



RICE UNIVERSITY

THE MEASUREMENT OF SMALL MAGNETIC FIELDS
NEAR A ROTATING SUPERCONDUCTOR

by

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TABLE OF CONTENTS

	Page
I. ABSTRACT	
II. INTRODUCTION	1
III. EXPERIMENTAL	8
A. Mechanical System	8
B. Detection Systems	11
C. Experimental Procedure	14
IV. EXPERIMENTAL RESULTS AND CONCLUSIONS	19
V. APPENDIX	20
VI. ACKNOWLEDGEMENTS	25

ABSTRACT

It was predicted by F. London¹ that a superconductor rotating with angular velocity ω would produce a magnetic field, H, in its interior given by

$$\underline{H} = \frac{2mc}{e} \omega$$

Order of magnitude calculations indicate that for speeds of 1000 radians/second the resulting magnetic field is about 10^{-4} gauss. This field has never been experimentally observed although attempts have been made.² In this investigation we are concerned first with rotation at high speeds, second with the problem of a detection system capable of measuring 10^{-4} gauss, and third with a measurement of the dependence of the London field on the rotational speed. We have chosen to use mechanical rotation since it is simple and eliminates the problem of unwanted magnetic fields encountered in a magnetic suspension and rotation scheme. Two separate detection systems have been evaluated, both depending on the flux changes produced by a rotating superconductor.

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INTRODUCTION

F. London¹ predicted that a superconductor brought into a state of rotation should produce a magnetic field quite independent of the initial conditions. Somewhat earlier Becker and coworkers³ had predicted a similar result, but in their treatment the field due to rotation was present only if the superconductor were set into rotation in zero external magnetic field after cooling below the superconducting transition temperature. That rotation should produce a magnetic field is demonstrated by the following argument: At the beginning of the rotation the lattice and the normal electrons will accelerate as a rigid body and leave the superelectrons behind. This means that there is a net circulation of positive charge and a changing magnetic field. This magnetic field creates an electric field by induction which accelerates the superelectrons and forces them to move almost everywhere exactly in phase with the body except very near the surface. Here they lag a little and thus generate a surface current which produces the magnetic field.

From the London equations for superconductors, the magnetic field produced by rotation can be derived. The

London equations are:

$$e \underline{\underline{E}} = m \dot{\underline{\underline{v}}} \quad (1)$$

$$\text{curl } \underline{\underline{v}} = \frac{-e}{mc} \underline{\underline{H}} \quad (2)$$

where $\underline{\underline{v}}$ is the velocity of the superelectrons, m their mass, and e the electronic charge. The net current is given by

$$\underline{\underline{j}} = ne (\underline{\underline{v}} - \underline{\underline{v}}_0) \quad (3)$$

where $\underline{\underline{j}}$ is the current density, n denotes the number of superelectrons/volume, and $\underline{\underline{v}}_0$ the velocity of the lattice and normal electrons. Using equation (3) and Maxwell's equation for the magnetic field we obtain

$$\text{curl } \underline{\underline{H}} = \frac{4\pi ne}{c} (\underline{\underline{v}} - \underline{\underline{v}}_0) \quad (4)$$

In this latter expression the displacement current has been neglected as it is small. For the case of a rotating sphere of radius R having a constant angular velocity $\underline{\underline{\Omega}}$, we find (using spherical polar coordinates):

$$\underline{\underline{v}}_0 = \underline{\underline{\Omega}} \times \underline{\underline{r}} \quad (5)$$

$$\text{curl } \underline{\underline{v}}_0 = 2 \underline{\underline{\Omega}} \quad (6)$$

$$\text{curl curl } \underline{\underline{v}}_0 = 0 \quad (7)$$

Eliminating \underline{H} from equations (2) and (4) the following differential equation for \underline{v} is obtained:

$$\text{curl curl } (\underline{v} - \underline{v}_0) = -\frac{4\pi n e^2}{m c^2} (\underline{v} - \underline{v}_0) \quad (8)$$

Outside the sphere the divergence and the curl of the field are both zero. Due to symmetry the field outside may be represented by a dipole field parallel to the axis of rotation, in this case the polar axis. The dipole field expressed in spherical polar coordinates is:

$$\left. \begin{aligned} H_r &= \frac{2A}{r^3} \cos \theta \\ H_\theta &= \frac{A}{r^3} \sin \theta \\ H_\phi &= 0 \end{aligned} \right\} r \gg R \quad (9)$$

where A is an arbitrary constant that is determined by the boundary conditions at $r = R$. As the expression for the velocity of the lattice and normal electrons (5) has only a ϕ component, we will assume that the velocity is a function of r and varies as the sine of θ :

$$v_\phi = \{ \omega r + f(r) \} \sin \theta \quad (10)$$

where $f(r)$ gives the correction to rigid body motion.

