NONLINEAR EFFECTS NEAR AN EXPLOSION

by

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ABSTRACT

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The purpose of this study is to find the dimensions of the shattered zone produced by the explosion of a spherical charge, in terms of the properties of the medium and the weight of the explosive charge; and then, to establish a relationship between the size of this nonlinear region and the frequency spectrum of the elastic signal.

The computation of the size of the shattered zone, shows that this region is bounded by a so called "Critical Radius," which occurs when the shock wave pressure caused by the explosion decreases in value until it equals the yield strength of the medium. Within this region, different values of the shock wave pressure for different values of explosive charges follow the Principle of Similarity.

The calculation of the frequency spectra, shows that for an ideally elastic medium, the shape of the frequency spectrum of the signal outside the shattered zone, changes as a function of both the critical radius and the P-wave velocity of the medium.
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I. INTRODUCTION

Of the three physical processes involved in oil prospecting namely, the generation of the seismic waves, their propagation as refracted, reflected or dispersed waves, and the recording of the motion produced by them at the surface, we have the least understanding of the generation process.

This is in spite of the circumstance that it has been recognized that the intensity, quality, and characteristics of the seismic records are strongly influenced by the conditions at the shot point.

Every study made toward the improvement of subsurface data from seismic techniques should include an investigation of the factors that influence the characteristics of the outgoing signal.

One of those factors is the mechanism that generates the particular shape of a seismic signal resulting from an explosive source. Theoretical discussions of the first motions from such sources (Burridge et al., 1964; Case and Colwell, 1967) have usually been based on the supposition that the medium is infinite in all directions, in such a way that the observed disturbances would not be influenced by different behaviours of the medium, or the proximity of any geological contact.

The purpose of this work was to find the dimensions of the nonlinear region around the shot hole produced by the ex-
plosion of a spherical charge. As postulated in the shock-wave theory of W. E. Peet (1960), such dimensions are shown to depend on the weight of the explosive charge and the properties of the particular medium in which the shot is fired.

A relationship was also established between the size of the nonlinear region and the frequency spectra of the signal outside the shattered zone. Seven different media were considered in this study, all having in common the fact that they are likely to be found at the usual depths to which the shot holes are drilled in oil exploration.
I. GENERAL DESCRIPTION

For simplicity, consider a spherical explosive charge buried in an infinite isotropic medium, which is initiated at its center.

Since the detonation wave is the fastest disturbance which can pass at a stable velocity through the explosive (Cole, 1948), the surrounding medium will be unaffected until the instant when all the explosive has been converted into a hot gas at high pressure.

The walls of the shot hole will then be subjected instantly to a pressure of the order of several hundred tons per cm$^2$, and a pressure pulse in the medium will be generated. The form of this pulse will be very nearly of the type of a Heaviside step function (Morris, 1950).

After this stage the pulse will spread out spherically about the center of the shot hole.

The stresses initially induced in the medium will greatly exceed the ultimate strength of the medium, and therefore will break it up, forming in this way a complicated crack system.

However, the medium will not move appreciably during the passage of the shock wave.

As the spherical pulse moves outwards, the stresses will fall due to the increasing volume occupied by the pulse, and due to the expenditure of energy in forming new surfaces of the crack system.
At some critical radius, (Peet, 1960), the stresses will just equal the elastic limit, \( \sigma_m \), of the medium. At that distance the pulse becomes an elastic pulse.

The initial energy of the shock wave is a function of the velocity of detonation of the explosive, (Cole, 1948), on the other hand the stresses at the critical radius are dependent upon the properties of the medium. Thus, if the explosive charge is increased, the value of the critical radius will be larger.
II. WAVE PHENOMENA IN THE NONLINEAR ZONE

Existence of a Shock Wave.— Consider a medium to be macro-homogeneous and macro-isotropic, and which behaves elastically up to a certain yield stress $\sigma_m$.

For stresses greater than the yield stress the medium is assumed to be shattered by the action of a shock wave.

The existence of such a shock wave in the high pressure zone near the charge is assumed to be caused by the filler fluid in the pores of a water-saturated porous medium, such as encountered in many areas at the depths to which shot holes are normally drilled.

Under high pressures the solid particles in the medium will become more compacted, and as the water can not be driven out of the formation, in a very short time the water will predominate as the medium in which the waves are transmitted.

Since water is a compressible fluid, the originally smooth wave sharpens its front to a step during the process of propagation because the high-pressure parts in a wave in this medium travel faster than the low-pressure parts, (Cole, 1948).

A wave of the type in Fig. 1 (c) is called a shock wave, and its steep front has a velocity of propagation that exceeds the speed of small amplitude sound waves.

As the shock wave zone is considered to behave as a compressible fluid, the phenomena in that region can be treated in a similar way to those occurring in underwater explosions.
The Differential Equations for Ideal Fluids.- As a first step in discussing the propagation of waves in fluids, the basic laws of mechanics should be stated in a suitable mathematical form. In this discussion it is assumed that the fluid is ideal in the sense that viscous stresses and effects of heat conduction may be neglected; it is assumed that there are no discontinuities of pressure, velocity of the fluid, or internal energy.

Conservation of Mass.- The simplest restriction on the motion of a fluid is the conservation of mass,

\[ \frac{\partial \rho}{\partial t} \, dt \, dx \, dy \, dz = \text{change of mass} \]

where \( \rho \) is the density.

If such a change occurs it must be as a result of motion in the fluid. Let the velocity of a point moving with the fluid
be described by its three components u, v, w, in the x, y, z, directions. The net transport of fluid in a time \( dt \), from motion in the x-direction is:

\[
[ (\rho u)_{x} - (\rho u)_{x+dx} ] dt \ dy \ dz = - \frac{\partial}{\partial x} (\rho u) \ dt \ dx \ dy \ dz
\]

The mass conservation then yields in vector notation

\[
\frac{\partial \rho}{\partial t} + \text{div} (\rho \ \vec{V}) = 0
\]

where \( \vec{V} \) is the velocity vector.

If the symbol \( d/dt \) is understood to mean differentiation at a point moving with the fluid rather than a fixed point, the equation may be expressed as:

\[
\frac{d\rho}{dt} = - \rho \ \text{div} \ \vec{V} - - - - - - - - - - - - - - - - - - - - (1)
\]

which is the original form developed by Euler.

**Conservation of Momentum.**- Equating the force and inertia terms on an element of volume, it is found for the x-component of motion:

\[
\rho \ \frac{du}{dt} = - \frac{\partial P}{\partial x}
\]

where \( P \) is the pressure; similar terms can be found for the other two components.

