RICE UNIVERSITY

Analyticity and Logical Truth

by

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A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF

Master of Arts

Thesis Director's signature:

Houston, Texas
May, 1965
The concept of analyticity is not one concept but many. Consequently if a distinction is to be made between statements which are claimed to be analytic and statements which are synthetic, how the distinction is to be made becomes a question of crucial importance. In the essay I review critically several representative formulations of the distinction that has traditionally been made between two kinds of truth: the Aristotelian distinction between essence and accident; Leibniz' effort to separate truths of reason from truths of fact; Hume's classification of relations; and the explicit formulation of Kant who originated the terminology "analytic-synthetic." Each sought to distinguish truths of experience from truths which can be known on logical grounds. But each understood logic differently and each formulated the distinction within the context of his own epistemology. It was Frege who first completed and systematized modern logical theory and distinguished between analytic and synthetic propositions within a formal language system where the distinction could be made apart from epistemological considerations.

Since Frege, there have been many efforts to express the distinction precisely, both within formal language, and in natural language as well. Tarski's formulation of the semantic concept of truth explicates the distinction for all first order uninterpreted languages but there has been no comparably successful explication for interpreted language.
In lieu of such a characterization some philosophers have sought to extend the defined concept of truth for formal language to natural language. However, though the concept of logical truth explicates one aspect of the notion of analyticity, it does not clarify the concept for natural language where additional factors must be considered. If traditional formulations of analyticity do represent efforts to clarify an intuitive notion, then perhaps attempts to characterize statements of natural language apart from epistemological and psychological considerations are misdirected.

The concluding Chapter focuses on two of the numerous contemporary attempts to explicate a concept of analyticity: the characterization of analytic statements as "definitional rules" developed by Rudolph Carnap, and the effort to understand analyticity as "empirical generalization." Though these two approaches are not fully representative of current studies, they do presuppose very different assumptions about the behavior of natural language, and suggest divergent solutions to the problem. I argue that Carnap's formulation involves serious ambiguities and distorts common linguistic usage; the analytic statements of natural language are dependent upon context and cannot be distinguished apart from the interpreter of the language.
Preface

Questions of meaning and truth are of perennial concern to philosophy. Thus it is hardly surprising that the problem of analyticity has become so prominent a concern in recent philosophical literature. For whether we can explicate a concept of analyticity, involves our answers to such questions as: "Under what conditions does a statement have meaning?" and "How do we find out whether a statement is true?" Some participants in the recent controversy over the analytic-synthetic distinction argued that the distinction belongs to a version of empiricism no longer viable; others maintained that the distinction was basic to the empiricist position per se. My interest in the problem originated in an effort to understand the interrelationships between analyticity and empiricism and between analyticity and other philosophical traditions. The foregoing essay reflects only one aspect of this problem, for I discovered that there are many concepts of analyticity involving many answers to questions of meaning and truth.

Of those books listed in the bibliography three have proven particularly helpful to me. In the absence of a comprehensive historical survey of the distinction, I have found William and Martha Kneale's The Development of Logic most useful. For its extensive consideration of the literature of the controversy and its original suggestions
for a development of a concept of analyticity, I am indebted to Alan Pasch's *Experience and the Analytic*. But Arthur Pap's exhaustive arguments in *Semantics and Necessary Truth* have contributed more than any other volume to the development of my own understanding of the problem.

However, most of all, I am indebted to Professor John Alan Robinson who has given me most generously of his time, when I requested it, and granted me freedom to flounder, when I chose that course. I am also grateful to the Chairman of the Department of Geology for his many services, but especially for his generous loan of library space. And to Chairman, J.S. Fulton and other members of the Philosophy Department my sincere thanks for their many expressions of interest and encouragement.
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INTRODUCTION

Much recent philosophical literature has been concerned with the reconsideration of a distinction which has traditionally been accepted as a mere part of the philosophical furniture. Since Aristotle—at least—philosophers have distinguished propositions which are true by reason, or necessarily true, from propositions which are only accidentally or factually true. Kant characterized the first class of propositions as "analytic" and the second as "synthetic." Though usages of "analytic" or of variant terms with similar intentions have fluctuated, not only among philosophers, but within the works of a given philosopher, nearly all have meant to indicate that such propositions were recognizable as "true on logical grounds."

Recently, however, some philosophers, in quest of a more precise characterization of analyticity, have associated the term "analytic" with the formally defined modern notion of "logical truth." This association, based, as we shall see, on an extension of the defined concept of logical truth has, not only failed to clarify the notion of analyticity, but has obscured the precision of the concept of logical truth, as well.

In the following essay I would like to show that the notion of analyticity is not the same as the notion of logical truth but is actually a very different notion which, originally formulated within the system of Aristotelian
logic, has subsequently undergone considerable modification. Reformulated within the context of modern logical developments, analyticity still resembles the historic notion of "truth on logical grounds" though it is perhaps today a more useful and basic notion for ordinary language philosophy than it was in Aristotle's or Kant's formulation. However, it is only for convenience that we refer to the notion of analyticity for there are actually many notions of analyticity. Each philosopher has formulated it somewhat differently, relating it to his own general philosophical exposition and to his particular understanding of such terms as "necessary" and "logical." It is the hope of some that it will eventually become possible to define analyticity with the precision and exactitude which Tarski gave to the concept of truth, though explication of so complex a concept awaits further logical development. I hope to show that, though we lack such an explication, the notion of analyticity is nonetheless well worth retaining despite the criticisms of those philosophers who label it "a dogma of empiricism."

The historical section of this essay can in no way be regarded as a comprehensive survey. In the course of reviewing the philosophical tradition it has become increasingly evident to me that virtually all the major contributors to the tradition at least imply a distinction between two kinds of truth. I have, however, chosen to discuss only
a few - Aristotle, Leibniz, Hume and Kant - who sought to make the distinction explicitly. The classification has played varying roles within each of these philosophical systems and though it would be of considerable interest to show the interrelationships between the ways in which they distinguish kinds of truth and the other features of their systems, this is another topic and deserves separate consideration. It will have to suffice for our purposes merely to suggest that there are such interrelationships. We shall focus instead on how Aristotle, Leibniz, Hume and Kant characterize the propositions\(^1\) which they assert to be true independent of matters of fact or circumstance - the propositions which we generally label "analytic" today.

Though the work of George Boole and Augustus De Morgan were fundamental to the development of modern logical theory, the systematic formulation of logical theory is most often credited to Gottlob Frege. It is not merely coincidental that Frege also marks the historic turning point in the formulation of the concept of analyticity. "Analytic" for Frege could only be understood with reference to his own logic. Thus in order to understand the change

\(^1\)We will, for the present, make no distinction between "sentence," "statement," "proposition," and "judgement." Though such a distinction is relevant to the material introduced in Chapters II and III, to introduce it now would only suggest falsely that the philosophers under discussion here actually employed such a distinction where, in actual fact, these terms were commonly employed synonymously until recently.
in the usage of the term "analytic" it becomes necessary to indicate the new complexities in logical form which, in the last century have formed the nucleus of a significant new body of theoretical knowledge. The concept of logical truth occupies a central position within this framework, and thus our distinction of logical truth from analyticity serves, not only to clarify confusion, but to point to interrelationships (as well as distinctions) between analyticity and particular logical forms. It should be emphasized expressly that the usage of "analytic" as a synonym for "logical truth" does not convey the traditional significance of the term at all.

The concluding Chapter will deal with contemporary efforts to clarify the notion of analyticity for statements of natural language. Though the directions of such efforts are almost as numerous as their individual authors, I have chosen to focus on two of the more general directions in which the discussion has pointed. I do not mean to preclude additional alternatives, for a general characterization of analyticity would have to take cognizance of them too. But the characterization of analytic statements as "definitional rules" has received much attention and involves, I am convinced, serious ambiguities. The understanding of analytic statements of natural language as "empirical generalizations" presupposes very different assumptions about the function of natural languages and, I believe, describes more accurately the traditional notion of analyticity. That our language
does contain evidence for what Gustav Bergmann called a "felt" difference between analytic and synthetic statements has in the last few years regained general acceptance. But whether or not it is possible to abstract from specific linguistic practices a notion of analyticity which can then be universally applied to all of them remains undecided.

CHAPTER I

The assertion that everything is the same as itself has been the starting point of philosophy since Parmenides. Aristotle noted that:

To say of what is that it is not, or of what is not that it is, is false while to say of what is that it is or of what is not that it is not, is true. *Metaphysics* LV. 7 2.

The apparent simplicity of the assertion, however, is deceptive. What is it that the relation of identity relates? Is it a relation between words? -- between objects? -- or between words and objects? Aristotle is claiming here that the relation of identity holds between what we say and what is, that it is on the basis of this relation that we are entitled to apply the terms truth and falsity. But under what conditions are we entitled to claim truth for our words? When we raise such questions the simplicity of the principle of identity gives way to complexities and ambiguities inherent in the philosophical task.

Aristotle applied the principle of identity to his doctrine of essence. He sought to resolve the confusion between statements of identity and statements of predication. Some philosophers who had not understood the distinction that must be made between uses of "is" had argued that no predication is possible other than identical predication. But actually, such assertions as: "The Apostles are twelve" differ in an important way from: "The Apostles are pious."
The grammatical structure of language obscures the distinction implicit in common usage where the verb "to be" does triple duty: it indicates temporal distinctions, it asserts existence, and it connects the subject to the predicate of a sentence. "The Apostles are twelve" asserts the identity of the number of Apostles with the number "twelve;" "The Apostles are pious" attributes to each of the Apostles the predicate "pious." The first statement affirms the existence of the subject of the sentence; the second predicates a property of the subject. Aristotle recognized the distinction and dismissed the notion that the only predication is identical predication as "absurd." (Physics 187a) For Aristotle predicates in affirmative statements were related to the subject in one (or more) of four ways:

1) The predicate of a statement may give the definition of the subject, i.e., signify its essence. A definition consists of the genus and differentia. Arguments about difference concern the sameness or difference of two "things;" thus to demolish a definition it need only be shown that they are not the same.

2) The predicate may give a property of the subject; something that is not the essence of the subject but belongs to it alone and is predicated convertibly of it. Thus it is the property of a man to be capable of learning grammar, and if he is capable of learning grammar, he is a man. (Topics 1 5 (102a 19-22))
Aristotle distinguishes this usage of "property" from other usages where we call predicates "properties," such as "being on the right hand side" which is a temporary property or "being two-footed" which is a relative property (a man is two footed relatively to a dog or a horse). But "being capable of learning grammar" is an absolute property and only they may be convertible predicates. "Being two-footed," unlike "being capable of learning grammar," is not a convertible predicate, because it does not necessarily follow that if something is two footed it is a man.

3) The predicate may be the genus of the subject; it is whatever may be appropriately answered to a generic question, such as "What is the object before you?" In the case of a man: "He is an animal" or "Is one thing in the same genus as another or in a different one?" To give the genus of a term is also to signify its essence; it is to predicate of a subject something that belongs to it essentially, but it is not a convertible predicate. "Being an animal" does not belong to "man" alone as "being a rational animal" does.

4) The predicate may be an accident - "Something which may possibly either belong or not belong to any one and the self-same thing." (102b (6-7)). Whiteness
is an accident, for the same thing may be white at one time and not white at another; comparisons of things are also accidents when they are derived from what happens (accidit) to be true of them. "Is the life of virtue or the life of self-indulgence the pleasanter?" The question then, is to which of the two does the predicate happen to belong more closely? An accident may also be a temporary or a relative property, but it is never a property absolutely.

This theory of predication is based upon two distinctions: the necessary-contingent distinction and the distinction between convertible and non-convertible predication. Where the predicate is convertible (it gives a definition or a property) we may argue from x is y to y is x, i.e., from "Socrates is white" to "There is a white thing which is Socrates." We will consider the second distinction in more detail later; but first let us try to clarify the necessary-contingent distinction. Aristotle's distinction between predicates which define and predicates which give a property is closely interrelated with the necessity-contingency question. A definition and a genus are always essential, but we may call a predicate a "property" if it is either essential or accidental. It may belong to the subject temporarily or absolutely, contingently or necessarily. But they must be distinguished. For the predicate "featherless biped" does not, for Aristotle, belong to "man" in the way "rational animal" does -- it belongs contingently, not
necessarily. But what are Aristotle's criteria for making this distinction? He never explicitly states them, though in the Prior Analytics the context suggests that the necessary may be defined as "the-not-possibly-not," the possible as "the-not-necessarily-not," and then the contingent may be defined in terms of the notions of necessity and possibility as "what is possible but not necessary." But whether Aristotle intended to confine the application of the term "necessary" to propositions whose denial is self-contradictory or to extend it beyond this usage was never fully clarified.

Aristotle then uses his predicate distinctions to clarify the concept of sameness or identity in order to explain what it means to assert that a predicate gives a "definition." Among the many senses of sameness he distinguishes three: numerical (the most literal kind of sameness), specific sameness, and generic sameness by which we can indicate the same object either essentially or accidentally. Later in the Topics after he has further enumerated the meanings ascribed to sameness and the difficulties in attributing sameness to any two things rather than to a single thing, he stipulates that things are not the same unless they are so in all senses including existence; for no two terms can be called the "same" unless there is no discrepancy between what can be predicated of each and unless whatever the one is a predicate of, the other is also.
"Again look and see if, supposing the one to be the same as something, the other also is the same as it; for if they are not both the same thing, clearly neither are they the same as one another. Moreover examine them in the light of their accidents; for any accident belonging to the one must belong also to the other, and if the one belongs to anything as an accident, so must the other also. If in any of these respects there is a discrepancy, clearly they are not the same." vii I (152a 31-38).

