A THEORETICAL STUDY OF RADON
DIFFUSION IN THE LUNAR REGOLITH

by

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ABSTRACT

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An investigation is made, via computer simulation, of the fate of radon gas produced in the lunar regolith by decay of uranium and thorium. The model used is that of radon atoms released and allowed to perform random walks within the lunar soil. Pressures based on results from Apollo lunar experiment packages indicate that the mean free path for such random walks will be the mean dimensions of void spaces within the soil rather than the (much longer) mean free path for collision with other gas molecules. Calculations are made, using various assumptions for mean void size, heat of adsorption of lunar material for radon, subsurface temperature profile, and depth of release, for the probability that a radon atom may escape through the regolith upper surface before it decays, to become part of the lunar "exosphere". Discussion is made of the implications such escape may have for lunar surface radioactivity and for lead/uranium dating of soil samples.
Table of Contents

I. Motivation. 1
II. Physical model. 2
III. Mathematical model. 6
IV. Results. 12
V. Discussion and implications. 13
Appendix I: Symbols used. 23
Appendix II: Fortran program. 24
   I. Temperature assignment subroutine. 24
      A. For lunar noon and midnight. 24
      B. For lunar sunset. 24
      C. For solar angle 135°. 24
   II. Main program. 25
Acknowledgements. 29
References. 30
Tables. 32
Figures. 37
I. Motivation

Studies of inert gases formed in lunar soil have led to the question of what the ultimate fate might be of radon generated in the lunar regolith in the decay chains of uranium and thorium to lead. If the radon is situated in mineral sites which permit it to reach void spaces between soil particles, it will, being a gas, conceivably diffuse out of the regolith into the lunar atmosphere. The question is: what chance does it have of leaving the regolith before it decays?

Once in the atmosphere, radon atoms perform ballistic "hops" from one point on the lunar surface to another. If they happen to land on the lunar night side, they may be adsorbed for a significant (with respect to their half-life) time on the lunar surface material. If a radon atom decays to polonium while in the atmosphere or adsorbed on the surface, there are two possibilities: (1) if the direction of recoil of the resulting polonium atom is toward the surface, it will of course be implanted into the surface material; (2) if the direction of the polonium atom's recoil is toward the skyward hemisphere, its recoil velocity exceeds the escape velocity, hence removes it entirely from the vicinity on the Moon.

This loss of half of the daughter atoms of atmospheric
radon might have an effect on the apparent ages of lunar soil samples obtained by lead/uranium dating: such ages may appear noticeably different from lead/uranium ages for typical rocks of the same lunar locality if this process is significant, especially if Rn\textsuperscript{222} has a significantly better chance or reaching the atmosphere before it decays than Rn\textsuperscript{219}, due to its longer half-life (3.82 days vs. 4.0 seconds).

The atmospheric and surface decay of Rn\textsuperscript{222} is also responsible for the formation of an alpha-radioactive deposit on the Moon's surface.

These questions of possible relative fractionation and changes in apparent age, and of alpha-active deposit formation, has prompted me to formulate a computer program to make theoretical predictions as to what might be the percentages of the three principal radon isotopes (Rn\textsuperscript{219}, Rn\textsuperscript{220}, and Rn\textsuperscript{222}) that would be able to reach the atmosphere under various assumptions of the values for heat of adsorption and "void dimension" between soil particles, and various subsurface temperature profiles.

II. Physical model.

The portion of the Moon of concern in this investigation is the regolith, which is the layer of finely divided
soil on the lunar surface. The regolith is believed to have an average depth of a few meters and to be underlain by bedrock, which may be solid or made up of large broken blocks.

The physical model on which the computer program is based is that of a radon atom released at some depth in the regolith and allowed to perform a random walk. The desired result is then the probability that this random walk will take the atom above the lunar surface before it decays.

Dr. John Freeman has informed me that the daytime surface pressures are of the order of $10^{-10}$ to $10^{-9}$ mm Hg (from the SIDE results of the ALSEP packages of Apollo 12 and 14). From these pressure values I conclude that the distance between collisions of radon atoms with other gas atoms exceeds by many orders of magnitude the mean "void dimension" between soil particles, so that this mean "void dimension" represents the true mean free path for this random walk. In the present work, the system of soil particles and interparticle void spaces will be represented by a system of randomly oriented baffles (the baffles being the surfaces of soil particles), and the radon atoms diffuse in this system by collisions with the baffles.

Birkebak and Dawson report lunar soil densities ranging from 1.3 gm/cm$^3$ to 1.8 gm/cm$^3$, values approximating half the density of solid rock from the same areas of the
Moon, so that the mean "void dimension" will be roughly the same as the mean soil grain size. Median grain sizes of 43 microns\(^5\) and 62 microns\(^6\) have been reported for Apollo 11 fines, 18 microns for Apollo 12 fines\(^7\), and a graphic mean of 49 microns for Apollo 14 fines\(^8\). Core samples taken on the Apollo 12\(^9\) and Luna 16\(^10\) missions show variation in grain size of soil with depth, but to simplify calculations, a constant void dimension at all depths is assumed in this work. Calculations have been made for mean free paths of 10 and 80 microns, which effectively "bracket" the values quoted; and to allow for the possibility that these reported values are based on particle mass rather than particle number, or are for some other reason too large, calculations have been made for a mean free path of one micron.

The other factor that will determine the diffusion coefficient, \(D\), for this random walk is the "effective speed" of the radon atom, which will in turn be determined by two quantities: (1) the mean thermal speed, \(\bar{v} = \sqrt{8RT/\pi M}\); and (2) the mean "residence time" during which a radon atom is adsorbed each time it encounters a particle surface, \(\tau = \tau_0 e^{Q/RT}\), where \(Q\) is the heat of adsorption for radon. No measured values for \(Q\) were available at this writing, so an attempt has been made to extrapolate from heats of
adsorption of other noble gases. If one plots the maximum potential energy for adsorption of neon, argon, krypton, and xenon on graphite\textsuperscript{12} vs. their respective atomic weights, on linear graph paper (see fig. 1), one sees that the values for Ar, Kr, and Xe lie nearly on a line, and that line extrapolates to a value of about 5.2 kcal/mole at atomic weight 222. On a log-log plot (see fig. 2) things appear to be less well ordered. While 5.2 kcal/mole appears to be the "best guess", calculations have also been made for 7 kcal/mole and 4 kcal/mole to check the effects varying this parameter would have.