The equations of the components are equivalent to a single vector equation:

\[
\frac{d\vec{V}}{dt} = - \frac{1}{\rho} \ \text{grad} \ P - - - - - - - - - - - - - - - - - - - - (2)
\]
In the derivation of the previous equations, the effect of dissipation processes has been neglected. If the properties of a specified small element of the fluid are described by these equations, a further condition on the state of this element is implied which has not been explicitly stated. If no dissipative processes take place in a given period of motion, no element moving with the fluid can exchange heat with any other element or its surroundings during this time. The changes in the physical state of the element must therefore take place at constant entropy, a situation that can be expressed by the relation $\frac{ds}{dt} = 0$, that is, changes of density due to applied pressure take place adiabatically. Therefore, for any point in the fluid at any time for which dissipative processes can be neglected, the pressure is a single valued function of density.
III. PRINCIPLE OF SIMILARITY

It can be shown that the principle of similarity follows from the basic equations describing the motion of a fluid:

\[
\frac{dV}{dt} = -\frac{c^2}{\rho} \text{ grad } P \quad \text{(a)}
\]

\[
\frac{dP}{dt} = -\text{ div } V \quad \text{(b)}
\]

where \( c^2 = \frac{dP}{d\rho} \), \( V \) = particle velocity, \( \rho \) = specific mass, \( s \) = entropy, \( P \) = pressure, \( c \) = velocity of propagation, and \( t \) = time.

Suppose that measurements of pressure have been made at a distance \( r \) from a charge of specified dimensions at a time \( t \) after it is initiated, and that a new experiment is arranged in which all the linear dimensions of the charge are changed by a factor \( K \).

The Principle of Similarity states that: If the linear dimensions of the charge and all the other distances are altered in the same ratio for two explosions, the shock wave formed will have the same pressures at corresponding distances scaled by this ratio, if the times at which pressure is measured are also scaled by the same ratio.

In order to examine the validity of the principle of similarity, it is observed that the differential equations \((3, a, b)\) are satisfied if the scales of measurement of both length and time are changed by a factor \( K \).
By substituting $r' = kr$, $t' = kt$, equation (3a) becomes:

$$
\frac{d}{dt} v (kr, kt) = - \frac{c^2}{\rho} (r', t') \text{grad'} P (r', t') - - - (4)
$$

where grad' indicates differentiation in the primed coordinates. Note that all the indicated derivatives are of the same order, therefore equation (4) may be written:

$$
\frac{d}{dt} v (kr, kt) = - \frac{c^2}{\rho} \text{grad} P (kr, kt),
$$

the scale factor $k$ cancelling out. Hence the same differential equation is satisfied by $V (kr, kt)$ as by $V (r, t)$. Similar results can be obtained with equation (3, b).

The validity of the principle of similarity depends, among other conditions, on the assumption that no external forces act upon the system. Gravity is such an external force and of course is always present, but it is unimportant compared with the internal forces involved in the generation and propagation of the shock wave. However, its effects can not be neglected in the later behavior of the explosion products, (Brinkley and Kirkwood, 1947; Jeffreys, 1931b).

Therefore, the principle of similarity, as stated above, does not apply to the phenomena following the shock wave (Cole, 1948).
IV. SHOCK WAVE EQUATION

The Form of the Shock Wave. - The shock wave from explosives is found in most cases to be a highly reproducible phenomenon which can often be represented at a given point to a first approximation by a discontinuous rise in pressure followed by an exponential decay with time (White and Sengbush, 1963). It is to be expected that charges which are not spherically symmetrical will give rise to a shock wave which is not symmetrical, and differences in the form of the wave at different points around the charge are in fact observed (Cole, 1948). As the shock wave spreads out from the charge its peak value decreases and its duration, as estimated for example from the time constant of exponential decay, increases gradually.

Parameters of the Shock Wave Equation. - A complete representation of the pressure time curves of the shock wave as experimentally measured at the Underwater Explosives Research Laboratory, Woods Hole, Mass. (Cole 1948), for any type of charge, would be an equation giving the pressure as a function of time $\tau$ after arrival of the initial peak, in terms of the charge weight and position of the point of measurement. Even for the simplest forms of charges, an exact result of this kind would be troublesome and not readily visualized.

Despite the limitations of the exponential decay as a description of shock waves, its analytical convenience is so great that parameters based on it are widely used, and in many cases
give a reliable description. In this approximation, for a spherical source in an infinitely extended compressible medium, the pressure as a function of time $t$ after arrival of the shock front is expressed as:

$$ P(t) = U(t) P_m e^{-t/\theta} $$  \hspace{1cm} (5)

where $t$ is the time measured from the start of the shock wave (Fig. 2), $U(t)$ is the unit step function, $P_m$ is the initial peak pressure and $\theta$ is the time constant of exponential decay. The accuracy of this first approximation in any particular case can be tested by plotting $\log P$ as a function of $t$, which should give a straight line with negative slope, $1/\theta$.

(a). The velocity of propagation $V_p = \sqrt{\frac{dP}{d\rho}}$; $P$ = pressure, $\rho$ = density. The plot $P(\rho)$ - $\rho$ is concave upward, so: $V_p$ increases with increasing compression.

(b). $P(t) = U(t) P_m e^{-t/\theta}$, where $U(t)$ is Heaviside's unit function defined as:

$$ \tau = t - \frac{r}{V_p} ; \quad U(t) = \begin{cases} 0 & t < \frac{r}{V_p} \\ 1 & t > \frac{r}{V_p} \end{cases} $$

Fig. 2. Shock wave propagation in compressible fluids. (after Peet, 1960)
In order to see more clearly the reliability and the restrictions of this first approximation, consider a spherical charge of explosive placed in an infinite elastic medium, as is shown in Fig. 3; let the radius of the charge be \( r_0 \), and assume a region bounded by spherical surfaces with radii \( r_0 \) and \( a \), in which shock wave phenomena occur. The matter within this region is considered to behave as a compressible fluid, so that the phenomena in that region can be treated in a similar way to those occurring in underwater explosions. The shattered zone is considered to behave as a compressible fluid as long as the maximum pressure in the wave diverging from the explosive charge exceeds a critical value \( \sigma_m \), which is the yield stress of the medium (Peet, 1960).

