small excited
fraction to alter the overall decay rate of the ensemble. The fraction of the ensemble in any given excited state is determined by the canonical distribution function. Alpha-decay dependence is approximated by the Gamow one-body alpha-decay formula.

Temperature enhanced alpha-decay will be most effective for isotopes which are unstable for "ordinary" alpha-decay (decay at the temperature of the laboratory, essentially $T_0 = 0$. Since "ordinary" alpha-emitters are found in nature to be generally heavier than iron, the most likely candidates for temperature dependent alpha-decay influence are the producers of the trans-iron isotopes. There are only two important nucleosynthesis processes which produce such heavy element abundances.

The r-(rapid-) process arises from the exposure of the iron abundance peak to a very large neutron flux. The neutron capture rate exceeds the intervening beta-decay rates until very neutron-rich isotopes are created. Upon termination of the flux the r-process products beta-decay back toward the valley of beta-stability.

It has been very difficult to explain the source of neutron fluxes of the magnitude required by the r-process. But whatever the source and the temperature of the event it
seems unlikely that temperature dependent alpha-decay can compete appreciably with such very fast neutron captures.

For smaller fluxes, believed associated with such events as hydrostatic carbon burning, neutrons are captured at rates much slower than intervening beta-decay. This slow neutron capture constitutes the s-(slow-) process whereby isotopic abundances along the path of beta-stability from iron to lead are created. As can be seen from the following diagram most s-process isotopes also receive part of their total abundances from the r-process via beta-decay. However, some s-process isotopes are shielded by other beta-stable nuclei from receiving r-process contributions. These beta-shielded isotopes are called "s-process-only" and are important sources of information about the s-process (Fig. 1).

The s-process neutron captures move initial iron "seed" nuclei along the beta-stability path. Assuming for the moment that the iron abundance remains fixed and that the neutron flux exposure time has been long enough to approximate a condition of equilibrium among the isotopic abundances then

$$\frac{d n(Z,A)}{dt} = 0 = \text{flux} \times (n\sigma_{Z,A}-1-n\sigma_{Z,A})$$
where \( n(Z,A) \) is the s-process abundance of isotope \((Z,A)\), flux is the neutron flux, and \( \sigma(Z,A) \) is the average neutron capture cross section of isotope \((Z,A)\) for neutrons with the thermal velocity distribution associated with the flux. This implies that in the equilibrium limit that

\[
\sigma_{Z,A-1} = \sigma_{Z,A} = \sigma_{Z,A+1} = \ldots.
\]

Thus, the equilibrium condition for the s-process chain of isotopes is that the "current,"

\[
C = \text{flux} \times n_{i} = \text{flux} \times n_{i+1} = \ldots
\]

of transmutation between adjacent pairs of isotopes must be equal everywhere along the s-process paths otherwise relative \( n \sigma \) values will be changing which contradicts the equilibrium assumption.

By ordering s-process isotopes with the index \( i \) the equality of isotopic current can be ideally illustrated as in Fig. 2.

If an isotope on the path, say \( i+3 \), has a temperature dependent alpha-decay rate \( \lambda(T_{\text{g}})_{i+3} \) comparable to its neutron capture rate, \( \text{flux} \times \sigma_{i+3} \), then a portion of the abundance of \( i+3 \) will alpha-decay rather than capture neutrons. Unless the alpha-decay daughter nucleus is shielded by a beta-stable isotope off the path it will ultimately beta-decay back onto the s-process path. Thus, a fraction of the abundance of isotope \( i+3 \) is being "recycled" through a portion of the s-process path rather than immediately continuing on to \( i+4 \).
by neutron capture, and the recycled fraction $= \frac{\lambda(T_g)}{\text{flux} \times \sigma}$.

Such an alpha-decay loop is illustrated by Fig. 3.

Because of the assumed equilibrium condition the currents into and out of the loop must be equal,

$$C = C' ,$$

otherwise the total abundance within the loop will change violating the equilibrium condition. And the abundances and $n_\sigma$ values for each isotope within the loop must also separately remain constant. Therefore, at isotope $i$ it must be true that

$$C + C^\alpha = C''$$

and at isotope $i+3$

$$C' = C'' + C^\alpha .$$

But since $C = C'$ the latter expression becomes

$$C'' = C + C^\alpha = C + n\lambda_{i+3} .$$
The alpha-decay rate is a positive definite quantity, therefore $C'' > C$ which implies that

$$n\sigma_{i+2} > n\sigma_{i+3}$$

because of the proportionality between the neutron capture current and $n\sigma$ at each isotope.

Only neutron capture currents act between $i$, $i+1$, and $i+2$ so that $n\sigma_i = n\sigma_{i+1} = n\sigma_{i+2}$. And from $C = C'$

$$n\sigma_{i-1} = n\sigma_{i+3}.$$ Thus, the above inequality leads to the conclusion that

$$n\sigma_{i-1} < n\sigma_i = n\sigma_{i+1} = n\sigma_{i+2} > n\sigma_{i+3} = \ldots \ldots .$$

It is seen that the $n\sigma$ values for the isotopes within the alpha-decay loop with the exception of the alpha-decaying isotope itself are increased by the alpha-decay competition with the neutron capture.

$$n\sigma \text{ (within the loop)} = n\sigma \text{ (outside)} + \frac{\lambda (T)n_{i+3}}{\text{flux}}$$
is the expression for the increase of \( n \sigma \) arising from the alpha-decay. It is interesting to note that the \( n \sigma \) value for the alpha-emitter \( i+3 \) is the same as it would have been without the decay since \( n \sigma_{i+3} = n \sigma_{i-1} \) in the equilibrium limit.