Values for $\tau$ for these three heats of adsorption, and for a $\tau_0$ assumed to be $10^{-13}$ second,\textsuperscript{13} are given in Table 1 for temperatures of 100$^\circ$K, 200$^\circ$K, and 400$^\circ$K. Figures 3 and 4 are plots of $D$ vs. $T$ for various values of $Q$ and mean free path, showing the effects of varying these parameters and $T$, and depicting well the "knee" where one changes from the regime where $D$ is governed by thermal speed (square root dependence on $T$) to that where $D$ is governed by residence time (exponential dependence on $T$).

Temperature profiles used in these calculations are based on those found in figures 5, 6a, and 6b, with the assumption of a constant 200$^\circ$K at all solar angles, below 12 cm. depth.\textsuperscript{14,15} The Apollo 15 Heat Flow Experiment now
indicates temperatures at 100 cm. depth are more nearly 250°K and are increasing with depth. This means below 12 cm. this work is using underestimates for D, and hence for escape rates.

III. Mathematical model.

If one considers radon atoms released at time \( t = 0 \) in a plane parallel to the lunar surface at some depth within the regolith one can compute the probability density at some later time \( t \) that one of these atoms is in one of the two planes parallel to the initial one at a distance \( x \) above or below it. On the basis of diffusion theory, one finds this probability \( P(x,t) = \frac{1}{\sqrt{\pi Dt}} \exp(-x^2/4Dt) \) if one ignores the radioactive decay, but when one takes the decay into account, \( P(x,t) = \frac{1}{\sqrt{\pi Dt}} \exp(-x^2/4Dt) \exp(-t \cdot \log_2/t_{1/2}) \) where \( t_{1/2} \) is the half-life of the isotope in question. One can then find the "probability flux" (flux from diffusion theory normalized to a single initial atom) \( J(x,t) = \frac{\partial P(x,t)}{\partial x} = \frac{x \cdot e^{-x^2/4Dt} - t \cdot \log_2/t_{1/2}}{2t \sqrt{\pi Dt}} \). This is the probability that the atom (at time \( t \)) is moving through the plane at depth \( +x \) in the positive direction, plus the probability that it is moving through the plane at depth \( -x \) in the negative direction. If one integrates \( J(x,t) \) from \( t = 0 \) to \( t = \infty \), one finds the probability that a radon atom released in the
regolith will reach a depth of at least \( x \) away from its initial depth before it disintegrates.

\[
\int_0^\infty \frac{x}{2t/\pi Dt} e^{-x^2/4Dt} - t \cdot \log_e 2/\sqrt{2} \, dt
\]

(let \( A = \frac{x^2}{4D}, B = \frac{t_{1/2}}{\log_e 2} \))

\[
= \sqrt{\frac{A}{\pi}} \int_0^\infty \frac{1}{t^{3/2}} e^{-A/t} - t/B \, dt
\]

\( \left( \frac{\partial}{\partial A} \right) \left[ \frac{1}{t^2} e^{-A/t} - t/B \right] = -\frac{1}{t^{3/2}} e^{-A/t} - t/B \)

\[
= \sqrt{\frac{A}{\pi}} \int_0^\infty \frac{-3}{\partial A} (1/t^{1/2} e^{-A/t} - t/B) \, dt
\]

\( \left( \frac{u^2}{2} = t, 2udu = dt \right) \)

\[
= -2\sqrt{\frac{A}{B}} \int_0^\infty \frac{e^{-u^2/B} - A/u^2}{u} \, du
\]

(let \( u = \sqrt{Bv}, \, du = \sqrt{B}dv \))

\[
= -2\sqrt{\frac{AB}{B}} \int_0^\infty e^{-v^2 - A/Bv^2} \, dv
\]

\[
= -2\sqrt{\frac{AB}{B}} \frac{\partial}{\partial A} \left[ \left( e^{-2\sqrt{AB/B}/\pi} \right) \right]
\]

\( = e^{-2\sqrt{A/B}} = e^{-x/\log_e 2/\sqrt{2}} = p_d(x) \)

for a given isotope.

The computer program used in this study operated in the following procedure:

1. Values were assigned for mean free path \( \lambda \) and heat of adsorption \( Q \).

2. Temperature was defined for the desired solar angle as a step function—constant over intervals of 1 centimeter each—approximating as well as possible
the values derived from figures 5, 6a, and 6b with a constant $200^\circ K$ assumed at depths greater than 12 cm.

3. Mean "residence time" of adsorption $\tau$ was computed from the assigned $Q$ and the temperature $T$ for each 1 cm. interval; likewise the mean thermal speed $\bar{v} = \sqrt{8RT/\pi M}$ was computed for each interval. From this, for each 1 centimeter interval was computed the "effective velocity" for a random-walk step, $v_{\text{eff}} = \lambda/\tau + \lambda/\sqrt{v}$ and then the diffusion $D = \frac{1}{3}v_{\text{eff}}$. (Calculation of $D$ was actually done as a single step; $v_{\text{eff}}$ was never calculated separately.)

4. The probability $P(x)$ was calculated, for a value of $x = 1$ centimeter, for each isotope $^{219}\text{Rn}$, $^{220}\text{Rn}$, and $^{222}\text{Rn}$ over each depth interval. The probability that a radon atom starting at a depth of $n$ centimeters will reach the regolith surface is then computed as the product $\prod_{k=1}^{n} P(d=k \text{ cm})(x=1)$. The factor $\frac{1}{2}$ is needed because I want to consider only the radon atoms that reach the surface, not those that moved downward into the regolith from the starting plane. The validity of this procedure can be seen by examining the case where temperature is the same at all depths and noting that one can vary
x, and consequently the number of "steps", without changing the value of the probability of escape from any assigned depth. That is to say, it works because the behaviour of the exponential function makes the product taken in this way mathematically equivalent to a direct integration over the corresponding number of centimeters.

5. For each isotope, probability of escape was calculated for each successive centimeter of depth, and printed out, until the probability of escape had dropped below 1% or a depth of 4 meters—which was assumed to be the depth of the regolith\(^{18}\)—had been reached, whichever came first.

6. The probability of escape over this depth interval of significant loss was averaged, and this average printed out.

In addition to the computer program, hand calculations were done for certain cases down to 10 meters, ignoring the diurnal temperature variations in the top 12 centimeters and assuming a constant 200°K temperature at all depths.

