Nuclear Spin Relaxation of He$^3$ in Liquid He$^3$ - He$^4$ Mixtures

by

Ephrem Philip Horvitz

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF ARTS

Thesis Director's signature:

Houston, Texas
May, 1965
ABSTRACT

The spin-lattice relaxation time, $T_1$, and the spin-spin relaxation time, $T_2$, of $^{3}$He nuclei in liquid mixtures of $^{3}$He and $^{4}$He at 5%, 14% and 35% $^{3}$He concentrations have been measured. $T_1$ and $T_2$ values were also obtained for pure $^{3}$He. The technique of adiabatic fast passage was employed to measure relative magnitudes of the magnetization.

The results show that $T_1$ and $T_2$ tend to decrease as the temperature decreases below the lambda point of the mixtures. They show also that $T_1$ and $T_2$ decrease as the concentration of $^{3}$He decreases. The latter implies that impurity effects become more important the lower the $^{3}$He concentration is. A model based on bulk and wall impurities was used to explain the results. This model was also used to compute the diffusion coefficient for $^{3}$He in the three mixtures. The calculations show that the diffusion coefficient increases with decreasing $^{3}$He concentration and with decreasing temperature. This implies that the superfluid component of the $^{4}$He behaves as if it were a vacuum.

Khalatnikov and Zharkov have calculated the diffusion coefficient of $^{3}$He in HeII on the basis of the Landau theory of superfluidity. The experimentally determined diffusion coefficients are in good agreement with this theory.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. THEORY OF MEASUREMENT</td>
<td>4</td>
</tr>
<tr>
<td>A. Adiabatic Fast Passage</td>
<td>4</td>
</tr>
<tr>
<td>B. Bloch Equations</td>
<td>8</td>
</tr>
<tr>
<td>III. EXPERIMENTAL APPARATUS</td>
<td>11</td>
</tr>
<tr>
<td>A. Gas Handling System</td>
<td>11</td>
</tr>
<tr>
<td>B. Nuclear Resonance Equipment</td>
<td>12</td>
</tr>
<tr>
<td>C. Cryogenics</td>
<td>13</td>
</tr>
<tr>
<td>IV. EXPERIMENTAL PROCEDURE</td>
<td>14</td>
</tr>
<tr>
<td>V. EXPERIMENTAL RESULTS</td>
<td>18</td>
</tr>
<tr>
<td>VI. DISCUSSION AND CONCLUSIONS</td>
<td>20</td>
</tr>
<tr>
<td>A. Spin-Lattice Relaxation</td>
<td>20</td>
</tr>
<tr>
<td>B. The Experimental Diffusion Coefficients</td>
<td>25</td>
</tr>
<tr>
<td>C. Theory of Khalatnikov and Zharkov</td>
<td>30</td>
</tr>
<tr>
<td>D. Spin-Spin Relaxation</td>
<td>33</td>
</tr>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>35</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>36</td>
</tr>
</tbody>
</table>
I. INTRODUCTION

The Bloembergen, Purcell, and Pound\(^2\) theory of nuclear relaxation in liquids is based on an independent particle model. The theory describes successfully the nuclear relaxation of liquid He\(^3\) above 1°K. There is not enough published data below 1°K to determine whether it is successful in that region.

Figure 1 is a plot of some of the spin-lattice relaxation \(T_1\) data in pure liquid He\(^3\) above 1°K. The most outstanding feature is the very large variation in the observed values of \(T_1\). Rorschach\(^4\) has shown that above 1°K, the data is well described by the BPP theory if one considers the net inverse relaxation time to be the sum of an inverse wall relaxation time and an inverse bulk relaxation time.

\[
\frac{1}{T_1} = \frac{1}{T_{1\text{wall}}} + \frac{1}{T_{1\text{bulk}}} = C_1 D + \frac{C_2 \rho}{D}, \tag{1}
\]

where \(C_1\) and \(C_2\) are constants, \(D\) is the diffusion coefficient and \(\rho\) is the density of He\(^3\). If there are no impurities or wall effects present, the relaxation time will be the true bulk time, and according to the BPP theory for short correlation times it should be given by a term of the form

\[
\frac{1}{T_{1\text{bulk}}} \propto \frac{\rho}{D}. \tag{1}
\]

Any relaxation at the wall will reduce \(T_1\), and if the walls are black for relaxation, \(T_1\) will depend on the diffusion time to the wall. This mechanism will give a contribution \(\frac{1}{T_{1\text{wall}}} \propto D\). A more detailed analysis\(^4\) shows that
Impurities in suspension in the liquid can affect the relaxation time in two ways. In one case a cluster of impurities can be almost stationary, thereby having the same effect as an impurity at the wall. In the case that the impurity diffusion coefficient is nearly equal to that of the He\(^3\), a bulk relaxation similar to that caused by paramagnetic ions in solution in ordinary liquids may then occur.

Romer\(^3\), using a Pyrex chamber, obtained a relaxation time closest to the true value for pure He\(^3\) for \(T > 2.3^\circ K\). Low and Rorschach\(^4\), using a nylon chamber, obtained the longest relaxation times for \(1.0^\circ K < T < 2.3^\circ K\).

The BPP theory predicts that for a liquid, the spin-spin relaxation time (\(T_2\)) should be equal to \(T_1\). Walters and Low\(^7\) measured \(T_1\) and \(T_2\) for pure He\(^3\) in a Pyrex bulb at 1.3\(^\circ\)K. They found \(T_2 \approx 200\) sec., a value slightly shorter than \(T_1\). Low and Rorschach\(^4\) have measured \(T_2\) for pure He\(^3\) in a nylon chamber and found it to be much shorter than and independent of the value of \(T_1\). They consider these short times to be due to impurities in suspension.

The initial purpose of the research described in this thesis was to measure \(T_1\) and \(T_2\) of He\(^3\) nuclei in liquid He\(^3\) and in mixtures of He\(^3\) and He\(^4\). However, the true times were not obtained as the relaxation was dominated by impurities. Nevertheless, the \(T_1\) results were used to compute the dif-
fusion coefficient of $\text{He}^3$ nuclei in three mixtures. The coefficients for two mixtures compare favorably with a theoretical estimate.
II. THEORY OF MEASUREMENT

Adiabatic fast passage is the theory underlying the measurement of the relative values of the magnetization. In such a case the spin system behaves as if it were non-interacting. The physical meaning of "adiabatic" and "fast" is presented with a quantitative criterion giving the limits for which the adiabatic fast passage solutions are valid.

