RICE UNIVERSITY

Nuclear Susceptibility of Helium Three Liquid and Gas

by

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ABSTRACT

The technique of adiabatic fast passage has been used to measure the relative magnetic susceptibility, $\chi$, of $\text{He}^3$ and mixtures of $\text{He}^3-\text{He}^4$ with an experimental r.m.s. deviation of $\pm 1\%$. Two types of measurements were made:

1) Measurement of $\chi$ versus temperature (from $1.96^\circ\text{K}$ to $0.93^\circ\text{K}$) of pure $\text{He}^3$ liquid, and $\text{He}^3-\text{He}^4$ mixtures containing 35%, 14%, and 5% $\text{He}^3$.

2) Measurement of relative $\chi$ versus density of pure $\text{He}^3$ gas at $2.99^\circ\text{K}$, with density varying from $1.1 \times 10^{-3}$ to $4.5 \times 10^{-3}$ moles/cc. Preliminary measurements of $\chi$ versus density, $\rho$, of $\text{He}^3$ atoms, for $2 \times 10^{-3} \leq \rho \leq 26 \times 10^{-3}$ moles/cc. at $1.96^\circ\text{K}$ were made by using liquid $\text{He}^3-\text{He}^4$ mixtures ranging from 5% to 100% $\text{He}^3$. The method of relating these individual measurements is explained.

The above results are compared to theory and other previous work. Experimental difficulties affecting precision measurements are discussed.
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I. INTRODUCTION

A. Helium Three

The $^3$He atom is a rare isotope of $^4$He but has remarkably different properties$^{1-3}$. $^3$He has a boiling point at 3.2°K. The greatest difference between $^3$He and $^4$He is that $^3$He is a fermion, having spin $1/2$ and thus being subject to Fermi-Dirac statistics. It has an intrinsic paramagnetic moment whose properties have been studied$^4-7$. Much of the interest in $^3$He concerns how it differs from an ideal Fermi gas due to interactions; its magnetic properties provide a method of studying this.

B. Theory

The distribution function for an ideal Fermi gas is given by

$$f(E) = \frac{1}{e^{(E-E_f)/kT} + 1} \quad (1.1)$$

The Fermi energy, $E_f$, is given by

$$E_f = \frac{\hbar^2}{8m} \left( \frac{3N_0N}{\pi V} \right)^{2/3} = kT_0 \quad (1.2)$$

where $T_0$ is the Fermi temperature and

$$k = \text{Boltzman's constant}$$

$$N_0 = \text{Avagadro's number}$$

$$N/V = \text{density in moles/cc.}$$

An ideal Fermi gas of particles having mass, $m$, equal to that of a $^3$He atom and a density, $N/V$, equal to that of
liquid He\textsuperscript{3} has a calculated degeneracy temperature of 5°K. Below this temperature many of the spins of the Fermi system would be paired anti-parallel, in accordance with the Pauli exclusion principle. As the temperature of the system is lowered, the number of paramagnetic moments available to contribute to the magnetic susceptibility of the sample is decreased.

Well above the degeneracy temperature the Fermi gas susceptibility approaches the classical Curie law dependence

\[ \chi_c = \frac{C}{T} . \tag{1.3} \]

An approximate expression\textsuperscript{8} for the Fermi susceptibility at low temperatures, \( T \ll T_0 \), is (where \( \mu \) is the Bohr magneton)

\[ \chi_f = \frac{3}{2} \frac{NN_0\mu^2}{kT_0} \left[ 1 - \frac{\pi^2}{12} \left( \frac{T}{T_0} \right)^2 \right] \tag{1.4} \]

while in the high temperature region,

\[ \chi_f = NN_0 \frac{\mu^2}{kT} \left[ 1 - \frac{2}{3(2\pi)^{1/2}} \left( \frac{T_0}{T} \right)^{3/2} \right] \tag{1.5} \]

where the term outside the brackets is the Curie susceptibility. Thus, in the low temperature limit, the ratio \( \chi_f/\chi_c \) goes to zero when \( T \) goes to zero. In the high temperature limit, \( \chi_f/\chi_c \) approaches unity as \( T \) becomes large.

It is also possible to obtain an expression for \( \chi_f \) as a function of the density of the gas in the high temperature limit. The Fermi susceptibility per unit volume is
where \( \rho = N/V \) = density of gas. But the Fermi temperature is defined in terms of the density, so the molar susceptibility is

\[
\chi_m = \frac{\chi}{\rho} = \frac{N_o \mu^2}{kT} \left[ 1 - \frac{2}{3(2\pi)^{1/2}} \left( \frac{T_0}{T} \right)^{3/2} \right]
\]  

which completes the desired representations for the ideal Fermi gas.

As will be seen in the following section, liquid He\textsuperscript{3} does not behave like an ideal Fermi gas. The exchange interaction is thought to be responsible for the deviation of He\textsuperscript{3} from the Fermi gas behavior\textsuperscript{7}. The energy of the system contains a term representing the exchange energy. This term is smallest when the spins are aligned. The wave function of any pair of particles with spins aligned will be spatially antisymmetric, thus minimizing the repulsive interaction.

C. Previous Experimental Results

Early experiments on liquid He\textsuperscript{3} from 2.8\textdegree to 1.2\textdegree K by Fairbank, Ard, Dehmelt, Gordy, and Williams\textsuperscript{9} indicated no deviation of the magnetic susceptibility from the Curie law; however, the data had a 10\% scatter. Thus, if He\textsuperscript{3} were to behave as a Fermi system, the degeneracy temperature would have to be below 1\textdegree K rather than the calculated \( T_0 \) of 5\textdegree K.
Later measurements by Fairbank, Ard, and Walters\textsuperscript{4} over the temperature range of 2° down to 0.23°K do show $\chi/\chi_c < 1$ below 1°K and dropping toward zero as T goes to zero. This data is shown in Figure 1. It was found that the later data showed Fermi behavior but with a degeneracy temperature of 0.45° rather than 5°K, showing the importance of exchange. The present research on the low temperature susceptibility of He\textsuperscript{3} is centered around this fact. To express it another way: the data shows the susceptibility remains much larger than expected. This indicates that there is a strong ferromagnetic interaction, probably an exchange interaction, which tends to increase the susceptibility and thus competes with the Pauli exclusion principle which would give a decreasing ratio, $\chi/\chi_c$, following an ideal Fermi gas.

A parameter which can be calculated from the susceptibility data is the "effective mass", $m^*$. This is the mass which must be attributed to the He\textsuperscript{3} atom to give the observed Fermi temperature. Setting

\[ T_0 = 5^\circ = \frac{h^2}{8m} \left[ \frac{3N_0N}{T} \right]^{2/3} \]

\[ T_0 \exp = 0.45^\circ = \frac{h^2}{8m^*} \left[ \frac{3N_0N}{V} \right]^{2/3} \]  \hspace{1cm} (1.8)

we find that

\[ \frac{m^*}{m} = 11 \]  \hspace{1cm} (1.9)
Figure 1. Susceptibility measurements of Fairbank et al.\textsuperscript{4,9}. The bottom diagram shows the high temperature data. The top diagram shows the measurements extended to lower temperatures where deviation from the Curie law is apparent.
The graphs depict the relationship between $\chi/\chi_c$ and $T$ in kelvin. The top graph shows a gradual increase of $\chi/\chi_c$ with $T$, while the bottom graph displays a percentage deviation of a certain parameter with respect to $T$.
It should be mentioned that Fairbank, Ard, and Walters\textsuperscript{4} attempted to normalize their data to the Curie law by use of their high temperature liquid measurements and a gas measurement at 4.2°K, the assumption being that He\textsuperscript{3} would behave as an ideal Fermi gas at this temperature (which later measurements have not verified). The problem of normalization has not yet been completely solved.

