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Measurements of the Trapped Flux
in a Long Hollow Superconducting Cylinder

by

Todd Iversen Smith

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Harold E. Reuschke

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Abstract

The recent discovery of quantized flux appeared to explain the extremely long lifetime of persistent currents in superconductors. The existence of the flux quantum indicated that the energy of a superconductor in a current carrying state is a local minimum with respect to current variations if the flux is an integer number of flux quanta. The stability is thus explained as a greatly reduced transition probability for the large current changes needed to lower the energy. Bloch and Rorschach have investigated the energetic stability of persistent currents in a long hollow superconducting cylinder, using the charged Bose gas model proposed by Schafroth. The criterion for stability was that the energy of the system could not be lowered by single particle transitions. They showed that the maximum stable magnetic field which could be trapped inside the cylinder was a function of the externally applied field and the sample dimensions, and could be less than the critical field $H_c$. In contrast to these results one would predict from Maxwell's equations, together with the infinite conductivity of a superconductor, that it should be possible to trap and maintain any field inside the cylinder as long as both the internal and external fields are smaller than
the critical field.

Measurements have been made of the maximum stable field inside a hollow tin cylinder (wall thickness $d = 1.5 \text{ mm}$, inner radius $r = 7.5 \text{ mm}$, length $L = 12.5 \text{ mm}$) as a function of the externally applied field. The results seem to be in qualitative agreement with the Bose gas theory, in that the maximum internal field is a function of the external field. However, the theory predicts that when the external field is zero the maximum internal field should be $d/r H_c = 0.2 H_c$, while the experimentally determined value is $0.6 H_c$. It is possible that these results are due to end effects, as the field at the cylinder walls is greater at the ends than at the center. The rate of decay of the internal field when the external field is changed indicates that these end effects may be important.
I Introduction

Until recently the apparently infinite lifetime of persistent currents in superconductors has been a puzzle. For every current-carrying state, there appeared to be a nearby state with an infinitesimally smaller current and a lower energy. If this were the case, one would expect the currents to decay due to the usual electron-phonon interaction. The decay might be very slow, but if the transition to a lower current is energetically possible, the decay should occur at a rate observable in the laboratory. The discovery of quantized flux appeared to solve the stability problem. The existence of the flux quantum suggested that the energy of a system would be a local minimum with respect to current variations if the flux were an integer number of flux quanta. Therefore the system cannot lose energy in infinitesimal amounts but must do so in finite steps, leading to a much reduced transition probability and very long lifetimes of the persistent currents.

Bloch and Rorschach have investigated the energetic stability of persistent currents in a long hollow superconducting cylinder, using the charged Bose gas model proposed by Schafroth. The criterion for stability was that the energy of the system could not be lowered by single
particle transitions. With this criterion they showed that the maximum stable magnetic field which could be trapped inside the cylinder was a function of the externally applied field and the sample dimensions. In particular, if \( d \) is the wall thickness and \( r \) the inner radius of the cylinder, then if \( d \ll r \) the maximum stable trapped field when the external field is zero is \((d/r)H_0\).

In contrast to these results one would predict from Maxwell's equations together with the infinite conductivity of a superconductor that it should be possible to trap and maintain any field inside the cylinder as long as both internal and external fields are smaller than the critical field.

Measurements taken of the maximum stable field inside a hollow tin cylinder \((d = 1.5 \text{ mm}, r = 7.5 \text{ mm}, \text{length} = 12.5 \text{ cm})\) as a function of the externally applied field are discussed in Section V. The results seem to be in qualitative agreement with the Bose gas theory. However because of the finite length of the cylinder the results could be due to end effects instead of the more fundamental reason suggested by the theory. In the appendix both this problem and a possible method of eliminating it will be discussed more fully.
II Theory

Inside a perfect conductor the electric field is identically zero. It follows immediately that the flux through any contour is a constant, as long as all portions of the contour are in the perfectly conducting body. If we denote the contour by $c$ and the area bounded by the contour by $a$ then we have

$$\oint \mathbf{E} \cdot d\mathbf{s} = \oint \nabla \times \mathbf{E} \cdot d\mathbf{a} = 0 = \frac{1}{c} \int_a \frac{d\Phi}{dt} \cdot d\mathbf{a} = \frac{1}{c} \frac{d\Phi}{dt}$$

therefore $\Phi_a = \text{const.}$

where $\Phi_a$ is the flux through the contour.