The heuristic arguments that are the basis of the Bloch equations are presented. These equations are transformed to a rotating coordinate system and two special solutions are obtained. These solutions form the theory of the measurement of $T_1$ and $T_2$.

A. Adiabatic Fast Passage

Consider a system of non-interacting spins with magnetic moment, $\mu$, in the presence of an external magnetic field $\vec{H}$. The expectation value of a quantum mechanical operator obeys the same equation of motion as does the corresponding classical variable. Therefore,

$$\frac{d\langle \vec{\mu} \rangle}{dt} = \gamma (\langle \vec{\mu} \rangle \times \vec{H})$$

where $\langle \rangle$ denotes the expectation value.

The independent magnetic moments can be summed over a unit volume to define a total magnetic moment, $\langle \vec{M} \rangle = \sum \langle \vec{\mu} \rangle$. 
Then \[ \frac{d}{dt} \langle \mathbf{M} \rangle = \gamma (\langle \mathbf{M} \rangle \times \mathbf{H}). \] (2)

From now on we drop the use of the \( \langle \cdot \rangle \) and always understand \( \bar{M} \) to mean the expectation value of the magnetization.

In this experiment we are interested in the motion of \( \bar{M} \) in a large external static field \( \mathbf{H} = H_0 \hat{k} \) on the order of 7,000 gauss and a small oscillating field \( 2H_1(t) = 2H_1 \cos \omega t \hat{i} \), whose amplitude \( 2H_1 \) is on the order of one gauss. The oscillating field can be decomposed into two fields, each of magnitude \( H_1 \) rotating in opposite senses with frequency \( \omega \). Only the component rotating in the same sense as the precession of \( \bar{M} \) has an appreciable effect. The effect of the other component, which we neglect, is to modify slightly the resonant frequency.

The solution of this problem is simplified if it is carried out in a coordinate system rotating around the z axis, \( \hat{k} \). The transformation from the laboratory to the rotating system of the time rate of change of the vector \( \bar{M} \) is \( \frac{d\bar{M}}{dt}_R = \frac{d\bar{M}}{dt}_L + \bar{\omega} \times \bar{M} \), where \( \frac{d\bar{M}}{dt}_R \) and \( \frac{d\bar{M}}{dt}_L \) are the time rate of change of the magnetization in the rotating and laboratory system respectively.

In the rotating frame the effective component of the oscillating field becomes a static field of magnitude \( H_1 \) and points along the x-axis. Then

\[ \frac{d\bar{M}}{dt}_R = \gamma \bar{M} \times (H_0 \hat{k} + H_1 \hat{i}) + \gamma \bar{M} \times \bar{\omega} = \gamma \bar{M} \times \mathbf{H}_{\text{eff}}, \]
where $\vec{H}_{\text{eff}}$ is the effective magnetic field in the rotating frame and is equal to $H_0 \hat{k} + H_1 \hat{\gamma} + \frac{\omega}{\gamma} \hat{\varphi}$. In the rotating frame the magnetization, $\vec{M}$, precesses about the effective field, $\vec{H}_{\text{eff}}$.

From equation (2) we see that $\frac{d}{dt} (M^2) = 2 \vec{M} \cdot \frac{d\vec{M}}{dt} = 0$, and therefore the magnitude of $\vec{M}$ is constant regardless of the time variation of $\vec{H}$.

We now make use of the adiabatic theorem which states that if the time variation of $\vec{H}$ is slow enough the angle between the magnetization and the effective magnetic field is preserved in the rotating coordinate system. The adiabatic condition can be expressed as follows:

$$\frac{dH_0}{dt} \ll \frac{\gamma H_{\text{eff}}^2}{\sin \theta}$$

where $\tan \theta = \frac{\gamma H_1}{\gamma H_0 + \omega}$. The condition is strictest at resonance when $\sin \theta = 1$,

$$\frac{dH_0}{dt} \text{ (resonance)} \ll \frac{\gamma H_{\text{eff}}^2}{\gamma H_0 + \omega}.$$  

If this requirement is satisfied and one starts with $H_0$ far above resonance and goes to far below resonance, the magnetization, initially aligned along $H_0$, will, at the end of the passage, point in the opposite direction. On its way the magnetization will always follow $H_{\text{eff}}$. Then, at resonance, $\vec{M}$ will have only a transverse component of magnitude $M_o$. 
Let us now imagine that the spin system is not isolated but can interact with itself and the lattice. Then the equations of motion for the magnetization must include the effects of the spin-lattice and spin-spin interaction. However, we may neglect these interactions in the special case that we pass through resonance fast enough so that the spin-lattice and spin-spin interactions are negligible. In such a case the solution above for the non-interacting spin system is sufficient. We call this case "Adiabatic Fast Passage."

We now give a quantitative criterion for "fast enough." The interaction of the spins with the lattice are characterized by $T_1$, where $T_1$ is the relaxation time describing the growth of $M_z$ from zero to $M_0$.

$$\frac{dM_z}{dt} = -\frac{(M_z-M_0)}{T_1},$$

and $T_2$ is the relaxation time describing the exponential decay of the transverse component of the magnetization due to spin-spin interactions,

$$\frac{dM_{x,y}}{dt} = -\frac{M_{x,y}}{T_2}.$$

For the relaxation effects to be negligible it is necessary that the time for passage through resonance, $\gamma$, be much less than $T_1$ and $T_2$. 
\[ \gamma \approx \frac{H_1}{dH_0/dt} < T_1, T_2 \]

The complete condition for adiabatic fast passage can be written:

\[ \frac{1}{T_1}, \frac{1}{T_2} < \frac{1}{H_1} \left| \frac{dH_0}{dt} \right| < \left| \gamma H_1 \right| . \]

It is now easy to compute the voltage \( V_x \), induced in a coil, which has its axis along the x-axis. \( V_x = -\mu_0 NA \beta \frac{dM_x}{dt} \), where \( N \) is the number of turns in the coil, \( A \) is the cross sectional area of the coil, and \( \beta \) is the filling factor.

Under the condition of adiabatic fast passage the solution for \( M_x \) is:

\[ M_x = M_0 \sin \theta \cos \omega t . \]

Then \( V_x = -\mu_0 NA \beta \omega M_0 \sin \theta \cos \omega t \).

This voltage is maximum at resonance when \( \sin \theta = 1 \), and is on the order of several millivolts.