More recent measurements have been made by Schwettman and Rorschach\textsuperscript{6,7,10}. The first results\textsuperscript{6} indicated a drop in $\chi/\chi_c$ of 10% at 1.2° compared with that of 2°K. However, later measurements\textsuperscript{7,10} show a drop of only 2% at 1.2° and a decrease of 5% at 0.9°K. These measurements in the high temperature range of 0.9 to 3°K show a scatter of only ±1%. In addition Schwettman and Rorschach measured the temperature dependence of the susceptibility of He\textsuperscript{3}-He\textsuperscript{4} mixtures. Their 81%, 70%, and 38% He\textsuperscript{3} mixtures show the same temperature dependence as their 100% He\textsuperscript{3}, being 5% less at 0.9° than at 2°K. Since the different He\textsuperscript{3}-He\textsuperscript{4} mixtures allow the separation between He\textsuperscript{3} atoms to be varied, they may lead to a better understanding of the Fermi and exchange effects.

Schwettman and Rorschach\textsuperscript{10,12} have tried to include the effects of exchange by use of the local field model of Stoner\textsuperscript{13}. Each spin sees an effective field

$$H' = H + \alpha M$$  \hfill (1.10)
where $\alpha$ is a local field constant depending on the exchange interaction. The resulting expression for the susceptibility is

$$\frac{\chi}{\chi_c} = \frac{\chi_f/\chi_c}{1 - \alpha \chi_f}$$

For $\alpha$ positive, this equation predicts that the ratio $\chi/\chi_c$ could have a maximum greater than unity and then approach the classical limit from above.

Fairbank and Walters\(^5\) reported results on $\chi$ versus $T$ for liquid He\(^3\) under its vapor pressure, and for pressures of 11 and 28 atmospheres, and for a higher pressure at which the sample solidified. The striking result of these measurements is that the degeneracy temperature decreased with increasing pressure; or, the susceptibility increased with increasing density (below about 0.6°K). This is an indication that the exchange effect is dominating the Fermi effect under these conditions.

Thomson and Meyer\(^{14}\) report similar results, the Fermi temperature decreasing as the pressure is increased on the liquid He\(^3\) sample at temperatures below 1°. Beal, et al.\(^{15}\) have measured $\chi$ versus $T$ of liquid He\(^3\) above 1° for various pressures. Their curve for 1 atmosphere is similar to that of Schwettman and Rorschach at the saturated vapor pressure, but the curves for higher pressure tend to show a higher Fermi temperature. Their other data shows no change in susceptibility as a function of pressure at 1.2°, 1.7°, and 3.1°, in
general agreement with the results of Fairbank or Thomson.

A different type of measurement has been carried out by Romer and by Luszczynski, Norberg, and Opfer. They have measured the relative molar susceptibility of gaseous He\(^3\) as a function of density (pressure) of the gas. Romer sought to determine if He\(^3\) gas at 3° could be described as an ideal Fermi gas, whose behavior as a function of density would be as shown in Figure 2.

Figure 2: Low density expansion of classical Curie law gas (1) and ideal Fermi gas (2).

Romer's data shown in Figure 3 appears to follow the ideal Fermi expansion, plotted for \(\frac{m^*}{m} = 1\), at the lower densities and then becomes relatively constant above \(3 \times 10^{-3}\) moles/cc.

Luszczynski, Norberg, and Opfer have measured the volume susceptibility of He\(^3\) gas at 3.4°K over a similar density range. From this the density of He\(^3\) gas was calculated and compared to the absolute measurements of Keller. The molar susceptibility appeared constant within their somewhat large experimental error of \(\pm 5\%\) below \(\rho = 3 \times 10^{-3}\) moles/cc. and \(\pm 2\%\) in the region 3 to 15 \(\times 10^{-3}\) moles/cc. A very recent report of their work over the density range 0.17 \(\times 10^{-3}\) to
Figure 3: Data of Romer showing molar susceptibility of He$_3$ gas at 3°K as a function of density. The solid line is the low density expansion of the ideal Fermi gas susceptibility, for m*/m = 1, with the ordinate fitted arbitrarily to the experimental data.
22 x $10^{-3}$ moles/cc. at 3°K states that the molar susceptibility was constant to within an experimental error of ±1%, in disagreement with the results of Romer. Thus the problem of normalization is still open.

The object of the research in this thesis was to measure accurately the susceptibility of liquid He$^3$ and He$^3$-He$^4$ mixtures, extending these measurements to a very dilute 5% He$^3$ sample. Also the molar susceptibility of He$^3$ gas as a function of density at 3°K was measured. Finally initial measurements normalizing the susceptibility of the liquid He$^3$ and He$^3$-He$^4$ mixtures to each other were made; this measurement also gives the molar susceptibility as a function of density at 1.96°K. When possible, parameters such as the effective mass have been calculated.
II. THEORY OF MEASUREMENT

A. Block Equations

Block has made use of the quantum mechanical principle that the time behavior of the expectation value follows the classical time dependence. Let us consider the classical equation of motion for a nucleus, of magnetic moment $\bar{\mu}$, in a magnetic field. The torque is $\bar{\mu} \times \bar{H}$ and the angular momentum is

$$\bar{L} = \frac{\bar{\mu}}{\gamma}$$  \hspace{1cm} (2.1)

where $\gamma$ is the gyromagnetic ratio. The equation of motion is

$$\frac{d\bar{L}}{dt} = \frac{1}{\gamma} \frac{d\bar{\mu}}{dt} = \bar{\mu} \times \bar{H}$$  \hspace{1cm} (2.2)

or, in terms of the magnetization

$$\frac{d\bar{M}}{dt} = \gamma \bar{M} \times \bar{H}$$  \hspace{1cm} (2.3)

If $\bar{H}$ is composed of a small oscillating magnetic field, $\bar{H}_1$, and a large steady state field, $\bar{H}_0$, perpendicular to it, then the components are

$$H_x = H_1 \cos \omega t, \quad H_y = -H_1 \sin \omega t, \quad H_z = H_0$$  \hspace{1cm} (2.4)

In equilibrium

$$M_z = \chi_0 H_0$$  \hspace{1cm} (2.5)

but for non-equilibrium, the spin system undergoes relaxation described by
for the $z$-component of $M$ and by

$$\frac{dM_{xy}}{dt} = -\frac{M_{xy}}{T_2}$$

(2.7)

for the transverse components. $T_1$ is the spin-lattice relaxation time and $T_2$ is the spin-spin relaxation time.