In principle, the variation of \( n \sigma \) within an alpha-decay loop should allow a measurable test of alpha-decay importance for the s-process. Looking for such \( n \sigma \) variations is unfortunately complicated by other factors. The s-process is actually best described as a quasi-equilibrium process so that the \( n \sigma \) values should not be equal along the path but rather smoothly varying. Also, the neutron capture cross sections usually have to be extrapolated from thermal energies up to the relatively high velocities associated with stellar fluxes\(^2\). The assumed low velocity cross sections for many of the s-process isotopes are considerably uncertain.

A study of the end products resulting from various neutron flux intensities by Seeger, Fowler, and Clayton has shown that a neutron flux exposure which predicts the s-process \( n \sigma \) values of Fig. 4 most closely fits the known solar system s-process \( n \sigma \) values\(^3\). The region around \( A = 142\text{-}148 \) which includes isotopes of Neodinium and Samarium is at the bottom of a ledge which begins to drop at \( A = 138 \). This region includes several alpha-unstable isotopes such as
Nd-144 and Sm 148 and is interesting in the light of the previous discussion. The possibility of no enhancement due to temperature dependent alpha-decay loops at Nd 142 and Sm 148 will be discussed explicitly because of their status as s-process only isotopes. The cross sections of Sm 148 and Sm 150, another s-process only alpha-unstable isotope, have been accurately determined for stellar velocity neutrons, unfortunately the same has not been done for Nd 142 which is probably the most sensitive isotope for possible appearance of no enhancement. The temperature dependent alpha-decay half-lives of other interesting isotopes will also be calculated.

A careful examination of low velocity neutron cross sections is necessary before a decision is justified on the question of alpha-decay influence. However, certain implications concerning temperature-flux relationships will be made with special emphasis placed upon hydrostatic carbon burning.
Calculation Procedure for Temperature Dependent Alpha-Decay

In order to make a general survey of temperature dependent alpha-decay half-lives it is computationally expedient to make certain approximations and assumptions which allow the development of an analytic expression for the half-life, $t_{1/2}(Z,A,T)$, as a function of the atomic number $Z$, atomic weight $A$, and the temperature $T$. Since decay from thermally excited nuclear states is the basis for the temperature enhancement of alpha-decay, the first assumption is in the form of a selection rule imposed on alpha-decay from excited states. This rule is derived from the behavior of ordinary alpha-decay in which the parent ground state decays to daughter states.

Alpha-decay proceeding from the parent ground state depends on many details of the individual nucleus, but generally there is a preference for daughter states which are members of rotational bands based on intrinsic states of the same angular momentum and parity, $J^P$, as the parent ground state. Decay to the other states of the daughter nucleus is usually so highly hindered as to be negligible. Figure 5 illustrates this approximate selection rule for ordinary
alpha-decay from even-even nuclei.

In analogy with ordinary alpha-decay, excited state alpha-decay will be assumed to obey the similar selection rule: "only excited states which are members of rotational bands based on intrinsic states of the same $J^P$ as the daughter ground state will be allowed to alpha-decay to that daughter ground state." Decay to other daughter states need not be considered because it will be less energetic, consequently, much slower. Temperature enhanced alpha-decay will be assumed to follow the selection rule as illustrated in Fig. 6. Some decay is probably ignored in such an assumption but the ensuing error will be more than offset by following approximations.

Even though the isotopes which are of most interest for possible alpha-decay influence upon the s-process are even-even, it should be noted in passing that in the case of non-even-even nuclei where ground state to ground state alpha-decay is usually hindered it can nevertheless be assumed that excited state decay is predominantly to the daughter ground state. Such decay has as parent states those excited states which satisfy the selection rule.

The alpha-decay rate of an ensemble of isotopic nuclei is equal to the sum of the decay rates from the populations
in the different nuclear levels. And since the mean-life is equal to the inverse of the decay rate the following formal expression giving the half-life of the ensemble is immediate:

\[
t_{\frac{1}{2}}(Z,A,T) = \left[ \sum \frac{F(E_i,T)}{t_{\frac{1}{2}}(Z,A,E_i)} \right]^{-1}
\]

\(F(E_i,T)\) is the fraction of the ensemble in state \(i\) at temperature \(T\), and \(t_{\frac{1}{2}}(Z,A,E_i)\) is the half-life of the \(i\)th state for decay to the daughter ground state. The summation is taken over all nuclear states of the parent nucleus.