B. Bloch Equations

The intuitive arguments for obtaining the Bloch equations are:

1. In a homogeneous magnetic field the magnetization obeys the equation of motion \( \frac{d\vec{M}}{dt} = \gamma \vec{M} \times \vec{H} \).

2. In a static field \( H_z = H_0 \) the magnetization's approach towards its equilibrium value is described by \( \frac{dM_z}{dt} = -\frac{(M_z - M_0)}{T_1} \).
3. Suppose that the magnetization is rotated so that it has a component perpendicular to the applied field $H_0$. This component will decay exponentially due to spin-spin interactions: \[ \frac{dM_x, y}{dt} = -\frac{M_x, y}{T_2}. \]

4. We assume that if there is a large applied static field, and a very small oscillating field, the motion of the nuclear spins due to relaxation can be superposed on the motion of the free spins. Combining these four statements gives the Bloch equations:

\[ \frac{d\vec{M}}{dt} = \gamma \vec{M} \times \vec{H} - \frac{M_x, y}{T_2} - \frac{M_z - M_0}{T_1} \hat{k}. \]

In the rotating frame these equations become:

\[ \frac{d\tilde{M}_x}{dt} = -\frac{\tilde{M}_x}{T_2} + \Delta \omega \tilde{M}_y, \]

\[ \frac{d\tilde{M}_y}{dt} = -\Delta \omega \tilde{M}_x - \frac{\tilde{M}_y}{T_2} - \omega_1 \tilde{M}_z, \]

\[ \frac{d\tilde{M}_z}{dt} = -\omega_1 \tilde{M}_y - \frac{\tilde{M}_z - M_0}{T_1}, \]

where $\tilde{M}_x$ and $\tilde{M}_y$ are the transverse components of $\tilde{M}$ in the rotating frame.

The steady state solution of these equations is:

\[ \tilde{M}_x = \frac{\Delta \omega \gamma H_0 T_2 M_0}{1 + (T_2 \Delta \omega)^2 + \gamma^2 H_1 T_1 T_2}. \]
If $\gamma H_1 T_2 \gg 1$ and the magnetic field is at resonance so that $\Delta \omega = 0$, then $\tilde{M}_x = \tilde{M}_y = \tilde{M}_z = 0$. It is also clear that if initially $\Delta \omega = 0$ and $M_z = 0$, then the solutions for $\tilde{M}_x$ and $\tilde{M}_y$ are:

$$\tilde{M}_x = \tilde{M}_x(0) e^{-t/T_2}$$

$$\tilde{M}_y = \tilde{M}_y(0) e^{-t/T_2}.$$ 

These are the two special solutions of the Bloch equations that are the basis of the measurements of $T_1$ and $T_2$. The measuring process is described in greater detail in Section IV.
III. EXPERIMENTAL APPARATUS

A. Gas Handling System

Figure 2 is a diagram of the gas handling system. The sample storage is either a one liter or one-half liter metal cylinder. It is separated from the Helium trap and glass manometer by Hoke valves.

The glass manometer is used both to measure the sample pressure and as a Toepppler pump to force the sample back into the storage when the run is over. When used as a pump, the mercury level in the right arm is raised and lowered by applying pressure or a vacuum to the left arm. The pressure is obtained from a nitrogen cylinder and the vacuum from a fore-pump separated from the left arm by a Hoke valve.

The Helium trap is used in an attempt to remove all condensable vapors from the sample. This is done in a separate run by collecting the sample in the sample chamber at 1.2°K after passing it through the Helium trap at 4°K. It is impossible to collect all of the sample by way of the trap due to its large effective volume at 4°K. After flushing the sample through the trap several times and then closing the valve which connects the trap to the sample chamber, additional sample can be forced into the sample chamber through the trap by-pass.

The vent is a small flange and O-ring that allows the system to be connected to a pump or a leak detector.
Figure 3 is a drawing four times actual size of the separate parts of the sample chamber. The nylon filler is first inserted into the nylon chamber. The brass plug is then force fit into the nylon chamber. To avoid super-leaks, it is essential that the surfaces of the brass plug and nylon chamber be smooth.

B. Nuclear Resonance Equipment

Figure 4 is a schematic diagram of the electronic equipment. An r.f. signal is provided by a crystal oscillator operating in the citizen's band at approximately 27.1 megacycles. This voltage provides the small oscillating field $2H_1 \cos \omega t$.

The tank circuit is made of 27.5 turns of number 24 Nyclad wire and a 20 μf capacitor. The coil is tapped at approximately one-half turn so that the impedance of the tank circuit at resonance will match the 150 ohm characteristic impedance of the one-eighth-wavelength transmission line.

The signal induced by the magnetization is equivalent to a change in complex impedance at the bottom of the transmission line, but appears as a change in resistance at the top of the line. That this is so can be shown by considering the expression for the impedance, $Z_T$, at the top of a transmission line of length $L$ in terms of the characteristic impedance of the line, $Z_0$, and the impedance of the tank.
circuit, $Z_L$.

$$Z_T = Z_o \left[ \frac{Z_L \cos \left(\frac{2\pi L}{\lambda} \right) + i Z_o \sin \left(\frac{2\pi L}{\lambda} \right)}{Z_o \cos \left(\frac{2\pi L}{\lambda} \right) + i Z_L \sin \left(\frac{2\pi L}{\lambda} \right)} \right]$$

but $Z_o = Z_L$ and $L = \lambda/8$

therefore, $dZ_T = \frac{Z_o}{Z_o + iZ_L} (1-i) \ dZ_L = -i \ dZ_L$

The signal is detected by a balanced detector and then amplified and recorded on a Model 320 Sanborn Recorder.

The large uniform magnetic field is provided by a twelve inch Varian Electromagnet. At resonance, the field is approximately 7800 gauss. In order to sweep several hundred gauss in the region of resonance, a battery driven S.I.E. integrator model SI 100 was connected to the sweep input of the magnet power supply.

C. Cryogenics

A 1.5 liter glass Helium dewar of a standard type and a brass nitrogen dewar were used to obtain a one inch working space in the 1-3/4" magnet gap. Temperatures down to 1.2°K were obtained by pumping on the Helium bath with a Kinney fore-pump. With the aid of an oil-diffusion pump, a temperature of 0.93°K could be reached. The vapor pressure was read to an accuracy of 10 microns with a Texas Instruments Precision quartz pressure gauge. This allowed a determination of the temperature to within approximately .001°K.
IV. EXPERIMENTAL PROCEDURE

The discussion in this section includes the method of making up the various mixtures, the procedure followed in cleaning and transferring the sample, and the procedure for making a run from 2°K to 1°K. A detailed discussion of the method of measuring $T_1$ and $T_2$ is also included.

