KANT'S MATHEMATICAL SYNTHESIS

by
Gary Martin Seay

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J.S. Fricton

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ABSTRACT

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In the Critique of Pure Reason, Kant advances the notion that there are certain kinds of judgment which are distinctly 'mathematical' in character. These 'mathematical judgments' are not confined solely to the realm of arithmetic and geometry, furthermore, but can in fact be discovered as part of every true judgment about the world: they are at the very core of the synthetic a priori judgments in which the world actually comes to be known to us, Kant believed, because the Transcendental Synthesis (the judgment considered as a whole) always takes 'mathematical judgments' as logically required in 'dynamical judgments.'

The purpose of this essay is to show that Kant regarded the mathematical judgment as basic in the Transcendental Synthesis, because judgments making use of the 'dynamical' categories do in fact presuppose judgments employing 'mathematical' categories. Furthermore, I hold that these two types of 'judgments'—which together
comprise the Transcendental Synthesis— are, according to Kant, synthetic in two different ways. The crucial distinction between mathematical and dynamical synthesis in the Kantian theory of judgment is treated at length, and mathematical synthesis is shown to be the more elementary of the two. The mathematical synthesis is a kind of simple composition from logically unrelated elements which, for Kant, represents the most fundamental sense of juxtaposition possible in the synthetic a priori judgment.

The program of this essay is to examine Kant's theory of judgment as presented in the Transcendental Analytic and his discussion of the 'mathematical method' in the Discipline of Pure Reason, and thereby to make clear his notion of the mathematical synthesis as a synthesis which is 'fundamental,' both in its manner of combination and in terms of the manifold which it combines.
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PART ONE
I. INTRODUCTION

In the collection of his lectures published in 1962 as Die Frage nach dem Ding, Heidegger makes some interesting observations about the character of synthetic judgments in the critical philosophy of Kant. Focusing his attention primarily on the Transcendental Analytic in the Critique of Pure Reason, Heidegger attempts to set out a systematic exposition of all the Principles of the Pure Understanding. The Principles are, of course, those judgments with which Kant intended to demonstrate the validity of the categories as those Concepts which alone make possible objective statements about the world. Heidegger's discussion is particularly interesting in its treatment of the Principles Kant named 'Mathematical': the Axioms of Intuition and the Anticipations of Perception. Kant makes clear that they are called 'mathematical' because of their application, not because of their content, and Heidegger attempts to show just what the 'mathematical' application of judgment is. The sort of synthesis going on in mathematical judgments is, as Heidegger rightly points out, altogether different from that which takes
place in the judgments under the 'dynamical' categories, and part of the purpose of this thesis will be to make this distinction plain.

Heidegger's remarks on the mathematical judgments in the context of his discussion of the Analytic of Principles leaves room for a more specialized investigation of the role of the mathematical synthesis in the Critique of Pure Reason: we shall take into account Kant's treatment of the mathematical form of judgment in the Transcendental Doctrine of Method as well as in the Transcendental Analytic itself. The mathematical synthesis deserves a more thorough explanation in terms of Kant's doctrine of judgment, and that will be the essential task of this essay.

For the sake of clarity and coherence in the main section of the discussion in Part Two, we shall have to spend the first part of the paper dealing with the program of the Critique of Pure Reason within which the theory of judgment arises: we shall be especially concerned with the related topics of intuition, concept, judgment, and objectivity. We shall have to have a
reasonably clear understanding of Kant's doctrine of judgment in general before proceeding to examine his use of judgments in the Transcendental Analytic, and before we can consider the special function of mathematical judgments in particular.

It is also desirable that something be said, at the outset, about the purpose and scope of the Critique of Pure Reason, since this has been a point of sharp disagreement among philosophers since Heidegger's publication of Kant and the Problem of Metaphysics in 1929. While having no desire to enter into the thick of the debate on the Heidegger thesis, I think that it would be safe to say that he has provided some possible alternatives to the philosophic interpretation, primarily in England and America (and among Continental Neo-Kantians), which has tended to see the first Critique as 'theory of knowledge' simply—and nothing more. Kant himself referred to the Critique of Pure Reason, in the Preface to the first edition, as metaphysics in the more general sense:

"It (metaphysics)... is nothing but the inventory, systematically arranged, of all our possessions through pure reason."
Kantian 'metaphysics,' in the new meaning of the term given in the *Critique of Pure Reason*, must be understood as being at once both epistemology and ontology. That there can be a 'Copernican Revolution' in philosophy indicates that, for Kant, questions of the sort, "What can I know?" are inseparable from questions about what there is.

Kant's project in the first *Critique* is to deal with both of these questions, as it were simultaneously, by formulating a coherent theory of judgment. The problem of being able to say what sorts of objective things (occurrences, facts) there are in the world— an ontological question— is in fact a problem of being able to make objective judgments about the world— for Kant, a logical question (or, more properly, 'transcendental'). On the solution of this latter problem rests our understanding of the possibility of objective human knowledge. Kant recognized that a new definition of the objective judgment itself might hold the key to this solution. Accordingly, he proposed his famous 'Copernican Revolution' in metaphysics: rather than assume, as previously,
that all (true) knowledge must conform to objects, we should instead consider objects as conforming to our knowledge. Thus, judgments about the world must themselves be the source of whatever objectivity our knowledge can possess. The 'object,' as a knowable fact, cannot exist before the 'objective' judgment. Objective judgments are, of course, those which are made in accordance with the a priori conditions of space and time, as expressed in the categories. The precise function of concepts in judgment, and particularly the role of the categories, we shall attempt to make clear in the course of our discussion. Clarity on this point will be especially important for our treatment of the Mathematical Principles in the Transcendental Analytic.

It should be noted, however, that this clarification will be incidental to our primary purpose, which is an examination of the mathematical synthesis. A thorough exposition of Kant's doctrine of judgment, as of his theory of concepts, would be a volume in itself. We shall try, then, only to obtain a sufficient clarity and consistency in our use of key terms in Kant's system.
to be able, to some extent, to discuss the mathematical synthesis in his own terms. Our discussions of space and time, intuition, concepts, and the forms of judgment will be to that end.

I hope to show in this thesis that the mathematical synthesis is significantly different from the dynamical, in terms of both (a) how it synthesizes and (b) what it synthesizes. Accordingly, mathematical judgments will be seen to perform a different kind of function in the Transcendental Analytic from dynamical judgments. We might, in a sense, talk about mathematical judgments as performing a synthesis on a different 'level' from the dynamical, and with a rather arbitrary character of juxtaposition which is quite unlike anything to be found in the realm of dynamical judgments.

Kant remarks in his Introduction to the Analytic of Principles that it is only because they are synthetic a priori judgments-- according to his classification-- that he places the mathematical judgments with the other Principles of the Pure Understanding instead of in the Transcendental Aesthetic. They will, in fact, be seen
to express the most basic sense in which space and time can be understood, namely, in terms of things. We never encounter 'space' or 'time' as such in the world, but only the spatiality of things. We understand spatiality and temporality first of all, therefore, in terms of measure—amount, extent, enumeration, degree. Mathematical judgments, in the most general sense, are 'measurable' judgments, and for this reason find numerical (or geometric) expression most convenient. The systematic composition of arithmetical quantities or geometric forms is a synthesis of bare, elemental units—numbers in arithmetic, lines and points in geometry—and this is the synthesis that Kant calls 'mathematical.' Judgments of mathematics are 'mathematical' because they synthesize in this way. Indeed any judgments which employ this form of synthesis will be, according to Kant's definition, mathematical judgments.

By comparing these judgments with the 'judgments of physics,' which Kant calls dynamical, it will be seen that the 'synthetic unity' in a mathematical judgment is fundamentally synthetic, in a way that the synthetic
unity in a dynamical judgment (e.g. one of causality) is not. It will become clear, at last, that the mathematical synthesis represents the most fundamental notion of synthesis expressed in the Critique of Pure Reason.

NOTES TO CHAPTER I


4 Kant, Critique of Pure Reason, A xx.

5 Ibid., B xvii.

6 That Kant implies just this in the Transcendental Doctrine of Method we hope to show later.

7 Kant, Critique of Pure Reason, A149/B189.
II. SPACE AND TIME

The first stage of our discussion will deal with the source of possible judgments about the world: with what Kant calls 'intuition.' In the next chapter a closer looks will be taken at the term 'judgment' itself. Kant's desire to distinguish his philosophy from Berkeley's idealism or Hume's skepticism is well known, but, as Professor Körner points out, he was equally apprehensive about being read as a Leibnizian. Certainly in his treatment of the origins of knowledge, he owes more to the Empiricist than to the Rationalist tradition. Time and again in the first Critique he returns to the Empiricist theme that all knowledge begins with experience.

Kant's notion of 'experience' cannot, of course, be understood simply as consisting of atomic, mutually unrelated 'impressions of sense'—as Hume would have it; rather, Kant believed that the necessary factors in 'experience' could be seen in two clearly distinguished modes of intuition (instead of the one form of 'sensible' intuition). More precisely, he hoped to draw a distinc-
tion between the material content of intuition and its 'form.' The former he named empirical intuition, the latter, pure a priori intuition. Of these, empirical intuition is the more commonly recognized. The sensible contents of intuition resemble the 'sense impressions' of Hume, and in this century have been confused with the popular notion of 'sense-data.' Kant referred to them, however, simply as the 'matter' of sensible things, as opposed to their 'form.' They are the barely sensory content of perception. Kant speaks of the field of sensory contents, from which empirical intuition presents itself to sensibility (the 'faculty of sensation,' in its various divisions), as a 'sensible manifold.' It is within this manifold of sensible contents that the undetermined objects of empirical intuition appear. These 'appearances' are often described by Kant in language which suggests that they are material contents which commonly appear together in empirical intuition. Kant seems to follow Hume in treating them as mere appearances, aggregates not obviously connected in any way. Appearances are 'un-
determined' objects of empirical intuition in that they are not fully comprehended as 'objects' in a definite sense-- that is, no universal validity as objects can be claimed for appearances-- they can, of themselves, be described only by statements about seeing, feeling, hearing, etc.. Thus, if I say, "This water seems hot," or "I feel the downward pressure of this stone against the palm of my hand," I report only what seems to me to be the case; I make no unqualified statement about whether the water is indeed hot, or the stone, in fact, heavy, such that the warmth of the water or the weight of the stone would be claimed as universally recognizable. The water's being hot, or the stone's being heavy is not expressed in the statements I have made about them. Such statements, about mere appearances, carry no commitment to unqualified truth; they are statements about 'what appears' or 'what seems,' not about 'what is.' That this is an important distinction for Kant will become obvious later, in our discussion of objectivity, though, at the same time, we shall have to bear in mind that Kant sometimes refers to the 'objects of the understanding'--
'objects' in a definite, unqualified sense— as 'appearances' (to be distinguished from what we have called 'mere appearances'), and for appearances in this sense Kant would want to claim a universal validity as objects of knowledge.

