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INFRARED RERADIATION FROM CIRCUMSTELLAR SHELLS

by

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ABSTRACT

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by

Susan Geisel

A catalogue of the stars known to emit excess infrared radiation is compiled, and a dust model is applied to 36 of these "infrared stars". In this model, the infrared excess is produced by thermally reradiating dust particles concentrated in a thin spherical shell around a central star. Photometric measurements of the observed continuum of an infrared star, and of the observed continuum of stars of similar spectral classification, are used to derive the energy distribution of the dust's infrared emission. The total energy emitted by the dust shell, the peak shell flux, and the temperature of the shell are computed from the derived shell energy distribution, and from these data the radius of the shell is determined by several means. The sizes of some of the stars are amenable to measurement, so that this dust model of infrared stars can be tested.
I. **INTRODUCTION**

In 1965 Neugebauer, Martz and Leighton announced the discovery of a "stellar" object in Cygnus whose spectral energy distribution peaked in the infrared beyond 2\(\mu\). Up to that time, all of the known "infrared stars" were thought to be either late-type (M6-M9) stars, which emit most of their energy at ~ 1\(\mu\), or stars heavily reddened by the interstellar medium, which appear to emit peak energy at about the same wavelength. Anomalously large amounts of interstellar extinction could not explain the very red colors of NML Cygnus, however, making this object the first star known to emit most of its energy at infrared wavelengths beyond 2\(\mu\). Subsequent photometric measurements out to 22\(\mu\) (Johnson, Low and Steinmetz 1965) confirmed this result, and placed the peak emission at 5\(\mu\).

Since the discovery of NML Cyg, many other galactic objects have been found whose redness cannot be explained by the effects of interstellar matter alone. All the objects found so far fall into two groups, on the basis of geometry, into stellar and extended infrared sources. The character of the optical and infrared energy distributions further subdivides these two groups. The stellar infrared sources include five classes:
A. Protostars gravitationally contracting from the interstellar medium, e.g., Becklin's Object.

B. Main sequence stars with circumstellar envelopes, e.g., Herschel 36.

C. Red giants which suffer mass loss, e.g., μ Cep.

D. Planetary nebulae, e.g., NGC 7027.

E. Supernovae, e.g., the Crab Nebula.

The known extended infrared sources are less numerous, and only two classes can be distinguished. These are:

A. Optically thick sources connected with pre-main-sequence evolution, e.g., the infrared nebula in Orion.

B. Optically thin sources, i.e., tenuous nebulae surrounding hot stars, e.g., the Trapezium and M17.

A comparison of the physical parameters associated with each class of infrared sources emphasizes their distinguishing features. To this end, Table 1 lists the infrared color temperature, luminosity, linear size, and nebular mass and age estimates for typical examples of each class; the solar system is included for comparison.
Despite differences in physical appearance, all the galactic infrared sources have been linked by a common physical mechanism; the supernovae, which emit characteristic synchrotron spectra, are the exception to this rule. First proposed by Low and Smith (1966) for R Mon, a star spectroscopically similar to T Tauri, the infrared mechanism is simply the degradation of hot stellar continua by dense circumstellar dust. In their treatment of R Mon, Low and Smith showed that the observed spectrum could be reproduced by a solar-type central star surrounded by a thick envelope of large ($\sim 10\mu$), black dust particles, which absorb the optical wavelength energy from the star and reradiate it at infrared wavelengths corresponding to the equilibrium temperature in the envelope.

Several dust models have since been proposed to account for the infrared emission from all the other galactic infrared sources. The models differ with regard to the chemical and physical properties of the particles and/or the evolution and physical characteristics of the envelopes into which they form. For example, in his examination of the dynamic evolution of a protostar in gravitational collapse, Larson (1969) found that the observed spectrum of Becklin's Object (Kleinmann and Low 1967) is compatible with the thermal emission of a thick
cloud composed of graphite particles with an Oort-van de Hulst size distribution. An optically thin cloud of graphite particles (sizes 0.02\(\mu\) to 1\(\mu\)) can produce infrared excesses observed in planetary nebulae (Krishna Swamy and O'Dell 1968; Gillett, Low and Stein 1967). A wide range of cloud thicknesses has been used in models of red giant and carbon stars, which form dust envelopes in the process of mass loss. Of particular interest concerning the circumstellar clouds around cool stars is the attempt by Stein, Gaustad, Gillett and Knacke (1969) to identify the chemical make-up of circumstellar dust; their mineral grain hypothesis may explain the 9-10\(\mu\) excesses observed in the red giants.

Extended infrared objects have also been studied in terms of the dust model. The difference between the infrared nebula in Orion and Becklin's Object can be treated as a difference of scale only; the infrared nebula may contain 1 to 100 massive protostars (Hartmann 1967). The optically thin nebulae, on the other hand, may resemble multiple planetary nebulae, according to a suggestion by F. J. Low; in both cases, H II regions around hot, O-type stars interact with the surrounding medium, producing or retaining significant masses of dust in the form of particles large enough to radiate efficiently in the infrared (Krishna Swamy 1968).
In every case in which the dust model has been applied so far, attention has been directed toward interpretation of a single object or class of objects. Data are now available on a wide selection of infrared objects, and all of the point sources can be treated using a uniform method for computation of the physical properties of the circumstellar envelopes. Such a procedure should point out the similarities and differences between stellar systems which emit excess infrared radiation, and lead to an understanding of the sources of the dust particles (if they exist), their chemical and physical properties, and the relationship between the physical conditions in circumstellar envelopes and the masses and ages of the central stars.

In order to carry out such a study, a method for definite selection of the stars which emit excess infrared radiation is presented. The test is applied to several stars to show that it can be determined whether or not a star is (1) highly reddened by the interstellar medium but not by a circumstellar envelope of dust; (2) intrinsically cool; or (3) reddened by circumstellar dust which emits infrared energy. Without this test, the term "infrared star" remains ambiguous, so that a single model would be inapplicable to all infrared stars.
Once a list of infrared stars is obtained, the dust model proposed by Low, Johnson, Kleinmann, Latham and Geisel (1969) is applied to all the stars for which there are sufficient observational data. In this model, infrared excess radiation is produced by dust distributed in the form of a thin spherical shell around a central star. Assuming that the dust is black or gray, and that the energy emitted by the dust is thermal, the temperatures and sizes of the shells have been computed by several means by separating the shell emission from the observed continuum. The density of dust particles and their radiative properties are discussed in terms of the agreement between the sizes computed by three different methods.

Computation of the sizes of infrared stars is of particular interest, because the validity of the dust model can be tested directly by comparing theoretical values with measurements of the stars' sizes. Although these measurements are now possible in only a few of the cases studied here, it is expected that the observational results will yield unambiguous evidence bearing on the existence and radiative properties of circumstellar dust.
II. Distingushing Features of "Infrared Stars"

A star with a large ratio of observed infrared luminosity to observed visual luminosity has (1) been reddened by the interstellar medium; or (2) is intrinsically cool, so that its energy distribution peaks in the near infrared around 1\mu (M, R and N stars); or (3) emits most of its energy at ultraviolet and visual wavelengths but is part of a compact system in which the higher-energy photons are degraded into infrared photons. Stars in group (2) can be distinguished from stars in group (3) by observation of their absorption spectra. It is more difficult to distinguish between stars in groups (1) and (3), however. In this section the properties of both groups of stars will be outlined, so that the observational differences between reddened stars and infrared stars can be determined.

A. Effects of Particulate Matter on Stellar Continua. Extinction.

1. Total Luminosity and Reddening Effects

When a star is surrounded by dust -- circumstellar or interstellar dust -- that star's continuum is extincted to some degree at all wavelengths. The extincted light is the sum of the light which is scattered
away from the observer and that which is absorbed by the dust particles. The scattered light can, in principle, be recovered by observing a sufficiently large field of view, for the case of a spherically symmetric scattering cloud. The absorbed energy, on the other hand, is shifted toward longer wavelengths, where it is thermally re-emitted by the absorbing dust particles. Both the re-emitted energy and the scattered energy can only be detected by observing the entire absorbing cloud. This limitation excludes any possibility of unambiguous measurement of the total energy of a reddened star, since interstellar clouds are typically large enough (\( \sim 1 \) pc) so that they scatter and absorb the light from many stars simultaneously. Therefore, all stars affected by interstellar dust appear under-luminous.