Finally, the equations of motion are

\[
\begin{align*}
\dot{M}_x &= \gamma \left[ M_y H_0 + M_z H_1 \sin \omega t \right] - \frac{M_x}{T_2} \\
\dot{M}_y &= \gamma \left[ M_z H_1 \cos \omega t - M_x H_0 \right] - \frac{M_y}{T_2} \\
\dot{M}_z &= -\gamma \left[ M_x H_1 \sin \omega t - M_y H_1 \cos \omega t \right] + \frac{M_o - M_z}{T_1}
\end{align*}
\]

(2.8)

A certain type of solution exists for the case called "adiabatic fast passage". This is the case where the magnetic field is swept through resonance fast enough that relaxation is negligible but slow enough that the magnetization can follow the field. The conditions to be satisfied are that the time of passage, $\tau$, be

$$\tau \approx \frac{H_1}{dH_0/dt} \ll T_1, T_2$$

(2.9)

and the adiabatic condition

$$\left| d\left( \frac{H_0}{H_1} \right) \right| = \left| \frac{\delta}{dt} \right| \ll \left| \gamma H_1 \right|$$

(2.10)

where $\delta$ is a convenient parameter in the solution of the
equations defined to be
\[ \delta = \frac{H_0 - H^*}{H_1}, \quad H^* = \omega \] (2.11)

Together, the conditions can be written
\[ \frac{1}{T_2} \ll \frac{1}{H_1} \left| \frac{dH_0}{dt} \right| \ll \left| \gamma H_1 \right| \] (2.12)

Bloch has shown that the solutions to the equations of motion are
\[ M_x = \frac{M}{(1 + \delta^2)^{1/2}} \cos \omega t \]
\[ M_y = \frac{M}{(1 + \delta^2)^{1/2}} \sin \omega t \] (2.13)
\[ M_z = \frac{M \delta}{(1 + \delta^2)^{1/2}} \]
when the adiabatic fast passage conditions are satisfied.

B. Adiabatic and Fast Passage Corrections

The observed resonance signal is proportional to the transverse component of the magnetization. Defining the z axis to be the polar axis, the maximum transverse component of the magnetization occurs when the magnetic field is swept through resonance, \( H_0 = H^* \), and \( \Theta = \pi/2 \). A first order correction to the magnetization for a sweep speed which does not quite satisfy the fast passage condition can be derived. The transverse equation of motion is
\[ \frac{dM_{xy}}{dt} = -\frac{M_{xy}}{T_2} \] (2.14)
An approximate correction can be derived using $M_{xy} \approx M_o \sin \theta$

where $M_o$ is assumed to remain constant.

Thus

$$\frac{dM}{d\theta} \frac{d\theta}{dt} \approx \frac{-M_o \sin \theta}{T_2}$$

(2.15)

$$\Delta M_{xy} = \frac{-d\theta}{dt} \int_0^{\pi/2} \frac{M_o \sin \theta}{T_2} d\theta = \frac{M_o}{T_2} \frac{d\theta}{dt}$$

(2.16)

but

$$\frac{d\theta}{dt} \approx \frac{\pi/2}{T/2} = \frac{\pi}{T}$$

(2.17)

so

$$\frac{\Delta M_{xy}}{M_o} = - \frac{\pi}{\pi T_2}$$

(2.18)

This correction is discussed more completely in appendix B, and the correct expression is

$$\frac{\Delta M_{xy}}{M_o} = \frac{1}{T_2} \Delta \left(1 + \Delta^2/2\right) \log \left[\Delta + (1 + \Delta^2)^{1/2}\right]$$

(2.19)

where the $\Delta$ on the right side is the equilibrium value of $\delta$

before the magnetic field, $H_o$, is swept. It is shown in

appendix B that under the conditions of this experiment that

the expression can be reduced to
The other limit on the sweep speed is the adiabatic condition. Abragam has shown that

\[ \frac{\Delta M_{xy}}{M_0} = \frac{-5.3}{T_2 |\delta|} , \quad \dot{\gamma} \approx \frac{H_0}{H_1} \approx H_0 \] (2.20)

Thus

\[ |\Delta M_z| \sim \left| \frac{M \Omega}{\gamma H_1} \right| \] (2.21)

where

\[ \Omega = H_1 \frac{\dot{H}_0}{H_e} \approx \frac{H_0}{H_1} \text{ at resonance} \] (2.22)

as

\[ H_e = (H_0 + \frac{\omega}{\gamma}) \hat{k} + H_1 \hat{i} \] (2.23)

Thus

\[ \left| \frac{\Delta M_{xy}}{M} \right| \approx \frac{H_0}{\gamma H_1^2} \] (2.24)

For He, \( \gamma \approx 2 \times 10^4 \); for our experiment, \( H_1 \approx 1 \) gauss. If this fractional change in the magnetization is to be less than 1%, the sweep speed must be less than 200 gauss/sec.
III. EXPERIMENTAL APPARATUS

A. Magnetic Resonance and Detection System

Figure 4 is a block diagram of the resonance and detection system. The system has good stability and signal to noise ratio, necessary for precision measurements.

The large field $H_o$ is produced by a 12 inch Varian electromagnet and power supply. The field can be swept several hundred gauss from its equilibrium value by means of an external sweep. To provide a uniform sweep, a Texas Research and Electronics S.I. 100 voltage integrator powered by wet-cell batteries is used. The sweep rate can be varied by controls within the integrator or by changing the battery voltage applied to it. The equilibrium field of the magnet is adjusted to be about 100 gauss above the resonance field, $H^*$. The choice of the proper sweep rate is a critical factor, as will be seen later.

The resonance and detection system consists of a crystal oscillator feeding into a balanced detector, one arm being the transmission line and resonant circuit. The signal from the detector is then amplified and recorded. The detection scheme has been described by Schwettman$^{11,12}$ and is shown in Figure 4.
Figure 4. Block diagram of the resonance and detection system, showing the balanced detector circuit.
The sample chamber has a coil wound around it and a capacitor in parallel, forming a resonant tank circuit. The tank circuit is matched to the characteristic impedance, $Z_0$, of the transmission line by a tap on the coil. The proper tap is determined by a Smith chart plot. The transmission line to the top of the dewar has a length of $\lambda/8$ which transforms the imaginary impedance change, $dZ_L$, in the tank circuit due to the induction signal into a real impedance, $dZ_T$, at the top of the line

$$dZ_T = -i dZ_L,$$

where $Z_T = Z_0$.

The crystal controlled r.f. oscillator operates at several fixed frequencies depending on the choice of crystal; fortunately the citizens band crystals at 10Kc intervals around 27.1 Mc form a convenient frequency range. The output signal is stable and relatively noiseless. Several crystals are necessary because the tank circuit may shift to a slightly different resonant frequency each time the apparatus is cooled to liquid helium temperatures. The oscillator chassis also contains a separate tunable r.f. oscillator which is convenient for determining the resonant frequency of the tank circuit at the beginning of each run.

The balanced detector has a null output except when the magnetic field is swept through the resonance of the He$^3$ nuclei.
The output is then a Lorentzian signal with amplitude proportional to the magnetization of the He\(^3\). The detector is based on a design by Thomas and Huntoon\(^2\)\(^3\). The signal then goes to a Sanborn Model 320 dual channel recorder where it is amplified in one channel and sent through a precision potential divider to the other channel where it is amplified again and recorded. The maximum amplification and minimum attenuation is used when the He\(^3\) gas measurements are made; under these conditions about 0.05 millivolts of noise is seen on the output of the balanced detector and approximately an equal amount is added by the amplifiers in the recorder. The potential divider serves to keep the amplitude of the recorded signal nearly constant so that non-linearities in the response of the mechanical recording pen will not affect the measurements.