Substituting this expression into the differential equation for \underline{v} given by (8) we obtain:

$$f'' + \frac{2}{r} f' - \left\{ \frac{2}{r^2} + \frac{4\pi ne^2}{mc^2} \right\} f = 0 \quad (11)$$

This differential equation has the following general solution:

$$f = \frac{B}{r^2} (\sinh \beta r - \beta r \cosh \beta r) + \frac{C}{r^2} (\cosh \beta r - \beta r \sinh \beta r) \quad (12)$$
$$\beta^2 = \frac{4\pi ne^2}{mc^2}$$

In this expression the constant C must be set equal to zero since this part of the solution becomes infinite at the origin. The solution valid inside the sphere is therefore

$$v_{\varphi} = \left\{ \sqrt{2} r + \frac{B}{r^2} (\sinh \beta r - \beta r \cosh \beta r) \right\} \sin \theta \quad (13)$$

where the constant B will be determined from the boundary conditions. Using equation (2) the expressions for \underline{H} can be found:

$$H_r = \frac{mC}{e} \left\{ 2\Omega + \frac{2B}{r^3} (\sinh\beta r - \beta r \cosh\beta r) \right\} \cos \theta$$

$$H_\varphi = 0 \quad (14)$$

$$H_\theta = \frac{mC}{e} \left\{ -2\Omega + \frac{B}{r^3} [(1+\beta^2 r^2) \sinh\beta r - \beta r \cosh\beta r] \right\} \sin \theta$$

From the boundary conditions at the surface of the sphere which require the continuity of \underline{H} we can determine the constants A and B:

$$A = \frac{mC}{e} \Omega R^3 \left\{ 1 - \frac{3}{\beta R} \coth \beta R + \frac{3}{\beta^2 R^2} \right\} \quad (15)$$

$$B = \frac{3\Omega R}{\beta^2 \sinh \beta R}$$

From these equations the magnetic field in the interior of the sphere is given by

$$\left. \begin{aligned} H_z &= H_r \cos \theta - H_\theta \sin \theta \\ &\cong \frac{2mC}{e} \Omega \end{aligned} \right\} R-r \gg \beta^{-1} \quad (16)$$

and represents a homogenous magnetic field (except for the thin surface layer of depth β^{-1}).

It is of interest to note that only by rotation can a uniform magnetic field exist inside a superconductor.

For a superconductor at rest, the magnetic induction B is zero.³ However, the occurrence of this uniform magnetization can also be understood if one transforms to a coordinate system rotating with the superconductor. If the superconductor is rotating with a constant angular velocity $\underline{\Omega}$, then velocities in a coordinate system rotating with velocity $\underline{\Omega}$, are given by

$$\underline{v}' = \underline{v} - \underline{\Omega} \times \underline{R} \quad (17)$$

and therefore the momentum is

$$\underline{p}' = \underline{p} - \frac{e}{c} \underline{A} - m \underline{\Omega} \times \underline{R} \quad (18)$$

where $e\underline{A}/c$ is the "potential" momentum. For an ordinary conductor in an external magnetic field the momentum of the electrons is given by

$$\underline{p}' = \underline{p} - \frac{e}{c} \underline{A} \quad (19)$$

where \underline{A} is the usual vector potential. Comparing equations (18) and (19) it is clear that (18) will be identical with (19) if the vector potential \underline{A} in equation (18) is replaced by an effective vector potential \underline{A}' :

$$\underline{A}' = \underline{A} - \frac{mc}{e} \underline{\Omega} \times \underline{R} \quad (20)$$

Then it follows that the effective magnetic field \underline{H}' is given by:

$$\underline{H}' = \text{curl } \underline{A}' = \underline{H} - \frac{2mc}{e} \underline{\Omega} \quad (21)$$

But for a superconductor the Meissner effect requires that the effective field \underline{H}' vanish, which means that

$$\underline{H} = \frac{2mc}{e} \underline{\Omega} \quad (22)$$

If the Lorentz force is expressed in terms of the effective field \underline{H}' we get

$$\underline{F}_L = \frac{e}{c} \underline{v} \times \underline{H}' = \frac{e}{c} \underline{v} \times \underline{H} + 2m (\underline{v} \times \underline{\Omega}) \quad (23)$$

and we see that this differs from the usual expression by $2m(\underline{v} \times \underline{\Omega})$ which is just the well known Coriolis force.

EXPERIMENTAL

A. Mechanical System

Rotation by mechanical means was adopted from the first as the simplest method as well as one that would eliminate unwanted magnetic effects. However, the requirement of high speeds presented several problems: The drive shaft must be extremely straight and be rigidly supported to eliminate the effects of vibration and friction; and the lower section of the drive shaft must contain the superconductor and operate satisfactorily in the liquid helium. The motor must be of constant and controllable speed and operate in a partial vacuum at low temperatures; it should be relatively far from the superconductor to eliminate stray fields. These problems were solved by using a long drive shaft, consisting of an upper section of 1/2" stainless steel tubing (about 12" long), a center section of 1/8" brass rod (13" long), and a 4" length of 3/16" brass rod, the end of which contained the tin sample. This drive shaft was supported by four beryllium-copper ball bearings (OD=0.2500", ID=0.1875") mounted on three circular brass plates which were supported by four symmetrically spaced 1/2" stainless steel tubes. (See Figure I). To insure that all bearing

surfaces were properly aligned, this complete assembly was mounted directly to the base plate of the motor. (See Figure I). The stainless steel tubes were light and very strong and provided a rigid framework. They were also excellent for the drive shaft as they were "straight" and not subject to whipping. They could not be used in the lower sections of the drive shaft since stainless steel is slightly magnetic, retaining a small magnetic moment. Finally, the overall length of the drive shaft allowed the motor to be placed "far" from the superconductor and to operate near room temperature. This extensive use of stainless steel tubing kept the heat leak at a minimum. Successful operation of the apparatus depended on the bearings and the amount of heat generated by rotation. It was found that there was only a very slight boil off of the liquid helium when operating at the maximum speeds, corresponding to an increase of a few millimeters in the vapor pressure of the Helium bath at 3.7°K.

