RICE UNIVERSITY

A Suprathermal Ion Accelerator

by

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To facilitate calibration of the Rice ALSEP/SIDE instrument a low energy (thermal to 4.0 kev) positive ion beam of variable current density (10^{-12} to 10^{-8} amp/cm^2) has been developed. A high perveance electron beam of 0.15 to 0.40 kev is formed using electrostatic lenses alone, without recourse to the customary confining axial magnetic field. The electron beam is introduced into a region where ionization occurs by electron collisions with any of several species of gas introduced into the vacuum system for this purpose. Ions are then extracted by an asymmetric electric field designed to produce a minimal increase in the width of their initial Maxwellian energy distribution. An investigation of the spreading of the ion energy distribution due to dispersion of the electron beam and inherent lens defects is carried out using arguments taken primarily from statistical mechanics and electrostatic field theory.

Once removed from the ionization region, ions are
focused into a beam and accelerated or decelerated to the desired energy by conventional ion optics.
FOREWORD

From the writings of Wang Yang-ming, Chinese philosopher, ca. 1150 A.D.

In former years I said to my friend Chien: "If to be a sage or a virtuous man, one must investigate everything under heaven, how can at present any man possess such tremendous power?" Pointing to the bamboos in front of the pavilion, I asked him to investigate them and see. Both day and night Chien entered into an investigation of the principles of the bamboo. For three days he exhausted his mind and thought, until his mental energy was tired out and he took sick. At first I said it was because his energy and strength were insufficient. Therefore I myself undertook to carry on the investigation. Day and night I was unable to understand the principles of the bamboo, until after seven days I also became ill because of having wearied and burdened my thoughts. In consequence we mutually agreed and said, "We cannot be either sages or virtuous men."
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Section 1
INTRODUCTION

One of the problems confronting the experimenter in space science is the need for accurate calibration of his flight instrument (which in all probability will not be available for re-examination once the experiment is over). This paper describes the calibration device used to determine the response of a suprathermal ion detector. The experiment is formally known as the ALSEP/SIDE (an acronym for Apollo Lunar Surface Experiments Package, Suprathermal Ion Detector Experiment) and is designed to measure the charge to mass ratio and energy distribution of ions near the moon's surface.

The design of the calibration system (called hereafter the "ion gun") draws heavily upon electron optics for its results. The field of low energy particle optics is oriented toward the development of power tubes such as klystrons, cathode ray tubes of various usage, and toward beams for use in charge exchange and other such experiments in physics or physical chemistry. The results from these fields are not always entirely applicable to the present experiment without some alteration. New results have been derived where necessary to bridge gaps which exist between the general body of knowledge and the particulars needed for the ion gun.
Section 2

DESIGN CRITERIA

Since the *raison d'être* of the ion gun is the calibration of the SIDE, it is expedient to define the gun requirements in terms of the expected performance of the detector.

a. The SIDE sensing device and associated accumulators are capable of a dynamic range of $10^6$ counts/sec with a maximum of $10^6$ counts/sec (where 1 count is taken to imply approximately 1 ion at the sensor. However, due to loss factors in the ion optics of the SIDE instrument an input of approximately $10^{11}$ particles/sec at the entrance aperture is needed to insure the maximum count rate. In terms of a current this is equivalent to $1.6 \times 10^{-8}$ amp or, since the SIDE entrance aperture is a 2 mm x 8 mm rectangle, a current density of $2.6 \times 10^{-9}$ amp/cm is required. In order to allow ample leeway, however, a maximum current density capability of $1.0 \times 10^{-8}$ amp/cm$^2$ will be sought in the ensuing calculations.

b. As mentioned previously the SIDE aperture is 2 mm x 8 mm so the ion beam must be homogeneous over this area. The beam was therefore designed to be circular in cross section with a 1.91 cm diameter.
The circular cross section is desirable to allow varying angular orientations of ion gun and SIDE to check beam homogeneity. For this same reason and so that alignment problems at calibration time would not be so critical, the relatively large beam diameter of 0.75 inches was chosen.

c. The entire experiment must operate in the space-like environment of $10^{-7}$ mm Hg. There are approximately $4 \times 10^9$ particles/cm$^3$ (at standard temperature) to be ionized at this pressure unless a pressure gradient is maintained in the gun.

d. The ion beam has a twofold energy requirement. First it must function from the thermal region (in which it has the minimum possible streaming velocity consistent with producing any "beam" at all) through the 4 kev region. Secondly, the beam must have a minimal width in its Maxwellian energy distribution. The minimum width possible is defined by the ambient temperature of the region in which ionization occurs plus any energy the ions might acquire in the ionization process. This will be treated more thoroughly later in the discussion.

e. Because the SIDE is designed to distinguish between particles of different species, the preceding requirements must hold uniformly for the following
species of ion: $\text{H}^+$, $\text{H}_2^+$, $\text{He}^+$, $\text{O}^+$, $\text{O}_2^+$, $\text{N}^+$, $\text{N}_2^+$, $\text{NO}^+$, $\text{A}^+$, $\text{Ne}^+$, $\text{Kr}^+$, $\text{CO}^+$, $\text{CO}_2^+$, $\text{H}_2\text{O}^+$. The method of ionization chosen must be flexible enough to meet the first four requirements regardless of the changes in cross section with different species.

To the above constraints on the ion gun is added the stipulation that the gun should utilize no magnetic fields that might in some way disturb the field distribution in the SIDE’s crossed electric and magnetic field velocity filter. This is mentioned because as will later be seen, a confining magnetic field can in some instances simplify gun design.
3.1 Methods of Ionization

Singly charged positive ions may be produced from a neutral gas in any of several ways. The simplest methods are photoionization, rf ionization and ionization by electron collision. Ionization by processes such as charge exchange or ion-neutral collision presuppose the existence of a second ion beam and hence are of no use here.

Photoionization occurs most simply when a photon of energy $h\nu_1$ interacts with the highest orbit electron in an atom or molecule whose first ionization potential is $V_1$ where

$$h\nu_1 \geq eV_1 \leq \frac{hc}{\lambda_1}.$$  

This gives the corresponding wavelength for ionization

$$\lambda_1 = \frac{hc}{eV_1}.$$  

For $V_1$ expressed in electron volts and $\lambda_1$ in angstroms

$$\lambda_1 = \frac{12,398}{V_1} \text{ Å}.$$  

For the above list of ion species $V_1$ varies from 9.5 eV for NO to 24.46 eV for He. This corresponds to wavelengths of 1300 Å for NO to 488 Å for He. Since photo-
ionization cross sections vary as a function of photon wavelength in a discontinuous manner, a photon source of tunable frequency as well as of tunable power would be necessary to insure a constant output of ions. Such a source is not available in any simple form if one exists at all.

Ionization can be produced as well by pumping longer wavelengths of electromagnetic radiation into a gas until the gas absorbs enough energy from the field to ionize or disassociate. This is a device commonly found in particle accelerators. The applied rf field is difficult to confine spatially, however, with the result that when an extraction field is applied across the ionization region the accelerated beam has a broad energy spectrum. It is possible that a device such as a waveguide could be used to confine the rf energy to a well defined region small compared to the extent of the extraction field, but this seems to be creating more problems than already exist.

Electron collision appears as the most feasible possibility for producing ionization within the design requirements. Fortunately relatively high fluxes of electrons of any desired energy may be produced in a high vacuum environment. There are several methods of electron production among them field emission or photoemission from
metals, but thermal emission from a heated cathode or filament is the simplest in terms of emitter configuration and associated paraphernalia. In addition, thermal emission from metals is a quite efficient process for producing the maximum amount of current vs. the amount of power put into the system. Emission currents as high as 10 amp/cm$^2$ are quite feasible and the cross section for producing single ionization is typically two or three orders of magnitude higher for a given electron flux than for the same number of photons per unit energy range.

3.2 Cathode Requirements

In order to produce a given number of ions per unit volume, a certain number of electrons at a specified energy will be needed. The number of electrons required is determined by the collision cross section and by the efficiency with which the electrons are transported from the cathode region to the ionization region. The transportation problem will be left until later. Table 1 gives the cross sections for species listed on page 3.

For a nearly monoenergetic electron beam carrying a current $i_0$ incident upon a thin target of neutral gas whose apparent cross section is $q_i$, then the resulting positive ion current is given by the approximation

$$i = i_0 N q_i x.$$  \hspace{1cm} (1)
Here $x$ is the thickness of the target and $N$ its number density. The term thin assumes that ionization is caused by primary electrons alone so there is at most one collision per electron. The assumption is justified at these pressures since

$$\text{mean free path } \sim \frac{1}{N\sigma_o}$$

The number density at $10^{-7}$ mm Hg is $4 \times 10^9$/cm$^3$ and the typical collision cross section is $10^{-17}$cm$^2$/particle. The mean free path is thus on the order of $10^7$cm for the electrons among the ions.

Equation (1) allows calculation of the approximate electron current needed in the ionization region to produce an ion current of $10^{-8}$ amp. Taking the previous value of the beam diameter to be 1.9 cm, then the smallest possible cross section (helium) requires an electron current of 0.012 amp. The largest cross section is that of argon which requires 0.0011 amp of electron current. Thus the cathode must be able to provide a variable source of current from a surface area which ideally is small compared to the region in which the ion beam is formed. The reason for the latter will be apparent further on in the discussion.

Examination of Figure 1 shows that because the
dimension 2R is fixed by the ion beam diameter, it remains only to determine the electron beam width, $2z_0$, in order to fix the electron current density required of the cathode. As will be shown later, $z_0$ should be as small as practicable to limit the width of the ion energy distribution. The maximum current density required of the cathode is thus (for the case of helium)

$$j_0(\text{max}) = \frac{6.5}{2z_0} \text{mA/cm}^2,$$

(1a)

where $z_0$ is given in cm. The above current density is not strictly the maximum required because of loss mechanisms present between cathode and ionization region.

The amount of current available from a cathode operated as one plate in a plane diode geometry can be found from statistical mechanics. If the work function of the metal at absolute zero is $\Phi_0$, then the current density of electrons at the surface of the emitter is given by

$$J_s = A(E_0, T)T^2 \exp \left[ -\frac{E_0}{kT} \right]$$

(2)

The function $A(E_0, T)$ varies from metal to metal as well as with temperature, although in a theoretical derivation
it is a constant. The discrepancy arises because the derivation does not take into account the change in the quantum mechanical barrier potential at the surface of a metal resulting from temperature variations. Experimental values of \( A \) are given in Table 2.

3.3 The Electron Lens

Once the problem of producing electrons in adequate quantities is overcome, some way must be found to accelerate them to the proper energy for ionization of a particular species of neutral atom or molecule. Low energies of about \( .1 \) kev are needed since it is here that the ionization cross section curves peak for most atoms. The desired electron acceleration is accomplished through use of an electrostatic "lens".

A lens is usually designed by specifying the shape of beam desired; its nominal energy; whether the lens is to focus the beam, to accelerate it, or both; the focal length needed; and the current density to be transmitted. Once the initial requirements are determined, there are a number of general results from the theory of electron optics that can be applied to define lens parameters.