The second necessary factor in intuition is the form of possible empirical intuition. Kant believed that this form (to use Kant's term) is 'primary' in all intuition in the way that the sensory contents themselves may be called 'primary': that is, the form of intuition is not mediatelly given, but, like the material contents, a fundamental dimension which presents itself directly in intuition. It is itself 'intuitive' in that it is not 'derived' in any way, but simply present in all instances of empirical intuition, and known in advance to be present as such because it is necessary for the possibility of empirical intuition. Thus Kant uses the term 'pure a priori intuition' to refer to the formal dimension of intuition, as contrasted with its material dimension (the 'material content' alone) which he calls 'empirical intuition.' And these two dimensions or 'modes' of intu-
ition can never occur separately, for they are both intuitions of the same thing. Kant believes that a priori intuition, as a necessary factor in experience, is necessary not in an empirical sense but in a logical sense: there can be no instances of empirical intuition which are without form, that is, which are not in space and time. The two types of pure a priori intuition are the 'transcendental conditions,' space and time.

The meaning of the term 'transcendental condition' can best be explained, Kant believes, by showing how our notions of space and time are not derived by abstraction from any specific instances of empirical intuition. They are, rather, the conditions under which sensory contents can present themselves in empirical intuition, and thus cannot be discovered by an examination of any particular instance of empirical intuition, but only by a consideration of the characteristics of empirical intuition as such. This is what Kant proposes to do in his 'metaphysical expositions' of space and time.

First, he attempts to show that space is form of a
priori intuition, and thus not a concept but the 'transcendental condition' upon which all spatial concepts depend for their meaningfulness. We cannot, he points out, have an acquaintance, in any instance of empirical intuition, with space 'by itself.' We can encounter it only in the spatiality of things expressed by concepts themselves. That is, Kant says that empirical concepts are, in a certain sense, representations of things in the world, and that it is inconceivable that any physical thing should not be spatially extended. We may well enough imagine a space in which there are no things, but we cannot conceive of a thing which is not in space. Kant clearly believes that some notion of spatiality will be a necessary condition for our being able to understand concepts as representations of things in space. It is an a priori condition in that it can be known of empirical concepts in advance.

Kant attempts further to distinguish space, as an a priori condition, from empirical concepts by pointing to the 'non-discursive' character of space: though we may speak of dividing space into parts, it makes no sense
to talk about composing space from parts. Our general notion of space cannot be constructed from spatial concepts, says Kant, because spatial concepts themselves presuppose space as an a priori intuition. The spatiality of everything that is spatial is part of one space; thus, Kant believes not only that space cannot itself be a concept, but also that it must be something more fundamental which makes spatial concepts possible—that is, pure a priori intuition.

Kant turns next to the 'metaphysical exposition' of time, proposing to show that, like space, time is not an empirical concept: it is not derived in any way from sensory intuition, but provides the condition under which appearances in the sensible manifold can be perceived as temporal. Time is a necessary condition for all empirical concepts. According to Kant, it is even more fundamental than space, for there are indeed things of which we may conceive which are in time but not in space. Feelings and sounds, for instance, are temporal but not spatial. Yet all spatial concepts, says Kant, are tem-
poral concepts too, because spatially extended things have a determinate existence in time, and it would be impossible to conceive of their being otherwise. Even their measure— their 'magnitude,' in Kant's terms, both quantitative and qualitative— can be shown to be a temporal determination. 11

That time cannot be a concept Kant believes is clear from our ordinary understanding of 'periods of time': any concept of a particular period of time— as a day, a month, a year— is understandable first of all in terms of its defining limits, its beginning and its end, which mark it off as a 'period of time.' For Kant, the notion of a period of time seems to presuppose a whole time— In contemporary language, we might say that one could not use the words, 'day' or 'year' unless he already had some idea of what 'time' was. For Kant, there can be no temporal concepts unless time is given a priori as a pure intuition. Thus time is not discursive: it is not formed by the combination of its parts, though it may be divided into parts.

Kant says that time is not only given a priori, but
that it is a "necessary representation that underlies all empirical intuitions." That is, it is a condition in terms of which all instances of empirical intuition are possible. This may be illustrated by a consideration of intuition in time as necessarily occurring either simultaneously or successively. It is impossible, Kant proposes, to imagine an instance of empirical intuition which does not have a certain duration in time, and which does not occur either before or after other such instances, or simultaneously with them. Thus time is a 'one-dimensional' condition presupposed by all concepts of 'duration' and by the sensible contents of empirical intuition in the order of their occurrence. Similarly, Kant points out that we cannot account for change, the "combination of contradictorily opposed predicates in one and the same object" without appealing to time as the condition under which alone such predication could be possible.

If time is, then, a necessary condition for the occurrence of the sensible contents of empirical intuition and for the possibility of their combination in concepts, it
cannot be the case that time is itself a concept. Kant concludes that it is a pure a priori intuition.

Kant causes some unnecessary confusion in the *Critique of Pure Reason* by his use of the term, 'pure intuition' in more than one sense. Its meaning in the Transcendental Aesthetic and the Transcendental Analytic is somewhat more restricted than, for instance, in the Transcendental Doctrine of Method. It will be helpful to say a few words about this before leaving the topic of intuition. Kant's reference, in section I of the Discipline of Pure Reason, to a 'non-empirical intuition' which is neither space nor time simply, but a 'pure intuition' whereby mathematical constructions are represented in the imagination, might seem to be a departure from his conventional use of 'pure intuition.' But in fact Kant's meaning here will be found to be compatible with his earlier, more precise use of the term. Although mathematical propositions (which are based on constructions) are always synthetic, still they do not require a discursive proof, as in the philosophical analysis of concepts, but are immediately—i.e. intuitive—
ly—recognizable as valid. That a perpendicular bisector to the hypotenuse of an isosceles triangle will bisect its right angle, for example, is, in Kant's terms, knowable by 'pure intuition' alone, not by empirical intuition. Kant understands mathematical constructions as the pure, logical expression of the a priori intuitions of space and time, without any appeal to empirical intuition. In this sense, one could be said to grasp a mathematical truth by pure intuition, since what would be grasped in this case would be simply an elementary determination of space and time.

We shall have the opportunity to explore the differences between mathematical and philosophical reasoning at greater length in Chapter Five. For our purposes at present, we shall be using the term, 'pure intuition' in the more specific sense of 'a priori intuition.' The two forms of a priori intuition, space and time, provide the framework within which the sensory content—empirical intuition—can be intelligible. And in fact, Kant's most common use of the term, 'intuition,' in the first Critique, is to refer to the empirical intuition of
undetermined objects in the sensible manifold.

Finally, it must be made clear that Kant's characterization of space and time, at the end of the Transcendental Aesthetic, as 'merely subjective conditions' of sensible intuition is not meant to imply a mitigation in any sense of their logical force as necessary conditions. They are 'subjective' to the extent that the sensible contents of empirical intuition are themselves subjective: for space and time can apply, within the realm of possible human experience, only to empirical intuition. But they hold as universally and necessarily true for all instances of such intuition, and thus are the a priori conditions of experience as such. All experience which takes as its 'matter' sensible intuition must be ordered in accordance with the absolute requirements of space and time. Further, if we follow Kant closely here and understand 'ordered' as an adjective and not as a transitive verb, then we will not make the mistake of attributing to Kant some notion of 'mental activity.' Kant should not be interpreted to mean that space and time are in some sense 'imposed' on the sensible
manifold by the mind, or that space and time would not exist if there were no intuiting beings. His position is, rather, that the idea of a thing not in space and time (or, at least, not in time) simply cannot be conceived, and that that is not a psychological fact but a logical truth. That it applies to human intuition, then, does not mean that the mind merely imposes space and time upon a manifold of sensible contents in intuition which, of themselves, have no necessity of conforming to the requirements of space and time; or that space and time are to be understood as purely contingent features of human mental processes. Space and time are the universal conditions under which appearances can present themselves to the mind. They are not a mental 'element' somehow infused into the sensible contents of intuition, but the necessary forms in which such contents must appear. These two modes of intuition, then, the pure intuition of space and time given a priori, and the empirical intuition of the sensible manifold, provide the two essential factors in 'experience,' and it is on the basis of this that Kant attempts to construct a
theory of judgment that will permit objective state-
ments about the world.

NOTES TO CHAPTER II


2 Kant, Critique of Pure Reason, Bl.

3 Or, more precisely, 'pure intuition knowable a priori.' See A43/B60.

4 Kant, Critique of Pure Reason, A20/B34.

5 Ibid.

6 The 'understanding' is the 'faculty of knowledge' (B137), as sensibility is the 'faculty of sensation.'

7 Or, at least, not in time, as we shall explain later.

8 Kant speaks of empirical concepts in A79/B105 as 'unities' of representations (i.e. of instances of empirical intuition) which serve as rules (A105) by which we come to know appearances as specific objects. We shall deal with this at greater length in the follow-
ing chapter.

9 Kant, Critique of Pure Reason, A104.

10 Ibid., A34/B51.

11 This topic will be dealt with in Chapter Four, in
the discussion of the Mathematical Principles and the schemata of Number and Intensity.

12 Kant, *Critique of Pure Reason*, A31/B47.

13 Moving in one direction. See A31/B47.

14 Kant, *Critique of Pure Reason*, B49.

15 See especially A21-22/B35-36.


17 Ibid., A39/B56.

18 Ibid., A21/B35.

19 Ibid., A49/B66.

20 Ibid., A20/B34.

III. CONCEPT AND JUDGMENT

That Kant regarded his Critique of Pure Reason not as 'theory of knowledge' simply, but also as metaphysics, is evident from the fact that he concerns himself as much with what can be said about the world as with what can be known about it. Kant's theory of knowledge is at the same time a 'theory of judgment': for Kant, all knowledge must be expressible in judgments—what we have also referred to as 'statements' or 'propositions.' Knowledge requires both the sensible contents of empirical intuition and the formal elements of space and time, in accord with which the sensible elements must present themselves. But it also requires that instances of empirical intuition in space and time fall under certain spatial and temporal concepts, which Kant calls Pure Concepts of the Understanding, and such 'falling under' can occur only in judgment. Only in judgment do the mere appearances of the sensible manifold, the 'objects' of empirical intuition, become knowable, in the strict sense, as objects of the understanding. And
only in this way can statements about the world be possible which will have a valid claim to 'objectivity.'

In this chapter we shall try to give some account of the doctrine of judgment which is central to the first Critique. In the discussion of this doctrine, various questions about the nature of synthetic judgments arise, and one of these will occupy our attention later as the principle topic of this essay. But any discussion of judgment in Kant's terms can scarcely avoid a consideration of concepts as well, for the two notions are intimately related in such a way that the one cannot be understood without an understanding of the other. It will be helpful, then, to begin by explaining how Kant understands the notion of a 'concept' and how concepts function in judgments.

"Thoughts without content are empty, and intuitions without concepts are blind."¹ With this famous dictum Kant points to the interdependence of concepts and intuition in judgments: knowledge must first have the 'matter' of intuition, but that matter is not knowable without the synthesizing unity provided by concepts.
Only in the 'unity' of concepts can an object be known; that is, the mere appearance of unconnected sensible contents can be comprehended as a universally recognizable object with spatial and temporal dimensions only when the sensible contents of intuition are unified in a concept. Knowing an object as a particular thing persisting in time requires bringing a series of temporally distinct instances of empirical intuition under one concept; and that is, in Kant's terms, to effect a 'synthesis.' A similar point can be made with regard to instances of empirical intuition which, though not temporally distinct, are nonetheless distinguishable: a thing's size is distinguishable from its shape, and both are distinguishable from its color. But all are understood together as an 'object' only when they are brought under one concept in a synthesis.