Hand calculations have also been done for certain cases to check the effect of the "reflection" that would be imposed on the random walk if, at the bottom of the assumed regolith depth of 4 meters, there were a solid base of
impermeable bedrock. In this case,

\[
\text{Probability of escape from n cm, with reflection) \sim}
\]

\[
\text{(Probability of escape from n cm, without reflection) +}
\]

\[
\text{(Probability of escape from 4 meters, without reflection)}^2
\]

\[
\text{(Probability of escape from n cm, without reflection)}
\]

The first term on the right hand side approximately represents the probability that a radon atom will go directly out through the regolith without going down to bedrock. The second term approximates the probability that the atom first goes down to bedrock, is reflected, and then goes back up and out; it is the product of two factors: (1) the proportion of those atoms starting at n cm which get down as far as the 4-meter regolith base, and (2) the probability of escape from regolith base with reflection. Factor 1 = the proportion of those atoms starting at the base which reach up as far as n cm depth = \( \frac{1}{2} \) the probability of escape from regolith base without reflection + the probability of escape from n cm without reflection. Since factor 2 = 2 x the probability of escape from regolith base without reflection, the final result is the formula given.

Integration over all time at a given temperature profile as was done here is somewhat unrealistic, because the rotation of the Moon causes the profile at any given location to vary with a 28-day period. In other words, the
radon atom is never allowed to diffuse under the chosen conditions for an infinite length of time. But let us briefly discuss the consequences by considering a radon atom that began to diffuse at lunar noon, when the temperatures in the first 12 cm are relatively high. All those Rn atoms which find themselves in the 12 cm level at dark will be slowed down in this layer compared to noontime conditions. However, consider also a radon atom that is released at the same starting depth at lunar midnight. If it arrives in the top 12 cm layer when the daytime warming sets in, it has an increased probability of escape compared to midnight conditions. The two effects tend to cancel.

It is necessary to make a comment about boundary conditions. The treatment here has considered that there is no change in physical conditions at the upper surface of the regolith; that is, that at far as the mathematical treatment is concerned, there might as well still be soil above this level. If one assumes, however, that all radon that leaves the surface is unable to reenter the regolith, this imposes the boundary condition that the concentration of radon at the surface is at all times zero, because there is no return flux downward. The effect of this boundary condition is to precisely double the net flux outward at the surface, for
the unreflected cases. All escape probabilities given here for unreflected cases would then have to multiplied by two. This factor follows from the solution of the diffusion equations for a radon atom released at some depth in a regolith which is infinite downward and which is bounded at the top by a plane where the probability of finding the radon atom is zero at all times.* All calculations for reflected cases would be totally invalidated.

The significant aspect to the problem is that of the interaction between radon in the regolith and in the atmosphere. If the assumption of no return flux is valid, one can assume that during a significant portion of the lunar day and night, a quasi-equilibrium is reached in which the number of Rn\(^{222}\) atoms decaying in the atmosphere equals the number entering it from the regolith. These quasi-equilibria are perturbed mainly at sunset and sunrise. Since the Rn atoms perform ballistic "hops" in the atmosphere, there is at all times a downward flux of these atoms striking the surface of the regolith. With thermal velocities of about 200 meters per second, there is in the quasi-equilibrium a downward flux of Rn\(^{222}\) roughly \(10^4\) to \(10^5\) times as great as the outward flux of Rn\(^{222}\) just enter-

*I wish to thank Dr. R. A. Wolf for calling my attention to the consequences of this boundary condition.
ing the atmosphere from the regolith. We cannot accurately treat the probability of reentry of an atmospheric \( \text{Rn}^{222} \) atom into the regolith, but the very large ratio (in quasi-equilibrium) of \( \text{Rn}^{222} \) atoms striking regolith surface/\( \text{Rn}^{222} \) atoms leaving the regolith suggests that the assumption of zero inward flux at the regolith surface may be unrealistic. This in turn suggests that the escape probabilities for \( \text{Rn}^{222} \) lie somewhere between the values given in figures 7-20 and tables 2 and 3 (for unreflected cases) and twice these values.

The assumption of zero return flux at the surface may be more justified for \( \text{Rn}^{219} \) and \( \text{Rn}^{220} \), since typical times of flight of ballistically hopping radon atoms in the atmosphere are of the order of one or two minutes, which exceeds the half-lives (4.0 seconds and 51.5 seconds) of these two isotopes. Because of these short half-lives, the ratio of atoms returning to the regolith, compared with the number leaving the regolith, will always be much less for these two isotopes than for \( \text{Rn}^{222} \).

IV. Results.

The objective was to calculate radon loss probabilities for a column of regolith at a fixed locality on the Moon and fixed temperature profiles (lunar noon, lunar midnight, and
a small number of other solar angles), and then to find the **average** losses over a lunar day for that locality.

Results of the computer program showing escape probabilities vs. depth for Rn$^{219}$, Rn$^{220}$, and Rn$^{222}$, for a 4 meter regolith depth, as well as results of hand calculations for effects of reflection imposed at 4 meters, are summarized in figures 7 through 20 and in Table 2. Results for calculations down to 10 meters (no reflection) are summarized in Table 3.

A simple arithmetic average of the noon and midnight losses is used in Table 2 because various "weighted averages", accounting for more detailed variations of solar angle, were tried for the case of the greatest day-to-night change in loss rate ($Q = 7$ kcal/mole, $\lambda = 10$ microns), and were found to yield results not greatly different from the simple arithmetic average. Since it was found to be a fairly good approximation and was simple to calculate, the arithmetic average was used throughout. A principle reason this simple average works so well appears to be that the loss rate profile changes more rapidly than the temperature profile (not surprising in view of the fact that one of the parameters determining the diffusion coefficient, $\tau$, is an exponential function of $T$).
V. Discussion and Implications.

The first observation from figures 7-20 is that the lunar regolith acts as a strongly preferential filter for the three radon isotopes: losses of Rn$^{219}$ will always be small, even under the most favorable conditions; a slight amount of Rn$^{220}$ may get out, if conditions are favorable; and Rn$^{222}$ losses will always far exceed either or the other isotopes. Losses of Rn$^{222}$ can be quite significant indeed: for a $Q$ of 5.2 kcal/mole and a $\lambda$ of 10 microns, Table 3 shows a loss of better than 20% even from 10 meters depth. In fact either the heat of adsorption of radon must be high, or the mean free path of the random walk must be very short (or both), to prevent significant losses of Rn$^{222}$ into the atmosphere. This preferential filtration will act upon any radon which is produced within the regolith, or which is made available at its base.

