RICE UNIVERSITY

ELASTIC SCATTERING OF ALPHA PARTICLES
BY C\textsuperscript{13}

by

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ABSTRACT

The cross-section for the elastic scattering of alpha particles by C\textsuperscript{13} was measured for $E_\alpha = 2.0$ MeV to $E_\alpha = 3.5$ MeV at center of mass angles 169.6°, 142.6°, 107.9°, and 54.7°. Analysis of these excitation curves allowed assignment of parity to 8 states of O\textsuperscript{17}. Relative parities from the C\textsuperscript{13}(α ,n)O\textsuperscript{17} angular distribution analysis allowed assignment of parity to two additional states. A discrepancy was noted in the spin assignment of one state ($E_x = 8.195$ MeV). Assignments of parity and of two possible spins were made to this state and to one state not observed in the C\textsuperscript{13}(α ,n)O\textsuperscript{17} cross-sections. The assignments made, together with the excitation energies in O\textsuperscript{17} of the states are 7.929, $\frac{1}{2}^-$; 8.071, $\frac{3}{2}^+$; 8.195, $\frac{3}{2}^-$; 8.317, (3/2, 5/2)-; 8.385, 5/2-; 8.450, 7/2+; 8.489, (3/2, 5/2)-; 8.685, 3/2-; 8.867, 3/2+; 8.875, (9/2, 7/2)-; 8.924, (7/2+).
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I. Introduction

There is considerable interest in the reactions produced by alpha particles incident on C\(^{13}\). Only two particle reactions occur for alpha-particle energies less than 5 MeV:

\[
\begin{align*}
C^{13} + \alpha & \rightarrow O^{17} \rightarrow C^{13} + \alpha \\
C^{13} + \alpha & \rightarrow O^{17} \rightarrow O^{16} + n + 2.2 \text{ MeV}
\end{align*}
\]

Above an alpha particle energy of 4.6 MeV a neutron group to the first excited state of O\(^{16}\) (6.06 MeV) occurs:

\[
C^{13} + \alpha \rightarrow O^{17} \rightarrow O^{16} + n, \quad -3.8 \text{ MeV}
\]

The neutron reaction has been the object of a number of experiments. Experiments were conducted in the energy region covered by this report by Frier, Fulk, Lampi, and Williams in 1950\(^1\), by R. E. Trumble in 1954\(^2\), and by Walton, Becker, Clement, and Zucker in 1955\(^3\). In 1956 work was done by M. G. Rushridge\(^4\), by Bonner, Kraus, Marion, and Schiffer\(^5\), and by Becker and Barschall\(^6\). Becker and Barschall were interested in the reaction as a source of neutrons, since the neutrons from this reaction are monoenergetic over a wide energy range. These experiments were mainly concerned with measuring the yield as a function of the energy of the incident alpha particles. Resonances were observed which were attributed to the excited states of O\(^{17}\). Most of the excited states of O\(^{17}\) in this energy region had been discovered by 1956.

Tentative spin assignments of \(3/2\) had been made to two O\(^{17}\) states at 8.18 and 8.71 MeV\(^4,6\). The assignments of spins to most of the O\(^{17}\) states in this energy region were made by analysis of the angular distributions of this reaction by Schiffer, Kraus, and Risser\(^7\) and by Walton, Clement, and Borelli\(^8\) in 1957.
These levels of O\(^{17}\) have been observed in other reactions leading to O\(^{17}\). The best known is the O\(^{16}(n,n)O^{16}\) reaction. The work before 1958 is found with references in BNL-325\(^9\). An analysis in 1961 of the total neutrons cross sections of O\(^{16}\) by Fossan, Walter, Wilson, and Barschall\(^10\) gives spin assignments in agreement with the analysis of the C\(^{13}(\alpha ,n)O^{16}\) angular distributions. Another reaction leading to the compound nucleus O\(^{17}\), the O\(^{16}(n,\alpha )C^{13}\) reaction, has been studied by V. Gierke in 1953\(^11\), by Seitz and Huber in 1955\(^12\), and by Walton, Clement, and Borelli in 1957\(^8\). Walton, et al., invoked the principle of detailed balancing to show good agreement between the excitation curves of this reaction and those of C\(^{13}(\alpha ,n)O^{16}\), but no analysis was made to obtain spins or parities of the O\(^{17}\) states from this reaction.

The O\(^{17}\) nucleus may be regarded from the viewpoint of the shell model as a single neutron outside closed neutron and proton shells. Thus one might expect the states of normal parity (the same parity as the ground state) to correspond to excitation of this single neutron. The normal parity states between the ground state and 6 MeV excitation energy agree with the predictions of a simple shell model. Above this energy the agreement is not as good, but one might expect the states to exhibit single particle character to some degree. If this is so, one would expect the reduced widths to be reasonably large fractions of the Wigner limit, since this is a measure of the extent to which a state consists of the excitation of a single nucleon. A shell model analysis of the mirror nucleus F\(^{17}\) has been made by Salisbury and Richards\(^13\) with some success. This paper also gives a comparison of the known energy states of O\(^{17}\) and F\(^{17}\). There is a one to one correspondence up
to about 5 MeV in excitation energy. The correspondence begins to break
down between 5 and 7 MeV. Above 7 MeV the F$^{17}$ levels have not been
identified. Therefore no inference from the levels of F$^{17}$ may be drawn
at present about the levels of O$^{17}$ in the range of this experiment.

As a result of the work on C$^{13}(\alpha, n)$O$^{16}$ and the other reactions
mentioned which lead to O$^{17}$ as a compound nucleus, much is known about
the energy levels of O$^{17}$. The energies and widths of the states are
reasonably well determined. The partial widths are known to about +
50%. Except for a few ambiguities the spins are well known. The
characteristics of the neutron reaction which make the analysis feasible,
however, prevent the determination of the parity of the states. The
incoming channel spin is 1/2 only since the ground state of C$^{13}$ is 1/2$^{-}$
and that of the alpha particle 0$^{+}$. For energies less than 5 MeV
neutrons are emitted only to the 0$^{+}$ ground state of O$^{16}$, so the outgoing
channel spin also has the value 1/2. Thus as $J = l + s$, a state of
the compound nucleus characterized by an angular momentum $J$ may be formed
with only one value of $l$ and decay with only one value $l'$. The
angular distributions are therefore of the form$^{7,14}$

$$\sigma(\theta) \sim \sum_{\nu_2} \mathcal{Z}(l, l', J, \nu_2, J) \mathcal{Z}(\ell', \ell', J, \nu_2, J) \rho(\cos \theta)$$