If a superconductor is a perfect conductor, then the above argument shows that the field inside a long hollow superconducting cylinder should be constant, since the field is uniform and proportional to the flux.

Thus, one would expect that the maximum stable field which can exist inside a long hollow cylinder as a function of a longitudinal external field would be as shown in figure 1. As long as $|H_e| \geq H_c$ the internal and external fields will be equal, since the material is in the normal state. However, for $|H_e| \leq H_c$ one would expect to be able to trap a field equal to $H_c$ inside the cylinder, since for any field less than $H_c$ the cylinder is superconducting. If, as in
figure 1, we plot $H_i \ vs \ H_e$ we find that the region of stability for a perfect conductor (range of possible internal fields ($H_e$) as a function of the external field) is a square with corners at $H_i = \pm H_e$, $H_e = \pm H_e$.

However, a closer look at the question of the energetic stability of the persistent currents necessary to maintain various fields inside the cylinder, shows that the conclusions reached above cannot be strictly true. In the first place, it is obvious that if any currents are flowing, the energy of the system is higher than if there were no currents. In this sense all persistent currents are at best only in metastable equilibrium. However, the discontinuous decay of the current would require a simultaneous transition of an extremely large number of electrons, and for all practical purposes this process can be neglected. On the other hand, if the energy of the system can be lowered by single particle transitions, i.e. an infinitesimal current change, then one might expect these transitions to occur in a length of time observable in the laboratory.

For a thin walled cylinder in which $R >> \lambda >> \delta$, where $R$ is the cylinder radius, $\lambda$ is the penetration depth, and $\delta$ is the wall thickness, the single particle transitions can be studied without further excessive simplifying assumptions, using the Bose gas model of superconductivity.

Consider a system at $T=0$ of $N$ Bose particles of charge $e$
in a thin cylinder of inner radius \( R \), wall thickness \( \ell \), and length \( L \). Then, considering only axial fields, we have:

\[
|A| = H_2 = \frac{\Phi(r)}{\pi r^2}, \quad \text{and} \quad |\vec{A}| = A_0 = \frac{\Phi(r)}{2\pi r}
\]

The Hamiltonian for a single boson is then given by

\[
\hat{H} = -\frac{\hbar^2}{2m} \left( \frac{1}{r} \frac{d}{dr} r \frac{d}{dr} + \frac{\ell^2}{r^2} \right) + \frac{1}{2m} \left( \frac{\hbar^2}{r} \frac{d}{d\phi} - \frac{e}{c A_0} \right)^2 + V(r)
\]

where \( V(r) \) is an electrostatic potential due to all of the other particles and the lattice. It can be determined by a self-consistent field calculation, but here we need notice only that it serves to make the ground state wave function nearly constant except near the cylinder walls. Otherwise there would be an accumulation of charge in the center of the wall and a high Coulomb energy.

We can find a solution to this Hamiltonian by separation of variables.

\[
\psi = \frac{1}{\sqrt{L \alpha}} e^{i \ell \theta} e^{i \hbar z f(r)}
\]

with the normalization

\[
\frac{1}{\alpha} \int_{R}^{R+\ell} |f_{\ell z}(r)|^2 2\pi r dr = 1 \quad ; \quad \alpha = \pi (R+\ell - R)
\]

and the boundary conditions

\[
f_{\ell z}(R) = 0 = f_{\ell z}(R+\ell)
\]

If we define the quantities

\[
\Phi_0 = \frac{\hbar c}{e} \quad ; \quad \alpha = \Phi_0 A_0 \quad ; \quad \ell^2 = \frac{2 m E}{\hbar^2}
\]

we have
As we are interested only in the lowest energy states of the particle, we see that we must take \( k = 0 \). We have already stated that \( V(r) \) serves to make \( f \) nearly constant and \( \approx 1 \) over the cylinder. Consequently, if we denote by \( f_0 \) the function for \( \alpha = 0 \) (as in this case \( l = 0 \) gives the lowest energy), and the nearby states by \( f_\ell \), then according to first order perturbation theory,

\[
\varepsilon_\ell \approx \varepsilon_0 + \int \frac{(\ell - \alpha)^2}{r^2} \psi^* \phi \psi = \varepsilon_0 + \frac{(\ell - \alpha)^2}{R^2}
\]

if we neglect the variation of \( \alpha \) over the range \( R + 2 \leq r \leq R \).