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2/14

LEGEND FOR FIGURE I

- A: Synchronous motor
- B: Stainless Steel drive shaft
- C: Circular brass bearing plate
- D: Miniature ball bearing
- E: Brass rod
- F: Tin sphere
- G: Detection coil
- H: Lead slug
- I: Speaker
- J: Brass support can

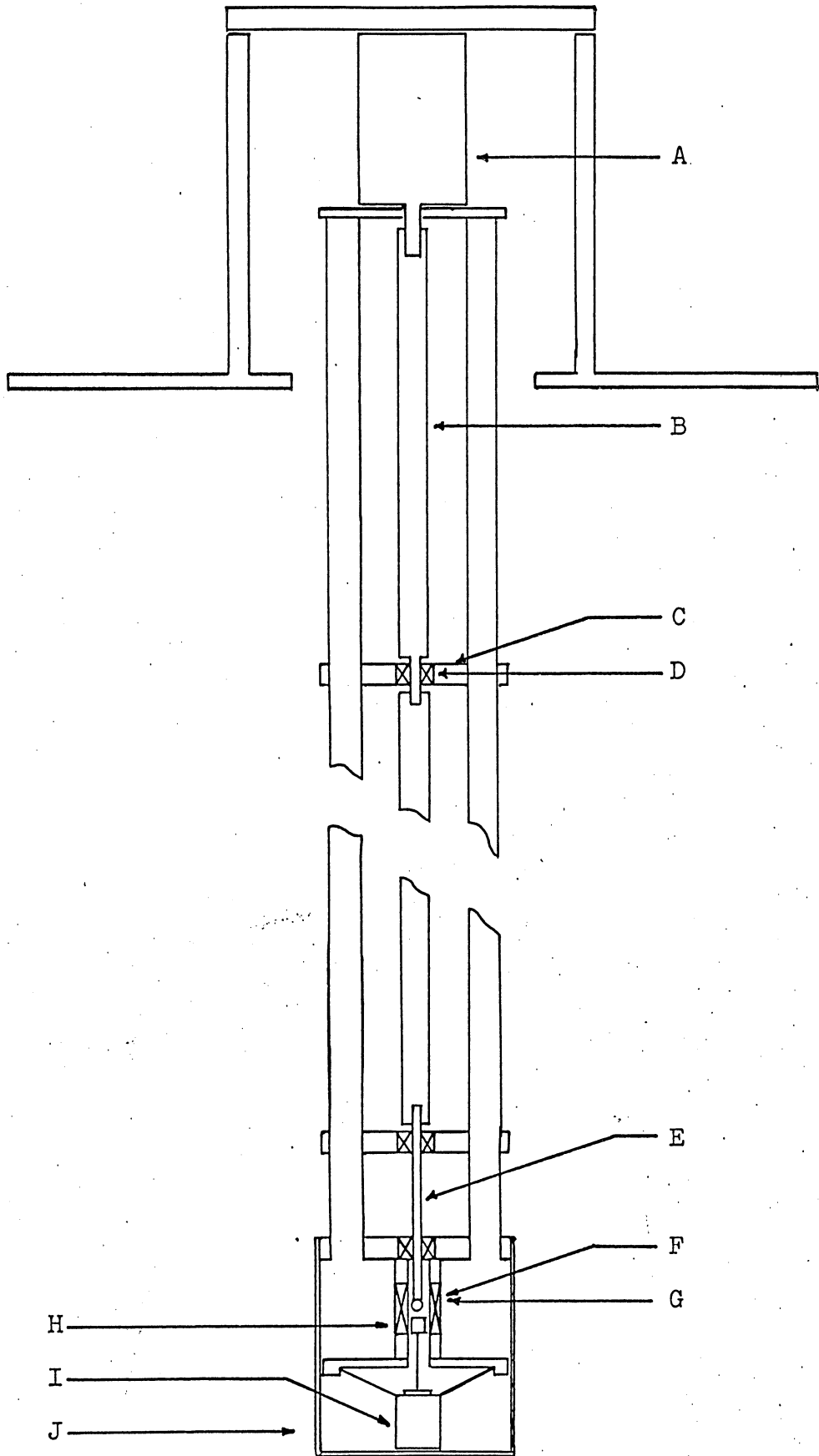


Fig. 1

B. Detection Systems

Two separate methods for the measurement of small magnetic fields have been tried. The first used a 5 mm diameter tin sphere. The sphere was attached to the end of the brass rod by first machining a spherical cavity in the brass rod just large enough for the tin sphere. With the sphere about 75% enclosed, the thin wall of the brass rod was crimped, thus securely holding the sphere. To detect the field, a pick-up coil (OD=2.2 cm, ID=1.0 cm, and length 4 cm) with 5000 turns of #40 copper enamel wire surrounded the sphere. This coil was attached to the lower bearing plate. Situated directly below the sphere was a lead slug which was attached to the cone of a small speaker and which vibrated with the speaker. (See Figure I). When the sphere was superconducting and rotating, the vibrating superconducting slug was to distort the flux lines from the sphere and induce a voltage in the pick-up coil. This coil was part of a bridge circuit (Figure II), and thus cancellation of signals due to coupling between the speaker coil and the pick-up coil was possible. To further reduce this coupling, the speaker was encased in lead foil, and for rigidity it was screwed to the bottom of a brass can which was attached to the lower bearing plate.