The effects of the interstellar medium on the spectral energy distribution of a star are shown in Figure II-1, where the continua of two stars are plotted, normalized to the peak flux in each case. \( \beta \) Ori is a nearby (270 pc) star and is not measurably obscured by interstellar matter. VI Cyg No. 12, in contrast, lies near the center of a heavily extincted stellar association. From the plot of the normalized spectra, it can be seen that the effect of the interstellar dust is to shift the
peak emission toward longer wavelengths; hence the term "reddening". By plotting the observed continua of the two stars on a scale such that both stars are at the distance of the VI Cyg association (500 pc), the effect of an interstellar cloud on the total luminosity of the reddened star can also be demonstrated (see Figure II-2). Note also in Figure II-2 that the continua of the two stars coincide at the longest wavelengths, i.e., the infrared colors of stars are unaffected by interstellar matter.

2. Measurements of Extinction

The wavelength dependence of a reddened star's extincted light can be measured by several independent methods. These methods are described in detail in the literature (Johnson 1968; Dufay 1968; Whiteoak 1966; Miller 1968). Only one method, the so-called color-difference method, applies to this study; it will be described here briefly.

As the name implies, the color-difference method is merely a comparison of the colors of a reddened star with those of an unreddened one. For the unreddened star, the color at any two wavelengths $\lambda_i$ and $\lambda_j$ is given by

$$m_i - m_j = M_i - 5 + 5 \log d - M_j + 5 - 5 \log d = M_i - M_j \quad (1)$$
where \( m_{i,j} \) = apparent magnitude at \( \lambda_{i,j} \)

\[ m_{i,j} = M_{i,j} = \text{absolute magnitude at } \lambda_{i,j} \]

and \( d \) = distance to the star in parsecs.

For the reddened star, the apparent color differs from its intrinsic (unreddened) color if the extincting medium scatters and absorbs light by an amount that depends upon the wavelength. Denoting the amount of extincted light in magnitudes by \( A_{\lambda} \), the apparent color of a star is

\[
m_{i} - m_{j} = M_{i} - 5 + 5 \log d + A_{i} - M_{j} + 5 - 5 \log d - A_{j}
= M_{i} - M_{j} + A_{i} - A_{j}.
\]

The color difference (color excess) is

\[
E_{i-j} = A_{i} - A_{j} = (m_{i} - m_{j}) - (M_{i} - M_{j})
\]

The intrinsic colors (= \( M_{i} - M_{j} \)) of a star of a given effective temperature and luminosity can be obtained by measuring stars which are not greatly affected by the interstellar medium because they are nearby or at high galactic latitudes, or in a region of space in which obscuring matter is sparse (e.g., between the spiral arms of the Milky Way). Johnson's (1966) average of the colors
of many (~200) bright stars, which were known or assumed
to be unreddened, provide the intrinsic colors for the
stars studied here.

Measurements of the colors of unreddened stars and
of a reddened star, along with the definition of color
excess, yield the wavelength dependence of interstellar
extinction, but not the total extinction at any given
wavelength. The total extinction can be obtained gra¬
phically, however. The convention is to plot $A_{\lambda}$ vs $1/\lambda$
(the so-called Whitford curve [Whitford 1958]) with an
arbitrary ordinate, and to determine the true zero of the
ordinate scale by extrapolation of the curve to $1/\lambda = 0$
and a physical assumption about the particles. Specifically,
this assumption is that there is no extinction at infinitely
long wavelengths, due to the finite size of interstellar
dust particles. The extinction curve for VI Cyg No. 12,
plotted in the Whitford manner, is shown in Figure II-3.

If the interstellar medium produces extinction which
is linearly dependent on the number density of dust parti-
cles, then extinction curves of different stars can be com-
pared by normalizing them to some color excess. The usual
practice is to plot $A_{\lambda}/E_{B-V}$, where $B$ and $V$ refer to
the UBVRI photometric system, defined by Johnson (1966).
(However, since so many of the stars studied in this paper
have peculiar emission spectra which contaminate the blue (B) filter measurements, the normalization to $E_{V-R}$ is used throughout. The difference between the two normalizations is negligible, since for typical interstellar dust, $E_{B-V} \approx E_{V-R}$. If the ordinate scale is set so that $A_V = 0$, then normalization to $E_{B-V}$ implies that the extinction curve intersects the vertical axis at $1/\lambda = 0$ at the point $A_V/E_{B-V} = R$, the ratio of total to selective absorption. Differences in the value of $R$ among extinction curves have been interpreted as differences in the chemical composition or size of the interstellar dust over the Milky Way (Sharpless 1963; Johnson 1965; Lee 1968). Alternatively, the non-normal extinction curves might imply local reddening and the presence of circumstellar dust. The following examination of the effects of circumstellar dust on the infrared continuum of a star supports the latter hypothesis.

B. Effects of Circumstellar Dust on Stellar Continua. Reradiation.

Unlike the interstellar case, circumstellar dust interacts with a radiation field dominated by one star only. Consequently, circumstellar dust absorbs and scatters incident stellar radiation with far different effects on the stellar continuum than interstellar dust produces. For
instance, there are no observable effects of circumstellar light scattering if (1) the circumstellar envelope is spherically symmetric; (2) the star plus envelope are contained within the field of view; (3) the particle density is low enough so that multiple scattering is negligible; and (4) circumstellar grains, like interstellar grains, are strongly forward scattering (Wickramasinghe 1968). In so far as all these conditions are satisfied, the scattering process in circumstellar dust envelopes does not affect the total luminosity of the infrared star, nor does it redden the central star's continuum. The circumstellar absorption process, like circumstellar scattering, has no effect on the total luminosity as long as the infrared star is observed over the entire wavelength range in which the star and the dust emit energy. However, the absorption process is an observable effect; the dust's re-emission of absorbed starlight at infrared wavelengths reddens the observed continuum.

It is expected that circumstellar extinction curves differ from interstellar extinction curves in two respects, corresponding to the peculiar effects of circumstellar dust on stellar continua as described above. First, the wavelength dependence of the scattering process is not observed, since scattering cannot be observed; circumstellar
extinction curves are, therefore, flatter than interstellar extinction curves at visual wavelengths. Second, the reddening of the stellar continuum by means of the dust's thermal reradiation produces a steep rise in the circumstellar extinction curve toward infinite wavelengths. Unlike the case of interstellar extinction, where the infrared colors of stars are unaffected by dust, and the extinction curve approaches zero slope at infinite wavelength (cf. Figure II-3), circumstellar dust strongly reddens a star's infrared colors, forcing its extinction curve toward infinite slope at infinite wavelength. Only at very long wavelengths, beyond which the circumstellar dust does not radiate appreciably, will the circumstellar extinction curve approach zero slope.

If the dust model is the correct interpretation of the infrared star phenomenon, the circumstellar extinction curves should be flatter at short wavelengths and steeper at long wavelengths than interstellar extinction curves. The expected long-wavelength rise in the extinction curves of infrared stars is merely a consequence of infrared emission, and is, therefore, a result of any model of infrared stars.
C. Selection Criteria for Infrared Stars

The infrared stars studied in this paper have, for the most part, been selected from reddened stars and intrinsically cool stars by means of the characteristic abrupt change in their extinction curves toward infinite slope at red and near infrared wavelengths. It must be emphasized that the basis for selection of infrared stars is the slope of the extinction curve at long wavelengths, and not the value of the point $R$. However, since even the infrared-radiating dust will not add measurably to the stellar continuum at infinite wavelengths, the slope of the extinction curve does approach zero eventually. Hotter envelopes show this effect at shorter (infrared) wavelengths. Bearing these limits of the test in mind, it might be said that the selection of infrared stars is really based on the color excess gradient in the range $0.7\mu \leq \lambda \leq 10\mu$. As such, the test is completely equivalent to the color-color plots which have been used by others (Johnson 1967a; Mendoza V. 1968a; Woolf and Ney 1969) to select infrared stars.