Electron optics is usually developed from mechanics by starting with a principle of least action. An equation of motion is then derived appropriate to the electrostatic
field. This equation does not usually contain the parameter time (see Appendix A). The equation of motion for electrons in the xz plane is conventionally written (in particle optics)

\[
\frac{d^2 z}{dx^2} = \frac{1 + \left(\frac{dz}{dx}\right)^2}{2V} \left[ \frac{\partial V}{\partial z} - \frac{dz}{dx} \frac{\partial V}{\partial x} \right]
\]

(A7)

where V is the electrostatic potential distribution giving rise to the field. The equation is non-linear and, in general, quite difficult to solve. Two simplifying assumptions can be made when this equation is used to derive particle trajectories in lenses. The first is that the particles enter the lens nearly parallel to the axis of the lens (the x axis in this case). This makes the first derivative small compared to one. The second assumption is to approximate the potential distribution everywhere in the lens by a distribution gotten by expanding the potential along the x axis. The result is that the above equation has the form (see Appendix A)

\[
\frac{d^2 z}{dx^2} = -\frac{dz}{dx} \frac{1}{2\phi} \frac{d\phi(x)}{dx} - \frac{z}{2\phi} \frac{d^2 \phi(x)}{dx}
\]

(A9)
where $\phi(x)$ represents the potential along the x axis. The above is called the paraxial ray equation. In many cases the ray equation is no easier to solve than its parent owing to the complicated form of the potential for most applications. One case which has been dealt with extensively in the literature is that of the unipotential or "einzell" lens. The lens is diagrammed in Figure 2 together with its defining parameters. Equations can be derived (see Appendix B) for the focal length of an einzell lens of rectangular cross section. The focal length is measured from the midplane of the einzell lens and is given by

$$\frac{1}{f} = \frac{11}{16L} \left( 1 - \bar{\phi} \right)^2$$

where

$$\bar{\phi} = \frac{V_0 - V_i}{L} \left[ \frac{1}{2} \left( \frac{z_o}{L} \right)^2 - L \right]$$

From the above relation (3) it is clear that this particular lens can be used to accelerate particles as well as
focus them (i.e. give the resultant beam a defined focal length). Furthermore, the two functions can be independent of one another. The outer electrode potential and aperture size may be fixed to control the size and net energy of the beam. Once this is done the inner electrode potential and aperture may be varied to change the focal length. This meets all requirements set forth previously except the requirement on beam dispersion.

3.4 Electron Beam Dispersion

Ignoring dispersion due to electron scattering in ionizing collisions the two main sources of spatial dispersion of the electron beam are the following:

a. Beam spreading due to the thermal distribution in velocities of electrons emitted from the cathode.

b. Spreading due to the self electric field created by electron caused space charge.

The distribution of electrons leaving the cathode gives them components of velocity in the off-axis (y and z) directions. Although the lens acts to accelerate them along the z axis to velocities many times greater than their thermal velocities, the velocity distribution still contributes an appreciable amount of spreading. Pierce⁴ and others have calculated the maximum current density available after a beam has been transmitted from the
cathode to the image area via an arbitrary type of lens. This limiting current density for a beam energy greater than 10 volts is (see Appendix C)

\[ j_{\text{max}} = j_0 \frac{2}{\sqrt{\pi}} \left( \frac{eV_I}{kT} \right)^{1/2} \sin \theta \]  

(4)

Where \( \theta \) is the half angle of the aperture as seen at the focal point of the lens, \( j_0 \) is the current density at the cathode and \( V_I \) is the potential difference between the point where the image (or crossover) is formed and the cathode. This is only the maximum which can be attained, the actual current density may be less depending on the magnification of the lens. A particle lens is analogous to a photon lens and can have a magnification which is defined in the same manner for both: the ratio of (apparent) object size to image size. If the current density at the cathode is thought of as being evenly redistributed to form the image of itself, then it becomes obvious that for a magnification greater than one the image must have a smaller current density associated with it. The relationship between cathode and image current densities is
where $\beta = M \sin \theta$ and $M$ is the magnification (see Appendix C). The magnification is given by the equation:

$$M = \frac{f_1 \cdot P_2}{f_2 \cdot P_1}$$

where $f_1$ and $f_2$ are focal lengths and $P_1$ and $P_2$ the corresponding principal plane distances. For an einzel lens the focal lengths and principal planes are identical on either side of the lens so the magnification is unity. It can also be shown (Appendix C) that $j \to j_{\text{max}}$ for a beam of long focal length ($\sin \theta \ll 1$) and unity magnification.

For a particular aperture size the above relations allow calculation of the current density at the cathode that will give the required beam density for a specific cathode temperature and lens potential.

The second effect tending to disrupt the electron beam is the beam's space charge field. This problem is
handled in the literature\textsuperscript{35,36,37} by considering the spread of a beam of charged particles in a region which is field free (with the exception of the beam field). It will later be seen that the ionization region through which the electron beam must travel cannot be field free if it is to facilitate extraction of the ions. This has led to an attempt by the author to evaluate the effects of space charge through calculation of the instantaneous force on beam electrons at each point in the combined space charge and extraction fields. Numerical evaluation of this equation of motion will then yield the particle trajectories.

The force due to space charge can be found by applying Gauss' Law to a "pill box" enclosing a section of the electron beam. No attempt is made to solve Poisson's equation for the redistribution of the fields in the extraction region. In other words the image fields of the beam are assumed to be small.

Figure 6 indicates the orientation of the Gaussian surface in the electron beam. From Gauss' Law

\[ 2\epsilon_0 \int_0^{\Delta x} \int_0^{\Delta y} E_z \, dx \, dy = \int_0^{\Delta x} \int_0^{\Delta y} \int_0^{\Delta z} \rho \, dx \, dy \, dz. \]
Since the beam spread is assumed to be small (i.e. the effect of space charge is a perturbation on the beam), then $\rho$ may be taken as constant through the beam and the above integrals reduce simply to the expression

$$E_z = -\frac{\rho \Delta z}{2 \varepsilon_0}.$$ 

The charge density in the one dimensional case is

$$j_x = \rho v_x.$$ 

The total beam current is

$$i = j_x \Delta y \Delta z$$

which gives for the charge density the quantity

$$\rho = \frac{i}{\Delta y \Delta z v_x}$$

The resultant electric field is therefore

$$E_z = -\frac{i}{2 \varepsilon_0 \Delta y v_x}.$$ 

To simplify notation let $I$ be the current per unit beam height in the $y$ direction. Then
The dependence of $E_z$ on the velocity $v_x = v_x(x, z)$
is the precise complication which makes the usual textbook methods inapplicable here.

The electron dispersion in coordinate space will be defined if the trajectories of the outermost electrons in the beam are known. This is due to the simplifying assumption that the trajectories do not cross one another. There then exists a bounding trajectory such that an electron on this trajectory experiences a total force

$$\mathbf{F} = -e \left[ \mathbf{E}_{\text{space}} + \mathbf{E}_{\text{extraction}} \right]$$

The fields are given by equation (6) and by

$$\mathbf{E}_{\text{extraction}} = -\nabla \phi(x, z)$$

The intricate form of the potential distribution in the ionization region dictates that a numerical solution be found to the set of equations
\[ \ddot{x} = \frac{e}{m} \frac{\partial}{\partial x} \varphi(x, z)_{\text{extr.}} \]

\[ \ddot{z} = \frac{e}{m} \left[ \frac{I}{2 \varepsilon_0 v_x} + \frac{\partial}{\partial z} \varphi(x, z)_{\text{extr.}} \right] \]

The most straightforward method of solution is the Runge-Kutta technique\textsuperscript{34} which is quite amenable to computer programming (see Appendix D).

Some of the computer results for the solution to the equations of motion are given in Figures 9 and 11. A discussion of the effects of these trajectories on the ion distribution in energies occurs on pages 34 through 37.
Section 4
THE IONIZATION REGION

Ions must be formed in a region of space which will neither hinder the passage of a coherent electron beam nor interfere with the removal of ions from the region. However, at the same time this region of space must possess an electric field capable of removing the ions and forming them into a beam. It is best to separate the functions of extraction and beam formation entirely since the two operations performed together do not permit electron flow in a beam-like configuration.

A first intuitive guess at a field configuration might be that Figure 4 which is a parallel plate capacitor with an electric field across it. This scheme is often used in cases where the kinetic energy of the electron beam is large enough to overcome the deflection effects of the field at right angles to the direction of beam flow. The equation of motion in this field is

\[ z = \frac{1}{4d} \frac{V_z}{V_x} x^2 + z_0 \]

where \( V_z/d \) is the field strength across the plates and \( V_x \)
is the acceleration potential of the electrons. Because of the parabolic trajectories of the electrons, ions exit the grid in Figure 4 with energies given by

\[ W = eV_z \left[ \frac{1}{4d^2} \frac{V_z}{V_x} x^2 + \frac{z_0}{d} + \frac{1}{2} \right] \]

which is dependent upon both \( x \) and \( z \). For the case where

\[ V_x \gg \frac{V_z x^2}{d^2} \]

some of the energy spread may be eliminated, but the finite width of the electron beam cannot be done away with although it may be minimized in its effect.

The crossed field can be reduced by placing the plates further apart but this will not cure a second problem which arises because of fringing fields at the edge of the plates. These fields must be taken into account because the electron beam must cross them to enter the ionization region. The plate which comprises one side of the electron lens can be used to provide a well defined equipotential upon which the capacitor plate's equipotential lines may be superposed to give a closed solution to the electric field problem. This proposal is dia-
grammed in Figure 5 with a terminating plate for collection of the electron beam after it has traversed the region. If the aperture in the wall is small compared to the dimensions of the box then the problem is a well defined one in electrostatics and is solved in Appendix E. for the general case. The resulting distribution function has the form

$$\Phi(x, z) = \sum_{i=1}^{4} \phi_i(x, z)$$

where, for example,

$$\phi_1(x, z) = \sum_{n=1}^{\infty} \frac{4V_1}{(2n - 1)\pi} \left[ \sin (2n - 1)\pi \frac{x}{a} \right] \left[ \frac{\sinh (2n - 1)\pi \frac{b - x}{a}}{\sinh (2n - 1)\pi \frac{b}{a}} \right].$$

This distribution function has been evaluated numerically by computer and is shown in Figure 5.

Solutions like the one to the above potential problem are too complex to allow any sort of intuitive guess about the trajectories of particles entering the field defined by the distribution. Thus numerical evaluation of the equations of motion will be carried out for most
of the cases of interest. The method used for the solution of the differential equations is the Runge-Kutta technique.

For purposes of comparison a series of trajectories was calculated for electrons in both the box and the capacitor configurations. For a 100 eV particle the deflection resulting from the capacitor field is five times that resulting from the box field, (for comparable geometries). Deflection of the beam in the box field will still have to be accounted for since its effect is far from negligible. This will be discussed later.
Section 5
THE ION BEAM

5.1 The Ion Lens

The field derived in Section 4 for the ionization region accelerates some of the ions which are created through the grid in the grounded plane in Figure 5. It then becomes necessary to form the ions into a beam, bring them to the desired energy and see them off in the proper direction toward the SIDE instrument. This discussion will ignore the place of origin of the ions in the lens field and assume that the grid exhibits the properties of an ideal cathode (i.e. there is no velocity distribution to deal with). Deviations from this assumption will be examined in the sections on energetic and spatial dispersion of the beam.

One of the best analyzed lens configurations is that of the two cylinder electrostatic lens with equal diameter tubes. This is diagrammed in Figure 7. The definition of various lens parameters are also included since the cylinder lens is one of a class of lenses known as "thick" lenses.

Again the first step in solving the lens problem is to first obtain a satisfactory solution to Laplace's equation, this time in cylindrical coordinates. Zworykin
uses as his model two semi-infinite cylinders separated by a distance small compared to their diameters so that they may be thought of as being terminated by a plane of constant potential equal to half the sum of the potentials on the cylinders. His result for the axial potential distribution is

\[ \Phi(z) = \frac{V_2 + V_1}{2} + \frac{V_2 - V_1}{\pi} \int_0^\infty \frac{\sin kz}{k} \frac{dk}{J_0(ikR)} . \]

Somehow it strikes home that the paraxial ray equation just might be difficult to solve for this distribution. Fortunately Hutter and others have found simple but close approximations to the distribution. For a tube of inner radius \( R \) with potentials \( V_1 \) and \( V_2 \) applied to them, the axial distribution takes the form

\[ \Phi(z) = \Phi_0 \exp \left( \frac{1}{\pi} \arctan \frac{z}{a} \right) . \]

Spangenberg has solved the paraxial equation for this distribution in terms of focal lengths and distances to the principal planes (his results are presented in quite useful form in terms of object and image distances). The resultant focal lengths of the lens are given by the expression (in this case \( i \) refers to the image side of the
lens, \( o \) to the object side).

\[
f_{i, o} = \pm R \frac{\exp \left( \frac{Q \pi}{w \sqrt{3}} \right)}{\sin \left( \frac{\pi}{w} \right)}
\]

and the distances to the principal planes by

\[
P_{i, o} = R \frac{\exp \left( \frac{Q \pi}{\sqrt{3} w} \right) \pm \cos \left( \frac{\pi}{w} \right)}{\sin \left( \frac{\pi}{w} \right)}
\]

where

\[
Q = \frac{\sqrt{3}}{4 \pi} \ln \frac{V_2}{V_1}
\]

\[
w = (1 + Q^2)^{1/2}
\]

\[
R = \text{the radius of the two lens tubes, which in this case are equal.}
\]

Because ions are extracted with a constant energy of 200 eV a correction must be made for this departure from the standard initial conditions of electron optics (zero initial kinetic and potential energy). The correction and a discussion of its consequences is given in Appendix G.
5.2 Ion Beam Dispersion (Spatial)

The preponderant cause of spatial dispersion of the ion beam is space charge. This is true to an even greater degree than for the electron beam just discussed. What makes space charge more important here is the cylindrical shape of the ion beam as opposed to the thin, flat geometry of the electron beam. The self field produced by a cylinder of charge is radially directed so that the dispersion occurs in essentially two dimensions neither of which can be neglected. This was not the case for the rectangular beam.