The values of $l$ and $l'$ are restricted to $J \pm 1/2$ by the single channel
spin. Further, due to the different parities of the incoming and out-
goings channels, $l'$ must be odd if $l$ is even and vice versa. Thus

$$l' = J + 1/2 \text{ if } l = J - 1/2 \text{ and } l' = J - 1/2 \text{ if } l = J + 1/2.$$ 

The properties of the $\mathcal{Z}$ coefficients are such that under these circumstances
the expression for the angular distribution reduces to

$$\sigma(\theta) \sim \sum_{\nu_2} \mathcal{Z}(l, J, l', J, \nu_2, J)^2 \rho(\cos \theta)$$
This is the same for either parity. Thus the parity cannot be assigned from this analysis of the angular distributions. Relative parities between adjoining states can sometimes be assigned from interference terms between the two states. A number of these relative parities are known from this type of analysis\(^7,8\).

The purpose of this experiment was to investigate the other of the two possible particle reactions mentioned, the elastic scattering of the alpha particles by the C\(^{13}\). Analysis of the excitation curves for this reaction may be made by means of an expression for the elastic scattering of charged particles with channel spin 1/2 using the form of the S matrix given by Blatt and Biedenharn\(^{14}\). The term representing the interference between the resonance scattering and the Rutherford scattering gives resonant shapes which are mainly characteristic of \(l\) and therefore of the parity, allowing the parities of the states to be extracted from the analysis of the data. The partial width \(\Gamma/\Gamma_l\) is also determined by this analysis and may be used to check the \(\Gamma/\Gamma_l\) determined from the C\(^{13}\)(\(\alpha\),n)\(^{16}\) angular distribution analysis. This reaction has been previously investigated by R. L. Steele in 1960\(^{15}\). In the present experiment the cross section was measured as a function of energy from 2.0 to 3.5 MeV incident alpha particle energy at four angles. Fits to these excitation curves allowed parity assignments to be made to eleven states of \(0^{17}\) and tentative spin assignments to be made to two states of \(0^{17}\). One state not seen in the C\(^{13}\)(\(\alpha\),n)\(^{16}\) data was observed. A calibration of the analyzing magnet was carried out.
II. Experimental Techniques

Single charge He\(^+\) ions were accelerated in the Rice University 5.5 MeV Van de Graaff accelerator. The small cylindrical scattering chamber used has been previously described \(^{15,16,17,18}\). It consists of two shallow cylinders. The lower one is fixed and contains the beam defining slits, the target holder, and the Faraday cup for integrating the incident beam. The upper cylinder rotates and has provisions for two counters fixed at 90° apart. These two counters are mounted at an angle of 15° to the place of rotation of the cylinder. This allows the chamber to reach laboratory scattering angles between 15° and 165°. A liquid air cold trap had been previously added to the beam tube of the chamber to reduce the C\(^{12}\) buildup on the targets\(^{19}\). For this experiment a small diffusion pump was added to the chamber to further reduce the buildup.

The targets were thin self-supporting foils on the order of 20 \(\mu\text{gm/cm}^2 (\sim 5 \times 10^{-6} \text{ cm})\) in thickness, made by the method of Kashy, et al.\(^{19}\), from either methane or methyl iodide enriched in C\(^{13}\). A mass-spectrometer analysis showed the methane to be enriched to 56.7% C\(^{13}\) and the methyl iodide to be enriched to 41.6% C\(^{13}\), to an estimated accuracy of 0.5%. The methane was cracked on a 2.5\(\times\)10\(^{-4}\) inch nickel foils. The nickel was etched away over a spot about 1 cm in diameter after the nickel foil was mounted on an aluminum target holder.

The scattered particles were detected with an Ortec 50A3 silicon detector, made of 300 ohm-cm silicon with an area of 50 square millimeters, using a bias voltage of 22.5 volts. This is a charged-particle detector, giving a pulse proportional to the energy of the detected
particle. The detector was mounted 13 centimeters from the target, and the solid angle was defined by a slit immediately in front of the detector. Most of the data were taken with a solid angle of $4.556 \times 10^{-4}$ steradians. Some of the $169.6^\circ$ excitation curve data were taken with a solid angle of $1.032 \times 10^{-3}$ steradians. The $54.7^\circ$ angle data were taken with a solid angle of $5.878 \times 10^{-5}$ steradians.

The provision for two counters on the chamber allowed data to be taken simultaneously at two angles. For the calibration points, in order to use the 256-channel analyzer at all angles for greater accuracy, one counter was used. These calibration points were taken early in the experiment. The early excitation curve data were taken at two angles simultaneously, using the 256-channel analyzer for one angle and a single channel analyzer for the other angle. The single channel was biased to accept pulses only between an upper and a lower cutoff voltage. This window of accepted pulses was made seven volts wide. A twenty-channel analyzer was used to monitor the incoming pulses and allowed the amplification of these pulses to be adjusted so the scattered alpha particle pulses were kept in the window of accepted pulses.

For the later excitation curve data, additional multichannel analyzers became available and were used with both detectors. A 400-channel was used as two 200 channel analyzers and a 1,024 channel analyzer was used as two 256 channel analyzers. This allowed the scattered alpha particles to be separated from the background much more accurately.
III. Analysis

A. Excitation Curves

Excitation curves were measured for the thin C$^{13}$ enriched targets, consisting of the number of elastically scattered particles as a function of energy at fixed angles. The target thickness of each target was measured before and after the data were taken. This thickness measurement was made by counting 3 MeV protons elastically scattered from the target to 165° (laboratory angle). The thickness was calculated using this number of counts and the known cross sections for elastic scattering of 3 MeV protons by C$^{12}$ and C$^{13}$ at this angle.

Alpha particles elastically scattered from the enriched target were counted at 54.7°, 107.9°, 142.6°, and 169.6°. The 169.6° and 54.7° angles were taken with the 256 channel analyzer and the 142.6° and 107.9° angles were taken with the single channel analyzer for the data from 2 to 2.9 MeV. Data were taken from 2 to 3.5 MeV at the 169.6° and 107.9° angles, and from 2.5 to 3.5 MeV at the 142.6° and 54.7° angles.