B. Gas Handling System

A diagram of the system used to handle the He\(^3\) sample is shown in Figure 5. The same system is used for both types of runs; those where the sample is liquified in the sample chamber and those where it is not.

A sample, either pure He\(^3\) gas or a He\(^3\)-He\(^4\) mixture, is stored in a 500 cc or 1000 cc steel bottle and can be let into the right side of a glass U-manometer through a Hoke valve. The manometer is used to measure the pressure of the sample let in and can also be used as a Toepppler
Figure 5. Diagram of the system for handling the sample gas.
pump to move gas in the system. Thus, it is necessary that the left side of the manometer be connected to either vacuum or pressure. The manometer is used with a liquid sample to determine when the sample chamber is filled and with a gas sample to determine its pressure and hence density in the sample chamber.

The sample is let from the manometer into a helium cooled trap, where an attempt is made to condense out other gasses besides helium which may be in the sample. Impurities shorten the relaxation time of the sample and in extreme cases also form a frozen plug above the sample chamber. From the trap a copper plated stainless steel capillary carries the sample through the dewar top and down to the sample chamber. This capillary also serves as the center conductor of the r.f. transmission line.

The sample chamber for the initial measurements was nylon while for the last few runs one of glass was used. The nylon chamber is force-fitted over a brass plug which in turn is soldered to the capillary.

C. Cryogenics

The sample chamber at the end of the transmission line is at the bottom of a 1.5 liter glass helium dewar, which in turn is surrounded by a liquid nitrogen dewar. The tails of these dewars fit in the 1-3/4 inch gap between the magnet's pole pieces.
The helium bath can be pumped by a Kinney Model KD 30 fore pump. During a run, the inner dewar is filled with liquid He$^4$ at 4.2°K and then by pumping on the bath with the Kinney pump, any desired temperature down to 1.2°K can be obtained.

A Consolidated Electrodynamics type KS200-01 diffusion pump allows temperatures of 0.93°K to be obtained. To carry this work to lower temperatures it will be necessary to construct a He$^3$ refrigerator.

The temperature of the bath around the sample chamber is measured in several ways. The vapor pressure of the He$^4$ bath gives the temperature at the bath's surface. When operating below the $\lambda$-temperature, the temperature of the bath is constant throughout. For operation above the $\lambda$-temperature, one must wait for equilibrium to be established so that the temperature in the bath at the sample chamber equals that of the surface. The pressure of the bath may be read with a mercury manometer, or more accurately, to about 10 microns (corresponding to 0.005°K at 1°K) with a Texas Instruments Precision Quartz Pressure Gauge. Below 2°K the temperature may also be read with a He$^3$ vapor pressure thermometer whose vapor pressure bulb is located in the bath just above the sample chamber. Finally as a check, when pure liquid He$^3$ is in the sample chamber, the sample itself can be used as a He$^3$ thermometer.
The use of the T.I. quartz pressure gauge deserves some elaboration. It is also used to provide automatic temperature control. Pressure inlets on the gauge are connected to the He$^4$ bath vapor and to a reference vacuum. The absolute pressure difference corresponds to a certain temperature. The gauge can also be set for this pressure and when the bath pressure equals the gauge setting there is a null electrical output. When the pressure drifts there is a corresponding positive or negative output. By connecting this output to a Brown Model 356413-3 amplifier and a Honeywell Model 362479-3 servo which controls a needle valve in the He$^4$ bath pumping line, automatic control is achieved, an important factor in improving the accuracy of the data.
IV. EXPERIMENTAL PROCEDURE

In order to obtain susceptibility measurements having an accuracy within ±1%, the technique of making the measurement is very important. It was found during this research that there are several factors which can effect the recorded susceptibility.

A. Liquid He\(^3\) Measurements

Described below are the methods for filling the sample chamber with a pure sample, tuning the resonance and detection system, changing temperature, and making a resonance measurement.

For measurements of \(\chi\) versus temperature, T, liquid samples of pure He\(^3\) and He\(^3\)-He\(^4\) mixtures of 35%, 14%, and 5% He\(^3\) were used. The 14% He\(^3\) sample had already been prepared and analyzed by mass spectroscopy. About 190 cc of 35% He\(^3\) ±1% was made in a manometer system designed by H. A. Schwettman for mixing gases. When the data on this mixture was completed, all but 20 cc was removed from the sample system and by adding 120 cc of He\(^4\), a 5% ±1% sample was obtained. This was likewise purified and susceptibility data taken. The sample was removed and a pure He\(^3\) sample let in, purified, and data taken. The samples are purified in about 20 cc amounts by passing the gas through a trap which is at 4.2°K. The amount the sample chamber is filled is monitored by the amplitude of the resonance signal and the pressure.
in the sample system.

The electronic resonance and detection system is tuned at the beginning of a run. In order to get optimum linearity the crystal whose frequency is nearest the center of the resonance of the tank circuit is chosen. That the tank circuit may have a slightly different resonant frequency each time it is brought to liquid helium temperatures was not realized during the earlier part of this research. Once a resonant frequency is chosen the detector is balanced by varying the fixed and variable capacitors in the bridge arm opposite to the transmission line. For most runs the detector was balanced so that there was no output signal when the oscillator was turned on; a different balance will be discussed in part B. The He$^3$ resonance signal also varies with oscillator power and will be discussed in part B.

Measurements of susceptibility were made at about 5 temperatures between 1.96°K and .93°K. The temperature must be lowered monotonically to avoid distillation effects in the He$^3$-He$^4$ mixtures. Since the susceptibility roughly follows the Curie law, $\chi \propto 1/T$, the signal increases as the temperature is lowered. To eliminate non-linearities in the recording pen mechanism a precision attenuator is installed between the first and second channels of the dual channel recorder. As the temperature is decreased, the attenuator is set to decrease the signal according to the ratio of the tempera-
tures. Thus the data shows the relative deviation from the Curie law. The data is corrected for change in density of the liquid sample as a function of temperature, using the data of Ptukha in Figure 6. In the case of gas measurements, the attenuator is set to decrease the signal as the density is increased.

The most important parameter is the sweep rate of the Varian magnet. Sweep rates of about 8 to 30 gauss/sec were used, depending on the sample and its relaxation times. It is necessary to satisfy the adiabatic and fast passage conditions. The effect of inadequate sweep resulting in spin-spin relaxation, due to the very short $T_2$ of some of the samples, was determined in this experiment.

The procedure in a resonance measurement is to set the equilibrium value of the magnetic field, $H_0$, at a chosen value, 100 gauss, above the resonance value $H_\ast$. The field is held here for 6 to 8 $T_1$'s to allow complete recovery of the signal (recovery after 5 $T_1$'s are within 1%). Using the integrator as an external sweep for the magnet's power supply, the field is then swept rapidly through the resonance of the He$^3$. The detected dispersion signal is recorded.

B. Gaseous He$^3$ Measurements

To measure the relative susceptibility of He$^3$ gas as a function of density, the procedure is to make fast passage
Figure 6. Data of Ptukha$^{24}$ showing mass density as a function of temperature for He$^3$-He$^4$ mixtures and pure He$^3$. 
measurements at several different pressures. The pressures are then related to the density by the data of Keller\textsuperscript{25}.

Because of the low density of He\textsuperscript{3} in the sample chamber and resulting low signal to noise ratio, about ten fast passage measurements are made at each pressure.