From one point of view-- and, I think, the more nearly correct one-- the use of such metaphors as 'bringing under a concept' is somewhat misleading (though the language is Kant's own, and not merely that of his interpreters), since it seems to imply some notion of
synthesis as mental act of synthesizing. As we shall see, however, it will not be necessary to resort to a doctrine of 'mental activity' in order to give an adequate account of the notion of synthesis. Of course, it is true that Kant sometimes speaks of synthesis as if it were a mental activity. In the Analytic of Concepts, he says that for a manifold of representations to be known it is necessary that they be "gone through in a certain way, taken up, and connected." This he calls 'synthesis.'

"By synthesis, in its most general sense, I understand the act of putting different representations together, and of grasping what is manifold in them in one knowledge."²

In the first edition version of the Transcendental Deduction— the second section of the Analytic of Concepts— Kant continues to speak of synthesis as if it were an activity of the mind (or, more precisely, of the imagination, a special faculty of the understanding³). However, in the rewritten version of the Deduction which appears in the second edition, Kant's language changes somewhat in his talk about synthesis; he adds to the metaphors of mental activity (e.g. synthesis described
in B130 as an 'act of the understanding') allusions to 'thinking a synthesis,' and to the synthetic unity of the manifold's being 'generated a priori.' Such subtle changes as these suggest that Kant was struggling to express by the term 'synthesis' something of a more nearly logical nature than psychological. If this is the case—and I think it is—then we could understand his talk of 'putting representations together' as a logical putting together, in the sense suggested by Jonathan Bennett. This seems clearly to be Kant's meaning in B151, where he speaks of an 'intellectual synthesis.' For our purposes in this paper, we shall understand the term 'synthesis' primarily in this logical sense and try to avoid any interpretation of it as a 'process of thought' occurring over a period of time. It will be convenient, however, to adopt Kant's technique of talking about synthesis in words—such as 'combination,' 'composition,' 'connection,' and 'construction'—which can be construed as reference to an activity as well as to a state of being. Similarly, in talk about concepts, an occasional use of the metaphors 'falling under' and 'bringing under' may be helpful,
but they should not be taken to indicate mental activity in any literal sense. But what is the peculiar significance which Kant attaches to the 'thinking of a synthesis' under a concept which makes the unity so formed 'objective' and thus different from a mere appearance? What is the nature of the objectivity possessed by judgments in which the matter of intuition is brought under spatio-temporal concepts? To deal with these questions, it will be necessary to distinguish between the Pure Concepts of the Understanding— which Kant calls the 'categories'— and ordinary empirical concepts.

The Pure Concepts are universal rules presupposed in the use of empirical concepts. Kant believes that he has discovered twelve Pure Concepts, which are not in any way derivable from empirical intuition (or empirical concepts) and yet without which 'objective' judgments about the world would be impossible. These Pure Concepts— which Kant calls 'functions of the understanding,' and which he believes apply a priori to the contents of intuition— he thinks of as somehow
uniquely expressing all the differentiable spatio-temporal features of the world. They are the twelve categories which all judgments about the world must employ. Objects in space and time are determined a priori by certain necessary spatial and temporal forms. Every judgment about objects in the world must, whatever else it may be, also be a judgment of Quantity, Quality, Relation, and Modality, and therefore must make use of at least one of the three categories under each of these subdivisions. Hence, all specific judgments which employ empirical concepts also employ the Pure Concepts of the Understanding, because in any use of empirical concepts, Pure Concepts are presupposed. The Pure Concepts, or 'categories,' Kant groups into the four subdivisions as follows: under the subdivision of Quantity are the categories of Unity, Plurality, and Totality; under that of Quality are the categories of Reality, Negation, and Limitation; under the subdivision of Relation are Inherence and Subsistence, Causality and Dependence, and Community and Reciprocity; finally, under the subdivision of Modality are the categories of
Possibility and Impossibility, Existence and Non-existence, and Necessity and Contingency.

The system of judgments which employ these Pure Concepts is what Kant calls Transcendental Logic. It differs from general logic by virtue of its not abstracting from all particular content of knowledge. That is, it concerns itself with more than simple internal consistency, as in formal logic. Although Kant accepts the analytic Principle of Contradiction— that "p cannot be both q and not-q"— as the 'universal condition of all judgments in general,' he specifies that this cannot be analytic if it incorporates the element of time, as in "p cannot at one and the same time be both q and not-q," for to do so would be to make it dependent on a transcendental condition, namely, time. Transcendental conditions carry the same force of necessity as the Principle of Contradiction, but are concerned with more than the internal consistency of a proposition, and function as logical requirements only in judgments about the world. Transcendental Logic, like formal logic, makes use of the Principle of Contradiction as a merely negative
criterion of truth, but, since Transcendental Logic proposes to make objective statements about the world, its judgments must further conform to the transcendental conditions of space and time. Kant clearly understands the force of logical necessity in Transcendental Logic as 'necessity in space and time,' and accordingly, Transcendental Logic will be, for Kant, the basis of all true, objective judgments from the contents of empirical intuition in space and time.

Since it is the basis for these judgments, and of itself contains no empirical element, Transcendental Logic seems rather to be removed from the realm of those judgments about the world which employ ordinary empirical concepts. Further, if the categories are the 'universal rules' for the employment of such concepts in accordance with the requirements of space and time, and if space and time are not, however, empirical concepts at all but a priori conditions, then how can it be possible for the Pure Concepts of the Understanding to have any logical force in ordinary empirical judgments? In order to deal with this problem, Kant saw the need for a third
element, which would be at once both intellectual and sensory, to 'bridge the gap,' as it were, between the a priori categories and the empirical concepts. The solution, he believed, was the 'transcendental schema' of time. Since intuition itself is possible only in time, Kant reasons, therefore time must be the factor which makes possible the use of categories in bringing the sensory content of intuition under a synthetic unity in judgments. Judgments employing particular empirical concepts always involve an employment of the categories as well, but only formally: that is, they do not explicitly contain the Pure Concepts of Quantity, Quality Relation, and Modality, but— in so far as they are true judgments about the world— they are necessarily Quantitative, Qualitative, Regulative, and Modal judgments. Kant believed that the Schematism of the categories would make clear their immediate relevance to ordinary empirical judgments. A 'schema;' in the general sense defined by Kant, is simply a "universal procedure of imagination in providing an image for a concept." The schema cannot be an image itself, but only a general
rule for forming images, since it is necessary in all variations of its possible images but not sufficient to all possible variations (just as the concept of a triangle in general is the necessary but not sufficient rule for constructing any particular triangle).

In order to be used, then, a category must be schematized, that is, combined with the transcendental schema of time to form the particular schema appropriate to the categories of that class. Thus each subdivision of categories—Quantity, Quality, Relation, and Modality—has its own particular schema. For categories of Quantity and Quality, with which we shall be concerned in the next chapter, the respective schemata are Extensive Magnitude (or Number) and Intensive Magnitude. The former is, according to Kant, derivable from the combination of the transcendental schema of time with the categories of Quantity, for Quantity can only be perceived temporally as a number of successive units; Intensive Magnitude, he says, is similarly derivable from the combination of the transcendental schema with the categories of Quality: for all sensations in time
necessarily have a certain intensity, and that intensity is itself always the manifestation of a possible 'scale of intensity' conceivable only temporally (though here the influence of the temporal factor is by no means obvious, and it will be our task in Chapter Four to make the connection more clear).

Finally, then, understanding how objective judgments about the world are possible, involves first of all seeing that the Pure Concepts of the Understanding (categories) are simply the necessary conceptual expression of all determinations of reality in space and time (and this Kant believes he has established in the Transcendental Deduction), and secondarily, seeing that the employment of empirical concepts in our everyday judgments about the world does (when those judgments can rightfully claim objectivity) in fact presuppose the employment of the schematized category. The next step, which we can now begin, is to attempt some explanation of Kant's notion of an 'empirical concept.'

To 'apply' a category is to talk in the way we ordinarily do about objects, for that is to make judg-
ments about objects. The way objects first come to be
for us is in our naming certain appearances of perceptual aggregates by assigning certain terms to those aggregates. They come to be 'ordered' under those terms, and the convention of the terms themselves is established through use. These terms Kant would call 'concepts,' but they can be concepts of objects only when the representations in the aggregate which they name possess a certain coherence or connectedness among themselves. The notion of an object, for Kant, is the notion of that which necessitates a definite sort of 'unity' in appearances.

"... The object is viewed as that which prevents our modes of knowledge from being haphazard or arbitrary, and which determines them being a priori in some definite fashion. For in so far as they are to relate to an object, they must necessarily agree with one another, that is, they must possess that unity which constitutes the concept of an object." But the knowable object with which we have to deal—the 'object of the understanding'—cannot be in any sense the object-in-itself, which is strictly unknowable; Kant specifies that the object of the understanding
can only be the 'synthetic unity in the manifold of representations,' that is, the appearance which is brought under a concept. And the concept itself—the 'unity of rule' which we use to identify certain aggregations of representations as objects—cannot simply 'arise' out of those representations, however coherent they may be, though their coherence, in some sense, makes a conceptual unity necessary for their being recognized. Rather, Kant indicates that it must be the unity of consciousness which is the source of the conceptual unity by which the appearances come to be designated as objects. A common illustration of the way in which a unity of consciousness is necessary for the employment of concepts is that in which a granular substance that both looks white and tastes sweet is identified as sugar. In order for both empirical representations to appear in a synthetic unity under the concept 'sugar,' it is necessary that there be one consciousness before which both can appear; and, as with any sensible contents of intuition, they must exhibit a certain rule-governed connectedness in order to be
capable of belonging to a single consciousness. The unity of the object and the unity of consciousness are interdependent, since without the unity of consciousness there could be no employment of concepts, and without the unity of the object there could be no appearances. Thus, while it is the unity of the object which requires a 'unity' as the concept, it is the unity of consciousness which makes that 'unity' as the concept possible.

Still, it may be objected that we have merely identified the object-- in the unqualified sense of 'object of knowledge'-- with the concept, as the unity of empirical representations, and not given an adequate account of wherein the 'objectivity' of judgments made under such concepts consists. If we believe that some of the ordinary sense of 'objectivity' can be preserved in Kant's notion of judgment, we must spell out still more clearly what that sense of objectivity can be for Kant which would allow a distinction to be made between 'what seems to me' and 'what is.' The need for making this distinction is, after all, the reason why Kant would insist on a need for the categories. Strawson
has attempted such an explanation in his recent book, The Bounds of Sense, and although he seems to be reading Kant's idealism in a narrower way than Kant would have approved, still Strawson's discussion of objectivity is useful in showing how Kant's notion of objectivity does indeed make use, to some extent, of the ordinary distinction between 'objective' and 'non-objective.' Strawson shows that in order to identify an experience as 'non-objective,' it is necessary to have the concept of an 'objective' experience, which itself entails having the concept of the distinction between experiences of reality, and those which are mere appearances only.

"'This is how things are (have been) experienced by me as being' presupposes 'This is how things are (have been) experienced as being'; and the latter, in turn, presupposes a distinction, though not (usually) an opposition, between 'This is how things are experienced as being' and 'Thus and so is how things are.'" 