The presence of C$^{12}$ and C$^{13}$ enriched targets presented the problem of separating the counts due to alpha particles scattered from C$^{12}$ from those due to alpha particles scattered by C$^{13}$. These particles were indistinguishable except at the backward angle, for the elastic process under consideration with the resolution obtained in this experiment. The difficulty was further complicated by the C$^{12}$ which was deposited on the face of the target by the beam while running on the target. This caused both the target thickness and the relative amounts of C$^{12}$ and C$^{13}$ to change during the run. Bombarding with alpha particles,
thin targets (20 \( \mu \text{gm/cm}^2 \)) had to be used to keep the energy lost in the target from being too much greater than the widths of the narrow resonances observed, which were as small as 4 keV. For these thin targets, the C\(^{12}\) buildup was an extremely serious problem. During a long excitation curve, it would nearly double the amount of C\(^{12}\) in the target.

A datum point at a particular energy of the incident alpha particles in the laboratory consisted of the number of alpha particles, per 30 microcoulombs of incident beam, which were elastically scattered to a specified angle. The number of incident alpha particles was given by the total charge divided by the charge per particle. The following notation will be used in describing the analysis.

\( C_{p1} \) = number of 3 MeV protons elastically scattered by the C\(^{13}\) enriched target to 165° (laboratory angle) for 30 \( \mu \text{coul.} \) incident beam in the initial target thickness measured.

\( C_{pf} \) = number of 3 MeV protons elastically scattered by the C\(^{13}\) enriched target to 165° (laboratory angle) for 30 \( \mu \text{coul.} \) incident beam in the final target thickness measurement, made after taking an excitation curve with the target.

\( C_T \) = total number of incident alpha particles of a particular energy elastically scattered to a particular angle by both the C\(^{12}\) and C\(^{13}\) in the C\(^{13}\) enriched target for 30 \( \mu \text{coul.} \) of incident beam during the \( n^{th} \) point run on the target.

\( P \) = total number of points run on the C\(^{13}\) enriched target.

\( \sigma_{12}(\sigma_{13}) \) = differential laboratory cross section for the elastic scattering of 3 MeV protons to 165° by C\(^{12}\) (C\(^{13}\)).

\( A_{12}(A_{13}) \) = atomic weight of C\(^{12}\) (C\(^{13}\))
The fractional enrichment of \( ^{13}\text{Cl} \) atoms in the target is given by:

\[
\frac{f}{1 - f} = \frac{\text{atoms}^{13}\text{Cl}}{\text{atoms}^{12}\text{C} + \text{atoms}^{13}\text{Cl}}
\]

The initial thickness of the enriched target (in gm/cm\(^2\)) is given by:

\[
M_i = \frac{A}{N_0} \frac{\pi c \rho}{\sigma \times I' \times d\Omega}
\]

where:

\[
N_0 = 6.02 \times 10^{23}
\]

\[
A = f \times A_{^{13}\text{Cl}} + (1 - f) A_{^{12}\text{C}}
\]

\[
\sigma = f \pi \sigma_{^{13}\text{Cl}} + (1 - f) \sigma_{^{12}\text{C}}
\]

\[
d\Omega = \text{Solid angle subtended by the detector in the laboratory}
\]

\[
I' = 30 \times 10^{-6}/1.6 \times 10^{-19} = \text{No. protons in 30 \mu coul. integrated beam.}
\]

The part of this which is \( ^{12}\text{C} \) is given by:

\[
M_{i12} = M_i \times \frac{(1 - f) A_{^{12}\text{C}}}{(1 - f) A_{^{12}\text{C}} + A_{^{13}\text{Cl}}}
\]

The buildup on the target was assumed to consist only of \( ^{12}\text{C} \). The mass built up (in gm/cm\(^2\)) is given by:

\[
\Delta M_{12} = M_{i12} \frac{C_p}{C_{^{12}\text{C}} \rho \times I' \times d\Omega}
\]

If the buildup is assumed to be a linear function of the number of points run on the target, the change in \( ^{12}\text{C} \) mass per point due to buildup may be written as:

\[
b = \frac{\Delta M_{12}}{P}
\]

Then the mass per centimeter squared of \( ^{12}\text{C} \) as a function of the point number may be written:

\[
M_{12} = M_{i12} + n b
\]

where \( n \) is the point number.

The number of alpha particles expected to be elastically scattered by the
C\textsuperscript{12} in the C\textsuperscript{13} enriched target was calculated from this thickness and the C\textsuperscript{12} cross section. The C\textsuperscript{12} cross section for the elastic scattering of alpha particles was calculated from the known\textsuperscript{21} phase shifts from 2.5 to 3.5 MeV. At 2 MeV, previous work\textsuperscript{22} and measurements taken while running the calibration points showed the value of the cross section was equal to that calculated from Rutherford scattering. Between 2 and 2.5 MeV, the Rutherford scattering cross section, linearly normalized to join smoothly to the known cross sections at 2.5 MeV, was used.

The number of counts due to C\textsuperscript{13} was calculated by subtracting the counts due to C\textsuperscript{12} calculated above from the total number of counts, C\textsubscript{T}. If the counts due to C\textsuperscript{13} are denoted by C\textsubscript{13}, the cross sections were calculated by

\[ \sigma_{\text{Lab}} = \frac{A_{13}}{N_0} \times \frac{C_{13}}{M_{13} \times I'' \times d} \]

where

\[ M_{13} = M_1 - M_{12} \]

\[ I'' = \frac{30 \times 10^{-6}}{2 \times 1.6 \times 10^{-19}} \]

= No. of $\alpha$ particles in $30 \mu$ coul.

In some cases at the backward angles, the C\textsuperscript{13} yield was so low that the counts due to C\textsuperscript{13} were less than one percent of the total counts, or the counts due to C\textsuperscript{12}. The statistical scatter was extremely large for the C\textsuperscript{13} counts calculated by the previous method. The C\textsuperscript{12} - C\textsuperscript{13} peaks were distinguishable on the pulse-height spectra, though not well separated. The procedure used for these points was to estimate the C\textsuperscript{13} counts directly from these pulse-height spectra. This procedure gave the shape of the excitation curve, though the actual cross section level may be off as much as 20\%. Pulse height spectra showing the separation of
of the alpha particles scattered from $^{12}\text{C}$ and $^{13}\text{C}$ at the backward angles are shown in fig. 1.