The fraction of the ensemble in state \(i\) is given by the canonical distribution:

\[
F(E_i,T) = \frac{e^{-E_i/kT}}{\sum_i e^{-E_i/kT}}
\]

If, however, the summation is taken over the energy levels rather than the states themselves, labelling the energy levels by the \(i\) index formerly used to label the individual states, then because of the magnetic quantum number degeneracy the fraction of the ensemble in energy level \(i\) is
The denominator in the expression for \( F(E_i, T) \) is the partition function and can be approximated by \((2J'+1)\), \( J' \) being the angular momentum of the parent ground state which is by far the most populated level. Actually, for any non-zero temperature the partition function will be larger than \((2J'+1)\) but not by much even for temperatures in the billions of degrees. This approximation serves to decrease slightly the calculated temperature dependent half-lives.

For the term \( \frac{1}{2} (Z, A, E_i) \), the alpha-decay half-life of an excited state at energy \( E_i \) above the ground state, the Gamow alpha-decay theory allows an analytic solution. The Gamow one body theory assumes the parent state to be a compound state made up of the daughter nucleus and the alpha particle. With respect to the daughter nucleus the alpha particle has energy \( E_i + Q \) where as before \( E_i \) is the excited level energy with respect to the parent ground state, and \( Q \) the alpha-decay energy between parent and daughter ground states. The alpha particle has an orbital angular momentum,
L, with respect to the daughter nucleus.

The effective potential barrier keeping the alpha particle within the compound nucleus is the sum of the Coulomb barrier of the daughter nucleus and the centrifugal barrier due to the orbital angular momentum, L, of the alpha particle. The one-body model is shown in Fig. 7.

Classically, when $E_i + Q$ is less than $V_L$ at the interaction radius $R$ the alpha particle is energetically constrained to remain within the compound nucleus. However, the actual quantum mechanical nature of the nucleus provides a possibility that the alpha-particle may escape by "tunneling" through the barrier. This tunneling effect is essential for many other important nuclear phenomenon (e.g. nuclear fusion energy generation within stars).

A common derivation of $t^{1/2}(Z,A,E_i)$ begins with

$$t^{1/2}(Z,A,E_i) = \frac{\hbar A \pi}{\Gamma_\alpha}$$

where $\Gamma_\alpha$ is the width of the compound state of the parent nucleus. And $\Gamma_{\alpha,L}$ can be written
\[ \Gamma_{n,L} = \frac{3\hbar v}{R} P_L \theta_L^2 \]

with the alpha particle velocity after escape given as \( v \); the interaction radius between the daughter nucleus and the alpha particle labelled \( R \); the penetrability factor, \( P_L \), a function of \( L \), \( Z \), and \( A \); and \( \theta_L^2 \) the "reduced width." \( P_L \) is commonly given as

\[
P_L = \left( \frac{E_B - E}{E} \right) \frac{1}{R_0} \exp \left[ -\frac{2\sqrt{2u}}{h} \int_{r}^{R_0} \left( \frac{E}{r} + \frac{E_R r}{2} - E \right) dr \right] \]

where \( E_B \) is the sum of \( E_c \) and \( E_L \), the Coulomb and centrifugal barriers; \( u \) the reduced mass of the alpha particle; and \( R_0 \) the radius at which the alpha-particle energy \( Q + E_1 \) equals the barrier energy \( E_B \). By using a WKB method solving for \( P_L \) in the case where \( E_c > E_L \) which allows the separability of the angular momentum term, the corresponding half-life has been solved:
\[ t_{1/2}(Z,A,E_i) = \frac{\hbar}{\theta^2} \left[ \frac{693}{33.3} \right] \left[ \frac{BR^3}{2(Z-2)} \right]^2 e^{\left[ 1.05 \left( \frac{BR(Z-2)}{2} \right) \right]^2} \]

\[ = \left[ \frac{31.28}{\sqrt{1000}} \right] \left( \frac{2(Z-2)}{E_{i+Q}} \right)^{1/2} \left( 1 - \frac{2(E_{i+Q})^{3/2}}{3\pi E_c^2} \right) \]

\[ \times e^{-\left[ \frac{7.62L(L+1)}{[BR(Z-2)]^2} \right]} \]

\[ \times R(L)^5 \]

where \( B = \frac{(A-4)x4}{A} \) a.m.u. is the reduced alpha mass;

\( R = 1.4 \left[ (A-4)^{1/3} + 4^{1/3} \right] \) fm. is the nuclear interaction radius between alpha particle and daughter nucleus;

\( E_c = \frac{(Z-2)}{(A-4)^{1/3} + 4^{1/3}} \) Mev is the Coulomb potential at \( R \); \( Q \) is just the alpha-decay Q-value between parent and daughter ground states; \( R(L) \) is the orbital angular dependence term

\[ \theta^2 \]

is the so called "dimensionless reduced width."

The major error in the Gamow formula comes from the assumption that the parent state can be represented by a compound nucleus which ignores much of the actual detailed nuclear nature of the state. This uncertainty is concentrated into a dimensionless number, the "dimensionless reduced width," \( \theta^2 \). Empirically it is found for nuclear states.
In a sense $\theta^2$ is a measure of how well the parent state can be described by such a compound nucleus. If the compound model does not resemble the parent state very well then $\theta^2$ will be very small\(^6\).