Strawson believes that this distinction between the objective and the non-objective is the basis on which Kant's arguments about objectivity rest— it is presupposed even by the doctrine of the unity of consciousness, for in order to conceive a single consciousness to which
diverse perceptions can appear, and by which they can come to be known as a single object, it is necessary, Strawson says, to assume that there is a conceptual difference between these mere perceptions and the experiences of objects.

Of course, Kant does assume that it should be possible to distinguish between the objects of empirical intuition, which he calls 'mere appearances,' and the objects of knowledge; otherwise there would be no sense in talking about objective judgments at all, and thus no need for a Transcendental Analytic. Thus Kant would surely agree that the unity of consciousness presupposes a distinction between objective and non-objective appearances, but he would also insist that any notion of objective experience— as opposed to non-objective— must allow for the employment of concepts in judgments, and that there can be no such 'unity of rule' in appearances unless there is a unity of consciousness to which such sensible contents of intuition can appear.

Finally, we must not too quickly conclude that the
distinction Kant wishes to draw between mere appearances and objects of knowledge is anything so simple as that between an 'exterior' object and a 'sense-datum experience.' Metaphors about 'inner' and 'outer' worlds are, in any case, foreign to Kant's vocabulary. My knowledge of an object, in the sense understood by Kant, is not knowledge by way of a sense-datum experience; no sense-datum stands between me and the object. The object which I know is, in Kant's words, 'empirically real'; that is, it is not simply a collection of ideas in my mind, but a real 'physical object' in the world. It is, however, 'transcendentaly ideal,' in the sense that the object can be known only in a certain way: in so far as the appearance is in accordance with the transcendental conditions of space and time-- i.e. in so far as the appearance is 'ordered' in terms of Quantity, Quality, Relation, and Modality. But what I know is always an appearance which is an object, not an appearance of an object (i.e. not a sense-datum).

While it is clear, on the one hand, that Kant would want to give an account of objective knowledge which
would make it distinct from mere perception, it is also true that such an account can only be in terms of what one might call the **degree of comprehension** by which appearances as objects of the understanding differ from appearances as objects of empirical intuition.

To be an object of knowledge is to be the object of a judgment in which categories are employed— it is not just to be perceived, but to be **understood**: it is to be an appearance which is fully recognized (and thus is **capable of being recognized**). Graham Bird, in his very helpful *Kant's Theory of Knowledge*, is pursuing a similar line in his remark that

"The contrast between the object of sense and understanding is, therefore, best expressed in such terms as those of indeterminate and determinate, indiscriminate and discriminated, or non-descript and described, appearances or objects."^19

This is not to deny that in order to be able to make statements about 'what seems,' we must have some concept of 'what is' (the 'concept of an object'); it is only to suggest that this ordinary sense of objectivity must be understood within the framework of Kant's Transcendental
Idealism, which would not hold that the 'object of knowledge' in any case is merely a 'sense-datum experience.'

At least this much is clear: that in Kant's view, any reference to mere appearances carries with it an implicit dependence upon the 'concept of an object.' In any particular case, to talk about something's seeming to be a particular object implies that one must already have an idea of (1) what it is to be an object, and (2) what it is to be this particular kind of object (i.e. to have a concept). But can we adopt the position— as Strawson appears to— that our criterion for having a certain concept will be our ability to use words in a certain way? Though this is no doubt an attractive position, it may prove to be one which can be only marginally useful in understanding Kant, if it can be used at all.

The recent philosophical trend toward linguistic analysis has yielded some interesting attempts at dealing with the Kantian notion of 'concept' as a rule. Unhappily, not all these attempts have proven significantly valuable. One of the less valuable is Jonathan
Bennett's; in his book, *Kant's Analytic*, Bennett ascribes to Kant a thoroughly Wittgensteinian view of concepts. He suggests that, although Kant often adopts the introspectionist attitude with regard to concepts in his discussion of the nature of analytic propositions, it is impossible to understand his meaning in the *Analytic of Concepts* unless we take him to mean by 'concepts' linguistic skills.

Kant insisted on an interpretation of concepts which made them meaningless apart from being used in judgments: thus only creatures that are capable of making judgments can possess concepts. On this much Kant and Wittgenstein are agreed. But while it seems to be true that, for Wittgenstein, 'having a concept' just is being able to make a certain kind of judgment, for Kant, 'having a concept' means having a certain mental entity which makes judgments of a certain kind possible; and therefore it will be misleading to try to give an account of Kant's notion of concepts exclusively in terms of linguistic skills. 'Having a concept' is simply not reducible in Kant, as Bennett thinks it is, to the ability
to use *words* in a certain way (though that ability is certainly *part* of Kant's meaning).

Kant's treatment of concepts deserves to be understood as, in some sense, mid-way between Descartes's and that of Wittgenstein. Richard Rorty, in a recent article, suggests that Kant and Wittgenstein represent two different ways of repudiating the Cartesian notion of mental entities as inevitably 'self-luminescent' (i.e. that whatever was *mental* could easily be *known*):

"The major difference between [The Kantian and Wittgensteinian revolts against Cartesian-ism] is that concepts are viewed in different ways. For Kant, a concept is a representation, a species of mental content, whereas for Wittgenstein it is a skill, a skill at linguistic behavior-- the ability to use a word."\(^{22}\)

Kant's attempt at rejecting Descartes's 'mental eye' concept of mind, that is to say, was only partially successful. His result was a new system of mental contents, in which the object of knowledge could be known only in the *judgment*: the synthesis of the sensible contents of intuition under a concept.

Rorty observes that in order to be correctly said to 'possess' a concept, one must have *applied* it at
some time or other in making a judgment.

"To use a concept is to be able to make a judgment, which involves having a thought expressible in complete sentences... If all one has in one's conceptual repertoire are adjectives... then one does not have anything in one's conceptual repertoire. For one is not yet, lacking substantives, in a position to use these adjectives to form a judgment, and if one cannot form judgments, one does not possess concepts."23

Thus it is impossible to understand Kant's notion of a concept without some understanding of his notion of predication in judgment, and it is likewise impossible to conceive of a judgment, in Kant's sense, without appealing to the notion of a concept.

In the course of our discussion so far, we have attempted to give an account of the doctrine of judgment in general, that is, to deal with Kant's judgments in terms of the synthetic nature which judgments about the world inevitably possess. Accordingly, we have tried to spell out Kant's notion of a concept in such a way as to make the meaning of his talk about synthesis more clear. Now we shall have to examine more closely the specific types of judgment which Kant believes are possible, and particularly the 'synthetic a priori
judgment.'

The traditional logic, which Kant inherited from Aristotle and the Rationalists, divided judgments into the necessary or 'self-evident,' which Leibniz called 'truths of reason,' and the merely contingent, which (when true) Leibniz called 'truths of fact.' Kant devised the terms 'analytic' and 'synthetic' to describe necessary and factual judgments respectively: analytic judgments were always those whose predicates were, in a sense, 'contained' in their subjects, so that the judgment could serve only as a 'clarification' of given knowledge and never as an extension of knowledge; synthetic judgments were those in which subject and predicate were originally distinct thoughts, related only in the judgment itself, and this conjunction (synthesis) produced a new fact of knowledge—Kant would have said 'object of knowledge'—different from either of the two original thoughts of which it was a combination.

Judgments of the first type are always recognizable as obviously true because their truth can be established through analysis (hence the name, 'analytic'), and thus
one could say that their truth was knowable 'in advance,' without appeal to any instances of factual evidence: that is, they are true a priori. But synthetic judgments, for Kant, represent a much more complex topic. While most philosophers of logic before Kant (and since) have held synthetic judgments to be knowable only upon examination of relevant evidence, that is, to be knowable a posteriori, Kant held that there were indeed some synthetic judgments which could be recognized as true a priori. These synthetic a priori judgments, Kant believed, were capable of claiming universal validity in true statements about the world. Such judgments are clearly not analytic (in the sense that we have defined) since they represent an 'extension of knowledge' and not merely a clarification; still, says Kant, they are the sorts of judgments we can know in advance to be true, because they are made in accordance with necessary determinations of space and time expressed in the categories. Thus all synthetic a posteriori judgments which express true statements about the world necessarily presuppose synthetic a priori judgments, says Kant,
because in order to express true statements, it is necessary for synthetic a posteriori judgments to employ implicitly (i.e. formally) the Pure Concepts which synthetic a priori judgments employ explicitly.

The project of the Critique of Pure Reason is to investigate the possibility of synthetic a priori judgments in mathematics, in science, and in metaphysics. Not surprisingly, it is that section of the first Critique called the Transcendental Analytic which has attracted the most interest historically, since it attempts to demonstrate that synthetic a priori judgments are possible in natural science, i.e. in statements about the world. The Transcendental Analytic is divided into two sections; the first, the Analytic of Concepts, contains Kant's argument for the use of categories; in the second, the Analytic of Principles, Kant attempts to prove that the twelve categories he prescribes are in fact the ones we use in making judgments. The 'principles' to which the title of this latter section refers are synthetic a priori judgments, called the Principles of the Pure Understanding, with which Kant attempts to
prove the validity of his twelve categories. The Principles fall into four classes corresponding to the four subdivisions of the categories. For the subdivisions of Quantity, Quality, Relation, and Modality, the Principles are, respectively, the Axioms of Intuition, the Anticipations of Perception, the Analogies of Experience, and the Postulates of Empirical Thought.

The Principles of the Pure Understanding are of particular interest because of their further division by Kant into two groups. The Axioms of Intuition and the Anticipations of Perception he calls the 'Mathematical Principles'; the Analogies of Experience and the Postulates of Empirical Thought he calls the 'Dynamical Principles.' This distinction clearly has something to do with the types of synthesis involved in mathematical judgments on the one hand and in dynamical judgments on the other. It also has to do, in either case, with what is synthesized: the manifold of empirical intuition. The synthesis in mathematical judgments Kant characterizes as the combination of a homogeneous manifold, while in dynamical judgments the synthesis is characterized
as the combination of a heterogeneous manifold. In the former case, the sensible contents of intuition are all alike: they could be characterized as elemental, undifferentiated instances of intuition which, just because they are alike in being elemental, can exhibit no 'affinity' among themselves— their being distinct instances depends solely on their being separated spatially or temporally— and hence Kant describes them as 'not belonging to one another.' Their combination, therefore, will have the character of composition, says Kant; that is, it will be a synthesis of units. This synthesis of units will be recognized as the nature of judgments in mathematics. In the synthesis of the heterogeneous manifold, the combination is called a connection of constituents which 'necessarily belong to one another,' as, for instance, a property to a substance or an effect to a cause (both of which Kant believes are necessary relations). This 'connection' of essentially different constituents which accordingly can display a natural 'affinity,' Kant sees as being the nature of judgments in 'physics' or 'dynamics' (i.e. physical science).
Dynamical judgments, therefore, will have the character of connection, says Kant, in contrast with the composition which characterizes mathematical judgments.

The terms 'mathematical' and 'dynamical' will be understood to apply in the first instance to forms of synthesis in judgments, and only secondarily to specific judgments of mathematics or physics. The reason for this, Kant insists, is that mathematical judgments and physical judgments deserve to be called 'mathematical' or 'physical' only because of the distinctive forms of synthesis they represent. It is therefore not with any 'basic principles' of mathematics or 'basic principles' of physics that Kant is concerned in the Principles of the Pure Understanding, but with the conditions for synthetic combination in judgment which make particular synthetic a priori judgments in mathematics and physical science possible. Kant makes it clear that it is because of their functioning in this way that he has named the Principles Mathematical and Dynamical, and not because of their content. It is clear that he would not want to deny the mathematical nature of the synthesis in the
judgments employing the categories of Quantity and Quality, and the dynamical nature of the synthesis in the judgments of Relation and Modality, but to say that the Principles set out the basic requirements for mathematical and dynamical statements is not to say that they are themselves mathematical or dynamical statements.

