HIGH FREQUENCY RADIO HEATING OF THE IONOSPHERE

by

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Abstract

In this work the effect of a high frequency radio wave incident upon the ionosphere is studied. The changes in the ionospheric parameters are derived. A particular case is examined, namely, the values found for conditions over the Arecibo Observatory. The radio source is a transmitter, with 160 kW, tunable from 5 to 25 MHz, using the dish antenna (305 meters of diameter) of the Arecibo Observatory. The uses of the ordinary or the extraordinary waves are discussed. The different conditions in the non deviative and deviative absorption regions are considered. The probing instruments are: the 430 MHz radar (at the Arecibo Observatory), photometers, and an ionosonde.
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1. THE IONOSPHERE

1.1 INTRODUCTION

The history of the ionosphere covers nearly a century, and begins with Balfour Stewart's (Stewart, 1878) postulating the existence of a conducting layer in the upper atmosphere, in order to explain the variations of the Earth's magnetic field. An electric current could flow in this conducting layer and the induced magnetic field would modify the total magnetic field at the surface of the Earth. To explain how Marconi succeeded in sending radio waves across the Atlantic, A. E. Kennelly (1902), and independently O. Heavyside (1902) suggested that the waves were reflected from some conducting layer at high altitude.

Only with the experiments of Appleton and Barnett (1926), in England, and Breit and Tuve (1926), in the United States, direct evidence for the existence of an ionized and conducting layer was found. Appleton and Barnett used a frequency
modulated transmitter and were able to receive the signal back, after the same was reflected from the ionosphere (the name given to the ionized conducting layer of the upper atmosphere). Breit and Tuve used a different system, they sent up radio pulses and measured the elapsed time until they returned. If the pulses had travelled with the speed of light, they would take \( t \) seconds to make the round trip, assuming they were reflected at an altitude \( h' \). The relation between \( c \) (the velocity of light in vacuo = \( 3.0 \times 10^8 \) m/sec), \( h' \), and \( t \) is:

\[
h' = 2ct
\]

As the velocity of a radio wave is not \( c \), but \( v = cn \), where \( n \) is the index of refraction of the atmosphere (function of altitude), \( h' \) is only a virtual height, not the real reflection height. Due to the presence of the magnetic field of the Earth, \( n \) has two values, and the radio wave is split into two modes, the ordinary and extraordinary waves. The value of \( n \)
is a function of the electron density, electron collision frequency, magnetic field, and the frequency of the wave, for each mode.

The wave is reflected when \( n \) becomes close to zero. By changing the carrier frequency of the exploratory pulse, and measuring the time it takes to return, it is possible to obtain a profile of \( h' \) versus the frequency (\( f \)) of the carrier (Budden, 1966). Such profile is called an ionogram, the apparatus that makes this plot is the ionosonde (Wright et al., 1957). From an ionogram the real reflection height can usually be determined (Budden, 1966). Today more sophisticated techniques are used to measure directly the real reflection height, like rocket probes and incoherent scatter radar.

The incoherent scatter technique is a very powerful tool for ground based measurements of the ionosphere. The electron density, electron and ion temperatures, plasma velocities
and composition of that portion of the atmosphere between about 50 and a few thousand kilometers (KM) of altitude, can be measured simultaneously from the ground.

In 1963 D. T. Farley (Farley, 1963) suggested that a high frequency (HF) transmitter of sufficient power could be used to modify some ionospheric parameters, as the electron density, and the electron temperature. In April 1970 an experiment was done in Boulder, Colorado, where the modifying wave was produced by a transmitter of nearly 2 MegaWatt (MW), fed into a 10 element ring array, and the probing instruments were ionosondes, photometers, and oblique path sounders. Some ionospheric modifications were reported in ten letters in the November first issue of the Journal of Geophysical Research (Space Physics). W. F. Utlaut (1970) describes in detail these experiments.

In October, 1970, and January, 1971, series of modification experiments were done at the Arecibo Observatory, Puerto Rico (AO), using a modifying wave produced by a 160 KW transmitter,
fed into the 305 meter (m) diameter antenna (this set-up has approximately the same aperture-power product of the set-up of the Boulder experiment); the diagnostic equipment consisted of photometers, an ionosonde, and the 430 MHz radar. The importance of this experiment is due to the use of the incoherent scatter radar, because important parameters such as the electron density, electron temperature, ion temperature, and others, can be measured simultaneously, and continuously, in almost real time!

For ionospheric heating ("heating" will be used in a broad sense, indicating that modification in temperature can be followed by other parameters modification), using HF frequencies, it is useful to divide the ionosphere into two regions, instead of the traditional D, E, F1, F2. The cited two regions are:
a) NDAR - non deviative absorption region; b) DAR - deviative absorption region (Davies, 1965). The first region is the lower part of the ionosphere (lower limit approximately at 50 KM of
altitude), in which the radio "rays" (Budden, 1966) are not
deviated from their path, while in the second region (lower
limit dependent on the frequency), the rays are bent (ordinary
in one way, extraordinary in a different way) due to the fact
that the real part of n is not close to unity as it was in the
NDAR (Davies, 1965). The deviation of the rays is the refraction,
as in geometric optics. At HF frequencies the radio waves
behave like rays, for most conditions.

It should be noticed that the ionospheric parameters, in
general, are highly variable. For the purpose of artificial
modification of the ionosphere, it is necessary to know about
the natural variations, in order to differentiate them from the
artificially induced variations. In the remainder of this
chapter the physics of the NDAR and DAR will be briefly discussed.
1.2 THE NON DEVIATIVE ABSORPTION REGION

The morphology of the NDAR will be discussed, and special attention will be paid to the parameters that can interact with a propagating HF radio wave. The parameters below are among these:

a) index of refraction, \( n = \mu - i\chi \);

b) electron density, \( N \);

c) temperatures, \( T_e \) (electrons), \( T_i \) (ions), \( T_n \) (neutrals);

d) collision frequencies, \( \nu \);

e) composition;

f) absorption, \( A \);

Typical values for these parameters will be discussed now. All the values are for the space above the Arecibo Observatory and nighttime, unless stated differently. In all this work the system of units to be used is the International System of Units (SI) (Paris and Hurd, 1969).

**INDEX OF REFRACTION** - This is the parameter chosen to
classify the ionosphere into NDAR and DAR, since different assumptions have to be made in dealing with these two parts of the upper atmosphere. The magnitude of the real part of \( \eta \), \( \mu \), is close to unity, so no appreciable deviation (refraction) occurs, in the NDAR. In the DAR \( \mu \) departs from unity (goes to zero at the height of reflection), the radio waves are split into the ordinary and extraordinary parts (O-wave, X-wave), and due to refraction, are bent in different directions. This analysis is for HF frequencies (Ratcliffe, 1962). At UHF (ultra high frequencies), like the 430 MHz radar at Arecibo, the refraction of the radio waves is negligible, in both ionospheric regions.

**ELECTRON DENSITY** - One of the more important parameters in ionospheric studies, electron density is used to classify the ionosphere into the classical regions: D, E, F1, F2. Figures (1.2) and (1.3) (Johnson, 1965) give extreme values for the electron density in daytime, and nighttime, respectively. Figure (1.4) shows an
electron density profile measured at the AO (Arecibo Observatory), at night, during the summer; it is a normalized plot, the values of \( N \) are relative to \( N_{\text{max}} \), which can be computed from the knowledge of \( f_N \), the plasma frequency, by means of the formula:

\[
f_N = \frac{1}{2\pi} \left[ \frac{N_{\text{max}} e^2}{m_e e_o} \right]^{\frac{1}{2}}, \quad N_{\text{max}} = \frac{f_N^2}{80.43}
\]

in MKS (rationalized) units, where \( m_e \) is the electron mass, \( e \) is the electron charge, and \( e_o \) is the permittivity of the free space.

The plasma frequency can be determined from an ionosonde, or from backscatter data (plasma line) (Evans, 1969). In figure (1.5) (Whitten and Poppoff, 1965), for comparison, some experimental results of the electron density in the NDAR are shown.

**TEMPERATURES** - The temperatures of the species in the ionosphere are due to the motion of the particles. In the NDAR the thermal conduction is high enough to maintain the thermal equilibrium among the species, \( T_e = T_i = T_n \). Figure (1.1) from Johnson (1965), is a plot of the extreme conditions for the neutral
temperatures.

**COLLISION FREQUENCIES** - Three different kinds of collisions are significant, electron collision frequency, $\nu_e$, ion collision frequency, $\nu_i$, neutral collision frequency, $\nu_n$.

Starting with $\nu_n$, it is seen that the collisions between neutrals is not very important, as far as the passage of an HF radio wave, because the electromagnetic field of the wave will not interact with neutral particles. The neutrals act as a heat sink, since their density is much larger than the other species. For instance, at 500 KM of altitude, the neutral's concentration is $7.3 \times 10^{13}$ (approximately), while the electron density is of the order of $10^{11}$. At its maximum (around 300 to 400 KM of altitude), the electron density never reaches $10^{13}$ (particles/m$^3$), while the neutral density is of the order of $10^{15}$ (particles/m$^3$), at that height.

The ion collision frequency is composed of 3 terms:

-10-
\[ v_i = v_{ii} + v_{ie} + v_{in} \]

\( v_{ii}, v_{ie}, v_{in} \) are respectively the frequencies of collision between ions and ions, ions and electrons, ions and neutrals. \( v_{ii} \) is unimportant, because the density of ions is much smaller than neutral density. The ions are so outnumbered by the neutrals that the probability that an ion would strike another ion is much smaller than the probability it would strike a neutral. If an ion collides with an electron, because of the difference of mass, the effect will be negligible, then \( v_{ie} \) is also unimportant. \( v_{in} \) is the term that dominates the ion collision frequency, \( v_i = v_{in} \). Chapman (1956) presented an expression for:

\[ v_{in} = 2.6 \times 10^{-5} (N_n + N_i) M^{-\frac{1}{2}} \]

where \( N_n \) is the neutral density, \( N_i \) is the ion density (same as electron density, since the ionospheric plasma is neutral), and \( M \) is the molecular weight of neutrals or ions, which are assumed
to have the same mass (Johnson, 1965). Note that accordingly to the above expression, even when $N_i$ goes to zero still a substantial number of collisions occurs. The explanation is that the formula in question holds only in a region in which the number of ions is large enough (above approximately 60 KM), for a small $N_i$ the formula does not hold.