In the expression for $t_{\frac{1}{2}}(Z,A,E_1)$ $\theta^2$ will be set equal to unity for excited states of the parent nucleus satisfying the selection rule and equal to 0. for all other parent states. The error in $t_{\frac{1}{2}}(Z,A,T)$ due to this assumption will most probably reduce the calculated half-lives.

In the formal expression for $t_{\frac{1}{2}}(Z,A,T)$ the summation over energy levels requires an exact treatment of the location of excited states which would preclude the possibility of a general analytic solution. However, the use of excited level density functions, such as the one developed by Gilbert, Chen, and Cameron from a nucleon gas model, makes possible such a solution. The introduction of a density function changes the summation of energy levels to an integration over the density function, $D(Z,A,E,J)$, of levels with angular momentum $J$ at energy $E$ above the ground state. The density of energy levels is taken from Gilbert \textit{et al.} to be
The parameters S, U, and Y have been individually determined for different isotopes in order to give the best possible agreement with experimentally determined level placements. This formula is accurate up to about 5-6 Mev beyond which the actual level densities increase at a slower rate than that given by \( D(Z,A,E,J) \). The density function of \( J = 0 \) levels for Sm 150 is given in Fig. 8.

The parity of the state does not enter explicitly into the expression for \( D(Z,A,E,J) \). By dropping the parity restriction in the selection rule the density function can be used as stated, and the ensuing error will again serve to decrease slightly the calculated half-lives. Also, all excited states of the same angular momentum \( J' \) as the daughter ground state will be assumed to be intrinsic base states for rotational bands. This assumption may again reduce slightly \( t_{1/2}^{1/2}(Z,A,T) \).

An "enhancement function," \( H(Z,A,E,T) \), which is an a priori measure of the effectiveness of excited states in increasing the decay rate, can be defined
\[ H(Z,A,E,T) = [F(E,T)/t_{1/2}^*(Z,A,E)] \times t_{1/2}^*(Z,A,E=0) \]

where \( t_{1/2}^*(Z,A,E) \) is the Gamow half life for a hypothetical state of \( J = 0 \) at energy \( E \) above the ground state and with \( R(L) \) set equal to unity. The enhancement function is a measure of how much faster such hypothetical states alpha-decay than does the ground state (disregarding orbital angular momentum dependence). Enhancement functions are shown for Sm 148 at \( T_g = 1 \) and 1.4 in Figs. 9 and 10.

The peak in the enhancement function curve arises from the dependence of \( H(Z,A,E,T) \) on the exponentially increasing energy term of the alpha-decay rate (\( 1/t_{1/2}^* \)) and on the exponentially decreasing energy term of the canonical distribution function. It is obvious from looking at the enhancement curves of Figs. 9 and 10 that the most important temperature enhancement of alpha-decay will come from nuclear levels within the limited energy range under the peak.

An important approximation can be made based on the general nature of the enhancement function curve. In general, for each intrinsic base state there is exactly one rotational
band member of each angular momenta $J'+2$, $J'+4$, $J'+6$, etcetera, and these band members lie at progressively higher energies above their base state. Because $D(Z,A,E,J)$ is a monotonically increasing function with respect to $E$ there is a higher density of states of $J'$ around the energy of the enhancement peak than below it. Therefore, if the density functions for all rotational band members are set equal to the density function of the base states (of angular momentum $J'$),

$$D(Z,A,E,J'+2N) = D(Z,A,E,J') \quad N = 0,1,2,\ldots$$

then a slightly higher density of states satisfying the selection rule will be assumed within the energy region of important enhancement than will actually be the case. The utility of this assumption is that $\mathcal{Q}(L)$ being separable in $t_{\frac{1}{2i}}(Z,A,E)$ can be taken outside of the integral which can now be integrated only over the density and canonical distribution of the base states (of angular momentum $J'$), so that
\[ t_{1/2}(Z,A,T) = \left[ \sum_{n=0}^{4} \frac{(2J'+1)}{R(J'+2n)} \right] \int_{0}^{\infty} \frac{D(Z,A,J',E)F(E,T)}{t_{1/2}^{*}(Z,A,E)} \, dE \]^{-1}. \]

\( R(L) \) has been exchanged by \( R(J) \) since for even-even nuclei \( L \) must equal the \( J \) of the parent state in order to satisfy conservation of angular momentum.

As shown by the upper limit \( N=4 \) on the summation, angular momenta greater than \( J'+8 \) are not included. The orbital angular dependence term, \( R(J) \), causes \( J'+10 \) and higher momenta states to contribute several orders of magnitude less to excited state decay than states of lower \( J \) at the same energy.

The functions \( D(Z,A,E,J) \), \( t_{1/2}^{*}(Z,A,E) \), and \( F(E,T) \) are well-behaved functions which vary smoothly over energy intervals of the order of 1 Mev. This allows the integration to be numerically calculated by electronic computer with the integral replaced by a step summation over intervals of .1 Mev. An upper energy limit of 10 Mev may be used for the calculation for temperatures up to \( T_9 \sim 2.5 \) because of the exponential decrease of the enhancement function beyond the
peak. The discrepancy due to the error in $D(Z,A,E,J)$ at energies over 5-6 Mev where the energy level density is overestimated is to decrease the calculated half-life.