The current density due to this particle can be written

\[
|j_{\ell}(r)| = j_{\ell}(r) = \psi^* \left( \frac{\hbar}{r} - \frac{e}{\varepsilon \kappa r} \right) \psi = \frac{\hbar}{2\pi m r} (\ell - \alpha) \psi \psi
\]

From equation (4 we see that for a given \( \alpha \) there is a particular value of \( \ell \) which minimizes the energy of the particle. Since the same equation holds for all \( N \) particles, it is reasonable to expect that at \( T = 0 \) all particles will be in this same quantum state \( \ell \), which minimizes \( (\ell - \alpha)^2 \).

If this is the case, the total current density is

\[
|\bar{\mathbf{J}}(r)| = J_0(r) = \frac{\pi}{N} |j_{\ell}(r)| = \frac{\hbar}{2\pi m r} (\ell - \alpha) N \psi \psi = \frac{n\hbar}{2\pi m r} (\ell - \alpha)
\]

where \( n = N \psi \psi \) is practically constant due to \( V(r) \). If we
wish to consider \( J_\phi (r) \) as independent of \( r \), then \( \alpha (r) \) must be nearly constant over the cross section of the cylinder.

We therefore require

\[ \Delta \alpha = \alpha (R+\ell) - \alpha (R) \ll \ell - \bar{\ell} \]  

(8)

where \( \bar{\ell} \) is the average value of \( \alpha \) in the wall.

From Maxwell's equations we know that

\[ J_\phi = \frac{\mathcal{C}}{\pi} \frac{H_\phi}{\ell} \]  

(9)

where \( H_\phi \) is the field produced by the Bosons, and we consider \( J_\phi \) to be constant.

Combining equations (7 and (9 we have

\[ (\ell - \bar{\ell}) = \frac{2 \pi m R}{e \hbar c} \]  

(10)

Calculating \( \Delta \alpha \), we have

\[ \Delta \alpha = \frac{1}{\varrho_0} \Delta \varphi = \frac{2 \pi}{\varrho_0} \int_{R}^{R+\ell} H(\ell) r d\ell = \frac{2 \pi}{\varrho_0} \int_{R}^{R+\ell} (H_e + H_\phi) r d\ell \]  

(11)

where \( H_e \) is the field due to external sources and which we assume is uniform. \( H_\phi \) of course is zero at \( r = R+\ell \).

Thus,

\[ \Delta \alpha \approx \frac{\pi R d}{\varrho_0} (2H_e + H_\phi) \]

The condition is

\[ \Delta \alpha = \frac{\pi R d}{\varrho_0} (2H_e + H_\phi) \ll (\ell - \bar{\ell}) = \frac{m R c}{2 \pi \hbar c e} H_\phi \]  

(12)

or

\[ \lambda^2 (1 + \frac{2H_e}{H_\phi}) \ll \lambda^2 \]  

(13)

where \( \lambda = \frac{1}{\varrho_0} \frac{mc^2}{e \hbar} \)