The He\textsuperscript{3} gas is purified and then either stored in some volume such as the trap, being let into the manometer-sample chamber system in steps of increasing pressure, or an amount of gas sufficient for the highest pressure is initially contained in the manometer and then let out in steps of decreasing pressure. The densities chosen were those used by Romer\textsuperscript{16} of 1 x 10\textsuperscript{-3} to 4 x 10\textsuperscript{-3} moles/cc. at 2.99°K. The pressure-density curve taken from Keller\textsuperscript{25} is shown in Figure 7. For pressures just beyond the range of the data provided by Keller, the density can be calculated using the first and second virial coefficients obtained by Keller from his data for 2.991°K. The coefficients in the expansion,

\[
\frac{PV}{N} = RT \left[ 1 + B \left( \frac{N}{V} \right) + C \left( \frac{N}{V} \right)^2 + \cdots \right]
\]

have been found to be

\[ B = -86.25 \text{ cc/mole} \]
\[ C = 1718 \text{ (cc/mole)}^2 \]
\[ RT = 186,640 \text{ mm-cc/mole} \]

The expansion is a cubic equation in \( \frac{N}{V} \).
Figure 7. Top: Data of Keller\textsuperscript{25} showing the density of He\textsuperscript{3} gas at 2.99°K as a function of the pressure on the gas.

Bottom: Relative observed resonance signal of He\textsuperscript{3} gas as a function of fraction of maximum oscillator voltage (maximum voltage is 3.4 V).
He$^3$ GAS AT 2.99°K

Pressure - cm Hg

Density - moles/cc $\times 10^3$

Relative Signal

Oscillator Voltage
which can be solved for \( \frac{N}{V} \). The accuracy of \( \frac{N}{V} \) using these coefficients is within \( \pm 5\% \).

For gas measurements, the oscillator and detector are tuned as they were for liquid measurements. However, for gas measurement No. 6, the detector was balanced for null output when small changes in the oscillator voltage about the operating voltage were made.

The effect of the sweep rate becomes even more pronounced for a gas because the relaxation times are quite short (as low as 1 sec. for \( T_2 \)). The initial slow sweep speed of 8 gauss/sec was found to be inadequate and was increased to 250 gauss/sec on the last run. This is still not adequate to give less than 1% relaxation of the signal. From eq. (2.20) we have

\[
\frac{\Delta M_{xy}}{M_0} = \frac{-5.3}{T_2 |\delta|}
\]

where \( \delta = \frac{\dot{H}_0}{H_1} \approx \frac{H_0}{H_1} \) as \( H_1 \approx 1 \text{ gauss} \).

For a 1% relaxation, taking \( T_2 = 1 \text{ sec} \),

\[ .01 = \frac{-5.3}{H_0} \]

so \( H_0 = 530 \text{ gauss/sec} \) is required. One way to avoid this is to use a purer sample having longer relaxation times.
C. Liquid He$^3$ versus Density Measurements

To extend the range of density of He$^3$ nuclei beyond that included in the gas measurements, various mixtures of liquid He$^3$-He$^4$ are used. At 1.96°K these give densities of $1.4 \times 10^{-3}$ mole/cc. He$^3$ for 5% He$^3$ to $26 \times 10^{-3}$ mole/cc. for pure He$^3$.

However, the susceptibility measurements from each run must be normalized to each other. The net electronic gain of the system leading to the recorded signal may vary from run to run. Conveniently, when a nylon sample chamber is used, the protons form a fixed sample which when measured at the same temperature provide a means of normalizing our results relative to each other. Absolute normalization to the Curie value has not yet been accomplished.

The resonance measurements of the He$^3$ nuclei are the same as those described in part A. The proton resonance is measured in a similar manner but particular characteristics of the proton require further investigation in order to obtain consistent measurements from run to run. The very long relaxation times of the protons mean that the magnetic field must be set, say, 100 or 200 gauss from resonance for about two or three hours $= 5T_1$. The same sweep rate is used for each run and satisfies the fast passage condition for solids

$$\frac{1}{T_1} \ll \frac{1}{\gamma} \ll |\gamma H_1|$$.
For the sweep speeds used, $\gamma \sim 1$ sec, $T_1$ is found to be 
$\sim 1800$ sec, and $\gamma H_1 = \frac{\omega}{H_0} H_1 \approx 1.4 \times 10^4$.

Thus $0.5 \times 10^{-2} \ll 1 \ll 1.4 \times 10^4$.

An important parameter is the oscillator voltage. It was discovered only toward the completion of the measurements that the r.f. field strength was not sufficient to be operating in the field-independent region.

The $p^+$ signal is recorded in the same manner as the He$^3$ signal and the attenuator is used to maintain a constant amplitude for signals at lower temperatures. The proton signal can be used$^7,12$ to check the linearity of the spectrometer as a function of temperature. Some difficulty was encountered during this research on this point; the ratio $\chi/\chi_c$ for the protons tended to decrease with temperature.
V. EXPERIMENTAL RESULTS

A. Liquid $\text{He}^3-\text{He}^4$ Versus Temperature

We wish to determine how the recorded resonance signal is related to the desired relative susceptibility. We had $M = \chi H$, where $M$ is the actual magnetization and $\chi$ is the actual susceptibility. However, during a given run we can only say that the resonance signals, amplified by the two channels of the recorder, are the relative magnetizations (as a function of temperature) in unknown units. But the recorded signal is attenuated by the precision attenuator $A$ to remove the general Curie law increase as temperature is lowered

$$M_{\text{rec}} = AM, \quad A = \frac{T}{T_L}$$  (5.1)

so

$$\chi = \frac{M}{H} = \frac{M_{\text{rec}}}{AH}, \quad \chi_c = C \frac{T}{T_L}$$  (5.2)

therefore

$$\frac{\chi}{\chi_c} = \frac{M_{\text{rec}} T}{(HC)A} = \frac{M_{\text{rec}} T}{(HC)\frac{T}{T_L}} = M_{\text{rec}} \frac{1}{C \frac{T}{T_L}} H$$  (5.3)

$$\frac{\chi}{\chi_c} \propto M_{\text{rec}}$$  (5.4)

To correct for changes in density of the liquid with temperature

$$\frac{\chi}{\chi_c} \propto M_{\text{rec}} \frac{d_{\text{mix}}}{d_{\text{i mix}}}$$  (5.5)
where \( d_{\text{mix}} \) = density of mixture at temp. = \( T \)
\( d_1 \) = initial temp. = \( T_1 \)

Relative to the first temperature we have

\[
\frac{\chi}{\chi_{\text{rel}}} = \frac{M_{\text{rec}}/d_{\text{mix}}}{M_1 \text{ rec}/d_1 \text{ mix}} \tag{5.6}
\]

The data is plotted in this form for 100\%, 35\%, 14\%, and 5\% He\(^3\) mixtures. The r.m.s. deviation of the data is within ±1\% and each point represents an average of three to twelve measurements. The data have been corrected for changes in density as a function of temperature as discussed above.

Two sets of measurements were made on the same 100\% He\(^3\) sample; both are shown. This data exhibits a 7\% deviation at 1.2° which is larger than the 4\% found by Fairbank, Ard, and Walters\(^4\) and the 2\% found by Schwettman and Rorschach\(^7\).

Two sets of measurements were made on the same 35\% He\(^3\) sample and both are shown. Run No. 2 exhibits a 4\% deviation at 1.2° which is larger than the 2\% deviation found by Rorschach and Schwettman for a 38\% mixture\(^10\). The inconsistency of run No. 1 will be discussed.