B. Cross Section Calibration Points

More accurate determination of the cross section was made at points widely spaced in energy as a check of the cross sections calculated from the excitation curves. These cross-section calibration points were taken at the same angles as the excitation curves. The energies at which the points were taken were chosen in energy regions where neither the $^{12}\text{C}$ nor the $^{13}\text{C}$ cross sections were changing too rapidly with energy, thus reducing the error in the measurements. Thicker targets (on the order of 100 gm/cm$^2$) were used and fewer points were taken to reduce the problems associated with the buildup. The 256 channel analyzer was used at all angles to increase accuracy. All measurements were made first on a $^{13}\text{C}$ enriched target and repeated later on a $^{12}\text{C}$ target. The treatment of the data was very similar to that for the excitation curves. The same notation will be used, with the quantities measured for the $^{12}\text{C}$ target denoted by the superscript 12. Let the $n'$ th point on the $^{12}\text{C}$ target be at the same angle and energy as the $n$ th point on the $^{13}\text{C}$ enriched target. The mass of $^{12}\text{C}$ in the $^{12}\text{C}$ target is given by

$$M_{12} = M_{12}^{12} + n'b_{12}^{12}$$

where

$$M_{12}^{12} = \frac{A_{12}}{N_0} \frac{C_p^{12}}{\sigma_{12}^{12} x I' x d\Omega}$$

and

$$b_{12}^{12} = \frac{\Delta M_{12}}{P_{12}}$$

$$\Delta M_{12} = \frac{A_{12}}{N_0} \frac{(C_{pf}^{12} - C_{pi}^{12})}{\sigma_{12}^{12} x I' x d\Omega}$$
\( \phi (\text{C.M.}) = 142.6^\circ \)

\( \phi (\text{C.M.}) = 169.6^\circ \)

FIGURE 1a
The number of counts $C^{12}$ due to $C^{12}$ in the enriched target is then given by the ratio of the mass of $C^{12}$ in the enriched target to the mass in the $C^{12}$ target times the number of counts from the $C^{12}$ target.

$$C_{12} = \frac{M_{12} + n_{12} b}{M_{12} + n_{12} b} C_T^{12}$$

The number of counts $C^{13}$ due to $C^{13}$ in the enriched target is given by

$$C_{13} = C_T - C_{12}$$

The cross section is calculated from $C_{13}$ by the formula given in the discussion of the excitation curves.

In calculating cross sections the assumption was made that the singly charged He$^+$ ions of the beam were stripped of another electron in passing through the target and were thus doubly charged when they entered the Faraday cup in the chamber. This assumption was checked as follows. The elastic scattering of the alpha particles from a $C^{12}$ target was measured at low energy and forward angles (1.5 and 2.0 MeV; 25°, 30°, and 40° lab). Using the calculated Rutherford cross section and the measured number of counts, the thickness of the target was calculated. This thickness was compared to the thickness measured by the elastic scattering of 3 MeV protons. The two methods agreed to within 7%. This is slightly higher than expected from results reviewed by Allison and Warshaw$^{32}$, but agrees within the experimental errors. Allison and Warshaw give a table of the percentage of alpha particles which are doubly charged after passing through thin foils. This is about 97% at 2.0 MeV, 98% at 2.5 MeV and 99% at 3.5 MeV. Thus the charge state correction would be one to two percent over most of the energy range covered.
The cross sections measured for the \( ^{13}\text{C}(\alpha,\alpha)^{13}\text{C} \) reaction from 2 to 3.5 MeV are shown in fig. 2. The small arrows at the 169.6° angle indicate the position of resonances in the \( ^{13}\text{C}(\alpha,n)^{16}\text{O} \) cross section.

The analysis of both the calibration points and the excitation functions were carried out by programs written for the IBM 1620 and 790 computers.

C. Error Analysis

To assign limits of error to these cross section, it is necessary to evaluate the error in both the total counts and in the counts subtracted as the \( ^{12}\text{C} \) contribution. The total number of detected particles was recorded by a pulse height analyzer. Aside from the statistical fluctuation, the principal error in this number comes from separating the pulses due to particles elastically scattered from carbon from other pulses. In the case of the data taken on the multichannel analyzers, the error in this separation is estimated to be no more than 2%. In the case of the single channel adjusted with the 20 channel, it is estimated to be no more than 5%.

The calculation of the portion of these counts due to \( ^{12}\text{C} \) must be treated in two parts. The amount of \( ^{12}\text{C} \) originally in the target was known as accurately as the thickness of the target and the percentage of \( ^{12}\text{C} \) in the target. The target thickness was measured to an estimated accuracy of \( \pm 3\% \). The percentage of \( ^{12}\text{C} \) was checked on a mass spectrograph to an accuracy of \( \pm 0.5\% \). From this amount of \( ^{12}\text{C} \), the number of counts assumed to be due to this part of the \( ^{12}\text{C} \) was calculated from the \( ^{12}\text{C}(\alpha,\alpha)^{12}\text{C} \) cross sections which are reported as
accurate to $2.6\%^{21)}$. Due to the problem of matching the two energy scales in subtraction, this calculation cannot be assumed to be accurate to more than $\pm 5\%$. Thus taking the RMS average of the errors, the error in this number of calculated counts should not exceed $\pm 6\%$. The second part of the $^{12}C$, that due to the $^{12}C$ deposited on the target by the beam, was not so well known. The total buildup was determined by the difference of two numbers of proton counts. A typical example was 6,009 proton counts originally and 7,966 counts after buildup. With an error of $\pm 2\%$ in each number of counts, this gives an RMS error of $\pm 10\%$ of the difference. This difference was used, together with the number of counts calculated to be due to $^{12}C$ in the original thickness measurement to calculate a percentage buildup of $^{12}C$ in the target of 86%. As the number of $^{12}C$ counts is accurate to about 10%, this buildup should be accurate to about $\pm 14\%$. The assumption that this buildup is a linear function of the running time on the target may increase the error in the buildup by as much as $\pm 30\%$. Thus the RMS error in the counts calculated for this part of the $^{12}C$ could be as high as $\pm 45\%$.