Ions will exit the ionization region with kinetic energy nominally equal to that of the electrons which in most cases will be about 100 eV. The limiting energy of operation of the beam at the low energy limit means that the space charge problem will occur primarily in the field free region between gun and SIDE. A second trouble spot of somewhat lesser magnitude will occur inside the tube as the ions are decelerated.

Calculation of the space charge dispersion for a cylindrical ion beam requires a solution of Poisson's equation and is more difficult than the one dimensional thin-beam case. The solution to the paraxial ray equation in these circumstances is worked out in detail in Appendix F.
with the final result being that

\[ z_w = \left[ \frac{2^{3/4} \pi \epsilon_0 \left( \frac{m}{e} \right)^{1/2}}{1/2} \right]^{1/2} \frac{\sqrt{V}}{r_0}. \]

\[ \cdot \exp \left[ \frac{-(R_0')^2}{2} \right] \int_0^{R_0} \exp(u^2) \, du. \quad (14) \]

This gives the distance from the end of the ion lens to the beam waist for a given focal length and beam current. The particular focal length chosen for the graph in Figure 8 is such that the minimum beam diameter occurs at the aperture of the SIDE. The results for the maximum required beam current of \(1.6 \times 10^{-8}\) amp are given in the figure for the range of ionic mass that will be encountered. At low energies the problem gets out of hand and the only solution is to decrease the ion gun to SIDE working distance.

There is a further limitation to be placed on the maximum current density available in the ion beam. Smith and Hartman\(^{32}\) have solved Poisson's equation for a cylinder of charge moving inside a hollow tube such as one of the cylinders in the lens just discussed. Their conclusion was that the maximum current that can be transmitted
down a tube of radius \( R \) by a beam of radius \( \rho \) for a constant current density \( j^+ \) is

\[
I_{\text{max}} = \frac{V_o}{6.07 \times 10^4} \left( \frac{m_e}{m_i} \right) \left( 1 - \frac{V_R}{V_o} \right)^{3/2} s_\rho^2
\]

where \( V_o = \) potential difference between the ion source and the potential on the cylinder.

\( V_R = \) potential difference between the center of the beam and the lens' cylinder.

\[
s_\rho = \rho^{1/2} \left[ \left( \frac{m_i}{2e} \right)^{1/2} 4\pi j^+ (V_o - V_R)^{-3/2} \right]^{1/2}
\]

(15a)

In the case where \( R/\rho = 2 \), then \( \frac{V_R}{V_o} = 0.78 \) for which the factor

\[
\left( 1 - \frac{V_R}{V_o} \right)^{3/2} s_\rho^2 = 0.35
\]

(15b)

for the maximum transmission of current. (These values were taken from graphs in the reference.) The expression
for $I_{\text{max}}$ reduces to

$$I_{\text{max}} = 1.35 \times 10^{-7} v_o^{3/2} \left( \frac{m_p}{m_i} \right)^{1/2}$$  \hspace{1cm} (16)

where $m_p$ and $m_i$ are proton and ion masses.

5.3 Ion Beam Dispersion (Energetic)

Knowledge of the energy dispersion in the ion beam when it arrives at the SIDE instrument is of prime importance in the calibration of the detector. Each element developed so far for the ion gun has been analyzed for its effect on the energy distribution of the ion beam. The net energy of individual ions in the beam will be

$$W_{\text{total}} = W_{\text{thermal}} + W_{\text{electron}} + W_{\text{electric}} \cdot \text{collision fields}$$

The field term can be conveniently broken into two parts

$$W_{\text{electric}} = W_{\text{extraction}} + W_{\text{lens}} \cdot \text{fields field}$$

Ideally the energy distribution of the entire ion beam can be found by the application of transport theory which is simplified in the beam itself because the beam is essentially collisionless. The solution to the problem lies in finding the conditional probability distribution
which gives the velocity distribution at a specified point in space, and in this instance at some point along the ion beam, presumably at the detector. Moments of this distribution can then be taken by the usual methods to determine the distribution width, mean energy, etc. Unfortunately, finding the particle distribution function \( f(r, v) \) entails solving the Boltzmann equation

\[
\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{r}} + \frac{\vec{F}}{m} \frac{\partial f}{\partial \vec{v}} = \left( \frac{\partial f}{\partial t} \right)_{\text{collisions}}
\]

which is fairly difficult when the forces on the particles arise from potential distributions such as those found in the ionization and lens regions. Methods exist for the solution of the distribution problem in a rigorous manner, however, they are outside the scope of this work. (For example, see Lindsay\textsuperscript{12}.)

Nevertheless, something can be done in the way of a quick zeroth order look at the total energy dispersion of the beam around its mean energy. Taking the sources in
order:

a. Thermal

The Maxwellian distribution in velocities is given by

$$f(v) = n \left( \frac{m_i}{2\pi kT} \right)^{3/2} \exp \left( -\frac{m_i v^2}{2kT} \right)$$

where $m_i = \text{mass of the } i^{\text{th}} \text{ ionic constituent}$. The resultant distribution in speeds is given by

$$F(v) = f(v<|v|<v + dv)$$

$$= 4\pi v^2 f(v) dv.$$

For a differential speed (or energy) distribution the modal value is given by solving

$$\frac{dF(v)}{dv} = 0 \text{ for } v \text{ (or } W).$$

The result is often called the most probable speed and is given by

$$\bar{v} = \left( \frac{2kT}{m_i} \right)^{1/2}$$  \hspace{1cm} (17a)
Thus the most probable energy is

$$\bar{W} = kT.$$  \hspace{1cm} (17b)

Approximately 98% of all particles will fall within $\bar{W} \pm \bar{W}$ in the Maxwellian distribution which gives a method of measuring the energy spread of the beam due to the thermal contribution. The mean energy of the beam is $\frac{3}{2}kT$ and so is quite easily related to experimentally identifiable parameter $\bar{W}$. (At room temperature $kT$ is about 0.04 eV.)

b. Electron Collision

The electron beam has a nominal energy of 100 eV and its corresponding velocity is $2 \times 10^3$ times the thermal velocities of the neutral gas. The center of mass for collisions can thus be considered to be located in the heavy particles' frame of reference. The mean energy gained by neutrals (ions) when averaged over all collision angles is

$$\bar{W}_{a_{coll.}} = \frac{2m_em_a}{(m_e + m_a)^2} \bar{W}_{el}$$

where $\bar{W}_{el}$ = mean energy of the electrons before collision and $m_e$ and $m_a$ are the electronic and atomic
masses respectively. For these masses, \( m_e \ll m_a \) so that

\[
\overline{W}_{a, \text{coll.}} = \frac{2m_e}{m_a} \overline{W}_{\text{el}}
\]  

(18)

For molecular hydrogen being ionized by the 100 eV beam the energy acquired by the hydrogen is about 0.05 eV. Comparison with its thermal energy at room temperature (0.04 eV) shows that the electron beam heats up the gas so that the ion temperature is about twice that of the neutrals. This effect is smaller for heavier particles and for krypton it is a negligible 0.003 eV.

c. Extraction Field

It is somewhat more difficult to define a mean energy associated with the electric fields. In general the energy of an ion emerging through the ground plane is

\[
W_{E, \text{extr.}} = e \Phi(x_i, z_i)
\]

where \((x_i, z_i)\) are the coordinates giving the location of the \(i^{th}\) ion at the point of ionization. To first
order the coordinates \((x^1, z^1)\) can be thought of as being distributed uniformly within the boundaries of the electron beam. These boundaries are determined by the forces on the electrons and are due primarily to the space charge created by the presence of the beam. A secondary effect for realistic electron currents is the extraction field which causes a slight bending of the electron beam away from the x axis.

A mean ion energy can be defined, however, by the set of electron trajectories which bisects the electron beam in the xz plane. The variation in this mean energy due to bending of the electron beam is shown by the dashed lines in Figure 9. It is on the order of 0.5 eV for a 100 eV beam and 1.25 eV for a 200 volt beam. The finite width of the electron beam causes a distribution in ion energies about this mean energy. This is shown qualitatively in Figure 10. The effect of thermal and collisional energies is not shown. Limits on the resultant ion energies are shown as heavy lines in Figure 9.

Space charge causes the primary disturbing force on the electron trajectories at beam currents \(\approx 10^{-5}\) amp. This is illustrated for comparison in Figure 9.
by the outermost dashed lines. These define the energy boundary for an electron beam of $10^{-5}$ amp. For larger beam currents the problem is much greater as can be seen from Figure 11.

The only methods of offsetting the dispersion effects of space charge are increases in electron energy and/or focusing of the beam. Table 1 indicates that electron beam can be accelerated to 200 or even 300 eV without serious loss in ionization efficiency. The improvement resulting from a more energetic electron beam is evident from Figure 11. The efficacy of the more energetic beams is not seen until realistic (with respect to total maximum ion current needed, see page 8) electron currents of $10^{-3}$ amp are reached. This is due to the overall increase in potential in the ionization region caused by increasing the einzel lens potential. Decreasing the focal length of the electron beam is helpful in narrowing the energy distribution, but is not effective without an increase in beam energy. This is due to the relatively large z component of velocity contributed by the strongly focusing einzel lens.

Ions leaving the extraction region are thus seen to have a spatially varying energy distribution.
Superimposed upon this are the thermal and collisional energies of the ions. However, these are quite small when compared to the energy dispersion caused by the electron beam (0.1 eV for collisions as compared to 5.0 eV for a $10^{-3}$ amp electron current) and will be neglected.

The final acceleration or deceleration of the ion beam is accomplished by the tubular lens already discussed. Here again rather large energy dispersions are encountered.

d. Lens Field

The ion lens will accelerate particles differentially according to their point of entry into the ion lens field. The potential along a radius is not constant, but is depressed toward the center of the beam because of the presence of space charge. As mentioned in Section 5.2 the solution to Poisson's equation under these circumstances has been obtained and from it, the potential depression. The depression along a radius is given by

$$\frac{V_s}{V_o} = \left(1 - \frac{V_R}{V_o}\right) \left[1 + \frac{s^2}{4} \left(\frac{r}{\rho}\right)^2\right].$$
All terms in the above equation are as previously defined for Equation (15) with the addition of

\[ V_s = V_o - V_R + V(r) \quad (19a) \]

which is the potential difference between the ion source and some point, \( r \), in the beam. Again this assumes that the current density does not depend upon \( z \), the distance along the tube axis. For the case where \( \frac{R}{\rho} \gg 1 \) the higher order order terms in the expansion may be dropped giving

\[ \frac{V_s(r)}{V_o} = \left(1 - \frac{V_R}{V_o}\right) + \left(1 - \frac{V_R}{V_o}\right)s^2 \left(\frac{r^2}{4\rho^2}\right) \quad (20) \]

For the maximum transmission of current, Equation (15b) stipulates that
\[
\left(1 - \frac{V_R}{V_0}\right) s^2 = 0.35.
\]

A further condition for maximum transmission of ion current is that
\[
\frac{V_R}{V_0} = 0.78
\]
for the case where \( \frac{R}{\rho} = 2 \) (see page 25). Substitution into Equation (20) gives
\[
\frac{V_s(r)}{V_0} \approx 0.22 - 0.088 \frac{r^2}{\rho^2} \quad (0 \leq r \leq \rho)
\]
(21)

For the most part this is a constant effect. It is independent of the beam current (so long as the beam current is the maximum possible) within the limitations imposed in the derivation. The net result is a 40% change in beam energy over half of the width of the beam. This is shown schematically in Figure 12.