SCHMATIC OF BRIDGE CIRCUIT

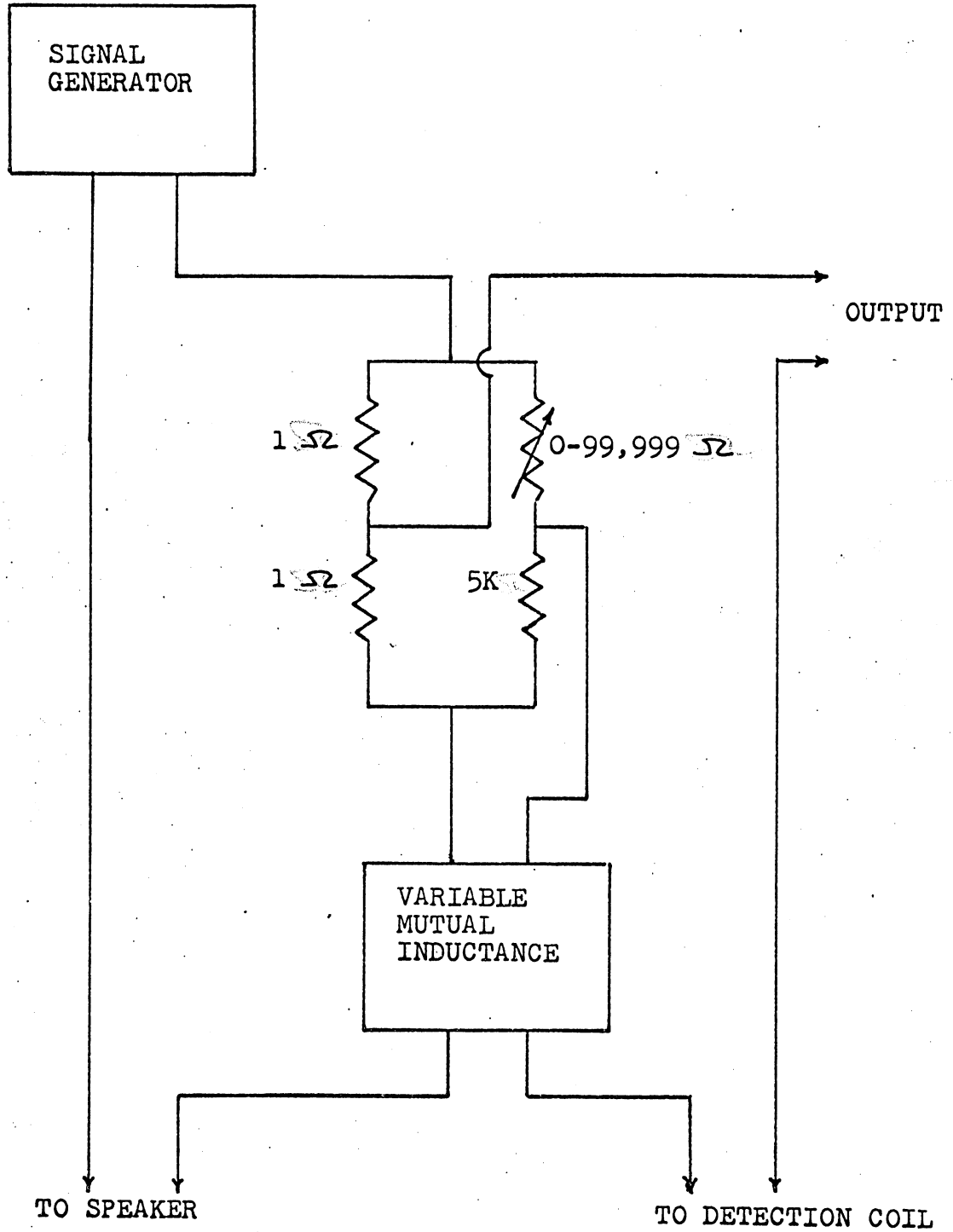


Fig. 2

Any imbalance in the circuit was amplified and detected with a phase sensitive detector, the reference being obtained from the oscillator that drives the speaker and generates the EMF for the bridge. The phase sensitive detector (Princeton Applied Research Model JB-4) has an extremely narrow bandwidth and thereby reduces the amplification of unwanted signals. By using the maximum RC time constant of 2 seconds it was possible to integrate the signal, further reducing the noise. Shielded double coaxial cables were used and grounded at a common point, care being taken not to have any ground loops.