For purposes of assigning a maximum probable error to the cross sections, one of the worst cases should be calculated. The worst error arises when the $^{12}C$ counts become a very large fraction of the total counts. At $169.6^0$, during the run for which the above buildup was calculated, the worst case came at 2.8 MeV where the total number of counts was 1,028. There were 277 counts from the original $^{12}C$ and 200 counts from the $^{12}C$ due to buildup, according to calculation. Thus, using the errors estimated, the number of counts due to $^{13}C$ was:

$$(\text{Total counts}) \quad 1028 \pm 20$$
(Original C^{12} counts) \(-277 \pm 17\)  
(Buildup C^{12} counts) \(-200 \pm 90\)  
\(551 \pm 94\)

where the RMS error has been taken. As the C^{13} counts are accurate to about 17%, the cross sections should be accurate to 19%. Similar calculations at the other angles give \(\pm 15\%\) for the 142.6° angle, \(\pm 16\%\) for the 107.9° angle, and \(\pm 15\%\) for the 54.7° angle.

For the calibration points, the buildup was held to essentially zero. The C^{12} counts were measured to an accuracy of about 2% and the relative thickness of the two targets to an accuracy of about 1%. With the 0.5% accuracy of the percentage of C^{12} in the enriched target, this should give the counts due to C^{12} to about \(\pm 2.2\%\). The total counts were measured to \(\pm 2\%\). As the C^{12} counts did not become more than 70% of the total counts, the calibration points should be accurate to about 10%. With these calibration points, it should be safe to place an accuracy on the cross sections of \(\pm 15\%\).

D. \(^{13}\text{C}(\alpha,n)\text{O}^{16}\)

The \(^{13}\text{C}(\alpha,n)\text{O}^{16}\) yield was taken at 0° simultaneously with the elastic scattering, using a long counter. A cross section calibration was made at 90° (laboratory), using a calibrated Pu Be source as a standard. This agreed quite well with cross sections measurements previously made at this angle \(^5,6,8\)\). The coefficients of the angular distributions, given by Walton, et al. \(^8\), were used to calculate the cross section at 0°. This was used to assign a cross section scale to the previously measured 0° neutron yields. The \(^{13}\text{C}(\alpha,n)\text{O}^{16}\) excitation
FIGURE 3
curve taken is shown in fig. 3. The amount of C\textsuperscript{13} in the target was known to about + 3%. Thus the principal error in the 90\textdegree cross section comes from the use of the Pu Be source. The neutrons from the reaction are about 5 MeV in the region calibrated. Those from the source range in energy from 0 to 10 MeV\textsuperscript{23}). As the long counter efficiency is a function of energy\textsuperscript{24}), this introduces errors in measuring the cross sections. This is probably less than 15% error. The cross sections should be accurate to ± 20%.

E. Dispersion Theory Analysis

Calculated curves were fitted to the excitation curves by means of the single level dispersion-theory form of the scattering matrix as given by Blatt and Biedenhorn\textsuperscript{11}). The derivation of the cross section formula is given by C. W. Reich\textsuperscript{22}) and by E. Kashy\textsuperscript{16}). Discussions of this type of analysis have been given by Kashy, et al.\textsuperscript{19,26}). The program used has been discussed by T. A. Belote\textsuperscript{17}). For a particle of spin \( i \) incident upon a nucleus of spin \( I \) the differential elastic scattering cross section is given by

\[
\frac{d\sigma}{d\Omega} = \sum_{S, M_S} \frac{1}{(2I+1)(2I'+1)} \left| \mathcal{F}_S^{M_S} (\theta) \right|^2
\]

where

\[
\mathcal{F}_S^{M_S} (\theta) = -\frac{n}{2k} \ csc^2 \frac{\theta}{2} \ \epsilon^{i \gamma \ln \ csc \frac{\theta}{2}} \ \chi^S_{M_S}
\]

\[
+ \sum_{S', M_{S'}, \ell, \ell'} \frac{k}{2} \sqrt{\ell(2\ell+1)} \ \gamma_{S, S'}^{\ell - \ell + 1} \ \chi^S_{M_S} \chi^{S'}_{M_{S'}}
\]

\[
\times \left[ J^S_{\ell} \ M^S_{\ell} \ M^S_{\ell} \right] \left[ S \ \ell \ \ell' \ M_S \ M_S \right] \left( e^{2i \alpha_1} \ \delta_{\ell \ell'} - S_{\ell \ell'} e^{-2i \alpha_1} \right)
\]

and

\[
S_{\ell \ell'} e^{-2i \alpha_1} = e^{-(\alpha_1 + \alpha_1' + \phi_k + \phi')} \left\{ \delta_{\ell \ell'} + \sqrt{\frac{\beta}{\gamma}} \left( e^{2i \beta \gamma} - 1 \right) \right\}
\]
The symbols are defined below. Writing $f_{\Delta}^{m_\Delta}(\theta)$ in this form assumes that the channel spin, $S$, is conserved. This is not a restriction when the channel spin is $1/2$ only, since the channel spin must be conserved in this case.

The expression actually programmed, after the general expression was reduced by inserting the parameters appropriate to channel spin $1/2$, was

$$\frac{d\sigma}{d\Omega} = |A|^2 + |B|^2$$

where

$$A = -\frac{n}{2k} \csc^2 \Phi e^{i\eta \ln \csc^2 \Phi} + \sum_{l} \sqrt{4\pi(2l+1)} \ e^{i(2\alpha_k + \phi)}$$

and

$$B = \frac{\alpha}{2k} \sum_{l} \sqrt{\frac{2l+1}{2l+1}} \ (K_k^+ - K_k^-) Y_k^l(\theta)$$

where

$$K_k^\pm = e^{2i(\alpha_k + \phi)} \sum_{r=0}^\infty \ \frac{\Gamma^{J'M}_k}{\Gamma^{J'M}_k} \ (1 - e^{2i\beta^{J'M}})$$

$\Gamma^{J'M}_k$ = total width of the $J'M$ state

$k$ = elastic scattering partial width for the $J'M$ state

$\eta = \frac{\beta_3 \delta e^+}{\hbar \nu}$

$\alpha_k = \frac{l}{\delta_1 + \delta_2} \ tan^{-1} \ \delta / \delta$

$\phi_k$ = hard sphere phase shifts

$\beta^{J'M}$ = resonant phase shift for the $J'M$ state

$$= tan^{-1} \left( \frac{\Gamma^{J'M}}{2|E_{res}^{J'M} - E|} \right)$$

$E_{res}^{J'M}$ = resonant energy of the $J'M$ state

A set of single level isolated resonance curves that were calculated to aid in the identification of the resonances are shown in fig. 4. They were calculated for a resonant energy of $2.7$ MeV.
FIGURE 4