The electron collision frequency is also divided into 3 parts:

$$v_e = v_{ee} + v_{ei} + v_{en}$$

where $v_{ee}$, $v_{ei}$, $v_{en}$ are, respectively, the frequency of collisions between electrons and electrons, electrons and ions, electrons and neutrals. $v_{ee}$ is not important, because due to the small electron density, in comparison with the neutral density, the probability that an electron collides with another is small, then $v_{ee} \ll v_{en}$. Comparing $v_{ee}$ with $v_{ei}$, it is seen that due to the small electron mass: $v_{ee} \ll v_{ei}$

Both $v_{ei}$ and $v_{en}$ are important in the electron collision
frequency, in the NDAR. At higher altitudes $v_{ei}$ becomes dominant.

The expressions for $v_{ei}$ and $v_{en}$ are:

$$v_{ei} = 38 \times 10^{-6} N \times T_e^{-3/2}$$

$$v_{en} = 8.9 \times 10^{-18} N_n \times T_e$$

from Thrane and Piggott (1966),

**COMPOSITION** - From ground to about 85 KM of altitude, the atmosphere is well mixed and its composition is approximately constant. Table 1 lists the atmospheric constituents, at sea level, and their densities (Davies, 1965).

**TABLE 1**

<table>
<thead>
<tr>
<th>Molecule</th>
<th>Mass (gr)</th>
<th>Percentage</th>
<th>Density ($m^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_2$</td>
<td>28.022</td>
<td>78.084</td>
<td>$2.098 \times 10^{25}$</td>
</tr>
<tr>
<td>$O_2$</td>
<td>32.009</td>
<td>20.946</td>
<td>$5.629 \times 10^{24}$</td>
</tr>
<tr>
<td>$A$</td>
<td>39.960</td>
<td>0.934</td>
<td>$2.510 \times 10^{23}$</td>
</tr>
<tr>
<td>$CO_2$</td>
<td>44.024</td>
<td>0.330</td>
<td>$8.870 \times 10^{21}$</td>
</tr>
<tr>
<td>AIR(average)</td>
<td>28.973</td>
<td>100.000</td>
<td>$2.687 \times 10^{25}$</td>
</tr>
</tbody>
</table>
While up to 85 KM the percentages of the constituents remain approximately the same, the density is reduced exponentially, according to:

\[ N_s = N_{so} \exp \left( \frac{h - h_0}{H} \right) \]

where \( N_s \) is the density of the specie \( s \) at a height \( h \), \( N_{so} \) the density of the specie \( s \) at a reference level \( h_0 \), \( H \) is the scale height, that is equal to \( kT/mg \) (\( T \) is the kinetic temperature of the specie, \( m \) the molecular weight of the specie, \( k \) the Boltzmann constant, and \( g \) the acceleration of the gravity).

Above 85 KM, the formula for \( N_s \) still holds locally, if the proper value of \( H \) is used. At higher heights photo-chemical processes do occur, resulting in a variation in the percentage of the constituents with height.

The molecular oxygen and nitrogen start to dissociate into their atomic form, in the NDAR. The main ions in this region are \( \text{NO}^+ \) and \( \text{O}_2^+ \). The main neutrals are \( \text{N}_2 \) and \( \text{O}_2 \).

**ABSORPTION** - The equation for a one dimensional plane wave travelling in a direction \( \mathbf{r} \) is given by:
where $E$ is the magnitude of the electric field of the electromagnetic wave at a distance $r$ from the origin, $E_0$ the same at the origin ($r = 0$), $\omega$ the angular frequency of the wave $= 2\pi f$, $t$ the elapsed time, $c$ the velocity of light, and $n$ the complex index of refraction. $E$ can be put in the form:

$$E = E_0 \exp(-\frac{\omega \chi}{c} r) \exp(i\omega t - i \frac{\omega}{c} \mu r)$$

where $\frac{\omega \chi}{c} = k$ measures the decay in amplitude per unit distance, and is called absorption coefficient. Here again $n = \mu - i\chi$ (Budden, 1966). If $\chi$ is large enough, absorption can occur in both NDAR and DAR. This concept is not restricted to plane waves, it can be extended to any wave with the same dependence of the above equations.
1.3 - THE DEVIATIVE ABSORPTION REGION

The same parameters discussed in the NDAR will be reviewed for the DAR. The DAR, in this work is associated with an HF radio wave around 6 MHz. For these frequencies the lower limit in the DAR can be set near 160 to 180 KM of altitude. Above these heights the HF radio waves are bent from their path.

INDEX OF REFRACTION - The ionosphere is a birefringent medium, an HF radio wave entering it is split into two components, the O-wave and the X-wave, in general (in special cases only one mode can be produced) (Ratcliffe, 1962).

The Appleton-Hartree theory (Budden, 1966) provides an expression for the complex index of refraction of the ionosphere:

$$n^2 = 1 - \frac{X(1 - X - iZ)}{(1 - iZ)(1 - X - iZ) - \frac{Y_T^4}{2}}$$

where $X = \frac{w_N^2}{w_m^2}$; $w_m^2 = \frac{N e^2}{m e_0}$; $e$ being the electron charge, $m_e$ the electron mass and $e_0$ the permittivity of free space.
\( Y = \omega_H / \omega; \omega_H = eB/m_e; B \) being the magnitude of the Earth's magnetic field (considered constant).

\( \theta \) being the angle between the ray and the magnetic field.

\( Y_L = Y \cos \theta; \quad Y_T = Y \sin \theta \)

\( Z = \nu \omega \). The electron collision frequency to be used here, \( \nu \), as shown by Sen and Wyller (1960), is 5/2 of the original \( \nu_e \). The physical explanation is that Appleton-Hartree considered collisions produced by mono-energetic electrons (the energy being the average energy of the electrons), while Sen-Wyller used a Maxwellian electron distribution, and the contribution of the electrons in the high energy tail is substantially bigger than the contribution for collisions from the electrons in the low energy tail or near the peak of the distribution.

Then the values of \( \nu \) used in the formula for \( n \) should be

\( \nu = 5/2 \nu_e \).

The double sign in the expression for \( n^2 \) reflects the fact that there are two components, O and X-waves, the upper sign
for the O-wave, the lower sign to the X-wave.

Figures (1.8A) and (1.8B) are plots of the real and imaginary parts of $n$, solving the Appleton equation for the following input parameters: $X$ variable, using the $N$ values of figure (1.3) (average profile); $Y$ constant, using the value for $B$ as the value for the AO ($B = 0.370 \times 10^{-4}$ Weber/m$^2$); $\theta$ constant, ($\theta = 40^\circ$), corresponding to a wave launched vertically over Arecibo; $Z$ variable, the value of $v$ being $5/2$ the values of $v_e$ from the figures (1.7) and (1.9).

**TEMPERATURES** - Due to the low density in the DAR, the thermal interaction among the species is small, then during daytime the $3$ temperatures can be different (Evans, 1969). However, $T_n$ is almost equal to $T_i$ (Evans, 1969). $T_e$ is the highest of the three.

Assuming that $T_n = T_i$ (Evans, 1969), and using a radar backscatter technique, the ratio $T_e/T_i$ can be directly measured. Figure (1.10) gives an example of values measured at the AO. It
shows the electron density profile and the ratio $T_e/T_i$, the last quantity measured by two different methods.

Figure(1.11) (Johnson, 1965) presents the temperature profile of the DAR (neutral temperature), for extreme conditions (general case, not only over AO). Figure(1.11A) shows the $T_e$ profiles to be used in the calculation for the examples in this work.

During the nighttime the three temperatures are approximately the same (Evans, 1969), for a given height.

**ELECTRON DENSITY** - Figures(1.2) and (1.3) already presented the extreme values of the electron density, for both NDAR and DAR. Figure (1.10) also presents some experimental values for the electron density, at the heights of the DAR, measured by radar backscatter.

**COLLISION FREQUENCIES** - The same comments made for the collision frequencies at the NDAR are still valid in the DAR. As the atmosphere is less dense at the heights of the DAR, the electron neutral collision frequency is small at DAR heights. Also, as there are many more ions at DAR heights, $\nu_{ei}$ is more
important than $v_{en}$. Then $v_e = v_{ei}$ (Johnson, 1965). Figure (1.9) is a plot of typical values for $v_e$ at DAR heights, nighttime, over AO, in summer.

**COMPOSITION** - The major neutral at the DAR is atomic oxygen, while the major ion is ionized atomic oxygen. Figures (1.12) and (1.13) show the theoretical concentration of particles versus altitude for daytime, maximum of sunspot cycle, and nighttime, minimum of sunspot cycle, respectively (Johnson, 1965).

**ABSORPTION** - The absorption of HF radio waves is more effective in the DAR (Davies, 1965), since the waves spend more time in a region in which $\mu$ is small, and $\chi$ is large (see Figure (1.8B).

If a wave has a frequency larger than $f_N$, it is partly absorbed, but is not reflected (for vertical incidence) back to the ground. If $f$ is smaller than $f_N$ the wave reflects on the ionosphere and returns to the ground. The absorption coefficient $k = \frac{u\chi}{c}$ is proportional to $\chi$, then the maximum absorption will be near
the peak of $\chi_o$ for the ordinary wave, and $\chi_x$ for the extra-
ordinary wave (Budden, 1966) (see Figure 1.8B).
Figure 1.1
Figure 1.2

ELECTRON DENSITY (ELECTRON/M³)

Minimum Of Sunspot Cycle

Maximum Of Sunspot Cycle

Figure 1.3

ELECTRON DENSITY (ELECTRON/M³)
Figure 1.5
Figure 1.7
Figure I.8A

REAL PART OF $\eta$

$\mu_0$

$\mu_x$
Figure 1.9

ALTIITUDE (km)

ELECTRON COLLISION FREQUENCY (COLL/SEC)
ELECTRON DENSITY (el/m$^3$)

Figure 1.10

ARUBA
10 AUG. 1966

- O ION SPECTRUM
- O PLASMA LINE
- ELECTRON DENSITY

ALTITUDE (KM)

0740 AST
1030 AST
1210 AST

Te/Ti

Figure 1.10
Figure 1.11

Temperature (°K)

Altitude (km)

Average Atmosphere

Daytime Maximum

Nighttime Minimum

Solar Cycle
Figure I.11A
Figure 1.12
Figure 1.13
2. MEASUREMENT TECHNIQUES

2.1 TEMPERATURE MEASUREMENTS

The ionospheric temperatures can be measured in a variety of ways, among them, the most widely used are: spectrographic techniques, rocket borne experiments, atmospheric drag, meteor trails, and radar incoherent scatter.