This result is particularly interesting in the light of the observations by Turkevich et al. of alpha radioactivity at Surveyor landing sites. They calculate from their observations an emission rate of Rn$^{222}$ well exceeding that of Rn$^{220}$, a conclusion nicely consistent with the calculations in this paper. They expected a still higher ratio of emissivity of Rn$^{222}$ to Rn$^{220}$ if the Th/U ratio is near the value which they expected of 3. However, the results of the
calculations here indicate that if the mean free path of the random walk is large, emission ratios of $\text{Rn}^{222}$ to $\text{Rn}^{220}$ as low as 7.8 can be obtained during the lunar day (see figure 13, table 2). The suggestions put forth by Turkevich et al. are possible alternative explanations of the low ratio they observed: (1) higher radon production in Mare Tranquilitatis than in surrounding regions coupled with escape of part of the $\text{Rn}^{222}$ before its decays can take place, (2) $\text{Rn}^{222}$ produced in mineral sites more retentive than those of $\text{Rn}^{220}$, (3) longer half-life of $\text{Rn}^{222}$ permitting it to be preferentially removed from the lunar atmosphere by other processes, e.g. solar wind, before decaying.

The amounts of alpha activity observed by Turkevich et al. seem to indicate that at the locations of Surveyors 5, 6, and 7, the majority of radon atoms produced by uranium and thorium are not in mineral sites which permit them to begin the sort of random walk described here. For example at the Surveyor 5 site, if one assumes a uranium concentration of .5 ppm, a soil density of 1.5 gm/cm$^3$, a heat of adsorption of 5.2 kcal/mole, a mean free path of 10 microns, a regolith depth of 4 meters, and a temperature profile characteristic of lunar noon, one finds that if all radon atoms produced were able to undergo the random walk, one would expect an $\text{Rn}^{222}$ emission rate of 2.2 atoms/cm$^2$sec.
The value derived by Turkevich et al. of $58 \times 10^{-3}$ atoms/cm$^2$sec is less than 3% of this. And if one further assumes a Th/U ratio of 3, one would bet, on the basis of $100\%$ Rn engaged in random walk, an emission rate for Rn of $9.9 \times 10^{-2}$ atoms/cm$^2$sec, against which Turkevich et al. see only $8 \times 10^{-3}$ atoms/cm$^2$sec, only 8% of this.

Such low emanation rates from production sites are consistent with results obtained by Heymann and Yaniv (1971), who measured emanation rates of Rn and Rn from various Apollo samples, and Soenner et al. (1971), who observed Rn in the Apollo 11 sample return container. The results so far of Heymann and Yaniv indicate that the percentage of Rn and Rn that is able to reach void spaces in the soil is less than 0.5-1%; Stoenner et al. find the fraction of Rn emanating to be $2.3 \times 10^{-3}$. Although Adams et al. have measured an emanation to production ratio for Rn of roughly 0.48 from Apollo sample 12070, less confidence is placed in these results, as this experiment was done in air, whereas the measurements by Heymann and Yaniv were done vacuum ($10^{-2}$ mm Hg), and hence more closely simulate lunar conditions.

If there are any regions on the Moon where a substantial percentage of the radon produced is able to participate in the random walk, there will be another noteworthy effect:
since escape probabilities derived from this random walk model for Rn\textsuperscript{219} are very small, while Rn\textsuperscript{222} escape probabilities may be quite respectable, the subsequent loss of half the decay daughter atoms can yield a large fractional loss, ultimately, of Pb\textsuperscript{206} vs. virtually no loss for Pb\textsuperscript{207}, especially in the case that bedrock provides a layer of "reflection" at some depth in the regolith.

(Note on the effectiveness of alpha-decay for generating 50% loss of daughter atoms of radon decaying in the atmosphere: recoil velocities will be upward of 270 kilometers per second, exceeding escape velocity by 2 orders of magnitude. This recoil energy is also sufficient to overcome a heat of adsorption of up to 7 kcal/mole with negligible reduction in velocity.)

Paulo Barretto\textsuperscript{23,24} has noted that lead fractionation due to radon mobility can change the apparent lead/uranium ages of lunar samples. Details of the effects such lead fractionation may have on the apparent lead/uranium ages can be seen in figure 21. A 5.2 kcal/mole heat of adsorption and a 10 micron mean free path would yield (for a 4 meter thick regolith, ignoring reflection), as in Table 2, a 40% escape rate of Rn\textsuperscript{222} into the atmosphere (if 100% of the radon produced is able to reach void spaces and start the random walk), which means a 20% loss of the ultimate Pb\textsuperscript{206}.  

If one has a sample which without losses would have yielded an age of 3.5 billion years, such a 20% loss would lower its apparent Pb$^{206}$/U$^{238}$ age to about 2.9 billion years. This will also be a very discordant age, because the Pb$^{207}$/U$^{235}$ age will still be near 3.5 billion years.

Take also the case where one had a set samples made from a mixture of 2 components; say one of 2 billion years and another of 4 billion years. Without lead losses, ages found for these samples would lie on a line forming a "chord" intersecting the concordia curve at the 2 billion year point and the 4 billion year point. If one then has the same 20% loss of Pb$^{206}$ as above, and if this loss is in proportion for all points of the chord, the resulting displaced points will form a new chord which will intersect the concordia near the 1.5 billion year point and near the 4.5 billion year point. (See fig. 21.) In other words, it still appears to be a mixture, but the older component will appear to be older, and the younger to be younger, than they would in the absence of losses.

These trends can be expected to hold if the region of interest on the Moon produces radon at a rate comparable to or greater than surrounding neighboring regions. If, however, the production rate of radon is much less than in the region's surroundings, the ballistic hopping of radon in
the atmosphere might bring in enough Rn\textsuperscript{222} so that even with the subsequent loss of half the decay daughters, the net effect is an increase of Pb\textsuperscript{206} in the region. The effects of such an increase can again be seen in figure 21. A 20% Pb\textsuperscript{206} gain in a sample of 3.5 billion years' actual age will raise its apparent Pb\textsuperscript{206}/U\textsuperscript{238} age to just over 4 billion years. A similar gain at all points of a 2 billion/4 billion year chord will raise that "chord" entirely above the concordia. This possibility of discordant ages is interesting in the light of apparent ages found by Wanless et al. for Apollo 11 type D fines sample 10084.\textsuperscript{25} They find a Pb\textsuperscript{206}/U\textsuperscript{238} age of 5.41 \times 10^9 years, but a Pb\textsuperscript{207}/U\textsuperscript{235} age of 4.885 \times 10^9 years, a combination that plots well above the concordia. Influx of Rn\textsuperscript{222} emitted via the random walk from the surrounding areas cannot be wholly responsible for these apparent ages, because even the Pb\textsuperscript{207}/U\textsuperscript{235} age is higher than can reasonably be expected, but the age difference suggests it may have been a contributing process.

This model may prove of interest for regions of the Moon where soils are similar chemically to the local rocks, but appear to have different Pb\textsuperscript{206}/U\textsuperscript{238} ages, since radon formed within a rock is unlikely to be near enough to its surface to enter the random walk, and since daughters of Rn\textsuperscript{222} that "immigrated" from neighboring regions can only be
implanted at the surface, making only insignificant contributions to large rocks. Thus a difference in Pb$^{206}/u^{238}$ ages between rocks and surrounding soil does not by itself preclude that soil from having formed from those types of rocks.