ENERGY, IN UNITS OF RESONANCE WIDTH $\Gamma$

\[ \phi (C.M.) = 169.6^\circ \]

\[ \phi (C.M.) = 142.6^\circ \]

\[ \phi (C.M.) = 107.9^\circ \]

\[ \phi (C.M.) = 54.7^\circ \]
The shape of two resonances with the same \( \ell \) value and different \( J \) values are very similar, differing only in amplitude. Since the amplitude varies with \( \frac{\Gamma}{\Gamma} \) also, the value of \( J \) and \( \frac{\Gamma}{\Gamma} \) cannot both be determined from the elastic data. The elastic data can therefore not be expected to resolve an ambiguity between the two values of \( J \) for a given \( \ell \). It is clear from the figure that the shapes for the curves of different parity are quite different, however, and the parity can be determined by the elastic scattering experiment. Since most of the \( J \) values and some relative parities are determined by the analysis of the \( \text{C}^{13}(\alpha,n)\text{O}^{16} \) angular distributions, the assignment of parities from the elastic scattering allows a fairly complete determination of the spins and parities of the states in \( \text{O}^{17} \) in this energy range.

The determination of \( \frac{\Gamma_{\ell}}{\Gamma} \) from the elastic scattering resolves ambiguities in the partial widths also. It is only necessary to know that \( \Gamma_{\ell} > \Gamma_{n} \) or \( \Gamma_{\ell} < \Gamma_{n} \) to resolve these ambiguities, since \( \Gamma_{\ell}^{\alpha} \Gamma_{n}^{\alpha} \) is determined from the \( \text{C}^{13}(\alpha,n)\text{O}^{16} \) analysis, and since \( \Gamma_{\ell}^{\alpha} + \Gamma_{n}^{\alpha} = \Gamma \) in this energy range. In most cases \( \Gamma_{\ell} \ll 1 \) and cannot be accurately determined from the elastic data. The knowledge that it is small, however, allows the choice to be made between the two possible pairs of \( \Gamma_{\ell} \) and \( \Gamma_{n} \) obtained from the \( \text{C}^{13}(\alpha,n)\text{O}^{16} \) analysis.

F. Discussion of Angular Momentum and Parity Assignments

The resonance observed in the cross sections are in general most prominent in the \( \text{C}^{13}(\alpha,n)\text{O}^{16} \) cross sections. These \( \text{C}^{13}(\alpha,n)\text{O}^{16} \) cross sections are shown in figure 3, where the resonant energies are identified by arrows. These energies are also indicated by arrows on
the $169.6^\circ C^{13}(\alpha,\alpha')C^{13}$ cross section in figure 2. The lack of structure in the elastic cross-section data at a number of the resonances is due to low partial widths ($\Gamma/\rho$). This reduces the amplitudes of the resonances. In addition, in the case of a large total width ($\Gamma$), the resonance is spread over a wide energy range, making resonant variation of the cross section comparable with the variation of the background potential scattering. This further obscures the resonance shape. In some cases interference effects between the states are all that allows fits to be made. In nearly all cases the results from the $C^{13}(\alpha,n)O^{16}$ angular distribution analysis are relied upon. Fitting the elastic scattering would be quite difficult if not impossible without these results.

From 2 to 2.5 MeV in both the neutron and elastic excitation curves (fig. 2, 3), there are three broad resonances, ranging from 80 to more than 150 keV wide. The spins of the $O^{17}$ states corresponding to these resonances are known from the $C^{13}(\alpha,n)O^{16}$ angular distributions, and they are known, also from the neutron analysis, to have alternating parities.

The lowest resonance in the elastic data was at 2.067 MeV ($E_{o17} = 7.929$ MeV). The corresponding state in $O^{17}$ has a spin of 1/2. The difference in cross section and shape between the assignments 1/2− and 1/2+ to this state was too small to permit a direct parity assignment from the elastic data, as the resonance is quite broad (154 keV) and rather weak ($\Gamma/\rho = 0.14$). Only the assignment of parities to the two adjacent states from the elastic data, plus the known parity of this state relative to these adjacent states, permits an assignment of 1/2−.
Figure 5

$^{13}\text{C}(\alpha,\alpha)^{13}\text{C}$

#### Energy (MeV)

- $\Phi (\text{C.M.}) = 169.6^\circ$
- $\Phi (\text{C.M.}) = 142.6^\circ$
- $\Phi (\text{C.M.}) = 107.9^\circ$
- $\Phi (\text{C.M.}) = 54.7^\circ$

**Cross Section (Barns/steradian)**

**Alpha Particle Energy (MeV)**
The resonances at 2.253 and 2.415 MeV alpha particle energy both have an angular momentum assignment of 3/2 and must be of opposite parity. These resonances correspond to states at 8.071 and 8.195 MeV in $^{17}\text{O}$. The combination of 3/2- for the 8.071 MeV state plus 3/2+ for the 8.195 MeV state gives only a very slight rise in cross-section at 169° between the two resonances. The combination of 3/2+ for the lower energy state plus 3/2- for the higher energy state gives a quite strong rise in cross section between the corresponding two resonances at 169°. A quite definite rise in the cross-section in the data in this region at this angle allows a choice of the second assignment, 3/2+ for the 8.071 MeV state and 3/2- for the 8.195 MeV state. The fits to the data are shown in fig. 5.

From 2.5 to 3 MeV there is a group of four narrow resonances, ranging from 4 to 24 keV wide.

For the 2.575 MeV resonance ($E_{Q} = 8.317$ MeV), Walton, et al. report 1/2+. The position of the dip at 169.6° with respect to the resonant energy (as determined by the neutrons which were taken at the same time) shows this to be a 3/2- or a 5/2- rather than a 1/2+.

The resonance at 2.668 corresponds to a 5/2+ state. The two possibilities fit equally well at the three forward angles, but the 5/2- gives a somewhat better fit at 169.6°, the 5/2+ giving a stronger peak at slightly too high an energy to fit the data.