![Diagram]

- \( c \) = center
- \( r_0 \) = initial radius of explosive
- \( r'_0 \) = shot hole radius after detonation
- \( r_0 - a \) = shock wave zone
- outside \( a \) = elastic medium

*Fig. 3. Spherical shot hole in infinite homogeneous elastic medium.*
It is generally characteristic of shock wave results for high explosives that peak pressure $P_m$ may be expressed by:

$$P_m = K_1 \left[ \frac{Q^{1/3}}{x} \right]^\alpha$$

where $Q$ is the amount of explosive and $\alpha$ is a constant. This power law gives a compact, reasonably precise method of representing the data, and is also a reasonably good representation of results from the theory of Kirkwood and Bethe on shock wave propagation (Cole, 1948).

Experimental values of the constants obtained at the Underwater Explosives Research Lab., Woods Hole, Mass., are summarized in Table 1. In quoting values of the constants, charge weights are taken in pounds, and distance $r$ from the charge in feet.

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<th>Explosive</th>
<th>$10^4 K_1$</th>
<th>$\alpha$</th>
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<td>TNT ($\rho=1.52$)</td>
<td>2.16 (2.60)</td>
<td>1.13 (1.21)</td>
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<tr>
<td>PENTOLITE ($\rho=1.60$) (50% PETN, 50% TNT)</td>
<td>2.25 (2.85)</td>
<td>1.13 (1.23)</td>
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**TABLE I**

Parameters of shock waves for two explosives. Values predicted from the Kirkwood-Bethe Theory are given in parenthesis, (after Cole, 1948).
The time constant $\theta$, can be approximated by:

$$\theta = K_2 \, r_o \, \log \left[ \frac{r}{r_o} \right]$$

(7)

where $K_2$ is a constant involving factors that depend on the kind of explosive material and the properties of the medium in which the charge is fired, and $r_o$ is the charge radius.

Equation (7) is not as a good an approximation as equation (6), and actually applies well only for the first part of the exponential decay (Cole, 1948).

Equations (6) and (7) are illustrated graphically in Fig. 4.

\[ P(T) = U(T) P_m e^{-T/\theta} \]

$P_m = K_1 \left[ \frac{\rho^{1/3}}{r} \right]^\alpha$ ; $\theta = K_2 r_o \log \left[ \frac{r}{r_o} \right]$

Fig. 4. Shock wave shape as a function of distance from center of charge (after Peet, 1960).
Critical Radius.- The medium in the zone near the charge is assumed to behave as a compressible fluid as long as it is sufficiently compacted by high pressures, (Peet, 1960), that is, as long as the pressure $P_m$ of the shock wave exceeds the critical value $\sigma_m$. The critical radius "a", which separates the nonlinear from the linear phenomena, occurs when $P_m$ equals $\sigma_m$.

As was mentioned in the General Description of the problem, any variation in the energy initially supplied to the pulse by the explosive will affect the value of the critical radius, but will not affect significantly the energy in the pulse beyond the shattered zone. It is thus clear that the dimensions of this radius should be determined for representative media.

At the radius "a":

$$P_m = \sigma_m$$

then from equation (6),

$$P_m = \sigma_m = K_1 \left[ \frac{Q^{1/3}}{a} \right]^\alpha$$

now, solving for "a"

$$\text{Critical Radius} = a = \left[ \frac{K_1}{\sigma_m} \right]^{1/\alpha} Q^{1/3} C(\text{mts}) \quad - \quad (9)$$

(The computation of this expression for different media and different changes is given in APPENDIX I).

Now, from equation (7), the time constant of exponential decay at the radius "a" is:

$$\theta_1 = K_2 r_o \log \left[ \frac{a}{r_o} \right]$$
with the aid of equation (9) and the fact that $Q = \text{constant}$

$$\theta_1 = MQ^{1/3}, \text{for which } M = \text{constant} \quad (10)$$

At the critical radius, the shock wave pressure is then given by:

$$P(\tau) = U(\tau)\sigma_m e^{-\tau/\theta_1} \quad (11)$$
V. EQUATION OF MOTION AFTER THE SHOCK WAVE PHENOMENA

As mentioned in the preceding sections, beyond the critical radius, the shock wave has degenerated into an elastic pulse. It is necessary, therefore, to state the equations which express the motion beyond the shattered zone.

The equation of motion of a compressional wave in an homogeneous elastic body is given by:

\[
\frac{\partial^2 \hat{\phi}}{\partial t^2} = V_p^2 \nabla^2 \hat{\phi} \quad (12)
\]

where \(\hat{\phi}\) is the compressional displacement potential, and \(V_p\) is the P-wave velocity in the medium.

Suppose that the compressional wave is generated by the sudden occurrence of a symmetrical pressure with respect to C, given by \(-AU(t)\), within a sphere of radius "a" centered at C. Let \(r\) denote the distance from C, \(U(t)\) the Heaviside's unit function such that \(U(t)=0\) for \(t < 0\), \(U(t) = 1\) for \(t > 0\), and \(A\) a constant.

Then assume there is zero displacement at all points for \(t < 0\), and that the sphere \(r=a\) is acted from inside by a symmetrical pressure (Bullen, 1965).

The above considerations may be approximated by saying that the radius "a" of the sphere in which the symmetrical pressure is assumed to act, is in fact the "critical radius" which bounds the shattered zone caused by the symmetrical pressure produced by the explosion of a spherical charge.
In Eq. (12), $v^2 \phi$ can be expressed as:

$$v^2 \phi = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial \phi}{\partial r})$$

or

$$v^2 \phi = \frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r} \frac{\partial \phi}{\partial r}$$

then

$$v^2 \phi = \frac{1}{r} \frac{\partial^2 \phi}{\partial r^2} (r \phi)$$

So Eq. (12) becomes:

$$\frac{\partial^2 \phi}{\partial t^2} = \frac{v^2}{r} \frac{\partial^2 \phi}{\partial r^2} (r \phi)$$

or

$$\frac{1}{v^2} \frac{\partial^2 \phi}{\partial t^2} = \frac{\partial^2 \phi}{\partial r^2} (r \phi)$$

or

$$\frac{\partial^2 \phi}{\partial r^2} (r \phi) - \frac{1}{v^2} \frac{\partial^2 \phi}{\partial t^2} (r \phi) = 0$$

Solving this partial differential equation by the separation of variables method, the following result is obtained:

$$r \phi = Te^{-qr}$$

where $p$ and $q$ are operators defined as:

$$q = \frac{p}{v}$$

$$p = \frac{\partial}{\partial t}$$

and $T$ is a function of the time only (Jeffreys, 1931a).
Further, if $u$ is the radial displacement;

$$\dot{\psi} = \nabla \cdot u = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u)$$

or

$$\dot{\psi} = \frac{\partial u}{\partial r} + \frac{2u}{r} - - - - - - - - - - - (17)$$

That is:

$$r^2 \dot{\psi} = \frac{\partial u}{\partial r} r^2 + 2ur = \frac{\partial}{\partial r} (r^2 u)$$

or

$$r^2 u = - \int_{r}^{\infty} r^2 \dot{\psi} \, dr - - - - - - - - - (18)$$

the limits being fixed so that $u$ shall vanish at any value of $r$ until $\dot{\psi}$ has become different from zero.