While this paper has made no consideration of any other processes by which lead or uranium may be lost or added to samples under lunar conditions, and which may bring about lead/uranium compositional oddities of their own, this model may still account for differences between rocks and soils of a given region, if their compositions are alike except for the Pb$^{206}/u^{238}$ ratio. One could then suppose that other process(es) operated in the initial formation of the mineralogy of the region to create the original "oddness", and that as the regolith formed, radon diffusion and consequent Rn$^{222}$ loss began to operate, providing the difference between soil and rock.

It could be pointed out that while this model can account for losses of Pb$^{206}$, or if there has been an influx of radon similarly produced in surrounding regions, for gains of Pb$^{206}$, if a sample appears to have an increased Pb$^{207}$ composition, such as do Apollo 11 fines analyzed by Silver$^{26}$, any radon contributing to this must have been produced by some entirely different process: volcanic
venting, for example, or some special event early in the history of the Moon. For no radon which undergoes the selective filtration which a slow "percolation" through the regolith, as in this model, will give it, will succeed in liberating major amounts of Rn\textsuperscript{219}. On the other hand, radon which is released by volcanic or other large-scale venting would probably be transported to the surface on a time scale markedly shorter than the hours or days typically required by this random walk model, opening the possibility for some Rn\textsuperscript{219} escape.

It should also be repeated that the question of lead/uranium dating will only be of importance for regions on the Moon, should any be found, where significant proportions of the radon atoms produced are able to initiate the random walk. In all other regions of the Moon the feature for which this model will have primary relevance will be the alpha-radioactive deposit on the regolith surface.

Among the measurements and experiments which this problem suggests, or which would be valuable for corroborating or refuting the results predicted, are:

1. Measurement of the heat of adsorption for radon, preferably a heat of adsorption on lunar samples, or on materials of similar composition.

2. Measurement of lunar surface and subsurface
temperatures vs. depth and solar angle. This is now being done by the temperature probe left at the Apollo 15 site, and will be supplemented by the same means on Apollos 16 and 17.

3. Better measurements of distributions of particle number vs. particle size, so that one might decide how meaningful the mean free path values chosen in this paper are.

4. More precise measurements of the percentages of radon atoms that can escape their production sites to enter the random walk in various uranium-bearing and thorium-bearing lunar minerals.
Appendix I: Symbols Used

λ = mean free path of random walk through regolith

x = depth interval

τ = mean "residence time" that a particle remains adsorbed when it encounters a solid surface in the regolith = \( \frac{Q}{RT} \)

\( \tau_o \) = constant, taken to be 10^{-13} second

Q = heat of adsorption

\( \bar{v} \) = mean thermal speed = \( \sqrt{\frac{8RT}{\pi M}} \)

M = gram-molecular weight of radon

t\(_{\frac{1}{2}}\) = half-life of whichever radon isotope is under consideration

\( v_{\text{eff}} \) = effective velocity for the random walk = \( \frac{\lambda}{\tau + \lambda/\bar{v}} \)

D = diffusion coefficient = \( \frac{1}{3} \lambda v_{\text{eff}} \)

\( J(x,t) \) = flux of radon of a given isotope away from its plane of origin at some depth \( x \) above and below its original depth = \( \frac{x}{2t\sqrt{\pi Dt}} \) \( e^{-x^2/4Dt} - t \cdot \log e 2/t_{\frac{1}{2}} \)

(Normalized to initial number density of 1.)

\( P_d(x) \) = probability that a radon atom of a given isotope has made it a distance \( x \) from an initial depth \( d = e^{-x/\log_e 2/Dt_{\frac{1}{2}}} \)

R = universal gas constant

T = temperature
Appendix II: Fortran Program for Calculating Probability of Escape of Radon from Lunar Regolith as Function of Depth

I. Temperature assignment subroutine.

A. For lunar noon and lunar midnight runs.

SUBROUTINE TFUNCT(T, DELTA, TSURF)

DIMENSION T(13)

DO 105 N=1, 12

DEPTH2=FLOAT(N)*DELTA

105 T(N)=200.+(TSURF-200.)*EXP(-DEPTH2/4.5)

T(13)=200.

RETURN

END

B. For lunar sunset runs.

SUBROUTINE TFUNCT(T, DELTA, TSURF)

DIMENSION T(13)

T(1)=192.; T(2)=243.; T(3)=267.; T(4)=277.; T(5)=279.;
T(6)=273.
T(7)=266.; T(8)=257.; T(9)=249.; T(10)=232.; T(11)=231.;
T(12)=227.

T(13)=200.

RETURN

END

C. For solar angle 135°.

SUBROUTINE TFUNCT(T, DELTA, TSURF)
DIMENSION T(13)

T(1)=126.; T(2)=154.; T(3)=175.; T(4)=196.; T(5)=217.;
T(6)=233.

T(7)=247.; T(8)=256.; T(9)=257.; T(10)=251.; T(11)=272.;
T(12)=233.

T(13)=200.

RETURN

END

II. Main program.

ALAMBD=.008

Note: this assigns $\lambda$ (in cm.) for a given run.
Alambd may be .008, .001, or .0001.

DELTA=1.

DIMENSION T(13), D(13), THALF(3), PDDEL(13)

TSURF=400.; CALL TFUNCT(T, DELTA, TSURF)

DO 16 I=1, 3

16 D(I)=1./3.*ALAMBD**2/(1.E-13*EXP(2.2E11/8.31E7/T(I))
+ALAMBD/SQRT(8)

Note: may have 1.7E11, 2.2E11, or 2.9E11,
depending on the value of Q for the run.

2.*8.31E7*T(I)/3.14159/222.))

THALF(1)=4.0; THALF(2)=3.3E5; THALF(3)=51.5

18 FORMAT(36H0PROBABILITY OF ESCAPE OF RN219 FROMF6.1,
6H CM ISF6.3,11

2H ON DAYSIDE)

20 FORMAT(36H0PROBABILITY OF ESCAPE OF RN222 FROMF6.1
6H CM ISF6.3,11
2H ON DAYSIDE)

22 FORMAT(36H0PROBABILITY OF ESCAPE OF RN220 FROM 6.1, 6H CM ISF 6.3, 11 6H CM)

2H ON DAYSIDE)

24 FORMAT(/59H0MAX DEPTH OF SIGNIFICANT LOSS OF THIS ISOTOPE ON DAYSIDE)

2D E=F6.1/43H AVERAGE LOSS PROBABILITY OVER THIS DEPTH =F6.3///

Note: formats were changed as appropriate to indicate Sunset and Solar Angle 135° runs.