There are two advantages to be gained by the extinction curve test. The first is practical: several color-color plots must be drawn to show all the information on an extinction curve. The other advantage is aesthetic:
extinction curves contaminated by infrared reradiation from circumstellar envelopes are physically more transparent than the color-color plots, so to speak. The extinction curve test suffers the disadvantage, however, of requiring knowledge of a star's intrinsic energy distribution (by way of its spectral classification) in order that it can be determined whether or not the star has an infrared excess. In those cases where no spectral class has been assigned, selection of stars must be made by means of color-color plots.

Mendoza V. (1968a) has listed many of the infrared stars found by means of color-color plots. A comparison of his list with the present one (Section III) shows that no new stars have been found from the same photometric data that Mendoza V. used, but some are not included in the present list. As a check on the consistency of the two tests, it is noteworthy that VI Cyg No. 12, a star with an extinction curve resembling the curves produced by typical interstellar dust, does not appear to be an infrared star on Mendoza V.'s color-color plot.
III. CATALOGUE OF INFRARED STARS

Table 2a lists all the infrared stars found by applying the selection criteria outlined in Section II to stars whose energy distributions have been measured over a wide range of wavelengths (≈ 0.3μ to ≧ 3.5μ) and whose spectral classifications have been published. All the measurements used here are wide-band photometric measurements, made with the UBVRIJHKLMNQ filter system defined by Johnson (1965, 1966) and Low et al. (1969). Along with the stars, the list includes the sources of the photometric data. The spectral types were generally supplied by the same sources, except for the T Tauri stars. Their spectral types were taken from Herbig (1962, 1966a, 1966b). Where the line spectrum of T Tauri object made it impossible to assign a unique spectral classification, it has been assumed that the star's intrinsic colors approximated a G V (i.e., solar-type) star. Statistically, this assumption is well-founded, since almost all T Tauri stars are late-type dwarfs (Kuhi 1966). Besides the T Tauri stars, there are other stars for which the spectral class and/or intrinsic continuum have not been determined. For each of these stars an (unreddened) comparison star or spectral type, also listed in Table 2, provides the intrinsic colors.
The Whitford-type extinction curves by which these infrared stars were selected are plotted in Figures III-1. These extinction curves may be compared (cf. Figure II-3) to a typical curve for a star reddened by interstellar, rather than circumstellar matter. Note the abrupt infrared increase in slope which distinguishes the infrared stars.

Stars with infrared excesses which could not be detected by means of their extinction curve characteristics (due to lack of knowledge of the intrinsic continua) are listed in Table 2b. These stars were selected by means of color-color plots.

There is a class of infrared stars for which extinction curves and color-color plots do not provide good tests for the presence of infrared excess radiation. These are the red giants, which are thought to emit excess energy in the region $9\mu \leq \lambda \leq 12\mu$ (Gillett, Low and Stein 1968; Woolf and Ney 1969). Since the intrinsic energy distributions of these stars are not well known at such long wavelengths, the infrared excess that was detected is an excess over the black-body curve which fits shorter-wavelength data. The amount of excess radiation is not yet determined, since the pertinent data are narrow-band filter wheel measurements for which the absolute calibration
remains uncertain (F. J. Low, private communication). The stars suspected of emitting infrared excess energy on the basis of the present calibration (Woolf and Ney 1969) are listed in Table 2c.

The detection of far infrared excesses in red giants by means of narrow band measurements implies that other classes of infrared stars may exist which cannot be distinguished by means of extinction curves or color-color plots. Consequently, it is expected that the present cataloguing of infrared stars will soon be outdated.
IV. APPLICATION OF THE DUST MODEL TO INFRARED STARS

Several measurable physical parameters of infrared stars may be computed from the application of a dust model to them. The present dust model assumes that the infrared excess radiation is produced by black particles which form into a thin spherically symmetric shell around a star. The particles absorb the ultraviolet and optical wavelength energy radiated by the central star and thermally reradiate the energy at infrared wavelengths corresponding to the physical temperature of the dust. Using this model, the distribution and flux level of the radiation emitted by a circumstellar shell can be separated from the continuum of the observed system. The distribution of shell radiation gives the temperature of the dust. From the shell temperature and the total circumstellar flux, the angular size of the shell can be determined. The sizes of the shells are of particular interest here, since the computed angular diameters may be compared directly to measured ones to test the validity of this dust model and the uniform application of it to all the infrared stars.
A. Separation of the Shell's Radiation from the Observed Continuum

If the intrinsic continuum emission from the central star in an "infrared star" were known, then the emission from the shell alone could be obtained by taking the difference between the intrinsic flux and the observed flux in the infrared. The intrinsic fluxes can be determined by scaling the energy distribution for stars of the correct spectral type to give the observed luminosity. In this section, the details of the calculation, as it was applied to the stars in Table 1, are outlined.

According to the previous discussion (Section II), the short wavelength energy absorbed by circumstellar dust is reradiated at infrared wavelengths, so that the observed luminosity of the star plus circumstellar shell equals the luminosity of the star alone. The total flux from each infrared star has been obtained by a numerical integration under the smooth curve defined by the UBVRIJHKLMNQ photometric measurements, plus extrapolations to zero flux outside the measured wavelength range. The fluxes obtained in this way are given in Table 3, along with the total luminosities for stars whose distances are known. Some stars, notably the CIT objects and some OB stars with small infrared excesses, have not been measured in a wave-
length range large enough to permit accurate computation of the total flux. These stars are, therefore, not included in the Table.

Having found the total luminosity of the central star, its intrinsic energy distribution can be obtained if (1) the absorption line spectrum of the central star permits definite assignment of a spectral classification; and (2) the energy distribution for unreddened stars of that class is known. These restrictions severely limit the number of stars in Table 1 to which the dust model can be applied. Thus the carbon stars and the planetary nebulae have been rejected from the study, even though color-color plots indicate infrared excesses for many of them, on the basis that their intrinsic energy distributions are not well known. Although the spectral classifications and intrinsic energy distributions of Becklin's star, HD 45677 (I.R.), and R CrA are unknown, these stars were not excluded from the study, since they are so luminous in the infrared that it can be assumed that all the observed energy originates in the shell, and no separation of stellar from shell flux is necessary. The observed continua of the infrared stars to which the dust model is applicable are plotted in Figures IV-1.
For the remaining infrared stars, the intrinsic energy distributions have been taken, for the most part, from Johnson's (1966) tables of intrinsic colors for stars of luminosity classes I, III and V and effective temperatures from ~ 3000°K to ~ 25000°K. Infrared stars with standard spectral types which are not included in Johnson's tables are compared to other unreddened stars of similar spectral type. The comparison spectral type or star is listed in Table 2.

With suitable comparison spectra selected, the intrinsic colors are converted into fluxes normalized to the peak flux. The flux level of intrinsic stellar radiation can then be computed from the condition that the observed flux equals the intrinsic total flux, so that

\[ \int F_{\text{observed}}(\lambda) \, d\lambda = F_\star(\lambda_{\text{max}}) \int \left( \frac{F_\lambda}{F_{\lambda_{\text{max}}}} \right)_\star \, d\lambda, \quad (4) \]

where the integral over the normalized flux from the comparison star, \( \int \left( \frac{F_\lambda}{F_{\lambda_{\text{max}}}} \right)_\star \, d\lambda \), has been performed numerically, in the same way that the integration was done for the observed flux. An example of the resulting intrinsic stellar continuum along with the observed infrared star is plotted in Figure IV-2, for VY CMa and its comparison star, α Her.
The radiation from the shell can now be obtained by subtracting the stellar continuum from the observed flux where the latter exceeds the former. The result is shown in Figure IV-3 for all the stars studied here. Note that this procedure warrants some caution. The observed flux at any wavelength is the sum of the transmitted starlight plus the reradiated light from the shell. The relative contribution of these two sources to the observed continuum varies with wavelength according to the following scheme: At short wavelengths, the reradiation by the dust is negligible and all the observed energy is produced by the central star. At long wavelengths, the stellar radiation is negligible and the dust reradiation accounts for the observed flux level. At intermediate wavelengths, an indeterminate amount of the flux is produced in the star and in the shell. Therefore, the maximum information that is available from this part of the curve is a lower limit to the amount of energy emitted by the circumstellar shell.