All approximations and assumptions have been made in the sense of decreasing the calculated $t_{\frac{1}{2}}(Z,A,T)$ except for the selection rule. However, with the exception of setting $\theta^2 = 1$, all ensuing errors probably add up to less than an order of magnitude uncertainty. The assumption regarding the "reduced width," however, probably causes the final calculations to be justifiable only as order of magnitude approximations.

Calculations of temperature dependent alpha-decay were made for the isotopes listed in Table 1. "P" status signifies creation by the little understood nucleosynthesis process responsible for the occurrence in nature of very small abundances of proton-rich isotopes found between iron and lead. The half-lives shown in Table 2 were calculated for these isotopes over a range of temperatures from $T_0 = 0$. ("ordinary" alpha-decay) up to $T_9 = 2.4$.

The dependence of $t_{\frac{1}{2}}(Z,A,T)$ on the $Q$-value is made apparent by ordering the isotopes according to descending $Q$ in Table 2. Hf 174 and Hf 176 have high $Q$-values which are, to a certain extent, offset by a higher atomic number,
Table 1: Isotopes used for temperature dependent alpha-decay half-life calculations showing Q-values, density function parameters, and nucleosynthesis process status.
Table 1

<table>
<thead>
<tr>
<th>Z</th>
<th>A</th>
<th>Q (Mev)</th>
<th>S</th>
<th>U</th>
<th>Y</th>
<th>Process Status</th>
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<tbody>
<tr>
<td>Nd 144</td>
<td>60</td>
<td>144</td>
<td>1.90</td>
<td>4.25</td>
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<td>.57</td>
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<tr>
<td>Sm 148</td>
<td>62</td>
<td>148</td>
<td>2.00</td>
<td>4.6</td>
<td>.7</td>
<td>.55</td>
</tr>
<tr>
<td>Sm 150</td>
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<td>150</td>
<td>1.48</td>
<td>5.1</td>
<td>-.08</td>
<td>.555</td>
</tr>
<tr>
<td>Sm 152</td>
<td>62</td>
<td>152</td>
<td>.26</td>
<td>5.2</td>
<td>-.08</td>
<td>.55</td>
</tr>
<tr>
<td>Dy 158*</td>
<td>66</td>
<td>158</td>
<td>.85</td>
<td>4.8</td>
<td>.07</td>
<td>.515</td>
</tr>
<tr>
<td>Hf 174*</td>
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<td>174</td>
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<td>4.55</td>
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<td>.475</td>
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<td>Hf 176*</td>
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<td>176</td>
<td>2.43</td>
<td>4.55</td>
<td>.16</td>
<td>.485</td>
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</table>
Table 2: Calculated temperature dependent alpha-decay half-lives in years for several isotopes at temperatures from $T = 0$ to $T = 2.4$ and ordered by descending $Q$-values.
Table 2
Calculated Temperature-Dependent
Alpha-Decay Half Lives (Yrs.)

<table>
<thead>
<tr>
<th>Isotope-Q Mev</th>
<th>$T_9$</th>
<th>$T_9$</th>
<th>$T_9$</th>
<th>$T_9$</th>
<th>$T_9$</th>
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<tr>
<td>Hf 174-2.56</td>
<td>$10^{11.8}$</td>
<td>$10^{11.8}$</td>
<td>$10^5.7$</td>
<td>$10^{-8}$</td>
<td>$10^{-5.9}$</td>
<td>$10^{-10.}$</td>
<td>$10^{-13.4}$</td>
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<tr>
<td>Hf 176-2.43</td>
<td>$10^{13.7}$</td>
<td>$10^{13.7}$</td>
<td>$10^6.0$</td>
<td>$10^{-6.6}$</td>
<td>$10^{-5.8}$</td>
<td>$10^{-10.}$</td>
<td>$10^{-13.4}$</td>
</tr>
<tr>
<td>Sm 148-2.00</td>
<td>$10^{12.4}$</td>
<td>$10^{12.3}$</td>
<td>$10^4.4$</td>
<td>$10^{-1.9}$</td>
<td>$10^{-6.6}$</td>
<td>$10^{-10.4}$</td>
<td>$10^{-13.5}$</td>
</tr>
<tr>
<td>Nd 144-1.90</td>
<td>$10^{12.4}$</td>
<td>$10^{12.3}$</td>
<td>$10^4.0$</td>
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<td>$10^{-6.8}$</td>
<td>$10^{-10.5}$</td>
<td>$10^{-13.5}$</td>
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<tr>
<td>Sm 150-1.48</td>
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<td>$10^{18.2}$</td>
<td>$10^6.7$</td>
<td>$10^{-6.6}$</td>
<td>$10^{-5.9}$</td>
<td>$10^{-10.}$</td>
<td>$10^{-13.2}$</td>
</tr>
<tr>
<td>Dy 158- .85</td>
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<td>$10^{28.4}$</td>
<td>$10^{12.}$</td>
<td>$10^{2.8}$</td>
<td>$10^{-3.6}$</td>
<td>$10^{-8.3}$</td>
<td>$10^{-11.9}$</td>
</tr>
<tr>
<td>Sm 152- .26</td>
<td>$10^{37.9}$</td>
<td>$10^{32.5}$</td>
<td>$10^{13.3}$</td>
<td>$10^{3.4}$</td>
<td>$10^{-3.}$</td>
<td>$10^{-7.9}$</td>
<td>$10^{-11.5}$</td>
</tr>
</tbody>
</table>
Z, creating a higher Coulomb barrier against alpha-particle penetration from the compound nucleus.