In order that the first order perturbation expression (5 be correct, the second order contribution of \( \frac{(\ell - \bar{\ell})^2}{r^2} \) must
be small compared to \( \frac{(l-\omega)^2}{R^2} \). That is,
\[
\sum_{n=2}^{\infty} \left| \int_{\phi} \frac{\psi_{n\phi}^*}{r^{\phi}} \psi_{n00} \, d\tau \right| \leq \frac{(l-\omega)^2}{R^2} \langle \nu \rangle \tag{14}
\]
But since \( \xi_{2\phi}^2 - \xi_{1\phi}^2 \leq \xi_{n\phi}^2 - \xi_{1\phi}^2 \), the above condition will be satisfied if
\[
\sum_{n=2}^{\infty} \left| \int_{\phi} \frac{\psi_{n\phi}^*}{r^{\phi}} \psi_{n00} \, d\tau \right| \leq \frac{(l-\omega)^2}{R^2} \langle \nu \rangle \tag{15}
\]
Using the closure property of the wave functions, this becomes
\[
\left| \int_{\phi} \frac{\psi_{1\phi}^*}{r^{\phi}} \psi_{100} \, d\tau - \int_{\phi} \frac{\psi_{n\phi}^*}{r^{\phi}} \psi_{n00} \, d\tau \right| \leq \frac{(l-\omega)^2}{R^2} \langle \nu \rangle \tag{16}
\]
As we have already required that \( (l-\omega) \) be nearly constant over the walls of the cylinder, the integrals in (16 can be approximated (assuming \( f_\phi \approx 1 \) over the walls and \( R \gg d \)).
\[
\frac{(l-\omega)^2}{R^2} \left| \frac{(1-\frac{2d}{R}) - (1-\frac{d}{R})}{\gamma R^2} \right| = (l-\omega)^2 \frac{d^2}{R^2} \leq \frac{(l-\omega)^2}{R^2} \tag{17}
\]
The condition then becomes,
\[
(l-\omega)^2 \ll \frac{d^2}{R^2} \tag{18}
\]
Using equation (10 we find that this becomes
\[
\nu^2 \ll \frac{d^2}{\rho \lambda^2} \tag{19}
\]
where \( \nu = \rho / \pi \lambda^2 \).

From equation (10 we can solve for \( l-\omega \) in terms of \( \nu \) and \( l \) where \( \nu = \nu_{l} + \nu_{c} \).
\[
l-\omega = \frac{(l-\omega_{c}) R}{1 + \frac{\nu_{l}}{\lambda^2}} \tag{20}
\]
If \( R \lambda \gg \lambda^2 \), then
If all of the particles are initially in the same state, given by \( l' \), then the energy is given by

\[
\frac{2mE}{\hbar^2} = \frac{\varepsilon_i}{N} - N\left(\frac{\varepsilon_i}{\hbar^2}\right)^2 = N\varepsilon_i + \frac{4\pi N}{s^2} \left(\frac{\varepsilon_i}{\hbar^2}\right)^2
\]

(22)

To investigate the stability of the system under single particle transitions, we look at the change in energy when one of the particles changes to the state \( l'' \). To first order the fields remain unchanged. Consequently the energy change is due primarily to the change in kinetic energy of the particle.

\[
\Delta E_{l''l'} = \frac{\hbar^2}{2m}\left[ (l'' - l')^2 - (l' - l')^2 \right] = \frac{\hbar^2}{2m}\left[ (l'' - l')^2 - 2(l'' - l')(l' - l') \right]
\]

(23)

This is positive for all \( |l'' - l'| \geq |l(l' - l')| \) if \( 1(l' - l') \ll 1 \). From equation (10 this condition becomes

\[
2(l' - l') = \frac{m\varepsilon_i}{\hbar c h d} H_0 = \frac{q}{m} \varepsilon_i H_0 < 1
\]

(24)

\[
H_0 < \frac{\hbar}{2c} \left( \frac{q}{m} \varepsilon_i \right) = \frac{\hbar}{2} H^*
\]

(25)

Since \( H_o = H_0 + H_c \), we have the result that persistent currents are stable with respect to single particle transitions if

\[
H_c < H_0 + \frac{\hbar}{c} H^*
\]

(26)

with the conditions, \( 1 + \frac{H_c}{H_0} \ll \frac{1}{2} \frac{\hbar^2}{m} \); \( \frac{H_c}{H_0} \ll \frac{\hbar}{2c} (2H)^2 \)

If \( \frac{\hbar}{2c} \gg 1 \), condition (13 becomes

\[
\frac{H_c}{H_0} \ll \frac{1}{2c}
\]

(27)

or, since \( H_c = H_0 + H_0 \).
Combining this with (25 we see that the maximum stable fields are

\[
\frac{H_z}{H_\lambda} \ll \frac{1}{\sqrt{\lambda}}
\]