The distinction between 'composition' and 'connection' as two very different forms of synthesis deserves closer inspection—with an eye to discerning whether one might be in some sense more fundamental than the other. In the Mathematical Principles, Kant is concerned with the conditions for the composition of appearances, while in the dynamical his intent is merely to set out the rules which define the relations in which contingent existences are connected. The a priori character of dynamical judgments, then, will be observable only in the necessary relation (e.g. causality) between, or modality of, existences which are empirical facts, and no question of the 'composition' (construction) of these appearances will arise at this level. Put a different
way, one might say that the physical occurrences or 'facts' about a relation or a mode of existence cannot be 'constructed' a priori; the objects which are related or which exist modally must themselves be already the product of a priori mathematical construction, but the facts of their relation or existence are necessarily empirical facts. Kant believed that it follows from the a priori nature of synthetic judgments about such empirical facts that they will inevitably be regulative in some sense, while the a priori character of the synthetic judgments in which the appearances are themselves constructed will be constitutive (i.e. mathematical). Our project in the second part of this paper will be to make clear the important difference between intrinsic 'construction' on the one hand, and extrinsic 'connection' on the other, and to show the former to be the more fundamental type of synthesis. It will, in fact, be seen to be the simplest form of juxtaposition which Kant's doctrine of judgment allows.
NOTES TO CHAPTER III

1 Kant, *Critique of Pure Reason*, B75.

2 Ibid., A78/B103.

3 Heidegger believes that Kant intends the imagination as belonging neither entirely to the understanding nor entirely to the sensibility, but as a separate 'faculty' somehow between the two. (*Kant and the Problem of Metaphysics*, p. 136.)

4 Kant, *Critique of Pure Reason*, B151.

5 Ibid., B134.


8 Ibid., A79/B105.

9 In Kant's vocabulary, 'analytic propositions' are those in which the predicate is 'contained in' the subject, as with Leibniz's 'truths of reason.'


13 Ibid., A105.

14 Ibid.

15 Or, 'object of the understanding.'
16 Strawson's use of the term 'experience' is not meant to imply Kant's technical meaning.

17 Strawson, The Bounds of Sense, p. 108.

18 Ibid., p. 97.


20 Kant, Critique of Pure Reason, A106.

21 Bennett, Kant's Analytic, p. 71.


23 Ibid.

24 As in the judgment, "All bachelors are unmarried," in which the predicate 'unmarried' is merely part of the meaning already expressed in the subject 'bachelors.'

25 Or by appeal to the metaphysical doctrines of the Schoolmen and the Rationalists— as, e.g. about God and His Creation— the sort of abstract metaphysical speculation Kant repudiated.


27 Ibid., B202.

28 Ibid., A165/B206.

29 Ibid., B202.

PART TWO
IV. THE MATHEMATICAL PRINCIPLES

Now that we have gotten some understanding of Kant's doctrine of judgment, and of the metaphysical program of the first Critique within which that doctrine arises, it will be our task in this section to examine a particular aspect of the theory of judgment and to determine how it can be seen as fundamental to the theory as a whole. Our special concern is with the notion of synthesis in synthetic a priori judgments, and with the distinction— to which we have already alluded— between the synthesis of the homogeneous manifold and the synthesis of the heterogeneous manifold— the syntheses Kant calls, respectively, 'composition' and 'connection.' If we can give an adequate explanation of this distinction, we should be able to see whether the mathematical synthesis is indeed a 'fundamental synthesis' and whether it is, as was suggested at the very outset, the most elementary form of synthesis which Kant proposes.

To accomplish this explanation, it will be helpful to follow Kant's procedure in the first Critique of
treating the synthesis in the mathematical judgments employing the categories first, and then examining the mathematical method itself more closely. With the exception of his brief reference to the non-analytic character of arithmetical and geometrical judgments in the Introduction to the second edition, Kant's discussions of the mathematical synthesis occur only in two sections of the *Critique*, the Analytic of Principles (in the Transcendental Analytic) and the Discipline of Pure Reason (in the Transcendental Doctrine of Method). We shall begin, in this chapter, then, with a comparison of the Mathematical and Dynamical Principles of the Pure Understanding, and with a closer look at the bases of mathematical judgment in the Axioms of Intuition and the Anticipations of Perception. In the next chapter we shall focus our attention primarily upon Kant's treatment of the mathematical method in the Discipline of Pure Reason, and determine how the synthesis in mathematical judgments makes them different, in a fundamental way, from dynamical judgments (and indeed from any synthetic judgment whose certainty can be established by
discursive proof).

Mathematical judgments are concerned with the 'constitution' of objects, while dynamical judgments are concerned with their 'regulation.' Perhaps we could get closer to Kant's meaning by saying that while in dynamical judgments we come know about objects, in mathematical judgments we first come to know objects. Concepts function in mathematical judgments as the unity of rule in the synthesis where the bare object—abstracted from its 'existence' in the world—comes to be recognized; but in dynamical judgments concepts serve as the unity of rule where the object in its relations and its mode of existence come to be recognized.

Furthermore, Kant would surely want to deny that two separate syntheses take place in any given judgment that we actually make about the world; for we never know objects without at the same time knowing something about them, as, for instance, their mode of existence in relation to our understanding and their relation to other objects. The Transcendental Synthesis, wherein the object comes to be known in terms of both its quantity
and quality, and in terms of its relations and mode of existence, can, however, best be understood by considering its mathematical and dynamical aspects separately. For although the two are always together in fact, nonetheless, they could be said to represent a synthesis on two different 'levels,' and deserve to be characterized as being 'synthesis' in two different senses, because they synthesize different kinds of manifold, and because they do, in fact, synthesize them in different ways (as will be made clear in this section). Kant's Transcendental Synthesis is two different syntheses at once, though the two are inseparable in their actual occurrence.

To begin with, let us make clear the distinction between the constitutive and regulative syntheses in synthetic a priori judgments. Particularly, we need to clarify the contrast between mathematical and dynamical judgments which Kant first defines in the Mathematical and Dynamical Principles of the Pure Understanding. Kant says that mathematical judgments will be characterized by an 'intuitive' certainty, whereas dynamical
judgments will admit of only a 'discursive' certainty. Mathematical judgments concern the intrinsic constitution of an object—its quantitatively measurable extension, on the one hand, and its degree of qualitative intensity, on the other—and so, one could say that to know an object at all (i.e. even abstracted from its mode of existence or its relation to other objects) is to know it mathematically, that is, as the synthetic unity of elementary constituents. To 'know' anything less is not to know an object. What is certain in a mathematical judgment, therefore, will be necessarily certain—and that is to say that a mathematical judgment is 'intuitive'—because a mathematical judgment contains only those minimal constituents of an object without which it would not be the particular object it is. The mathematical judgment, then, says nothing about the object at all; and since its statement is thus not 'factual,' there is no way its truth can be contingent: if it is true, its truth will be necessary, and its certainty, therefore, intuitive.

While the certainty of mathematical judgments is in-
ternal, and thus apodictic, that of the dynamical judgments is dependent upon the particular spatial or temporal relation of the object to other objects, or its mode of existence in space and time. Dynamical judgments, then, are propositions whose certainty is knowable only discursively. A discursive proof is one which moves by deductive steps from premises to conclusion, as in the hypothetical syllogism ("If p, then q; if q, then r; therefore, if p, then r"). The certainty of any judgment employing the dynamical categories can be seen only in such a proof (though in practice the proof is usually taken for granted). A dynamical judgment which makes use of the relational category of Causality and Dependence is, in Kant's terms, a Hypothetical judgment: e.g. "If p comes into proximity with r, and r moves, then p caused r to move." Similarly, a dynamical judgment employing the relational category of Inherence and Subsistence will be, for Kant, a Categorical judgment, as, for example, "The building I saw on this corner yesterday was low, gray, and angular; the building I see on this corner today is also low,
gray, and angular; therefore, it is the same building (which has endured in time)." The certainty of relational judgments which is disclosed in this discursive fashion will always be "the representation of a necessary connection" between appearances. Such 'connections' are, as Hume pointed out, not given in perception; nevertheless, we must be able to give some account of duration in time, cause and effect, and reciprocal relationship between objects, in order for science to be possible. This is the basis for the arguments for the relational categories which Kant advances in the Analogies of Experience.

The judgments of those Principles entitled Postulates of Empirical Thought are intended by Kant as arguments for the three categories of Modality, the categories employed in Problematic, Assertoric, and Apodictic judgments about appearances (objects) in the world. The dynamical judgments of Modality—like those of Relation—will also admit of only a discursive certainty, but here that certainty will consist in the object's mode of being present to our understanding, and not in an object's
relation to other things. This mode of being present to our understanding must involve a judgment about the extrinsic 'regulation' of an object (as an existence in the world) which is either possible, actual, or necessary; and this suggests that (if true) its truth could not be known intuitively— as one could be said to know the mathematical constituents of an object intuitively— but only upon reflection, that is, discursively.

This point will become more clear if we take care to distinguish, as Kant seems to, between two senses of 'object of knowledge.' Kant's word 'object' (Gegenstand) can, in this sense, mean either a 'thing' (Ding) or an 'appearance' (Erscheinung). If we understand the object of a mathematical judgment exclusively as a thing, and the object of a dynamical judgment in the somewhat broader sense of appearance, then it will become obvious in what sense dynamical judgments can be said to be judgments about objects, i.e. judgments about things. As we have suggested earlier, all objects are also appearances— including the objects of mathematical
judgments which we call 'things'—but the objects of
dynamical judgments are always objects in the sense of
'apparances,' and never in the sense of 'things.' The
objects of dynamical judgments are always certain dis-
tinctive 'appearances of things,' as, for example, a
thing's appearance as a 'persisting thing' in time
(in relation to other things), a thing's appearance as
a thing causally related to another, or a thing's ap-
pearance as actually existing in the world.

Things are never encountered except as situated in
the world, bearing a certain objective relation to other
things and having a certain objective mode of being
present to our understanding; that is, mathematical
judgments do not 'occur' alone but always with dynamical
judgments. But this does not mean that mathematical
judgments logically require dynamical judgments: that
the object of a mathematical judgment is always encountered
as situated in the world is a fact about the way things
are apprehended, not a necessary truth. A thing (the
object of a mathematical judgment) can indeed be con-
sidered in the abstract, that is, apart from what may
be said about it (the object of a dynamical judgment),
but the object of a dynamical judgment always presupposes the objects of mathematical judgments, for there can be no judgments about things unless there are things.