Summarizing the procedures described in this section, the shell continuum radiation has been obtained for all the stars in Table 3 by subtracting the intrinsic stellar radiation from the observed continuum. According to the previous discussion, only a lower limit to the actual shell emission at any wavelength can be determined.
However, inspection of Figure IV-2 shows that the uncertainty in the reradiation from the shell is considerable only for short wavelengths, around 1μ in the case of VY CMa, and that at longer wavelengths the error caused by the unknown contribution of stellar radiation is negligible.

B. The Temperatures and Sizes of the Circumstellar Shells

According to the dust model presented here, the infrared reradiation takes place in a thin shell of dust at a distance from the central star which is large enough so that the temperature gradient is negligible across the thickness of the shell. The characteristic temperature of the shell is indicated by the Planck curve which best fits the shell continuum radiation. The curve-fitting process is more accurate than the uncertainties in the data allow, however, so the temperatures of the stars studied here have been obtained from the position of the peak flux according to Wien's displacement law,

\[ \lambda_{\text{max}} T = \text{const} = .29 \text{ cm}^\circ \text{K} \].

These temperatures are listed in Table 3.

The angular sizes of all the shells have been computed by several methods which are not entirely independent. Three limiting cases are considered: (1) the shell is optically thick at all wavelengths; (2) the shell
is optically thick at the peak wavelength, i.e.,
\[ \varepsilon_{\lambda_{\text{max}}} = 1 \] ; and (3) the shell is optically thin and gray (\( \varepsilon = 0.5 \)) and the particles reradiate equally in all directions (if such is not the case, then the computed sizes are too small by a factor of \( \sqrt{2} \)).

In case 1, the circumstellar shell radiates energy like a perfect black body at the characteristic temperature of the shell. No starlight can be seen. Examples for which this limiting case is probably the most accurate description are Becklin's Object and HD 45677 (I.R.).

Balancing the observed total flux with the total energy radiated by a black body at the shell's temperature gives the size of the shell:

\[
4\pi d^2 \int F_{\text{shell}} \, d\lambda = 4\pi r^2 \sigma T_{\text{shell}}^4
\]
or
\[
\left( \frac{\theta}{2} \right)^2 = \frac{\int F_{\text{shell}} \, d\lambda}{\sigma T_{\text{shell}}^4}
\]

where \( \theta = \) angular diameter of the shell,
\( d = \) distance to the star in parsecs, and
\( r = \) radius of the shell.
If the flux is given in units of $10^{-15}$ W/cm$^2$, the temperature in 100°K, and $\theta$ in seconds of arc, the expression may be simplified:

$$\theta \approx 0.548 \frac{\sqrt{F}}{T^2}.$$

The corresponding linear radius is given in astronomical units by

$$r = 0.5 \theta d$$

when $\theta$ is given in seconds of arc and $d$ is given in parsecs.

For case 2, the size of the disk which emits the peak flux in the shell continuum has been computed, assuming that the particles have unit emissivity at the peak wavelength. A black body at the shell temperature would emit the observed peak flux if its angular size were

$$\left(\frac{\theta}{2}\right)^2 = \frac{F_{\lambda_{\text{max}}}}{\pi I_{\lambda_{\text{max}}, T}}$$

(6)

where

$$I_{\lambda_{\text{max}}, T} = \frac{2hc^2}{5 e^{hc/k_{\lambda_{\text{max}}}T} - 1}.$$  

(7)
With $F_{\lambda_{\text{max}}}$ in units of $10^{-15}$ W/cm$^2$Å, T in 100°K
and $\lambda_{\text{max}}$ in Å, $\theta$ is given in seconds of arc by

$$\theta \approx 8.07 \times 10^{-4} \sqrt{F_{\lambda_{\text{max}}}}.$$  

Finally, the sizes of thin gray shells are computed. In this case, the observed luminosity is the sum of the gray-body luminosity of the shell and the transmitted stellar energy:

$$\int F_{\text{observed}} (\lambda) \, d\lambda = \frac{\varepsilon A_{\text{shell}} \sigma T^4}{4\pi d^2} + \frac{(1-\varepsilon) A_*}{4\pi d^2} \int F_*(\lambda) \, d\lambda$$

where $A = \text{total surface area}$. 

Separating the observed flux into its components, the flux from the shell, and the transmitted emission from the central star, the preceding relation can be simplified to obtain:

$$\int F_{\text{shell,obs.}} (\lambda) \, d\lambda + (1-\varepsilon) \int F_{*,\text{obs.}} (\lambda) \, d\lambda$$

$$= \frac{\varepsilon A_{\text{shell}} \sigma T^4}{4\pi d^2} + \frac{(1-\varepsilon) A_*}{4\pi d^2} \int F_*(\lambda) \, d\lambda$$
or

\[ \int F_{\text{shell,obs.}}(\lambda) \, d\lambda = \frac{\varepsilon A_{\text{shell}} \sigma T^4}{4\pi d^2} = \varepsilon \left(\frac{\theta}{2}\right)^2 \sigma T^4. \]  

(8)

However, it was shown previously, that the method of separation of the shell's radiation from the observed continuum gives only a lower limit to the total energy emitted by the shell. The upper limit to the shell radiation is given by the observed total energy. Therefore, the limits to the sizes of the shell are:

\[ \frac{2}{T^2} \sqrt{\int \frac{F_{\text{obs.}}(\lambda) \, d\lambda}{\varepsilon \sigma}} > \theta > \frac{2}{T^2} \sqrt{\int \frac{F_{\text{shell,obs.}}(\lambda) \, d\lambda}{\varepsilon \sigma}}. \]

Assuming that \( \varepsilon = 0.5 \), the lower limit is just a factor of \( \sqrt{2} \) larger than the thick shell size obtained in case 1. Both limits have been computed and listed in Table 3.

C. Estimated Errors in the Results

Besides the assumptions explicitly stated in the description of the dust model, e.g., spherical symmetry, thin-shell geometry, etc., further idealizations of the physical situation are required. Since ideal conditions do not (by definition!) exist, systematic errors are
introduced into the results of the computations. These are listed below, roughly in order of magnitude.

1. **Neglect of Interstellar Reddening**

   It has been assumed throughout the computations that the observed flux represents the total output from the star and its shell. However, the presence of any interstellar dust makes this an unwarranted assumption, since some of the energy of the infrared star will be extincted by the interstellar medium. Because the wavelength dependence produced by scattering can only be observed for interstellar reddening, any color excess can be attributed solely to interstellar extinction. Therefore, the effects of interstellar extinction on observed stellar continua can be computed by fitting a VI Cygni extinction curve to the V-R excess of a reddened star and integrating the total flux from the "unreddened" star. Applying this method to RY Tau, one of the most heavily reddened infrared stars studied here, shows that the error in the total luminosity from neglecting interstellar extinction is ~40%, but the error in the shell luminosity is negligible. The result is that the uncertainty in the upper limit of the size of the shell for the optically thin case is ~-20%. Note that this number gives only an
estimate of the order of magnitude of the largest errors expected when interstellar extinction is not taken into account.

2. **Peculiar Continua**

Spectroscopic studies of many of the stars studied here indicate that the stars have peculiar intrinsic continua. That is to say, these stars would not have colors resembling those of any normal star with a standard spectral type. Peculiar continua introduce errors into the computations of the shell sizes by causing miscalculation of the intrinsic stellar energy distribution, and, therefore, the shell energy distribution. The sizes of these errors cannot be estimated until further observational data determine the intrinsic energy distribution of the peculiar stars.