(28)

The above calculations, as well as those of Bloch and Rorschach, are based on the Bose gas model of superconductivity. While the quantitative predictions may be incorrect due to an over-simplified model, one would expect the qualitative behavior to be as indicated. That is, the persistent currents necessary to maintain a field inside the cylinder are energetically unstable with respect to single particle transitions, if the field exceeds a critical value.
III Experimental Apparatus

All of the measurements were made on hollow cylinders of tin which were machined out of spectroscopically pure tin stock. Stock was not readily available with a large enough diameter for machining so that it was necessary to melt several smaller pieces of tin in a vacuum oven to form one large cylinder. No attempt was made to control crystal formation while the tin was solidifying.

The cryogenics of the experiment were straightforward. A glass nitrogen dewar surrounds a glass helium dewar with a two liter capacity and a 2" inner diameter. The helium dewar was connected to a Welch vacuum pump via a 1 3/8" brass line containing a ball valve and a needle valve in parallel. The needle valve was used to control the pumping speed and thereby regulate the pressure of the helium gas and the temperature of the bath.

The pressure of the helium was measured by a mercury manometer. The pressure was sensed at a fixed level below the ring seal. There is a small correction in pressure due to the difference in height between the helium level and the sensing level, but for measurements above 1°K it is small enough to be ignored.

Outside the nitrogen dewar was a long solenoid which
produced the external field. The field produced by this solenoid was uniform to \( \pm 0.04\% \) over a distance of 12 cm along the axis. The magnitude of the field was \( 17.55 \pm 0.1 \) gauss/ampere (Huench).

Several methods of measuring the field inside the tin cylinder were tried. We first tried to make use of the magneto-resistance of bismuth. The change in resistance is proportional to \( \vec{H}^2 \) for fields up to at least several kilogauss. Although we were able to measure a definite effect, it was not reproducible, even when the system was not thermally cycled, and the effect observed was an order of magnitude smaller than observed by others. This could easily be due to small amounts of lead impurity in the bismuth.

We next integrated the emf produced in a 13,000 turn coil by flux changes in the center of the sample. The integral should be proportional to the flux change in the coil, and indeed it was, for short times. However, the experiment required a minimum of two hours at each temperature, and the stability of the amplifiers used to perform the integration was not good enough to keep the zero drift to an acceptable level for this length of time.

A method which gave reasonably good results was one in which we used the Meissner effect in a superconducting lead slug to deform the uniform field produced by the cylinder and the external solenoid. By causing the slug to move in
and out of a sensing coil, the amount of flux through the coil is alternately increased or decreased. If the amplitude of the oscillations is constant, then the signal induced in the coil is proportional to the field being measured. The apparatus is shown schematically on figure 2. The lead slug is connected via a long shaft to one end of the armature of the driving solenoid at the upper end of the dewar. The other end of the armature is connected to the top plate of the system by a spring. If the frequency of the current energizing the driving solenoid is near one half of the resonate frequency of the spring-armature system, the slug will move up and down with an amplitude of about 1 mm, at twice the driving frequency. As the size of the slug must be small enough that the resultant field distortion at the cylinder walls is negligible, the signal from the coil is quite small. (The slug used was 1 cm long, and .5 mm in diameter.) However, by using phase sensitive detection the signal could be easily measured.

When testing this apparatus it was noticed that the signal output was not strictly proportional to the field as supposed, but rather acted as though an imperfect Meissner effect were causing some sort of hysteresis effects in the lead slug (frozen in moments, etc.). The effect was not large, and the data could be corrected to eliminate it. Consequently we obtained accuracies of about .2 gauss (.1
ampere of current in the external solenoid) or $\pm 2\%$ whichever was larger. The data plotted on figures 4, 5, 6 were obtained from this method.