Two sets of measurements were made on the same 14\% mixture. Measurement No. 1 was the first made during this research and the technique of measurement was not yet developed. Run No. 2 has a small r.m.s. deviation but shows no deviations with temperature from 1.96° to 1.2°K. The results of Schwettman and Rorschach\(^10\) show perhaps a 1-1/2\% deviation.
Four sets of measurements were made on a dilute 5% He$^3$ mixture. The first measurement was made on one sample, the remaining three on another sample. Run No. 1 shows no deviation from 1.96° to 1.38°. The gain was accidentally changed between the 1.38° and 1.2° measurements. Run No. 2 shows a large deviation and will be discussed later. Runs No. 3 and No. 4 are dubious and are included in the separate appendix and discussed there.
Figures 8 to 14. Deviation of $\chi/\chi_c$ versus temperature (relative to $\chi/\chi_c$ at 1.96°K) for liquid 100%, 35%, 14%, and 5% He$^3$ mixtures.
B. Gaseous He\textsuperscript{3} as a Function of Density

The measurement of relative susceptibility of He\textsuperscript{3} gas as a function of density of the gas is similar to the measurement as a function of temperature. However, the customary form for plotting is relative molar susceptibility versus density.

As before, the recorded signal represents the magnetization in unknown units. But as the density of the gas is increased during a run, the signal is attenuated by the precision attenuator, A, keeping the recorded signal approximately constant.

\[ M_{\text{rec}} = AM, \quad A = \frac{\rho}{\rho} \]

where \( \rho = \frac{N}{V} \) = density of He\textsuperscript{3} gas in moles/cc.

so

\[ \chi = \frac{M}{H} = \frac{M_{\text{rec}}}{AH} = \frac{M_{\text{rec}}}{H} \]

The molar susceptibility is defined as

\[ \chi_{\text{molar}} = \frac{\chi}{\rho} = \frac{M_{\text{rec}}}{pH} \]

Thus

\[ \chi_{\text{molar}} \propto M_{\text{rec}} \]

Relative to one of the densities measured we have

\[ \chi_{\text{molar}}|_{\text{rel}} = \frac{\chi_{\text{molar}}}{\chi_{1\text{molar}}} = \frac{M_{\text{rec}}}{M_{1\text{rec}}} \]
The data is plotted in this form. Six runs were made. The first three runs were inconsistent due to experimental effects; this data is plotted and discussed in the separate appendix A.

Measurements No. 4 and No. 6 are shown here; they cover a density range of about $1 \times 10^{-3}$ to $4.5 \times 10^{-3}$ moles/cc. Run No. 4 differs from run No. 6 because of sweep speeds used, as is discussed in the discussion. The last measurement, No. 6, agrees roughly with the results of Romer, though our data shows a less pronounced deviation as a function of density. However, even No. 6 is possibly affected by the sweep speed (as will be discussed). Recent work of Opfer, Luszczyński, and Norberg reports no deviation, within experimental error, over the wide range of $0.17 \times 10^{-3}$ to $22 \times 10^{-3}$ moles/cc. We did not have a sufficient sample of He to condense to a liquid at $3^\circ$, thus extending the range of density measured at this temperature.

The effective mass will be calculated from run No. 6 in the discussion.
Figure 15. Relative molar susceptibility as a function of density of He$^3$ gas at 2.99°K. (Each run is plotted relative to one of the susceptibilities measured during that run.)
Measurement № 4

Measurement № 6

DENSITY - moles/cc
C. Liquid $\text{He}^3$ as a Function of Density

The technique of measurement of relative susceptibility of $\text{He}^3$ in the liquid state as a function of density of $\text{He}^3$ is identical to the measurement in part A as a function of temperature. In addition, the 100%, 35%, and 5% $\text{He}^3$ mixtures provide various densities of liquid $\text{He}^3$ and relating these individual measurements gives the relative susceptibility as a function of density, analogous to part B for gaseous $\text{He}^3$.

If we wish only to compare the molar susceptibility, at say 1.96°, then the recorded signals need only be corrected for relative density. But we will want to plot the entire $\chi$ vs. T curves for the various mixtures normalized to one another when $\chi$ as a function of density is determined, so a general expression should also contain the Curie dependence.

In part A we had

$$\frac{\chi}{\chi_c} \propto M_{\text{rec}} \quad \text{(5.12)}$$

where $M_{\text{rec}} = AM$, $A = \frac{T}{T_1}$

so that the Curie dependence is removed.

Next,

$$\frac{\chi_{\text{molar}}}{\chi_c} = \frac{\chi/\rho}{\chi_c} \propto \frac{M_{\text{rec}}}{\rho} \quad \text{(5.13)}$$

where
\[
\rho = \rho_{\text{He}^3} = \frac{d_{\text{mix}} \text{ (g/cc.)} \times \% \text{ He}^3 \times 0.01}{\text{3 (g/mole)}}
\]

\text{(5.14)}

and \(d_{\text{mix}}\) is a function of temperature given by \(P_{\text{tujapha}}\). The amplification factor for the separate measurements can be determined from the respective proton signals. One measurement can be chosen and the others related to it by

\[
\frac{\chi_{\text{molar}}}{\chi_c} = \frac{M_{\text{rec}}}{\rho} \frac{p_1^+}{p^+}
\]

\text{(5.15)}

where \(p_1^+\) = proton signal at 1.96° when the He\(^3\) signal \(M_{\text{rec}}\) was taken at 1.96° (5% He\(^3\) mixture was chosen for this reference.)

\(p^+\) = proton signal at 1.96° when the He\(^3\) signal \(M_{\text{rec}}\) was taken at 1.96° (35% and 100% in this case).

The molar susceptibility can now be calculated relative to that of the 5% He\(^3\) mixture by

\[
\frac{\chi_{\text{molar}}}{\chi_c} |_{\text{rel}} \propto \left(\frac{M_{\text{rec}}}{\rho}\right) \frac{p_1^+}{(M_{\text{rec}}/\rho_1)}
\]

\text{(5.16)}

Figure 16 is a plot of the relative molar susceptibility at 1.96° as a function of density of He\(^3\), covering the range 2 to 26 \(\times\) 10\(^{-3}\) moles/cc. He\(^3\) and p\(^+\) resonance signals were taken for 5%, 35%, and 100% He\(^3\) and plotted relative to the susceptibility of the 5% solution. An almost linear increase with density of the molar susceptibility is found, being 31% greater for the 100% mixture than for the 5% mixture. Later
another set of measurements were made on the 5% and 100% He\textsuperscript{3} mixtures, using an improved technique to take the p\textsuperscript{+} signal. These are also plotted relative to each other. This data would not agree with the results of Opfer, Luszczynski, and Norberg\textsuperscript{18} who find no change in $\kappa$ vs. $\rho$ for He\textsuperscript{3} gas at 3\textdegree K.

Figure 17 shows the $\kappa$ vs. T curves of part A normalized to each other according to their relative molar susceptibility at 1.96\textdegree. The later set of p\textsuperscript{+} signals are used for this and the point for 35% He\textsuperscript{3} is estimated from the line between the 5% and 100% measurements. Use of the earlier measurements would give similar but larger separations. The susceptibility of 5% He\textsuperscript{3} is plotted at the Curie value; however, it could also be suggested that the 100% curve should fall just below the Curie value and the other curves below that.