2.1.1 Spectrographic techniques

From aurora and airglow radiations, the temperature of the emitting atmos can be determined, based on Doppler effect. The half width of the emission lines, mainly due to the kinetic energy of the atom are:

$$\Delta \lambda = \frac{\lambda}{c} \frac{\sqrt{2kT}}{m_a}$$

where $\lambda$ is the wavelength of the emitted radiation, $c$ is the velocity of light in free space, $k$ is the Boltzmann constant, $m_a$ is the mass of the atom, and $T$ the temperature of the atom. The main emitter atom is oxygen (OI), in the bands of 5577A ($1 \text{A} = 10^{-10} \text{ m}$), and 6300A. At twilight the doublet line of
sodium (D line) can be observed (Barbier, 1962). Some rotation bands also can be observed, especially those of \( N_2^+ \), \( \text{HO} \), \( N_2 \) and the 8645A line of \( O_2 \) (Barbier, 1962). Figure (2.1) (Barbier, 1962) shows temperatures measured by these techniques. In that figure, the rotational band of \( N_2^+ \) measurements are denoted by initials.

### 2.1.2 Rocket borne experiments

As the velocity of sound in the air is proportional to the square root of \( T \) (average air temperature):

\[
  v_A = \sqrt{\frac{3RT}{M}} \tag{2.2}
\]

where \( R \) is the universal gas constant \( = 8.354 \) joule/mole °K, \( M \) is the average mass of the air, it is possible to measure the air temperature if the velocity of sound is known (\( R \) and \( M \) generally are well known).

In the "project grenade" a rocket is flown, and at regular intervals grenades are released in the air to explode. On the ground 5 microphones record the sound, and from the time the
sound takes to reach the ground, the vector velocity of sound is inferred. From this data the velocity of sound, and consequently the air temperature, are measured, from 30 to about 95 KM of altitude (De Mendonca et al., 1969). As an example figure (2.2) gives the results of air temperature measurements over a period of two years, over Natal-RN, Brazil, near the equator, using the grenade experiment technique (De Mendonca et al., 1969).

2.1.3 Atmospheric drag

If the atmosphere density is known, $\rho_o$, at a reference level (ground), the average neutral temperature can be inferred (Johnson, 1965) from:

$$\frac{\rho_s}{\rho_o} = \exp\left(-\int_o^h \frac{M g \rho_o}{kT} \mathrm{d}h \right)$$

$$\frac{\rho_s}{\rho_o} = \exp\left(-\int_o^h \frac{M g \rho_o \gamma}{kT(R_0+h)} \mathrm{d}h \right)$$

24
where $\rho_s$ is the average particle density at level $s$, $h$ height above the ground, $R_o$ the radius of the Earth, $k$ the Boltzmann constant, $M$ the average particle mass, $g_o$ the acceleration of gravity at ground level.

The principal techniques to measure the air density are: vacuum gauges, observation of falling spheres (released by rockets), observation of satellite rate or orbit decay (produced by atmospheric drag force).

The drag force is given by the expression:

$$D = \frac{1}{2} \rho v^2 C_D A$$  (2.6)

where $C_D$ is the drag coefficient (dependent on the geometry of the body), $A$ is the area exposed to drag, $\rho$ the density of the medium, and $v$ the velocity of the body.

The falling spheres method is good until about 90 KM of altitude, while the satellite drag is good from 150 to about 1500 KM of height. An example of a satellite used for drag measurements is the ECHO I.
2.1.4 Meteor trails

When a meteor enters the Earth's atmosphere, it produces an ionized trail, because as it collides with air neutral particles, the meteor kinetic energy is transferred to the air particles, ionizing them, since this energy is of the ionization potential of the air particles (atoms or molecules) (Ratcliffe, 1960).

According to the size of the meteor, and its velocity, the ionization is correlated with the atmosphere particle density. Greenhow and Lowell (Ratcliffe, 1960) present a good description of the techniques used in these measurements and the obtained results. The neutral temperature and scale height, at the heights of meteoric trails can be inferred from these measurements. Generally the trails are formed around 80 to 120 KM of altitude (Ratcliffe, 1960).

2.1.5 Incoherent scatter

A method that can determine the ionospheric ion and electron
temperatures, from ground measurements, is the incoherent scatter technique. A radar pulse well above the plasma frequency (430 MHz at Arecibo) is sent up, the electrons will scatter the signals, whose phases are varying with time and which bear no relation one to another due to the random motion of the electrons. At the receiver the signal powers will add (the voltages will cancel due to the random phase), and on the average the cross section per unit volume (finite in a practical experiment) will be simply the product of the electron density and the cross section per electron. As the ionospheric plasma contains ions also, their presence narrows the effective spectrum width. The above argument, first stated by Gordon (1958), gave origin to the term "incoherent scatter".

By measuring the shape of the spectrum of the returned signal, $T_i$, $T_e/T_i$ (and then $T_e$) can be measured, if the ionic composition is known (Gordon, 1967). Fortunately the ion composition is well known, and at the heights of the DAR (for
instance) the majority ion is $0^+$ (Johnson, 1965). Figure (2.3) gives the ion composition in the DAR (Johnson, 1965).

The height resolution is limited by the signal to the noise ratio. It can be improved when the integration time is not limited, but if the integration takes too much time, the ionospheric parameters could change and the error instead of being in the height resolution would be in the value of the parameter itself.

The scatterer power consists of two components: an electronic, and an ionic. The electronic component contributes with:

$$\left\{ 1 - \frac{1}{kD^2 + 1} \right\}$$

the ionic component contributes with:

$$\frac{1}{[k^2D^2 + 1][k^2D^2 + 1 + T_e/T_i]}$$

times the power reflected back by the free electrons. Here $k = \frac{4\pi}{\lambda}$, where $\lambda$ is the operating frequency (430 MHz for Arecibo), and D is the Debye length ($= 69\sqrt{T_e/N}$ meters). Note that the value of $k$ is twice the usual, $k = \frac{2\pi}{\lambda}$. The physical
reason is that the signal has to make a round trip.

In the DAR, $kD \ll 1$ or $D \ll \lambda$, then the ratio of the scattered power (total) to the power scattered by free electrons is due only to the ionic component, when $T_e \approx T_i$, as at night, and the value of the ratio is (Evans, 1969):

$$\frac{P(tot)}{P(electrons)} = \frac{1}{1 + \frac{T_e}{T_i}} \quad (2.7)$$

When $T_e \neq T_i$ the approximation is still good (Farley, 1966), except near the equator.

The half width of the spectrum of the backscattered signal is proportional to $(T_i/m_i)^{\frac{1}{2}}$, where $m_i$ is the ion mass (Fejer, 1961)(Moorecroft, 1964), then $T_i$ can be calculated if $m_i$ is known, by:

$$\Delta f = \frac{1}{\lambda} \left(\frac{8kT_i}{m_i}\right)^{\frac{1}{2}} \quad (2.8)$$

The ratio $T_e/T_i$ determines the wing to the center amplitude, it depends only on that ratio, and allows its calculation.

Figure (2.4) gives an example of this dependence, for three different ions, $O^+$, $He^+$, and $H^+$ (Gordon, 1967). In the DAR,
the curve for $O^+$ holds, because $O^+$ is the majority ion in that region.
2.2 ELECTRON DENSITY MEASUREMENTS

2.2.1 Ionosondes

The electron density, \( N \), can be measured by ionosondes, for it plots virtual height, \( h' \), versus \( N \). After the ionograms have been reduced, the value of \( N \) versus real height, \( h \), can be found. Nevertheless this gives only the values of \( N \) below the maximum of the electron density.

2.2.2 Rockets

Another way of measuring \( N \) is by direct measurement, with rockets. The problem with them (in addition of the high cost) is the relatively short time they stay in the air. As \( N \) is highly variable, it is desirable to have a continuous track of it (electron density). The electron density, for the diurnal variation, would require at least 24 rockets (1 by hour) to be launched.

2.2.3 Incoherent scatter technique

The incoherent scatter technique can be used to measure
N, with the advantage of giving a continuous real time coverage of N versus real height, h. There are three ways to determine N using this method: power profile, Faraday rotation, and the plasma line.

2.2.3.1 Power Profile

The backscattered power profile:

\[ P(h) = C \times \frac{N(h)\sigma(h)}{h^2} \quad (2.9) \]

is proportional to \( N(h) \), where \( C \) is a constant, \( \sigma(h) \) is the total cross section attributable to the ionic component (its value is the area under the curve of figure (2.4)). The problem is the determination of \( \sigma(h) \). The ratio of \( \sigma(h) \) to \( \sigma_e(h) \) (the total cross section attributable to the electron component) is:

\[ \frac{\sigma(h)}{\sigma_e(h)} = \frac{1}{(1 + k^2D^2)(1 + k^2D^2 + \frac{T_e}{T_i})} \quad (2.10) \]

This expression is valid only for a small range of \( \frac{T_e}{T_i} \).

Figure (2.5) gives the values of \( \frac{\sigma(h)}{\sigma(h)} \), even outside the range \((1 \leq \frac{T_e}{T_i} \leq 3)\), for the case \( \lambda >> 4\pi D \), as in the DAR. The values for \( \frac{\sigma(h)}{\sigma_e(h)} \) out of the validity range of the expression
(2.10) were calculated by Moorecroft (1963). In practice the value $T_e/T_i$ lies within the range $1 \leq \frac{T_e}{T_i} \leq 3$, and for the portion of ionosphere of interest ($kD << 1$). If the value of $N(h)$ can be determined by an independent method at a point $h_o$, then by the relation:

$$\frac{P(h)}{P(h_o)} = \frac{N(h)}{N(h_o)}$$

(2.11)

the altitude profile can be determined.

2.2.3.2 Faraday rotation

It is known that a signal propagating in the ionosphere, with frequency greater than the plasma frequency, due to the Earth's magnetic field, it splits into two components, circularly polarized in opposite directions, with different phase velocities. At a given instant the net result is a rotation of the wave electric field with respect to the initial plane containing the electric field. The amount of rotation (Browne et al., 1956) is:

$$\Omega = \frac{1}{4\pi} x \frac{e^3 \mu_o}{m_e f^2 c \varepsilon_o} \int_0^R N(r)H(r) \cos \theta(r) \, dr$$

(2.11)

where $\mu_o =$ free space permeability, $\varepsilon_o =$ permittivity of free
space, \( N(r) = \) electron density at range \( r \), \( H(r) = \) magnetic excitation \( (H = \mu B) \), \( \theta (r) = \) angle between the ray and the magnetic field, at range \( r \), \( c = \) the velocity of light.