The final method used what is commonly known as a "flux gate" magnetometer. It has the advantages of no moving parts and high accuracy. The magnetometer consists of a sliver of high permeability material with two coils wound around it. If a sine wave is applied to one coil, the sliver will be saturated first in one direction and then the other. Consequently a clipped sine wave will be induced in the second coil. If there is no dc component present in the field seen by the sliver then the clipping in the second coil will be symmetric, and no even harmonics will be present. However, if there is a dc component, then the clipping will be asymmetric, and even harmonics will appear. If the circuitry is arranged so that one of the coils can be used to produce a dc field as well as function as the primary or secondary of the transformer, then the detector can be used as a null device. The current through the coil, when adjusted for minimum second harmonic in the output, is directly proportional to the field.

The circuitry is indicated in block form on figure 3. The oscillator drives the primary of the magnetometer at a frequency $f$ (5 kc). The output of the secondary passes through two twin-T filters, one tuned to frequency $f$ and one to $3f$. Then it passes into a tuned amplifier set for
2f, and finally into a phase sensitive detector driven at 2f. The driving frequency is obtained by driving a diode with the oscillator and amplifying the second harmonic in the output. The dc output of the detector is amplified and fed into the primary coil around the silver. The polarity of the gain of the system is adjusted so that the feedback is negative. Thus the dc field produced by the coil is such as to null out the field being measured. Since the field produced by the coil is proportional to the current flowing, an ammeter in series with the coil can be calibrated in terms of the field being measured.

With this method, the accuracy obtained was limited by the ammeter used, and was better than ±1%. The data plotted on figures 7, 8, 9, 10 were obtained by this method.

Measurements were made on three cylinders. The wall thickness of all three was 1.5 mm and the inner radius was 7.5 mm. The first cylinder was 10 cm long while the last two were 12.5 cm long. We had intended to make all measurements on one cylinder (#1), the 10 cm one, but it was held by a nylon holder which apparently contracted at low temperatures and caused the cylinder to crack after several thermal cycles. Large regions on the second cylinder (#2) turned into grey tin after only two curves were made. This transition was probably aggravated by the presence of the zinc in the brass which we used to hold the tin in order
to eliminate the nylon holder. The third cylinder (#3) was kept from contact with the brass by a thin sheet of teflon around the brass parts, and lasted for the rest of the experiment.
IV Experimental Technique and Results

The curves shown in figures 4-10 represent the measured value of the maximum field which could be maintained in the cylinders as a function of the external field. Each curve was made at a constant temperature.

The procedure followed was to cool the cylinder down to some temperature, and then to maintain that temperature while we measured the maximum stable internal field at various values of the external field. The cylinder was then cooled to a new temperature and more measurements made. During a given run the temperature was always monotonically lowered, to assure that thermal gradients would be a minimum.

Once the cylinder reached a particular temperature, enough current was put through the external solenoid to produce a field greater than the critical field at that temperature. The current in the solenoid was then reduced by about one ampere (17.5 gauss), and the field inside the cylinder was measured. After the field inside had stopped changing, we reduced the current again, etc. When the current reached zero it was reversed and then increased in steps until the field produced exceeded $-H_c$. The current was then lowered in steps to zero, reversed, and then increased until the field was the same as that with which we
had started. Thus the external field goes through a complete cycle, from $H_e > H_c$ to $H_e = 0$, to $H_e < -H_c$, to 0, to $H_e > H_c$.

When the tin was in the normal state and the external field was changed, the internal field changed with a time constant of about a second, indicating a very low normal resistance of the tin at low temperatures. However, when the tin was superconducting the time constant was about 30 seconds. Thus about two minutes was required for the internal field to come to equilibrium at each value of the external field.

The curves in figures 4 and 5 were obtained from the first cylinder ($d = 1.5 \text{ mm}$, $r = 7.5 \text{ mm}$, $L = 10 \text{ cm}$). The two sets of curves do not agree with each other, although the experimental conditions seemed to be the same. However, examination of the cylinder showed several longitudinal cracks in the wall. These cracks were probably the cause of the difference between the two figures, but at the same time they cast some doubt as to the validity of the first results, since there were probably severe strains present in the tin, even if it were not cracked.

The curves in figure 6 were obtained from the second cylinder ($d = 1.5 \text{ mm}$, $r = 7.5 \text{ mm}$, $L = 12.5 \text{ cm}$). Since large portions of this cylinder converted to grey tin some time after the curves were obtained, it was of no further use. However, we believe that the data presented were
obtained before the appearance of the grey tin.