Aristotle here makes it quite obvious that he is thinking of the applicability of the term "same" to things not to linguistic entities. But these requirements for sameness having been established, it is difficult to understand how any two distinct things could be regarded as satisfying them. In the following passages Aristotle discusses the considerable difficulty involved in establishing a predicate which gives a definition, and thereby signifies the essence of the subject. After we have considered his reflections on sameness, his predicate classification no longer seems arbitrary for we see the problem he sought to solve in order to explain how definition is possible at all.

In the appendix to the Topics, De Sophisticis Elenchis, Aristotle makes the following remarks in the course of a discussion of arguments that depend on accident:

There is no necessity for the attribute which is true of the thing's accident to be true of the thing as well. For only to things that are indistinguishable and one in essence is it generally agreed that all the same attributes belong. 24 (179a 37-41).

Kneale comments¹ that Aristotle's observations here express

a version of the principle of the transitivity of identity, the principle of the indiscernibility of identicals, and the principle of the identity of indiscernibles, all of which were later reformulated by other philosophers and were to play more predominant roles there than in Aristotle's discussion of identity. Because the development of these principles was so significant for the problem Aristotle posed, it will be useful if we formulate them here:

1) The principle of the **transitivity** of identity:
   
   If $a = b$ & $b = c$ then $a = c$.

2) The principle of the **symmetry** of identity later employed by Frege and sometimes referred to as the indiscernibility of identicals:
   
   If $a = b$ then $b = a$.

3) The principle of the **identity of indiscernibles** generally credited to Leibnitz, who developed it on the basis of Aristotle's monadic logical form:

   If for all $P$, $P(a) \iff P(b)$ then $a = b$.

   That is, if for all properties $P$, the properties of $a$ are equivalent to the properties of $b$, then $a = b$.

Within Aristotle's reflections on the nature of the relation of identity is the germ of much later logical development. Though imbedded in the subject-predicate logical form, its range of applicability extends far beyond the formal restrictions of the Aristotelian logic. And Aristotle's difficulties in applying these insights to his logical form reflect the ambiguities inherent in the form itself. I would like to discuss two problems which, though not explicitly recognized by Aristotle, have become significant problems for his interpreters.
First, we have already suggested that Aristotle's use of language is often equivocal. Was he classifying linguistic expressions or what the expressions signify? He did take great pains to refute Plato's thesis that each term has an appropriate "Form" and thus a single signification or definition. Aristotle showed that there were predicates which could be applied in all categories (not only to substance), predicates such as "one", "being", "same", "other", and "opposite" which have no one single definition. (A point, incidentally, which G.E. Moore has also considered and to which we will refer again later.) Kneale$^1$ concludes that Aristotle's concern was with the classification of things signified by terms. The quotation cited on page 11 tends to confirm this conclusion. Aristotle was interested in showing that a man differs from a time of day or a virtue in another way than he differs from a dog or another man. By this means he sought to differentiate intra-category distinctions from inter-category distinctions. He was trying to distinguish things and the properties which may be applied to things rather than terms and what might be predicated of them.

Second, we have seen that Aristotle expressed his essence-accident distinction in terms of his subject-predicate logic, but in order to recognize certain ambiguities in this formulation we must look more closely at his logical form. He recognized three forms of affirmative statements which

$^1$Ibid., p. 29.
affirm a predicate of a subject:

1) The singular - Socrates is white.
2) The universal - Every man is white.
3) The particular - Some man is white.

Thus general statements and singular statements share the same form, but general statements have a complexity not shared by singular statements, a difference which Aristotle partially acknowledged by distinguishing contraries of general statements from contradictories. For "everyman" and "some man" function differently in a statement than does a singular subject. To substitute "Plato is white" for "Socrates is white" yields a result which differs significantly from the substitution of "everyman" for "Plato". The symbols function differently in the two cases. But to employ the same subject-predicate form for the singular proposition as for the general is to ignore the differences. Two of them are particularly outstanding: One function of singular statements and of particular statements is to affirm the existence of something. To say that Plato or Aristotle is white is to say that there exists someone named Plato or Aristotle; but to say "Everyman is white" is to affirm a property of a class of things. Whether or not there actually exists anything belonging to the class is another question. But Aristotle did not distinguish these two questions and merely assumed that a general statement has existential import. This ambiguity compounds the problem of distinguishing essential from accidental predicates.
Another difficulty encountered in Aristotle's logical form involves his classification of predicates into those which are convertible and those which are not. We noted above that the predicates which give a definition and those which give a property were considered by Aristotle to be "convertible," so that we could argue from "x is y" to "y is x" without loss of sense. It would seem then that here would be a test for partially distinguishing essential from accidental predicates. But the statement: "Every man is white," for instance, cannot be so converted; its subject and predicate terms play different roles. This is also true of terms in particular negative statements. Yet Aristotle failed to take adequate account of this difference. Though he sought to distinguish the senses of "is" that we discussed above, he did not fully understand the distinction that must be made between statements of identity and statements of predication. It was not until a logic of classes was developed that this distinction could be expressed with simplicity. A statement of identity asserts that two classes \( a \) and \( b \) are equal where every member of \( a \) is the same as every member of \( b \); thus \( a = b \Rightarrow b = a \). A statement of predication expresses a different relation between \( a \) and \( b \). Clearly, if \( a \) belongs to \( b \) then it is not the case that \( b \) belongs to \( a \); thus \( a \in b \Rightarrow \sim b \in a \).

This ambiguity in Aristotle's explanation of predication created further problems, too. He did not notice that by maintaining that statements of definition and
property are convertible, he is also assuming that the predicate terms as well as the subject terms must have application. For particular affirmative statements there is no problem. To assert that: "Socrates is white" is equivalent to asserting that there is at least one white thing, that is, Socrates. But "Every man is white" is not equivalent to "Every white thing is a man." Though it is possible to argue from "No man is white" to "No white thing is a man," the assertion that "Some white thing is not a man" does not follow from the first two, though Aristotle had assumed that it did. The last statement assumes not only the existence of at least one white man, but of other white things as well, an assumption which, if true, is true independently.¹

It seems then that what appeared at first blush to be a very clear-cut distinction between two readily distinguishable kinds of predicates is, on inspection, really rather ambiguous. Aristotle offers us no criterion of essentiality at all. In the last analysis it even becomes doubtful whether there are any essential predicates. The sameness of form of singular and general statements and the convertibility property of two classes of predicates blurs the intended distinction between essential and accidental predication. If making a statement purporting to

¹For this observation I am indebted to Kneale's analysis of Aristotle's logic.
predicate something of a subject essentially commits us to the existence of an object satisfying that predicate, then the statement is not in fact asserting something which must essentially be the case at all -- for it is the case only if there actually happens to be such an object accidentally.

The leap from Aristotle to Leibniz may at first glance seem presumptuous, but in significant respects Leibniz' logic bridges the span between Aristotle's and modern logical concepts. Like Aristotle he was interested in the relation between identity and predication. By comparing their formulations we may gain further insight into what it means to say of a proposition that "it is true on logical grounds."

Leibniz had been interested in Locke's vision of a new logic as a third branch of science. It was to be called "semiotics" or the doctrine of signs. It would concern the nature of signs of ideas which the mind uses in the absence of the thing it is considering in order to communicate thoughts to others or retain them for its own use. Thus for Locke, semiotics would - through the consideration of ideas and words - proceed to the discovery of truth. Locke had lacked the mathematical ability to develop his theory. Leibniz, however, established the importance of good symbolism. His discovery of the infinitesimal calculus simplified mathematical notation and his advocacy of a common

\[\text{\textsuperscript{1}}\text{Locke, John: Essay Concerning Human Understanding. xxi. 4.3.}\]
scientific language emphasized the need for formalization in order to attain increased rigor both in the communication of thought and in thinking, itself.

Despite certain logical advances over Aristotle, Leibniz still clung to the subject-predicate form with similarly unfortunate consequences. We have noted that Aristotle did not make a fundamental distinction between primary substance (a particular man or horse) and secondary substance (the species man or animal) and so thought of general propositions in a way appropriate only to singular propositions. Kneale comments¹ that Leibniz, too, sometimes assimilated universal propositions to individual propositions but more commonly committed the opposite error of trying to treat propositions about individuals as though they were like laws expressibly in universal statements. Thus for Leibniz every true affirmative statement ascribes to the thing which its subject term denotes an attribute which really inheres in the thing. He spoke as though each individual has a concept or essence which necessarily involves all the attributes that are predicable of that individual. Kneale claims that it was this assumption that was the origin of the principle of the identity of indiscernibles. But if an individual is defined as an aggregate of attributes, how can it possibly possess an attribute at one time (e.g., being asleep) and lack that attribute at

another time, yet still be the same individual? Not only did Leibniz' notion of individuals preclude a distinction which is necessary in order to maintain a theory of personal identity, but it also hindered the development of his system of symbolism. There was no means of indicating how the same individual might at different times or in different relations possess different properties. An individual for him was more like a general concept defined by a compound monadic predicate than a particular object which could be defined by a name. Had he understood this distinction, he would have been able to clarify many of the ambiguities he had inherited with the subject-predicate logical form. But he took similarities in grammatical structure to indicate similarities among things and what it is possible to say of things. Consequently, he was unable to adequately distinguish an individual from a property of the individual and developed no means of expressing relational properties, e.g., being a father or being warmer.

Much as attributes inhere in a thing for Leibniz, and thereby express what is necessarily true of it, so a "real" definition guarantees the necessary truth of that which it defines. The problem is merely to distinguish "real" definitions from merely conventional and nominal definitions. Leibniz tried to sort out definitions in this way in order to determine the axioms of a deductive system. He regarded the principle of identity as the only undemon-
strable axiom. The proof of a proposition then proceeds by a substitution of equivalents and demonstrates that its predicate concept is contained in its subject concept. The proof, then, merely consists in the analysis of the two concepts. In this way a chain of definitions is produced, and by referring to it we can see that the proposition is, if true, an identity. Leibniz considered that all necessary truths would thereby be guaranteed by the definitions of their terms. Though Leibniz assumed that such a proof could be constructed with reference to no axiom other than the axiom of identity, he actually employed additional axioms which he failed to acknowledge. Frege has pointed out, for instance, that Leibniz' proof that 2 and 2 are 4 contains a gap which is concealed by the omission of brackets. He constructs his proof in the following way:

Definitions: (1) 2 is 1 and 1  
(2) 3 is 2 and 1  
(3) 4 is 3 and 1  

Axiom: If equals be substituted for equals, the equality remains.  
Proof: 2 + 2 = 2 + 1 + 1 (by Def. 1) = 3 + 1 (by Def. 2) = 4 (by Def. 3).  
∴ 2 + 2 = 4 (by the Axiom).

Frege notes that to be strictly accurate we should have to write:

\[ 2 + 2 = 2 + (1 + 1) \]
\[ (2 + 1) + 1 = 3 + 1 = 4. \]

What is missing is the proposition


2*New Essays* iv. 10 (Erdmann edn., p. 363).
\[2 + (1 + 1) = (2 + 1) + 1,\]

which is a special case of
\[a + (b + c) = (a + b) + c.\]

By assuming this law every number can be defined in terms of its predecessor. But because Leibniz failed to observe that he was employing this principle, his characterization of logical truth was oversimplified. Had he distinguished additional logical principles involving binary as well as monadic relations, it would have been possible to characterize logical truths of much greater complexity and diversity.

His account of definition presupposed, too, that in every affirmative truth the predicate does inhere in its subject (an assumption which is in turn based on the suppositions that all complexity arises out of the conjunction of attributes and that every term has just one adequate definition). A "real" definition for Leibniz contains an implicit assertion of the possibility of what is defined, a possibility which is not established merely by the conventions governing the use of words but is, in a sense, pre-established.

But what Leibniz means by "possible" is never fully explained. Since a "necessary" truth for him is one whose negation would be self-contradictory,\(^1\) it might be assumed that possible means something like "free from formal contradiction." But, if nothing is impossible but what is

excluded by the principle of contradiction, then we cannot say for instance that A includes in its nature C-ness which is incompatible with B-ness where neither is the simple negation of the other. It would seem that what is possible for Leibniz is necessarily possible.

And further, since he claims that all true propositions are virtual identities, how can Leibniz account for the distinction he makes between truths of reason and truths of fact? They are, for Leibniz, only in a sense distinguishable. Truths of reason are necessary truths guaranteed by definition to be identities. Truths of fact though are guaranteed too, but by the principle of sufficient reason, which assures us that nothing happens without a ground. Thus they too are identities, but they differ from the truths of reason in that the demonstration of factual truth involves reference to the actual world. This is the world which God chose out of all possible worlds. For us this world is a posteriori, but for God all worlds including the actual world are a priori. In a sense then we might say that out of the truths of reason, which are true for all possible worlds, God selected certain truths to become actual. Thus they are only in a sense contingent in that they depend upon God and hold only for the actual world. Yet they are derivative from the truth of all possible worlds, which is based on the principle of identity. Truth then, for Leibniz, is most basically a property of that which is possible; the impossible is only the self-contradictory.
The distinction here between the two kinds of truths is deliberately minimized. Though the distinction bears certain resemblances to the essence-accident distinction in Aristotle, no attempt is made to develop it as a significant part of the system. The difficulties which we have pointed out - Leibniz' preoccupation with the conjunction of attributes and his failure to give a satisfactory account of existential significance - hindered both the development of his logic and the establishment of a meaningful distinction between truths based on reason and truths based on fact. The situation is in some respects analogous to Aristotle's. The logic of both was limited by the subject-predicate form and by the confusion between singular and general propositions. Though Aristotle took the distinction more seriously, neither was able to provide a criterion by which propositions could actually be so distinguished. However, Leibniz' formulation of the principle of the identity of indiscernibles and his attempt to characterize analyticity as "truth in all possible worlds" suggested rich possibilities for the development of a logic which Leibniz had only anticipated. We shall return later in our discussion of the work of Tarski and Carnap to the role of these significant insights in later formulations of logical truth.