The rapidly decreasing half-lives with increasing temperature of Nd 144 and Sm 150 raise the possibility of alpha-decay enhanced ησ values for the s-process-only isotopes Nd 142 and Sm 148 as in Figs. 11 and 12 (an alpha-decay loop originating at Sm 148 will not affect its own ησ value or that of any other s-process-only isotope). Before examining measured ησ values, low energy neutron capture behavior will be discussed. For heavy elements neutron capture occurs through the many wide overlapping levels of the compound nucleus created by the neutron capture. Thus, neutrons of energy KT, the most probable Maxwell-Boltzmann energy of the flux, may be assumed responsible for all neutron captures. It is found experimentally that the neutron capture cross section is approximately inversely proportional to the neutron velocity or to the square root of the neutron energy (including s- and p-wave neutron capture)⁹):

\[ \sigma(T) \propto V^{-1} \propto E^{-1/2} \propto T^{-1/2} \]
This allows neutron capture cross sections determined at a single low energy (e.g. 25 Kev corresponding to $T_g \approx .3$ neutrons) to be extrapolated to other energies associated with stellar temperatures:

$$\frac{\sigma(T)}{\sqrt{T}} \approx \frac{\sigma(T_g = .25, 25\text{Kev})}{\sqrt{T_g \cdot .25}}$$

Therefore, the ratio of the cross sections for two isotopes is constant for any given stellar temperature:

$$\frac{\sigma_i(T_g)}{\sigma_{i+1}(T_g)} \approx \frac{\sigma_i(.25, T_g / \sqrt{.25})}{\sigma_{i+1}(.25, T_g / \sqrt{.25})} = \frac{\sigma_i(25\text{ Kev})}{\sigma_{i+1}(25\text{ Kev})}$$

$\sigma$ ratios are accordingly not affected by the temperature dependence of low energy neutron cross sections. This procedure results in serious error only for heavy elements with magic numbers of neutrons. In these cases the cross section is strongly influenced by the location of specific resonances and the thermally averaged cross section has a more complicated temperature dependence. Table 3 shows $\sigma$ values for some s-process-only isotopes using 25 Kev cross sections.
Table 3: $n_\sigma$ values for some s-process-only isotopes showing possible alpha-decay enhancement at Nd 142. The general decrease of $n_\sigma$ with increasing mass number is expected because of the quasi-equilibrium nature of the s-process. Taken from Clayton, Fowler, Hall, and Zimmermann.\textsuperscript{10}
TABLE 3

S-Process-Only nσ Values

<table>
<thead>
<tr>
<th>s-only isotope</th>
<th>n: abundance per $10^6$ Si Atoms</th>
<th>$\sigma$(25 Kev)</th>
<th>nσ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nd 142</td>
<td>.21</td>
<td>150 mb.*</td>
<td>30</td>
</tr>
<tr>
<td>Sm 148</td>
<td>.282</td>
<td>213 mb.</td>
<td>6</td>
</tr>
<tr>
<td>Sm 150</td>
<td>.185</td>
<td>324 mb.</td>
<td>6</td>
</tr>
<tr>
<td>Gd 154</td>
<td>.079</td>
<td>710 mb.</td>
<td>5.6</td>
</tr>
<tr>
<td>Dy 160</td>
<td>.089</td>
<td>430 mb.</td>
<td>3.8</td>
</tr>
<tr>
<td>Hf 176</td>
<td>.090</td>
<td>400 mb.*</td>
<td>3.6</td>
</tr>
</tbody>
</table>

*considerable uncertainty in $\sigma$(25 Kev)
The decrease in $n\sigma$ values along the s-process path is generally attributable to the quasi-equilibrium nature of the process. $n\sigma$ enhancement of Sm 148 by temperature dependent alpha-decay from Sm 150 does not appear to have taken place as can be seen from the recently determined

\[
\frac{n\sigma(\text{Sm 148})}{n\sigma(\text{Sm 150})} = 1.02 \pm 0.06
\]

based on Samarium cross sections for low energy neutrons measured at Oak Ridge by Macklin, Gibbons, and Inada\textsuperscript{11}. This implies that the temperature dependent alpha-decay half-life of Sm 150 must have been at least an order of magnitude greater than its neutron capture time during the s-process:

\[
t_{\frac{1}{2}}(62,150,T) > 10 \times \text{Neutron Capture Time of Sm 150}
\]

The $n\sigma$ value for Nd 142 is seen in Table 3 to be five times greater than for Sm 148. Unfortunately the cross section for Nd 142 is not adequately measured, so this discrep-
ancy may be due to error in the estimated $\sigma (25 \text{ keV})$ of Nd 142. The need for measurement must be stressed, inasmuch as Nd 142 has the magic neutron number $n = 82$. If the assumed cross section is approximately correct, however, the Seeger, Fowler, and Clayton calculations predict $n\sigma$ (Nd 144) should approximately equal $n\sigma$ (Sm 148). Thus, it is possible that temperature enhanced alpha-decay competition with s-process neutron capture has increased the Nd 142 $n\sigma$ value by around a factor of 5. Whether or not such is actually the case cannot be determined until low energy neutron capture measurements for Nd 142 are made. Not only does $n\sigma$ depend explicitly on the cross section, $\sigma$, the predicted value does also. For example, the predicted $n\sigma$ ledge dropoff at Ba 138 will have to be moved forward to include Nd 142 if the Nd isotopes are found to have smaller cross sections than used by Seeger, Fowler, and Clayton in their calculations.