The second detection scheme represented a completely different approach to the problem. For this the sample was a tin cylinder (6 mm by 2 mm). It was glued with Eastman "910" in the lower brass drive shaft in a hole which made an angle of 45° with respect to the axis of the drive shaft. The pick-up coil was mounted so as to enclose the sample and was fixed at an angle of 45° with the shaft axis. (See Figure III). As the sample rotated, its position relative to the detection coil changed from a "parallel" to a "perpendicular" orientation and back again. In Appendix I it is shown that the rotation should induce a voltage in the detection coil which varies with the frequency of rotation of the shaft. This produces an ac

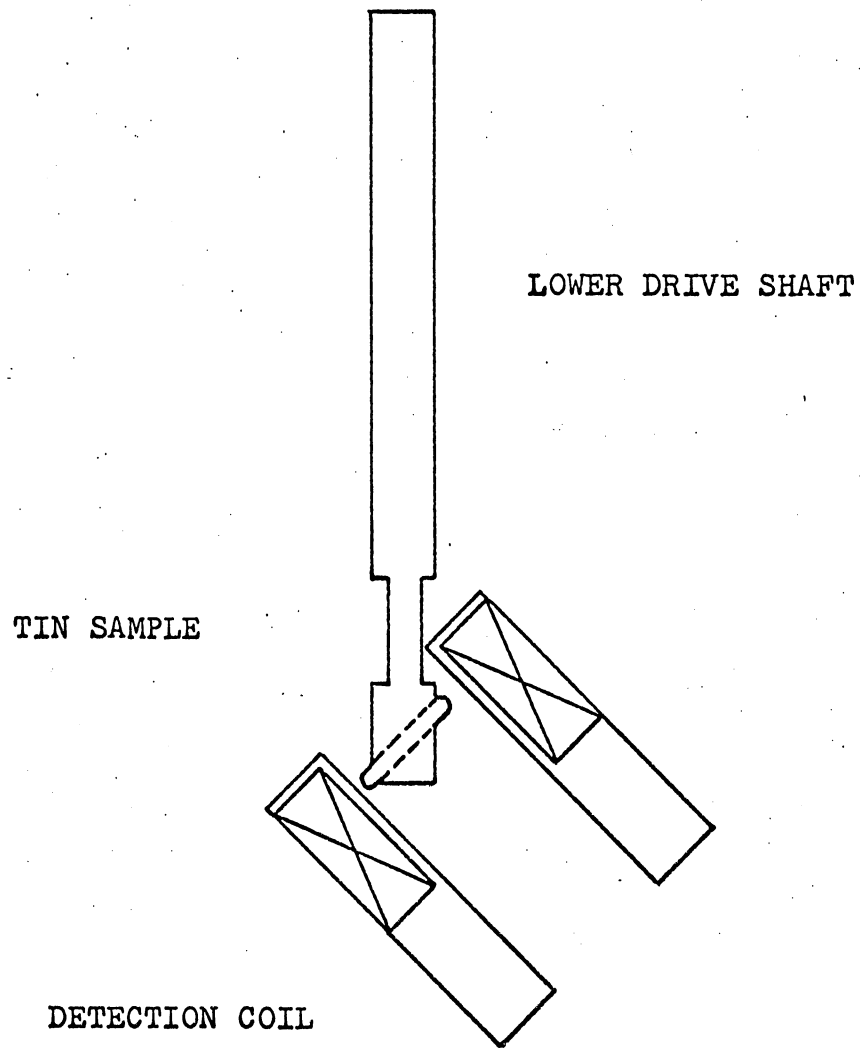


Fig. 3

signal at the rotation frequency which was amplified and detected using the phase sensitive detector. The reference frequency for the phase sensitive amplifier was taken directly from the rotating shaft in the following manner: A photoresistor and a flashlight bulb were mounted next to each other and directly opposite the stainless steel drive shaft. A piece of black electrical tape was cut in the shape of a sine wave and attached to the shaft. Thus as the shaft turned, the amount of light reflected varied sinusoidally, and hence the resistance of the photoresistor also varied sinusoidally. Thus a signal with the frequency of rotation of the shaft was generated. This scheme was necessary and favorable as the motor tended to run just a few cycles per second less than the input frequency, and thus pick-up at the motor drive frequency was reduced.