To Locke's disparagement of all truths of reason as "trifling propositions" Leibniz had replied with a defense and concluded:
This leads us finally to the ultimate ground of truths, viz: to the Supreme and Universal Mind, which cannot fail to exist, whose understanding, to speak truly is the region of eternal truths...And in order not to think that it is unnecessary to recur to this, we must consider that those necessary truths contain the determining reason and the regulating principle of existences themselves, and, in a word, the laws of the universe. Thus necessary truths being anterior to the existence of contingent beings, must be grounded in the existence of a necessary substance. Here it is that I find the original of the ideas and truths which are graven in our souls, not in the form of propositions but as the sources out of which application and occasion will cause actual judgements to arise.1

Hume's rejoinder stands in stark contrast to Leibniz' apologetic:

The ideas of internal sentiment, added to those of the external senses, compose the whole furniture of the human understanding, we may conclude, that none of the materials of thought are in any respect similar in the human and in the divine intelligence.2

How was Hume able to distinguish the ideas of internal sentiment from those of the external senses? For Hume

...nothing is ever really present with the mind but its perceptions or impressions and ideas, and... external objects become known to us only by those perceptions they occasion. To hate, to love, to think, to feel, to see; all this is nothing but to perceive.3

1Ibid., pp. 516-517.


For the interpretation of Hume in this section I am much indebted to Professor Chappell's illuminating introduction and to Professor John Alan Robinson's Ph.D. dissertation: Causation, Probability and Testimony, (unpub.) and his article "Hume's Two Definitions of "Cause,"" Philosophical Quarterly, 12 (1962), 162-171.

The perceptions are the constituents of our experience, the objects of thought and sense, and the basis of all knowledge. From this starting point Hume seeks to discover what is true about perceptions and claims that he has discovered empirically certain distinctions. He concludes that simple ideas are derived from impressions to which they correspond and which they represent. Then he examines the relation between ideas and claims to have found certain groups of associated ideas. He distinguishes three different relations which hold between them: Resemblance, contiguity and cause and effect. These he calls the natural relations because the association of ideas is derived from a relation between things or events. Only after he has clarified the nature of ideas is he ready to consider knowledge. To know means to be aware of a relation holding between ideas. The object of knowledge, or - for Hume - of belief, too, is not the ideas but the relation which holds between the ideas. The mere having of an idea is not knowing or believing. Hume here implies a distinction between having an idea and assenting to an idea. (We might note that there need be no proposition as the object of the belief or knowledge though later in the Inquiry Hume does discuss propositions which assert relations among ideas.) The having of an idea does not involve a relation necessarily, but assenting to an idea does. Though the having of an idea is a consequence

1For a more complete discussion of Hume's theory of natural relations see Robinson paper cited above.
only of its "vivacity" and does not involve belief in the strong sense we indicated above, Hume does speak of "belief" in another sense also. The mere "entertaining" of a simple idea, that is, an idea which involves no relation, is in a weak sense a form of belief. But in the strong sense in which we used the term above, "belief" involves a complex idea and a relation and is distinguishable from "knowing" only in that it is less certain. In this sense belief involves a proposition as well as a noun. That it is less certain is in part determined by the class of relation involved. In order then, to clarify his concept of knowledge, Hume attempts to classify kinds of relations.

He distinguishes seven different kinds of relations and to set his use of the term "relation" apart from the casual use, he calls them all "philosophical relations." Then the seven kinds of relations are divided into two classes. We mentioned above the three principles of association; these he calls the natural relations because the establishment of the relation depends on the natural association of the ideas. The relation can never be determined by reasoning or reflection alone. But in addition to these three relations, Hume claims to have discovered four other relations, and it is these relations alone which he maintains can be objects of certain knowledge. Though the natural relations necessarily involve information from experience, the establishment of the remaining four depends solely on the ideas we compare together. Unlike the natural
relations which may change without any change in the ideas, the other four relations depend only upon the ideas we are comparing together. Let us for convenience call them the "internal" relations. We will list them together below followed by the three natural relations:

1) **Resemblance.** Hume remarks that it is essential to all relation, but it is not of itself sufficient to produce an association of ideas.

2) **Quantity** and **number.**

3) Degrees of quality which relate objects, e.g., heavier, warmer, redder.

4) **Contrariety.**

5) **Identity.** He specifies that this relation applies to objects which are constant and unchanging. It is the most universal of relations, for it is common to everything that exists through time.

6) **Space** and **time.** They are the ideas of the manner and order in which objects exist, or more accurately impressions occur.

7) **Cause** and **effect.**

His arguments for this classification are not altogether convincing. Though his classification of the causal relation among the natural relations is probably the most philosophically controversial (and generally misunderstood) and occupies a principle position in Hume's philosophy, other items in the classification are also in need of further scrutiny. To call space and time an observable relation,
as he does and then to describe the relation in terms of the notion of manner or order, which itself is neither an object nor an impression but yet is a source of ideas, involves Hume in an inconsistency he is not able to resolve.

In other respects as well, his distinction between the classes of relations is not clear-cut. Hume wants to argue that the "internal relations" are logical in that they hold between ideas independent of matters of fact or observation. The relation is internal to the terms and cannot change unless the ideas themselves change. But in a natural relation the relation and the ideas may change independently of one another. In the first case we have knowledge *a priori*; when we know what the ideas are, then we either know automatically that the relation holds between them or we apply reasoning and are able to *deduce* the relation logically. But where the relation is natural we know nothing on a purely *a priori* basis; the mere *belief* in the ideas or *opinion* about them does not suffice to establish a relation between them. We must also consult experience, and there, Hume claims, we can only know that the relation holds with varying degrees of probability. Reasoning here is only *inductively* related to its premises and the conclusions of an inductive inference are dependent on experience for their truth.

When Hume tries to apply the theoretical distinction to the three natural relations and the four logical relations, it becomes evident that the relations overlap and
the same relation may be either internal or natural, though the criterion by which this is determined is never made fully explicit. The need for further qualification is obvious. Though Hume seems to regard mathematical reasoning as the model of the internal relation, resemblance, identity, qualitative comparisons, and contrariety may all be natural as well. That red azaleas resemble pink azaleas more than they do white azaleas is true merely by virtue of the way we describe color properties, but that the azaleas in my garden resemble the azaleas in your garden more than they do those in our neighbor's is true only factually.

Similarly the natural relation of identity is in need of further distinction. For a waiter may set a table with 6 knives to the right of 6 plates and though the number of plates and knives are identical the objects are not. Hume does not explain fully what it means to claim that objects are identical. He suggests, where he remarks that identity is common to every being whose existence has duration, a concept of identity similar to Leibniz' identity of indiscernibles, though it is not elaborated in Hume.

The relation which Hume calls "proportions of quantity and number" is similarly an internal relation only when the objects of the ideas are themselves numbers. The relation "greater than" is internal where it is asserted that 2 is greater than 1 but it is external where it is claimed that the population of St. Louis is greater than the population of Lyon. (We shall return to this distinction
later in connection with Frege's logic.)

Hume's classification of relations does not, then, group them homogenously at all. Among the internal relations are some which are necessary (in an a priori sense) and others which are clearly contingent. But the internal relations do differ from the natural relations in that the latter group are never knowable a priori; experience is a necessary condition for the establishment of the relation. But as we have seen, it may not be sufficient. There is a confusion in Hume's account of relation that calls for further distinction. It resembles, in some respects, Aristotle's confusion in distinguishing essential from accidental predication. Essentiality for Aristotle was present in all categories, and though he claimed that it was separated from the accidental, he presented no criterion whereby they actually could be so distinguished. His confusion of singular with general terms only compounded the problem. Hume also, fails to distinguish adequately between singular terms which have existential import and possess properties only accidentally and general terms which do not name any existing thing but may be true of a class of things that embodies its properties logically. It is logically true, for instance, that all bachelors are unmarried males because the class of bachelors is the same as the class of unmarried males. Whether or not there may actually exist such a creature is accidental and irrelevant to the truth of the assertion. However, Hume, like Aristotle, failed to perceive this distinction.
And, like Aristotle, he had difficulty accounting for the relation of identity. Since, for Hume all ideas must have their origin in experience, it is difficult to understand what is claimed to be identical with what. We have pointed out that he fails to distinguish between the identity of numbers as objects and the identity of other kinds of objects and that identity can only be understood as an internal relation where the objects are numbers. Hume had a difficult time here, for he observed rightly that an object conveys the idea of unity, not of identity. He concluded that identity was then only invariableness and uninterruptedness of an object through a supposed variation in time. But he was in fact discussing not one but two different concepts, and his inability to clarify the concept of identity was related to his inability to adequately distinguish logical and empirical relations.

However, Hume's attempt to distinguish two kinds of relations involves another distinction and in clarifying the second distinction he made it possible to better understand the first. He is distinguishing both between two kinds of relational ideas and between two kinds of knowledge. Though both kinds of knowledge involve a complex idea with a relational component, the one is certain, the other only probable, the one is a priori, the other a posteriori. The distinctions are never made in such a way that the four components are made explicit. If we apply Kant's terminology to Hume's analysis, it would appear that all a priori
knowledge is analytic and all a posteriori knowledge is synthetic. The possibility of an analytic a posteriori, deductive reasoning with empirical conclusions would not have occurred to Hume. Also since the distinction between the two kinds of relations is originally expressed as a relation between ideas (rather than between linguistic entities as is more commonly done today), the four-fold aspect of the distinction is never developed.

Given Hume's account of the empirical origin of all ideas, and his location of the two kinds of truths in the relation between ideas, it is an interesting question whether he could have distinguished a priori and a posteriori classes of knowledge from his classification of relations. For the epistemological distinction is really dependent upon the distinction between relations. There is really only one source of knowledge, experience, and it is the relations that "sort out" the ideas derived from experience into "certain knowledge" where the relation depends solely on ideas and "probable knowledge or belief" where an external relation is involved. Kant, however, beginning from a different supposition about the origin of knowledge made the four-fold distinction explicitly. The a priori for him was synonymous with the non-empirical and a posteriori knowledge only was derived from experience. We shall see in the next section how the epistemological and the logical distinctions were interrelated in Kant's philosophy.
Kant, dissatisfied with Hume's account of causation, reformulated the distinctions Hume had made between kinds of knowledge and kinds of relations. Hume had claimed to find the origin of the relation of causation in a natural relation between impressions of two objects (or events) where one object (or event) occurs next to the other in space and time. This accounts for our impression of spatial contiguity and temporal succession. But this explanation accounts only for particular objects or events, not for an entire class or collection of objects. Philosophers had traditionally argued that to account for the causal relation generally we must assume that there is a necessary connection between events which is a priori and analytic. Hume argued that if this were so then the denial of a proposition of the form "All events have causes" would be self-contradictory. But actually the ideas of objects or events may remain the same while the relation between them varies, which, for Hume, was a sure sign of an external relation. The supposed "necessary connection" Hume explained as merely an empirical inference which we make when we observe that constant conjunction and invariable succession hold between types or classes of events.

The distinction which Hume made between internal and external relations (or later between propositions which assert relations between ideas and those which assert re-

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1Treatise, Bk. I, Pt. III, Sect. III.
lations between matters of fact) was retained by Kant, who coined the terms "analytic" and "synthetic" to characterize the distinction and applied the terms to judgements. He thereby removed a certain vagueness attached to Hume's characterization, which referred to ideas. The adoption of the judgement as the locus of meaning rather than the idea made it possible to formulate the problem more explicitly than had been the case for Hume's imagist theory of meaning.

For Kant as for Hume, too, the analytic judgement is one whose denial is a contradiction, or which is "logically necessary," or as Kant also sometimes expressed it, "its negation is logically impossible." All these formulations were essentially equivalent for Kant. Any judgement which does not meet this criterion is synthetic. However, for both Hume and Kant this was merely a traditional means of formulating the distinction and played only a subordinate role in their characterizations of analyticity. Neither developed this criterion in conjunction with his other means of specifying the distinction; for both it was principally a secondary consideration. (Hume's account of necessity though, unlike Kant's, was based on "imaginability.") We shall return to Kant's discussion of contradiction later in

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1Kant seems to have used the term "judgement" interchangeably with "proposition" and from his actual manner of applying the terms we can conclude that judgements were not solely psychological for him but both psychological and linguistic.
another context but first we must note the distinction for which Kant is most renowned.