DO 43 I=1, 3

FLUP=1.; PDQ=0.

DO 35 J=1, 13

PDDEL(J)=EXP(-DELTA*SQRT(ALOG(2.)/D(J)/THALF(I)))

FLUP=FLUP*PDDEL(J); PD=FLUP/2.; PDQ=PDQ+PD; DEPTH=FLOAT(J)*DELTA

GO TO (32, 33, 34), I

32 PRINT 18, DEPTH, PD; GO TO 10034

33 PRINT 20, DEPTH, PD; GO TO 10034

34 PRINT 22, DEPTH, PD

10034 IF(PD<.01)GO TO 41

35 CONTINUE

36 DEPTH=DEPTH+DELTA; FLUP=FLUP*PDDEL(13); PD=FLUP/2.; PDQ=PDQ+PD

IF(DEPTH>400.)GO TO 41

GO TO (37, 38, 39), I

37 PRINT 18, DEPTH, PD; GO TO 40
38 PRINT 20, DEPTH, PD; GO TO 40
39 PRINT 22, DEPTH, PD
40 IF (PD = .01) GO TO 36
41 AVE = PDQ / DEPTH
   PRINT 24, DEPTH, AVE
43 CONTINUE

   Note: program stopped at this point for Sunset and Solar Angle 135° runs.

   TSURF = 100.; CALL TFUNCT(T, DELTA, TSURF)
   DO 47 I = 1, 13
47 D(I) = 1. / 3. * ALAMBD**2 / (1.E-13 * EXP(2.2E11 / 8.31E7 / T(I)) + ALAMBD / SQRT(8
   2. * 8.31E7 * T(I) / 3.14159 / 222.))
48 FORMAT(36H0PROBABILITY OF ESCAPE OF RN219 FROM 6.1,
   5H CM =F6.3, 13H
   2 ON NIGHTSIDE)
50 FORMAT(36H0PROBABILITY OF ESCAPE OF RN222 FROM 6.1,
   5H CM =F6.3, 13H
   2 ON NIGHTSIDE)
52 FORMAT(36H0PROBABILITY OF ESCAPE OF RN220 FROM 6.1,
   5H CM =F6.3, 13H
   2 ON NIGHTSIDE)
54 FORMAT(61H0MAX DEPTH OF SIGNIFICANT LOSS OF THIS ISOTOPE ON NIGHT
   2SIDE =F6.1/43H AVERAGE LOSS PROBABILITY OVER THIS DEPTH =F6.3///)
FLUP=1.;PDQ=0.

DO 65 J=1,13
PDDEL(J)=EXP(-DELTA*SQRT(ALOG(2.)/D(J)/THALF(I)))
FLUP=FLUP*PDDEL(J);PD=FLUP/2.;PDQ=PDQ+PD;DEPTH=
FLOAT(J)*DELTA

GO TO(62,63,64),I

62 PRINT 48,DEPTH,PD;GO TO 10064
63 PRINT 50,DEPTH,PD;GO TO 10064
64 PRINT 52,DEPTH,PD

10064 IF(PD<.01)GO TO 71

CONTINUE

65 DEPTH=DEPTH+DELTA;FLUP=FLUP*PDDEL(13);PD=FLUP/2.;
PDQ=PDQ+PD

IF(DEPTH>400.)GO TO 71

GO TO(67,68,69),I

67 PRINT 48,DEPTH,PD;GO TO 70
68 PRINT 50,DEPTH,PD;GO TO 70
69 PRINT 52,DEPTH,PD
70 IF(PD=.01)GO TO 66

71 AVE=PDQ/DEPTH

PRINT 54,DEPTH,AVE

73 CONTINUE

STOP

END
Acknowledgments

I wish especially to thank my advisor, Dr. Dieter Heymann, for the suggestion that led to this investigation, and for the many fruitful discussions we have had. I also wish to thank Mrs. Jewell White for her typing.

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References


3. John W. K. Freeman, private communication


15. D. O. Gough, private communication


Table 1

Sticking time $\tau = \tau_0 e^{Q/RT}$ as function of $Q$ and $T$, for $\tau_0 = 1 \times 10^{-13}$ sec.

<table>
<thead>
<tr>
<th>$T$</th>
<th>$100^\circ K$</th>
<th>$200^\circ K$</th>
<th>$400^\circ K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 kcal/mole</td>
<td>$5.25 \times 10^{-5}$ sec</td>
<td>$2.29 \times 10^{-9}$ sec</td>
<td>$1.52 \times 10^{-11}$ sec</td>
</tr>
<tr>
<td>5.2 kcal/mole</td>
<td>$2.51 \times 10^{-2}$ sec</td>
<td>$4.90 \times 10^{-8}$ sec</td>
<td>$7.0 \times 10^{-11}$ sec</td>
</tr>
<tr>
<td>7 kcal/mole</td>
<td>$2.04 \times 10^{-2}$ sec</td>
<td>$4.57 \times 10^{-6}$ sec</td>
<td>$6.6 \times 10^{-10}$ sec</td>
</tr>
</tbody>
</table>

$R = 8.31 \times 10^7$ erg (g mole)$^{-1}$ deg$^{-1}$

4 kcal/mole = $1.67 \times 10^{11}$ erg/mole

5.2 kcal/mole = $2.18 \times 10^{11}$ erg/mole

7 kcal/mole = $2.93 \times 10^{11}$ erg/mole
### Table 2

Average escape probabilities for assumed regolith depth of 4 meters

#### Radon 222

<table>
<thead>
<tr>
<th>Q</th>
<th>λ=1μ</th>
<th>10μ</th>
<th>80μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0 kcal/mole</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>.436 (.77)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>.434 (.77)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>.435 (.77)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.2 kcal/mole</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>.187</td>
<td>.420 (.72)</td>
<td>.476 (.91)</td>
</tr>
<tr>
<td>M</td>
<td>.085</td>
<td>.389 (.67)</td>
<td>.471 (.90)</td>
</tr>
<tr>
<td>A</td>
<td>.136</td>
<td>.405 (.69)</td>
<td>.474 (.905)</td>
</tr>
<tr>
<td>7.0 kcal/mole</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>.225</td>
<td></td>
<td>.441 (.78)</td>
</tr>
<tr>
<td>M</td>
<td>.024</td>
<td></td>
<td>.335 (.60)</td>
</tr>
<tr>
<td>A</td>
<td>.125</td>
<td></td>
<td>.388 (.69)</td>
</tr>
</tbody>
</table>

Figures in parentheses are based on assumption of impermeable bedrock at 4 meters depth.