3. **Bright Lines**

In addition to the wide-band peculiarities in some stars' continua, intense emission lines contaminate the photometric measurements, so that some stars appear to have peculiar continua. Emission lines cause uncertainty in the calculation of stellar radii through the integration of the observed flux. The U and B filters are most commonly the line-contaminated measurements. Smak (1964)
and Aveni (1966) have made estimates of the true U and B continuum points for the T Tauri stars. Kuhi (1969) has also considered the problem for T Tauri stars; he estimates that the contamination of the R filter measurements by H\(\alpha\) emission forces an underestimate of the flux integral by \(\sim 10\%\). Although the errors introduced by line effects in the photometric measurements have only been quantitatively considered for T Tauri stars, there are uncertainties due to emission lines in the calculated physical parameters for all the emission stars.
V. DISCUSSION

A. Temperatures of the Shells

In the sample of 36 infrared stars studied here, there is a range in shell temperatures from 500°K to 3600°K. There appears to be some clustering among these computed temperatures. The clustering effect is, however, merely apparent and not real. It is the result of the wavelength spacing of the measurements; that is, the temperature clustering corresponds to the response functions of the photometer filter.

The stars whose shell temperatures are higher than 3000°K are η Tau, ζ Per, κ Dra, and all of the Sword, Belt and Trapezium Orion stars. These are all OB stars, and an interpretation of this result might be that the hottest stars have the hottest shells. However, Herschel 36 (07), υ Sgr (B6), and NGC 6530/65 (BO IV) all have temperatures below 2000°K, indicating that there is no such general rule about hot stars. These stars seem to defy application of the dust model to them, since it is implausible that dust exists in equilibrium at temperatures as hot as 3000°K. It has been suggested (Lee 1968) that the infrared excesses in these stars are not the result of circumstellar reddening and reradiation at all, but
the result of an unusual interstellar extinction law. Specifically, Lee proposes that $R > 3$, possibly as large as 5.5, in some regions of Orion. The Trapezium region, for example, would suffer "non-normal" ($R > 3$) extinction, but stars just outside, like : Ori, would be extinguished by "normal" ($R \sim 3$) interstellar dust. It seems unusual that the interstellar medium changes so abruptly in a region of only $\sim 0.05$ sq. degrees. Note also that $\epsilon$ Per emits "hot" excess infrared radiation, and other stars in its vicinity do not have peculiar extinction curves. These considerations support the argument that the infrared excess is caused by emission. The emission mechanism may, however, not be thermal reradiation by circumstellar dust particles. Definite evidence barring either the possibility of non-normal interstellar extinction or the possibility of an emitting dust shell does not exist at present.

The values of the shell temperatures found for the stars studied are also of interest. Note that no shell temperature is low enough to admit the possibility that circumstellar dust is composed mostly of ice particles. However, almost all the stars measured at 22$\mu$ exhibit excess far infrared radiation, more than the emission expected from one shell. It has been suggested (Low et al.,
1969) that the $22\mu$ excess indicates the presence of an additional, colder shell where ice might radiate the observed flux. Assuming that infrared stars are preplanetary systems, the entire infrared system, star plus warm shell plus very cold shell, might be compared to the sun, the earth or Mars, and Jupiter. Like the observed infrared stars, the earth might have formed out of the refractory materials which made up a warm shell around the Sun early in its evolutionary history, and Jupiter may have evolved from an icy shell.

B. Sizes of the Shells

The sizes of the circumstellar dust shells have been computed by three methods. In Case 1, it was assumed that the shell radiates like a black body at all wavelengths. Next, it was assumed that the shell radiates like a black body at its peak wavelength (Case 2). Finally, the sizes of the shells were computed on the basis that the shell emits energy like a gray body of emissivity $\epsilon = 0.5$. Upper and lower limits have been computed for the gray body case (Case 3), corresponding to the upper limit of the shell luminosity (total energy in the system) and its lower limit (total energy of the separated shell radiation). Any disagreement between the sizes computed
by all these methods can be explained in terms of the methods used to derive the physical parameters of the shells, and the properties of the dust.

It might at first appear as if the sizes computed for Cases 1 and 2 should agree automatically, since both radii pertain to black body radiation. Inspection of the computed sizes shows, however, that these sizes do not agree for each star, and that the sense of the discrepancy is not the same for all stars. Such effects can be understood using the following argument. The equivalence of Cases 1 and 2, and, hence, the agreement of the corresponding radii, can be expected only if the integral of Equation 6 gives Equation 5. Two conditions must be met in order to perform the integration. First, it must be assumed that the emissivity is constant and equal to unity for all wavelengths, so that the size of a shell is independent of the wavelength at which it is observed. Then

\[ \int_0^\infty \left( \frac{\theta}{2} \right)^2 \pi I_{\lambda,T} \, d\lambda = \left( \frac{\theta}{2} \right)^2 \int_0^\infty \pi I_{\lambda,T} \, d\lambda = \left( \frac{\theta}{2} \right)^2 \sigma T_{\text{shell}}^4. \]

If this assumption is not valid, then the shell sizes computed for Cases 1 and 2 will differ, in the sense that Case 1 sizes will be smaller (assuming the emissivity
peaks at the shell's peak wavelength). The second condition that must be met to equate Case 1 and Case 2 sizes is that the total energy emitted by the shell in all wavelengths must be equal to the derived total shell energy. According to the discussion in Section IV-A, if the shell transmits any starlight, then only a lower limit to the total shell energy is available; consequently, it is expected that Case 1 sizes should be smaller than Case 2 sizes by an amount given by

$$\sqrt{\frac{\text{Derived Total Shell Energy}}{\text{True Total Shell Energy}}}$$

It is evident, therefore, that Case 1 is equivalent to Case 2 only if the following two independent conditions can be satisfied: (1) the emissivity of the shell is independent of wavelength, and (2) the shell's derived total energy is its true total energy. If either condition is not met, the Case 1 size should be smaller than the Case 2 size. The fact that, for five stars, Case 1 sizes are the larger, reflects the uncertainties in the data and the computations.
The upper and lower limits to the gray body sizes (Case 3) disagree for one reason only, and that is the openness of the shells. The lower limit is less than the upper limit by a factor of

\[ \frac{\sqrt{\text{Derived Total Shell Energy}}}{\sqrt{\text{Total Energy of the Infrared Star}}} \]

This difference is considerable for all the stars but three, Becklin's Object, NML Cyg, and HD 45677 (I.R.). That is, most of the stars do not have closed shells.
VI. CONCLUSION

A dust model similar to the model proposed by Low et al. (1969) has been applied to 36 infrared stars. Several physical parameters of the stars, including total excess flux, temperature of the dust, and size of the dust shell, were computed based on this model and the available photometric data and spectral types of the stars.

The sizes of the dust shells were computed by three different means, corresponding to the conditions of
(1) optically thick, black shells, (2) shells with unit emissivity at the peak flux, and (3) optically thin, gray shells with emissivity ≈ 0.5. For Case 3, upper and lower limits were computed; the disagreement between these figures, for all but a few stars, implies that most circumstellar shells are open, i.e., that starlight penetrates the dust. The sizes of shells computed for Cases 1 and 2 also disagree for many of the stars. The difference between the computed Case 1 and Case 2 sizes is produced by two non-separable effects: the fact that only a lower limit to the shell luminosity is available, and the possibility that the emissivity of the shell is not constant with wavelength.
Both the model used here and the conclusions it leads to can be tested by measurement of the stars' radii. Possible methods of measurement include resolution by a circular aperture, Michelson interferometry, and lunar occultation.

Circular apertures resolve stars as small as $2.44 \frac{\lambda}{(\text{diameter of the telescope objective})}$. With the 200" telescope at Mt. Palomar, this disk is 0.25 at 5μ; with this resolution, it would be possible to measure the diameters of the three largest stars, VY CMa, NML Cyg and R Agr.