Dynamical judgments, therefore, always necessarily presuppose mathematical judgments. As an example, let us consider a dynamical judgment in which the object of the judgment, in this case, let us say, a thing's being causally related to another, is in question. I might say either, (1) "Billiard ball A struck billiard ball B, causing it to move," or (2) "Billiard ball A came into proximity with billiard ball B, and billiard ball B then moved." Because judgment (1) employs the category of Causality and Dependence, its object— that A was the cause of B's moving (or that B moved because of A)— is fully comprehended as an 'object of knowledge.' But judgment (2) does not make use of the category, and thus its object can be nothing more than an 'object of empirical intuition,' namely, the mere appearance: A's coming into proximity with B, and B's moving. In both judgments, objects (i.e. appearances), as things, are
presupposed. Each judgment is a proposition about billiard balls, and in neither proposition are the billiard balls themselves in question; the only appearance in question (in judgment (2)) is the appearance of A's causality in relation to B and B's dependence in relation to A. Thus it is obvious that in neither judgment are the 'things'—the objects of mathematical judgments—dependent upon the objectivity of the judgments made about them: A's being a billiard ball is not dependent upon whether A is a cause in relation to B, and B's being a billiard ball is not dependent upon whether B is an effect in relation to A. On the contrary, things are a necessary (though not sufficient) condition, in this case, for the possibility of a dynamical judgment. Kant would insist that the dynamical judgment presupposes mathematical judgments.

The logical priority of the 'mathematical object' in all judgments of physical science suggests that a closer examination of the general characteristics of mathematical judgments may reveal the synthesis in those judgments to be synthetic in a fundamental way. We have seen that
in dynamical judgments involving the appearance of relation between objects, of the appearance of an object's being either possible, actual, or necessary, the postulation of causality, necessity, duration, etc., has been necessary to account for 'what happens'—that is, to **effect the synthesis:** to provide the concept of an object which, as it were, 'holds the synthesis together.'

But what is the analogous factor that holds a mathematical synthesis together? Can we use any of the mathematical categories to 'account for' the appearance of synthetic unity in a mathematical judgment? Must we conclude that mathematical synthesis is simply an arbitrary juxtaposition which occur in judgments employing the six quantitative and qualitative categories?

To answer these questions, we shall first have to look at the arguments advanced in the Axioms of Intuition and the Anticipations of Perception for the six categories of Quantity and Quality which are employed in all mathematical judgments; then, in the next chapter, we shall try to spell out explicitly what the mathematical synthesis is— and what it is not. The Mathematical
Principles, to which we now turn, are of particular importance to our investigation because they are Kant's statement of the general requirements for mathematical judgments. They show how it is that all mathematical judgments can be said to be constructive judgments; they set out the principles of construction in judgment. In this way, they serve as the general Principles which make the special principles of the science of mathematics possible. 10

Because the Mathematical Principles are at the same time, however, intended as 'arguments' for the mathematical categories, they cannot themselves be mathematical judgments; Kant realized that in arguing for the mathematical categories, mathematical judgments would be of little help, since their certainty is intuitive. A proof, in order to be effective, must be discursive. The Axioms of Intuition and the Anticipations of Perception, then, are discursive proofs for the categories of Quantity and Quality respectively. Their purpose is to show that all combination (synthesis) in the synthetic a priori judgments Kant named 'mathematical' must make
use of the 'mathematical categories': Unity, Plurality, Totality, Reality, Negation, and Limitation. Kant believes that to show this it will be sufficient to demonstrate that all such combination, which he later calls 'construction,' \[12\] is in accordance with the universal condition of time, so that the constructive synthesis which presupposes time \[13\] will necessarily presuppose the mathematical categories (through the employment of their schemata, Extensive Magnitude and Intensive Magnitude).

The first of the Mathematical Principles to be dealt with by Kant are the Axioms of Intuition, which take as their principle that "All appearances are, in their intuition, extensive magnitudes." \[14\] That is, the recognition of an object in time (or in space and time) is possible only as the schema of Extensive Magnitude is employed, and the object, in order to be recognizable, must be distinct: it must be determinate, in the sense of having a certain measurable (or numerical) extent in space and time (or in time alone). Of course, it is true, as Paton observes, that it is not necessary actually to measure an object in order to recognize it as an object,
but "in regarding it as an object we presuppose that it can be measured."\(^{15}\)

All quantitative extension, for Kant, can be understood in some sense of 'extension in time.' Not only the numerical quantity of arithmetic, or the clearly non-spatial quantity of time, but even the spatial quantity of geometry can be seen to presuppose time because they are all instances of construction, and construction can occur only in time. Yet this is not to imply that Kant's notion of construction relies upon a doctrine of mental activity. On the contrary, if time is simply a 'universal condition' under which appearances must present themselves to the mind, then it will not be necessary to understand the 'construction of appearances in time' as some sort of psychological process.

Let us take, for example, construction in arithmetic. An extensive magnitude in arithmetic would, of course, be a number, such as '15.' Kant would insist that the concept of the number 15 presupposes time because it is impossible to conceive of constructing the number 15 except by the successive addition of units. This is
not to say that we actually perform some mental process constructing the number whenever we use the concept, only that the number is a composition of 15 units, and that if we were to compose the number from such units, then it could only be accomplished over a period of time.

That spatial construction also presupposes time can be seen in Kant's specification that to conceive a line is to conceive it as drawn. The geometrical notion of a line as a successively generated series of points may be said to 'contain' within it the element of time, for 'successive generation' is a temporal notion. This must not, however, be read as a merely psychological statement; the psychological is irrelevant to Kant, and it is clear from his conception of time as a pure a priori (i.e. non-empirical) intuition that he is concerned here not with an empirical statement, as a psychological statement of my mentally drawing a line would be, but with a statement about a priori construction--construction which is presupposed. My concept of a drawn line is of a constructed magnitude, as indeed all magnitudes are
necessarily constructed magnitudes. And that is to say that any geometrical construction, if it were to be carried out, would be possible only over a period of time. Paton observes that to recognize a triangle is not to take the time mentally to construct each side, but that if there were any doubt about its being a triangle,

"We should test the figure by considering whether it could be constructed in this way... A successive synthesis is the basis on which our recognition rests. Indeed, if Kant is right, to recognize this figure as a triangle is to recognize, however 'obscurely,' that it is the product of such construction."

Kant believes that it is a fact about the nature of all appearances considered as magnitudes that they must be, implicitly, the product of temporal construction. This fact becomes even more obvious in the case of time segments. Our conception of any period of time (e.g. a day, a month, a year) carries with it the necessity of having been constructed from what can only be successive moments. That is, we conceive of a period of time as measurable (as an extensive magnitude) because it represents a composition of smaller units of time.
But does this not present a contradiction with the notion of space and time as *non-discursive* presented in the Transcendental Aesthetic? If we understand all knowable time as 'periods of time' and all knowable space as 'segments' of the same sort as our drawn line, and if the periods and segments are constructions from "manifolds of antecedently given parts," then must we not conclude that space is discursive? Robert Paul Wolff, in his *Kant's Theory of Mental Activity*, cites this as a clear contradiction of the Aesthetic. But a closer look will, I believe, prove Wolff mistaken: the knowable, discursive space and time of the Analytic are only parts of the intuitive space and time of Aesthetic. In the Aesthetic, space and time are not 'knowable' as such; they are the pure a priori intuitions which make knowledge possible. On this level, as Justus Hartnack rightly points out, space and time are indeed not composed of parts, but may be divided into parts; and it is with these parts—which are themselves discursive compositions—that Kant is concerned in the Transcendental Analytic. Heidegger expresses this distinction somewhat more clearly
in *Die Frage nach dem Ding* by showing that Kant's understanding of 'magnitude' includes the notions of both *quantum* and *quantitas*. Magnitude understood as *quantum* is 'the sizable,' but magnitude understood as *quantitas* is 'the measurement of the sizable.' That is, space or time as *quantum* is the continuous non-discursive space or time of the *Aesthetic*. And this *quantum* is always presupposed by *quantitas*, so that we can say of any measure of space or time that it requires a 'sizable' space or time to be measured. Thus space and time, as the a priori forms of intuition, provide the basis for the determination of all the sensible contents of intuition.

All appearances which are extensive magnitudes, says Kant, are "intuited as aggregates." These aggregates he characterizes as "complexes of previously given parts," and not as constructions. The appearance, in this case, then, will have to be taken as a *mere* appearance, for "only through successive synthesis of part to part in the process of its apprehension can it come to be known." But Kant must not be taken to mean that appearances are
first mere aggregates (in intuition) and then constructions (in the synthetic judgment, where they come to be known). Rather, the intuition of the appearance as mere aggregate and the synthesis of the appearance as object of knowledge always occur together; they are simply two aspects of apprehension, not successive steps in a mental procedure. We do not recognize objects in the world by first encountering instances of empirical intuition and then putting those instances together to form an object: the object is present from the beginning in intuition (specific 'instances of intuition' are marked off by us), and the synthesis of the object does not follow intuition but is simultaneous with it.

Still, how are we to understand the 'putting together'? Kant's talk of a 'successive synthesis' seems to lend credibility to interpretations of the constructive synthesis as a mental activity. Kemp Smith's insistence on inserting the words 'the process of' in his translation of the passage quoted above makes it appear (as he contends) that the synthesis is effected in a temporal sequence carried out by the imagination as it apprehends
the aggregate. In his Commentary, Kemp Smith suggests that Kant did understand this synthesis as a mental act of composition from units, and that this view of apprehension was later proved false by empirical psychology.

But this, I believe, represents a misreading of Kant to begin with. Kant does not refer to the mathematical synthesis in the Axioms of Intuition as a 'process.' Elsewhere in the Analytic of Principles, where he does refer to it in this way, he is careful to qualify his use of the word 'process' by specifying that construction is merely a 'possible' process. It is a process that is 'implicit' in any notion of constructed magnitude; the noun 'construction,' when used, as in geometry, to refer to the object of a mathematical synthesis, carries with it the implication that that object could be constructed over a period of time: that its construction by a successive synthesis is possible. In this sense the constructive synthesis is 'temporal,' but the actual apprehension of the synthesis will always be, as it were, in a single moment. For Kant the temporal synthesis of the manifold must always be a priori, and
not empirical, because *time*, as an intuition, is itself a priori and not empirical. To ask about the temporal nature of synthesis in construction is thus not to ask anything about the activity of the mind; it is, rather, to ask what sorts of requirements (for the synthetic object) are presupposed in our talk about 'construction.'

The fact that to recognize any magnitude as a construction is to presuppose that it could have been the product of a constructive process in time is a truth which holds necessarily for all synthetic a priori judgments. In particular, it is a principle for all those judgments which represent the synthesis of a homogeneous manifold—the judgments Kant calls 'mathematical judgments.'

Kant's discussion of the Mathematical Principles turns next to the Anticipations of Perception, synthetic a priori judgments which are intended as discursive proofs of the categories of Quality. In the same way that judgments employ the quantitative categories of Unity, Plurality, and Totality by virtue of being judgments of extensive magnitude, so judgments employ the qualitative categories of Reality, Negation, and Limitation by virtue
of being judgments of intensive magnitude. **Intensive Magnitude** is the schema of the categories of Quality. And as with judgments of Quantity, judgments of Quality are constructions; therefore, they can be shown to presuppose time.