It would be unwise at this time, therefore, to expound s-process alpha-decay influence on the basis of the Nd 142 abundance. However, since the Nd 142 cross section has probably been, if anything, overestimated it is safe to say as a limit that the actual Nd 142 $n\sigma$ value is at most 10 times larger than the Sm 148 $n\sigma$. Assuming this increase in $n\sigma$ is due solely to the presence of an alpha-decay loop from Nd 144
the following inequality may be written:

\[ 10 \times t^{1/2}(60,144,T) > \text{Neutron Capture Time of Nd 144} \]

Noting that the neutron capture time of isotope \( i \) is given by

\[ [\text{neutron capture time of } i] = \frac{1}{\text{flux} \times \sigma_i(T)} \]

and remembering that

\[ \sigma(T_g) \approx \sigma(T_g = .25) \frac{\sqrt{T_g}}{\sqrt{.25}} \]

the flux-temperature restriction shown in Fig. 23 may be made using the calculated temperature dependent half lives and assuming \( \sigma(144) \) at 25 Kev = 170 mb. and \( \sigma(150) \) at 25 Kev = 324 mb.

If future cross section measurements of Nd 142 indicate an unusually high \( n\sigma \) value (enhanced over predicted value by \( \approx \) factor of 5) and if this \( n\sigma \) enhancement is assumed to arise from the alpha-decay loop originating at Nd 144 then the half-
life calculations may be used in the expression

\[ 5 \times t^{1/2}_{1/2}(60,144,T) = \text{neutron capture time of Nd 144} = 1/\sigma(144) \times \text{flux} \]

to give a more restricted flux-temperature relation. Using \( \sigma(T_9 = .25) = 170 \text{ mb. for Nd 144 results in the relation shown in Fig. 14.} \) By superimposing the Sm 150 s-process flux limit from Fig. 16 it can be seen that \( T_9 \approx 1.6 \) is indicated as an upper temperature limit for s-process events which were responsible for solar system abundances. This same temperature limit can be deduced from the Sm 150 flux limit alone if it is noted that a flux \( \gtrsim 10^{25} \text{ neutrons/cm}^2 \text{-sec.} \) impinging upon iron seed will result in an r-process event rather than an s-process event. Thus, the more conservative assumptions used in making the Sm 150 s-process flux limit can be used to predict the same upper temperature limit for the s-process as the more uncertain assumption of Nd 142 alpha-decay enhanced \( n\sigma \).

It is of interest to compare the predicted flux-temperature relations with actual neutron fluxes believed resulting from constant temperature carbon burning which may
be largely responsible for s-process nucleosynthesis in large
stars. Figures 15 and 16 show neutron fluxes associated with
non-explosive carbon burning models made by Arnett and
Truran with superimposed flux-temperature restrictions from
Fig. 13). The predicted fluxes are generally consistent
with the alpha-decay derived relationships except after long
carbon burning times. But, it is at these long carbon burn-
ing times where the constant temperature assumption of the
model probably breaks down for actual stellar conditions
due to consumption of most of the carbon fuel and the pos-
sible onset of new nuclear burning stages.

Temperature dependent half-life calculations may also
be used to deduce a temperature-time relationship for the
maximum amount of time s-process abundances remain within
any star after flux termination. Obviously, the Sm 148
abundance did not undergo any appreciable temperature de-
pendent alpha-decay after s-process termination or its $\sigma$
value would not agree so well with that of the slower decay-
ing (calculated) Sm 150. In order to maintain the measured
$\sigma$ equality within 10% between Sm 148 and Sm 150 the follow-
ing may be used:

$$\frac{t_{\frac{1}{2}}(62, 148, T)}{\text{time}} \geq 10$$
This relation results in the time limit shown in Fig. 17.

In summary, the calculated temperature enhanced half-lives due to decay from thermally excited nuclear states may be used to gain certain information on necessary s-process neutron flux strengths and the time between s-process termination and interstellar injection. These relationships can be strengthened if low energy neutron capture cross section measurements of Nd 142 and Nd 144 show alpha-decay influence on the Nd 142 abundance. The calculated flux-temperature restrictions seem to be consistent with constant temperature carbon burning fluxes which are believed to be important sources for s-process events.
References


Acknowledgments

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This portion of the s-process path shows R-process contributions and s-process-only isotopes Sm$^{148}$ and Sm$^{150}$ shielded from R-process contamination by the beta-stable isotopes Nd$^{148}$ and Nd$^{150}$. 