The two resonances at 2.749 MeV and 2.800 MeV must be considered together, as they interfere strongly. They correspond to states at 8.150 MeV and 8.489 MeV. Schiffer, et al. assigned the 8.489 MeV state as 5/2 or 3/2. Walton, et al. assign the 8.150 MeV state as
7/2+ and the 8.489 MeV state as 3/2+, with the same relative parity. Of the four possibilities allowed by these assignments, the 7/2-, 5/2- pair gives the best fit at 169.6°, but this is definitely incorrect at 107.9°. In order to reproduce the shape at this angle, opposite parities had to be allowed, the best fit being obtained with 7/2+, 5/2- or 7/2+, 3/2-.

Above 3 MeV there are two broad states (90 and 150 keV) followed by two relatively narrow ones (20 and 30 keV).

The two states at 8.685 MeV (E α = 3.056 MeV) and 8.867 MeV (E α = 3.294 MeV) form another 3/2, 3/2 pair of opposite relative parity. The fitting follows from the same arguments as in the 8.071 MeV and 8.195 pair, with the 3/2-, 3/2+ pair necessary to fit the data.

The resonance observed in the C<sup>13</sup>(α,n)O<sup>26</sup> yield at 3.294 MeV is 150 keV wide and has been shown to correspond to a J = 3/2+ state. This is the state assigned above. At 3.305 MeV, however a narrow peak with the strong backward peak of an l = 3 or 4 is observed. Fitting at the other angles, particularly the very characteristic resonant shape at 54.7° shows this to correspond to a 7/2- or 9/2- state. This state has not been observed in the C<sup>13</sup>(α,n)O<sup>16</sup> cross-section. R. L. Steele observed it in the C<sup>13</sup>(α,α')C<sup>13</sup> cross-section but assigned it as a 7/2+.

The fit to the resonance at 3.368 MeV does not justify an assignment. The parity of the corresponding 8.294 MeV state relative to the 8.867 MeV state is known from the C<sup>13</sup>(α,n)O<sup>16</sup> analysis. There is some possibility of this relative parity assignment being questionable due to the previously unknown state at 8.875 MeV. This present relative
parity assignment would give positive parity to the 8.294 MeV state.

These assignments are shown in a portion of the level scheme of $^{17}\text{O}$ in fig. 6.

G. Resonance Parameters

These fits shown in fig. 5 were calculated using the following parameters.

<table>
<thead>
<tr>
<th>$E$ (MeV)</th>
<th>$\Gamma_{\text{lab}}$ (keV)</th>
<th>$\Gamma_{1}\Gamma$</th>
<th>$J^\pi$</th>
<th>$l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.067</td>
<td>154</td>
<td>0.14</td>
<td>1/2-</td>
<td>0</td>
</tr>
<tr>
<td>2.253</td>
<td>92</td>
<td>0.09</td>
<td>3/2+</td>
<td>1</td>
</tr>
<tr>
<td>2.415</td>
<td>78</td>
<td>0.08</td>
<td>3/2-</td>
<td>2</td>
</tr>
<tr>
<td>2.575</td>
<td>24</td>
<td>0.10</td>
<td>3/2-</td>
<td>2</td>
</tr>
<tr>
<td>2.668</td>
<td>10</td>
<td>0.05</td>
<td>5/2-</td>
<td>2</td>
</tr>
<tr>
<td>2.749</td>
<td>10</td>
<td>0.70</td>
<td>7/2+</td>
<td>3</td>
</tr>
<tr>
<td>2.800</td>
<td>4</td>
<td>0.50</td>
<td>5/2-</td>
<td>2</td>
</tr>
<tr>
<td>3.056</td>
<td>90</td>
<td>0.06</td>
<td>3/2-</td>
<td>2</td>
</tr>
<tr>
<td>3.294</td>
<td>150</td>
<td>0.22</td>
<td>3/2+</td>
<td>1</td>
</tr>
<tr>
<td>3.305</td>
<td>20</td>
<td>0.70</td>
<td>7/2-</td>
<td>4</td>
</tr>
<tr>
<td>3.368</td>
<td>30</td>
<td>0.03</td>
<td>7/2+</td>
<td>3</td>
</tr>
</tbody>
</table>

These fits also included the effect of two states lower in energy than the data taken.

Ajzenberg and Lauritsen\textsuperscript{27} give these states as follows:

<table>
<thead>
<tr>
<th>$E_x$ in $^{17}\text{O}$ MeV</th>
<th>$J$</th>
<th>$\Gamma_{\text{c.m.}}$ (keV)</th>
<th>Decay</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.676</td>
<td>5/2</td>
<td>22</td>
<td>$\alpha, n$</td>
</tr>
<tr>
<td>7.560</td>
<td>7/2</td>
<td>4</td>
<td>$\alpha, n$</td>
</tr>
</tbody>
</table>
There is also a 750 keV wide state at 7.72 MeV, but this decays by neutrons only and was not included. The next lower state is < 2 keV wide and its effect should be negligible as it is more than 280 x \( \Gamma \) away from the first resonance in the data. The spins, parities, and partial widths of these states were arbitrarily picked, within the limits allowed by the known parameters, to give the best fit to the data. The parameters used were:

<table>
<thead>
<tr>
<th>( E ) (MeV)</th>
<th>( \Gamma_{c.m.} ) (keV)</th>
<th>( \Gamma / \Gamma )</th>
<th>( J^\pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.748</td>
<td>22</td>
<td>.5</td>
<td>9/2-</td>
</tr>
<tr>
<td>1.595</td>
<td>4</td>
<td>.5</td>
<td>7/2+</td>
</tr>
</tbody>
</table>

The phase shifts \( \phi_k \) used are given by the following quadratic functions of the energy:

\[
\phi_0 = -2.5203E^2 + 1.1911E - 2.0768, \quad E > 2 \text{ MeV}
\]
\[
\phi_1 = -2.6743E^2 + 1.3288E - 1.9265, \quad E > 2 \text{ MeV}
\]
\[
\phi_2 = 0, \quad E < 2.5 \text{ MeV}
\]
\[
\phi_3 = -1.4881E^2 + 6.7875E - 7.6706, \quad E > 2.5 \text{ MeV}
\]
\[
\phi_4 = 0, \quad E < 2.5 \text{ MeV}
\]
\[
\phi_5 = -4.3890E^2 + 1.9238E - 2.0861, \quad E < 2.5 \text{ MeV}
\]
\[
\phi_l = 0, \quad l > 3
\]