Hence:

$$r^2 u = - \left( \frac{r}{q} + \frac{1}{q^2} \right) Te^{-qr} - - - - - - - (19)$$

Now, from the stress-strain relation equation,

$$P_{ij} = \lambda \dot{\varepsilon}_{ij} + 2\mu \epsilon_{ij} - - - - - - - (20)$$

The radial stress is given by:

$$P_{rr} = \lambda \dot{\varepsilon} + 2\mu \frac{\partial u}{\partial r}, \text{ but by Eq. (17) can be written as:}$$

$$P_{rr} = (\lambda + 2\mu) \dot{\varepsilon} - 4\mu \frac{u}{r}, \text{ and by Eq. (16) and Eq. (19), this can be expressed as:}$$

$$P_{rr} = \left[ (\lambda + 2\mu) + 4\mu \left( \frac{1}{qr} + \frac{1}{q^2 r^2} \right) \right] \frac{T}{r} e^{-qr} - - \quad (20a)$$
and when $r=a$, by assumption $P_{rr} = -AU(t)$, then equating (20A) and (19):

$$
2^ru = \frac{(qr+1)a^3 Ae^{-q(r-a)}}{\lambda+2\mu} \frac{2^u}{a^2 + 4\mu(qa+1)} U(t)
$$

or

$$
2^ru = \frac{a^3 A}{2\pi i} \int \frac{kr+1}{L} \exp\{\gamma t-k(r-a)\} \frac{dy}{y}
$$

(21)

where now $V_k = \gamma$, and the path of integration is a line from $-i\infty$ to $+i\infty$ on the positive side of the imaginary axis (Jeffreys, 1931b). Suppose that Poisson's relation $\lambda = \mu$ holds, then (21) becomes:

$$
2^ru = \frac{a^3 A}{2\pi i\mu} \int \frac{kr+1}{L} \exp\{\frac{\gamma t-\gamma}{\lambda+2\mu} \frac{kr+1}{k^2 a^2 + 4ka+4} e^{k(V t-r+a)} \frac{dk}{k}
$$

(22)

This is rigorously zero until $V t = r-a$, since by assumption $u=0$ for $t<0$; the integrand has poles at $k=0$ and $k= -\frac{2}{3}(1\pm i\sqrt{2})$.

Hence if $V t > r-a$, that is, outside the shattered zone, the radial displacement is given by:

$$
u_r = \frac{a^3 A}{4\mu r^2} [1+\exp(-\frac{2}{3} \frac{V t-r+a}{a}) \{ (\frac{r}{a} - \frac{1}{2}) 2\sqrt{2} \sin(\frac{2\sqrt{2}}{3} \frac{V t-r+a}{a})

-Cos(\frac{2}{3} 2\sqrt{2} \frac{V t-r+a}{a}) \}]
$$

(23)

The displacement at distance $r$ therefore starts at zero at time $(r-a)/V_p$ and then oscillates about a steady value, the amplitude of the oscillation dying down exponentially.
The sine factor vanishes for a second time after an interval of about $3.4a/V_p$; meanwhile the exponential factor has decreased to $e^{-2}$. (Jeffreys, 1931b; Bullen, 1965).
VI. EXPRESSION FOR FREQUENCY SPECTRA 
OUTSIDE THE CRITICAL RADIUS

Equation (23) expresses the displacement as a function of time, when the original shock wave has degenerated into an elastic pulse. Its frequency spectrum is given, to a constant factor, by the Fourier Transform of:

\[
\exp\left[-\frac{2}{3} \frac{V}{p} \frac{t-r+a}{a}\right] \left\{ \sin\left(\frac{2\sqrt{2}}{3} \frac{V}{p} \frac{t-r+a}{a}\right) - \cos\left(\frac{2\sqrt{2}}{3} \frac{V}{p} \frac{t-r+a}{a}\right) \right\}
\]

which can be written as:

\[
\exp\left[-\frac{2V}{3a} \frac{p}{p} \left(t+\frac{a-r}{V}\right)\right] \left\{ \sin\left(\frac{2\sqrt{2}}{3a} \frac{p}{p} \left(t+\frac{a-r}{V}\right)\right) - \cos\left(\frac{2\sqrt{2}}{3a} \frac{p}{p} \left(t+\frac{a-r}{V}\right)\right) \right\}
\]

knowing that the Fourier Transform of a sum, is the sum of the Fourier Transforms, Eq. (23A) can be split into the following expressions:

\[
\exp[-A(t+\frac{a-r}{V})] \sin B (t+\frac{a-r}{V})
\]

\[
\exp[-A(t+\frac{a-r}{V})] \cos B (t+\frac{a-r}{V})
\]

where

\[
A = \frac{2V}{3a}; \quad B = \frac{2\sqrt{2} V}{3a}; \quad B = \sqrt{2} A
\]

The initial conditions for the solution are:

\[
u_r = \begin{cases} 
0 & \text{for } t < 0, \ t \leq \frac{r-a}{V} \\
u_r(t) & \text{for } t > \frac{r-a}{V}
\end{cases}
\]

(26)
The Fourier Transform of Eq. (24) is:

\[
F(\omega) = \int_{-\infty}^{\infty} e^{-A(t+\frac{a-r}{P})} e^{-i\omega(t+\frac{a-r}{P})} \sin B(t+\frac{a-r}{P}) dt
\]

which can be written as:

\[
F(\omega) = \int_{-\infty}^{\infty} e^{-A(t+\frac{a-r}{P})} e^{-i\omega(t+\frac{a-r}{P})} \sin B(t+\frac{a-r}{P}) dt
\]

\[
+ \int_{-\infty}^{\infty} e^{-A(t+\frac{a-r}{P})} e^{-i\omega(t+\frac{a-r}{P})} \sin B(t+\frac{a-r}{P}) dt
\]

Because of the initial conditions, the first integral is zero, and the second one can be written in a more suitable form by a change of variable, \( T = (t+\frac{a-r}{P}) \), \( dT = dt \), and changing the lower limit. Then, the Fourier Transform of Eq. (24) becomes:

\[
F(\omega) = \int_{0}^{\infty} e^{-AT} e^{-i\omega T} \sin BT \, dT - - - - - - (27)
\]