#### Radon 220

<table>
<thead>
<tr>
<th>Q</th>
<th>λ=1μ</th>
<th>10μ</th>
<th>80μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0 kcal/mole</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>.022</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>.014</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>.018</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.2 kcal/mole</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>.0057</td>
<td>.019</td>
<td>.061</td>
</tr>
<tr>
<td>M</td>
<td>0</td>
<td>0</td>
<td>.030</td>
</tr>
<tr>
<td>A</td>
<td>.0029</td>
<td>.010</td>
<td>.046</td>
</tr>
<tr>
<td>7.0 kcal/mole</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>.010</td>
<td></td>
<td>.030</td>
</tr>
<tr>
<td>M</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>.005</td>
<td></td>
<td>.015</td>
</tr>
</tbody>
</table>

N = lunar noon, M = lunar midnight, A = arithmetic average
Table 2 (Continued)

Average ratio of escape probability, $\frac{R_{n\ 222}}{R_{n\ 220}}$.

<table>
<thead>
<tr>
<th>Q</th>
<th>$\lambda=1\mu$</th>
<th>10$\mu$</th>
<th>80$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0 kcal/mole</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>20:1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>31:1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>24:1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.2 kcal/mole</td>
<td>33:1</td>
<td>22:1</td>
<td>7.8:1</td>
</tr>
<tr>
<td>M</td>
<td>?</td>
<td>?</td>
<td>16:1</td>
</tr>
<tr>
<td>A</td>
<td>45:1</td>
<td>41:1</td>
<td>10:1</td>
</tr>
<tr>
<td>7.0 kcal/mole</td>
<td>23:1</td>
<td>15:1</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>?</td>
<td>?</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>25:1</td>
<td>23:1</td>
<td></td>
</tr>
</tbody>
</table>
Table 3

Probabilities of escape for Radon$^{222}$ from various depths, down to 10 meters, without reflection and assuming a constant regolith temperature (no diurnal variation) of 200°K.

$Q = 4\text{kcal/mole}, \lambda = 1\mu$

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>.186</td>
</tr>
<tr>
<td>5</td>
<td>.146</td>
</tr>
<tr>
<td>6</td>
<td>.114</td>
</tr>
<tr>
<td>7</td>
<td>.089</td>
</tr>
<tr>
<td>8</td>
<td>.069</td>
</tr>
<tr>
<td>9</td>
<td>.054</td>
</tr>
<tr>
<td>10</td>
<td>.042</td>
</tr>
</tbody>
</table>

$Q = 4\text{kcal/mole}, \lambda = 10\mu$

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>.380</td>
</tr>
<tr>
<td>5</td>
<td>.355</td>
</tr>
<tr>
<td>6</td>
<td>.332</td>
</tr>
<tr>
<td>7</td>
<td>.320</td>
</tr>
<tr>
<td>8</td>
<td>.289</td>
</tr>
<tr>
<td>9</td>
<td>.270</td>
</tr>
<tr>
<td>10</td>
<td>.252</td>
</tr>
</tbody>
</table>
Table 3 (Continued)

\[ Q = 4 \text{kcal/mole}, \lambda = 80\mu \]
\[ P(4m) = .454 \]
\[ P(5m) = .443 \]
\[ P(6m) = .432 \]
\[ P(7m) = .422 \]
\[ P(8m) = .412 \]
\[ P(9m) = .402 \]
\[ P(10m) = .392 \]

\[ Q = 5.2 \text{kcal/mole}, \lambda = 1\mu \]
\[ P(4m) = .045 \]
\[ P(5m) = .025 \]
\[ P(6m) = .014 \]
\[ P(7m) = .007 \]

\[ Q = 7 \text{kcal/mole}, \lambda = 80\mu \]
\[ P(4m) = .376 \]
\[ P(5m) = .35 \]
\[ P(6m) = .326 \]
\[ P(7m) = .303 \]
\[ P(8m) = .282 \]
\[ P(9m) = .263 \]
\[ P(10m) = .245 \]
Heats of adsorption vs. atomic weight for noble gases.

Figure 1
Heats of adsorption vs. atomic weight for noble gases. (Log-log plot.)
Diffusion coefficient $D$ vs. temperature, mean free path = 10 microns.
Figure 4

Diffusion coefficient $D$ vs. temperature, mean free path = 1 micron.
Surface and subsurface temperatures, $T$, at points in the lunar thermal equator, computed as a function of the phase. The numbers for the individual curves refer, in each case, to the depth below the surface for which the computation was made. From unpublished calculations by D. O. Gough.
Figure 6b
(Derived from Figure 6a.)

Temperature profiles vs. depth at various solar angles. Numbers in parentheses are values of solar angle.
Figures 7-20 are plots of probability of escape of radon from lunar regolith vs. depth of origin, under the conditions specified in each figure, using various representative points from the computer printouts. (Plus hand calculations for cases of reflection.)
Assumed mean free path $\lambda=10$ microns  
Assumed heat of adsorption $Q=4$ kcal/mole

Figure 8

$Rn^{222}$ corrected for "reflection" at 4 meters
$Rn^{222}$ uncorrected for "reflection"
Figure 9

Assumed mean free path $\lambda = 1$ micron
Assumed heat of adsorption $Q = 5.2$ k.cal/mole
Figure 10

Assumed mean free path $\lambda = 1$ micron
Assumed heat of adsorption $Q = 5.2 \text{k.cal/mole}$
Figure 11.

Assumed mean free path $\lambda = 10$ microns
Assumed heat of adsorption $Q = 5.2$ k.cal/mole

$\text{Rn}^{222}$ corrected for "reflection" at 4 meters

$\text{Rn}^{222}$ uncorrected for "reflection"

$\text{Rn}^{219}$

$\text{Rn}^{220}$
Assumed mean free path $\lambda = 10$ microns
Assumed heat of absorption $Q = 5.2$ k.cal/mole

Figure 12

Assumed mean free path $\lambda = 10$ microns
Assumed heat of absorption $Q = 5.2$ k.cal/mole

Rn$^{222}$ corrected for "reflection" at 4 meters
Rn$^{222}$ uncorrected for "reflection"
Figure 13

Assumed mean free path $\lambda = 80$ microns
Assumed heat of absorption = 52 k.cal/mole

- Rn$^{222}$, corrected for "reflection" at 4 meters
- Rn$^{222}$, uncorrected for "reflection"

Lunar noon
Assumed mean free path $\lambda = 80$ microns
Assumed heat of adsorption $Q = 5.2 \text{ k.cal/mole}$