The $^{14}$ mixture and the pure He$^3$ were previously prepared and analyzed by mass spectroscopy. The $^{35}$ mixture was prepared in an apparatus designed to measure and mix gases. Volumes were calculated using the gas laws and an accuracy of 1% was easily attained. The $^{5}$ mixture was prepared in the following manner. After the run on the $^{35}$ mixture was over, all but 20 cc was forced back into the metal storage. The Helium trap was pumped out through the vent and then filled to a pressure of several atmospheres with He$^4$. The valve between the manometer and the trap was slowly opened and 120 cc of He$^4$ was allowed to flow into the right manometer arm. The resulting 140 cc mixture of $^{5}$ He$^3$ was more than enough for one run.

The procedure for purifying and transferring all the mixtures, except that for the $^{5}$ which is to be explained last, was as follows. The metal storage containing the mixture was first connected to the apparatus. The manometer, the Helium trap, and the sample chamber were pumped down to $10^{-5}$ mm Hg. pressure. The $1 \text{ mm}$ of exchange gas, which
had cooled the cryostat and 1.5 liter dewar to 77°K, was pumped out and the dewar filled with liquid He\textsuperscript{4}. The Kinney fore-pump pumped on the Helium bath until a temperature of 1.2°K was attained. After making sure that no superleak was present, the sample chamber, the manometer, and the trap were closed off from each other. The Helium trap which had been previously cooled to 77°K was filled with liquid He\textsuperscript{4}. The Hoke valve on the metal storage was opened and about 20 cc of sample were let into the right manometer arm. The sample storage was closed off and the trap was opened to the manometer and to the sample chamber. To help the sample collect in the sample chamber, the right arm of the manometer was raised by opening the left arm to the atmosphere. This procedure was repeated until the sample chamber was full.

For the 5\% mixture, there was initially 140 cc in the manometer. It was cleaned and transferred at one time. Since only 80 cc of this mixture was required for a run, the excess was admitted to the Helium trap.

After a known mixture had been cleaned, a run was made to measure $T_1$ and $T_2$. The following procedure was adhered to during each run. One mm of air was admitted to the He\textsuperscript{4}-N\textsubscript{2} exchange space. After two and one-half hours, the cryostat and the dewar had cooled to 77°K. The exchange gas was pumped out for ten minutes with a forepump before the
liquid He$^4$ was transferred to the dewar. The Kinney fore-pump was used to go monotonically downwards in temperature from 2°K to 1.2°K. Temperatures from 1.2°K to 0.93°K were reached with the aid of an oil-diffusion pump.

At each temperature, $T_2$ was measured as follows: The magnetization was adiabatically rotated by 90 degrees so that it was in the plane perpendicular to the external field $H_0$. This was accomplished by starting with $H_0$ far above resonance and slowly reducing $H_0$ to its value at resonance $H_0 = \omega_0/\gamma$. It was shown in section II that if initially $M_z$ is zero at resonance then it will remain zero, and $M_x$ and $M_y$ will decay exponentially with the time constant $T_2$.

$$
M_x = M_x(0) e^{-t/T_2} \cos \omega t
$$

$$
M_y = M_y(0) e^{-t/T_2} \sin \omega t
$$

$$
M_z = 0
$$

In our experiment, the coil has its axis along the x-axis and therefore the recorder's response is proportional to $M_x$. $T_2$ is determined from the amplitude of the decay curve at two different times.

After five $T_2$'s have elapsed $M_x = M_y = M_z \approx 0$. The magnetic field $H_0$ is rapidly moved from 100 to 150 gauss off resonance and the magnetization approaches its equilibrium value exponentially:
At some time during the magnetization's recovery a fast passage is made to determine its magnitude. The equilibrium value $M_0$ is determined by making a fast passage after waiting at least five $T_1$'s, enough time to insure 1% accuracy. These two values of the magnetization are then sufficient to determine the relaxation time $T_1$. 

$$M_z = M_0(1-e^{-t/T_1}) .$$
V. EXPERIMENTAL RESULTS

In Figs. 5, 6, and 7 the measured $T_1$ and $T_2$ values are plotted against temperature for the 5%, 14%, and 35% mixtures. The pure He$^3$ results are in Fig. 8. The curves in these figures are drawn through the average values.

The average $T_1$ values for the three mixtures and pure He$^3$ are plotted together in Fig. 9. The average $T_2$ values are plotted in Fig. 10. These average values are also included in Table 1.
5% \text{He}^3

Spin-lattice relaxation time - \( T_1 \)
Spin-spin relaxation time - \( T_2 \)

**FIGURE 5**

**Graph:**
- **Y-axis:** Relaxation time (seconds)
- **X-axis:** Temperature (degrees Kelvin)
- Curves labeled \( T_1 \) and \( T_2 \)
- Data points plotted on the graph.
14% He$^3$

Spin-lattice relaxation time - $T_1$
Spin-spin relaxation time - $T_2$

**Figure 6**
35% $\text{He}^3$

Spin-lattice relaxation time - $T_1$
Spin-spin relaxation time - $T_2$

![Graph showing relaxation times $T_1$ and $T_2$ as a function of temperature.](image-url)

**Figure 7**
100% He³

Spin-lattice relaxation time - $T_1$
Spin-spin relaxation time - $T_2$
SPIN-LATTICE RELAXATION TIME VS TEMPERATURE

FIGURE 9
RELAXATION TIME

SPIN-SPIN RELAXATION TIME

VS

TEMPERATURE

FIGURE 10
### TABLE 1

#### 5% He³

<table>
<thead>
<tr>
<th>T⁰K</th>
<th>T₁ Ave.</th>
<th>T₂ Ave.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>19.1</td>
<td>2.35</td>
</tr>
<tr>
<td>1.20</td>
<td>16.2</td>
<td>2.30</td>
</tr>
<tr>
<td>1.38</td>
<td>23.8</td>
<td>4.6</td>
</tr>
<tr>
<td>1.66</td>
<td>33.7</td>
<td>9.7</td>
</tr>
<tr>
<td>1.96</td>
<td>69.9</td>
<td>23.5</td>
</tr>
</tbody>
</table>