For acceleration potentials large compared to the extraction potential the dispersion in energy due to the depression in the lens field will dominate.
In this case the size of the SIDE aperture will determine the energy dispersion seen by the detector. If the orientation of the slit is as shown in Figure 12, then for the relative beam and aperture sizes given in Section 2b, the energy dispersion will have the limits

$$\Delta W_x \approx 0.0\% \Delta W_0$$

$$\Delta W_y \approx 2.3\% \Delta W_0$$

Thus at ion acceleration energies on the order of 1 kev and less the aforementioned dispersion due to the electron beam will dominate. The size of the ion energy dispersion which results will depend heavily upon the amount of ion current required.

Assume that the ions are extracted from the center half of the ionization region as indicated by the vertical dashes in Figure 11. For a $10^{-9}$ amp ion beam the maximum electron current required is about 2 ma which fixes limits on the smallest possible extraction energy dispersion at

$$0.6 \text{ eV} \leq \Delta W_{\text{extr}} \leq 1.8 \text{ eV}.$$
Figure 13 shows the relation of the extraction and acceleration potentials to the final beam energy. There is an obvious problem encountered when $V_o \to 0$ because the lens ceases to function when space charge forces become comparable to the lens forces. An estimation of the potential at which this occurs can be obtained by equating the radial forces due to the lens with the forces caused by space charge.

The radial force near the center of the ion lens can be found by expanding the axial potential in the lens in a Taylor series about the axis. The first three terms in this expansion are

$$\phi(z, r) = \phi(z) - \frac{r^2}{4} \phi''(z) + \frac{r^4}{64} \phi^{(IV)}(z) - \cdots$$

If the first two terms are kept then the substitution of $\phi(z)$ from Equation (11) gives for the resultant acceleration

$$a_{lens} = 0.35 \frac{r}{R^2} (V_2 - V_1) \text{sech}^2 \left( \frac{1.318z}{R} \right) \cdot \frac{1}{R} \cdot \text{tanh} \left( \frac{1.318z}{R} \right).$$  \quad (22)
The acceleration of the ions away from the axis due to space charge is given by Equation (Fl)

\[ a_{\text{sp. chg.}} = \frac{I}{4\sqrt{2}\pi\epsilon_0\eta^{1/2}v^{3/2}\frac{1}{r}} \quad \text{(Fl)} \]

in (22) \( V_1 \) and \( V_2 \) represent the first and second potentials on the two lens tubes so that \( V_2 \) is total accelerating potential of the lens. The value of \( V_1 \) may be determined from Spangenberg and is \( 0.2 V_2 \).

The value of \( V \) is that of the potential at the edge of the beam and was shown to be \( 0.31 V_2 \) (see page 39).

Using these conditions, evaluation of (22) and (Fl) gives as the critical potential for the loss of the lens effect the expression

\[ V_2 \bigg|_{\text{crit.}} = V_c = \left[ 5.59 \times 10^7 \left( \frac{m_1}{m_p} \right)^{1/2} I \right]^{2/5} \quad \text{(23)} \]

The critical voltage turns out to be 2 volts or less over the range of ion masses and currents to be encountered. When the potential on the lens drops
to this value the ion beam current will drop to zero. The solution to this problem is to shift the extraction energy to another value so that $V_c$ will shift to another point on the $V$ vs. $W_{ion}$ curve. This makes it possible to calibrate the SIDE around the critical energy.
Section 6
EXPERIMENTAL DESIGN

Sections 2 through 5 serve as the theoretical basis upon which the laboratory model of the ion gun was built. These same sections will also be useful later in analyzing the experimental results.

The following list is an attempt to trace each physical parameter which must have a definite value in order for the gun design to be consistent with theory. Each parameter is given along with the source of its defining value(s).

List of ion gun parameters:

1. Ion beam diameter
   This was defined to be 1.91 cm = 0.75 in, see page 3.

2. Maximum cathode current density
   This was determined in Equation (1a). Since the type of cathode has been selected it is possible to say that the cathode will have to operate in the neighborhood 1000 °C. The construction materials for the gun will have to be able to withstand this quantity of heat, which restricts useful materials to stainless steel for conductors and ceramics for insulators. The amount of power needed for the filament to heat
the cathode to this temperature is about 200 watts.

3. Einzel electron lens
The potential on the outer electrodes ($V_0$ in Figure 14) will be 100 to 300 volts, see Table 1. The inner electrode potential ($V_i$) must be adjusted so that a maximum amount of current will be transmitted across the ionization region. For a given mechanical geometry the potential can be predicted, see page. The mechanical geometry is determined by setting $z_0$ equal to 0.02S, see Figure 5. The distance $z_i$ may then be set arbitrarily at $2z_0$. Since the lens will be made of stainless steel of 0.063 in. thickness, the distance between lens apertures, $L$, must be at least 0.63 in., see Reference 30.

4. The extraction region
The width of the region is on the order of the ion beam diameter and large enough to avoid edge effects, but otherwise arbitrary. It was chosen to be 2.00 in. which makes the nominal length of the region ($S$) equal to 4.00 in. The grids used in the ionization region should be of maximum possible transmission, but sufficiently fine so that peek-through fields are minimal. Since the field will fall off roughly as the inverse of the wire separation, a grid cloth of
140 \times 140 \text{ mesh made from } 0.0013 \text{ dia. wire was chosen.}

5. The ion lens

Equations (18a) and (25) give the defining relation so that the lens tube diameter is twice that of the desired beam diameter or 1.50 in. The cylinder length is, within reason, arbitrary and may be chosen as 8 tube radii. Cylinder spacing is on the order of 0.1 tube diameters, see Reference 13. The potentials applied to the lens are again determined by the governing lens equations. Final ion energy is determined by \( V_2 \) (Figure 14), but \( V_1 \) must be adjusted to compensate for space charge and varying image distances.

In order for the ions leaving the gun to have the proper energy when they arrive at the SIDE ground plane, the final potential surface through which they pass must be at the SIDE ground level. This necessitates floating the ion gun system ground to whatever final acceleration potential is desired. This poses no real difficulty other than the very great inconvenience of insulating and isolating the ion gun for voltages as high as 10 kv.

It is conceivable that mechanical parameters calculated in theory will not be exact. For this reason the gun was designed so that many dimensions could be changed
by as much as 100%. Because of the interrelation of many electrical and mechanical parameters this mechanical flexibility should relieve some of the electrical design difficulties.

Flexibility of design was sought because this gun was the first of its genre within the author's experience and corrections to the theory of design are to be expected. By building in the capability of later refinement it is hoped that a savings can be realized in terms of costly machined parts and electrical circuitry.
Section 7

EXPERIMENTAL RESULTS

The ion gun can be thought of as being made up of an "electron gun" with an adjoining neutral gas target area and ion accelerator. It was found convenient to carry out the experimental work on the ion gun in two stages based on the relative independence of the electron gun from the ion gun. The electron beam was first made to function properly. Once this was accomplished work began on the ion accelerator proper without any qualms over the performance of the electron beam.

After determining that current was flowing from the cathode to the first slit plate ($S_1$ in Figure 14), an attempt was made to measure the electron current to the electron collector (EC). It was found that the collector had to be biased 100 v positively with respect to $S_1$, $S_2$ and $P_2$ (Figure 14). That this measure was sufficient was shown by the fact that the curve of $V_{EC}$ vs. $I_{EC}$ saturated for $V_{EC} \sim V_0 + 50$ v. This would be expected from the secondary electron spectra at these energies. A typical value of the cathode to $S_1$ current for $V_0 = 200$ v was 25 mA. With $V_I = 0.0$, a current of 0.20 mA was transmitted to the electron collector. It was found that if
was varied the current to the electron collector changed and had a definite maximum. This is shown in Figure 16 for several values of beam current and energy.

An attempt was made to predict this result (i.e. the maxima) from the theory set forth on pages 16 through 19. The voltage ratio $\frac{V_i}{V_0}$ was determined via computer for a given lens geometry so that the focal properties of the lens would nearly produce a crossover in the electron beam. The results for several values of beam current and energy are given in Table 3.

The close agreement between computed and experimental results argues strongly for the hypotheses advanced on pages 18 and 19 as well as for the particular methods used for numerical determination of the trajectories. In contrast, it was found that the "universal beam spread curve" of Pierce and others would not yield the correct results in this case. This is no real failure on the part of that theory because in reality it is inapplicable to non-field-free regions as was previously pointed out.

Another consideration in the experiment is the overall efficiency of the electron lens. The total plate current at $S_1$ was measured over an area approximately
equal to the area of the emissive cathode face. The current density at the plate in the area of the lens slit was therefore about 6.3 mA/cm². The slit in plate S₁ had an area of 0.5 cm² so that approximately 3.1 mA of electron current could be expected to pass through the slit. Since 1.01 mA reached the electron collector, about two-thirds of the beam was lost enroute.

Much of this current loss can be accounted for in terms of the thermal dispersion of the electron beam. This was discussed on pages 13 through 15, and Equation (5) was shown to represent an expression for the limiting image current density which is

\[
\frac{j}{j_{\text{lim}}} = \frac{1}{M} \text{erf} \left[ \frac{\beta^2 \left( \frac{eV}{kT} \right)}{1 - \beta^2} \right]^{1/2}
\] (5)

For the case mentioned above, the 1.01 mA measured at the electron collector can be assumed to be collected over a 0.5 cm² area when the electron beam is operating under optimal focusing conditions (i.e. it has approximately the same cross section on either side of the ionization region, see Figure 8). The image current
density is thus about $2 \text{ mA/cm}^2$. Using the proper experimental data in Equation (5) gives the result

$$j_0 = 2.8 \text{ mA/cm}^2.$$  

This accounts for roughly half of the measured electron beam loss. However, due to the crude nature of the current density estimate for the entrance of the electron lens, this must be taken as an order of magnitude confirmation of Equation (5). In reality Equation (5) must account for all of the beam current loss except that due to electron-particle collisions.

One or two further points about the experimental results might be made before leaving the subject of the electron lens.

First, the cathode efficiency for a given temperature was lowered by as much as fifty percent each time the ion gun was brought up to atmospheric pressure. This was consistent with data furnished by the cathode manufacturer and is due to the adsorption of several monolayers (per cycle to atmosphere) of contaminants by the porous tungsten. The surface contaminants have the effect of raising the work function of the cathode electrons with a subsequent loss in emission efficiency (see Equation (2)).

A second contributor to cathode deterioration is ion
contamination or poisoning. This is caused by bombardment of the cathode by large numbers of ions created when surrounding surfaces outgas. The outgassing is caused by the high cathode temperatures which are sufficient to heat the walls of the vacuum system and thus raise the system pressure from $10^{-7}$ mm Hg to $10^{-5}$ mm Hg. The ions formed by the electron beam in the cathode region are then driven into the cathode by the approximately 200 volts of potential difference between cathode and first slit plate of the electron lens.

The design developed on pages 22 and 23 and in Appendix E for the ionization region proved to be quite workable. Figure 17 shows the dependence of the electron collector current on the potential applied to plate $P_1$. While there is some deflection of the electron beam for a $1.5 : 1.0$ ratio of the walls bounding the ionization region, there is no significant deflection for a ratio of $2.0 : 1.0$. Presumably this would also be true for ratios greater than $2.0 : 1.0$, but this has not been verified by experiment.

At present, of the several results predicted for the ion lens, not all can be verified with the laboratory apparatus immediately at hand. In particular, the space charge limitations on ion beam current density and focal
length cannot be measured properly without what might be described as an ion-optical bench. Such a bench for the ion gun would require an ion collector capable of motion along the axis of the ion lens. What is more difficult, however, is finding a vacuum system capable of holding the entire experiment. Such a system is not presently available to the author. In addition, spatial dependence within the ion beam of the particle energy distributions cannot be deduced experimentally without mechanical fixtures and vacuum feed-thru's capable of two degrees of freedom so that a detector can be used to examine different portions of the beam in a plane perpendicular to the lens axis. The preparation of such a fixture is out of the scope of the present work.

Experimental results have been obtained, however, which are pertinent to the aforementioned ion lens theory.