SCHEMATIC OF DETECTION SYSTEM

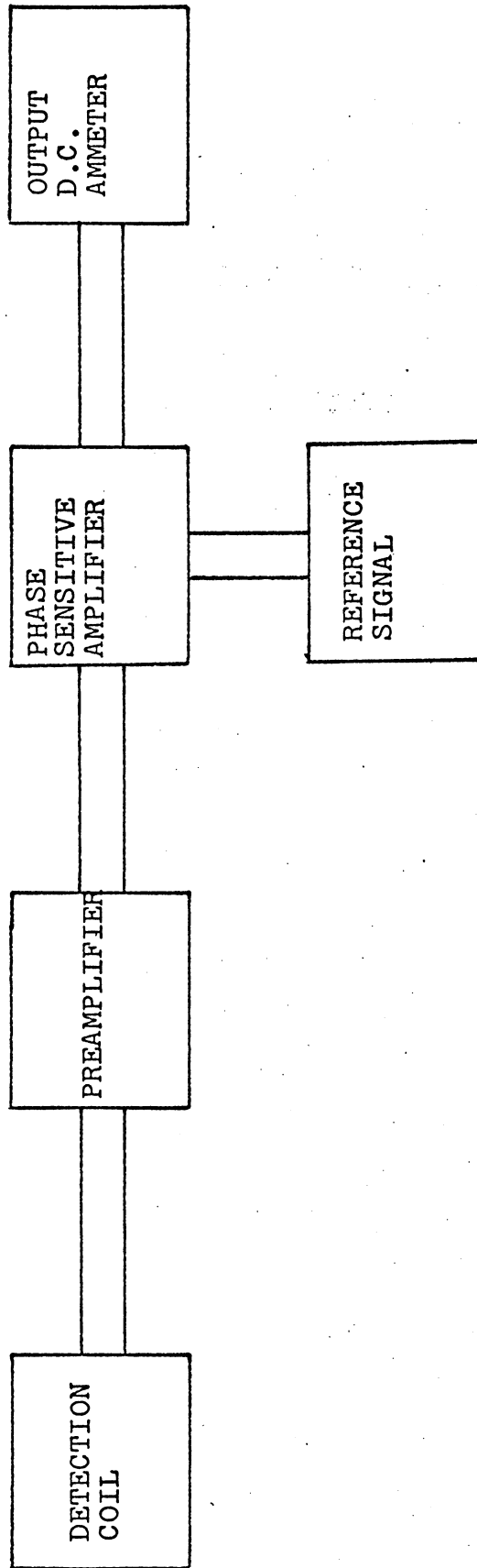


Fig. 4

C. Experimental Procedure

As the field produced by rotation was predicted to be on the order of 10^{-4} gauss for a speed of about 10,000 revolutions per minute, the object of the initial investigations have been to determine the ultimate sensitivity of the various detection systems. To this end the two detection systems previously described were evaluated.

In the first trial using the tin sphere and the vibrating lead slug, calibration of the detection system was done as follows: The apparatus was cooled to liquid nitrogen temperature by putting helium exchange gas in the helium dewar and liquid nitrogen in the outer dewar. Then liquid helium was added using a bottom filling technique. Usually about 1-1/2 liters of helium were needed to cool the unusually large mass of metal. Nevertheless the boil off of the liquid due to rotation and heat leaks was never excessive, and the helium lasted for about 12 hours. With the apparatus at 4.2°K , the lead slug was superconducting and the earth's field was excluded from its volume (Meissner effect). This resulted in an emf being induced in the detection coil when the lead slug was vibrating. This signal was nulled using the mutual inductor and resistance box in the bridge circuit. With the bridge balanced, the temperature was reduced until the critical temperature of the tin (3.72°K) was reached. Passing

through the superconducting transition, the earth's magnetic field was excluded from the interior of the tin sphere. The vibration of the lead slug distorted this excluded field and produced an emf in the detection coil. This ac signal was amplified and detected with a preamplifier and a phase sensitive amplifier. The reference frequency for the "lock-in" detector was taken from the oscillator which drove the speaker and supplied the emf for the bridge. With the tin sphere superconducting, an external magnetic field was applied with the Helmholtz coils which were calibrated, and they produced a known magnetic field at the sample. Any resulting signal should be proportional to this field. In this manner it was possible to get an approximate idea of the minimum magnetic field that could be detected. Measurements were made with the tin sphere at rest and indicated that a field of approximately 4 gauss (determined by the shift in the critical field curve) produced an input signal of approximately 4 microvolts at the detection coil. This meant that the maximum sensitivity was about 10^{-3} gauss, since signals on the order of 10^{-9} volts could be detected. Accurate calibration was difficult as there was a magnetic field of a few gauss present near the sample due to the permanent magnet in the speaker, and the magnetic field present at the sample was not accurately

known. Also rotation of the sphere produced some vibration which led to a changing coupling between the detection coil and the speaker coil which produced an unwanted signal. Even enclosing the speaker in lead foil did not adequately shield the speaker. Therefore the required sensitivity could not be obtained with this detection system.

The second detection system has the advantage of simplicity. It involves only the rotating tin cylinder, the detection coil, and the phase sensitive amplifier. (See Appendix I). The calibration of this system was effected by first cooling below the transition temperature of tin and then rotating the cylindrical sample. If the tin becomes superconducting in a known magnetic field, then due to the Meissner effect this field will be excluded from the tin and the rotation will produce a distortion of the external field and thereby induce a voltage in the detection coil which can be observed. If there were no London field due to the rotation of the superconductor, but only a small residual magnetic field present at the sample, then the Meissner effect will produce a signal varying linearly with the frequency of rotation:

$$\text{EMF} \propto A \omega + \text{Noise}$$

where "A" is proportional to the external magnetic field at the sample when it becomes superconducting. If the London field (which is proportional to ω) is present, then there will be an ω^2 term appearing:

$$\text{EMF} \propto A \omega + B \omega^2 + \text{Noise}$$

Therefore there will be a deviation from a linear dependence. In practice, the "A ω " term is made as small as possible by compensating the earth's field using Helmholtz coils. Calibration measurements have been made, and the results indicating the sensitivity of this method are shown in Figure 6. All measurements were made with the earth's field compensated to within < 0.01 gauss (determined by a rotating coil fluxmeter). There are five sets of points. The lower set represents the maximum signal observed when the sample was rotating in the normal state. This signal is due to the vibration of the detection coil in the residual magnetic field. The upper set corresponds to measurements taken in the superconducting state. The observed signal is seen to increase as the rotation frequency is increased up to approximately 183 cps. At higher frequencies, the signal behaves in a rather erratic manner. One should observe a linear increase of the observed signal with increasing frequency of rotation. The fact that this is not