For vertical propagation, the two way rotation between the ground and \( h \) is:

\[
\Omega = \frac{0.94 \times 10^{-2}}{f^2} \int_0^h N(h)H(h) \cos \theta(h)dh
\]

or

\[
\Omega = \frac{0.94 \times 10^{-2}}{f^2} \langle H \cos \theta \rangle \int_0^h N(h)dh
\]  \hspace{1cm} (2.14)

\[
N(h) = \frac{1}{C_1 \langle H \cos \theta \rangle} \times \frac{d\Omega}{dh}
\]  \hspace{1cm} (2.15)

Then the determination and differentiation of \( \Omega \) gives an electron density profile, if \( H \cos \theta \) is known (generally it is) (Millman et al., 1961). \( C_1 \) is a constant (the frequency \( f \) is included in the constant).

2.2.3.3 Plasma line

Another way to determine the altitude profile of electron density is using the plasma line. The plasma line is a weak
line near the electronic plasma frequency, enhanced by photoelectrons. Only in daytime can it be observed with the present radars in operation (Gordon, 1967), (Evans, 1969). For \( kD \ll 1 \) the energy in the electronic component of the spectrum appears as a pair of lines, displaced from the transmitter frequency by a value near the plasma frequency \( f_N \). Varying the receiver frequency, and observing the height of the plasma line, it is possible to determine the altitude profile of the electron density (Evans, 1969). Figure (2.6) gives an example of the application of this method for Arecibo. The plasma line gives the value of \( N \) with good precision (1%)! Another advantage is that no reference value is needed.
2.3 OTHER MEASUREMENTS

Many parameters could be measured in the ionosphere, but due to the passage of a HF radio wave, the parameters likely to change appreciably in the DAR are: \( T_e, N, v_r \) (radial electron drift velocity).

The best way to measure ionospheric parameters during a heating experiment is by means of an incoherent scatter technique, for with a single instrument \( N, T_i, T_e \), and \( v_r \) can be continuously monitored, from the ground, and with accuracy enough to distinguish variations produced by the passage of a strong HF radio wave.

By measuring the spectrum carefully, the drift velocity of the plasma can be measured (Behnke, 1968) (Evans, 1969) (Behnke, 1970). If \( v_r' \) is the drift velocity of the plasma as a whole, the entire spectrum will be shifted, without any change in shape, by:

\[
\Delta f = \pm \frac{2v_r'}{\lambda} \text{ Hz} \quad (2.16)
\]
if the electrons have a different velocity from the ions, the
Figure (2.7) gives an example on how this asymmetry can be
converted in \( v_d \sqrt{v_e} \), where \( v_d \) is the drift velocity, and \( \overline{v_e} \)
is the average thermal electron velocity:

\[
\overline{v_e} = \sqrt{\frac{2kT_e}{m_e}}
\]

The line of sight velocity (radial), \( v_r \), can be measured, but no information about the vector \( v_r \) is gathered from these measurements.

There are several schemes that can be used to measure the
vector \( v_r \). The first that one can think of is the straightforward
scheme: two complete incoherent scatter radar stations (receiver
and transmitter), and one station composed only of a receiver,
located in the base of a tetrahedron, a couple of hundred kilo-
meters apart, as seen in figure (2.8). This solution is rather
expensive, and does not exist presently such set of radars so
suitably located.

A simpler scheme using only one steerable receiving and
transmitting station, has been used at the AO (Behnke, 1970), assuming that the volume scanned by the radar was relatively small, and that the drift velocity did not vary too much within this volume, and within the time it takes to make the measurements (about 20 minutes). The idea is shown in figure (2.9). The scanned volume, unfortunately is large for the heating experiment.

A method to determine the ion composition of the ionosphere, by means of incoherent scatter, is this: compare the returned spectrum with a "library" of previously calibrated spectra, by choosing the shape of the spectrum that matches the one returned. The ion composition can be immediately determined. This procedure is done automatically by an electronic computer (Evans, 1969).

Information about the ion-neutral collision frequency also can be inferred from the shape of the returned spectrum (Evans, 1969).
Figure 2.1
Figure 2.2
Figure 2.3
Figure 2.4

RELATIVE POWER SPECTRA DENSITY

\[ \lambda \left( \frac{m_i}{8KT_e} \right)^{1/2} \]

\( T_e / T_i = 3.0 \)
\( T_e / T_i = 2.0 \)
\( T_e / T_i = 1.0 \)
\[ \sigma = \text{Observed Electron Cross Section} \]
\[ \sigma_e = \text{Radar Cross Section} \]

\[
\frac{\sigma}{\sigma_e} = (1 + \frac{T_e}{T_i})^{-1}
\]

Numerical Integration Of Spectrum

Figure 2.5
Figure 2.7

Graphs showing the Doppler shift ($\Delta f_e$) as a function of power ($W^{\theta}/N^{\theta/2}$) for different values of $T_e/T_i$, $v_d/v_e$, and $\alpha$. The graphs are logarithmic and illustrate the relationship between these parameters and the Doppler shift.
Figure 2.8
ASSUMPTION:
\[ \nu \approx \nu_A \approx \nu_B \approx \nu_C \]

Figure 2.9
3. ARECIBO OBSERVATORY FACILITIES

3.1 INSTRUMENTS

This work is done based in the Arecibo Observatory capabilities, that is a good place to make artificially induced modifications in the ionosphere (mainly DAR). Its 305 meter spherical reflector is the largest in the world, and can be used for transmitting or receiving purposes. The HF antenna is located in the focus of the reflector, and the HF array has a half power beamwidth of about 10 degrees, the radiation is sent toward the zenith.

The transmitter for high frequency modification has power up to 160 kW, it is tunable from 4 to 25 MHz, and the type of the emission can be pulsed or CW (continuous wave). In the case of this work, CW emission only will be considered.

Gordon and LaLonde (1961), Gordon (1964), Carlson (1965), Evans (1969), Showen (1969), among others have described the facilities and capabilities of the Arecibo Observatory, except
for the new HF transmitter, now in use.

Directly involved with the modification experiments are the following instruments: HF transmitter, HF antenna, antenna reflector, 430 radar transmitter, 430 radar receiver, interface, CDC 3300 electronic computer, ionosonde, photometers.

The HF antenna has the capability of sending the power in any of the magnetoionic modes (ordinary or extraordinary), by delaying $90^\circ$ in the phase of one of the crossed dipoles that constitute the HF antenna.

The radar antenna (430 MHz) can be steered up to 20 degrees from the zenith, in any azimuth.

At 160 KW the aperture power of the HF array is:

$$AP = E_x \times \frac{\pi}{4} \times D^2 \times p$$

$$AP = 0.6 \times 10^4 \text{ MW} - \text{m}^2$$

The modifying wave is the HF one, the radar diagnoses the modification in the ionosphere, while the photometers and the ionosonde provide additional information.
The radar is computer controlled, and as seen in chapter 2 can measure simultaneously the electron and ion temperatures, and the electron density. With more time available also the line of sight electron velocity can be measured in addition (Behnke, 1970).
3.2 DIAGNOSTIC MEANS

In order to diagnose the ionospheric modifications, the parameters $T_e$, $T_i$, $N$, and $v_r$, should be measured. As seen in chapter 2, the radar backscatter can be used to measure all these variables continuously. The 430 MHz radar at Arecibo, with 10 minutes half-power beamwidth, is able to make this measurement, before, during, and after the modification experiment.

An ionosonde also is available in order to gather additional information on the electron density. Photometers are in operation at Arecibo and are used to detect variations in the airglow emission, and indirectly information over the modification on the neutral temperature, and ion temperature.

The photometers can detect temperature variations, but they cannot resolve it in altitude, since they measure the integrated emission instead of the emission of each height. The emission comes mainly from 200 to 250 KM of altitude, for photometers, specially from the 6300A oxygen line.
4. MODIFICATION OF THE IONOSPHERE DUE TO THE PASSAGE OF HIGH FREQUENCY RADIO WAVE

4.1 ASSUMPTIONS

The problem may be reduced to calculating the power deposited (absorbed from the wave), and inferring how this power modifies the NDAR and DAR parameters.

Initially some assumptions need to be made:

4.1.1 The Incident Radio Wave

The frequency of the wave (modifying wave) will be in the HF (high frequency) region of the electromagnetic spectrum. This wave will be generated by a 160 KW (KiloWatt) transmitter, using as antenna a pair of crossed dipoles in the focus of the 305 meter reflector dish at Arecibo. The radiation will be directed into the zenith. The equipment has the capability of sending the wave in the ordinary or extraordinary modes.

4.1.2 Place

Arecibo Observatory (AO), Arecibo, Puerto Rico, 00612
Latitude: 18° 20' N

Longitude: 66° 45' W

4.1.3 Magnetic Field

The magnetic field in the region of interest (50 to 400 KM) will be considered constant. A value of \( B \) (magnetic induction), applied to the AO is: magnitude \( B = 0.37 \) gauss \((3.7 \times 10^{-5} \text{ Wb/m}^2)\); dip angle = 50°.

4.1.4 Time of the Experiment

Nighttime. The choice of a night experiment, in the initial phase, is dictated mainly by the facts: a) the three temperatures are approximately equal at night, any \( T_e \) modification is more easily detected; b) the absorption in the NDAR is small at night, then more power can reach the DAR (Davies, 1965). In the near future experiments will be done during the daytime also.

4.1.5 Electron Density Profile

The radar backscatter at the AO has the capability of measuring the electron density profile simultaneously with the electron
and ion temperatures, then, instead of assuming an analytical profile, as a Chapman layer, for instance, in this work the computer programs are designed to handle an electron density profile given by a table, based on actual data.

4.1.6 **Index of Refraction**

This quantity, a complex number, will be computed, by means of computer program ONE, according to the Appleton-Hartree theory (Budden, 1966), with the Sen-Wyller improvement (Sen and Wyller, 1960). The magnetic field and collisions will be taken into account. No approximations are being used. The real and imaginary parts of the index of refraction \( n \) are presented in chapter one, figures (1.8A) and (1.8B), for an incident frequency of 6 MHz, electron density profile given by figure (1.3), (average profile), magnetic field as in section (4.1.3), and collision frequency given by figures (1.7 and 1.9). The collision frequency used is the electron collision frequency, as calculated in program TWO.
4.1.7 Limit Between the DAR and the NDAR

It is placed at 160 KM arbitrarily, based on the values of the real part of the index of refraction. Note that in the lower part of the DAR, $\mu$, is almost unity, for either mode, then very small deviation occurs, giving, then, more accuracy on calculating the deviation on the DAR, because it insures that in the NDAR the deviation is very small indeed (see program ONE in the appendix).
4.2 NDAR MODIFICATION

The main effect is the absorption of the wave in the region, that is neglected in other works, but as the computation below indicates, it is not negligible. The power deposited into the region also will be calculated.