#### 14% He³

<table>
<thead>
<tr>
<th>T⁰K</th>
<th>T₁ Ave.</th>
<th>T₂ Ave.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.20</td>
<td>30.5</td>
<td>8.70</td>
</tr>
<tr>
<td>1.46</td>
<td>35.3</td>
<td>12.8</td>
</tr>
<tr>
<td>1.69</td>
<td>52.7</td>
<td>16.9</td>
</tr>
<tr>
<td>1.93</td>
<td>87.5</td>
<td>28.5</td>
</tr>
</tbody>
</table>

#### 35% He³

<table>
<thead>
<tr>
<th>T⁰K</th>
<th>T₁ Ave.</th>
<th>T₂ Ave.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.97</td>
<td>46.6</td>
<td>12.3</td>
</tr>
<tr>
<td>1.24</td>
<td>59.0</td>
<td>23.2</td>
</tr>
<tr>
<td>1.38</td>
<td>63.8</td>
<td>27.1</td>
</tr>
<tr>
<td>1.66</td>
<td>95.5</td>
<td>44.0</td>
</tr>
<tr>
<td>1.96</td>
<td>93.0</td>
<td>34.0</td>
</tr>
</tbody>
</table>

#### 100% He³

<table>
<thead>
<tr>
<th>T⁰K</th>
<th>T₁ Ave.</th>
<th>T₂ Ave.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>59.3</td>
<td>26.4</td>
</tr>
<tr>
<td>1.20</td>
<td>65.8</td>
<td>32.4</td>
</tr>
<tr>
<td>1.38</td>
<td>75.4</td>
<td>35.2</td>
</tr>
<tr>
<td>1.66</td>
<td>66.6</td>
<td>27.9</td>
</tr>
<tr>
<td>1.96</td>
<td>67.0</td>
<td>23.7</td>
</tr>
</tbody>
</table>

* Relaxation times are in seconds.
VI. DISCUSSION AND CONCLUSIONS

In this section we present the BPP theory of spin-lattice relaxation. This is followed by a qualitative discussion of some aspects of bulk impurity and wall relaxation phenomena. Using the published values for the diffusion coefficient of pure He$^3$, we were able to compute the coefficients $C_1$ and $C_2$ of Eq. (1) for the pure He$^3$ and thus to determine the wall and bulk relaxation times. Having obtained the values of $C_1$ and $C_2$ we were then able to calculate the diffusion coefficient and hence the wall and bulk relaxation times for the three mixtures. The diffusion coefficients predicted by Khalatnikov and Zharkov$^1$ are calculated for the 5% and 14% mixtures and then compared with the experimentally determined coefficients.

The spin-spin relaxation time is discussed in terms of the BPP theory, and an explanation is presented for the experimental fact that $T_2$ is much shorter than $T_1$.

A. Spin-Lattice Relaxation$^9$

The spin-lattice relaxation time is the number that describes the magnetization's exponential approach to equilibrium. It can be shown that $T_1 = \frac{1}{W}$ where $W$ is the probability of a spin $1/2$ nucleus making a transition between the two states $m = \pm 1/2$. In classical liquids each of the molecules executes Brownian motion. It is the Fourier
component of the fluctuating magnetic field, produced at
the site of one nucleus by another nucleus, that causes
these transitions.

We can write the interaction between the $i$'th and $j$'th
nucleus each with magnetic moment $\mu$ in terms of $r_{ij}, \theta_{ij},$
and $\phi_{ij}$, where $\vec{r}_{ij}$ is the vector between two atoms and the
angles $\theta_{ij}$ and $\phi_{ij}$ specify the orientation of $\vec{r}_{ij}$.

\[
V_{ij} = \frac{\delta^2 \hbar^2}{r_{ij}^3} \left[ \vec{r}_{ij} \cdot \vec{I}_j - 3(\vec{r}_{ij} \cdot \hat{r}_{ij})(\vec{I}_j \cdot \hat{r}_{ij}) \right]
\]

where $\vec{r} = \gamma \hbar \vec{I}$. If $\epsilon_1$, $\epsilon_2$ and $\epsilon_3$ are the direction cosines
of $\vec{r}_{ij}$ then $\epsilon_1 = \sin \theta_{ij} \cos \phi_{ij}$, $\epsilon_2 = \sin \theta_{ij} \sin \phi_{ij}$ and
$\epsilon_3 = \cos \theta_{ij}$ and one can write $V_{ij}$ as

\[
V_{ij} = \frac{\delta^2 \hbar^2}{r_{ij}^3} \left[ A + B + C + D + E + F \right]
\]

where $A = I_{zi}I_{zj} (1-3 \cos \theta_{ij})$

\[ B = -\frac{1}{4} \left[ (I_{x1}iI_{y1})(I_{xj}+iI_{yj}) + (I_{x1}+iI_{y1})(I_{xj}i-I_{yj}) \right] (1-3 \cos^2 \theta_{ij}) \]

\[ C = -\frac{3}{2} \left[ (I_{x1}+iI_{y1}) I_{zj} + (I_{xj}+iI_{yj}) I_{zi} \right] \sin \theta_{ij} \cos \theta_{ij} e^{-i\phi_{ij}} \]
For a liquid $r_{ij}$, $\theta_{ij}$, and $\phi_{ij}$ are random functions of the time, and if the Fourier spectrum of $C$ is finite for $\omega_0 = -\gamma H_0$ then $C$ can induce a transition $\Delta m_i = +1$. If $D$ has a Fourier component at $\omega_0 = \gamma H_0$ then it can cause a transition $\Delta m_i = -1$. $E$ and $F$ can produce transitions if they have a finite spectrum at frequencies $\pm 2\omega_0$. The terms $A$ and $B$ do not produce transitions and consequently they do not affect the relaxation time.

Let

$$Y_{1j} = \sin \theta_{ij} \cos \phi_{ij} e^{i\phi_{ij}} r_{ij}^{-3}$$

and

$$Y_{2j} = \sin^2 \theta_{ij} e^{2i\phi_{ij}} r_{ij}^{-3}$$

The intensity of their Fourier spectra is

$$J(\gamma) = \int_{-\infty}^{\infty} d\tau e^{i\omega \tau} \frac{Y(t)Y(t+\tau)}{Y(t+\tau)} = \int_{-\infty}^{\infty} d\tau e^{i\omega \tau} K(\gamma).$$

We assume $K(\gamma)$ to have the form $K(\gamma) = K(0) e^{-|\gamma/\tau_c|^2}$ where $\tau_c$, the correlation time, has the physical significance of being the time during which the magnetic field at nucleus $i$ due to nucleus $j$ is constant.