Figure 14

- $\text{Rn}^{222}$, corrected for "reflection" at 4 meters
- $\text{Rn}^{222}$, uncorrected for "reflection"

Lunar midnight
Figure 15

Assumed mean free path $\lambda = 10$ microns
Assumed heat of adsorption $Q = 7$ k.cal/mole

Probability of escape

---

Rn$^{222}$, corrected for "reflection" at 4 meters

Rn$^{222}$, uncorrected for "reflection"

---

depth
Assumed mean free path $\lambda = 10$ microns
Assumed heat of adsorption $Q = 7 \text{k.cal/mole}$
Figure 17

Assumed mean free path $\lambda = 10$ microns
Assumed heat of adsorption $Q = 7\, \text{kcal/mole}$

solar angle 135°
(45° after sunset)

$\text{Rn}^{222}$ uncorrected for "reflection"
Assumed mean free path $\lambda = 10$ microns
Assumed heat of adsorption $Q = 7$ kcal/mole

$^{222}\text{Rn}$ curves, "reflected" and "unreflected", are very nearly same as lunar noon.
Figure 19

Assumed mean free path $\lambda = 80$ microns
Assumed heat of adsorption $Q = 7 \text{k.cal/mole}$

- $^{222}\text{Rn}$ corrected for "reflection" at 4 meters
- $^{222}\text{Rn}$ uncorrected for "reflection"

depth

meters

probability of escape
Assumed mean free path $\lambda = 80$ microns
Assumed heat of adsorption $Q = 7$ k.cal/mole
Figure 21

Condordia plot of $\frac{\text{Pb}^{206}}{\text{U}^{238}}$ ratio vs. $\frac{\text{Pb}^{207}}{\text{U}^{235}}$ ratio, showing effects of Pb gains and losses on apparent ages.
Figure 22a

Decay of $^{238}\text{U}$ to $^{206}\text{Pb}$

$^{238}\text{U} \rightarrow \alpha + ^{234}\text{Th}$

$^{234}\text{Th} \rightarrow ^{234}\text{Pa} \rightarrow \beta - + ^{234}\text{U}$

$^{234}\text{U} \rightarrow \alpha + ^{230}\text{Th}$

$^{230}\text{Th} \rightarrow ^{226}\text{Ra} \rightarrow ^{222}\text{Rn} \rightarrow ^{220}\text{Rn} \rightarrow \alpha + ^{218}\text{Po}$

$^{218}\text{Po} \rightarrow \alpha + ^{214}\text{Pb}$

$^{214}\text{Pb} \rightarrow \beta - + ^{214}\text{Bi}$

$^{214}\text{Bi} \rightarrow ^{214}\text{Po} \rightarrow \alpha + ^{210}\text{Pb}$

$^{210}\text{Pb} \rightarrow \beta - + ^{210}\text{Tl}$

$^{210}\text{Tl} \rightarrow ^{210}\text{Bi} \rightarrow \alpha + ^{206}\text{Pb}$
Figure 22b

Decay of U\textsuperscript{235} to Pb\textsuperscript{207}

\begin{align*}
92\text{U}^{235} & \rightarrow \alpha + 90\text{Th}^{231} \\
90\text{Th}^{231} & \rightarrow \beta^- + 91\text{Pa}^{231} \\
91\text{Pa}^{231} & \rightarrow \alpha + 89\text{Ac}^{227} \\
99\% \\
89\text{Ac}^{227} & \rightarrow \beta^- + 90\text{Th}^{227} \\
90\text{Th}^{227} & \rightarrow \alpha + 88\text{Ra}^{223} \\
88\text{Ra}^{223} & \rightarrow \alpha + 86\text{Rn}^{219} \\
86\text{Rn}^{219} & \rightarrow \alpha + 84\text{Po}^{215} \\
99\% \\
84\text{Po}^{215} & \rightarrow \alpha + 82\text{Pb}^{211} \\
82\text{Pb}^{211} & \rightarrow \beta^- + 83\text{Bi}^{211} \\
83\% \\
83\text{Bi}^{211} & \rightarrow \alpha + 81\text{Tl}^{207} \\
81\text{Tl}^{207} & \rightarrow \beta^- + 82\text{Pb}^{207}
\end{align*}

\begin{align*}
1\% \\
89\text{Ac}^{227} & \rightarrow \alpha + 87\text{Fr}^{223} \\
99\% \\
87\text{Fr}^{223} & \rightarrow \beta^- + 88\text{Ra}^{223} \\
87\text{Fr}^{223} & \rightarrow \alpha + 85\text{At}^{219} \\
97\% \\
85\text{At}^{219} & \rightarrow \alpha + 83\text{Bi}^{215} \\
83\text{Bi}^{215} & \rightarrow \beta^- + 84\text{Po}^{215} \\
.005\% \\
84\text{Po}^{215} & \rightarrow \alpha + 85\text{At}^{215} \\
85\text{At}^{215} & \rightarrow \alpha + 83\text{Bi}^{211} \\
17\% \\
83\text{Bi}^{211} & \rightarrow \beta^- + 84\text{Po}^{211} \\
84\text{Po}^{211} & \rightarrow \alpha + 82\text{Pb}^{207}
\end{align*}
Figure 22c

Decay of Th$^{232}$ to Pb$^{208}$

\[ 90\text{Th}^{232} \rightarrow \alpha + 88\text{Ra}^{228} \]
\[ 88\text{Ra}^{228} \rightarrow \beta^- + 89\text{Ac}^{228} \]
\[ 89\text{Ac}^{228} \rightarrow \beta^- + 90\text{Th}^{228} \]
\[ 90\text{Th}^{228} \rightarrow \alpha + 88\text{Ra}^{224} \]
\[ 88\text{Ra}^{224} \rightarrow \alpha + 86\text{Rn}^{220} \]
\[ 86\text{Rn}^{220} \rightarrow \alpha + 84\text{Po}^{216} \]
\[ 84\text{Po}^{216} \rightarrow \alpha + 82\text{Pb}^{212} \]
\[ 82\text{Pb}^{212} \rightarrow \beta^- + 83\text{Bi}^{212} \]

\[ \text{65\%} \]
\[ 83\text{Bi}^{212} \rightarrow \beta^- + 84\text{Po}^{212} \]
\[ 84\text{Po}^{212} \rightarrow \alpha + 82\text{Pb}^{208} \]

\[ \text{35\%} \]
\[ 83\text{Bi}^{212} \rightarrow \alpha + 81\text{Tl}^{208} \]
\[ 81\text{Tl}^{208} \rightarrow \beta^- + 82\text{Pb}^{208} \]