4.2.1 Absorption

In order to calculate the absorption in the NDAR, a 6 MHz radio wave will be used, and the altitude range from 50 to 160 KM considered.

From Maxwell's equations (Jackson, 1962), that can be put into a form involving the scalar, \( \phi \), and the vector, \( \mathbf{A} \), potentials, one is able to show that the set of equations:

\[ \nabla^2 \phi - \mu_0 c \frac{\partial^2 \phi}{\partial t^2} = \rho/\varepsilon \]  \hspace{1cm} (4.1)

\[ \nabla^2 \mathbf{A} - \mu_0 \varepsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\frac{\partial \mathbf{J}}{\partial t} \]  \hspace{1cm} (4.2)

where:

\[ \mathbf{E} = -\nabla \phi - \frac{\partial \mathbf{A}}{\partial t} ; \quad \mathbf{B} = \nabla \times \mathbf{A} ; \quad \mathbf{B} = \mu_0 \mathbf{H} \]
with \( \phi \) and \( \mathbf{A} \) connected by the Lorentz condition:

\[
\nabla \cdot \mathbf{A} = \mu_0 \varepsilon \frac{\partial \phi}{\partial t}
\]

is an alternative form of the said equations (Paris and Hurd, 1969). \( \varepsilon \) is the medium permittivity, \( \mu_0 \) is the permeability, \( \rho \) is the volumetric charge density, and \( \mathbf{J} \) is the current surface density. In antenna problems, \( \phi \) is not really necessary, since \( \mathbf{H} \) \((\mathbf{B} = \mu_0 \mathbf{H})\) can be obtained from \( \mathbf{A} \), and then, the Maxwell's equation:

\[
\nabla \times \mathbf{H} = \varepsilon \frac{\partial \mathbf{E}}{\partial t}
\]

(4.3)

can be integrated with respect to time to give:

\[
\mathbf{E} = \frac{1}{\varepsilon} \int \nabla \times \mathbf{H} \, dt
\]

(4.4)

Then only the solution of the equation (4.2) is needed, this solution is:

\[
\mathbf{A} = \frac{\mu_0}{4\pi} \int_{\text{volume}} \frac{\mathbf{J}(t-r/v)}{r} \, d\mathbf{V}
\]

(4.5)

The calculation of the antenna fields is reduced then to the evaluation of equation (4.5), if the antenna current distribution is known. Then from \( \mathbf{B} = \nabla \times \mathbf{A} \) and \( \mathbf{E} = \frac{1}{\varepsilon} \int \nabla \times \mathbf{H} \, dt \) the
fields can be determined.

Generally the solution of the equation (4.5) is a complicated task, and approximations are used. The approximation for the far-field is different from the one for the near-field. In the particular example of the AO antenna (quasi-parabolic reflector), using the criterion given in the ITT Reference Data for Radio Engineers (5th Edition), the near-field (Fresnel) is located from the antenna to a distance given by: \( d = \frac{A}{2} \), where \( A \) is the effective area of the antenna aperture, and the wavelength. For Arecibo the values are:

\[
\lambda = 50 \text{m} \quad (f = 6 \text{ MHz})
\]

\[
A = 0.5 \times \frac{\pi}{4} \times (305)^2 = 3.7 \times 10^4 \text{ m}^2
\]

Then \( d = 370 \) m. The Fresnel field approximation, then holds till 370 m from the antenna only.

Using the same reference as above, the criterion for the far-field (Fraunhofer) is: the approximation holds for distances larger than \( d' \), where \( d' = \frac{2D^2}{\lambda} \), \( D \) is the diameter of the
aperture of the antenna (305 m for Arecibo), \( \lambda = 50 \) m. Then the value of \( d' = 3.75 \) KM. The Fraunhofer approximation (far-field) is good after 3.75 KM from the antenna.

In the case of this work only the Fraunhofer (far-field) region will be considered, because both NDAR and DAR are well above 3.75 KM.

The \( E \) and \( H \) fields, produced by a current element, are (Paris and Hurd, 1969):

\[
E = \frac{E_0}{r} \exp \left( -i \frac{\omega}{c} r \right) \quad (4.6)
\]

\[
H = \frac{1}{\eta} k \times E \quad (4.7)
\]

where:

\[
\eta = \sqrt{\frac{\mu_0}{\varepsilon}}
\]

As seen in chapter 1, the velocity of propagation of a radio wave in the ionosphere is not \( c \) (velocity of light in free space), but \( v = c/n \); as \( n = \mu - i\chi \), equation (2.6) becomes:

\[
E = \frac{E_0}{r} \exp(-i \frac{\omega n}{c} r) = \frac{E_0}{r} \exp(-i \frac{\omega}{c} r) \exp\left(-i \frac{\chi}{c} r\right)
\]
the real exponent ($-\omega x r/c$) is a measure of the absorption of
the wave. From now on, instead of $E$ and $H$, only the $E$ field
will be used, since $E$ and $H$ are in phase and related by the
equation (4.7).

The next step is to calculate the absorption that the
wave suffers in the NDAR.

The absorption that a wave will have, between 50 and 160 KM
of altitude is given by the value of the exponent ($-\omega x r/c$).
Calling $\omega x/c = k(h)$, for $x$ is a function of the altitude, the
value by which a wave is reduced in amplitude travelling from
50 to 160 KM is given by:

$$A_{o,x} = \exp \left( \int_{50}^{160} K_{o,x}(h) \, dr \right)$$

(4.8)

$k_{o,x} = \omega x_{o,x} / c$, where $x_{o,x}$ are the values of the imaginary part
of the index of refraction for the $0$- and $X$-modes, as calculated
in chapter 1 (see appendix for computer program ONE). The ray
path will be an almost straight line, for $\mu_{o,x}$ (real part of $n$)
is approximately equal to 1 in the whole NDAR.

The absorption below 50 KM is negligible, because $\chi_{o,x}$ is almost zero. Nevertheless, in the NDAR, while being small, $\chi_{o,x}$ is not negligible, and the integral (4.8), after being solved in the RICE B-5500 computer (see appendix for program THREE), gives the following values for 160 KM of altitude:

- **ORD WAVE ( $\chi_o$ )** $A_o = 0.967$
- **EXT WAVE ( $\chi_x$ )** $A_x = 0.964$

The values above calculated were for nighttime conditions. If expressed in decibels, as usual, the total absorption in the NDAR becomes:

- $A_o(\text{db}) = 20 \log (0.967) = -0.3 \text{ db}$
- $A_x(\text{db}) = 20 \log (0.964) = -0.5 \text{ db}$

The extraordinary wave is slightly more absorbed than the ordinary, but the absorption is small for both modes.

### 4.2.2 Heat Deposited

The power is deposited into the region by means of Ohmic
losses, through the volumetric Ohm's law:

\[ Q = \frac{1}{2} \text{Re}\{\sigma\} E^2 \]  

(4.9)

where \( Q \) is the power deposited by volume, \( \sigma \) is the conductivity tensor, and \( E \) is the RMS (root-mean-square) value of the electric field, supposed to be sinusoidal (the \( \frac{1}{2} \) factor arises from this supposition).

If the square of the angular gyrofrequency (\( \omega_H \)) is much smaller than the square of the angular frequency of the heater wave (\( \omega \)), then the equality below holds, following argument after Meltz and LeLevier (1970):

\[ Q = \frac{1}{2} \text{Re}\{\sigma\} E^2 = \eta S \text{Re}\{\sigma\} \]  

(4.10)

For the present case, \( Y = 0.178 < 1 \), then \( Y^2 = 0.0314 \ll 1 \).

In equation (4.10) \( \eta \) is the local impedance, as seen by the wave (that is the same as the free space impedance, for the NDAR heights = 377 Ohms), and \( S \) is the power density and can be calculated by the radar range equation:

\[ S_0 = \frac{PA G}{4 \pi r^2} \quad S_x = \frac{PA G}{4 \pi r^2} \]  

(4.11)
here $P$ is the power sent into space by the antenna, $G = G(\psi)$ is the antenna gain, and $r$ is the range. The expression for the gain is:

$$G(\psi) = \frac{4\pi A}{\lambda^2} g(\psi)$$  (4.12)

being $A_e$ the effective area of the reflector (for the AO antenna, in the HF region it is approximately $\frac{1}{2}$ of the physical area of the antenna aperture), $g(\psi) = \cos^2 k\psi$, is an approximated expression for the directivity of the antenna (ITT-Reference Data for Radio Engineers); the half power of the antenna is:

$$\frac{\lambda}{D} = \frac{50}{305} = 0.164 \text{ rad} = 9.5^\circ \quad \psi_{hp} = 4.75^\circ$$

$$\cos^2 (k\psi_{hp}) = \frac{\pi}{4} \quad k\psi_{hp} = \frac{\pi}{4} \quad k = 9.6$$

then $k = 9.6$.

The conductivity tensor is the following:

$$\sigma' = \begin{pmatrix} \sigma_1' & 0 \\ 0 & \sigma_2' \end{pmatrix}$$

$\quad \sigma = \begin{pmatrix} \sigma_2 & \sigma_1 \\ 0 & 0 \end{pmatrix}$

where $\sigma_0'$ is the conductivity along the magnetic field, $\sigma_1'$ is the conductivity perpendicular to $\mathbf{B}$ (Pedersen conductivity)
and when there are $B$ and $E$ fields, the current produced by
different ion and electron collision frequencies in the
$E \times B$ direction is due to the Hall conductivity, $\sigma'_2$.

The expressions for $\sigma'_0, \sigma'_1$, and $\sigma'_2$ are (Whitten and Poppoff, 1971):

$$\sigma'_0 = \varepsilon_0 \left[ \frac{i \omega_e^2}{\omega + i \nu_e} + \frac{i \omega_i^2}{\omega + i \nu_i} \right]$$  \hspace{1cm} (4.14)

$$\sigma'_1 = \varepsilon_0 \left[ \frac{i \omega_e^2 (\omega + i \nu_e)}{(\omega + i \nu_e)^2 - \omega_H^2} + \frac{i \omega_i^2 (\omega + i \nu_i)}{(\omega + i \nu_i)^2 - \omega_iH^2} \right]$$  \hspace{1cm} (4.15)

$$\sigma'_2 = \varepsilon_0 \left[ \frac{-\omega_e^2 \omega_H}{(\omega + i \nu_e)^2 - \omega_H^2} + \frac{\omega_i^2 \omega_iH}{(\omega + i \nu_i)^2 - \omega_iH^2} \right]$$  \hspace{1cm} (4.16)

where $\omega_e$ is the plasma angular frequency for electrons, $\omega_i$
the plasma angular frequency for ions, $\omega_H$ the angular gyro-
frequency for electrons, $\omega_iH$ the angular gyrofrequency for ions,
$\nu_e$ the electron collision frequency, $\nu_i$ is the ion collision
frequency, and $\omega$ is the angular frequency of the heating wave.
These components of the conductivity tensor are for the case of a two component plasma (electrons and positive ions), under alternating driving fields (AC conductivity).