The spectral intensity for one neighbor $j$ is
\[
J_j(y) = \int_{-\infty}^{+\infty} d\tau e^{i\omega\tau} e^{-|\tau|/\tau_c} |Y(t)|^2 = \frac{|Y_j|^2}{\tau_c^2} (1 + 4\pi^2\gamma^2 \tau_c^2)^{-1}. \tag{23}
\]

The total intensity for all neighbors is
\[
J(\psi) = \sum_j J_j(\psi) = \sum_j \frac{|Y_j|^2}{\tau_c^2} 2 \tau_c (1 + 4\pi^2\gamma^2 \tau_c^2)^{-1}.
\]

In each of these equations we must insert 1 or 2 when we are considering a particular one of the groups \(Y_1\) or \(Y_2\).

The transition probabilities due to the terms C through F are evaluated using first order perturbation theory. After calculating each term and adding the probabilities one finds the total transition probability for nucleus 1 to go from \(m_1 + 1\) to \(m_1 - 1\) or from \(m_1 - 1\) to \(m_1 + 1\) to be
\[
W = \frac{3}{4} \hbar^2 I(I+1) \left[ J_1(\nu_0) + \frac{1}{2} J_2(2\nu_0) \right].
\]

Then
\[
\frac{1}{T_1} = 2W = 3/2 \hbar^2 I(I+1) \sum_j \left[ \frac{2\tau_c |Y_{1j}|^2}{1 + 4\pi^2\gamma^2 \tau_c^2} + \frac{\tau_c |Y_{2j}|^2}{1 + 16\pi^2\gamma^2 \tau_c^2} \right].
\]

This can be written
\[
\frac{1}{T_1} = 0.3 \hbar^2 b^{-6} \left[ \frac{\gamma_c}{1 + \omega_0^2 \gamma_c^2} + \frac{2 \gamma_c}{1 + 4\omega_0^2 \gamma_c^2} \right]
\]
where \(b\) is the average interspin distance. In our experiments \(\omega_0 \gamma_c \ll 1\), which allows \(T_1\) to be simplified to
\[
\frac{1}{T_1} = 0.9 \hbar^2 b^{-6} \gamma_c.
\]
For liquids with a diffusion coefficient \( D(\gamma_c) = \frac{\pi N_0 b^6}{3a \gamma_c^{24}} \), this becomes

\[
\frac{1}{T_1} = 0.3 \pi \delta^4 h^2 N_0/aD
\]

where \( a \) is the atomic radius and \( N_0 \) is the number of magnetic moments per unit volume.

There are two other mechanisms that are important in determining the relaxation time.

1. **Bulk Impurity Relaxation.** This is the relaxation of \( \text{He}^3 \) atoms by paramagnetic impurities that are in suspension in the liquid. The magnetic moment of the electron is approximately 1000 times that of the \( \text{He}^3 \) nucleus and consequently one part per million of paramagnetic impurity would cause a relaxation effect equal to that of the inherent \( \text{He}^3 \) relaxation. The BFP theory gives for the bulk impurity relaxation:

\[
\frac{1}{T_1} \text{imp} = \frac{1}{T_1} \text{He}^3 \left( \frac{\mu_{\text{imp}}^2}{\mu_{\text{He}^3}^2} \right) \left( \frac{N_{\text{imp}}}{N_0} \right),
\]

where \( N_{\text{imp}} \) is the number of impurity atoms with magnetic moment \( \mu_{\text{imp}} \). Thus the total inverse bulk relaxation

\[
\frac{1}{T_1} \text{bulk} = \frac{1}{T_1} \text{He}^3 + \frac{1}{T_1} \text{imp} = \frac{C_2 \rho}{D}
\]

is inversely proportional to \( D \).

2. **Wall Relaxation.** Any paramagnetic impurities adsorbed on the wall will cause the \( \text{He}^3 \) nuclei to relax whenever they hit the wall. If the nuclei relax as soon as they hit the wall, then the wall relaxation depends only on the
diffusion coefficient and the diameter of the sample chamber. From dimensional analysis one arrives at

\[ \frac{1}{T_1^{\text{wall}}} \propto \frac{D}{d^2} = C_1 D \]

Thus the inverse wall relaxation is proportional to D.

Adding the inverse bulk and the inverse wall relaxation times we obtain the net inverse relaxation time for our model.

\[ \frac{1}{T_1^{\text{net}}} = \frac{1}{T_1^{\text{wall}}} + \frac{1}{T_1^{\text{bulk}}} = C_1 D + \frac{C_2 \rho}{D} . \]

B. The Experimental Diffusion Coefficients.

Figure 11 is a plot of \( \frac{10^{-6} \rho}{D^2} \) vs. \( \frac{10^{-1}}{DT_1} \) for pure He\(^3\). The diffusion coefficient was taken from Gaines et al.\(^{10} \), the density values are from Ptukha\(^{11} \) and relaxation times are those determined by us. The intercept gives \( C_1 = 75 \) and from the slope we find that \( C_2 = 9.1 \times 10^{-6} \). Using these values of \( C_1 \) and \( C_2 \) one finds for the pure He\(^3\) that the wall relaxation and the bulk relaxation are both important for all temperatures. The diffusion time to the wall is approximately 60 seconds and is shorter than the wall relaxation time given by the \( C_1 \) term. These results are listed in Table 2.

The short bulk relaxation times are due to bulk impurities. This causes a difficulty in computing the diffusion coefficient for the other mixtures using these values of \( C_1 \) and \( C_2 \) since it is not clear how to treat the bulk
\[ \frac{10^{-6}}{D^2} \rho \] vs \[ \frac{10^{-1}}{DT_1} \]

**Figure 11**

\[ \frac{10^{-6}}{D^2} \rho \left( \text{cm}^2 \right) \]

\[ \frac{10^{-1}}{DT_1} \left( \frac{1}{\text{cm}^2} \right) \]
### TABLE 2*

**Pure He^3**

<table>
<thead>
<tr>
<th>$T^\circ K$</th>
<th>$T_1$-Exp.</th>
<th>$T_1$ - Wall</th>
<th>$T_1$ - Bulk</th>
<th>$T_1$ - BPP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>59.3</td>
<td>202</td>
<td>89.5</td>
<td>160</td>
</tr>
<tr>
<td>1.20</td>
<td>65.8</td>
<td>185</td>
<td>97.5</td>
<td>205</td>
</tr>
<tr>
<td>1.38</td>
<td>75.4</td>
<td>173</td>
<td>104</td>
<td>245</td>
</tr>
<tr>
<td>1.66</td>
<td>66.6</td>
<td>155</td>
<td>118</td>
<td>320</td>
</tr>
<tr>
<td>1.96</td>
<td>67.0</td>
<td>139</td>
<td>135</td>
<td>410</td>
</tr>
</tbody>
</table>

*Relaxation times are in seconds.*
impurity relaxation. Fortunately the bulk relaxation becomes unimportant for the lower percentage mixtures. Even with an order of magnitude change in $C_2$, the relaxation time is determined by the wall for the 5% and 14% mixtures. Thus any reasonable assumption made about $C_2$ does not affect the results for these two mixtures. Consequently $C_2$ was treated as though the He$^3$ were described by the BPP theory. The same assumptions were made in the case of the 35% mixture even though it was recognized that the bulk relaxation, although small, is not negligible.