Since

\[ e^{-i\omega T} = \cos \omega T - i \sin \omega T \]

then:

\[
F(\omega) = \int_{0}^{\infty} e^{-AT} \cos \omega T \sin BT \, dT - i \int_{0}^{\infty} e^{-AT} \sin \omega T \sin BT \, dT
\]
Performing the integration, the following expression is obtained:

\[ F(w) = \left\{ \begin{array}{c}
\frac{e^{-AT}}{2[A^2 + (B-w)^2]} \left[ -A \sin(B-w)T - (B-w) \cos(B-w)T \right] \\
+ \frac{e^{-AT}}{2[A^2 + (B+w)^2]} \left[ -A \sin(B+w)T - (B+w) \cos(B+w)T \right] \\
- \frac{i}{2[A^2 + (B-w)^2]} \left[ (B-w) \sin(B-w)T - A \cos(B-w)T \right] \\
- \frac{i}{2[A^2 + (B+w)^2]} \left[ (B+w) \sin(B+w)T - A \cos(B+w)T \right] \right\} \]

Taking the limits of integration, this changes to:

\[ F(w) = \left\{ \begin{array}{c}
\frac{-(B-w)}{2[A^2 + (B-w)^2]} - \frac{(B+w)}{2[A^2 + (B+w)^2]} \\
- \frac{i}{2[A^2 + (B-w)^2]} + \frac{A}{2[A^2 + (B+w)^2]} \right\} \]

which can be expressed as:

\[ F(w) = \frac{B[(A^2 + B^2 - w^2) - 2iAw]}{[A^2 + (B-w)^2][A^2 + (B+w)^2]} \] \quad (28)

The Fourier Transform of expression (25) is:

\[ F(w) = \int_{-\infty}^{\infty} e^{-A(t+\frac{a-r}{v})} e^{-i\omega(t+\frac{a-r}{v})} \cos B(t+\frac{a-r}{v}) dt \]

which yields in a similar manner:

\[ F(w) = \int_{0}^{\infty} e^{-AT} e^{-i\omega T} \cos BT dT \] \quad (29)
or:

\[ F(w) = \int_{0}^{\infty} e^{-AT} \cos \omega T \cos B T \, dT - i \int_{0}^{\infty} e^{-AT} \sin \omega T \cos B T \, dT \]

Integrating, the following expression is obtained:

\[
F(w) = \left\{ \frac{e^{-AT} [(w-B) \sin (w-B) T - A \cos (w-B) T]}{2[A^2 + (w-B)^2]} \right. \\
+ \frac{e^{-AT} [(w+B) \sin (w+B) T - A \cos (w+B) T]}{2[A^2 + (w+B)^2]} \\
- \frac{i[e^{-AT} [-A \sin (w-B) T - (w-B) \cos (w-B) T]}{2[A^2 + (w-B)^2]} \right. \\
+ \frac{e^{-AT} [-A \sin (w+B) T - (w+B) \cos (w+B) T]}{2[A^2 + (w+B)^2]} \bigg\} \bigg|_{0}^{\infty}
\]

Taking limits, this changes to:

\[
F(w) = - \left\{ \frac{-A}{2[A^2 + (w-B)^2]} + \frac{-A}{2[A^2 + (w+B)^2]} \\
- \frac{i[-(w-B)]}{2[A^2 + (w-B)^2]} + \frac{i[-(w+B)]}{2[A^2 + (w+B)^2]} \right\}
\]

which can be expressed as:

\[
F(w) = \frac{A(A^2 + B^2 + w^2) - i w(A^2 + w^2 - B^2)}{[A^2 + (w-B)^2] [A^2 + (w+B)^2]} - \quad - \quad - \quad - \quad - \quad (30)
\]

Thus the Fourier Transform of expression (23A) is:

\[
F(w) = \frac{(B[A^2 + B^2 - w^2] - A[A^2 + B^2 + w^2]) - i(2ABw - w[A^2 + w^2 - B^2])}{[A^2 + (w-B)^2] [A^2 + (w+B)^2]} \quad (31)
\]
and the amplitude frequency spectrum is given by:

\[
|F(\omega)| = \frac{\sqrt{\left[B^2 + B^2 - \omega^2\right] - A(\omega^2 + B^2) + 2^2\left[2AB - (A^2 + \omega^2 - B^2)\right]^2}}{[\omega^2 + (\omega - B^2)] [\omega^2 + (\omega + B)^2]}
\]

\[\text{(32)}\]

This expression then, establishes a relationship between the size of the shattered zone produced by an explosion, and the frequency spectrum of the resulting elastic signal.

The graphs obtained from this expression, are found in APPENDIX II.
VII. DISCUSSION OF RESULTS

Variation of the Critical Radius upon the explosive charge.- The critical radius was computed by Eq. 9 for seven different media (see APPENDIX I).

The characteristics given in the following discussion can be observed in the case presented in Fig. 5, which delineates the manner in which the critical radius behaves in all cases treated.

The behavior of the Critical Radius as the explosive charge increases, is represented by a curve with a rather abrupt rise in the first part, and then a slower increase as the charge goes to higher values.

It was also found that the relation, explosive charge to critical radius value, follows closely the Principle of Similarity as expressed by the relation:

\[ R_p = p R_o \]

\[ (33) \]

where \( p \) is the ratio by which the linear size of the originally given charge is increased or reduced, \( R_p \) is the critical radius obtained with the altered charge, and \( R_o \) is the critical radius produced by the originally given charge; i.e., if the linear size of a given charge is reduced by half, which corresponds to a reduction of the weight to 1/8, then the critical radius obtained will also be reduced by half.
Fig. 5. Representative Critical Radius curve.
In choosing the explosive charge values to be used in the computation of the frequency spectra, particular attention was given to the region of the critical radius curves at which the maximum reduction of the shattered zone is obtained with the minimum reduction in charge weight, this zone certainly occurs at the steepest region of the critical radius curves.

This was taken into consideration since it is within the nonlinear zone where most of the energy produced by the explosion is dissipated (Howell and Budestein, 1955), and having a reduced shattered zone in relation with a small variation of the charge, a bigger percentage of energy will be transmitted into the elastic medium. As a consequence, the values from 5 to 30 lbs. were chosen to compute the frequency spectra.

The results of the calculation of the frequency spectra are shown in APPENDIX II, and will be discussed in the following section.