He regarded Hume's account of causation as inadequate, yet he was forced to accept Hume's argument that it was not logically contradictory to deny that "all events have a cause." Thus, he concluded, there must be another sense of "necessary" which is what we mean when we call the causal relation "necessary." This "necessity" Kant considered synonymous with "universality," i.e., "It does not admit of the possibility of an exception."¹ It is necessary not for all possible worlds in the Leibnizian sense, but it is true only for this world and in this way it could be regarded as a "contingent necessity" for its range of applicability is limited to possible experience. However, in another sense it is not contingent because it does not derive from experience - thus it is a priori. In this way Kant was able to attribute to empirical knowledge a certainty which had not been possible for Hume, and the Herculean effort that was the Critique of Pure Reason was massed in defense of the proposition that synthetic a priori knowledge was in fact possible.

Now let us examine Kant's characterization of analytic judgements more closely.

In all judgements in which the relation of a subject to a predicate is thought (I take into consideration affirmative judgements only, the subsequent application to negative judgements being easily made),

this relation is possible in two different ways. Either the predicate B belongs to the subject A, as something which is (covertly) contained in this concept A; or B lies outside the concept A, although A does indeed stand in connection with it. In the one case I title the judgement analytic, in the other synthetic. Analytic judgements (affirmative) are therefore those in which the connection of the predicate with the subject is thought through identity; those in which this connection is thought without identity should be entitled synthetic. The former, as adding nothing through the predicate to the concept of the subject but merely breaking it up into those constituent concepts that have all along been thought in it, although confusedly, can also be entitled explicative. The latter, on the other hand, add to the concept of the subject, a predicate which has not been in any wise thought in it, and which no analysis could possibly extract from it; and they may therefore be entitled ampliative. If I say, for instance, "All bodies are extended," this is an analytic judgement. For I do not require to go beyond the concept which I connect with "body" in order to find extension as bound up with it. To meet with this predicate, I have merely to analyze the concept, that is, to become conscious to myself of the manifold which I always think in that concept. The judgement is therefore analytic. But when I say, "All bodies are heavy," the predicate is something quite different from anything that I think in the mere concept of body in general; and the addition of such a predicate therefore yields a synthetic judgement.1

In the 2nd edition Kant then adds that judgements of experience are all synthetic and contrasts these judgements with the proposition that all bodies are extended, where the predicate is extracted from the subject in accordance with the principle of contradiction. In the 1st edition Kant specifies his remarks somewhat differently and does not refer to the principle of contradiction. Here he explains the ampliative character of the synthetic judgement by way

1Ibid., B 11.
of a "something else," an X, which is present in addition to the concept of the subject which is fundamental to understanding that the predicate does belong to the subject. In empirical judgements the X is the complete experience of the object which is thought through the concept A. The X, however, lies outside the concept and only through it is the synthesis of the concept with the predicate possible - and through such synthesis knowledge is extended. But in analytic judgements there is no X which connects the concept with the predicate, no unknown gives support to the understanding in order to discover the relation between the concept and the predicate for the connection is already present in the concept.

Subsequent studies of Kant's distinction have invariably made note of two obvious limitations:

1) The explanation refers only to judgements of the subject-predicate form.

2) It makes use of a metaphorical notion, "containment" which is more suitable to discussion of three dimensional physical objects than to a discussion of judgements. (The metaphor has, however, been put to good use, sometimes by its harshest critics who have proceeded from a criticism of this use of language to a discussion of how we might "unpack" the subject concept.)

Let us consider the second objection first. The metaphorical use of language here is really quite useful for it points to an inadequacy that has been common to many characterizations of analyticity, that is, the primacy in them of the subjective psychological character of meaning. The most commonly used word in the quotation we have just cited is "thought."
But if I can only know whether a judgement is analytic through what I happen to think in it, would not then the same judgement possibly be analytic for some, synthetic for others, and perhaps not constitute a judgement at all for those who use language in an uncommon way? Clearly everyone does not identify the same predicates with the same concept as everyone else. Even in the case of such a common concept as "body," unless we made explicit the predicates which we actually do think through identity, there would be no means of knowing whether our psychological meanings coincided. As long as the criterion remains an implicit one, as long as it is characterized by meanings "in mind" the ambiguity is irremovable. Kant, however, appears to have assumed naively that sense meaning was universally the same and provided a sort of "guarantee" that would enable us to distinguish analytic statements of subject-predicate form absolutely.

The first limitation of Kant's explanation, the confinement of the notion of analyticity to judgements of the subject-predicate form, is susceptible to many of the same criticisms as Aristotle's or Leibniz' formulation, and in some aspects, Hume's as well. The distinction, in this form, is applicable only to universal affirmative judgements for in singular existential judgements there is no subject concept. However, in his discussion of the X, the unknown that synthesizes the predicate with the subject, there is a suggestion that the distinction is applicable to the whole field of possible knowledge. But if he had extended the
definition to hypothetical judgements (e.g., If the moon is a body, then it is extended), or to disjunctive judgements (e.g., This body is heavy or this body is not heavy) he would most likely have regarded such forms as analytic as well. But if he had so extended the definition he would have had to look beyond the containment theory as well. Had he done so, it is unlikely that he would have characterized arithmetic judgements (e.g., \(7 \times 5 = 12\)) as synthetic. (We shall return to this point in the discussion of Frege's formulation of analyticity.)

Kant suggests once in the passage we quoted that in analytic judgements such as "All bodies are extended" the predicate is extracted from the subject by the principle of contradiction. This is merely suggested here by way of illustration but later (though only in one place) he develops this suggestion:

The proposition that no predicate contradictory of a thing can belong to it, is entitled the principle of contradiction, and is a universal, though merely negative, criterion of truth. For this reason it belongs only to logic. It holds of knowledge, merely as knowledge in general, irrespective of content; and asserts that the contradiction completely cancels and invalidates it.

But it also allows of positive employment, not merely, that is, to dispel falsehood and error (so far as they rest on contradiction) but also for the knowing of truth. For, if the judgement is analytic, whether negative or affirmative, its truth can always be adequately known in accordance with the principle of contradiction. ...The principle of contradiction must therefore be recognized as being the universal and completely sufficient principle of all analytic knowledge; but beyond the sphere of analytic knowledge it has
as a sufficient criterion of truth, no authority and no field of application.\textsuperscript{1}

This criterion of analyticity seems more suitable to Kant's original purposes for it would at least provide a definition by which all true judgements could be classified as "analytic" or "synthetic" whether or not they happened to be universal. But then there would be classed among the synthetic judgements some for which the determination of truth did not require appeal to experience. Kant overlooked the fact that there are judgements true on logical grounds which are not derived solely from the principle of contradiction. Kneale points out\textsuperscript{2} that the principle of the Aristotelian syllogism of the 1st mood of the 1st figure:

\begin{align*}
\text{If every M is L and every S is M then every S is L} \\
\text{(where M is the middle term, S is the minor, and L is the major term and the major premise is universal and the minor affirmative)}
\end{align*}

is certainly a true principle on logical grounds alone. Though we can, by negating a statement of this form obtain mutually incompatible consequences, we must first assume the principle of the syllogism. Had Kant taken such judgements into consideration it is difficult to see how he could possibly have classed them as synthetic yet they are not derivable from the principle of contradiction. If he had nonetheless held to his contradiction criterion and consequently classed such judgements as synthetic, what sense

\textsuperscript{1}Ibid., B 190-191.

\textsuperscript{2}Op. cit., p. 357.
would it have made to ask his famous question: "How are synthetic a priori judgements possible?" For in the sense of a priori (as a synonym for non-empirical) which Kant popularized, the question would have been a trivial one.
Chapter II

Gottlob Frege begins his essay "Über Sinn und Bedeutung" with the observation:

The idea of Sameness challenges reflection. It raises questions which are not quite easily answered. Is sameness a relation? A relation between objects? Or between names or signs of objects?¹

Frege's efforts to answer such questions profoundly affected the development of modern logical theory. His insight into the problem of identity and his reformulation of the notion of analyticity are bound up with his theory of general logic, commonly acknowledged to be the first comprehensive logical theory. His elaboration of a deductive system developed out of his reflections on the concept of number. In his earlier work Die Grundlagen der Arithmetik he had asked:

...(the letter) "a" does not mean some one definite number which can be specified, but serves to express the generality of general propositions. If, in "a+a-a = a," we put for "a" some number, any we please but the same throughout, we get always a true identity. This is the sense in which the letter "a" is used. With one, however, the position is essentially different. Can we, in the identity 1 + 1 = 2 put for 1 in both places some one and the same object, say the Moon? On the contrary, it looks as though, whatever we put for the first 1, we must put something different for the second. Why is it that we have to do here precisely what would have been wrong in the other case? Again, arithmetic cannot get along with "a" alone but has to use further letters besides (b, c and so on), in order to express in general

form relations between different numbers. It would therefore be natural to suppose that the symbol "1" too, if it served in some similar way to confer generality on propositions, could not be enough by itself. Yet surely the number one looks like a definite particular object, with properties that can be specified, for example that of remaining unchanged when multiplied by itself? In this sense "a" has no properties that can be specified, since whatever can be asserted of "a" is a common property of all numbers, whereas $1^1 = 1$ asserts nothing of the Moon, nothing of the Sun, nothing of the Sahara, nothing of the peak of Teneriffe; for what could be the sense of any such assertion?¹

Frege's was not merely the dilemma of the mathematician but he shared a dilemma common to all efforts to reason abstractly. He was motivated by the same fundamental problem that had motivated the enquiries of Parmenides, of Aristotle, and of Leibniz. He was enquiring into the basic nature of the thinkable and the assertable. But the equipment he was able to bring to bear on his problem differed appreciably from that of his predecessors. His solution involved the definition of numbers without reference to any notions other than those involved in an interpretation of a logical calculus. It was not a new proposal. We have already seen that Locke and Leibniz had suggested it too; but Locke had lacked even the mathematical ability to develop it, and Leibniz had been limited by his subject-predicate logic and his confusion of singular and general terms. Following Leibniz, for much of the 17th and 18th centuries,

logic was dominated by epistemology. Then in the 19th century the work of men such as Augustus De Morgan, George Boole and, to a lesser extent, C.S. Peirce freed logic from epistemology and brought about the organization and formulation of some of Leibniz' logical insights, particularly the resemblances he had noted between mathematical and non-mathematical languages (e.g., disjunction and conjunction of concepts and addition and multiplication of numbers). Boole used this formulation as the basis of an abstract calculus of logic capable of various interpretations. He had collected from previous work two significant discoveries:

1) that there could be an algebra of entities which were not in the ordinary sense numbers.

2) that laws which hold for types of numbers need not all be retained together in a system which is only applicable to some of them.

And he derived a view of logic which he expressed in this way in his *Mathematical Analysis of Logic*.

They who are acquainted with the present state of the theory of Symbolic Algebra, are aware that the validity of the processes of analysis does not depend upon the interpretation of the symbols which are employed, but solely upon the laws of their combination. Every system which does not affect the truth of the relations supposed, is equally admissible, and it is thus that the same processes may, under one scheme of interpretation represent the solution of a question on the properties of numbers, under another, that of a geometric problem, and under a third, that of a problem of dynamics or optics...We might justly assign it as the definitive character of a true Calculus, that it is a method resting upon the employment of Symbols, whose laws of combination are known and general, and whose results admit of a consistent interpretation. That
to the existing forms of Analysis a quantitative interpretation is assigned, is the result of circumstances by which these forms were determined, and it is not to be construed into a universal condition of Analysis. It is upon the foundation of this general principle, that I purpose to establish the Calculus of Logic, and that I claim for it a place among the acknowledged forms of Mathematical Analysis, regardless that in its objects and in its instruments it must at present stand alone. ¹

Boole saw language, not merely as a collection of signs but as a system of "expressions" whose elements represent laws of thought. Though he did not complete the separation of logic from epistemology, his insight into the structure of mathematical language was basic to the later development of a semantic theory of logical truth for an uninterpreted formal language. The methods which Boole introduced into logic - his theory of what became known in Frege's system as "truth-functions" and their expression in disjunctive normal form - provided a mechanical means of expressing by a process of algebraic calculation every kind of reasoning recognized in traditional logic. In his logic of classes it was possible to express Aristotle's syllogistic logic by the reduction of two class equations to one, then eliminate the middle term and proceed to the solution for the subject term of the conclusion.

But Boole's algebra of classes was limited to the assertion of principles which were all equations. He lacked a basic theory of how propositions might be derived from

¹Quoted by Kneale, op cit., p. 405. Underlined terms not underlined in original.
other propositions. However, the success of his efforts to construct an algebra which included traditional logic motivated others to develop a logic of relations analogous to Boole's logic of classes. C.S. Peirce, especially, tried to work out arguments which depended on relational properties. Earlier Augustus De Morgan had noted that syllogistic reasoning is only a special case of the theory of the composition of relations which states the transitive and convertible character of the relation of identity (i.e., if \( a = b \) and \( b = c \) then \( a = c \) and if \( a = b \) then \( b = a \)). We noted Aristotle's effort to work out the relation of convertibility within his subject-predicate logic and its relevance for his essence-accident distinction. But his failure to adequately distinguish singular from general terms inhibited the development of a general theory of relations. Peirce, aided by De Morgan's insight, sought to extend Boole's logic of absolute terms to the logic of relations. What Peirce developed was a logical symbolism adequate for the whole of logic. It was not an algebra in the Boolean sense, for it also included notation for quantifiers, which Peirce realized were essential for the general theory of relations. Aristotle's logic had lacked an unambiguous device for designating the applicability of terms; therefore, his formulation of the convertibility relation had confusing consequences. Peirce's very simple notation for the operators with the meanings of "some" and "every" removed this ambiguity and provided a ready device for the
distinction of existential and universal quantification of terms. The use of quantifiers to bind variables reduced to a very simple notation a concept which had previously been expressed by unduly cumbersome and confusing devices.