In expressions (4.14), (4.15), and (4.16) the second terms of the right hand sides are much smaller than the first terms, because while the denominators are comparable the numerators are orders of magnitude smaller in the second terms, due to the fact that the ion plasma angular frequency is \( \omega_i = \left( \frac{N_e^2/\varepsilon_o m_i}{\omega_{pe}^2} \right)^{1/2} \) much smaller than the electron plasma angular frequency, \( \omega_e = \left( \frac{N_e^2/\varepsilon_o m_e}{\omega_{pe}^2} \right)^{1/2} \). The components of the tensor become:

\[
\sigma_0' = \frac{i \varepsilon_o \omega^2}{\omega + i \nu_e} \tag{4.17}
\]

\[
\sigma_1' = \frac{i \varepsilon_o \omega^2 (\omega + i \nu_e)}{(\omega + i \nu_e)^2 - \omega_H^2} \tag{4.18}
\]

\[
\sigma_2' = \frac{-\omega^2 \omega_H \varepsilon_o}{(\omega + i \nu_e)^2 - \omega_H^2} \tag{4.19}
\]

Only the real part of \( \sigma \) contributes to heat deposition.

The real parts of \( \sigma_0', \sigma_1', \) and \( \sigma_2' \) are:
\[ \sigma_0 = \text{Re}\{\sigma_0^i\} \quad \sigma_1 = \text{Re}\{\sigma_1^i\} \quad \sigma_2 = \text{Re}\{\sigma_2^i\} \]

In this work both \( \sigma_0 \) and \( \sigma_1 \) will be considered. Generally in other works only \( \sigma_0 \) is considered, projected into the direction of interest.

In the ionosphere the heat deposited is the sum of two parts, one due to \( \sigma_0 \), and the other due to \( \sigma_1 \), projected along the range.

The value of the deposited heat, for the two modes, in the NDAR is given by the expressions:

\[ Q_0(r, \psi) = \eta_0 \times \frac{PA^2 A e^{2(9.6\psi)}}{\lambda^2 r^2} \times (\sigma_0 \sin I + \sigma_1 \cos I) \quad (4.20) \]

\[ Q_x(r, \psi) = \eta_0 \times \frac{PA^2 A e^{2(9.6\psi)}}{\lambda^2 r^2} \times (\sigma_0 \sin I + \sigma_1 \cos I) \quad (4.21) \]

The result of the computations are shown in figures (4.1) and (4.2), the first presents the ordinary case, and the second the extraordinary one.
4.3 MODIFICATION IN THE DAR

The situation in the DAR is not so simple as in the NDAR, because the magnitude of the index of refraction is not anymore near unity, so that the radio ray is bent, and absorbed, while passing into the region.

The parameters likely to change due to the passing wave are: $T_e$, $T_i$, $N$, $v_r$. $T_e$ is supposed to increase (Farley, 1963; Gurevich, 1967). Due to the electron collisions, the absorbed energy is used to increase the kinetic energy of the electrons, thus raising their temperature, rather than reradiating coherently the energy into the wave. By similar reason $T_i$ should also increase, but as the mass of the ions (mainly $O^+$, in the DAR) is much bigger than the electron mass, the energy needed to increase the ions kinetic energy by about the same percentage is much higher. The effect on the ion temperature is then negligible compared with the effect on the electron temperature.

The electron density is likely to decrease (Meltz and
LeLevier, 1970; Farley, 1963). The motion of the electrons, along the $B$ field lines should increase due to thermal expansion, then the line of sight velocity measured in the radar, $v_r$, should present an increase, specially in the upper part of the DAR, (Meltz and LeLevier, 1970), because of the small neutral density.

The absorption of the radio wave will be presented now, followed by the deposition of power in the DAR. After that the equations for the plasma behavior in the DAR will be shown. As a simple example, in chapter 5, a calculation for the Arecibo conditions is done.

### 4.3.1 Absorption

In calculating the power deposition, Meltz and LeLevier (1970) neglected the absorption until the height of maximum deposition (near the electron density peak, in the DAR). Even being small, as seen in this section, the absorption is not negligible, away from the peak.
The absorption along the range, for a vertically launched wave, for Arecibo conditions, is shown in figure (4.3).

The relation between the range (along the ray path) and the altitude is given by:

$$dh = dr \cos \alpha$$ (4.22)

where $\alpha$ is the angle between the ray and the vertical. The difference between altitude and range is small in the case of a 6 MHz wave, for from actual measurements in Arecibo (Gordon et al., 1971) at 300 KM of altitude the deviation is of the order of 30 KM, making the difference between height and range smaller than 10%. In the deposition calculations the altitude was used as range.

The absorption calculations were made in the RICE B-5500 computer by means of program THREE (see appendix).

4.3.2 Power Deposition

The power deposition in the DAR is calculated similarly to the way it was done in the NDAR. Again the formula below
holds (following argument by Meltz and LeLevier (1970)):

$$Q = \eta \sigma S$$ \hfill (4.23)$$

the conductivity, as well as the local impedance, now have different values for the different modes.

For the ordinary wave the quasi-transversal approximation is used (QT-approximation, see Budden, 1966), then the conductivity to be used is $$\sigma_1$$, and $$\eta_{\text{ord}}$$ is given by:

$$\eta_{\text{ord}} = \frac{\eta_0}{\sqrt{1 - X}}$$

For the extraordinary wave the QL-approximation is used (QL means quasi-longitudinal), and the values of $$\sigma$$ and $$\eta_{\text{ext}}$$ are:

$$\eta_{\text{ext}} = \eta_0 / \sqrt{1 - \frac{X}{1 - Y}}$$

$$\sigma = \sigma_0$$

The use of these approximations is justified by the fact that the ordinary wave tends to be perpendicular to the magnetic field near the maximum of the electron density profile, while the extraordinary wave tends to be parallel to $$\mathbf{B}$$ at the extraordinary reflection level. In this work the computations are...
made using the assumption that the ordinary wave barely penetrates, while the extraordinary is reflected. The difference between height and range being smaller than 10%, the altitude is used as range.

The value of $S$, calculated in the same way as in the NDAR, is:

$$S_{o,x} = \frac{pA^2 A \cos^2(k\gamma)}{\lambda x e^{2 \pi}} \frac{\lambda^2 x \gamma^2}{2 \lambda r^2} \quad (4.24)$$

Then, in the DAR the formulas for heat deposition by volume become:

$$Q_o = \frac{\eta_o}{\sqrt{1-x}} x \frac{pA^2 A \cos^2(k\gamma)}{\lambda^2 x \gamma^2} \frac{\lambda^2 x \gamma^2}{2 \lambda r^2} \times \sigma_1 \quad (4.25)$$

$$Q_x = \frac{\eta_o}{\sqrt{1-\frac{x}{1-y}}} x \frac{pA^2 A \cos^2(k\gamma)}{\lambda^2 x \gamma^2} \frac{\lambda^2 x \gamma^2}{2 \lambda r^2} \times \sigma_0 \quad (4.26)$$

for the ordinary and extraordinary waves, respectively. Note that $Q$ is a function of the range and of the zenith angle at
the launching point. From equations (4.25) and (4.26), it is seen that the deposition has a maximum for the ordinary wave (ordinary caustic) at the maximum of the electron density profile \((X = 1)\), and a maximum for the extraordinary wave at altitude in which it is reflected \((X/(1-Y) = 1)\). These facts reinforce the assumptions used in deriving equations (4.25) and (4.26).

The transmitter sends the power into a volume within the pattern of the antenna, then the power deposited in a particular point can only be determined exactly if one knows the ray path of the heater wave, for each zenith angle of interest at ground. Determination of the ray path is a complicated task, but there are special programs (computer programs) that can handle it. A complete reference can be found in the ray tracing issue of Radio Science ([3], 1968), and from Jones (1968). In this work nevertheless, as the range is not too different from the altitude (10%), the height is used as range. The simplicity of this pro-
procedure, and the fact that the result is not modified by more than 16% justify this procedure.

If the angle between the ray and the vertical is known (by means of a ray-tracing computer program) the horizontal displacement can be determined by:

\[ D = \int_{\text{ground}}^{r} \sin \alpha (r) \, dr \]

where \( r \) is the range and \( \alpha \) is the angle between the ray and the vertical.

Figure (5.1) presents, out of scale for clarity, the two chosen field lines (A and B), the approximated deposited power, under the above assumptions, and the ionospheric regions: NIR - non ionized region; NDAR - non deviative absorption region; DAR - deviative absorption region.

The power deposited by volume for the ordinary wave varies with the zenith angle at ground along the field line A, for the extraordinary wave it varies with the range (approximately equal
to the altitude), along the field line B.

The contour maps, for the range and zenith angle, of the heat deposited in the DAR is shown in figures (4.4) and (4.5). The values of the depositions were calculated by means of the computer program FOUR (see appendix). The power deposited is a function of range, as well as of the zenith angle at ground. The computations were made for ranges from 50 to 450 KM, for each 5 KM interval, and for zenith angles from zero to 9.5 degrees, for each half degree interval.

4.3.3 Plasma Equations for the DAR

The equations governing the phenomena in the DAR will be stated, under the assumption that the plasma is allowed to expand and contract only along the magnetic field lines (Farley, 1963) (Meltz and LeLevier, 1970). The space coordinate is given by x. The relation between x and h is given by:

\[ h = x \sin I \]

this is because x is along the direction of the Earth's magnetic
field, that is considered constant, with a dip angle given by I. Only the meridian plane in the North-South magnetic direction will be taken into account.