Variation of the Frequency Spectra upon the Critical Radius and Velocity of the media.— The frequency spectra of the elastic signal outside the critical radius for seven different media, computed according to Eq. (32), are shown in APPENDIX II. Typical examples are given in figures 6 and 7.
Fig. 6. Representative Frequency Spectrum.
As can be seen from Eq. 25A and Eq. 32, the determining factors in the shape of the spectra are the critical radius and the P-wave velocity of the media. It is known from the preceding section that the critical radius is determined by the amount of explosive charge, and the medium, therefore, for a given medium, the determining factors of the spectra are: Explosive Charge and Velocity.

In Figures 6 and 7 it can be observed that the spectra begins with a high value, which soon reaches a maximum, then decreases to a minimum, and rises again to a second maximum; from there it decreases rather smoothly.

The celerity with which the spectrum reaches the first maximum, the minimum and the second maximum, is in an inverse relation with the hardness of the medium and in a direct relationship with the value of the explosive charge.

It was found in all cases, in which the frequency interval chosen made it possible to measure, that the second maximum has a value of 47% of the value of the first maximum.

It can also be observed that in each medium the spectrum charges its shape as a function of the explosive charge. In the frequency range chosen, small charges give a relatively flat spectrum with the first maximum at a rather
high frequency, whereas for larger charges that maximum
shifts to lower frequencies.

The spectra in Figures 8 and 9, represent typical
cases of the results obtained in APPENDIX IIB, here again,
the shape of the frequency spectra is similar to the one
of the variable charge spectra.

In this case, the celerity with which the spectrum
reaches the first maximum, the minimum, and the second
maximum, is in an inverse relation with the hardness and
the velocity of the medium.

It is also observed that in each medium, the spectrum
changes its shape as a function of the velocity: high
velocities give a relatively flat spectrum with the first
maximum at a rather high frequency, whereas for smaller
velocities that maximum shifts to lower frequencies.

Influence of the Shock Wave on the Frequency Spectra out¬
side the Critical Radius.— In order to know whether or not
the duration of the shock wave influences the frequency
spectrum outside the critical radius, a multiplication of
the frequency spectra of the explosion wave and the elastic
wave was made, which is equivalent to the convolution of
both waves in the time domain.

As can be inferred from the expression for the shock
Fig. 8. Representative Frequency Spectrum.
Fig. 9. Representative Frequency Spectrum.
wave \( P(t) = U(t) P_m e^{-t/\theta} \), the crucial factor that determines the influence that the wave will have in a convolution process, is the time constant of exponential decay \( \theta \), which as stated before can be approximated by: \( \theta = K r_o \log(r/r_o) \), or \( \theta/r_o = K \log(R) \), where \( K \) is a constant involving the properties of the explosive and the medium in which the shot is fired, and \( R \) represents the ratio of the distance at which \( \theta \) is to be known to the charge radius; in this case the distance "r" equals the critical radius.

The given expression for \( \theta \), derived from the Kirkwood-Bethe shock wave propagation theory, is fairly accurate for distances between 10 and 100 charge radii, which is the interval in which the critical radius value ranges. From calculations made for 30 different explosive compositions by Kirkwood, Brinkley, and Richardson in 1943, (Cole, 1948), the values of \( \theta \) for distances between 1 to 100 charge radii are of the order of \( 10^{-6} \) sec. It was found from the Fourier Transform of the shock wave expression, which is:

\[
|F(\omega)| = (\theta^2 + \omega^2)^{-1} \left[ 1/\theta^2 + \omega^2 \right]^{1/2},
\]

that in order for the shock wave spectrum to have a significant influence on the elastic signal spectra, the minimum value of \( \theta \) had to be of the order of \( 10^{-4} \) sec.; if this is not the case, the decay of the shock wave spectrum would be so small, that no appreciable influence will be noticed even over a range of 500 cps.
So for the particular explosive and media considered in this work, the value of $\theta$ is so small that no change in the frequency spectra's shape of the elastic signal was noticed; in the case of hard media and small explosive charge, however, it was observed that the relative amplitude of the spectra was reduced by 15 percent, even though the shape of the spectra was unchanged.
VIII. CONCLUSIONS

The following conclusions can be reached:

Around the shot hole there exists a region in which nonlinear phenomena take place, and the volume of this region is proportional to the volume of the charge. Within this region, the Principle of Similarity holds true for different values of the shock wave pressures produced by different weights of explosive charges.

This region is bounded by the so-called Critical Radius, which occurs when the shock wave pressure caused by the explosive decreases in value until it equals the yield strength of the medium.

For an ideally elastic medium, the shape of the frequency spectrum of the signal outside the shattered zone, changes as a function of both the critical radius and the P-wave velocity of the medium.

Considering the results given in APPENDICES I and II, and since most of the energy produced by the explosion is dissipated within the nonlinear region, it would constitute an interesting problem, to find out the minimum critical radius value produced by the adequate explosive charge in a given medium, such that, producing a minimum shattered zone value, a larger percentage of energy would be transmitted to the elastic medium in order to improve the
signal that reaches the recording devices used in oil prospecting. This should be done keeping in mind the effects that a larger or a smaller charge has on the frequency spectrum of the signal, and considering the value of the particular frequency that is intended to improve or destroy. In such a problem, since cylindrical charges are used in reality, the departure from spherical symmetry should be taken into account.
IX. BIBLIOGRAPHY


APPENDIX I

COMPUTATION OF THE SIZE OF THE SHATTERED ZONE FOR SOME MEDIA, WITH CHARGE VARIATION FROM 1 TO 100 LBS.

In computing the Critical Radius in meters equation 9 was used. The parameter $\sigma_m = \text{yield stress}$ on equation 9, is defined as the stress at the knee of the stress-strain curve for rocks under high confining pressure, and it is usually indefinite owing to the lack of a marked break in the curve.

According to Brace (1960), "... the average character of the deformed rock under indentation appears to be similar to that deformed in compression tests at moderate to low confining pressure and room temperature. From the agreement between indentation hardness and compressive strength it is concluded that hardness has the same meaning as one point on the stress-strain curve in compression and, as such measures the compressive strength of the rocks..."