CALIBRATION POINTS

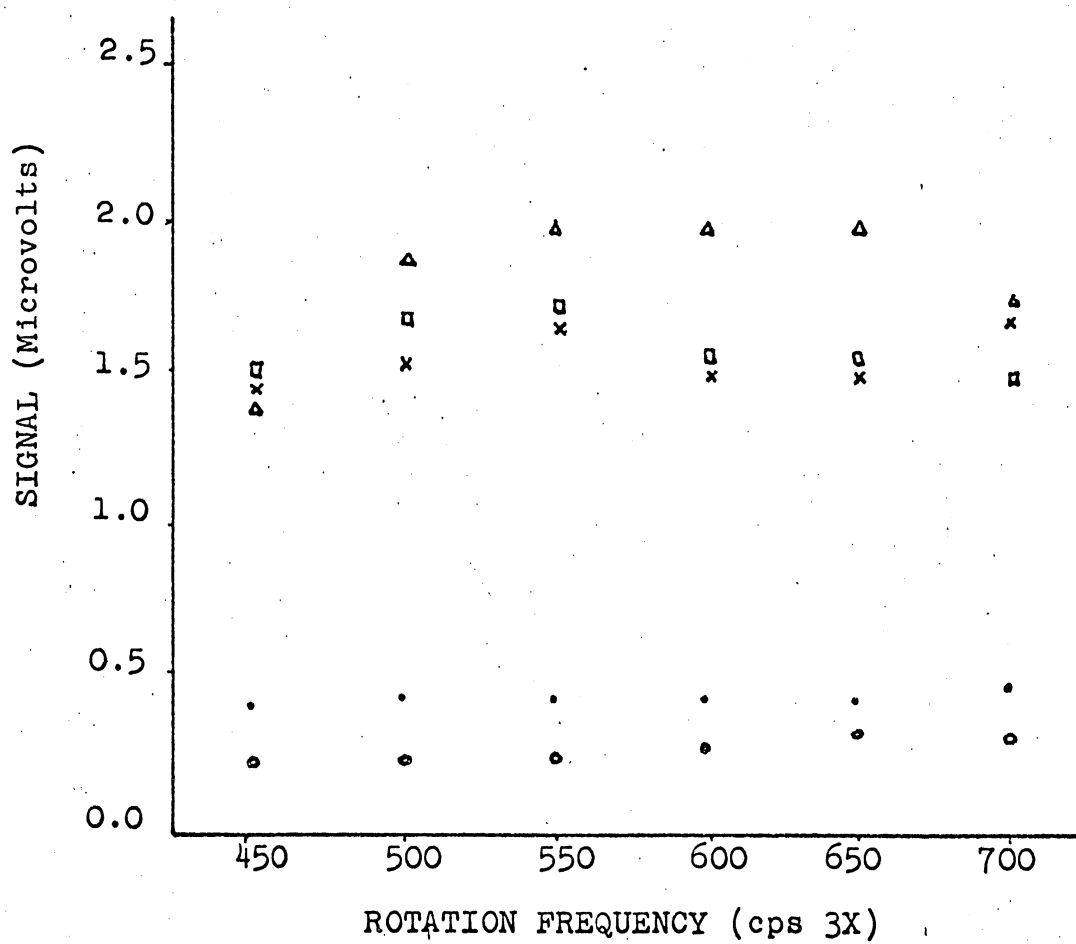


Fig. 6

observed may be due to changes in the gain characteristics of the amplifier at different frequencies, or to small changes in the degree of compensation of the earth's field (on the order of a few milligauss). However, one can say with certitude that the observed signal in the superconducting state is a factor of 5-10 larger than the signal in the normal state. A signal of 30 to 40 divisions on the meter corresponds to an induced emf in the detection coil of a few microvolts. If this signal were due to a residual field of $< 10^{-2}$ gauss, then with better compensation of the earth's field, one can hope to observe the London field. Thus far, the sensitivity has been limited by vibration of the detection coil in the residual external field due to inherent imbalance in the drive shaft. In the future, we plan to use a double coil; that is, two identical coils wound in opposition, one containing the superconductor, the other nothing. Then any signals due to vibration should be induced in both coils and will cancel, while signals due to rotation will not.

EXPERIMENTAL RESULTS AND CONCLUSIONS

Two detection schemes for small magnetic fields have been evaluated for detection of the London moment, the first depending on flux changes produced by a vibrating lead slug, the second on flux changes produced by a rotating superconductor. To date the best working sensitivity is 10^{-3} gauss; however, with some improvements it is hoped that the present system using the tilted tin cylinder can be used to detect the London field, since the sensitivity appears sufficient if the unwanted signals can be eliminated. We then plan to measure the magnitude of the field versus the frequency of rotation to verify the predictions of the London theory. If sufficient sensitivity can be obtained, then effects associated with quantized flux and vorticity in the electronic fluid may be investigated.