The curves in figures 7-10 were obtained from a third cylinder \( (d = 1.5 \, \text{mm}, \, r = 7.5 \, \text{mm}, \, L = 12.5 \, \text{cm}) \). The curves in figure 7 were obtained with sharp corners at the ends of the cylinder. In order to investigate end effects the curves in figure 8 were obtained with the ends rounded off. In figure 9 the ends had been squared off again, and in figure 10 the ends were still square, but the cylinder had been annealed at 200°C in vacuum for 24 hours to relieve any strains which might have appeared due to the thermal cycling.
V Conclusion

All of the curves in figures 4, 6-10 are similar in appearance, although they do not agree in detail. In general, as the external field is reduced from $H_c$, $H_1$ at first remains constant and equal to $H_c$. In this region we seem to have almost complete flux trapping, and the cylinder behaves as though it were perfectly conducting. As $H_e$ becomes smaller, the internal field begins dropping, although not as fast as the theory predicts. By the time $H_e = 0$, $H_1$ is down to about $0.6H_c$. Then as $H_e$ is increased in the opposite direction, $H_1$ continues to decrease, and finally when $H_e = -H_c$, $H_1 = H_e$. If we reverse the cycle, we of course get the same results as above, with all fields being negative. ($H \rightarrow -H$)

The theory predicts that when the external field is zero, the internal field should be $d/r H_c = 0.2H_c$. Experimentally we find that the value is about $0.6H_c$. The value varies slightly with temperature, becoming larger as the temperature decreases, and depends somewhat on the cylinder, but it is always between 0.50 and 0.66.

The theory of reference 3 was carried out for $T = 0$. Consequently any differences between theory and experiment should become smaller as $T$ becomes smaller. However, in
the range of temperatures used, the differences became larger as the temperature was lowered.

There are problems introduced by possible end effects, but, as will be discussed in the appendix, the end effects should make the internal field even smaller than the theory predicts.

Thus, assuming that the end effects do not predominate, the theory appears to be qualitatively correct. The measured maximum stable fields seem to be a function of the applied field, and the critical field cannot always be frozen in, as we would expect if the cylinder were a perfect conductor. That the theory appears to be quantitatively incorrect is not surprising, as the theory is based on the Bose gas model of superconductivity, a very unrealistic and oversimplified model.

Figures 7-10 were made with one cylinder, the different figures having been made in an attempt to determine the importance of the ends. Figure 7 was made with square ends on the cylinder. The data were obtained not long after the cylinder had been machined (two or three days). There is a considerable difference between figures 7 and 8, which at first was attributed to the fact that the data presented in figure 8 were obtained with the ends of the cylinder rounded. Consequently it appeared that end effects were rather important.

In figure 9 the ends had been squared off again (with
a loss of about 3 mm length), but the curves are not much different from those in figure 8 and are quite different from those in figure 7.

The curves in figure 10 were obtained after annealing the cylinder for 24 hours at 200°C, in order to reduce any strains which may have been present. The curves are almost identical to those in figure 9.

As figures 7, 9, and 10 were obtained with identical ends (square), the difference between figure 7 and figures 9 and 10 is possibly due to the fact that the cylinder had been severely strained by the machining and that the strains had been reduced greatly by the time the data for figure 9 was obtained.

Figure 11 is a plot of the field produced by the cylinder ($H_{cyl} = H_i - H_e$) as a function of the external field. Both curves represent the same cylinder and the same temperature ($T = 1.93^\circ K$), but one (solid line) comes from figure 7, while the other (dashed line) comes from figure 9. The solid curve is very interesting as it is very nearly even in $H_{cyl}$ and $H_e$. This indicates that the field which the cylinder can maintain is a function of the magnitude of the external field, and does not depend on the relative directions of the external and internal fields. However, the dashed line, which we believe comes from a relatively unstrained cylinder (preceding paragraph), displays no such symmetry.
The hump in the dashed line comes from the wiggle which appears in the curves on figure 9. Similar wiggles appear in figures 4, 8, and 10. These were not expected, and as yet are unexplained. They seem to be real, and do not seem to be due to the measuring apparatus.