The method of calculation was the following: $C_1$ and $C_2$ were found for the pure He$^3$ as explained above. For each of the other mixtures the correct He$^3$ density was used together with the experimentally determined relaxation time to give a quadratic equation for the diffusion coefficient. Only one of the roots ever had any meaning so that there was no difficulty in deciding which one to keep. The results of the calculations are shown in Table 3. Figure 12 is a plot of $D$ vs. temperature. The pure He$^3$ data was taken from Gaines, et al. 10, and the 0.01% data was taken from Beenacker et al. 12

Using these computed values of the diffusion coefficient we computed the wall relaxation time, $1/C_1D$, and the bulk relaxation time, $D/C_2\tau$, for the three mixtures. These results are also presented in Table 3.
**TABLE 3**

5% He³

<table>
<thead>
<tr>
<th>T° K</th>
<th>T₁ - Exp.</th>
<th>D x 10⁻⁴</th>
<th>T₁ - Wall</th>
<th>T₁ - Bulk</th>
<th>T₁ - BPP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>19.1</td>
<td>7.0</td>
<td>19.0</td>
<td>14,000</td>
<td>2360</td>
</tr>
<tr>
<td>1.20</td>
<td>16.2</td>
<td>8.2</td>
<td>16.3</td>
<td>16,500</td>
<td>3010</td>
</tr>
<tr>
<td>1.38</td>
<td>23.8</td>
<td>5.6</td>
<td>23.8</td>
<td>11,200</td>
<td>3600</td>
</tr>
<tr>
<td>1.66</td>
<td>33.7</td>
<td>3.9</td>
<td>34.2</td>
<td>7880</td>
<td>4700</td>
</tr>
<tr>
<td>1.96</td>
<td>69.9</td>
<td>1.9</td>
<td>70.2</td>
<td>3820</td>
<td>6030</td>
</tr>
</tbody>
</table>

14% He³

<table>
<thead>
<tr>
<th>T° K</th>
<th>T₁ - Exp.</th>
<th>D x 10⁻⁴</th>
<th>T₁ - Wall</th>
<th>T₁ - Bulk</th>
<th>T₁ - BPP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.20</td>
<td>30.5</td>
<td>4.30</td>
<td>31.0</td>
<td>3140</td>
<td>1100</td>
</tr>
<tr>
<td>1.46</td>
<td>35.3</td>
<td>3.73</td>
<td>35.8</td>
<td>2720</td>
<td>1450</td>
</tr>
<tr>
<td>1.69</td>
<td>52.7</td>
<td>2.46</td>
<td>54.2</td>
<td>1800</td>
<td>1750</td>
</tr>
<tr>
<td>1.93</td>
<td>87.5</td>
<td>1.40</td>
<td>95.2</td>
<td>1020</td>
<td>2180</td>
</tr>
</tbody>
</table>

* Relaxation times are in seconds.

D is in units of cm²/sec.
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
$T^\circ K$ & $T_1$ - Exp. & $D \times 10^{-4}$ & $T_1$ - Wall & $T_1$ - Bulk & $T_1$ - BPP \\
\hline
0.97 & 46.6 & 2.70 & 49.4 & 850 & 340 \\
1.24 & 59.0 & 2.10 & 63.5 & 660 & 477 \\
1.38 & 63.8 & 1.90 & 70.2 & 597 & 556 \\
1.66 & 95.5 & 0.96 & 139 & 302 & 720 \\
1.96 & 93.0 & 1.00 & 132 & 317 & 930 \\
\hline
\end{tabular}
\caption{35\% He$^3$}
\end{table}

* Relaxation times are in seconds.

$D$ is in units of cm$^2$/sec.
DIFFUSION COEFFICIENT
VS
TEMPERATURE

FIGURE 12
C. Theory of Khalatnikov and Zharkov.

Khalatnikov and Zharkov\(^1\) have calculated the diffusion coefficient for dilute solutions of He\(^3\) in HeII on the basis of the Landau theory of superfluidity. In this theory liquid He\(^4\) below the lambda point, HeII, is treated as a gas of excitations, phonons and rotons, moving in a superfluid background. The phonon energy, \(\varepsilon_p\), is a linear function of the momentum \(\varepsilon_p = cp\), where \(c\) is the velocity of sound in HeII. The roton energy, \(\varepsilon_r\), has a momentum dependence as follows,

\[
\varepsilon_r = \Delta + \frac{(p - p_0)^2}{2\mu_r}
\]

where \(\Delta/k = 8.9^\circ K\), \(p_0/h = 1.95 \times 10^8 \text{ cm}^{-1}\), and \(\mu_r\), the effective mass of the roton is \(1.7 \times 10^{-24} \text{ gms}\).

Landau and Pomeranchuk\(^13\) have shown that He\(^3\) dissolved in HeII combines with the normal component and does not participate in the superfluid motion. From second sound experiments Khalatnikov\(^14\) has determined the spectrum of the excitations associated with the He\(^3\), \(\varepsilon_{He^3}\), to be

\[
\varepsilon_{He^3} = \frac{p^2}{2m_3^*} + \Delta
\]

where \(m_3^*\) is the effective mass of the He\(^3\).

Kinetic equations for the excitations, now considered as localized, interacting packets, can be derived. The distribution functions for the excitations can be determined from the solution of these equations and using these
functions one can obtain the dependence of the diffusion coefficient on temperature and He\textsuperscript{3} concentration.