Peirce's logical notation was adequate for the expression of all logical principles which Frege elaborated; though, unlike Frege, Peirce never organized his work into a comprehensive system. His theory of relations though made it possible to understand that the assertion "Every man is mortal" entails the universal closure of the functional expression "If x is a man, then x is mortal." The understanding of this entailment makes explicit the logical properties of language that Aristotle had tried to generalize in his essence-accident distinction and also distinguishes the properties of language from the properties of things.

Frege's systematization of logic frees logic from its dominion by the grammar of ordinary language and lays bare the relational properties within the language. Logic for Frege is connected with grammar only insofar as it deals with the basic patterns every language must be able to express, patterns which had been obscured in the traditional formulation of logic.

His rigorous and systematic exploration of the logical basis of mathematics led him to a reformulation of the distinction between analytic and synthetic propositions and the distinction between a priori and a posteriori knowledge. Until the nineteenth century mathematics had generally been
regarded as an a priori body of knowledge which could be established as true without appeal to experience for information about particular objects. Hume's empiricism of meaning had not affected the traditional characterization of mathematical knowledge, but J.S. Mill applied the tenets of empiricism to all knowledge and maintained that arithmetic, too, rests on inductive generalization from facts about particular groups of things. Frege's queries about the nature of the identity relation, which we quoted at the beginning of this Chapter, developed out of his reaction to Mill's inductive explanation of arithmetic. Frege argued that the procedure of induction itself cannot be regarded as primitive. It is not a mere process of habitation but seems to require some general arithmetical propositions without which it has no power of leading to the discovery of truth, but is merely of subjective interest. Induction needs to be based on a theory of probability which presupposes arithmetic laws. He contrasted Mill's view with Leibniz' concept of arithmetic truth according to which necessary truths (such as those of arithmetic) must be based on principles whose truth does not depend on the evidence of the senses. Mill held that the identity \(1 = 1\) could be false on the basis of the observation that one pound weight does not always weigh precisely the same as another. He also maintained that the symbol of addition

\[\text{Op. cit. } \S 9, 10.\]
expresses the relation between parts of a physical body. Frege argued that the meaning of the proposition $5 + 2 = 7$ is not expressed in the operation of pouring 2 unit volumes of liquid into 5 unit volumes of liquid. This is only a particular application of the arithmetic proposition which, incidentally, only yields 7 unit volumes of liquid under certain chemical conditions. (For instance, if 5 volumes of alcohol are added to 2 volumes of water, because the densities are not simply additive, the result of the addition is less than 7 unit volumes of alcohol and water.) He also noted that "plus" can be used to designate other relations as well as relations between parts of physical bodies. We use it in a logical sense, for instance, when we assert that tyrannicides are a part of murder as a whole. Thus Frege concludes, the laws of addition cannot be part of the laws of nature.

The laws of arithmetic then are not empirical but a priori. The fact that we might make observations in order to be conscious of the content of a proposition does not justify the conclusion that the truth of the proposition depends on experience. By a priori Frege means that the proof of the proposition is derived from general laws whose necessity does not depend on appeal to facts. The a priori-a posteriori distinction for Frege refers to the justification for making the judgement. Where there is no justification the distinction cannot be made. He comments that an
a priori error is as utter nonsense as, say, a blue concept.\footnote{Ibid., \S 3.}
It makes sense to say that we might be mistaken about the properties of things; but number, unlike color, is not an empirical property, and so it makes no sense at all to talk about an a priori falsehood.

The analytic-synthetic distinction, too, refers for Frege to the justification for making the judgement.

When a proposition is called a posteriori or analytic in my sense, this is not a judgement about the conditions, psychological, physiological and physical which have made it possible to form the content of the proposition in our consciousness; nor is it a judgement about the way in which some other man has come, perhaps erroneously, to believe it true; rather, it is a judgement about the ultimate ground upon which rests the justification for holding it to be true.\footnote{Ibid., \S 3.}

The question then is removed from the context of psychology and framed in an objective context, which, if the question is mathematical, may, for Frege, be either analytic or synthetic. If, when we find the proof of a proposition and follow it back to the primitive truths which imply it, we find as primitive truths only general logical laws and definitions then the truth is analytic. But if the proof rests on truths that are not of a general logical nature, then it is synthetic. A proposition would apparently be a synthetic truth for Frege if the premises on which it depended contained empirical generalizations. But it would also be a priori in that its
proof is general; i.e., it contains no assertions about particular objects and thus is constructed without appeal to facts. The distinctions then, for Frege, look something like this:

- **a priori**
  - deductive proof
  - rules of inference derived from general laws which are necessary i.e., do not need nor admit of proof.

- **a posteriori**
  - inductive proof
  - involves appeal to facts to construct proof

- **analytic**
  - premises non-empirical
  - primitive truths are all general logical laws and non-empirical definitions

- **synthetic**
  - empirical premises

The analytic-synthetic distinction then, answers the question of the premises upon which a proposition is asserted to be true: are they logical or empirical? The a priori-a posteriori distinction concerns the procedure whereby the truth of a proposition is proved; does it involve appeal to facts? Both distinctions then concern the justification, not the content of the proposition.

Frege applies his characterization of the distinction to arithmetic and argues that in terms of his definition of analytic truth, arithmetic propositions are analytic and not synthetic, as Kant had claimed. He is careful to distinguish the grounds on which he bases the distinction from Kant's. Kant's conception of logic was based primarily on the logic of Aristotle. Though he had tried to connect analytic truth with the principle of contradiction he had
never developed this relation, but his containment characterization of analyticity had remained primary. The Aristotelian subject-predicate logic, on which the containment concept is based, is applicable only to universal affirmative judgements and consequently Kant's containment concept too, is not applicable to singular and existential judgements. Kneale remarks that Kant seems to have thought of the definition of a complex concept as a mere list of characters.¹ Though Leibniz' concept of analyticity more closely resembled Frege's than did Kant's, Leibniz, too, thought of concepts and propositions primarily as aggregates. As we have seen, he confused singular with general propositions and had no conception of relational properties at all. Though Boole's logic of classes marked a considerable advance over Leibniz' logical developments, Boole had only studied very simple structural patterns. Frege, independently of Peirce and more significantly, was able to specify the distinctions which had eluded his predecessors. His conception of analyticity was based on the logical form which he was able to develop. It possessed a complexity inconceivable to any of his predecessors. Consequently his classification of arithmetic must be distinguished from Kant's classification on two counts. 1) Arithmetic for Frege was not synthetic but analytic. 2) The analytic-synthetic distinction was based on a very different conception of what

it means to say of a proposition that it is "true on logical grounds." In the Conclusion of the Grundlagen Frege sums up his differences with Kant and reaffirms his conclusion that the laws of arithmetic are analytic judgements and are therefore a priori.\(^1\) Arithmetic is simply a derivative development of logic. The application of arithmetic to the physical sciences is merely the application of logic to observation, though observation, Frege adds by way of a footnote, "already includes within it a logical activity."\(^2\) Calculation then becomes deduction. But the laws of number, Frege reiterates, are not applicable to the external world; they are not laws of nature, but laws of the laws of nature.

In the whole of space and all that therein is, there are no concepts, no properties of concepts, no numbers. The laws of number, therefore, are not really applicable to external things; they are not laws of nature...They assert not connections between phenomena but connections between judgements; and among judgements are included the laws of nature.\(^3\)

Thus Frege claims Kant defined analytic judgements too narrowly, i.e., as universal affirmative judgements, and thereby underestimated them. That is why his division of judgements into analytic and synthetic is not exhaustive. His sense of defining a concept by an unordered list of characteristics is, for Frege, one of the least fruitful ways of forming concepts. Frege's logical definitions and the really fruit-

\(^1\)We will discuss later the necessary connection which Frege maintains holds between analyticity and the a priori.

\(^2\)Ibid., \(\&\) 87.

\(^3\)Ibid.
ful mathematical definitions (such as that of the continuity of a function) conform to a very different pattern, in which every element in the definition is intimately connected with the others. A fruitful definition draws boundary lines which were never previously given at all. It is by no means merely taking out of a box what was just put into it. The conclusions drawn, however, do extend our knowledge and on Kant's view are thereby synthetic, but Frege has shown that they can be proved by purely logical means and are as such analytic. They are, in a sense, contained in their definitions, but not as beams are contained in a house; rather in the way plants are contained in their seeds. We often require several definitions to prove a proposition and the proposition is not contained in any one of them but follows logically from all of them together.

Frege concludes his criticism of Kant's reflections on arithmetic with a statement of the extent of his agreement with him:

I consider Kant did great service in drawing the distinction between synthetic and analytic judgments. In calling the truths of geometry synthetic and a priori, he revealed their true nature... If Kant was wrong about arithmetic, that does not seriously detract, in my opinion, from the value of his work. His point was, that there are such things as synthetic judgements a priori; whether they are to be found in geometry only, or in arithmetic as well, is of less importance.¹

¹Ibid., 89.
Frege's redefinition of the analytic-synthetic and a priori-a posteriori distinctions was developed for only three of the four possible ways of characterizing propositions:

a priori analytic
a priori synthetic
a posteriori synthetic

He did not provide for the possibility of an a posteriori analytic proposition. However, considering the particular interpretation he gave to these terms, such a characterization would not, in any event, conform to the more common contemporary usage of a posteriori analytic. He was, of course, primarily concerned with the application of the distinctions to mathematics, and the attention which he devoted to this philosophical problem was justified for him by his success in thereby divorcing mathematics from any taint of psychologism. In the Introduction to the Grundlagen he had enunciated three fundamental principles for his enquiry, the first of which stated his intent to always sharply separate the psychological from the logical, the subjective from the objective.¹

...All these phases of consciousness (sensations, mental pictures, etc.) are characteristically fluctuating and indefinite, in strong contrast to the definiteness and fixity of the concepts and objects of mathematics. It may, of course, serve some purpose to investigate the ideas and changes of ideas which occur during the course of mathematical

¹Ibid., X.
thinking; but psychology should not imagine that it can contribute anything whatever to the foundation of arithmetic...Never let us take a description of the origin of an idea for a definition, or an account of the mental and physical conditions on which we become conscious of a proposition for a proof of it...a proposition no more ceases to be true when I cease to think of it than the sun ceases to exist when I shut my eyes...this account makes everything subjective, and if we follow it through to the end, does away with truth.1

Part of his critique of Kant's claim that arithmetic propositions are synthetic was directed at Kant's reliance on intuition rather than on proof to determine the truth of arithmetic propositions. Frege regarded Kant's usage of the term "intuition" as inappropriate. Kant had associated it with what he apparently regarded as a fundamental distinction between small numbers, where the formulae are immediately self-evident, and large numbers, where a proof is required. Frege takes pains to show the ludicrousness of such an "absolute" distinction. Further, the aim of a proof for him is not merely to place the truth of a proposition beyond all doubt, but also to afford an insight into the dependence of truths upon one another. Kant's logic, of course, had not made this interdependence apparent.

Kant's characterization of the distinction differs in another fundamental respect from Frege's. Kant had not been as exclusively interested as Frege in the application of the distinction to mathematics, but had intended it to

1Ibid., VI-VIIe.
serve as a more general characterization of propositions within the context of natural language. Frege acknowledged the "logical activity within observation," but was not so much interested in specifying this logical manifestation as in generalizing about the fundamental character of logical activity per se. Frege carefully distinguished the results of observation from the psychological activity of observing, and then abstracted, from the results, their logical foundations. But he acknowledged, too, that logical activity was imbedded already within the observing itself.

Frege's characterization of analyticity then, was a development of only one aspect of the intuitive sense of analyticity. This aspect had been implicit in the traditional logic and by freeing logic of psychologism and revolutionizing its form, Frege was able to clarify and give precision to this sense of the word "analytic." But in its traditional meaning "truth on logical grounds" also characterized a group of statements within ordinary language where the distinction cannot be made on a strictly a priori basis but is imbedded within a context of experience. The traditional distinction is not merely a logical distinction but epistemological as well. The analytic-synthetic distinction suggests two possible answers to the question: how do you know? Frege provided a structural model for an answer, but his model was strictly applicable only in relation to a formal language system. Where there is a logical syntax and a semantic interpretation of that syntax,
the four way distinction Frege specified is exhibited in all its clarity.

Aristotle's distinction was attempted as a characterization both in abstraction from the content of language and within the content of language. His formulation of the principle of non-contradiction belongs to a part of logic more fundamental than his theory of the syllogism, which was essentially designed as a tool for more effective discursive argument. His essence-accident distinction was developed to give ground rules for debating procedures. In the Topics when he has stated the distinction he comments that a definition is the most difficult part of an argument to establish but also the most difficult to overthrow, whereas an accidental property, the simplest to establish, can also be overthrown with equal ease. Aristotle's main task was not the establishment of a logic independent of epistemology but the exhibition of the logical properties within the context of ordinary language.