These results are examined in the discussion.
Figure 16. Relative molar susceptibility of mixtures of liquid He$^3$-He$^4$ at 1.96°K as a function of the density of He$^3$.

Figure 17. $\chi/\chi_c$ versus temperature curves for liquid 5%, 35%, and 100% He$^3$ mixtures (Figures 9, 11, and 13). They are plotted relative to each other using the information in Figure 16 (see text). The susceptibility of 5% He$^3$ has arbitrarily been set equal to the Curie value (see text).
VI. DISCUSSION AND CONCLUSIONS

In the previous section the data were presented and described in detail. Also, some comparison was made to the results of Schwettman and Rorschach, and to Romer.

In this section some conclusions are drawn from the results. The effect of sweep speed on the results is evaluated quantitatively. The effective mass, \( m^* \), is evaluated for He\(^3\) gas at 2.99\(^\circ\)K.

**Liquid He\(^3\) - He\(^4\) versus Temperature:**

The two 100\% He\(^3\) runs agree well with each other but differ from the results of Schwettman and Rorschach\(^7,10\). During these runs the proton signal was measured at the first two temperatures and found to drop below the Curie law; this possibly could mean a non-linearity in the He\(^3\) signal. However, the deviation of \( \chi/\chi_c \) for 100\% He\(^3\) seen in Figures 8 and 9 may be due to properties of this sample, such as a large impurity content.

Measurement No. 2 on 35\% He\(^3\) shows a relative decrease in \( \chi/\chi_c \) of 3 or 4\% which is comparable to the results of Schwettman and Rorschach. The large deviation in run No. 1 was caused by a slow sweep speed. From equation (2.20) we have

\[
\frac{\Delta M_{xy}}{M_0} = \frac{-5.3}{T_2 |\delta|} \tag{b.1}
\]

Now we use the values of \( T_2 \) found by Horvitz\(^27\) for this
mixture and the sweep rate recorded during this experiment to find

\[
\begin{array}{|c|c|c|c|c|}
\hline
T^\circ\text{K} & T_2\text{ sec} & \text{Sweep gauss/sec} & \Delta\left(\frac{\lambda}{\lambda_c}\right)_{\text{calc.}} & \text{Relative to 1.96}\degree\text{C} & \Delta\left(\frac{\lambda}{\lambda_c}\right)_{\text{exp.}} \\
\hline
1.96 & 34 & 13 & -1.2\% & 0 & 0 \\
1.38 & 27 & 8 & -2.5 & -1.3 & -1 \\
.97 & 12 & 5 & -9 & -8 & -10 \\
\hline
\end{array}
\]

The sweep voltage supplied by a dry cell decreased with use; in subsequent runs larger wet cells were used.

Measurements No. 2 and 3 on 5% He\textsuperscript{3} show large sweep speed effects.

\[
\begin{array}{|c|c|c|c|c|}
\hline
T^\circ\text{K} & T_2\text{ sec} & \text{Sweep gauss/sec} & \Delta\left(\frac{\lambda}{\lambda_c}\right)_{\text{calc.}} & \text{Relative to 1.96}\degree\text{C} & \Delta\left(\frac{\lambda}{\lambda_c}\right)_{\text{exp.}} \\
\hline
1.96 & 24 & 8 & -3\% & 0\% & 0 \\
1.66 & 10 & 8 & -7 & -4 & -4 \\
1.38 & 5 & 8 & -13 & -10 & -10 \\
1.2 & 2 & 8 & -33 & -30 & -13 \\
\hline
\end{array}
\]

These runs led to the realization that sweep speed effects were important due to change in $T_2$ over the temperatures measured. Run No. 4 shown in Appendix A was made with a sweep speed of 27 gauss/sec which is still not adequate and would cause a -3% deviation in $\lambda/\lambda_c$ at 1.38\degree and -10% at
1.1° which agrees fairly well with the results shown.

**Gaseous He³ versus Density:**

The final gas run, No. 6, was made using a sweep speed which was not quite fast enough to make the data completely free of sweep speed effects if the relaxation time, T₂, should change over the range at densities measured. However, if we accept this data, the effective mass can be calculated for T = 3°K. From equation (1.7)

\[ \chi_{\text{molar}} = \frac{\chi_f}{\rho} = \frac{N_0 \mu^2}{kT} \left[ 1 - \frac{2}{3(2\pi)^{1/2}} \left( \frac{h^2}{8\pi kT} \right)^{3/2} \frac{3N_0}{\pi} \rho \right] \]

so

\[ \frac{d \chi_{\text{molar}}}{d \rho} = - \frac{N_0 \mu^2}{kT} \left[ \frac{2}{3(2\pi)^{1/2}} \left( \frac{h^2}{8\pi kT} \right)^{3/2} \frac{3N_0}{\pi} \rho \right] \]

The slope of the line drawn through the data is \( \frac{\Delta \chi}{\chi} \) versus \( \Delta \rho \). If the measured susceptibility is represented as a Fermi gas but with m replaced by \( m^* \), then

\[ \chi_{\text{molar}} = a \left[ 1 - b(m^*) \right] \]

\[ \frac{d \chi_{\text{molar}}}{d \rho} = - ab(m^*) \]

and

\[ \text{slope} = \frac{1}{\chi} \frac{d \chi}{d \rho} \approx - b(m^*) \left[ 1 + b(m^*) \right] \]

From the data, slope = \( 5\% / 4 \times 10^{-3} = 12.5 \) and
Thus we have

\[ \text{slope} = -12.5 = -20.6 \left( \frac{m}{m^*} \right)^{3/2} - (20.6)^2 \left( \frac{m}{m^*} \right)^3 \times 2.5 \times 10^{-3} \]

The solution is

\[ m^* = 1.4 \ m \]

which would indicate that the He\(^3\) gas does not differ greatly from an ideal Fermi gas at this density and temperature.

It may be useful to note that very short relaxation times, \( T_2 \), can be measured using the correction to the magnetization due to slow sweep speeds. For example, during a measurement on He\(^3\) gas, a signal \( M_0 = 0.5 \ mV \) on the oscilloscope was seen for \( H_0 = 250 \ \text{gauss/sec} \); within a few percent this is proportional to the true \( M_0 \). At 32 gauss/sec, the signal 0.45 mV was seen, thus

\[ \frac{\Delta M}{M_0} = -0.1 = \frac{-5.3}{T_2 H_0} \]

so \( T_2 = 1.65 \)

while the estimated \( T_2 \) from the decay curve was 1.5 sec.

**Liquid He\(^3\)-He\(^4\) versus Density:**

The results for relative susceptibility as a function of
the density of He$^3$ atoms in liquid He$^3$-He$^4$ mixtures are shown in Figures 16 and 17. The interesting feature is that the order of increasing molar susceptibility is reversed from that expected for an ideal Fermi gas. The Fermi theory would predict a greater molar susceptibility for 5% than for 100% He$^3$. Thus the data indicates that the exchange increase in susceptibility is larger than the Fermi decrease in this temperature range. As has been mentioned previously, the use of the proton signal to relate these measurements deserves further investigation. However, the data shown in Figure 16 represents 3 independent measurements, all showing a similar increase in molar susceptibility with increasing density.