APPENDIX I

The Calculated EMF Induced in the Detection Coil by the Rotating Cylinder

Consider a cylinder rotated about an axis making an angle of 45° with respect to the cylinder axis. From pages 6 and 7, this rotation is equivalent to placing the cylinder in an external magnetic field $H_e = \frac{2mc}{e} \omega$ parallel to the rotation axis. If this field is decomposed into components parallel and perpendicular to the cylinder axis, it can be shown⁵ that the magnetizations per unit volume produced by this field are

$$I_{\parallel} = - \frac{H_e \cos \theta}{4\pi} \quad I_{\perp} = - \frac{H_e \sin \theta}{2\pi}$$

where θ is the angle between the field and the cylinder axis. Thus the moment parallel to the cylinder axis is half the transverse moment. This implies that the equivalent field due to ~~the moment~~ ^{the moment} is not along the axis of rotation, but is in general at an angle θ with respect to the rotation axis, θ in this case being approximately 18° . (See Figure 5). Thus as the cylinder is rotated through 180° , the orientation of this magnetic field changes from a "perpendicular" to a "parallel" orientation with respect to the detection coil axis. Therefore there is a varying flux linkage due to the rotation. As this field is not ever completely

ORIENTATION OF THE MAGNETIC FIELD
DUE TO ROTATION

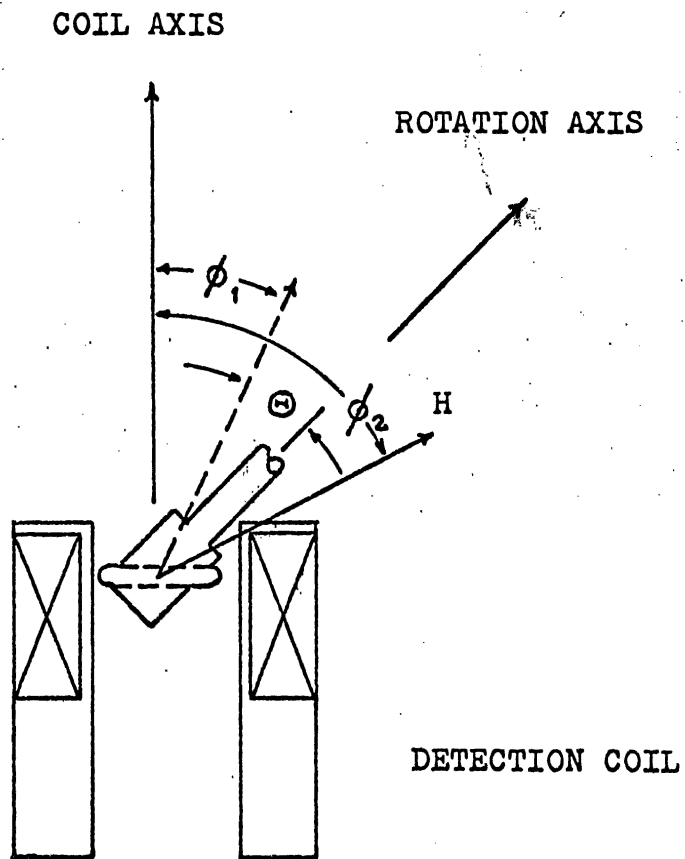


Fig. 5

"parallel" or "perpendicular" to the coil axis, the maximum flux change is reduced by the factor $(\cos\phi_1 - \cos\phi_2)$ where ϕ_1 and ϕ_2 are the minimum and maximum angles between the field and the coil axis.

The field outside the rotating superconductor is a dipole field and as a first approximation one can replace the rotating cylinder by a single turn of wire of radius "a" equal to the radius of the cylinder. The detection coil is replaced by an infinitely long solenoid with "n" turns per unit length. Denote the single turn of wire by "1" and the detection coil by "2". Then the flux linking circuit "2" produced by "1" is

$$\phi_{21} = \int \underline{H}_1 \cdot d\underline{S}_2, \quad \underline{H}_1 = \nabla \times \underline{A}_1$$

$$\phi_{21} = \int \nabla \times \underline{A}_1 \cdot d\underline{S}_2 = \int \underline{A}_1 \cdot d\underline{l}_2 = \frac{\mu_0}{4\pi} \int \frac{i_1 d\underline{l}_1 \cdot d\underline{l}_2}{r_{12}}$$

Now if we interchange the subscripts, we have the converse problem; namely, that of computing the flux linking circuit "1" produced by a current "i" flowing in circuit "2". But the field produced by an infinitely long solenoid is known to be

$$H = \mu_0 n i$$

And the current required to produce a field "H" at the center of a loop of wire is

$$i = \frac{2a}{\mu_0} H$$

Therefore the flux linking the solenoid due to a current "i" flowing in the single turn at the center of the solenoid is the same as the flux linking the single turn of wire produced by the same current "i" flowing in the solenoid multiplied by the area of the loop of wire:

$$\begin{aligned} \phi &= \text{Area} \times H_{\text{effective}} \\ &= 2\pi r a^3 H_{\text{effective}} \end{aligned}$$

But the loop is rotating with an angular frequency ω . Thus the maximum value of the induced EMF is

$$V_{\text{MAX}} = \frac{d\phi}{dt} = 2\pi r a^3 \omega H_{\text{effective}}$$

The order of magnitude of the parameters involved is:

$$a = 10^{-3} \text{ m}$$

$$H_{\text{effective}} \cong 4 \times 10^{-9} \text{ webers/m}^2$$

$$\omega = 1.5 \times 10^8 \text{ /second}$$

$$r = 5 \times 10^5 \text{ turns/m}$$