The plasma conservation equations are:

**CONSERVATION OF MASS**

\[ \rho \, dx = dM = \text{constant} \quad (4.30) \]

**CONSERVATION OF MOMENTUM**

\[ \frac{Du}{Dt} = - \left[ \frac{1}{\rho} \times \frac{\delta p}{\delta x} + \nu \, \sin \theta \, u + \frac{1}{\rho} \, \delta \rho \, g \, \sin \theta \right] \quad (4.31) \]

**CONSERVATION OF ENERGY FOR ELECTRONS**

\[ \frac{DE}{Dt} + \frac{p}{e} \frac{D}{Dt} \left( \frac{1}{\rho_e} \right) = \left( \frac{1}{\rho_e} \right) \left\{ Q - L_e + \frac{\partial}{\partial x} \left( \frac{\partial T_e}{\partial x} \right) \right\} \quad (4.32) \]

**CONSERVATION OF ENERGY FOR IONS**

\[ \frac{DE_i}{Dt} + \frac{p_i}{e} \frac{D}{Dt} \left( \frac{1}{\rho_i} \right) = \left( \frac{1}{\rho_i} \right) \left\{ -L_i \right\} \quad (4.33) \]

Along with these equations also the perfect gas laws are supposed.
to hold. $\rho$ is the plasma mass density, while $\rho_e$ and $\rho_i$ are the electron and ion mass density respectively. The ions are supposed to be $O^+$. $u$ is the plasma bulk velocity, $p$ is the pressure.

\[ P_i = N k T_i \quad P_e = N k T_e \quad P = P_i + P_e \]

$E_i$ and $E_e$ are the specific energies of the ion and electron gases, respectively:

\[ E_e = \frac{3}{2} k T_e / m_e \quad E_i = \frac{3}{2} k T_i / m_i \]

$m_i$ and $m_e$ are the ion and electron masses respectively. They are related to the densities by:

\[ \rho_i = m_i N \quad \rho_e = m_e N \quad \rho = (m_i + m_e) N \]

$g$ is the acceleration of gravity (assumed constant). $\delta T_e$ is the variation in electron temperature, $\delta T_e = T_e - T_{e0}$. $T_{e0}$ being the temperature of the electrons before the heating.

$Q$ is the heat (power) deposition, while $L_i$ and $L_e$ are the losses due to collisions in the ion and electron gases. The collisional losses of ions are due mainly to the elastic col-
collisions between ions and neutrals and between ions and electrons.

The expression for $L_1$ is:

$$L_1 = \frac{3}{2} Nk \left\{ \nu_{ei} \delta_{ei} (\delta T_i - \delta T_e) + \nu_{in} \delta T_i \right\}$$

Collisional losses of the electron gas is due to elastic collisions between electrons and neutrals, electrons and ions, and inelastic collisions between electrons and $O^+$ (Dalgarno and Degges, 1968). The expression for $L_e$ is:

$$L_e = \frac{3}{2} Nk \left\{ \nu_{ei} \delta_{ei} (\delta T_e - \delta T_i) + \nu_{en} \delta T_e + (\nu_0 + \delta T_e) \right\}$$

the last term is due to the fine-structure transitions in atomic oxygen. Other electron losses are possible as the excitation of the rotational levels of $N_2$, electronic excitation of $O(1D)$, rotational, vibrational, and electronic excitation of $O_2$. However as the concentration of $N_2$ and $O_2$ is small in the DAR, these losses generally are not taken into account in the DAR.

$\delta T_i$ in analogy with $\delta T_e$ is given by $\delta T_i = T_{i0} - T_{ie}$. $\delta_{ei}$ and $\delta_{en}$ are the fractional losses in each collision, they are
\[ \delta_{ei} = \frac{2m_e}{m_i} \quad \delta_{en} = \frac{2m_e}{m_n} \quad m_n = \text{neutral mass (0)} \]

The energy is transported away from the heated region mainly by electron conduction, since the thermal conductivity of the ion gas is smaller than the electron thermal conductivity \( K_e \). The expression for \( K_e \) is (Whitten and Poppoff, 1971):

\[ K_e = 1.23 \times 10^{5} T_e^{5/2} \]

The thermal energy invested in expansion or gained in compression is shown in the terms (Meltz and LeLevier, 1970):

\[ p_i \frac{D}{Dt} \left( \frac{1}{\rho_i} \right) ; \quad p_e \frac{D}{Dt} \left( \frac{1}{\rho_e} \right) \]

The operators \( D/Dt \) are used to indicate the total derivative:

\[ \frac{D}{Dt} = \frac{\partial}{\partial t} + u \cdot \nabla \]

Momentum changes are determined by equation (4.28), while electron and ion temperature changes are determined by equations (4.29) and (4.30).

It is possible to approximate the set of 2nd order, non-linear...
equations representing the plasma behavior, in order to get an idea of the electron temperature variation in the DAR. In spite of the approximations, the computed values are in fairly good agreement with the experimental results (Gordon et al., 1971).

To evaluate the variation in $T_e$, equation (4.29) will be used. Considering that a steady state is reached, $DE_e/Dt = 0$. Supposing that the losses can be approximated by the losses to the ions (elastic losses), as $\delta T_i \ll \delta T_e$, $L_e$ becomes (Meltz and LeLevier, 1970):

$$L_e = \frac{3}{2} Nk \delta T_e \upsilon_{ei} \frac{m_e}{m_i} = 3Nk\upsilon_{ei} \frac{m_e}{m_i} \delta T_e$$

The pressure work (energy used for expansion or compression) is small, since the heat supplied is small, then (Meltz and LeLevier, 1970):

$$P_e \frac{D}{DT}(\frac{1}{e}) = 0$$

As small variations are expected, $K_e$ can be considered as a constant, then equation (4.29) becomes:

$$K_e \frac{d^2 \delta T_e}{dx^2} - 3Nk\upsilon_{ei} \frac{m_e}{m_i} \delta T_e = -Q \quad (4.34)$$
The solution of the equation (4.34) represents the variation in electron temperature, after the steady state has been reached, for each altitude (or range). The solution is presented in chapter 5, for nighttime conditions over the AO.
Figure 4.1

\[ \Psi = 0° \]

\[ \Psi = 3° \]

\[ \Psi = 6° \]

\[ \Psi = 9° \]

O - WAVE

\[ T_{eo} = 1100°C \]

OBS. : Figures Out Of Scale
Figure 4.2
Figure 4.4
Figure 4.5

OBS: Figure Out Of Scale

Field Line

X-WAVE

$\Psi = 9^\circ$

$\Psi = 6^\circ$

$\Psi = 3^\circ$

$\Psi = 0^\circ$

DAR

$T_{e0} = 1100^\circ K$

DISTANCE (Km)

RANGE (Km)
5. CALCULATION OF THE ELECTRON TEMPERATURE VARIATION

The purpose of this chapter is to present, as an example, the solution of the equation:

\[
\frac{d^2 \delta T}{dx^2} - \frac{3NKn}{e^2} m_i \frac{e}{m_i} \delta T = -Q
\]  

(5.1)

for the electron temperature variation, along selected magnetic field lines. Two field lines are used, the one passing by the point of maximum deposition due to the ordinary wave, the ordinary caustic, that will be called field line A; and the field line that passes through the point of maximum deposition due to the extraordinary wave, the extraordinary caustic, that will be called field line B.

As the ordinary wave is perpendicular to the magnetic field, near the ordinary caustic, the deposition of heat, along field line A, is a function of \( \psi \), the zenith angle at ground, approximately, keeping the range constant.

For the extraordinary wave, as it is parallel to the Earth's
magnetic field, near the extraordinary caustic, that happens to be also the extraordinary height of reflection, the heat deposited is a function of the range, keeping $\psi$ constant at zero degrees.

It should be noted that the frequency used (6 MHz) is such that the ordinary wave barely penetrates; and the extraordinary wave is reflected at the extraordinary caustic level. Then no power is deposited outside the radiation pattern, for the case of field line A, and no power is deposited, along field line B, for heights larger than the extraordinary caustic. Figure (5.1) shows this problem.
5.1 SOLUTION OF THE EQUATION (5.1)

Starting from the equation (5.1), repeated here for easy reference:

\[ K_e \frac{d^2 \delta T_e}{dx^2} - 3Nk_v e \frac{m_e}{m_i} \delta T_e = -Q \]  

(5.1)

the first step is to identify the functional dependence of the parameters, and the boundary conditions. \( K_e \) is the electron heat conductivity, given by the expression (Whitten and Poppoff, 1971):

\[ K_e = 1.23 \times 10^{-11} T_e^{5/2} \]

As \( K_e \) is only dependent on the electron temperature, it can be written:

\[ K_e = C_1 T_e^{5/2} \]  

(5.2)

where \( C_1 = 1.23 \times 10^{-11} \). If no large change is expected in the electron temperature ("large" here means more than about 50\%), according to Farley (1963), and Meltz and LeLevier (1970), \( K_e \) can be considered approximately constant. \( \delta T_e = T_e - T_{eo} \)
where $T_{e_0}$ is the electron before heating, and $T_e$ after it.

The independent variable in equation (5.1) is computed along the magnetic field lines, it is related to the altitude by:

$$x = \frac{h}{\sin \theta}$$

(5.3)

The second term in the left hand side can be put in the form:

$$3Nk_\alpha \frac{m}{m_i} \delta T_e = C_2 N^2 T_{e_0}^{-3/2} \delta T_e$$

where $C_2 = 8.6 \times 10^{-31}$ (again for $T_e = T_{e_0}$), $\nu_{ei} = 3.8 \times 10^{-5} N^2 T_{e_0}^{-3/2}$.

$Q(h, \psi)$ is taken from the calculations of chapter 4, as a function of the range (for the extraordinary wave) and of the zenith angle (for the ordinary wave).

Only the steady-state solution is being considered (electron temperature variation after equilibrium has been reached).

The height dependence of $N$ is given from figure (1.3), it is an input parameter in the computer programs.

It is time now to prepare the equation (5.1) to be integrated by numerical methods.
As an advantage of the proposed solution, the input parameters are given by tables, which permits fast recalculation of the electron temperature variation for different sets of inputs, like the same wave incident upon an ionosphere with different electron density profile, and/or different electron collision frequency, and/or different $T_{eo}$. This makes the computer programs very flexible.

The input parameters are:

a) $T_{eo}$ = undisturbed DAR electron temperature ($900^\circ$ or $1100^\circ$K);

b) $N(h)$ = electron density profile, considered variable with altitude, but almost constant in time, can be taken from actual measurements, instead from the use of an idealized mathematical function;

c) $Q(h, \psi)$ = power deposited by volume, as calculated in chapter 4, or by other suitable method (it is a function of the altitude and of the zenith angle at the launching point); for
more precise calculations a ray-tracing program has to be used, and the altitude has to be changed by the ray-path.

The constants $C_1$ and $C_2$ are independent of the ionospheric parameters. The electron density is supposed to vary much less than the electron temperature (one order of magnitude less) (Farley, 1963), then $N = N_0$ (electron density before heating).

Equation (5.1) can then be put in the form:

$$
\frac{5}{2} C_1 T_{eo}^{5/2} \frac{d^2 \delta T_e}{dx^2} - C_2 N^2 T_{eo}^{-3/2} \delta T_e = -Q(h, \Psi)
$$

(5.4)

here, as in section (4.3.3), the relation between altitude $h$, and $x$ (distance along the field line) is given by:

$$
h = x \sin I
$$

where $I$ is the dip angle (= 50°).