But construction in judgments of intensive magnitude is somewhat different from that in judgments of extensive magnitude. While the latter is characterized by Kant as an 'aggregation,' the former is described as a 'coalition.' That is, the synthesis of the homogeneous in quantitative judgments (aggregation) is a synthesis of units, each of which bears its own unique relation to every other unit in the construction, but the synthesis of the homogeneous in qualitative judgments (coalition) is a synthesis of units in which every unit is related to every other unit in exactly the same way. A judgment of intensity will be a synthesis of 'successive' units only implicitly because such successive synthesis is implied in the notion of 'construction,' and because the 'intensity' of any given sensation can be understood only as an 'intensive magnitude,' i.e. as a construction.
A qualitative judgment is, then, a judgment in which we come to know an object in terms of its degree of intensity—the degree of intensity with which the sensible contents of intuition may be present in any given appearance. Clearly these sensible contents are always contingent factors in intuition; sensation can be of many different kinds, and its particular variations obviously cannot be anticipated in all perception generally. Thus it might seem that no a priori judgment could be made about the 'sensible character' of intuition. But there is at least one statement which can be said to be knowable a priori of the sensible contents of any intuition: that they must be present in a certain measurable intensity. Kant says, "In all appearances, the real that is an object of sensation has intensive magnitude, that is, a degree." And to that factor of intensity in sensation—considered as sensation in general—which can be known a priori without regard to the kind of sensation, Kant gives the name 'anticipation.' For to say that the fact of having a certain intensity can be known in advance about any
particular sensation is to say that it can be anticipated. Intensive magnitude, as the necessary conceptualization in which sensation is understandable, can be anticipated in all perception.

But how is intensive magnitude a temporal conceptualization? In what way does any concept of intensive magnitude presuppose time? We can best begin to answer this question by spelling out the sense in which an intensive magnitude can be said to be a constructed magnitude. To say that any object of sensible intuition must have a 'certain degree' of intensity is to say that its intensity can be understood only as a magnitude which is removed to a particular measurable degree from 'zero-intensity,' for, as Kant observes in the \textit{Prolegomena}, "There is no perception which can prove an absolute absence."\textsuperscript{29} For any given degree of intensity, therefore, there must be a continuous scale of possible degrees between that degree and zero. Furthermore, since it is impossible for a sensation to be of more than one intensity at a time, it is thus necessary to understand a particular degree of sensation (a particular intensive
magnitude) as, in some sense, the product of a successive addition (i.e. construction) of 'units of intensity' over a period of time. But this is not to say that in apprehending any given degree as a construction we somehow mentally pass through all possible degrees from zero up to that actual degree. It is sufficient to regard the particular degree of intensity as a construction which, if carried out, would then involve a temporal process of passing through all intermediate degrees and combining them in a synthesis. That is, any given intensive magnitude must be capable of being constructed in this way, by the synthesis of intermediate degrees as a process in time. Kant's point seems to be that the notion of 'construction' carries with it an implicit reference to such successive putting together: to understand what 'construction' is, is to have some conception of putting units together as a procedure which takes time.

To say that any object, then, considered as an intensive magnitude, represents a synthesis of all possible degrees of intensity between itself and zero, is simply
to say that the possibility of being constructed from intermediate degrees is presupposed in any such notion of an object. In the same way, intensive magnitude is a temporal determination, by virtue of being a construction which presupposes a successive synthesis of degrees, a synthesis which could occur only over a period of time.

We must therefore guard against any attempt to interpret the temporal nature of intensity as having to do, in some way, with the apprehension of the synthesis by the mind. As we have shown, Kant's talk of a 'process' of synthesis should be understood as reference to a 'possible process'\(^{32}\) (implied in the notion of construction) and not to a mental 'process of apprehension,' which would involve the mind's successively synthesizing successively given degrees. It is precisely in this latter sense that Kemp Smith misconstrues Kant's intentions in the Anticipations of Perception, reading Kant's discussion of the synthesis, once again, as an unsuccessful attempt at psychology.\(^{33}\) But Kant's insistence that the apprehension of the synthesis must occupy but a single instant of time\(^ {34}\) leads one to suspect that the discus-
sion of the synthesis in the Anticipations of Percep-
tion is definitely not intended as a treatise on the
psychology of mental processes. To Kemp Smith, this
insistence on an 'instantaneous' apprehension of the
synthesis seems paradoxical in light of Kant's ref-
erences to 'successive synthesis,' synthesis as a
'process,' etc., and therefore Kemp Smith tries to
explain it as a vagary which is incidental to the notion
of synthesis as a mental activity (which Kemp Smith
believes was Kant's actual position). If we understand
the synthesis, however, as a possible process entailed
by the notion of a 'construction,' as suggested above,
then it is not necessary to resort to an interpretation
of the synthesis in terms of mental activity which would
make the requirement of instantaneous apprehension appear
inconsistent. As we have shown, to apprehend any degree
of intensive magnitude as a construction is to apprehend
it as a temporal synthesis-- i.e. as one which could be
put together only in time-- and therefore such apprehen-
sion need not itself be a successive process occurring
over a period of time in order for the synthesis to be
'temporal.' The apprehension of an object in a certain degree of intensity will be in a single instant (which is, of course, the way we do ordinarily perceive degrees of intensity).

Kant's strategy in the Anticipations of Perception, then, is essentially the same as in the Axioms of Intuition: if it can be shown that any judgment of intensive magnitude— or, in the Axioms, any judgment of extensive magnitude— is necessarily a construction, then it will follow that such a judgment presupposes time, for nothing can actually be constructed (i.e. put together) except in time. In the next chapter we shall have a closer look at 'construction' itself, which appears to be the characteristic mode of mathematical synthesis. We have already seen that in every instance of dynamical judgment the mathematical judgment will be presupposed, and it should thus be possible to infer that for Kant the fundamental synthesis on which all judgment ultimately depends will be a constructive synthesis.
NOTES TO CHAPTER IV

1 Kant, *Critique of Pure Reason*, B15-17.

2 Kant's term is 'Dasein.'

3 To say the certainty is 'only discursive' is not to say it is incomplete.

4 Indeed, one cannot know anything less, according to Kant.

5 It is not, however, an analytic certainty. See A161/B200.


7 Or it may be both actual and necessary.

8 A particular spatially and temporally distinct individual. Not to be confused with the thing-in-itself (Ding-an-Sich).

9 This is a point advanced by Eugene Gendlin in his analysis of Heidegger's *What is a Thing?*, p. 278.


11 Ibid., A734/B762.

12 Ibid., A714/B742.

13 Some presuppose space as well, but all presuppose time.


NOTES TO CHAPTER IV (continued)

16 Kant, *Critique of Pure Reason*, A163/B204.
17 Hartnack, *Kant's Theory of Knowledge*, p. 70.
18 Paton, *Kant's Metaphysic of Experience*, p. 117.
21 These are 'parts' in the sense of 'divisions,' not of 'components.'
24 Kant, *Critique of Pure Reason*, A163/B204.
26 Kant, *Critique of Pure Reason*, B208.
27 Ibid., B202.
28 Ibid., B208.
30 Ibid., II., xxvi.
NOTES TO CHAPTER IV (continued)

31 Ibid.

32 Kant, Critique of Pure Reason, B208.

33 Kemp Smith, A Commentary to Kant's 'Critique of Pure Reason,' pp. 353-54.

34 Kant, Critique of Pure Reason, A168/B210.

35 Ibid., A169/B211.
V. WHAT IS MATHEMATICAL?

From our discussion of the principles of mathematical judgment, it should be clear that in all such judgments the synthesis will be a construction of appearances, not a connection of appearances. It will be the synthesis of a manifold which is homogeneous, and will therefore have the character of a simple composition of undifferentiated units. This simple composition from units is what is meant by 'construction,' and it is in this sense that the mathematical synthesis can be said to be a constructive synthesis.

In the mathematical synthesis, a whole is constructed from logically unrelated parts. These parts, or 'units,' must be bare units considered abstractly as having no characteristics (other than their spatial and temporal separation) which could allow one to be distinguished from another. For a given manifold, that is to say, they must be all alike, as units in arithmetic or points in geometry are all alike. This is the sense of Kant's insistence that the manifold of intuition (whether pure
or empirical) which is combined in the mathematical synthesis must be homogeneous. The synthesis must furthermore be a combination of units which are 'originally' unrelated: in other words, there must be no logical necessity of their being combined. Thus the whole which is formed by the combination of units is not derivable by implication from them. In arithmetic we can obtain the number 12 by adding 7 and 5 together, and in geometry we can combine two isoceles right triangles in such a way as to form a square, but it is surely wrong to say that the notions of 7 and 5, taken together, imply 12, or that the concept of a square is somehow derivable from the concepts of two triangles. ¹ The object of a mathematical judgment, then, will be 'underived.' Mathematical objects, unlike dynamical objects, will not be inferred but constructed.

The role of the empirical concept in a mathematical judgment, therefore, will have to be different from its function in a dynamical judgment. While it is true—as was suggested earlier—that the concept must always serve as the 'unity of rule' in synthetic judgments
whereby we can come to know aggregates of representations in certain appearances as objects, it is also true that the unity of rule will not have the same function in the dynamical synthesis of the connective judgment that it will have in the mathematical synthesis of the constructive judgment. Let us consider as an example a judgment involving the dynamical synthesis— a judgment, say, of a thing's duration in time. I judge that the building I see on a particular street corner today is the same building that I saw on that corner yesterday because the building I now see appears to be like the other in virtually all respects, and I can account for the similarity of the two appearances if the judgment I make about them employs a concept which presupposes the category of Inherence and Subsistence. In a dynamical judgment of this sort, the concept clearly has a connective function: the high degree of similarity between the two appearances can be explained if they can be connected by saying that they are appearances of the same thing (which has endured in time). Here the duration of the object can be inferred from the recurrence of appearances
which are highly similar only because of the employment of the category of Inherence and Subsistence. Furthermore, such an inference—in which the concept of objective duration is employed—is necessary in order to give an account of the object that appears.

But no such inference is possible in the mathematical judgment: as we have indicated above, the mathematical object cannot be derived from the combination of representations in an appearance—just as the concept of 12 is not derivable from the concepts 7 and 5. Therefore, the role of an empirical concept in the mathematical synthesis cannot be connective; its function is not to 'give an objective account' of the relation between appearances (i.e. between objects) or of the mode of existence of an object in the world. It serves simply as a convenient unifying term (or designation) under which perceptual aggregates which exhibit a certain internal coherence in an appearance can be understood as an object. In the mathematical synthesis there is nothing for the concept to 'account for,' only a composition of representations to be comprehended as a unity. For instance,
to 'recognize' a certain coherent appearance as a 'billiard ball' is to effect a synthesis of certain representations of extensive magnitude (a certain measurable size and geometric shape) and intensive magnitude (a certain measurable intensity of weight and color, hardness, etc.) in a mathematical judgment.

The object of this mathematical judgment, then, is a simple 'construction' which is not in any way implied in the sensible manifold of representations; and the synthesis in which the object comes to be known is just the understanding (or recognition) of an appearance as a composition, rather than as a mere aggregate.

Since it is of fundamental importance to our discussion of judgment that the mathematical synthesis be understood as a construction, it is obvious that we must be clear about what 'construction' is for Kant. For the sake of further clarification on this point, then, it will prove helpful to outline the distinction between constructive and discursive reasoning which Kant draws in the Discipline of Pure Reason in its Dogmatic Employment, which is the first major subdivision within the
Transcendental Doctrine of Method. This distinction can best be made, he believes, by comparing the type of reasoning in mathematics with the type of reasoning in traditional formal logic. The one he calls 'mathematical reasoning,' the other, 'philosophical reasoning.'