Hume's distinction between relations which depend solely on ideas and relations which also depend on matters of fact was also a classification within an already established epistemological context. Arthur Pap has commented\(^1\) that Hume was never aware of the distinction that must be made if the question of a priori synthetic knowledge is to have

any sense. Though he certainly distinguished between a priori (necessary) and empirical (contingent) truth, the question of whether a priori truths were all analytic never occurred to him. But Pap argues that Kant erroneously attributed to Hume the contention that all a priori truths are analytic because he was misled by Hume's use of the word "contradiction." Hume's sense of "contradiction" included not only the formally contradictory propositions but whatever is "unimaginable" as well. Pap contends that Hume's division of relations is not successful as such (the propositions of quantity and number fall into neither class) but is merely a division of the objects of knowledge into necessary and contingent propositions. Pap concludes that Hume confused logical possibility with imaginability and wrote indiscriminately of the "possibility," "conceivability," "imaginability," and "non-contradictoriness" of the negation of an empirical proposition. For him these were criteria for a proposition's being empirical though not necessary. Thus we cannot legitimately read back into Hume a distinction we owe to Kant.

We have seen though that Kant's distinction was more an intended distinction than an actual inclusive division of propositions. It remained for Frege to provide the logical precision that Kant had lacked. But it was another manifestation of Kant's peculiar genius to see that propositions contained both logical and epistemological properties and that they were potentially distinguishable.
The formal language system that Frege developed -- for which his classification of propositions is defined -- contains all the essentials of modern logic. Though there have been many significant innovations in logical form since Frege, they often were developed out of difficulties suggested by Frege's work. Russell and Wittgenstein were influenced by him and recently his philosophy of logic has received appreciable consideration, especially by Rudolph Carnap and Alonzo Church.

The proof procedure which Frege worked out was the result of an effort to place the truth of a proposition beyond all possibility of doubt and to exhibit the interrelationships among true propositions. Since Frege, there have been many efforts to clarify the notion of truth for formal language systems. Recently some logicians have tried to differentiate "levels" of language in order to formulate within a semantic "meta-language" a concept of logical truth for the syntactic "object-language." Carnap claims that he has provided a general definition of logical truth applicable to all formal language systems. Within such a system the logically true statements (which he also calls "analytic") are those statements of the uninterpreted language which remain true under all interpretations of the non-logical terms, or in Carnap's terminology, in all "state-descriptions." (A logical term is a constant such as "not," "or," "and," "if-then;" they are never interpreted.) Tarski's formulation of the semantic notion of truth resembles Carnap's but
utilizes the notion of "satisfaction" to define truth.

Such efforts to clarify the concept of logical truth, though often heralded as "general" or "absolute," are all relative to the structure of formalized languages - commonly first-order languages. Tarski claims that:

The problem of the definition of truth obtains a precise meaning and can be solved in a rigorous way only for those languages whose structure has been exactly specified.¹

In this context the assertibility of sentences depends only on their form and is independent of non-linguistic factors. Tarski's explication of truth is designed to avoid the kind of problems typified by Russell's famous "liar" paradox, where language is used to talk about itself. Thus a linguistic structure is specified and then properties of the language are discussed within a semantic meta-language which specifies:

1) rules governing well-formed formulae

2) the terms which are to be taken as primitive and undefined

3) rules of transformation - if the language is intended for purposes of deduction

Truth is defined in terms of "satisfaction," which for Tarski is a relation between arbitrary objects and certain linguistic expressions. The expression "true" denotes a class of sentences within the object language. (The "object" language is so designated because it is the object which

¹"The Semantic Conception of Truth" reprinted in Feigl-Sellars, p. 58.
the semantic metalanguage is talking about. Tarski's use of the term "object" both to denote the syntactic language and arbitrary objects which might only happen to be assigned to that language is needlessly confusing.) According to Tarski's definition: "A sentence is true if it is satisfied by all objects, and false otherwise."¹ Tarski's conception of truth explicates the traditional notion of formal necessity including Leibniz' formulation of it as "truth in all possible worlds." The "true" sentences of Tarski's object language remain true regardless of interpretation.

Though the clarification of the concept of truth has been very fruitful for an understanding of the properties of formal languages, especially related concepts such as consequence, synonymy, and meaning; there has been no successful explication of truth that ranges over all formal languages, and considering this limitation, the repeated demand for an explication of truth that ranges over all of natural language is justifiably regarded as "exorbitant."² However, some philosophers have carelessly applied the term "logical truth" to the analytic statements of ordinary language overlooking the fact that though the two terms may be interchanged in first-order formal languages, they are by no means synonymous outside this context.

¹Ibid., p. 63.

W.V. Quine in an article which has provoked considerable rethinking of the problem of analyticity in natural language has explored some of the ramifications of such an extension of the term "logical truth." He defines a logically true statement in this way:

Those (analytic) statements which may be called logically true, are typified by:

(1) No unmarried man is married.

The relevant feature of this example is that it not merely is true as it stands, but remains true under any and all reinterpretations of "man" and "married." If we suppose a prior inventory of logical particles, comprising "no," "un-," "not," "if," "then," "and," etc., then in general a logical truth is a statement which is true and remains true under all reinterpretations of its components other than the logical particles.\(^1\)

Quine's characterization of logical truth differs in one very significant respect from Tarski's conception of truth. To assert that "No unmarried man is married" is a logical truth is to cast no reflection on the meanings of "man" and "married" at all. The particular descriptive terms are not relevant to the logical truth of the sentence in any way. We could just as well have written "No unplunk is a plunk" or "No non-A is an A" and the logical truth of the sentence would in no way be affected. The terms "plunk" and the letter "A" do not possess any significance in natural language, but "man" and "married" are meaningful terms. Quine's insertion

\(^1\)"Two Dogmas of Empiricism" reprinted in From a Logical Point of View (Cambridge: Harvard Univ. Press, 1964).

\(^2\)Ibid., pp. 22-23.
of such terms in a statement which purports to define the notion of logical truth obscures the simplicity of the concept, which bears no fundamental relation to the particular meanings words happen to have, but is a property of syntactical structure. It is relevant to natural language only insofar as a formal language which possesses this property exhibits certain syntactical characteristics which any language must reflect. Logical truth pertains to the patterns of language which are a precondition of the possibility of meaning. Because of these patterns some statements come out true in all interpretations. The only successful explication of the concept of logical truth is relativized to a formal uninterpreted language and does not define truth for natural language at all. Logical truth is a property of sentences within natural language only insofar as they are regarded as interpretations of the true sentences of the uninterpreted language. But sentences of natural language do not possess this property in separation from the formal context.

Quine's example of a "logically true statement," "No unmarried man is married" is not merely a syntactical arrangement of signs, for the terms "married" and "man" already have an interpretation, which Quine himself acknowledges by substituting the word "reinterpretation" for "interpretation" in his definition of logical truth. Such a redefinition obliterates the distinction between syntax and semantics and obscures the distinction between logical
and descriptive terms. Both distinctions are fundamental to the definition of logical truth.

The application of the term "logically true" to sentences such as "No unmarried man is married" is undisputably a valid application of the concept but the meanings of the descriptive constants are incidental to the truth of the sentence. They are convenient for Quine's purposes, however, since he wishes to discuss a second class of analytic statements typified by:

(2) No bachelors are married.

In this case, the meanings of the constituent terms are essential to the truth of the sentence. Quine continues:

The characteristic of such a statement is that it can be turned into a logical truth by putting synonyms for synonyms; thus (2) can be turned into (1) by putting "unmarried man" for its synonym "bachelor." We still lack a proper characterization of this second class of analytic statements, and therewith of analyticity generally, inasmuch as we have had in the above description to lean on a notion of "synonymy" which is no less in need of clarification than analyticity itself.¹

The question then becomes: what does the word "analytic" mean in an interpreted language? The talk of turning such a statement into a logical truth merely confuses the issue. No doubt it has sometimes been confused in this way, but since we already have a satisfactory explication of logical truth for most formal languages, it

¹Ibid., p. 23.
seems a happier proposal would be that we confine the term "logical truth" to the sentences of formal language which possess the defined property, taking such sentences to be a subset of all analytic sentences, and then ask the question of how we are to clarify the meaning of analyticity as a property of certain sentences within the context of natural language. Such an approach to the problem neither obscures the contribution of modern logic to the clarification of analyticity nor minimizes the traditional intuitive notion of analyticity. The relationships between the two groups of statements can then be explored without danger of prematurely reducing the one to the other.

The relation of synonymy in a formal language to the notion of synonymy in a natural language is but one of many possible approaches to the problem. "All bachelors are unmarried males" is the kind of statement Aristotle called "essential." "Bachelor" and "unmarried male" are, as Aristotle would say, "convertible," a property which resembles the structural property we now term "symmetry" though it is not identical with it as some have contended. Symmetry is generally expressed in modern logic as a property of certain binary relational predicates (e.g., compatriot, cousin, spouse) so that if the individual x bears that relation to the individual y then y also bears it to x. Or to express it symbolically:

\[(1) (x)(y) \ (Fxy \rightarrow Fyx)\]
Aristotle, as we noted, often neglected to distinguish between singular and general terms and assumed the relation held indiscriminately between a singular term such as "Socrates" and a property such as "white" as well as between general terms (e.g., "man" and "rational animal").

The development of the logic of classes resolved the singular-general term confusion and the logic of relations has clarified the properties of such relational predicates (not only the relation of symmetry, but the other binary relation reflexivity and also the three-termed relation which Aristotle's syllogistic logic had sought to define, the relation of transitivity). Symmetry can also be expressed in terms of class membership so that:

(2) \((x)(y)((x \in A \& y \in A \& xRy) \rightarrow yRx)\)

Thus for all \(x\) and for all \(y\) if \(x\) belongs to the class \(A\) and \(y\) also belongs to \(A\) and further if \(x\) bears relation \(R\) to \(y\), then \(y\) also bears that relation to \(x\). But for both (1) and (2) the relation is defined as holding between individuals. However, the symmetry of "bachelor" and "unmarried male" is a relation between predicates, and predicates which exhibit this relation necessarily exhibit other relations as well, which is not the case for individuals. We might illustrate this distinction most simply by formalizing the notion of "convertibility" in modern logical notation:

(3) \((x) (Fx \rightarrow Gx \& Gx \rightarrow Fx)\) or equivalently \((x)(Fx \leftrightarrow Gx)\)

(4) \((x) (x \in A = x \in B)\)
Though formulations (3) and (4) differ in that (3) expresses this relation in terms of predicate notation, and (4) in terms of notation indicating class membership, both express the same relation, equivalence. Sentence (3) may be interpreted as stating that every individual who has the property "bachelor" also has the property "unmarried male" and (4) that every individual who belongs to the class of bachelors also belongs to the class of unmarried males. But symmetry is only one of the relational properties holding between equivalent predicates or classes; transitivity and reflexivity also hold. Thus the identification of Aristotle's notion of convertibility with the modern notion of symmetry is misguided, because symmetry applied to predicate terms does not imply the same logical properties as in application to singular terms. The comparison, however, is helpful in that it exhibits logical relations other than equivalence which justify the claim that a proposition is true on logical grounds. But though the problem of analyticity in natural language can be approached by different roads, synonymy is still the main highway and formalizations of synonymy explicate this aspect of analyticity only partially.

Because the classes "bachelor" and "unmarried male" contain exactly the same members, we are entitled to call them "synonymous." But though they are extensionally synonymous, i.e., the extensions of the two predicates are the same, the meanings of the terms "bachelor" and "unmarried male" are not. For the two terms to be synonymous in all respects,
it would at least have to be the case that knowing that x was a bachelor implied knowing that x was an unmarried male. Knowledge of synonymy relations then is necessary to determine the truth of the statement "All bachelors are unmarried males." But our definition of extensional synonymy makes no provision for this.

Frege reflected on this problem of substituting identities in non-extensional contexts and developed a theory of meaning which, though it by no means resolved all the problems it raised, threw the question of identity into bold relief. Frege noted that concepts may have the same extension without themselves coinciding. Consider 6 plates and 6 knives. There are just as many of one as the other. The extension of the concept "number of plates on table" may be identical with the extension of the concept "number of knives on table," but this does not mean that the concepts "number of plates on table" and "number of knives on table" are themselves identical. They are obviously distinct. But we are then committed to saying that the same number is two different concepts.

Frege approached the problem in a number of different ways. He made several successive efforts to explain how the sentence, "The morning star is identical with the evening star" differs from the sentence, "The morning star is identical with the morning star." He first sought to define the difference in terms of "content" but abandoned this and later made a new distinction between "sense" (Sinn) and "reference"
(Bedeutung). According to this distinction then the morning star and the evening star are **identical** in that the two names have the **same reference**, i.e., the planet Venus; but they are **different** in that the sense of the two names are not the same. We can, for instance, **know** that the name "morning star" refers to Venus without knowing that the name "evening star" does also. Knowledge of the one does not entail knowledge of the other but requires additional empirical inquiry; it is not solely a linguistic question but a matter of extra-linguistic fact. Thus "morning star = evening star" differs from "2 + 2 = 4" in at least one fundamental respect; we may know the equality of the name "2 + 2" with the name "4" without reference to experience, but we cannot know that the name "morning star" designates the same object as the name "evening star" without first looking to the world of experience. Though both sentences express extensional equalities, they point to intensional inequalities. The equality of "2 + 2" and "4" is **a priori** and analytic, but the second sentence is known empirically and can be said to be true not on logical grounds but only on empirical grounds. In both cases the object is, as Frege says, "given in different ways" so that the **names** differ. But in the one case we can know that they name the same object without experience but not in the other. The knowledge of what is named **a priori** is exhaustive; thus its meaning can be completely specified but empirical knowledge is always partial and therefore **extensional** equivalence is
but one aspect of the meaning of such terms. To specify the meaning of the latter group would involve specification of each of the senses in which the word is used which implies exhaustive knowledge of linguistic usage.