The boundary conditions were chosen as $\delta T_e = 0$ at the altitudes of 200 KM, and 455 KM.

In the particular case of this work $T_{eo} = 1100^\circ K$, then:

$$
C_1 T_{eo}^{5/2} = C_3 \quad C_2 T_{eo}^{-3/2} = C_4
$$
and the equation (5.4) can be simplified further, to:

$$\frac{d^2y}{dx^2} - c_5^2 y = - \frac{Q}{c_3}$$

where \( y = \delta T_e \), \( c_3 = 3.89 \times 10^{-4} \); \( c_4/c_3 = c_5 = 6.95 \times 10^{-32} \)

Finally:

$$\frac{d^2y}{dx^2} - c_5^2 y = - \frac{Q}{c_3} \quad (5.5)$$

Now equation (5.5) will be solved by finite difference techniques, for a constant \( \Delta h = 10 \) KM, then \( \Delta x = 13.6 \) KM, for magnetic field lines A and B.

The variable coefficients in equation (5.5), for each \( x \) are chosen to correspond to the same altitude. Table (5.1) presents the relations for the altitudes of interest. The value \( x = 0 \) was chosen to correspond to the altitude of 200 KM, for both field lines.

Equation (5.5), after applying the finite difference method becomes:

$$y_{i+1} - 2y_i + y_{i-1} - (\Delta x)^2 \frac{c_5^2 y_i}{c_3} = -Q(\Delta x)^2/c_3$$
where the second order derivative was replaced by:
\[
\frac{d^2 y_i}{dx^2} = \frac{y_{i+1} - 2y_i + y_{i-1}}{(\Delta x)^2}
\]

The solution of equation (5.5) is then reduced to the solution of the system of 25 equations below, that after putting the boundary conditions \(x = 0, y_0 = 0; x = 339, y_{26} = 0\), become:

\[
y_2 - 2y_1 + y_0 + [-(\Delta x)^2 c_5 N_1^2]y_1 = -Q_1 (\Delta x)^2 / c_3
\]

System

\[
y_3 - 2y_2 + y_1 + [-(\Delta x)^2 c_5 N_2^2]y_2 = -Q_2 (\Delta x)^2 / c_3
\]

\[
ym_{26} - 2y_{25} + y_{24} + [-(\Delta x)^2 c_5 N_{25}^2]y_{25} = -Q_{25} (\Delta x)^2 / c_3
\]

or

\[
y_2 + y_1[2 - (\Delta x)^2 c_5 N_1^2] + y_0 = -Q_1 (\Delta x)^2 / c_3
\]

System

\[
y_3 + y_2[2 - (\Delta x)^2 c_5 N_2^2] + y_1 = -Q_2 (\Delta x)^2 / c_3
\]

\[
ym_{26} + y_{25}[2 - (\Delta x)^2 c_5 N_{25}^2] + y_{24} = -Q_{25} (\Delta x)^2 / c_3
\]
where:

\[ y_0 = 0, \text{ and } y_{26} = 0 \]

The system above (System (5.2)) was solved by means of the Gauss-Jordan algorithm, using the computer program FIVE (see appendix). In the present case calculations are for the magnetic North-South meridian plane, for the field lines A and B.

The value of \((\Delta x)\), corresponding to a difference in altitude of 10 KM is \((\Delta x) = 13.6 \text{ KM}, \text{ and } (\Delta x)^2 = 170.41 \text{ KM}^2\).
5.2 RESULTS AND CONCLUSIONS

The equation (5.1) was solved for four cases:

a) field line $A, v_{ei} = 0$, $O$-wave.

b) field line $B, v_{ei} = 0$, $X$-wave.

c) field line $A, v_{ei} = 3.8 \times 10^{-5} \text{m} \times T_{eo}^{-3/2}$, $O$-wave.

d) field line $B, v_{ei} = 3.8 \times 10^{-5} \text{m} \times T_{eo}^{-3/2}$, $X$-wave

Figures (5.2), (5.3), (5.4), and (5.5) present the results of these calculations, done by means of computer program FIVE (see appendix).

Cases (a), and (b) are the limits of maximum $T_e$ variation, under the conditions stated, heat deposition as done in chapter 4, nighttime conditions, absorption included in the heat deposition, $T_{eo} = 1100^\circ K$, but considering $v_{ei} = 0$ at the field lines. In a real situation $v_{ei} \neq 0$, then the term $C_5 \times N^2(x)^2$ in system (5.2) is bigger than zero, and $\delta T_e$ should be smaller than the values presented in figures (5.2) and (5.3).
In cases (c) and (d) the term $C_5^2N^2(\Delta x)^2$ is considerably bigger than zero, producing a $\delta T_e$ considerably smaller (one order of magnitude smaller) under the same conditions as in cases (a) and (b), except for $C_5^2N^2(\Delta x)^2$ that was calculated for $v_{ei} = 3.8 \times 10^{-5} N^2 T_{eo}^{-3/2}$.

By looking at the equation (5.1) it is seen that due to the various possible parameters variations, $\delta T_e$ can vary in a large range. Among the parameters that can vary, $T_{eo}$, $N$, $v_{ei}$ are the more prominent.

It should be pointed out that these calculations were made using all the simplifications shown in section (4.3.3), for a time after a steady-state was achieved. This time is of the order to tens of seconds according to Heltz and LeLevier (1970), for the electron temperature.

Comparing the maximum $\delta T_e$ of these calculations with the values obtained at Arecibo by Gordon et al. (1971) ($\delta T_e$ around 200°K), it is seen that the experimental values are well within
the range predicted by this theory. The small $\delta T_e$ values of figures (5.4) and (5.5) are due to the inclusion of the absorption in the calculation of $Q(h,\Psi)$, and to the high value of the collision frequency (that can be 3 times smaller, according to Showen, (Showen, 1969)). It was used as a high value of the collision frequency in order to underestimate the $\delta T_e$, when $v_{ei} \neq 0$, and then, set values for the limits of the $\delta T_e$. The variation of $T_e$ with the electron temperature (undisturbed) is such that for 200 degrees variation in $T_{eo}$, when $v_{ei} \neq 0$, less than 2% variation in the maximum of $\delta T_e$ occurs. The accuracy of the radar permits the detection of a 5% variation. The higher the $T_{eo}$, the higher $\delta T_e$, for the same deposition. The higher $N$, the smaller $\delta T_e$, for the same deposition.

From the values it is seen that the ordinary mode is more efficient than the extraordinary mode in producing electron temperature variations, keeping the other parameters the same.
<table>
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<th>$\frac{Q X^* (\Delta x)^2}{C_3}$</th>
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Figure 5.2
Figure 5.3
Figure 5.4
Figure 5.5

EXTRAORDINARY WAVE

$\nu_{ei} \neq 0$

$T_{ei} = 1100^\circ K$

$\delta T_e$ ($^\circ K$)

Field Line (B)

I = 50°
APPENDIX

COMPUTER PROGRAMS
Objective: compute the real and imaginary parts of the index of refraction of the ionosphere.

Results: Figures (1.8A) and (1.8B).

Explanation of the symbols:

\( Y = \) angular gyrofrequency/angular frequency of the wave \((W)\)

\( \Theta = \) angle between the normal of the wave and the magnetic field.

\( E_D = \) electron density

\( N_D = \) neutral density

\( T_E = \) electron temperature

\( C_F = \) electron collision frequency

\( X = \) \( N_e^2 / m_e \epsilon_0 \)

\( Z = \) \( C_F / N \)

\( N_\phi = \) ordinary complex index of refraction

\( N_X = \) extraordinary complex index of refraction

\( \Re N_\phi = \) real part of \( N_\phi \)
MUX = real part of NX
QUIØ = imaginary part of NØ
QUIX = imaginary part of NX

Block diagram: next page
START

Y, W, THETA

DØ 400
I = 1, N

READ
ED, ND, TE

CALCULATE
CF = f(ED, TE, ND)

CALCULATE
X = f(ED)
Z = f(CF)

CALCULATE
NØ = COMPLEX (f(X, Y, Z, THETA))
NX = COMPLEX (f(X, Y, Z, THETA))

PRINT
HØ
MUX
QUIO
QUIX

400
STOP
Objective: compute the collision frequencies (electron-neutral, electron-ion, electron collision frequency, and Appleton-Hartree collision frequency).

Results: electron collision frequency in figures (1.7) and (1.9).

Explanation of the symbols:

ED = electron density

ND = neutral density

TE = electron temperature

CFEN = electron-neutral collision frequency

CFEI = electron-ion collision frequency

CF = CFEN + CFEI

CFAH = 5/2*CF

Block diagram: next page
Objective: compute the absorption suffered by the electric field of the waves, for each altitude.

Results: Figure (4.3)

Explanation of the symbols:

\[ B0 = \int \frac{w}{c} \chi \, dr \]
\[ BX = \int \frac{w}{c} \chi \, dr \]
\[ A0 = \exp(-B0) \]
\[ AX = \exp(-BX) \]

Block diagram:
Objective: compute the heat deposited by volume in the ionosphere due to the passage of a high frequency radio wave.

Results: Figures (4.1), (4.2), (4.4), and (4.5).

Explanation of the symbols:

\[ \begin{align*}
HT &= \text{range along the ray} \\
\eta_0 &= \text{local impedance as seen by the wave (ordinary)} \\
\eta_x &= \text{local impedance as seen by the extraordinary wave} \\
Q_0 &= \text{heat deposited due to a zero degrees zenith angle} \\
&\quad \text{at the launching point, for the ordinary wave} \\
Q_x &= \text{heat deposited due to a zero degrees zenith angle} \\
&\quad \text{at the launching point, for the extraordinary wave} \\
Q_0 &= \text{heat deposited by volume, taking into account the} \\
&\quad \text{radiation pattern of the antenna, for the ordinary wave} \\
Q_x &= \text{heat deposited by volume, taking into account the} \\
&\quad \text{radiation pattern of the antenna, for the extraordinary wave} \\
\end{align*} \]

Observation: \( Q_0 \) and \( Q_x \) are functions of the range along the
ray and the zenith angle at the launching point.

Block diagram:
Objective: solve a system of linear equations, up to 50 equations, using the Gauss-Jordan algorithm. In this work it was used to solve the differential equation (5.5)

Results: Figures (5.2), (5.3), (5.4), and (5.5)

Explanation of the symbols:

- $A(I,J)$ = augmented matrix of coefficients
- EPS = minimum allowable magnitude for a pivot element
- $N$ = number of equations in the system
- TV = temperature variation of the electrons

Block diagram:
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