Michelson interferometry far surpasses the circular aperture technique, and measurements of stellar diameters at $\sim 0.44\mu$ accurate to within $\pm 0.001$ have been reported by Pease (Hanbury Brown 1968). Since the size of an object completely resolved by interferometer mirrors a distance $D$ apart is $\frac{\lambda}{D}$, Pease's measurements imply that accuracies of $\pm 0.01$ are feasible in the infrared. Comparable accuracies can also be obtained by means of lunar occultation measurements (Hanbury Brown 1968). Nearly half the stars studied here can, in principle, be resolved by these techniques.
## Table 1

Physical Properties of Infrared Sources in the Galaxy

<table>
<thead>
<tr>
<th>Source</th>
<th>$T_c$ (°K)</th>
<th>$L/L_\odot$</th>
<th>$R$/a.u.</th>
<th>$M/M_\odot$</th>
<th>$t$(yrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Becklin's Object</td>
<td>700</td>
<td>$10^{3.1}$</td>
<td>2</td>
<td>3</td>
<td>$10^{3.4}$</td>
</tr>
<tr>
<td>Herschel 36</td>
<td>$\dagger, 5$</td>
<td>$10^{3.5, 6}$</td>
<td>$\dagger, 6$</td>
<td>$10^{-1.7}$</td>
<td>$10^{4.7}$</td>
</tr>
<tr>
<td>$\mu$ Cep</td>
<td>8</td>
<td>$10^{4.\dagger\dagger}$</td>
<td>$10^{2.9}$</td>
<td>$10^{-6.9}$</td>
<td>$10^{9}$</td>
</tr>
<tr>
<td>NGC 7027</td>
<td>10</td>
<td>$10^{3.11}$</td>
<td>$10^{4.10}$</td>
<td>$10^{-1.12}$</td>
<td>$10^{9}-10^{10}$</td>
</tr>
<tr>
<td>Crab Nebula</td>
<td>---</td>
<td>$10^{5.13}$</td>
<td>$10^{6.14}$</td>
<td>15</td>
<td>$10^{3}$</td>
</tr>
<tr>
<td>BNKL Nebula</td>
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<td>$10^{5.1}$</td>
<td>$10^{4.1}$</td>
<td>$10^{2-10^{3.1}}$</td>
<td>$10^{3.4}$</td>
</tr>
<tr>
<td>Trapezium</td>
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<td>$10^{2.15}$</td>
<td>$10^{4.15}$</td>
<td>15</td>
<td>$10^{4.16}$</td>
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<td>Solar System</td>
<td>6300</td>
<td>1</td>
<td>17</td>
<td>$10^{-3.17}$</td>
<td>$10^{9.17}$</td>
</tr>
</tbody>
</table>

$\dagger$ This paper

$\dagger\dagger$ Indicated by spectral type

1 Kleinmann and Low 1967  
2 Low et al., 1969  
3 Larson 1968  
4 Hartmann 1967  
5 Johnson 1967b  
6 Walker 1957  
7 Woolf 1961  
8 Stein et al., 1969  
9 Gillett, Low and Stein 1968  
10 Krishna Swamy and O'Dell 1968  
11 Gillett, Low and Stein 1967  
12 O'Dell 1963  
13 Becklin and Kleinmann 1968  
14 Trimble 1968  
15 Ney and Allen 1969  
16 Sharpless 1966  
17 Allen 1963
TABLE 2

Catalogue of Infrared Stars

A. **Stars Selected by Extinction Curve Test**

1. **T Tauri Stars**

<table>
<thead>
<tr>
<th>Name</th>
<th>HD or BD</th>
<th>Sp</th>
<th>Comparison</th>
<th>Source</th>
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</thead>
<tbody>
<tr>
<td>RY Tau</td>
<td>283571</td>
<td>dGOe</td>
<td>GO V</td>
<td>15</td>
</tr>
<tr>
<td>T Tau</td>
<td>284419</td>
<td>dG5e</td>
<td>G5 V</td>
<td>12</td>
</tr>
<tr>
<td>DF Tau</td>
<td></td>
<td>dMOe</td>
<td>MO V</td>
<td>15</td>
</tr>
<tr>
<td>SU Aur</td>
<td>282624</td>
<td>G2ne III</td>
<td>G2 III</td>
<td>15</td>
</tr>
<tr>
<td>RW Aur</td>
<td>240764</td>
<td>dG5e</td>
<td>G5 V</td>
<td>15</td>
</tr>
<tr>
<td>CO Ori</td>
<td>+11° 809</td>
<td>Gpeα</td>
<td>G5 V</td>
<td>13</td>
</tr>
<tr>
<td>GW Ori</td>
<td>244138</td>
<td>dK3e</td>
<td>K2 V</td>
<td>15</td>
</tr>
<tr>
<td>EZ Ori</td>
<td></td>
<td>F9:e V</td>
<td>F8 V</td>
<td>13</td>
</tr>
<tr>
<td>P 2305</td>
<td></td>
<td></td>
<td>G5 V</td>
<td>13</td>
</tr>
<tr>
<td>V380 Ori</td>
<td>- 6° 1253</td>
<td>A1:e</td>
<td>AO V</td>
<td>15</td>
</tr>
<tr>
<td>FU Ori</td>
<td></td>
<td></td>
<td>F5 V</td>
<td>15</td>
</tr>
<tr>
<td>R Mon</td>
<td>+ 8° 1427</td>
<td>A I:</td>
<td>F1 V</td>
<td>12</td>
</tr>
<tr>
<td>Lk Hα 120</td>
<td></td>
<td></td>
<td>G5 V</td>
<td>15</td>
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TABLE 2 - Continued

A.

2. OB Stars

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<th>Source</th>
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<tr>
<td>η Per</td>
<td>10516</td>
<td>B2pe</td>
<td>B2 V</td>
<td>6</td>
</tr>
<tr>
<td>u Sgr</td>
<td>18615,6</td>
<td>(B6)</td>
<td>B6 V</td>
<td>11</td>
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<tr>
<td>η Tau</td>
<td>23630</td>
<td>B7 III</td>
<td>B7 V</td>
<td>6</td>
</tr>
<tr>
<td>ε Per</td>
<td>24760</td>
<td>B0.5 Vp</td>
<td>B0.5 V</td>
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</tr>
<tr>
<td>δ Ori</td>
<td>36486</td>
<td>09.5 II</td>
<td>09.5 Ib</td>
<td>6</td>
</tr>
<tr>
<td>θ¹C Ori</td>
<td>37022</td>
<td>06p</td>
<td>05-7 V</td>
<td>6</td>
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<tr>
<td>θ²A Ori</td>
<td>37041</td>
<td>09.5 Vp</td>
<td>09.5 V</td>
<td>9</td>
</tr>
<tr>
<td>θ²B Ori</td>
<td>37042</td>
<td>B0.5 Vp</td>
<td>B1 V</td>
<td>9</td>
</tr>
<tr>
<td>τ Ori</td>
<td>37043</td>
<td>09 III</td>
<td>08-9 V</td>
<td>6</td>
</tr>
<tr>
<td>ε Ori</td>
<td>37128</td>
<td>BO Ia</td>
<td>BO Ia</td>
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<td></td>
<td>37140</td>
<td>(B5)</td>
<td>B5 V</td>
<td>9</td>
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<tr>
<td>ζ Ori</td>
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<td>09.5 Ib</td>
<td>09.5 Ib</td>
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<td></td>
<td>37903</td>
<td>Bl.5 V</td>
<td>B1 V</td>
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<td>M78A</td>
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<td>(B5)</td>
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<td>NGC 2244/5</td>
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<td>04</td>
<td>05-7 V</td>
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<td>α Leo</td>
<td>87901</td>
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<td>B7 V</td>
<td>6</td>
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<tr>
<td>κ Dra</td>
<td>109387</td>
<td>B5 IIIe</td>
<td>B5 V</td>
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<td>NGC 6530/65</td>
<td>164906</td>
<td>BO IV</td>
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<td>-5° 1318</td>
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<td>B2</td>
<td>B V</td>
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<tr>
<td>48 Per</td>
<td>25940</td>
<td>B3 Vp</td>
<td>B3 V</td>
<td>6</td>
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<tr>
<td>Herschel 36</td>
<td></td>
<td>07</td>
<td>05-7 V</td>
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TABLE 2 - Continued