Measurements of ion current densities were made with a Faraday cup designed by the author and used for calibration purposes on a different experiment. A schematic of the cup is presented in Figure 13. The estimated grid transmission is 45% based on the fact that each grid is 82% transmissive (optically). The insulating material used is Kel-F, a plastic, which possesses extremely high bulk and surface resistivity so that leakage currents for
typical applied fields are less than $10^{-13}$ amp.

Through use of the Faraday cup it was determined that the ion lens of Figure 14 would not function in the desired manner. The maximum ion current obtained (after correction to standard pressure and electron current and allowance for grid transmission) was $1.3 \times 10^{-10}$ amp. This is two orders of magnitude below the SIDE requirements. Some deliberation led to the hypothesis that the field caused by the steep potential gradient between $P_3$ and $L_1$ (Figure 14) was causing a lens effect to take place in this region rather than in the region $L_1$ and $L_2$. This idea was strengthened when the effect of $V_2$ was found to be negligible even when $V_2 < V_1$, at which point the lens effect should have vanished under normal circumstances$^{39}$. Confirmation of ion current loss to the first lens tube came when a current of approximately $10^{-8}$ amp was found to be flowing through the tube circuit to ground.

Reference to the literature$^{33}$ revealed that the near null result of this experiment might have been anticipated beforehand. It is ironic that one of the first results obtained in the field of particle optics was the discovery of the single aperture lens which is one of the very few divergent particle lenses that can be devised. The lens is divergent when the particle source is at ground poten-
tial and the single aperture is raised to some accelerating potential with respect to the source. Thus the field region formed by $P_3$ and $L_\perp$ merely duplicates this result in a slightly different way. Some lessons are dearly learned.

A simple change was made in the ion lens to test this hypothesis. The lens tube ($L_\perp$) was reconstructed with a length equal to half the original. An asymmetry was introduced into the lens by this action, but by removing $L_\perp$ from the ground plane ($P_3$) it was hoped that the steepness of the potential gradient would be reduced with beneficial effect. In particular, it was desirable to reduce the radial component of the field so that the ions would have a correspondingly greater chance of obtaining enough velocity parallel to the axis to reach the far side of the lens.

The limited goal of the experiment was realized and a corrected current of $2.4 \times 10^{-9}$ amp was measured at the Faraday cup. This is twenty times more than was available under similar conditions in the first experiment.

Based on the latter experiment a modification of the existent ion lens is proposed which will give a purely axial field between $P_3$ and $L_\perp$. Diagram 19 shows the insertion of a so-called drift tube between the plate and
the lens tube.

This segmented "tube" will serve two functions.

First, because the apertures are several times the diameter of both the ion beam and the aperture spacing, the field in the beam region will be nearly parallel to the lens axis and quite uniform. This brings the ions up to the first lens potential without imparting a radial velocity component. This should refine the partially successful result produced by cutting the tube L1 in half.

Secondly, the resistor values used to drop the voltage across the apertures can be selected to allow the potential to vary as the four-thirds power of the axial distance. Smith and Hartman\(^{31}\) have shown that this potential distribution matches the boundary conditions for the solution of Poisson's equation for cylindrical current flow.

Conclusion

The ion gun did not function in the manner described in the opening half of this paper. It should be noted, however, that despite the error in experimental design which led to ion lens failure, the electron lens and the ionization-extraction regions performed quite well and were adequate confirmation of the theory developed in
Sections 3 and 4.

A second ion gun will be built, or, more correctly, the present design will be modified as outlined in the preceding few pages. The initial success of the theory offers reason for optimism concerning the eventual functioning of the gun.
Appendix A

The Paraxial Ray Equation

Electron optics can be developed most easily from the principle of least action as stated for systems with time independent constraints and conservative forces. This has the form\(^1\)

\[
\delta \int T \, dt = 0 \quad (A1)
\]

where \(T\), the kinetic energy, is derivable from the Hamiltonian of the system. Using the conventional definition of the kinetic energy

\[
T = \frac{mv^2}{2} = \frac{m}{2} \left( \frac{ds}{dt} \right)^2 .
\]

The differential length of time may be solved for

\[
dt = \left( \frac{m}{2T} \right)^{1/2} ds . \quad (A2)
\]

Substitute (A2) into (A1) and replace \(T\) by \(H-V\) to get

\[
\delta \int (H - V) \left[ \frac{m}{2(H - V)} \right]^{1/2} ds = 0 \quad (A3)
\]
In the case of cartesian coordinates the two dimensional arc length is

\[ ds = \left[ (dx)^2 + (dz)^2 \right]^{1/2} \]

which can be written in terms of the first derivative

\[ z' = \frac{dz}{dx} \]

and then substituted in (A3) to give the following form of the variational principle

\[
\delta \int_{x_1}^{x_2} \left( H - v \right)^{1/2} \left( \frac{m}{2} \right)^{1/2} \left[ 1 + (z')^2 \right]^{1/2} \, dx = 0
\]

(A4)

The variational operator \( \delta \) indicates partial differentiation with respect to a parameter \( \alpha \) which serves to select a particular path in space. The path taken by the particle is the one for which the integral is minimal between the fixed end points \( x_1 \) and \( x_2 \). Make the operational change

\[ \delta \rightarrow \frac{\partial}{\partial \alpha} \]
and rewrite the functional dependence of \((A4)\) so that

\[ J(x, z, z') = \delta J(\alpha) = \frac{\partial J}{\partial \alpha} d\alpha \]

and also

\[ z = z(x, \alpha) \]

\[ V(x, z) = V(x, z(\alpha)) \]

\[ z' = z'(x, \alpha) \]

Make the above replacements in Equation \((A4)\) and carry the differentiation through the integral

\[
\frac{\partial J}{\partial \alpha} d\alpha = \int_{x_1}^{x_2} \left\{ (H - V)^{1/2} \left[ 1 + (z')^2 \right]^{1/2} \left[ z' \frac{\partial}{\partial \alpha} \left( \frac{\partial z}{\partial x} \right) \right] \right. \\
\left. - \frac{1}{2} (H - V)^{-1/2} \left[ \frac{\partial V}{\partial z} \frac{\partial z}{\partial \alpha} + \frac{\partial V}{\partial x} \frac{\partial x}{\partial \alpha} \right] \right\} dx d\alpha. \tag{A5}
\]

Since \(x\) is not a function of \(\alpha\), \(\frac{\partial x}{\partial \alpha} = 0\) in the above.

Do the first integral in \((A5)\) by parts.
\[ \int_{x_1}^{x_2} (H - V)^{1/2} \left[ 1 + (z')^2 \right]^{-1/2} \left[ \frac{\partial}{\partial x} \left( \frac{\partial z}{\partial \alpha} \right) \right] \, dx \, d\alpha \]

\[ = \left. \left\{ (H - V)^{1/2} \left[ 1 + (z')^2 \right]^{-1/2} \frac{dz}{dx} \frac{\partial z}{\partial \alpha} \right\} \right|_{x_1}^{x_2} \]

\[ - \int_{x_1}^{x_2} \frac{d}{dx} \left[ (H - V)^{1/2} \left[ 1 + (z')^2 \right]^{-1/2} \right] \frac{\partial z}{\partial \alpha} \, d\alpha \, dx \]

(A6)

The first term on the R.H.S. is zero at the end points because the differential change in \( z \) with respect to \( \alpha \) must be zero at those two points by definition of \( \alpha \). Placing the second term in (A6) into (A5) gives the result

\[ \frac{\partial J}{\partial \alpha} \, d\alpha = \int_{x_1}^{x_2} \left\{ -\frac{d}{dx} \left[ (H - V)^{1/2} \left[ 1 + (z')^2 \right]^{-1/2} \right] \frac{\partial z}{\partial \alpha} \right\} \, d\alpha \, dx \]

\[ - \frac{1}{2} (H - V)^{-1/2} \left[ 1 + (z')^2 \right]^{1/2} \frac{\partial V}{\partial z} \frac{\partial z}{\partial \alpha} \, d\alpha \, dx . \]
But the operation \( \frac{\delta z}{\delta \alpha} \) \( d\alpha \) is the total variation \( z, \delta z \).

The fact that the variation over the integral is zero means that the integrand must be zero i.e.

\[
\frac{d}{dx} \left[ (H - V)^{1/2} \left[ 1 + (z')^2 \right]^{-1/2} \right] + \frac{1}{2} (H - V)^{-1/2} \left[ 1 + (z')^2 \right]^{1/2} \cdot \frac{\delta V}{\delta z} = 0 .
\]

By carrying out the indicated operations and clearing of fractions, the following result is gotten

\[
\frac{d^2 z}{dx^2} = \frac{1 + (z')^2}{2(H - V)} \left[ z' \frac{\delta V}{\delta x} - \frac{\delta V}{\delta z} \right] . \tag{A7}
\]

Equation (A7) is equivalent to results stated in texts such as Harmon\textsuperscript{16} or Zworykin\textsuperscript{17} if the Hamiltonian is taken to be zero everywhere along the path. This is often the case in electron optics where the electron is assumed to start from rest at a point of zero electric potential such as a cathode. This assumption is not strictly true since the electron always possesses thermal energies at temperatures above absolute zero, as well as having potential energy while confined in the potential well created by the lattice atoms in the metal. In Appendix G the effect of ion kinetic energies on the ray
equation when ions emerge from an extraction field is discussed.

The general form of the equation is quite difficult to solve though it is nothing more than a contorted form of Newton's law. The advantage of Equation (A7) is that it may be simplified and then solved analytically in a few special cases which are of interest in electron optics without having to eliminate the parameter time to obtain spatial trajectories. Since only the spatial path is of interest, the additional information provided by time of flight is of little use.

As a first step toward simplification of (A7) expand the electric potential in two dimensions about an axis of symmetry which can conveniently be chosen as the x axis. The expansion in terms of z involves only even powers of z for the reason that there are no forces in the z direction because of the symmetry condition, so there can be no terms in the distribution function which can give rise to forces. The expansion has the form

\[ \phi(x, z) = \sum_{n=0}^{\infty} A_n(x) z^{2n} \]  

(A8)

If this is substituted into Laplace's equation the result is
The recursion relation for the above is

\[ A_n(x) = - \frac{1}{2n(2n - 1)} A''_{n-1}(x). \]

Writing out the first few terms indicates the general form of the recursion relation

\[ A_n(x) = (-1)^n \frac{1}{(2n)!} \phi^{(2n)}(x). \]

Substitution of this into (A8) yields for the potential

\[ \phi(x, z) = \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!} \phi^{(2n)}(x) z^{2n}. \]

An examination of (A7) shows that if particles are considered to enter the lens on trajectories nearly parallel to the x axis then it is true that

\[ (z')^2 \ll 1. \]
By definition the potential energy of a charged particle at any point in space is related to the electrostatic potential by the charge on the particle so that the forces in (A7) can be expressed in terms of the expansion just given. There results

\[ \frac{\partial V}{\partial x} = -e \frac{\partial \phi(x, z)}{\partial x} = -e \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!} \phi^{(2n+1)}(x) z^{2n} \]

\[ \frac{\partial V}{\partial z} = -e \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!} \phi^{(2n)}(x) z^{2n} - 1 . \]

The first terms of which are

\[ \frac{\partial V}{\partial x} = -e \left[ \phi'(x) - \frac{1}{2} \phi''(x) z^2 + \ldots \right] \]

\[ \frac{\partial V}{\partial z} = -e \left[ 0 - \frac{1}{2} \phi''(x) 2z + \ldots \right] . \]

Keeping terms up to the second derivative and applying the assumption that $\frac{dz}{dx}$ is small allows (A7) to be written for $H = T + V = 0$ as

\[ z'' + z' \frac{\phi'(x)}{2 \phi} + z \frac{\phi'(x)}{2 \phi} = 0 . \]
This is the paraxial ray equation used to find the so-called principal rays of a lens. These in turn allow calculation of the macroscopic parameters of the lens such as focal length and magnification.
Appendix B
Corrections to the Einzel Lens Equations