Comparing figure 8 with figures 9 and 10, we see that there is a difference between the square and rounded ends, but that it is not very much. Unfortunately, this still tells us very little about the importance of the ends. It is entirely possible that the results for a cylinder without ends (infinite length) would be considerably different. (Appendix)

The time dependence of the internal field is quite interesting. When the external field is suddenly decreased, the internal field remains constant for about 5 seconds, and then begins to decay more or less exponentially to its new equilibrium value with a time constant of about 30 seconds. One possible explanation of this behavior is that the time delay before the internal field begins to decay is due to time required for the flux lines to escape from the cylinder by leaking through the walls, starting at the ends and working in towards the center. If this is the case, then end effects are important.
VI Appendix

As mentioned several times in previous sections, the finite length of the cylinder may have a large effect on the measurements. This is due to the fact that near the ends of the cylinder the magnetic field lines are diverging. Consequently the field near the walls at the ends is greater than in the center of the cylinder. If the field in the center is $H_0$, then the maximum field at the ends will be greater than $H_0$, and the superconductor will go into the intermediate state.

Just what will happen to the field at the walls is somewhat uncertain. It is possible that the system will allow flux lines to escape through the walls until the field at the ends is $H_c$, at which point the field in the center will be less than $H_c$. This effect will give results somewhat like those predicted by Bloch and Rorschach, in that one would not be able to trap the critical field inside the cylinder.

On the other hand, it is possible that the flux lines might be trapped on an impurity or other crystal defect, and thus be prevented from escaping. It is also possible that the structure of the intermediate state in the case of the cylinder might be such as to prevent this flux leakage.
In either case, the field in the center would be unaffected by the ends.

Thus it is difficult to predict whether or not the ends are important. An attempt was made to determine their importance by changing the shape of the ends. Figure 8 was made with round ends and figure 9 with square ends. Although there is some difference, it is small and shows only that the ends do have some effect. A large change in the curves would have shown that the ends were important. However, the small change shows only that the effect is not strongly dependent on the shape of the ends.

To be certain that the measurements are not affected by the ends, they must be effectively eliminated. As the problem stems from the divergence of the field near the ends of the cylinder, if we could keep the field uniform throughout the superconductor the ends would be no problem.

This can probably be accomplished by winding short solenoids of the same radius as the cylinder, and placing one at each end of the cylinder. If the current density in the end coils is the same as that in the superconductor, the magnetic field produced by the end coil-superconducting cylinder system will be the same as that produced by a long solenoid. As the cylinder is in the center of the effective solenoid, the field in the neighborhood will be uniform, and the field at its ends will be the same as the field in
If the superconducting cylinder is long compared to its radius, the field in the center will be nearly independent of the current in the end coils. If this is the case the field produced by the cylinder will be equal to the internal field minus the external field \( H_{\text{cyl}} = H_i - H_e \), and the current density in the walls will be proportional to \( H_{\text{cyl}} \). Consequently, the current density in the end coils can be automatically controlled by supplying the current from the output of an amplifier whose input is proportional to \( H_i - H_e \). A block diagram of such a system is shown on figure 12.
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Bibliography

Region of stable internal fields as a function of external fields for an infinitely long hollow superconducting cylinder. Assuming that the superconductor is a perfect conductor, the region of stability is the entire square. Following Bloch and Rorschach, the region of stability is the shaded area. ($%\approx+%$)
Figure 2

Block diagram of field measuring apparatus using vibrating superconducting lead slug.
Figure 3

Block diagram of "flux gate" magnetometer.
Figure 4

Cylinder #1, ends square
Figure 5

Cylinder #1, ends square, but wall cracked
Figure 6

Cylinder #2, ends square
Figure 7

Cylinder #3, ends square
Figure 8

Cylinder #3, ends rounded
Figure 9

Cylinder #3, ends square
Cylinder #3, ends square, annealed at 200 °C for 24 hrs.
Figure 11

$H_{cyl}$ vs $H_e$

The dashed line is from the 1.93$^\circ$K curve on figure 9.

The solid line is from the 1.93$^\circ$K curve on figure 7.
Figure 12

Block diagram of possible means of eliminating end effects