Since the media are considered at zero confining pressure, and the explosion effect is more likely to be as the one produced by an indentation rather than by a compression, in Eq. 9 the ultimate strength of the medium will be used instead of the yield stress, in the knowledge that in this form a fair approximation will be obtained.
The ultimate strength values at zero confining pressure for the media used, were taken from Handin (1956). The following media were used:

<table>
<thead>
<tr>
<th>Medium and Location</th>
<th>Ultimate Strength in lbs/in^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limestone &quot;Austin Chalk&quot; Texas</td>
<td>462</td>
</tr>
<tr>
<td>Limestone &quot;Danby Marble&quot; Vermont</td>
<td>1418</td>
</tr>
<tr>
<td>Sandstone (70% sand, 30% carbonate silica cement, and water saturated) Alberta</td>
<td>1617</td>
</tr>
<tr>
<td>Dolomite Gloriete, New Mexico</td>
<td>1666</td>
</tr>
<tr>
<td>Dolomite Luning, Nevada</td>
<td>1980</td>
</tr>
<tr>
<td>Limestone &quot;Chico&quot; Texas</td>
<td>2545</td>
</tr>
<tr>
<td>Dolomite &quot;Fusselman&quot; Texas</td>
<td>4884</td>
</tr>
</tbody>
</table>

The type of explosive chosen was PENTOLITE (50% PETN, 50% TNT) with density of 1.60; and the range of variation of the explosive charge value was selected from 1 to 100 lbs. in order to have a better understanding of the behavior of the shattered zone, and to be able to determine the most appropriate charges to be used in the computation of the frequency spectra.

The program used in the computation of the critical radius and the results obtained are shown in the following listing and graphs.
FILE 5 = ANSA, UNIT = READER
FILE 6 = PRINT, UNIT = PRINT

C COMPUTATION OF THE CRITICAL RADIUS FOR VARIABLE CHARGE.
C MADE IN 7 DIFFERENT MEDIA AND CHARGE VARIATION FROM 1 TO 100 LBS.

START OF SEGMENT ********** 1

DIMENSION RC(100), TIT(10), NA(100)
READ (5,100) CHAR, NBLA
READ (5,102) ISIGM, TIT

C RC(J) IS THE COMPUTED CRITICAL RADIUS IN METERS.

DO 1 K=1,7
READ (5,101) VK, ALFA
DO 2 J=1,100
VQ=J/(1./3.)*C
VF=(VK/ISIGM)**VA
2 R(J)=VF*VQ
C RC(J) IS THE COMPUTED CRITICAL RADIUS IN METERS.
WRITE (6,104)

100 FORMAT (2AI)
101 FORMAT (3F10.0)
200 FORMAT (50/50 PETN-TNT "#17#
300 FORMAT (95X,25H EXPLOSIVE CHARGE IN LBS.)

DO 3 I=1,100
DO 5 J=1,100
5 NA(J)=NBLA
DO 6 L=1,100
6 CONTINUE
IF (DIFE)=1.6263
IF (DIFE)=DIF/2.1662.62
62 NA(L)=CHAR
6 CONTINUE
WRITE (6,120) VRAD, CHN(M), M=1,100
110 FORMAT (1H,6F15.7)
4 VRAD=VRAD+DIF
WRITE (4,120) M=5,100,5
120 FORMAT (50X,20I3,12I2)
WRITE (4,300)
300 FORMAT (95X,25H EXPLOSIVE CHARGE IN LBS.)
1 CONTINUE
STOP
END
50/50 PETN-TNT  
CRITICAL RADIUS IN MTS.

1418 LIMESTONE DANNY MARBLE

16.333391  
16.0718792  
15.8103593  
15.5488394  
15.2873195  
15.0257996  
14.7642797  
14.5027598  
14.2412399  
13.9797200  
13.7182001  
13.4568802  
13.1951603  
12.9336404  
12.6721205  
12.4106006  
12.1490807  
11.8878609  
11.6260410  
11.3645211  
11.1030112  
10.8414813  
10.5796614  
10.3184415  
10.0569216  
9.7954017  
9.5338818  
9.2723619  
9.0108420  
8.7498221  
8.4878022  
8.2262823  
7.9647624  
7.7032425  
7.4417226  
7.1802027  
6.9186828  
6.6571629  
6.3956430  
6.1341231  
5.8726032  
5.6110833  
5.3495634  
5.0880435  
4.8265236  
4.5650037  
4.3034839  
4.0419640  
3.7804441  
3.5189242

EXPLOSIVE CHARGE IN LBS.
50/50 PETN=TNT 1666  DOLOMITE GLORIETE NEW MEXICO
CRITICAL RADIUS in METERS
14.1622013  
13.9354452  
13.7086892  
13.4819331  
13.2551770  
13.0284209  
12.8016649  
12.5749088  
12.3481527  
12.1213967  
11.8946406  
11.6678845  
11.4411284  
11.2143724  
10.9876163  
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10.5341041  
10.3073481  
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4.6384663  
4.4116902  
4.115941  
3.9581781  
3.7314220  
3.5046659  
3.2779098  
3.0511538  
5...10...15...20...25...30...35...40...45...50...55...60...65...70...75...80...85...90...95...***
EXPLOSIVE CHARGE IN LBS.
EXPLOSIVE CHARGE IN LBS.

50/50 PETN=TNT
CRITICAL RADIUS IN MTS.

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**EXPLOSIVE CHARGE IN LBS**
APPENDIX II

COMPUTATION OF THE FREQUENCY SPECTRA FOR SOME MEDIA, WITH VARIABLE CHARGE AND DIFFERENT VELOCITIES.

The frequency spectra of the elastic signal in each medium were computed according to equation 32.

A.- In the first part of this APPENDIX, spectra for several values of explosive charge in each medium were computed, the values for the explosive charge were taken according to the results of the calculations of the critical radius, the frequency range was chosen to be from 1 to 500 c.p.s., and a fixed P-wave velocity was taken for each medium. The results are presented in the form of graphs, and in each medium the spectra are compared to each other by means of a normalized amplitude axis, and are placed in the plot in such a manner that, the value of the explosive charge used in the calculation of each spectrum is decreasing downward.

The numbers appearing in the upper right hand corner of the plots, have the following meaning from left to right: the first digit represents how many spectra are computed, that is, the number of different charges that are used for that particular medium, the second number is the P-wave velocity of the medium in Km/sec., the following
number represents the ultimate strength of the medium in lbs/in$^2$, the next is the explosive charge that corresponds to the one used in the calculation of the first spectrum, and the last number represents the decrement to the initial charge value, which is to be done as many times as necessary to satisfy the first number.