To solve the kinetic equations it is necessary to know the interactions between the excitations. At high temperatures, $T > 0.5\,^\circ\text{K}$, the region in which we are interested, the phonons can be neglected but one must take into account roton-roton, roton-He\textsuperscript{3}, and He\textsuperscript{3}-He\textsuperscript{3} interactions. Following Landau and Khalatnikov's\textsuperscript{15} treatment of roton-roton collisions, the authors chose a delta function of position as the roton-He\textsuperscript{3} and He\textsuperscript{3}-He\textsuperscript{3} interaction. They determined the diffusion coefficient for two limiting cases—where the relative number of He\textsuperscript{3} atoms is much smaller than the number of rotons and where the number of He\textsuperscript{3} atoms exceeds the number of rotons. They interpolated between these limits and obtained, for high temperatures,

$$D(K,Z.) = 2.8 \times 10^{-6} e^{\frac{\Delta}{kT}} \left(\frac{\rho_{no}}{\rho_n}\right)^2,$$

where $\rho_n$ is the density of the normal component of the solution, $\rho_n = \rho_{no} + \rho_{He^3}^*; \rho_{no}$ is the density of rotons and $\rho_{He^3}^* = \rho_{He^3} \frac{m^*}{m_3}$ is the effective normal density associated with the He\textsuperscript{3} atoms. $\rho_3$ is the ordinary density of He\textsuperscript{3} and $m_3$ is the mass of a He\textsuperscript{3} atom.

The normal density, $\rho_n$, has been experimentally determined by Dash and Taylor\textsuperscript{16} for concentrations 0-11\% He\textsuperscript{3} and for temperatures $1.3^\circ\text{K} < T < T_{\lambda}$. We used these data for the
5\% mixture for 1.3^\circ K < T < T_\lambda \) and by extrapolation obtained \( \rho_n \) at 1.0^\circ K. Dash and Taylor\(^\text{16} \) present a formula for \( \rho_n \) for all concentrations 0-11\% and temperatures 1.3^\circ K < T < T_\lambda \) in terms of the He\(^3\) concentration and a reduced temperature \( T^* = \frac{T}{T_\lambda} \). We have extended this formula in obtaining \( \rho_n \) for the 14\% mixture.

Having thus obtained the normal densities it is necessary to know the effective mass of the He\(^3\) in order to calculate \( \rho_{no}/\rho_n \). However the effective mass in the temperature range 1.0^\circ K < T < T_\lambda \) has not been experimentally determined. Nevertheless, we know that at 0^\circ K it is 2.8 m\(_3\) and that it decreases to m\(_3\) at T\(_\lambda\).

For an effective mass of 1.78 m\(_3\) the theory reproduces very closely the diffusion coefficients evaluated from the T\(_1\) data, D(T\(_1\)), for the 5\% mixture. For the 14\% mixture the theory gives a diffusion coefficient that has approximately the same value and temperature dependence as D(T\(_1\)). This mixture is in the interpolation region of the Khalatnikov and Zharkov theory.

Calculations show that an effective mass greater than 1.78 m\(_3\) gives a diffusion coefficient, D(K.Z.), smaller than D(T\(_1\)) and an effective mass less than 1.78 m\(_3\) gives a diffusion coefficient, D(K.Z.), greater than D(T\(_1\)) for the 5\% solution. Nevertheless, for this solution, any effective mass, m\(_3\) < m* < 2.8 m\(_3\), for 1.4^\circ K < T < 2.0^\circ K, gives a diffusion
coefficient whose value is within 50\% of D(T_1) and that is in good agreement with the temperature dependence of D(T_1).

Figure 13 is a plot of the diffusion coefficients D(T_1), denoted by \( \Theta \), and D(K.Z.) for \( m_3^* = 1.78 \text{ m}_3 \), denoted by 0, against temperature for the 5\% and 14\% solutions. The values of \( \rho_n \) used and the results of the calculations are in Table 4.

D. Spin-Spin Relaxation

If the spin-spin relaxation is determined by the same dipole interaction that determines T_1 and if the correlation time \( \tau_c \ll \frac{1}{\omega_0} \), the BPP theory states that \( T_2 = T_1 \). Since \( \tau_c \approx 10^{-12} \) seconds for liquid He\(^3\), \( T_2 \) should be equal to \( T_1 \). From Figs. 5 through 8, however, it is clear that \( T_2 \) is always from 1/3 to 1/8 the value of \( T_1 \). Thus it is important to find the mechanism that will explain the short \( T_2 \) times. A possible explanation is one proposed by Bloembergen\(^{17} \). He shows that in some liquids the short \( T_2 \) times are the result of an impurity spin-exchange interaction with a very long correlation time.
DIFFUSION COEFFICIENT

vs

TEMPERATURE

\[ 10^{-5} \text{cm}^2/\text{sec.} \]

\[ 5\% \text{He}^3 \]

\[ 14\% \text{He}^3 \]

\[ D(KZ) \]

\[ D(T_1) \]

FIGURE 13
TABLE 4

5% He$^3$

<table>
<thead>
<tr>
<th>T°K</th>
<th>$\rho_n$ gms/cc</th>
<th>D ($10^{-5}$ cm$^2$/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.02</td>
<td>0.143</td>
<td>19.9</td>
</tr>
<tr>
<td>1.84</td>
<td>0.075</td>
<td>27.0</td>
</tr>
<tr>
<td>1.42</td>
<td>0.027</td>
<td>60.5</td>
</tr>
<tr>
<td>1.21</td>
<td>0.017</td>
<td>82.5</td>
</tr>
<tr>
<td>1.00</td>
<td>0.012</td>
<td>69.0</td>
</tr>
</tbody>
</table>

14% He$^3$

<table>
<thead>
<tr>
<th>T°K</th>
<th>$\rho_n$ gms/cc</th>
<th>D ($10^{-5}$ cm$^2$/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.95</td>
<td>0.131</td>
<td>17.0</td>
</tr>
<tr>
<td>1.62</td>
<td>0.062</td>
<td>22.0</td>
</tr>
<tr>
<td>1.44</td>
<td>0.047</td>
<td>24.0</td>
</tr>
<tr>
<td>1.19</td>
<td>0.034</td>
<td>23.8</td>
</tr>
</tbody>
</table>
ACKNOWLEDGMENTS

I wish to express my appreciation to Professor H. E. Rorschach, Jr., for his guidance. I also wish to thank Mr. Belcher, Mr. Surles, and Mr. Van der Henst for their technical assistance.
REFERENCES

2. N. Bloembergen, E. M. Purcell, and R. V. Pound, Phys. Rev. 73, 679 (1948), hereafter referred to as BPP.