Frege's doctrine poses a host of problems. The theory of language he develops takes naming to be the primary function of language and subordinates all other uses of language to this. Though the implications of this theory are well worth exploration, for our present purposes we must focus attention on the applicability of his analytic-synthetic distinction to his theory of language. Is the distinction applicable to a non-extensional language at all? And even if we confine it to extensional languages, we could apply the distinction only where the interpretation was known a priori. Frege's definition of analyticity specifies that the primitive truths from which the proposition is derived are all general logical laws and definitions. How this concept might be extended to a natural language which is not solely a priori is never developed by Frege.

He comments in his conclusion to the Grundlagen, where he contrasts his conception of analyticity with Kant's, that a really fruitful definition does not merely use lines already given but draws boundary lines which were not previously given at all. Frege's redefinition of analyticity drew new boundary lines in such a way that the traditional application of the term "analyticity" was restricted. Though
Frege acknowledged that observation includes logical activity, such activity was not described in his notion of analyticity. The problem of clarification of analyticity within a context of observation remains. In the concluding Chapter we shall discuss two possible contemporary approaches to the problem.
CHAPTER III

We commented in our very sketchy summation of Boole's contribution that he freed logic from its 17th and 18th century domination by epistemology and revived it as an independent enquiry. Boole showed by example that logic could be studied without any reference to mental processes. Kneale points out that he was dealing not with the laws of thought, but with the laws of thinkables.¹ Frege's work completed the separation of logic from psychology and epistemology. In the Introduction to the Grundlagen Frege specifies, as we noted, the sharp separation of the psychological from the logical, the subjective from the objective, and the concept from the object. What we know is separated from how we come to know it. And his specification of the a priori-empirical distinction and the analytic-synthetic distinction is relativized to a formal language system. But if we are not to abandon the traditional concept of analyticity altogether, we must reconsider its relation to epistemology. In this Chapter we will consider briefly two possible characterizations of the notion Frege called "the logical activity within observation." They are not necessarily either mutually exclusive or exhaustive, but represent divergent conceptions of the relation of formal to natural language.

The first is the approach by way of definitional rules which has been most fully explored by Rudolf Carnap.

The second I shall call by Arthur Pap's term, empirical generalization. The characterization of analyticity via either approach is still provisional and incomplete; they are not fully developed conceptions but essentially programatic. Neither has been able to fully distinguish the logical from the epistemological characteristics of natural language.

Carnap commented in Testability and Meaning:

Epistemology in the form it usually takes—including many of the publications of the Vienna Circle—is an unclear mixture of psychological and logical components. We must separate it into its two kinds of components if we wish to come to clear, unambiguous concepts and questions. I must confess that I am unable to answer or even to understand many epistemological questions of the traditional kind because they are formulated in the material idiom.

Carnap enters a plea for the translation of philosophical questions into sentences of the formal idiom where it is possible to distinguish clearly between logical and empirical (e.g., psychological) questions. He then proceeds to sketch a model of a methodological investigation in which a logical question can be expressed by a concept belonging solely to logical syntax; a descriptive or non-logical question cannot be exclusively expressed in this context; it involves sens-

1 In Semantics and Necessary Truth, Pt. II, this approach to the explication of analyticity is comprehensively developed. Though the characterization sketched here owes much to Pap's argument, it is not intended as a summary of it and expresses primarily my own views.

sentences which are not solely analytic, i.e., their truth cannot be determined by logic alone but requires reference to non-logical rules or to facts outside the (formal) language in order that its truth may be determined. The development of such a methodological investigation proceeds for Carnap by the construction of definitions. A definition can be constructed in two ways. To introduce new terms into a language which has already been constructed, a sentence in the material idiom is reduced to the formal idiom and the reduction is replaced by a definition, which may be either an explicit definition, if the reduction sentence has the form of an equivalence and is analytic \( Q_3 \equiv Q_2 \), or a conditional definition if the reduction sentence does not have this simple form but is of the general form \( Q_1 \rightarrow (Q_3 \equiv Q_2) \). Though its antecedent may be empirical, such a definition is also analytic because whatever follows from it as a consequence is a logical truth.

Where our interest is the construction of a formal scientific language such an extension of the term "analytic" can be most useful if properly qualified. In such a context the language system must consider the empirical significance of its extra-logical terms. The interpretation may be given either by partially assigning a meaning to a term or by completely assigning the meaning.

Let us consider in more detail the conditions necessary to formulate a definition. In some cases, such as disposition terms, that express the disposition of a body
to react to conditions in a certain way, e.g., visible, smellable, soluble, a reduction cannot be replaced by a definition. A definition fixes the meaning of a new term once and for all, and in stating a definition it is necessary to make an arbitrary decision concerning any undetermined cases. We might have to revoke the definition later, when additional cases had been confirmed by presently undetermined empirical knowledge. Thus to introduce a new term into the language we must decide whether we wish to fix its meaning. If so, then a definition is the appropriate form but, if we wish to determine the meaning at the present time for only some cases, the method of reduction is the more suitable form. Then we are still free to consider other interpretations of the term as more empirical data become available. Then when all cases have been determined (where this is possible), the set of reduction sentences can be reduced to a definition which introduces a new predicate into the system. The definition fixes the meaning of the predicate so that certain interpretations are ruled out. For instance where \( Q_1 \) and \( Q_2 \) indicate properties of space-time points and a new predicate \( Q_3 \) is introduced by a definition of the form, \( Q_1 \rightarrow (Q_3 = Q_2) \), then for points of the class \( -Q_3 \) the predicate has no meaning. The intention of the maker of the language is decisive here. He may chose to define \( Q_3 \) explicitly, asserting it to be equivalent to a formulation consisting of primitive terms; in this case it can be eliminated from the sentence and the validity (either
logical or empirical) of the new sentence can be determined by the elimination of the defined terms. For instance, the definition, \( Q(x) = \ldots x \ldots \), is analytic, for once it has been stated as a valid sentence, by eliminating the predicate \( Q \), we get, \( \ldots x \ldots = \ldots x \ldots \), which is analytic.

But Carnap also would like to extend the applicability of the term "analytic" to include sentences where the defined term \( Q \) cannot be eliminated from the sentence. Such sentences are only conditional definitions; they are only partially reducible. We cannot assert exhaustively under exactly what range of conditions we would apply to term \( Q \); thus we introduce \( Q \) by a set of conditional sentences which fixes the meaning only under a limited range of conditions. Such sentences are not analytic for Carnap in the same sense as a sentence \( S \) which:

1) does not contain any descriptive symbols.

2) contains only primitive (undefined) descriptive symbols.

3) contains a defined descriptive symbol \( Q \) which is eliminatable.

They constitute, for him, a fourth class of analytic sentences in which a sentence \( S \):

4) contains a descriptive symbol \( Q \) introduced by a set \( R \) of pairs of reduction sentences.

Though \( Q \) cannot be eliminated, such sentences are nonetheless regarded as analytic in that they are a logical consequence of the set \( R \) of reduction pairs. The implication sentences containing:

1) The conjunction of the sentences of \( R \) followed by
2) the sentence $S$

is analytic in that every sentence resulting form this implication sentence - where we replace $Q$ by a symbol of the same type in the language which includes the new conditional definition - is a valid sentence in this language.¹

Carl Hempel in an article: "The Concept of Cognitive Significance"² has discovered some problems in this extension of analyticity within an interpreted formal language-system. He is critical of the extension of the term "analytic" to include sentences which are not logically true but only conditionally true. If we accept Carnap's concept of analyticity and regard sentences which include observational predicates as analytic as long as their consequences are truths of formal logic, we encounter certain difficulties. Assuming Carnap's new criterion for analyticity, if we take the sentence:

\[(1) \ (x) \ (P_1 x \rightarrow (Qx = P_2 x))\]

as a partial definition of $Q$, and then also introduce into the system the sentence:

\[(2) \ (x) \ (P_3 x \rightarrow (Qx = P_4 x))\]

where $P_3$ and $P_4$ are additional observational predicates, then on the basis of Carnap's criterion, we would have another analytic sentence. But we get the confusing result that the

¹Ibid., p. 453.

²Proceedings of the American Academy of Arts and Sciences, 80 (July, 1951), 61-77.
two sentences (1) and (2) taken together entail non-analytic consequences, i.e.,

\[(3) (x) \left( \sim (P_1x \& P_2x \& P_3x \& \sim P_4x) \& (\sim (P_1x \& \sim P_2x \& \sim P_3x \& P_4x) \right) \]

Hempel concludes then, that if the concept of analyticity is to be applied to sentences of an interpreted deductive system, it will have to be relativized:

1) With respect to the theoretical context at hand. For example, \(S_1\) might be considered analytic relative to system \(T\) whose remaining postulates do not contain \(Q\), but synthetic relative to \(T\) which also contains \(S_2\).

2) With regard to the rules of the language at hand. For the language determines what observational or other consequences are entailed by a given sentence. Hempel is emphasizing here the contextual character of analyticity in interpreted languages. Many devices have been suggested for extending the term "analyticity" to such contexts. Some involve a restriction on the material conditional, so that it is qualified relative to a given interpreted language. Though the problems of such an extension are by no means simply resolved, the statement of the problem clarifies certain aspects of the question of analyticity for ordinary language as well. The suggestion that an analytic statement might be regarded (relative to a given language) as a conditional statement with an empirical antecedent but whose consequences are formal logical truths, has been
explored for natural language in a number of recent papers. John L. Pollock\(^1\) distinguishes between immediate implication where the meaning relation is always a perceived or perceivable relation arising from what he calls "semantical intuitions" and derived implications which are not immediate. The recommendation that the implication sign be used in a qualified way is here specified and explored.

But Carnap's characterization of such sentences as "definitions" creates other difficulties as well, especially where the formal language is to be given an interpretation in a language (unlike most languages of theoretical physics) which is not strictly quantitative. Moreover, can the analytic statements of natural language be regarded as definitions at all? Quine has criticized Carnap's original identification of analyticity for its failure to take account of a language which includes "extra-logical synonym pairs" (e.g., bachelor and unmarried male). Carnap, in reply, has suggested other means by which analyticity could be explicated for such contexts. Here, too, the construction of a theoretical framework by means of definition is basic to his explication. In a paper "Meaning Postulates"\(^2\) Carnap attempts to show that for a language where the truth of statements depends on the meanings of descriptive terms, it is possible to explicate the concept of analyticity.


\(^2\)Philosophical Studies, III (5. October, 1952), 65-73.
(defined as: "truth based on meaning") in the framework of a semantical system by the use of "meaning postulates" — a method Carnap claims is suggested by "common-sense reflection." The explication, however, does not refer to natural language but to semantical language systems; in this respect Carnap parallels his explication to Tarski's explication of truth. (He comments that it seems to him that the problems of explicating such concepts for natural language are of an entirely different nature.)

He specifies a semantical language system L containing the customary connectives, individual variables with quantifiers, and as descriptive signs, individual constants (a, b, etc.) and primitive descriptive predicates (B, M, etc. where B signifies the property "bachelor" and M "married"). First he assumes that no rules of designation are given for the descriptive constants (hence the meanings of B, M, etc. have not yet become part of the system). He defines the logical truth (in the narrow sense) of a sentence of L for which rules have already been given which determine the concepts involved. Then in order to provide for the truth of:

\[ \neg (x) \ (Bx \rightarrow \neg Mx) \]

which is not guaranteed by the definition of L-truth, he specifies that it is a meaning postulate. He explains that no rules of designation for the predicates B and M are necessary, for the postulate states as much about the meanings of the predicates as is essential for analyticity, that is, that the two properties are incompatible. In order to
explicate analyticity Carnap stipulates that all logical relations, for instance implication or incompatibility, which hold between the intended meaning of primitive predicates within a system, be stated as postulates. Then when the meaning postulates have been accepted into the system $L$, analyticity for $L$ can be explicated by defining "$L$ (logically) true with respect to postulate $P"$. He suggests several alternative formulations for the right side of the definition such as:

- $S_i \text{ is } L\text{-implied by } P \text{ (in } L\text{)} \text{ and } P \rightarrow S_i \text{ is } L\text{-true (in } L\text{)}$

After he has explained the mechanics of his device for the explication of analyticity by means of definitional rules, he considers several related philosophical problems, principally the question of epistemological assumptions:

Suppose the author of a system wishes the predicates "B" and "M" to designate the properties "Bachelor" and "Married" respectively. How does he know that these properties are incompatible and that therefore he has to lay down postulate $P$? This is not a matter of knowledge but of decision. His knowledge or belief that the English words "bachelor" and "married" are always or usually understood in such a way that they are incompatible may influence his decision if he has the intention to reflect in his system some of the meaning relations of English words.²

Carnap is suggesting here a particular position, both toward the structure and function of formal language systems and the relation between formal and natural language. Thus in order to understand why he considers it appropriate that analyticity

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¹Ibid., p. 69.

²Ibid., p. 68.
be explicated by definitional rules, we must consider this position in more detail.

The distinction between knowledge and decision which Carnap expresses here is not merely relevant to the problem of meaning postulates but is a general characteristic of his philosophical viewpoint. He reiterates it in his article: "Empiricism, Semantics, and Ontology,"¹ where he defends this position with regard to all linguistic frameworks. For him, the acceptance of certain linguistic forms is not in need of any theoretical justification other than expediency and fruitfulness. Such questions are non-cognitive and cannot legitimately be regarded as epistemological questions at all.