A. 3. Late-Type Stars

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<th>Comparison</th>
<th>Source</th>
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<tbody>
<tr>
<td>VY CMa</td>
<td>58061</td>
<td>M6-7 Ia</td>
<td>α Her</td>
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<tr>
<td>R Scot</td>
<td>173819</td>
<td>GOe Ia-KOop Ib</td>
<td>GO I</td>
<td>6</td>
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<td>RY Sgr 180093</td>
<td>(GO Ib)</td>
<td>GO I</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>S Per  14528</td>
<td>M4e Ia</td>
<td>M3 I</td>
<td>4</td>
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### TABLE 2 - Continued

#### B. Stars Found by Color-Color Plots

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<th>Name</th>
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<th>Source</th>
</tr>
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<tbody>
<tr>
<td>R And</td>
<td>1967</td>
<td>S6,6e</td>
<td>14</td>
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<tr>
<td>R CrA</td>
<td>13027</td>
<td>F0:</td>
<td>15</td>
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<tr>
<td></td>
<td>45677(IR)</td>
<td>M6-7 Ia</td>
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<tr>
<td>Y CVn</td>
<td>110914</td>
<td>C5,4</td>
<td>6</td>
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<tr>
<td></td>
<td>113801</td>
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<td>gM3ep</td>
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<td>R2</td>
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<td>2° 3336</td>
<td>R2</td>
<td>16</td>
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<td></td>
<td>8° 2654</td>
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<td>16</td>
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<td>17° 3325</td>
<td>R0</td>
<td>16</td>
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<td>T Cnc</td>
<td>20° 2243</td>
<td>C4,5</td>
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<td>30° 3639</td>
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<td>T Lyr</td>
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<td>5</td>
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<td>V CrB</td>
<td>40° 2929</td>
<td>C6,2e</td>
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<td>U Cyg</td>
<td>47° 3077</td>
<td>C7,2e - C9,2</td>
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<td>RY Dra</td>
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### C. Stars Selected by Narrow-Band Measurements

<table>
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<th>Sp</th>
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<tr>
<td>o Ceti</td>
<td>14386</td>
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<td>FU Ori</td>
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<td>Flare</td>
<td>A1:e</td>
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<td>.002</td>
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<td>06p</td>
<td>B1 V</td>
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<td>Angular Diameter (&quot;)</td>
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<td>Linear Radius (a.u.)</td>
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<td>BO IV</td>
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<td>.0003</td>
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<td>Linear Radius (a.u.)</td>
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<td>C Per</td>
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**TABLE 3**

- Continued
ACKNOWLEDGMENTS

I am greatly indebted to Dr. Frank J. Low, who proposed the problem and guided this research. Dr. Douglas E. Kleinmann graciously provided his assistance in obtaining much of the data used in this paper.

This work was supported by the National Aeronautics and Space Administration.
REFERENCES


Mendoza V., E. E. 1968a, Pub. Departamento de Astronomia, Universidad de Chile, 7, 106.


FIGURE CAPTIONS

Figure II-1  Spectral energy distributions for β Ori and VI Cyg No. 12, normalized to peak flux. These data are taken from Johnson, Mitchell, Iriarte and Wisniewski (1966) for β Ori, and from Wisniewski, Wing, Spinrad and Johnson (1967) for VI Cyg No. 12.

Figure II-2  Spectral energy distributions for β Ori and VI Cyg No. 12, normalized to the distance of VI Cyg No. 12, i.e., 500 pc.

Figure II-3  The Whitford extinction curve for VI Cyg No. 12, which shows typical interstellar extinction (cf. Johnson 1965). These data were taken from Wisniewski et al. (1967).

Figure III-1  The Whitford extinction curves for:
18. δ Ori, 19. θ¹C Ori, 20. θ²A Ori,
21. θ²B Ori, 22. η Ori, 23. ε Ori,
24. HD 37140, 25. ζ Ori, 26. HD 37903,
27. M78A, 28. NGC 2244/5, 29. NGC 2244/3,
30. α Leo, 31. κ Dra, 32. NGC 6530/65,
33. BD -5° 1318, 34. 48 Per,
35. Herschel 36, 36. VY CMa, 37. R Scuti,
38. RY Sgr, 39. S Per.

Figure IV-1 Observed spectral energy distributions for:
1. RY Tau, 2. T Tau, 3. DF Tau, 4. SU Aur,
5. RW Aur, 6. CO Ori, 7. GW Ori, 8. EZ Ori,
9. P 2305, 10. V380 Ori, 11. FU Ori,
12. R Mon, 13. Lk Hα 120, 14. R CrB,
15. δ Per, 16. υ Sgr, 17. η Tau, 18. θ¹C Ori,
19. θ²B Ori, 20. HD 37140, 21. HD 37903,
22. M78A, 23. NGC 2244/5, 24. κ Dra,
25. NGC 6530/65, 26. BD -5° 1318,
27. Herschel 36, 28. RY Sgr, 29. R Scuti,
30. VY CMa, 31. Becklin's Object, 32. NML Cyg,
33. HD 45677(IR), 34. R Aqr, 35. R CrB,
36. S Per. These data were taken from the
sources listed in Table 2.
Figure IV-2  The observed continuum of VY CMa, and the continuum of a star of similar spectral type, α Her, normalized to the same total energy.

Figure IV-3  The spectral energy distribution of the infrared excess radiation from all the stars listed in Table 3. The numerical labels are the same ones given under Figure IV-1. It was assumed in the calculations that no starlight penetrates the shells around R CrA, Becklin's Object, and HD 45677 (IR). Therefore, the "excess" radiation from these stars is just the observed continuum, given in Figure IV-1. The spectral distribution shown for NML Cyg was obtained by subtracting the reddening effects of the interstellar medium (Low et al., 1969). The excess shown here for R Aqr is the star's excess emission compared to the emission expected from a black body normalized to the 1-3μ flux level.
FIG. II - 3
FIG. III - 1

$A \lambda / E_{V-R}$ vs $1/\lambda (\mu^{-1})$
FIG. III-1

$A_{\lambda}/E_{V-R}$ vs. $1/\lambda (\mu^{-1})$
FIG. III - 1
FIG. III - 1
FIG. III - 1

1/\lambda (\mu^{-1})

A_{\lambda} / E_{V-R}
\[ \frac{A_\lambda}{E_{V-R}} \]

\[ \frac{1}{\lambda} (\mu^{-1}) \]

FIG. III - 1
\[ \frac{A \lambda}{E_{V-R}} \]

\[ \leftarrow \frac{1}{\lambda} (\mu^{-1}) \]

FIG. III - 1
$A_\lambda / E_{V-R}$

$\leftarrow 1/\lambda (\mu^{-1})$

FIG. III-1
FIG. III-1
FIG. III - 1

$A_\lambda / E_{V-R}$ vs. $1/\lambda (\mu^{-1})$
FIG. III-1
\[ \frac{A_\lambda}{E_{V-R}} \]

vs.

\[ \frac{1}{\lambda} (\mu^{-1}) \]

**FIG. III-1**
FIG. III - 1

\[ \begin{array}{c}
\frac{A_\lambda}{E_{V-R}} \\
\hline
-14 & -12 & -10 & -8 & -6 & -4 & -2 & 0 & 2 & 4 \\
\hline
\end{array} \]

\[ \begin{array}{c}
\frac{1}{\lambda} (\mu^{-1}) \\
\hline
2.8 & 2.4 & 2.0 & 1.6 & 1.2 & 0.8 & 0.4 & 0.0 \\
\hline
\end{array} \]