The program used in the computation of the spectra and the results obtained are shown in the following listings and graphs.
```
**FILE** 5, NAME, UNIT = READ

**C** THIS PROGRAM COMPUTES THE FREQUENCY SPECTRUM OF THE SIGNAL

**C** ROUTINE TO THE RADIAL WAVE WHICH SEPARATES THE

**C** NON-LINEAR DYNAMICS IN THE LINEAR PHENOMENA, IN AN EXPLOSION.

**DIMENSION** X, Y(5,101), F(2,100), T(111), N(1)

**READ** (5,101) X

**FORMAT** (4,3,4,4)

**100 FORMAT** (1,10,10,1)

**AMX** 1,5/6,

**IF** (R1,2,3) **OR** 1

**2 FORMAT** (5,101) T

**101 FORMAT** (3,4,4)

**N** 1 [x, y, z] = 3

**READ**

**N** 2 [x, y, z] = 3

**AMX** 1,5/6,

**A** 1,2,3/6,

**B** 1,2,3/6,

**C** AICOMPUTES THE CRITICAL WAVE 14 I = 2, 3, 4, 5,

**AMX** 1,2,3/6,

**B** 1,2,3/6,

**READ**

**N** 1 [x, y, z] = 3

**FORMAT** (1,10,10,1)

**F** = 1,2,3,4,5,6,7,8,9

**G** = 1,2,3,4,5,6,7,8,9

**H** = 1,2,3,4,5,6,7,8,9

**IF** (1,2,3) **OR** 1

**C** F(3) IS THE FREQUENCY SPECTRUM OF THE SIGNAL

**C** ROUTINE TO THE RADIAL RADIUS.

**5** READS,

**IF** (1,2,3) **OR** 1

**AMX**

**READ**

**AMX** 1,2,3/6,

**G** = 1,2,3/6,

**H** = 1,2,3/6,

**READ**

**IF** (1,2,3) **OR** 1

**C** READS

**16** READS

**IF** (1,2,3) **OR** 1

**C** READS

**IF** (1,2,3) **OR** 1

**C** READS

**IF** (1,2,3) **OR** 1

**C** READS

**IF** (1,2,3) **OR** 1
```

---

This appears to be a Fortran program snippet that computes the frequency spectrum of a signal. The code includes variable declarations, data input, and calculations for computing the spectrum. The `READ` statements are used to input data, and the `IF` statements are used to control the execution flow based on conditions. The program seems to be part of a larger scientific or engineering application.
200 FORMAT (12H RELATIVE AMPLITUDE, 10X, 1AH VARIABLE CHARGE) 
   AMF=1,
   DD 10 INT50
   PRINT 101 AMF=(%11.11fM3199)
   103 FORMAT (1H AF12,3X)
   14 AMFWV=AMI4
   PRINT 112 AMW=(%10.400,50)
   104 FORMAT (14X,$10.10CTR*1353
   PRINT 100
   300 FORMAT (G7X,17H FREQUENCY IN CPS)
   GO TO 15
1 CONTINUE
STOP
END

R 0210
R 0210
R 0210
R 0210
R 0216
R 0219
R 0239
R 0240
R 0240
R 0240
R 0257
R 0257
R 0260
R 0261
R 0261
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</table>

FREQUENCY IN CPS
B. In the second part of this APPENDIX, spectra for several values of P-wave velocity were computed, the range of values for the P-wave velocity in each medium was taken from "Hand Book of Physical Constants" (1966), the frequency range was chosen to be from 1 to 500 cps. and a fixed explosive charge value was taken for each medium. As in the first part of this APPENDIX the results are presented in the form of graphs, and in each medium the spectra are compared to each other by means of a normalized relative amplitude axis, and are placed in the plot in such a manner that, the value of the P-wave velocity used in the calculation of each spectrum, increases downward.

The numbers appearing in the upper right hand side of the plot, have the following meaning from left to right: the first digit represents the number of spectra computed, that is, the number of different velocity values used in that particular medium, the second number is the fixed explosive charge value in pounds, used in the calculation of the spectrum, the following number represents the ultimate strength of the medium in lbs/in², the next is the velocity in Km/sec. that corresponds to the one used in the calculation of the first spectrum, and the last number represents the increment to the first velocity
value, which is to be done as many times as necessary to satisfy the first number.

The program used in the computation of the spectra and the results obtained are shown in the following listings and graphs.

All calculations of APPENDICES I and II, were made using a Burroughs 5500 computer at Rice University.
FILE 5 = ANAM UNIT = READER

THIS PROGRAM COMPUTES THE FREQUENCY SPECTRUM OF THE SIGNAL OUTSIDE THE CRITICAL RADIUS WHICH SEPARATES THE NON LINEAR FROM THE LINEAR PHENOMENA IN AN EXPLOSION.

START OF SEGMENT ***********

DIMENSION NA(50,100),FW(100),TIT(13),MN(6)

READ (9) MN
105 FORMAT (6A1)
15 READ (5,100) NA(I),VF,SI,VF,VV
100 FORMAT (15,F10.0)
00 AMIA=1./49.
00 IF (N.EQ.0) GO TO 1
2 READ (9) TIT
101 FORMAT (13A6)
DO 3 I=1,50
3 NA(I,J)=TIT(I)
Q=QF
VF=VF
DO 6 I=1,N
A2=1./1.13
A3=0.**11./3.)
A4=(A1/SI)**A2
A6=A6*A3*0.3
C A6=COMPUTED CRITICAL RADIUS IN METERS.
A*(2.*VF*10.**3.)/(3.*A6)
R= A2**.0.5
W0=。
DO 5 J=1,100
F0=1.J=(A6*A6+(W-B)**2.)/(A6*A6+(W-B)**2.))
F02=(B*AMAX1CAMA*FWCU))
5 AMAX=AMAX1CAMA
F=FWCU)*FW(J)/AMA
VMAX=VMAX-AMIA
DO 9 K=1,100
DIFsVMAX-FWCU)
IF (DIF+AMIA/2.)
10 CONTINUE
9 VMAX=VMAX-AMIA
4 VF=VF+VV
PRINT 102*TIT,NA,VF,SI,VF,VV
102 FORMAT (1H1,13A6,*5*4F8,2)
PRINT 200
200 FORMAT(19H RELATIVE AMPLITUDE*10x,18H VARIABLE VELOCITY)
     AME=1,
     GO 14 I=1,50
     PRINT 103 AME=(N(A(I,J)=J=1,100)
103 FORMAT (1H *F10.3*4H ,*100A1)
14 AME=AME-AME
     PRINT 104*(I=1=50*500*50)
104 FORMAT (14X*10(F7.4,A3))
     PRINT 300
300 FORMAT(97X,17H FREQUENCY IN CPS)
     GO TO 15
1 CONTINUE
    STOP
    END

SEGMENT 1 IS 283 LONG
LIMESTONE CHICAGO, ILL. (90% CHLORITE, 10% CACITE, 20% QUARTZ)
RELATIVE MOTTLE

FREQUENCY

5 25.00 500.00 2.80 0.90

FREQUENCY IN CPS