But has Carnap demonstrated here that even the minimal conditions of expediency and fruitfulness are satisfied? Is the device he suggests a possible solution to the problem Quine has posed? Has he shown that the technique of "meaning postulates" is useful in any way for clarification of the concept of analyticity as it applies to synonym pairs? The distinction Carnap makes between knowledge of linguistic usage and decision with regard to usage is so ambiguous and so contrary to common usage that understanding is difficult. From the context it seems evident that he is applying the term "knowledge" in a restricted sense. Matters

of knowledge and matters of decision are generally interre¬
lated in experience. My knowledge that the term "bachelor" is generally used incompatibly with "married man" certainly influences my decision regarding the application of the term. But Carnap seems to want to reserve the term "knowledge" for a special use. He wishes to distinguish between what is involved in making up one's mind to apply a term in a certain way in the context of a formally specified language (supposedly a non-cognitive activity) and what is involved in making up one's mind to apply a term in natural language (an activity that involves belief). In the first case, "knowledge" is irrelevant, but in the second, "beliefs concerning facts of the world" are involved.

Yet such a simple resolution of the ambiguity is unsatisfactory on several grounds. For is not natural language a linguistic framework as well as formal language? Why, then, should decisions with regard to one be taken to be cognitive but with regard to the other non-cognitive? And then too, Carnap's discussion tacitly implies that there is a relation between the way meanings are determined in natural language and in his language system. Otherwise of what significance is the problem of synonym pairs at all? He makes no effort to show that it is of interest for formal systems per se. His discussion of the application of the "meaning postulate" device to relational predicates lends further support to the argument that he is actually implying a relation between formal and natural language which he does not explicitly
Suppose that among the primitive predicates there are also some with two or more arguments designating two- or more- place relations, and that one of these predicates possesses some structural properties in virtue of its meaning. For example, let "W" be a primitive predicate designating the relation Warmer. Then "W" is transitive, irreflexive, and hence asymmetric in virtue of its meaning.

Carnap then lays down meaning postulates with respect to which a statement asserting these properties of "W" would be taken as L-true. But is it merely a coincidence that the property Warmer possesses the same structural properties in natural language? If the determination of properties depends solely upon the "intention" of the language-maker, as Carnap claims, then why chose these particular properties which are not arbitrary but already express meaning in natural language? Since Carnap has told us virtually nothing about the criteria on which a language-maker bases such decisions (supposedly expediency and fruitfulness, but with regard to what?) and since he denies that the constructed language is intended to reflect what is known about natural language, would it not make just as much sense to interpret Warmer as, say, intransitive, reflexive, and symmetrical?

But then why employ terms which already have a meaning at all? If one's sole interest were the illustration of the power of a language system to formalize structural properties, one might just as well employ meaningless symbols.

1Ibid., p. 70.
Instead Carnap utilizes language which has already been endowed with meaning and then directs us to disregard the way that meaning has been determined and to regard the same meaning in accordance with his epistemology. Meaning is no longer to be regarded as a generalization of linguistic convention but as a definitional rule codified by Carnap.

Further Carnap tells us that:

If we admit the form of semantical rules which we have called meaning postulates, we find that other customary kinds of rules may be construed as special kinds of meaning postulates. This holds, for example, for explicit definitions (if written as statements in the object-language with " " or " = ") and for contextual definitions...Further, the reduction-sentences which I proposed earlier for the introduction of disposition predicates may be construed as meaning postulates. (See Testability and Meaning paper above)\textsuperscript{1}

Carnap then takes note of the limitation which Hempel had observed and specifies that a bilateral reduction sentence:

\[(x) \ (Q_1x \rightarrow (Q_3x = Q_2x))\]

may be taken as a postulate specifying the meaning of \(Q_3\) since it has no synthetic consequences in terms of the original predicates \(Q_1\) and \(Q_2\). But for most formulas of reduction-pairs this is not possible since:

\[(x) \ (Q_1x \rightarrow (Q_2x \rightarrow Q_3x)) \text{ and } (x) \ (Q_4x \rightarrow (Q_5x \rightarrow \sim Q_3x))\]

together imply the synthetic statement:

\[(x) \sim (Q_1x \ & \ Q_2x \ & \ Q_4x \ & \ Q_5x)\]

Here Carnap specifies we must take as a postulate the weaker statement:

\textsuperscript{1}Ibid., p. 71.
\[ S_3 \rightarrow S_1 \& S_2 \]

which has no synthetic consequences.

But is it not then doubly confusing that Carnap has defined analyticity for these sentences by an extension of the term "logically true" to include them also. Such an extension of an otherwise fully explicated term to include a group of statements which are clearly not true in all interpretations not only fails to clarify the latter group of statements, but obscures the simplicity of the initial characterization as well. To apply the concept of logical truth to the definitions of descriptive terms in an interpreted language masks the precision of the concept without in any way illumining the concept of analyticity for interpreted language. Tarski's explication of truth ranges over all first order formal languages, but the redefinition Carnap suggests here must be relativized to each particular language. Moreover Tarski's concept and Carnap's original definition of logical truth is not dependent on the decision of the language-maker but characterizes structural properties without which language would have no expressive power. The confusion of such syntactical properties with arbitrary definitions not only tells us no more about the analyticity of synonym pairs, but obscures our understanding of analyticity for uninterpreted language as well.

The usefulness of definitional rules for certain interpretations is not open to question. But Carnap's application of the term L-true to them merely confuses their
role in the language. It is difficult to account for Carnap's sacrifice of clarity here on any other grounds than his insistence that "knowledge" be separated from "decision," a concern which seems to have a prior claim on him. But has he succeeded in showing that this group of statements can be characterized in abstraction from knowledge of linguistic practices? Can the meaning of such statements be regarded as merely a matter of definitional rules legislated by decision? Carnap set out to explicate "truth based on meaning" but he seems to have succeeded only in explicating "truth based on definitional rules."

In the Testability and Meaning paper he specified that the construction of a definitional rule involves reducing a sentence from the material to the formal idiom and then introducing the sentence into the formal language system. By this means psychological and epistemological questions are presumably separated from semantical ones and truths based on logic are distinguished from factual truths. But are such "synonym-pairs" as "bachelor" and "unmarried male" reducible at all? The reducibility of terms would at least imply that the necessary and sufficient conditions for the application of the term can be specified. But we do not always know what properties a thing must have for a term to be applicable to it, nor do we necessarily know under what range of conditions we would apply the term. Though "bachelor" and "unmarried male" are comparatively
fully specified terms, all descriptive terms in natural language possess a degree of intensional vagueness. The meaning of a descriptive term is relative to the conditions under which a user of the language is disposed to apply the term. To Kant it was intuitively obvious that "All bodies are extended" is an analytic statement. But in the light of post-Newtonian physics the applicability of the term "body" lacks the precision it previously had. Is an electron, for instance, a body? Yet within a certain range of meaning we would still regard Kant's example as analytic. So with "All brothers are male siblings," though the term "brother" is also applicable to lodge brothers or fraternity brothers or The Brothers of St. Francis. The equivalence of such synonym-pairs is contextual. We might substitute one for the other only within a certain range of contexts which are only more or less specified and for which there is generally no formula. There are then, no "real" definitions in natural language.

G.E. Moore exhaustively explored the question of definition; he sought after meanings in natural language that would settle the vexing problems about what a word really means. He asked, for instance, what does the word "real" really mean? Does the fact that we are able to say that "This book is bound in real leather" and "A vacation in Bermuda would be a real rest" and "Completing this chore would be a real accomplishment" and "The Easter bunny is a real animal" imply that all these "real" things have a
common property? Are we asserting that leather, rest, accomplishment, and the Easter bunny have some common ingredient? - that we can define the term "real" for all interpretations? Moore sought a definition beyond the immediate context and therein neglected to notice that we might paraphrase each of these assertions in a way that exhibits the many senses in which we use the term "real": that some books are not bound in leather but this one is; that such a vacation would indeed be restful; that completion of the chore would be a significant accomplishment; and that it is said that the Easter bunny is not merely a mythical animal. What is initially confusing and at first glance suggests that all these things do have one common property is the use of the same "token" word in so many different contexts so that we are tempted to generalize that they all represent the same "type" or "class." Aristotle, capitalizing on Plato's errors, had tried to avoid this temptation and explicitly denied that a single word has an appropriate Form and thus only one single meaning in all contexts. Aristotle's categories reflect an attempt to classify entities according to what can be significantly said about them. Predicates which can be applied to things in all categories (such as "one," "same," "other") do not have a single meaning. Their meaning is contextual -- it depends on how the word is used.

For some terms linguistic convention provides a "guarantee" of applicability. That "Nothing can be red and
green all over" is guaranteed by the conventional meaning of color words. But the range of applicability of many terms -- such as "religion" or "culture" or "language" -- is highly dubious. A recent report commented that in Southeast Asia there are 2790 different languages according to one authority and 4000 languages according to another. Obviously the authorities differ considerably on what is to count as a language. Within certain cognitive contexts it might be possible to specify the meaning of such terms, but in the context of ordinary language meaning is relative to the beliefs of the particular authority.

Meaning then, within natural language is, as Arthur Pap noted, a three termed relation, involving the sign, the object which the sign designates, and the interpreter. The relative bearing of each of these components depends upon such factors as the degree of vagueness of the term, the conditions under which the object is experienced, and the knowledge of the interpreter. Meaning cannot be fixed unless each component is specified, yet specification of meaning in any interpreted language is always a matter of degree. But if meaning in natural language must always remain a question of degree, and analyticity depends on meaning, then is not analyticity, too, a matter of degree? Does not analyticity depend upon such factors as the interpreter's choice of context or reference frame as well as on his beliefs and dispositions regarding the applicability of a term?
However, if this is the case, then the analytic statements of natural language cannot be translated from the material to the formal idiom without loss of meaning. Is it possible, then, within the material idiom to distinguish between statements true on logical grounds and statements whose truth depends on experience? Is not this distinction, too, a matter of degree and is it not interrelated with the distinction between a priori and empirical truth?

If the distinction can be made at all -- and observation of language seems to support the assertion that we do in fact make it -- then the analytic statements of natural language must be empirical generalizations whose truth is dependent upon what we know about linguistic practices. In contrast to a definitional rule, which, Pap notes, states a proposal (and is neither true nor false), an analytic statement is a generalization of what is the case; it is not a recommendation that it ought to be the case. But then how are such statements distinguishable from other factually true statements? They are in a sense contingent, for we must presuppose knowledge of contingent linguistic habits in order to teach that an analytic statement contains necessary relationships. But unlike other contingent statements, those we term "analytic" are not immediate observables.

Leibniz had made a similar distinction:

Someone in danger needs a pistol-ball, and lacks the lead to pound it in the form he has; a friend says to him: remember that the silver you have in
your purse is fusible! This friend will not teach him a quality of the silver but will make him think of a use he may make of it, in order to have pistol-balls in his pressing need.\footnote{New Essays Concerning Human Understanding, Langley trans. (New York, Macmillan, 1896), p. 492.}

Leibniz adds that such truths based on reason teach us nothing but they make us think at the right time of what we know. Leibniz' distinction between truths of reason and truths of fact is reflected in several ways in the characterization of analyticity as "empirical generalization." For both:

1) The distinction is relativized to a context.

2) The meaning of the term "silver" involves both the properties of the metal and the knowledge of the interpreter.

3) The distinction depends in turn upon the distinction between teaching someone something he has not known before and reminding someone of something he already knows. So that the same statement which is analytic in this context would be synthetic if we were teaching someone the properties of silver.

4) The distinction between a priori and empirical is not precise. Leibniz' distinction is made within the context of experience. Even if we translated it into the formal idiom and discussed, instead of the properties of silver, the predicates applicable to the term "silver," would we be justified in regarding such a statement as a priori? Certainly not in the...
sense that $2 + 2 = 4$ is a priori.

Then does the contemporary characterization of analyticity differ significantly from the traditional intuitive notion of analyticity? I think it does in at least one important respect. The questions of meaning and truth raised within modern semantics have at least clarified the pre-semantic notions of meaning and truth. The attention focused on the role of the language user in relation to language usage and the effort to understand the relation between the interpreter, the language he interprets, and the experience he is interpreting, have made a distinction explicit which in traditional philosophy had been concealed. Thus an explication of analyticity is possible today which:

1) Ranges over sentences of much greater diversity of logical form than had been possible within the subject-predicate formulation of analyticity.

2) Avoids ambiguities due to confusion of the roles terms play in the grammar of language. Though there is no guarantee that a speaker will not confuse singular with general terms, for instance, it is possible within a context to clarify this distinction.

3) Distinguishes analyticity for formal uninterpreted language from analyticity for natural language. This makes it possible to resolve many traditional confusions, for instance, how form is related to content and how the concept of necessity is to be understood.
(Is it a formal concept, or psychological, or material?).

However, does such a modern characterization of analyticity preserve the traditional intuitive notion of analyticity? I think we have shown that our characterization of analyticity as "empirical generalization" both preserves the claims of Aristotle, Leibniz, Hume and Kant that natural language reflects logical relational properties; and reflects, as well, the distinction Frege insisted upon between analyticity in a formal syntactical language and the logical activity within observation. But nonetheless, analyticity remains not one concept, but many concepts; though they are somehow related to one another, the relation is inadequately understood. Whether we must content ourselves to live with this rather unsatisfactory state of affairs or whether it may at some time become possible to gather all the many concepts of analyticity into one general inclusive concept, remains undetermined. Perhaps increased understanding of linguistic behavior may make such an explication possible; it should, at least, clarify the problem.
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