Philosophical reasoning is a technique of moving by steps from premises to conclusion: that is to say, it is 'discursive' reasoning. But mathematical reasoning is not the logical deduction of a conclusion from premises; it is a composition of units, all of which are essential. In other words, it is a 'constructive' reasoning. Kant observes that although in natural science these two forms "do indeed go hand in hand," they are nevertheless completely different, and that when logic, for instance, attempts to 'imitate' the mathematical method (as Kant believed Rationalist metaphysics had tried to do), it can lead only to error. Constructive reasoning has the advantage of a certain exactness which cannot be duplicated in philosophical reasoning. This exactness is the result of its employ-
ment of definitions, axioms, and demonstrations; Kant maintains that "none of these, in the sense in which they are understood by the mathematician, can be achieved by the philosopher."  

Mathematics, says Kant, always begins with definitions; moreover, its definitions are simply stated as elementary, without any attempt to derive them from anything else. Philosophical 'definitions,' such as occur in metaphysics and the 'philosophy of nature,' can only be judgments which are meant as expositions of other judgments (which show how the former can be derived from the latter); and since the thoroughness of such expositions is always open to question, these 'definitions' can never be apodictically certain. Kant would say they are not true definitions at all. But the judgments which serve as definitions in mathematical reasoning are 'constructions'; they represent what Kant calls an arbitrary synthesis of units. And since these judgments are fundamentally synthetic in this sense, they contain nothing but what is put into them (and nothing more can be deduced from them) and thus no question of the 'thor-
oughness' of these definitions can arise; therefore, mathematical definitions can never be in error. 7

Mathematical reasoning is always 'intuitive' reasoning. The principles upon which it proceeds are the necessary conditions for construction in space and time: what is knowable in a principle of mathematical reasoning will be knowable immediately, since it can be nothing other than an elementary determination of space and time; it must therefore be true that such principles can be known by pure intuition. 8 Only in mathematical reasoning, says Kant, can a synthetic judgment be possible which will be an intuitive principle—i.e. a proposition whose certainty is immediately evident, as in the geometrical proposition that three points always lie in a plane— and thus only mathematics can have axioms. 9

The principles of philosophical reasoning, by contrast, will always be 'discursive' principles: judgments in philosophy will be judgments deduced in some way from other judgments. The truth of a philosophical judgment, then, will not be immediately evident, but will require a certain inference, as when a relation of causality and
dependence is judged to obtain between two objects.
Judgments involving this sort of synthesis, which Kant earlier describes as 'connection,' will always rely upon an inference; their certainty will be that of an inference. Judgments of construction, on the other hand, require no such inference in order for their certainty to be recognized; in true mathematical judgments the certainty will be intuitive. Because mathematical reasoning alone proceeds upon intuitive principles, only mathematical reasoning can be said to possess axioms.

In a similar way, Kant would insist that only intuitive reasoning can make use of demonstrations. Thus mathematics can have demonstrations but philosophy cannot; the expositions advanced by way of analysis in discursive reasoning are not demonstrations. To illustrate this distinction, Kant compares the characteristic mathematical and philosophical methods of reasoning about the notion of a triangle. Given the problem of explaining what a triangle is, the philosopher can only begin with the concept of a triangle and attempt
to discover what is implied in that concept. He will find that the concept of a triangle presupposes the concepts of a 'straight line,' of 'three,' of an 'angle,' etc., and that each of these notions can in turn be seen to presuppose still more elementary concepts. No matter how extensive his exposition may be, however, he will never be able to avoid the explanation of concepts in terms of other concepts; the explanatory force of his exposition, then, must depend finally upon the intelligibility of certain concepts, and such an explanation, says Kant, cannot be called a demonstration.

For a mathematician, on the other hand, explaining what a triangle is involves nothing more than showing how a triangle can be constructed. He can construct the figure, bisect its angles, measure its sides, etc.; and, of course, the certainty of the mathematical truths expressed in the mathematician's exposition is not derived. Since mathematical (constructive) reasoning is in terms of particular examples— and not of general concepts, as with philosophical (discursive) reasoning— the mathematical exposition need not involve a successive
procedure of inference. The mathematical exposition is, on the contrary, evident in its every aspect; every part of the particular triangle which the mathematician constructs is evident in its relation to every other part; each step in the construction and what follows from it can be, as it were, laid out before us. Because only constructive reasoning can make use of such ostensive expositions, only mathematics can be said to employ demonstrations; according to Kant, no genuine demonstrations are possible in philosophy. 12

Kant maintains that a genuine demonstration will be intuitive. But its intuitive nature does not consist simply in its being an obvious construction in any given example where the construction is literally carried out (as in drawing a triangle on a blackboard); rather, what enables us to recognize intuitively the validity of any such ostensive demonstration is nothing but the fundamental simplicity of juxtaposition which characterizes all mathematical construction. Construction in mathematics involves a kind of putting together which is basically simple among all possible kinds of putting
together which are allowable in Kant's view of the world.

This is, of course, to say nothing about the possible complexity of any particular construction in geometry or arithmetic. However elaborate a given construction may be, the basic mode of synthesis which characterizes it throughout will, in Kant's view, be of one simple kind--the kind he refers to on one occasion as the 'mathematical synthesis.' The mathematical synthesis, particularly obvious in the constructions of mathematics, is a putting together in which there is never any logical necessity, but only the transcendental necessity in the a priori conditions of space and time. In other words, the possibilities of combination in any construction of mathematics is 'governed' only--and directly--by the transcendental requirements of space and time. Consequently, to grasp a geometrical relation is to know 'space' in the most immediate way it can be known. And to count a numerical series of units so as to arrive at an arithmetical sum--and thus to grasp the sense of succession--is to know 'time' in the most immediate way it can be known. To see the validity of a demonstration in math-
ematics is simply to grasp a mathematical truth, and to grasp a mathematical truth is nothing more than to recognize a certain truth about space and time (or time alone). For this reason, Kant calls such a demonstration *intuitive*; it is knowable by pure intuition. In Transcendental Logic a mathematical truth will be a logical truth; that is not to say that it is analytic, however, only that it is apodictic. 17

Kant's notion of 'construction,' then, as expressed in the discussion of mathematical method, can be seen to rely heavily upon a notion of *synthesis* which is 'fundamental' in character: it is a putting together of undifferentiated parts (or units) which, of themselves, have no necessity of being combined at all, and in this sense, it is an 'arbitrary' synthesis; and yet, since this combination is simply *constructed* of units in space and time, and not inferred, it is also capable of being known *intuitively* as nothing more than a logical expression of the transcendental requirements of space and time.

This fundamental synthesis seems to be central to Kant's
understanding of constructive reasoning in general, and so it may properly be called a 'mathematical synthesis.' But it is clear that in the first *Critique* Kant's concern with 'mathematical' judgments is not limited solely to considerations of the science of mathematics, and that, as we have already seen, a mathematical synthesis is essential not only to the construction of sums and triangles, but to the construction of appearances as well. While in the former case the construction will be literally carried out as a 'process'—either mentally or, e.g., on paper— in the latter case the recognition of an object as a magnitude (i.e. as a construction) will merely presuppose the possibility of its having been constructed by a process in time. But the 'synthesis of representations' in an appearance—whereby an object comes to be known—is, in any case, no less the synthesis of a *homogeneous manifold* than is any arithmetical composition of units. Of course, the homogeneous manifold in a synthesis of representations will necessarily be a sensible manifold.

As we have already suggested, Kant's theory of syn-
thetic judgment will allow of no combination more fundamental than that in which the categories of Quantity and Quality are employed. The combinations of quantitative representations and of qualitative representations— the distinctly measurable aspect of appearance— is effected in a 'mathematical synthesis.'\textsuperscript{18} From our discussion in this chapter of the nature of construction, it should now be clear why quantitative and qualitative judgments, as constructive judgments, must be, in a sense, 'fundamental' judgments— and the synthesis in each case a fundamental synthesis. The kind of juxtaposition involved in Kant's understanding of construction (or 'composition') is a simple putting together in which there is no necessity other than that expressed in the transcendental requirements of space and time. Judgments effecting such an elementary juxtaposition are judgments which, if true, will be intuitively certain. Since for Kant all judgments of quantity and quality are judgments of extensive and intensive magnitude respectively, and since such judgments will, as we have seen, presuppose construction in any case, thus— Kant would
argue—such judgments are capable of intuitive certainty. They are fundamentally simple in that the possibilities of their combination are defined only by the 'transcendental necessity' in space and time. Not without reason does Kant call the synthesis in these judgments mathematical.

The mathematical synthesis, then, is a fundamental synthesis in its form; but it can also be called fundamental in regard to its place in the theory of judgment. As we have seen in the last chapter, dynamical judgments always depend upon mathematical judgments, because there can be no judgments about an appearance's (i.e. an object's) mode of existence in the world, or about its relation to other appearances, unless there are appearances to begin with, and these 'bare appearances' are the objects of mathematical judgment: only in the mathematical synthesis of representations can the appearance first be conceivable as a 'thing'—as an object with extensive and intensive dimensions. Of course, it is only 'conceivable,' in the sense that it is not logically impossible to imagine an object ab—
stracted from its 'existence in the world' (its relations and mode of existence), but Kant would quickly add that no such 'abstract object' can ever actually be encountered. Kant's position seems to be that in any dynamical judgment about an object the mathematical object is always presupposed.

A dynamical synthesis, for Kant, always presupposes a mathematical synthesis; therefore, in the Transcendental Synthesis itself, where the object comes to be known in both its mathematical and dynamical aspects, the mathematical synthesis could be called 'primary' and the dynamical synthesis 'secondary,' since the dynamical is, as we have shown, logically dependent upon the mathematical. The mathematical synthesis, in Kant's view, will be primary in any specific synthetic judgment that can be made about the world.

We shall have to conclude, at last, that the theory of judgment which Kant advances in the Critique of Pure Reason accords a fundamental role to the mathematical synthesis. It is a fundamental synthesis both in the manner of its combination and the manifold which it
combines. And since the dynamical judgments which complete the Transcendental Synthesis must assume mathematical judgments— as mathematical judgments are actually contained (i.e. incorporated) in dynamical judgments— therefore the combination which is most elementary in the Transcendental Synthesis will be the mathematical. In this sense the mathematical synthesis can, to use a temporal metaphor, be discovered at the very beginning of the Kantian judgment.
NOTES TO CHAPTER V

1 Kant, Critique of Pure Reason, B202.
2 Ibid., A714/B742.
3 Ibid., A726/B754.
5 Ibid., A730/B758.
6 Ibid.
7 Ibid., A732/B760.
8 Ibid., A166/B207.
9 Ibid., A733/B761.
10 Ibid., A735/B763.
11 Ibid., A717/B745.
12 Ibid., A735/B763.
13 Ibid., A734/B762.
14 Ibid., A161/B200.

15 And, of course, there is the necessity of its conforming with the analytic Principle of Contradiction.

16 In arithmetic, Kant would say that it is governed by the requirement of time only.

17 I.e. it is "necessary in space and time."

18 Kant, Critique of Pure Reason, A161/B200.
NOTES TO CHAPTER V (continued)

19. This point will be recognized from the preceding chapter.

20. The priority suggested here is logical, not temporal.

21. This sense of 'logical' is meant to include Kant's transcendental requirements of space and time.
BIBLIOGRAPHY


BIBLIOGRAPHY (continued)