Direct integration of the paraxial ray equation (A9) is an analytical impossibility for an einzel lens of this type due to the complexity of the potential distribution. However, since a nearly parallel beam is desired \((f \gg L)\) then the entire three aperture system may be treated as a thin lens to good approximation. A more stringent requirement on the thin lens treatment is that the ratio of the radial field strength within the lens to the field strengths on either side of the central aperture must be very much less than one. Then nearly all of the change in the \(y\) component of velocity will occur near the central aperture and the assumptions made to arrive at the paraxial equation will be valid. The field strength at the aperture in the \(y\) direction is nearly \(V_i / D\) while the field strengths on either side are \(V_i - V_o / L\) for the einzel lens. Therefore, the above requirement can then be written

\[
\left| 1 - \frac{V_o}{V_i} \right| \ll \frac{L}{D}.
\]
In order to solve the lens problem, make a change of variables in (A9) by the substitution

\[ Y = y v^{1/4}. \]  

Then

\[ y' = \frac{-1}{4} y v^{-5/4} v' + y' v^{-1/4} \]  

and

\[ y'' = \frac{5}{16} y v^{-9/4} (v')^2 - \frac{1}{2} y' v^{-5/4} v' \]

\[ + \frac{1}{4} y v^{-5/4} v'' + y'' v^{-1/4}. \]

Making the appropriate replacements in (A9) and rearranging the results leads to

\[ Y'' + Y \left[ \frac{7}{16} \left( \frac{v'}{V} \right)^2 + \frac{1}{4} \frac{d}{dx} \left( \frac{v'}{V} \right) \right] = 0. \]

Integration between regions immediately on either side of the central aperture with the assumption that \( Y \) constant gives
\[ Y' \bigg|_0^L = - Y \left[ \frac{7}{16} \int_0^L \frac{(V')^2}{V'} \, dx + \frac{1}{4} \int_0^L \frac{d(V)}{V} \right]. \]  

(B2)

From Figure 2 there are the relations

\[ V' = \frac{V_1 - V_0}{L} \]

and

\[ V = V_0 + \frac{V_1 - V_0}{L} x. \]

Substitution into (B2) followed by evaluation of the integral and simplification of the result yields

\[ Y' \bigg|_0^L = - Y \left[ \frac{11}{16} \left( \frac{V_1 - V_0}{LV_1V_0} \right)^2 \right] \]

Using the value of \( Y' \) given in (B3a) gives for the L.H.S. of the above

\[ Y'(L) - Y'(0) = - \frac{1}{4} Y(L) \left( \frac{V'(L)}{V(L)} \right) + y'(L) \left( \frac{V(L)}{V(L)} \right)^{-1/4} \]

\[ + \frac{1}{4} Y(0) \left( \frac{V'(0)}{V(0)} \right) + y'(0) \left( \frac{V(0)}{V(0)} \right)^{-1/4} \]  

(B3)
Assuming that the incident trajectories are parallel to the $x$ axis implies that $y'(0) = 0$. In addition $V'/V$ does not change appreciably over $L$ so that the first and third terms in the above expression cancel out. The reason for this assertion is that

$$\frac{V'}{V} = \frac{1}{V} \frac{dV}{dx} = \frac{d}{dx} (\ln V)$$

and since $\frac{dV}{dx}$ is assumed to be constant over the region of integration, the derivative of $\ln V$ might be said to be even more constant over the region. Replacing what remains of $(B3)$

$$y'(L)\left(\frac{V(L)}{V_1}\right)^{-1/4} = \frac{11}{16} \frac{y(L)\left(\frac{V(L)}{V_1}\right)^{-1/4} (V_1 - V_0)^2}{V_1 V_0}.$$ 

But since $y'(L) = y(L)/f$ the above relation becomes

$$\frac{1}{f} = \frac{11}{16L} \frac{(1 - \frac{V_1}{V_0})^2}{\frac{V_1}{V_0}}.$$ 

Equation $(B4)$ may be compared with a result derived by Pierce$^{19}$ for thin lenses of circular cross section in which the numerical coefficient is $3/8$ rather than $11/16$. This is to be expected since the single rectangular
aperture forms a thin lens which is exactly twice as strong as its circular counterpart\textsuperscript{20}. The focal length given in (B4) is measured from the principal planes of the lens which in the weak lens case coincides with the central aperture.

The above derivation can be made more exact if instead of the potential applied to the electrode, the potential along the axis is used. Grivet\textsuperscript{21} discusses the results first derived by Regenstreif\textsuperscript{22} for an einzel lens of circular symmetry using the axial potential rather than the lens potential. The focal length for such a lens is

\[
\frac{1}{f} = \frac{3}{8L} \left( 1 - \frac{\Phi}{\Phi_0} \right)^2 \tag{B5}
\]

where

\[
\Phi = \frac{\Phi_1(z = 0)}{\Phi_0(z = 0)} = \frac{V_0 - V_1}{1 + \frac{L}{R_1} \arctan \left( \frac{L}{R_1} \right)} \tag{B6}
\]

\[
V_0 + (V_i - V_0) \left[ \frac{R_0}{2R_i} \right] \left[ 1 + \frac{L}{R_1} \arctan \left( \frac{L}{R_1} \right) \right]
\]

This rather complicated result was achieved by the laborious task of integrating numerically the paraxial ray equation in
cylindrical coordinates using the complete axial distribution function. Since the form of (B5) was derived in a very simple manner by Pierce and (B4) was obtained in an equally straightforward fashion there is ample evidence that the more exact result can be achieved (B4) without the necessity of performing the numerical integrations.

The procedure is to first secure an expression for the axial potential distribution in the vicinity of a rectangular slit. The distribution can be most easily derived from the theory of conformal mapping and in particular from the Schwarz–Christoffel transformation since a slit is essentially a polygonal boundary. It is comparatively easy to derive the transformation but relatively difficult to obtain the inverse transform in closed form. Zworykin gives the inverse transform without identifying either source or method. The transformation is

$$F(w) = A(B + w^2)^{1/2} + Cw + D.$$ 

The real part of the inverse transform (i.e. the potential distribution) is
\[ \varphi(x, z) = \frac{1}{\sqrt{8}} (E_1 - E_2) \left\{ \left[ \left( \frac{D}{2} \right)^2 + x^2 - z^2 \right]^2 + 4x^2z^2 \right\}^{1/2} \]

\[ \left( \frac{D}{2} \right)^2 + x^2 - z^2 \right\}^{1/2} - \frac{1}{2} (E_1 + E_2)x + V_0. \]

Along the \( x \) axis this reduces to

\[ \varphi(x, 0) = \frac{1}{2} (E_1 - E_2) \left\{ \left( \frac{D}{2} \right)^2 + x^2 \right\}^{1/2} - \frac{1}{2} (E_1 + E_2)x + V_0. \]

For the central electrode located at the origin the potential is

\[ \varphi_1(0, 0) = V_1 + \frac{V_0 - V_1}{L}. \]

At either of the two outer electrodes for negligible fields outside the lens the potential becomes

\[ \varphi(L, 0) = V_o + \frac{V_1 - V_0}{2L} \left\{ \left[ \left( \frac{D_0}{2} \right)^2 + L^2 \right]^{1/2} - L \right\}. \]

So for the case of the rectangular slit

\[ \Phi_r = \frac{\varphi_1}{\varphi_o} = \frac{V_1 + \frac{V_0 - V_1}{2L/D_1}}{V_o + \frac{V_1 - V_0}{2L/D_1} \left\{ \left[ \left( \frac{D_0}{2D_1} \right)^2 + \left( \frac{L}{D_1} \right)^2 \right]^{1/2} - \frac{L}{D_1} \right\}. \]
The extra factor of $D_1$ has been introduced into both numerator and denominator to bring out the essential similarities between the distribution used by Regenstreif (B6) and the one just derived. The terms relating aperture spacing to aperture opening appear in the same relationship to the potentials in both expressions. An expansion of the second term in the denominator of (B7) indicates that the factor in brackets has the same simple form as the factor $R_o/2R_i$ in (B6). These arguments have been intended to show that the potential ratio of equation (B7) can be validly used in (B4) in place of the simple aperture potential ratio $V_i/V_o$ in order to gain a more accurate expression with the saving of much labor.
Appendix C

Effect of the Velocity Distribution in the Electron Beam

The problem created by the thermal distribution in velocities of electrons leaving the cathode and passing through some arbitrary type of electrostatic lens may best be attacked with the aid of Figure 15 which is due to Pierce\textsuperscript{24}. The grid appearing in this drawing is used to accelerate electrons from cathode to lens and is assumed to accelerate them only in the $x$ direction, i.e. it does not change their velocity component in the $z$ direction. Electrons emitted normal to the cathode surface and a distance $\pm w_1/2$ from the axis intersect the axis at $L$. However, through the same points pass electrons with velocity components in the $\pm z$ directions and are bent through nearly the same angle as the normal trajectories. These trajectories meet at a distance $L \pm l$ on the axis so that the beam has a finite width $w_2$ at $L$. (This corresponds to chromatic aberration in photon optics.) If this did not occur then, among other things, it would be possible to have an arbitrarily large current density at the crossover ($L$).

The maximum amount of current density which can be attained at crossover (or at the image) was first calculated in a rigorous manner by Pierce\textsuperscript{4} although Langmuir
had obtained the same result earlier by an intuitive approach using photon optics\textsuperscript{25}. Pierce's result for the case of rectangular apertures is

\[
j_{\text{max}} = j_0 \frac{2}{\sqrt{\pi}} \Phi_P^{1/4} \cdot \exp(\Phi_P^{1/4}) \cdot (1 - \text{erf} \Phi_P^{1/4}) \sin \theta \quad \text{(C1)}
\]

where \( \Phi_P \) is the reduced particle potential energy at L, \( j_0 \) is the current density at the cathode, and \( \theta \) is the half angle defined by the aperture as seen from L. Since

\[
\Phi_P \equiv \frac{eV}{kT} = \frac{1.16 \times 10^4 \, V}{T}
\]

then for \( T \sim 10^3 \text{°K} \) and for a potential difference between cathode and image on the order of 100 volts, \( \Phi_P \sim 10^3 \) is a typical value for this application. As a result of the large value for the reduced potential the second term in (C1) is zero so that

\[
j_{\text{max}} = j_0 \frac{2}{\sqrt{\pi}} \left( \frac{eV}{kT} \right)^{1/2} \sin \theta \quad \text{(C2)}
\]
The maximum available current density is thus limited both by thermal dispersion and, as might be expected, the distance from image to lens. This limiting value cannot always be attained, however, because the location and size of the image defines a certain allowed angle of arrival of electrons from the cathode. This is due again to the distribution in velocities in the \( z \) direction and is shown in Figure 15 for an image formed at \( L' \). The limiting angle of arrival \( \theta \) is such that particles having a larger angle of arrival do not intersect the image. The angular limit at the image infers a limit on the velocity distribution at the cathode and can be used to derive the actual current at an image for the case of perfect focusing. Pierce\(^4\) gives this as

\[
    j = \frac{j_0}{M} \left\{ \text{erf} \left( \frac{2 \phi_p}{1 - \beta^2} \right)^{1/2} + \beta \exp(\phi_p) \left[ 1 - \text{erf} \left( \frac{\phi_p}{1 - \beta^2} \right)^{1/2} \right] \right\}.
\]

Here \( M \) is the magnification of the lens and

\[
    \beta = M \sin \theta.
\]

For the case where \( V > 10 \) volts the second term is zero and the above simplifies to
Since a weak einzel lens has principal planes coincident with the center plane of the lens as well as equal focal lengths the magnification as defined by

\[ j = \frac{j_0}{M} \text{erf} \left[ \frac{\beta^2 \Phi_p}{1 - \beta^2} \right]^{1/2} \]  \hspace{1cm} (C3)

is unity\(^26\). As a result

\[ j = j_0 \text{erf} \left[ \frac{\sin^2 \theta \Phi_p}{1 - \sin^2 \theta} \right]^{1/2} \]

For the case where the focal length of the lens is very much greater than the aperture and \( \sin^2 \theta \ll 1 \). Equation (C3) results in

\[ j = j_0 \text{erf}(\sin^2 \theta \Phi_p)^{1/2} \]

If it is true that \( \sin^2 \theta \Phi_p \ll 1 \), then the error function can be expanded\(^3\) to give
\[ j = j_o \left\{ \frac{2}{\sqrt{\pi}} \left[ \sin \theta \frac{\Phi_p^{1/2}}{p^{1/2}} - \frac{1}{3} \left( \sin^2 \theta \frac{\Phi_p}{p} \right)^{3/2} + \ldots \right] \right\}. \]

Keeping only the first term in the series leaves

\[ j = j_o \frac{2}{\sqrt{\pi}} \frac{\Phi_p^{1/2}}{p^{1/2}} \sin \theta. \]

which is the same result as (C2), i.e. \( j \rightarrow j_{\text{max}} \) as the potential difference between cathode and image decreases. This could be anticipated from Figure 15 since the potential difference decreases as \( \Lambda' \rightarrow \Lambda \) which at the same time places the image at the crossover so that \( j \rightarrow j_{\text{max}} \).
Numerical solutions to the second order differential equations encountered in this paper were carried out on an SDS 910 computer. The method used was the Runge-Kutta approximation process.