16
FIG. III - 1
FIG. III - 1
FIG. III-1
$A_{\lambda}/E_{V-R}$

$1/\lambda (\mu^{-1})$

FIG. III - 1
FIG. III-1
FIG. III - 1
FIG. III-1
$A_{\lambda}/E_{V-R}$

$\frac{1}{\lambda} (\mu^{-1})$

FIG. III-1
$A\lambda / E_{V-R}$

$1/\lambda (\mu^{-1})$

FIG. III - 1
FIG. III - 1
$\frac{A\lambda}{E_{V-R}}$ vs $1/\lambda (\mu^{-1})$

FIG. III-1
FIG. III-1

Plot of $A/\lambda E_{V-R}$ versus $1/\lambda$ (in $\mu^{-1}$). The graph shows a curve starting from the origin and increasing to the right, with a marked point at $A/\lambda E_{V-R} = 32$. The x-axis represents $1/\lambda$ (in $\mu^{-1}$).
\[ \frac{A\lambda}{E_{v-R}} \]

\[ \sim \frac{1}{\lambda} (\mu^{-1}) \]

**FIG. III-1**
$\frac{A\lambda}{E_{v-R}}$

$\leftarrow \frac{1}{\lambda} (\mu^{-1})$

FIG. III - 1
$A_{\lambda}/E_{\nu-R}$

$\rightarrow 1/\lambda (\mu^{-1})$

FIG. III - 1
FIG. III-1

The graph shows the relationship between $A\lambda / E_{V-R}$ and $1/\lambda$ ($\mu^{-1}$).
FIG. IV-1
FIG. IV - 1
FIG. IV-1
FIG. IV-1
FIG. IV - 1

$F(\lambda)$ (W/cm$^2$$\mu$) vs $\lambda$ ($\mu$)
\[ F(\lambda) \text{ (W/cm}^2 \mu \text{)} \]

\[ \lambda(\mu) \rightarrow \]

\[ \times 10^{-14} \]
\[ \times 10^{-15} \]
\[ \times 10^{-16} \]
\[ \times 10^{-17} \]
\[ \times 10^{-18} \]

FIG. IV-1
$F(\lambda) \ (W/cm^2\mu)$
FIG. IV-1
FIG. IV-1

$F(\lambda) \text{ (W/cm}^2\mu\text{)}$

$10^{-14}$

$10^{-15}$

$10^{-16}$

$10^{-17}$

$10^{-18}$

$\lambda (\mu)$
FIG. IV-1
Figure IV-1

$F(\lambda) (\text{W/cm}^2 \mu)$

$\lambda (\mu) \rightarrow$

$10^{-14}$
$10^{-15}$
$10^{-16}$
$10^{-17}$
$10^{-18}$

UBVRIJHKLMNQ
\[ F(\lambda) (W/cm^2 \mu) \]

FIG. IV-1
$F(\lambda) \ (W/cm^2\mu) -$
FIG. IV-1
FIG. IV-1

\[ F(\lambda) \text{ (W/cm}^2 \mu \text{)} \]

\[ \lambda (\mu) \]

\[ 19 \]
FIG. IV-1

\[ F(\lambda) \, (W/cm^2 \mu) \]

\[ \lambda(\mu) \rightarrow \]

UBVRJHKLMNQ

0.3 0.5 1.0 3.0 5.0 10.0 20.0 30.0

10^{-18} 10^{-17} 10^{-16} 10^{-15} 10^{-14}
FIG. IV - 1
$F(\lambda) \text{ (W/cm}^2\text{m})$

$10^{-12}$

$10^{-13}$

$10^{-14}$

$10^{-15}$

$10^{-16}$

$\lambda(\mu) \rightarrow$

UBVRIJHKLMNQ

0.3 0.5 1.0 3.0 5.0 10.0 20.0 30.0

FIG. IV-1
FIG. IV-1

Graph showing the relationship between $F(\lambda)$ (W/cm$^2\mu$) and $\lambda(\mu)$.

- The graph plots $F(\lambda)$ on the y-axis and $\lambda(\mu)$ on the x-axis.
- The y-axis is labeled with $10^{-18}$ to $10^{-14}$.
- The x-axis is labeled with $\lambda(\mu)$ values from 0.3 to 30.0 with spectral indices U, B, V, R, I, J, H, K, L, M, N, O.
- The data points are connected by a smooth curve.
- Figure IV-1 is referenced at the bottom right of the graph.
FIG. IV-1
$F(\lambda) \ (W/cm^2 \mu)$

$\lambda(\mu)$

FIG. IV - 1
FIG. IV-1
FIG. IV-1
Fig. IV-1

\[ F(\lambda) \text{ (W/cm}^2\text{m}) \]

\[ \lambda(\mu) \rightarrow \]

UBVRIJHKLMNQ

0.3 0.5 1.0 3.0 5.0 10.0 20.0 30.0

10^{-18} - 10^{-14}
$F(X) \left( \text{W/cm}^2 \right)$

$10^{-12}$

$10^{-13}$

$10^{-14}$

$10^{-15}$

$10^{-16}$

$\lambda(\mu)$

FIG. IV - 1
\[ F(\lambda) \left( \text{W/cm}^2 \mu \right) \]

**FIG. IV-1**
$F(\lambda) \, (W/cm^2\mu)\,$

$10^{-12}$

$10^{-13}$

$10^{-14}$

$10^{-15}$

$10^{-16}$

$\lambda (\mu) \, \rightarrow$

UBVRIJHKLMNQ

0.3 0.5 1.0 3.0 5.0 10.0 20.0 30.0

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FIG. IV-1
FIG. IV-1
FIG. IV-1
FIG. 9-2

$F_{\lambda} \times 10^{-15}$ W/cm$^2$-m$^{-2}$

$\lambda$ ($\mu$m)

$\alpha$ Her

VY CMa
FIG. IV-3
FIG. IV-3
$F(X) \ (W/cm^2/t)$

FIG. IV-3
FIG. IV-3
FIG. IV-3
$F(X) \ (W/cm^2_u)$

$FIG. \ nr-3$

$F(\lambda) \ (W/cm^2_\mu)$

$\lambda(\mu) \rightarrow$

$10^{-14}$ $10^{-15}$

$10^{-16}$ $10^{-17}$ $10^{-18}$

$UBVRIJHKLMNQ$

$0.3 \ 0.5 \ 1.0 \ 3.0 \ 5.0 \ 10.0 \ 20.0 \ 30.0$

$FIG. \ \textsc{iv}-3$
FIG. IV-3
FIG. IV-3
FIG. IV-3
$F(\lambda) \text{ (W/cm}^2 \mu \text{)}$

$\lambda(\mu) \rightarrow$

FIG. IV-3
$F(\lambda) \text{ (W/cm}^2 \text{)}$
FIG. IV-3
FIG. IV - 3
$F(\lambda) \ (W/cm^2 \mu)$

FIG. IV - 3
FIG. IV-3
$F(\lambda) \text{ (W/cm}^2\mu \text{)}$

$\lambda(\mu) \rightarrow$

FIG. IV-3
FIG. IV-3

- Diagram showing the relationship between $F(\lambda)$ (W/cm$^2\mu$) and $\lambda(\mu)$.

- Axes: $x$-axis represents $\lambda(\mu)$, ranging from 0.3 to 30.0.
- $y$-axis represents $F(\lambda)$, ranging from $10^{-18}$ to $10^{-14}$.

- The graph includes data points labeled with the number 26.

- The data points are plotted along the curve, indicating the variation of $F(\lambda)$ with $\lambda(\mu)$.
FIG. IV-3
FIG. IV-3

$F(\lambda) \ (W/cm^2 \mu)$

$10^{-14}$

$10^{-15}$

$10^{-16}$

$10^{-17}$

$10^{-18}$

$\lambda (\mu)$

UBVRJHKLMNQ

0.3 0.5 1.0 3.0 5.0 10.0 20.0 30.0

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FIG. IV - 3

$F(\lambda) \left( \text{W/cm}^2 \mu \right)$ vs $\lambda (\mu)$

$10^{-14}$

$10^{-15}$

$10^{-16}$

$10^{-17}$

$10^{-18}$

$0.3$ $0.5$ $1.0$ $3.0$ $5.0$ $10.0$ $20.0$ $30.0$

UBVRIJHKLNMQ
F(\lambda) \; (W/cm^2\mu)
FIG. IV-3
$F(\lambda) \ (W/cm^2\mu)$