Figure 1: This portion of the s-process path shows R-process contributions and s-process-only isotopes Sm$^{148}$ and Sm$^{150}$ shielded from R-process contamination by the beta-stable isotopes Nd$^{148}$ and Nd$^{150}$. 


Figure 2: An idealized s-process path which shows the equilibrium neutron capture current insuring static $\sigma$ values. $\sigma$ equality comes from equal neutron capture currents between isotopes.
Figure 3: An idealized s-process path with alpha-decay loop showing equilibrium current conditions $c = c', c + c^\alpha = c'', c'' = c^\alpha + c'$ which insures static $n\sigma$ values. The $n\sigma$ values for isotopes within the loop are increased by a factor of $\frac{c^\alpha}{c'}$. 
Figure 4: Theoretical nσ curve computed for a neutron flux exposure which gives the best agreement with measured solar system s-process values which are shown by solid dots. Notice the ledge structures such as at Ba 138.\(^3\).
Figure 5: An approximate selection rule for "ordinary" alpha-decay which goes to daughter states belonging to rotational bands based on intrinsic states of the same $J^\pi (0^+) \text{ as the parent ground state (even-even nuclei).}
$0^+\rightarrow (Z+2, A+4)$

PARENT NUCLEUS

$6^+$

$4^+$

$2^+$

$0^+$

$(Z, A)$

DAUGHTER NUCLEUS
Figure 6: The assumed selection rule for excited state alpha-decay which limits alpha-decay to parent states which belong to rotational bands based on states of the same $J^T(0^+)$ as the daughter ground state (even-even nuclei).
(Z+2, A+4)

PARENT NUCLEUS

(Z, A)

DAUGHTER NUCLEUS
Figure 7: An illustration of the one body alpha-decay compound nucleus model which approximates the parent state as the daughter state plus an alpha-particle; $E_B$ is the effective barrier and $R$ the radius of interaction$^5$.
Potential Well of Daughter Nucleus
Figure 8: The $J = 0$ excited level density function for Sm 150 showing a logarithmically increasing behavior. The ground state ($J = 0$) is not indicated by the formula\textsuperscript{7).}
Figure 9: The enhancement function of Sm 148 at Tg = 1 showing the limited important energy range for excited state decay from 1.7 to 2.7 Mev above the ground state.
Figure 10: The enhancement function for Sm 148 at $T_g = 1.4$ showing a shift in the energy range of important enhancement to 3.5-5 MeV above the ground state.
ENHANCEMENT FUNCTION $H(Z, A, E, T)$

ENERGY (MeV) ABOVE GROUND STATE

$T_9 = 1.4$

ENHANCEMENT FUNCTION $H(Z, A, E, T)$

ENERGY (MeV) ABOVE GROUND STATE

$T_9 = 1.4$

IMPORTANT ENERGY
Figure 11: Possible alpha-decay loops at Sm 148 and Sm 150—the latter capable of enhancing the $\nu_\sigma$ value of the s-process-only isotope Sm 148.
**Figure 12:** Possible temperature dependent alpha-decay loop from Nd 144 to Ce 140 which would enhance the no value of the s-process-only isotope Nd 142.
Figure 13: S-process neutron flux limits based on calculated temperature dependent half-lives of Sm 150 and Nd 144 through the relations

\[ \frac{1}{2} t_{\frac{1}{2}}(62,150,T) > 10 \times \text{neutron capture time of Sm 150} \]

and

\[ 10 \times \frac{1}{2} t_{\frac{1}{2}}(60,144,T) > \text{neutron capture time of Nd 144} \]
Figure 14: Predicted s-process neutron flux based on the calculated temperature dependent half-life on Nd $^{144}$ and assumed no enhancement of Nd $^{142}$ by a factor of 5 due to such alpha-decay. The relationship comes from the equation

$$5 \times t^{\frac{1}{2}}(60,144,T) = \text{neutron capture time of Nd }^{144}.$$ 

The Sm 150 flux limit from Fig. 13 is superimposed showing a derived upper temperature limit for the s-process at $T_9 \sim 1.6$. 
Figure 15: Comparison of calculated neutron flux of Arnett's constant temperature carbon burning model at $T_9 = 0.8$ with the Nd 144 and Sm 150 flux limits from Fig. 13 showing general consistency except at long carbon burning times.$^{12}$
$T_9 = 0.8$

![Diagram showing a logarithmic graph with a line labeled ND 144 Limit and another labeled SM 150 Limit. The x-axis represents $\log_{10}(t/$Sec$)$ ranging from 6 to 12, and the y-axis represents $\log_{10}$ Flux $(cm^2/sec)$ ranging from 0 to 30.](image-url)
Figure 16: Comparison of calculated neutron flux of Arnett's constant temperature carbon burning model at $T_g = 1$. with the Nd 144 and Sm 150 flux limits from Fig. 13 showing general consistency except at long carbon burning times.
Figure 17: Relationship of time limit of s-process products at stellar temperatures after s-process termination based on Sm 148 and Sm 150 no equality from the relationship

\[ \frac{t^{1/2}(62,148,T)}{\text{time (yrs.)}} \geq 0. \]