Briefly, the technique relies on a Taylor series expansion about successive points each of which estimates the point which follows it. The process is initiated through a knowledge of the boundary conditions and is continued in a bootstrap manner until enough of the solution is obtained.

Integration of a second order differential equation of the form

\[
\frac{d^2y}{dx^2} = F(x, y, \frac{dy}{dx})
\]

subject to the initial conditions

\[
y = y_0, \quad y' = y'_0 \quad \text{at } x = x_0
\]

can be most easily accomplished by artificially breaking it into two first order equations. These would have the form
The solution to third order accuracy in the spacing interval can then be written in terms of the two series

\[ \frac{dy}{dx} = F(x, y, y') \]

and

\[ y = \frac{dy}{dx}. \]

The terms in the series have the following values

\[ k_1 = hy', \]
\[ k_2 = h(y' + \frac{1}{2}k'_1), \]
\[ k_3 = h(y' + 2k'_2 - k'_1) \]

and

\[ k'_1 = hF(x_k, y_k, y'_k) \]
\[ k'_2 = hF(x_k + \frac{1}{2}h, y_k + \frac{1}{2}k_1, y'_k + \frac{1}{2}k'_1) \]
\[ k'_3 = hF(x_k + h, y_k + 2k_2 - k_1, y'_k + 2k'_2 - k'_1) \]
The error involved in this approximate solution is given by

$$E \approx \frac{y^{(1)} - y^{(2)}}{2^N - 1}.$$ 

Where $y^{(1)}$ is the ordinate computed using a spacing of $h$, and $y^{(2)}$ is the result of using a spacing of $2h$. The number of terms in the series is called the "order" and is designated by $N$.

The above discussion is taken from Hildebrand and allows straightforward computer application via the Fortran II language. The methods of path integration discussed by Zworykin, Pierce and other texts are more cumbersome for the same degree of accuracy obtainable from the R-K method.
Appendix E
The Potential Distribution in the Ionization Region

A grid has been introduced in Figure 5 to allow the electrons to escape after passing through the ionization region. This minimizes the possibility of a second and indeterminate ion source due to accelerated secondary electrons. The potential distribution in this two dimensional "box" can be found from a solution of Laplace's equation

\[ \nabla^2 \phi = 0 \]

provided that the current carried in the electron beam and resultant ion beam is not so large as to alter the distribution.

The general form of the solution to Laplace's equation in two dimensional cartesian coordinates is given by

\[ \phi(x, z) = \sum_{k=1}^{\infty} \left( a_k \sin kz + b_k \cos kz \right) \cdot \left( c_k \sinh kz + d_k \cosh kz \right) . \]

(El)

With reference to Figure 5, the complete general solution for potentials of \( V_i \) on the sides can be found by a
superposition of solutions found by setting one side at a time at a fixed potential while the remaining three walls are kept at zero potential. An outline of the procedure for solution follows:

Set the potentials \( V_2 = V_3 = V_4 = 0 \) so that the boundary conditions are

1. \( \phi_1(o, z) = 0 \)
2. \( \phi_1(a, z) = 0 \)
3. \( \phi_1(x, b) = 0 \)
4. \( \phi_1(x, o) = V_1 \)

The boundary conditions may be used to arrive at the values of the constants in (El) in the usual manner. The results of each condition are

1. \( b_k = 0 \)

2. \( k = \frac{m \pi}{a} \) for \( m = 0, 1, 2, 3, \ldots \)

3. \( c_m = -d_m \coth \frac{m \pi b}{a} \).

These three values are replaced in (El) to give the result

\[
\phi_1(x, z) = \sum_{m=1}^{\infty} A_m \sin \frac{m \pi x}{a} \left[ \frac{\sinh m \pi (b - z)/a}{\sinh m \pi b/a} \right]. \tag{E2}
\]
The fourth condition on the boundary results in

$$\sum_{m=1}^{\infty} A_m \sin \frac{m x}{a} = V_1$$

The coefficients $A_m$ are evaluated by using the orthogonality property of the sine function over the interval zero to $a$. Thus

$$4. \Rightarrow A_m = \frac{2V_1}{m\pi} (1 - \cos m\pi) \quad \text{for } m = 0, 1, 2, \ldots$$

This is zero for $m$ even, so make the substitution

$$m = 2n - 1, \text{ for } n = 1, 2, 3, \ldots$$

This results in

$$4. \Rightarrow A_n = \frac{4V_1}{(2n - 1)\pi} \quad \text{for } n = 1, 2, 3, \ldots$$

Substitution of this into (E2) gives the final solution

$$\varphi_1(x, z) = \sum_{n=1}^{\infty} \frac{4V_1}{(2n - 1)\pi} \sin(2n - 1)\pi \frac{x}{a}$$

$$\left[ \frac{\sinh (2n - 1)\pi (b - z)/a}{\sinh (2n - 1)\pi b/a} \right]. \quad (E3)$$
A similar procedure may be carried out for each of
the sides in turn. The complete solution is given by the
superposition of the four distributions

\[ \Phi(x, z) = \sum_{i=1}^{4} \phi_i(x, z). \]  (E4)
Appendix F

Space Charge Effects in the Ion Beam

The following derivation is an elucidation of the origins of Figure 8 which is based upon work done in Pierce\textsuperscript{9}. The paraxial ray equation in cylindrical coordinates can be reworked to take into account space charge. Poisson's equation can be combined with the paraxial equation to give the result

\[
\frac{d^2 r}{dz^2} = \frac{I}{\frac{4\sqrt{2} \pi \varepsilon_0 \eta^{1/2}}{V^{3/2}} r} \tag{F1}
\]

where \( n = \frac{e}{m_{\text{ion}}} \)

\( V = \) accelerating potential of the beam.

Make the following change in variables, let

\[ R = \frac{r}{r_0} \]

where \( r_0 = \) the initial beam diameter upon exiting the lens.

Also let

\[ Z = \left[ \frac{1}{2\sqrt{2} \pi \varepsilon_0 \eta^{1/2}} \frac{I}{V^{3/2}} \right]^{1/2} \frac{z}{r_0} \tag{F2} \]
where \( z \) is the distance along the beam axis measured from the end of the lens. Then

\[
R' = \frac{dR}{dZ} = \frac{dR}{dz} \frac{dz}{dZ}
\]

\[
R' = \frac{dr}{dz} \left[ \frac{1}{2} \ln \frac{1}{\sqrt{\frac{1}{V^{3/2}}} \frac{I}{\sqrt{\pi \varepsilon_0}}} \right]^{-1/2} \quad \text{(F3)}
\]

Differentiating this a second time w.r.t. \( Z \) and substituting into (F1) gives the result

\[
R'' = \frac{1}{2R}.
\]

The first integral is

\[
(R')^2 = \ln R + (R_0')^2.
\]

Integration of this yields

\[
Z = \int R \frac{dR}{\left[ \ln R + (R_0')^2 \right]^{1/2}}.
\]

Make the substitution \( u = R' \) with the result

\[
Z = \exp\left[ - (R_0')^2 \right] \int_{R_0'}^{\infty} \frac{\left[ \ln R + (R_0')^2 \right]^{1/2}}{\exp(u^2)} \, du.
\]
If the integral is performed over the region from the end of the lens where \( Z = 0, R = 1 \) and \( R' = R'_o \) to the point of minimum beam diameter where \( Z = Z_w \) and \( R = R_m \) and \( R' = 0 \) then

\[
Z_w = \exp\left[ - (R'_o)^2 \right] \int_0^{R'_o} \exp(u^2) \, du \quad (F4)
\]

which is Dawson's function and is usually written \( D R'_o \).

Substitute for \( Z_w \) from (F2) to get the actual distance

\[
z_w = \left[ \frac{I}{2^{1/2} \pi \epsilon_o \sqrt{3/2}} \right]^{-1/2} r_o D[R'_o] \quad (F5)
\]

Then from (F3)

\[
R'_o = \frac{dr}{dz} \bigg|_{z=0} \left[ \frac{I}{2^{1/2} \pi \epsilon_o \sqrt{3/2}} \right]^{-1/2}
\]

The initial slope of the particle trajectory can be determined from the focal length of the lens. From Equation (12) this is given by
Corrections to (12) due to initial ion kinetic energy are made in Appendix G. The essential effects resulting from space charge can be shown with (12). The additional ion energy will serve to lengthen the distance from the lens to beam minimum and bring the minimum closer to the beam focal point.

The following statements can be made about the lens (see Section 6):

Object distance = 4 lens diameters
Magnification = 1.0

Then from Figure 7, for an image distance of 5.5 diameters

\[ \frac{V_2}{V_1} = 6.5 \]

Putting in the above values leads to the result

\[ f = 32.8 \, r_o \]

Since

\[ \left. \frac{dr}{dz} \right|_{z = 0} = \frac{r_o}{f} \]
then Equation (F5) can be written

\[ z_w = \frac{r_o}{1.14 \times 10^3 m_r^{1/4}} \frac{v^{3/4}}{I^{1/2}} \frac{1}{3.41 \times 10^4 m_r^{1/4}} \left[ \frac{1}{3.41 \times 10^4 m_r^{1/4}} \right] \]

where \( m_r = \frac{m_{\text{ion}}}{m_{\text{prot}}} \). The maximum current required of the gun is \( I = 1.6 \times 10^{-8} \) amp so the final version of the beam spread equation is

\[ z_w = 7.64 r_o \left( \frac{v^3}{m_r} \right)^{1/4} D \left[ 0.212 \left( \frac{v^3}{m_r} \right)^{1/4} \right] \]

This function is plotted in Figure 8 for various species of ion.
Appendix G

Corrections to Ion Lens Equations

Hutter's work\(^7\) gives the analytical solutions for fields resulting from lenses with two cylinders. Careful examination reveals that a similar solution for the initial condition of finite particle kinetic energy is not possible. This can be inferred from a new derivation of the ray equation in which the Hamiltonian is non-zero.

Equation (A7) has the same form in cylindrical or cartesian coordinates and may be written

\[
\frac{d^2 r}{dz^2} = \frac{1 + (r')^2}{2(H - V)} \left[ \frac{dr}{dz} \frac{dV}{dr} - \frac{d^2 V}{dr^2} \right]
\]

The \(z\) axis and ion lens axis coincide. The Hamiltonian and potential energy are as defined in Appendix A. For a particle with initial kinetic energy \(T_0\) (such as an ion leaving the extraction region) in a potential field \(\varphi(r, z)\) then (A7) becomes:

\[
\frac{d^2 r}{dz^2} = \frac{1 + (r')^2}{2\left[\frac{T_0}{e} - e\varphi(r, z)\right]} \left[ \frac{dr}{dz} \frac{d\varphi(r, z)}{dr} - \frac{d\varphi(r, z)}{dz} \right]
\]

\((G1)\)
In order to follow a procedure similar to that used to derive (Blc) define

\[ R = r(\varphi - \tau)^{1/4} \]

where \( \tau = \frac{T^0}{e} \)

and \( \varphi = \varphi(r, z) \)

When these are substituted in (G1) there results the equation

\[ \frac{d^2R}{dz^2} - \frac{3}{16} \left( \frac{\varphi'}{\varphi - \tau} \right)^2 = 0. \quad (G2) \]

In this same notation, the ray equation used by Hutter to derive his formulation of particle optics is

\[ \frac{d^2R}{dz^2} - \frac{3}{16} \left( \frac{\varphi''}{\varphi} \right)^2 = 0. \quad (G3) \]

Of the several potential distributions discussed by Hutter, only the distribution given in Equation (11) will yield an analytical solution to (G3). This fortuitous course of events does not extend to (G2) and direct numerical integration by computer offers the only alternative.