An expression due to Stoner for the susceptibility of a Fermi system with exchange has been discussed by Rorschach and Schwettman\textsuperscript{7}; it is

$$\frac{\chi}{\chi_c} = \frac{\chi_f/\chi_c}{1-\alpha \chi_f}$$

where $\alpha$ is a positive constant depending on the exchange. A feature of this expression is that it requires $\chi/\chi_c$ to have a maximum greater than unity and then to approach the Curie value from above. A maximum of this type has not been observed experimentally. However, from our data relative values of $\alpha$ can be calculated to show its behavior as a function of density. Solving for $\alpha$ we have
\[ \alpha = \frac{1}{\rho} \left( \frac{\rho}{\chi_c} \right) \frac{1}{\chi_f/\chi_c} - \frac{1}{\chi/\chi_c} \]

where \( \rho = \rho_{\text{He}^3} \) = density of \( \text{He}^3 \) atoms (moles/cc)

\[ B = \rho/\chi_c \]

For \( T = 1.96^\circ\text{K} \), using the tables of McDougall and Stoner\(^{26} \)
for \( \chi_f/\chi_c \), we find

<table>
<thead>
<tr>
<th>% He(^3)</th>
<th>He(^3)</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>( 2.5 \times 10^{-3} )</td>
<td>4B</td>
</tr>
<tr>
<td>35</td>
<td>( 14 \times 10^{-3} )</td>
<td>62B</td>
</tr>
<tr>
<td>100</td>
<td>( 26 \times 10^{-3} )</td>
<td>46B</td>
</tr>
</tbody>
</table>

**Conclusions:**

Of the wide variety of measurements which have been presented, part of them have shown the effect of experimental conditions. Most of these conditions have been evaluated and eliminated. That liquid \( \text{He}^3 \) does not behave as an ideal Fermi gas has been shown by this and previous research. Furthermore, there does not appear to be a great difference in the temperature behavior of the susceptibility for the various \( \text{He}^3-\text{He}^4 \) mixtures.

Continuation of this research should lead to some interesting results. For future measurements on \( \text{He}^3 \) gas at \( 3^\circ\text{K} \), there is now a sufficient quantity of sample available to condense the sample to a liquid, thus extending the range of
density appreciably. The use of the proton signal to relate measurements on He$^3$-He$^4$ mixtures deserves further investigation. When the above measurements are successfully completed, a comprehensive set of data giving susceptibility as a function of density for temperatures from 1° to 3°K will be obtained, perhaps making possible a quantitative theoretical description.
APPENDIX A
Additional Data

Figures 18 and 19 show additional data which were strongly affected by experimental conditions. However, the appearance of the data due to these effects provide a useful reference for future measurements.

Figure 18: Data No. 3 and 4 on liquid 5% He$^3$. While taking the measurements at the last temperature 1.2°K, on run No. 3, it was discovered that the sweep speed of 8 gauss/sec was not adequate, since a sweep speed of 30 gauss/sec gave a much larger susceptibility, although 8 gauss/sec appeared adequate at 2°K (see the change in $T_g$ for 5% He$^3$, taken from Horvitz$^2$7, shown in Figure 14). Run No. 4 on 5% He$^3$ was made using a sweep speed of 27 gauss/sec for all temperatures. This sweep speed still causes some decrease in $\chi/\chi_c$. An additional possibility is that the temperature measured was not the true temperature; on previous runs the minimum temperature, using the Kinney pump only, was 1.2° while 1.1° is indicated here. If the true temperature was 1.2°, then $\chi/\chi_c$ would be as indicated, showing a decrease compared to 1.96° of 3%. However, the decrease in $\chi/\chi_c$ can also be explained as a sweep speed effect as discussed in part VI of this thesis. Clearly, better measurements are needed checking for sweep speed and temperature effects.
Figure 19: Data No. 1, 2, and 3 on gaseous He\textsuperscript{3} at 2.99°K. These measurements were made using a very slow sweep speed, 8 gauss/sec, compared to that which is required for sweep-independent measurements. The curves appear fairly smooth; the reason for opposite directions of deviation is not known but could be due to, say, a small change in the sweep rate.
APPENDIX B

Correction to Fast Passage Magnetization

Low and Rorschach\(^{22}\) have given the solution to the Block equations when the equilibrium value of the magnetic field, \(H_{qs}\), differs from the resonance field \(H^*\), by a finite amount. If

\[
\delta = \frac{H_o - H^*}{H_1} \tag{B.1}
\]

then let the equilibrium value before \(H_o\) is swept be

\[
\Delta = \frac{H_{qs} - H^*}{H_1} . \tag{B.2}
\]

Then Low and Rorschach get for the transverse magnetization

\[
M_{xy} = M_0 \sin (\omega t) \frac{\Delta}{\Delta^2 + T_1/T_2} \left[ \frac{1 + \Delta^2}{1 + \delta^2} \right]^{1/2} \tag{B.3}
\]

We can now use this expression to derive the correction to the fast passage magnetization. Let

\[
\delta = \frac{H_o - H^*}{H_1} = \frac{H_o - \omega / \gamma}{H_1} = \cot \theta \tag{B.4}
\]

Using

\[
\frac{dM_{xy}}{dt} = - \frac{M_{xy}}{T_2} \tag{B.5}
\]

\[
\frac{dM_{xy}}{d\delta} \frac{d\delta}{dt} = - \frac{M_{xy}}{T_2} , \quad \delta = \text{const.} \tag{B.6}
\]
then
\[ \frac{dM_{xy}}{dt} \delta = - \frac{M_0}{T_2} \frac{\Delta}{\Delta^2 + T_1/T_2} \left[ \frac{1 + \Delta^2}{1 + \delta^2} \right]^{1/2} \] \hfill (B.7)

Next we integrate over half of the resonance signal
\[ \theta: \ 0 \rightarrow \pi/2 \]
\[ H: \ H_{os} \rightarrow H^* \] \hfill (B.8)
\[ \delta: \ \Delta \rightarrow 0 \]
so
\[ \Delta M = \int_{\Delta}^{0} \frac{dM_{xy}}{d\delta} \ d\delta = - \frac{M_0}{T_2 \delta} \frac{\Delta(1+\Delta^2)}{\Delta^2 + T_1/T_2} \int_{\Delta}^{0} \frac{d\delta}{(1+\delta^2)^{1/2}} \] \hfill (B.9)

The resulting change in magnetization is
\[ \Delta M = \frac{M_0}{T_2 \delta} \frac{\Delta(1+\Delta^2)^{1/2}}{\Delta^2 + T_1/T_2} \log \left[ \Delta + (1 + \Delta^2)^{1/2} \right] \] \hfill (B.10)

The above expression can be simplified under the experimental conditions of this thesis. We have \( H_1 \approx 1 \) gauss and \( (H_{os} - H^*) \approx 100 \) gauss, so \( \Delta \approx 100 \), and
\[ \frac{\Delta M_{xy}}{M_0} = \frac{1}{T_2 \delta} \frac{\Delta^2}{\Delta^2 + T_1/T_2} \log (2 \Delta) \] \hfill (B.11)

Finally
\[ \frac{\Delta M_{xy}}{M_0} \approx -5.3 \frac{\Delta^2}{T_2 |\delta|} \] \hfill (B.12)
where \[ \delta = \frac{\dot{H}_0}{H_1} \approx \dot{H}_0. \]

This result shows the fractional change in the magnetization at resonance due to an inadequate sweep speed.
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