It must be noted, however, that the present experi-
mental apparatus requires that the ion collector be placed within a few centimeters of the ion lens tube due to vacuum system limitations. Variation in focal lengths (which are much longer than the ion tube length) will have little effect on measured beam currents. In the region where the ion kinetic energy is large compared to the lens potential it would be expected that the lens would exhibit longer focal lengths than predicted. This, however, is a moot point since no measurement is possible. The lens certainly will not be divergent for particles entrant parallel to the lens axis and this is a primary experimental object. Loss of lens effectiveness will decrease with higher acceleration potentials.

Computer results are not yet available to evaluate the result of the term $\tau$ in Equation (G2). Experimental results seem to indicate that the expected amount of ion current will be available (see page 55) despite the inability of the theory to determine the exact lens parameters.
Captions for Figures

Figure 1
Under ideal conditions this is the relationship of the ionizing electron beam to the ion beam produced.

Figure 2
This is the einzel electron lens cross section in the xz plane. The apertures are rectangular when viewed in the yz plane. Electric field vectors are indicated by arrows in the upper part of the drawing. The principal rays are idealized electron trajectories used to establish the optical parameters of the lens. The upper principal ray is traced by assuming that a particle enters the lens from the left parallel to, and slightly displaced from, the lens' axis. This path may be obtained analytically, but can be confirmed qualitatively by noting that as an electron enters the lens field from the left with energy $eV_0$, it is initially decelerated between $S_1$ and $S_2$ while experiencing a force directed away from the axis. As the electron passes the central electrode it has its smallest velocity and the inward directed forces cause a large deflection of the trajectory toward the lens axis. As the electron approaches aperture $S_3$ and regains its lost kinetic energy it experiences another outward directed
force, but since the electron is travelling faster here, the force has little time to take effect. The combined result of these fields is a net convergence of the electron beam, the amount of which is determined by the potential $V_i$ on the center aperture. An electron entering from the right experiences the same phenomena because of the symmetry of the lens about its midplane.

Figure 3

A schematic representation of electron beam dispersion due to space charge is shown. Focusing by the einzel lens causes the beam to be initially convergent, space charge causes beam divergence for $x > x_{\text{min}}$. Over-focusing will cause the beam to form a crossover which will result in the additional beam spread represented by the shaded area.

Figure 4

This is a hypothetical ionization-extraction region formed by a parallel plate capacitor. The ideal extraction area (shaded) is drawn with the assumption that ions are created with zero energy in the extraction field.

Figure 5

Final design of the ionization-extraction region with adjoining einzel lens. Equipotentials are drawn in to show
the nearly field free "corridor" down which the electron beam is intended to flow. This particular potential distribution results from a wall ratio of 2:1 and the applied voltages which are shown.

Figure 6

A so-called Gaussian "pillbox" is shown. It is used to derive an expression for the space charge fields due to the electron beam.

Figure 7

Schematic view of the ion lens showing the principal rays which are derived in the same manner as for the einzel electron lens. This lens is likewise convergent regardless of the direction in which it is traversed despite the assymmetry in the lens field.

Figure 8

This graph shows the limitations on focal lengths of the ion lens due to space charge dispersion of the ion beam. Giving the beam a finite focal length by way of the ion lens adds a small velocity component to the ions which is directed toward the axis. Space charge, however, produces an outward directed force which is active over the entire beam length and will eventually dominate the velocity component. Because of this the effective focal
length of the lens is shortened, though not so much as shown here for reasons discussed in Appendix G.

Figure 9

These are trajectories in "potential space" of electrons travelling through the ionization-extraction region. Only trajectories which define either side of the beam boundary are shown. Distance across the ionization region is normalized to 1. Because most ions are formed within the bounding electron trajectories, this graph shows the amount of dispersion in ion energy which can be expected due to the finite electron beam width (heavy lines) and due to the space charge carried by the beam (lighter outer lines in the bottom drawing in Figure 9). The central curve in each case represents the trajectories of electrons located in the center of the beam.

Figure 10

A three dimensional view of ion beam energy dispersion due to spatial dispersion of the electron beam in the ionization-extraction region. Since the SIDE detector cannot differentiate spatially with respect to particle energy, the distribution in Figure 10 will appear to be
squashed into the $\frac{dN}{dW_{\text{ion}}}$ plane so that the step-function distribution at $x = r_o$ will be completely distorted. This is shown in the bottom of the figure.

Figure 11

A computer was used to trace out electron trajectories in the ionization region for varying beam currents and energies. The ordinant is the effective ion energy dispersion and is similar to the result which could be obtained by subtraction of the lower from the upper curves in Figure 8.

Figure 12

A potential depression is caused within the ion beam by space charge. This is a graph of the resultant energy dispersion as seen by the SIDE detector.

Figure 13

Here the abscissa is the final accelerating potential of the ion lens while the ordinant is the effective ion energy at the beam center. The abscissa intercept is the minimal ion beam extraction energy. The ordinant intercept falls below the non-space charge value of 100 volts (or whatever the extraction potential happens to be).
Figure 14
The relationship of ion and electron lenses with idealized beam paths are shown in this diagram.

Figure 15
This illustrates the image formation in the electron lens when the thermal distribution in particle velocities is taken into account.

Figure 16
Experimental result showing the effect of the einzel lens in confining the electron beam to the electron collection region. The curves connecting the data points are a "visual aid" and are not to be taken as meaningful.

Figure 17
Experimental data showing the efficacy of a 2:1 wall-length ratio for the ionization region as opposed to a smaller ratio such as the 1.5:1 ratio shown in the lower curve.

Figure 18
This version of the Faraday cup is used to measure beam currents of either polarity down to \(10^{-13}\) amp at which point leakage currents from the collector plate to other points becomes appreciable. The grounded grids
isolate the retarding and suppression grids from each other, the incoming beam, and particularly from the high impedance point represented by the electrometer attached to the current collector. The retarding potential grid provides a way of measuring the energy spectrum of the incident beam. Grid transmission experiments are planned in which one of the grids is removed to determine the amount of current intercepted by a single grid.

Figure 19

A drift tube is shown in position between the grounded plate terminating the ionization-extraction region and the first ion lens tube. The series of apertures provides a very uniform beam directed along the lens axis.
FIGURE 2

EINZEL ELECTRON LENS
FIGURE 3

SPREADING OF BEAM DUE TO Crossover
FIGURE 4

REGION OF ION FORMATION
FIGURE 7

ION LENS

PRINCIPAL PLANE

MID-PLANE

P1

P2

f1

f2

D/5

LENS AXIS

FOCAL POINT

E

FOCAL POINT
Figure 8: Distance to beam minimum in beam radii vs. ion beam energy (eV). The graph shows four curves labeled $m_1 = 2$, $m_1 = 20.2$, $m_1 = 39.9$, and $m_1 = 83.8$. The graph is labeled 'Nominal Focal Length'.
Figure 9

200 eV Electron Beam

\[ \phi(x, z) \text{ (Volts)} \]

Reduced Distance Across Ionization Region \((x/a)\)
FIGURE II

ION BEAM ENERGY DISPERSION ($\Delta W$) IN eV

- $10^{-2}$ Amp, 300 eV
- $10^{-3}$ Amp, 100 eV
- $10^{-3}$ Amp, 200 eV
- $10^{-3}$ Amp, 300 eV
  $f/r_0 = 10$
  (ALL OTHERS, 40)
- $10^{-4}$ Amp, 100 eV
- $10^{-5}$ Amp, 100 eV

REDUCED DISTANCE ACROSS IONIZATION REGION ($X/A$)
FIGURE 13

\[ V = \frac{e}{2} \left( \frac{2}{p} \right) \]

\[ V = 0.250 \times 880.0 \]

\[ W_{\text{ion}} (\text{eV}) \]

\[ V (\text{VOLTS}) \]
FIGURE 14

ION "GUN"

TO FILAMENT SUPPLY
CATHODE

IONIZATION-EXTRACTION REGION
ELECTRON BEAM

ION BEAM

P₁
P₂
P₃

ELECTRON COLLECTOR

V₀ + Vₑc

V₁

S₁
S₂
S₃

CATHODE GROUND SIDE GROUND

V₂
CURRENT TO ELECTRON COLLECTOR (mA)

WALL RATIO = 2/1

WALL RATIO = 1.5/1

POTENTIAL ON REAR PLATE OF IONIZATION REGION (VOLTS)

0 100 200 300 400

0 10 1.5

FIGURE 17
FIGURE 18
FARADAY CUP

ENTRANT BEAM
OF
CHARGED PARTICLES

RETARDING GRID
GROUND Grid
ELECTRON
 Suppressor Grid

COLLECTOR PLATE

CONDUCTOR
KEL-F INSULATOR
### Table 1

Cross sections for ionization by electron impact.\(^{39}\)

<table>
<thead>
<tr>
<th>Species</th>
<th>Electron energy at maximum cross section in eV</th>
<th>Maximum cross section in (\text{cm}^2/\text{atom} \times 10^{-17})</th>
<th>Cross section at 100 eV in (\text{cm}^2/\text{atom} \times 10^{-17})</th>
<th>Cross section at 200 eV in (\text{cm}^2/\text{atom} \times 10^{-17})</th>
<th>Cross section at 300 eV in (\text{cm}^2/\text{atom} \times 10^{-17})</th>
</tr>
</thead>
<tbody>
<tr>
<td>He</td>
<td>125</td>
<td>3.5</td>
<td>3.5</td>
<td>3.3</td>
<td>2.8</td>
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<tr>
<td>Ne</td>
<td>175</td>
<td>8.5</td>
<td>7.5</td>
<td>8.5</td>
<td>7.5</td>
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<td>A</td>
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<td>36</td>
<td>36</td>
<td>30</td>
<td>25</td>
</tr>
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<td>H(_2)</td>
<td>70</td>
<td>10</td>
<td>10</td>
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<tr>
<td>N(_2)</td>
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<tr>
<td>O(_2)</td>
<td>140</td>
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<td>29</td>
<td>27</td>
<td>21</td>
</tr>
<tr>
<td>CO</td>
<td>100</td>
<td>30</td>
<td>30</td>
<td>26</td>
<td>21</td>
</tr>
</tbody>
</table>
Table 2
Emissivity and work function of various metals.³

<table>
<thead>
<tr>
<th>Metal</th>
<th>A in Amp/cm²( K)²</th>
<th>E in eV</th>
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</thead>
<tbody>
<tr>
<td>Mo</td>
<td>55</td>
<td>4.27</td>
</tr>
<tr>
<td>Ni</td>
<td>30</td>
<td>4.84</td>
</tr>
<tr>
<td>Pt</td>
<td>32</td>
<td>5.29</td>
</tr>
<tr>
<td>Ta</td>
<td>37</td>
<td>4.12</td>
</tr>
<tr>
<td>W</td>
<td>70</td>
<td>4.50</td>
</tr>
<tr>
<td>BaO + W</td>
<td>2.5</td>
<td>1.67</td>
</tr>
</tbody>
</table>
Table 3
Experimental results for the einzel electron lens.

<table>
<thead>
<tr>
<th>Cathode to plate (S1) current in mA.</th>
<th>Electron collector current at maximum in mA.</th>
<th>$\frac{V_i}{V_o}$ Experimental values</th>
<th>$\frac{V_i}{V_o}$ Computed values</th>
<th>Electron beam energy in eV.</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.63</td>
<td>0.40</td>
<td>0.42</td>
<td>200</td>
</tr>
<tr>
<td>25</td>
<td>1.01</td>
<td>0.26</td>
<td>0.25</td>
<td>200</td>
</tr>
<tr>
<td>37</td>
<td>1.11</td>
<td>0.33</td>
<td>0.30</td>
<td>250</td>
</tr>
<tr>
<td>42</td>
<td>1.34</td>
<td>0.30</td>
<td>0.33</td>
<td>300</td>
</tr>
</tbody>
</table>
ACKNOWLEDGMENTS

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