Reduced Widths

Using the resonance parameters obtained from the fitting of the excitation curves reduced widths for the alpha particle channel were calculated from the definition

\[
\nu_\alpha^2 = \frac{\Gamma_\alpha (c.m.)}{2 k_\alpha \nu_\alpha}
\]
where

\[ \mathcal{U}_d = \left[ \frac{1}{F^2 + G^2} \right]_{k_d a_d} \]

The interaction radius used was

\[ a_d = 1.45 \left( A^M + A^V \right) \times 10^{-13} \text{ cm.} \]

\[ = 5.71 \times 10^{-13} \text{ cm.} \]

The penetrabilities were calculated by a computer program written by R. W. Harris.\(^{28}\)

The ratio of the reduced widths to the Wigner limit were also calculated. The Wigner limit is given by

\[ \mathcal{V}^2_{\alpha \alpha} = \frac{3 \hbar^2}{2 \mu \alpha a_{\alpha}} \]

\[ = 3.58 \times 10^{-10} \text{ keV cm} \]

where \( \mu_{\alpha} \) is the reduced mass.

The resulting parameters are given below.

<table>
<thead>
<tr>
<th>( E ) MeV</th>
<th>( E_x ) in ( ^{17}O ) MeV</th>
<th>( \mathcal{V}^2_{\alpha \alpha} ) keV cm ( \times 10^{-10} )</th>
<th>( \mathcal{V}^2_{\alpha \alpha} / \mathcal{V}^2_{\beta \beta} ) x100</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.067</td>
<td>7.929</td>
<td>.3416</td>
<td>9.5</td>
</tr>
<tr>
<td>2.253</td>
<td>8.071</td>
<td>.1429</td>
<td>4.0</td>
</tr>
<tr>
<td>2.415</td>
<td>8.195</td>
<td>.1907</td>
<td>5.3</td>
</tr>
<tr>
<td>2.575</td>
<td>8.317</td>
<td>.0500</td>
<td>1.4</td>
</tr>
<tr>
<td>2.668</td>
<td>8.388</td>
<td>.0873</td>
<td>2.4</td>
</tr>
<tr>
<td>2.749</td>
<td>8.450</td>
<td>.1096</td>
<td>11.4</td>
</tr>
<tr>
<td>2.800</td>
<td>8.489</td>
<td>.0268</td>
<td>.7</td>
</tr>
<tr>
<td>3.056</td>
<td>8.685</td>
<td>.0417</td>
<td>1.2</td>
</tr>
</tbody>
</table>
As may be seen, the largest percentage of the Wigner limit for a state of normal parity is 11.4\%, and most of the percentages are much smaller. Thus one sees very little single particle character in the states in this energy range.
IV Alpha Particle Energy Calibration

The energy of the charged alpha particles is determined by a 90° bending magnet and defining slits. The magnetic field is measured by a magnetometer, which measures a probe frequency that is proportional to the magnetic field. As the particle energy is proportional to the square of the field strength, we may write \( E = kf^2 \). For energies below about 0.8 MeV, this \( k \) may be determined by measuring the frequency of a known threshold or resonance. Then the energy of the alpha particle corresponding to a probe frequency of the magnetometer is calculated by \( E = kf^2 \). Including the relativistic correction makes this formula

\[
E = m_0c^2 \left( \sqrt{1 + \frac{2kf^2}{m_0c^2}} - 1 \right)
\]

The magnetometer has two probe signals which may be used to determine the magnetic field, a lithium signal and a proton signal. The choice of which signal is used depends on the strength of the magnetic field. A recent survey on energy calibration\(^{29} \) recommends the Li\(_7^\)\( (p,n)\)Be\(_7^\) threshold at 1.881 MeV as a primary standard. This gives \( k_{pH} \) where the first subscript refers to the bombarding particle and the second subscript refers to the probe. The magnetic field strength was so low for this threshold that the hydrogen signal on the magnetometer was observed rather than the lithium signal. At the same field strength (frequency) an alpha particle would have an energy

\[
E_\alpha = \frac{\left( \frac{e_m}{e_m} \right)_p}{\left( \frac{e_m}{e_m} \right)_\alpha} E_p
\]

\[
= 0.25175 E_p
\]
or

\[ k_{a, H} = 0.25175 \ k_{\rho, H} \]

As the ratio of the two signal frequencies is inversely proportional to the constants are related by

\[ k_{a, Li} = \left( \frac{\mu_{a, H}}{\mu_{a, Li}} \right)^2 \ k_{a, H} \]

\[ = (2.5733)^2 \ k_{a, H} \]

\[ k_{a, Li} = 6.62162 \ k_{a, H} \]

Above 0.8 MeV alpha particle energy the field of the magnet becomes inhomogeneous and the field is no longer simply proportional to the square of the frequency. A correction must be made for this inhomogeneity. This correction is negligible to about 2 MeV for alpha particle energy, however.

The correction was empirically determined as follows. A \( k_{\rho, H} \) was determined from the \(^{13}\text{C}(p,n)\) threshold where the magnet was still homogeneous. Then the frequencies of several thresholds and resonances at higher magnetic fields where the magnet was no longer homogeneous were determined. These are shown in fig. 7. Using the \( k_{\rho, H} \) from the nonsaturated threshold, energies were calculated from the frequencies of these resonances and thresholds. Comparing these to known energies of the thresholds and resonances gave a \( \Delta E = E \) (known) - E (calculated). The \( \Delta E/E \) was plotted versus the frequency of the lithium probe. This is shown in fig. 8. This correction was then used in calculating the energy of alpha particles. The energy for a given frequency was first calculated using the non-saturated \( k \) and then
\[ C^{13}(\alpha, n)O^{14} \]

\[ C^{13}(p, n)N^{13} \]

\[ Li^{7}(HH^*, n)Be^{7} \]

\[ Li^{7}(\alpha, n)B^{10} \]

FIGURE 7
multiplied by \((1+a)\) where \(a\) is the correction factor for the frequency.

In practice it was convenient to measure the frequency of the \(^{13}\text{C}(\alpha,n)^{16}\) resonance at 2.800 MeV before taking data. Inhomogeneity had already set in at this point. The procedure was to calculate a \(k\) at this point and correct the \(k\) to what it would have been had there been no inhomogeneity. This \(k\) was used to calculate the energy which was then corrected by the correction curve discussed before.

The correction of the \(k\) was carried out as follows: Assume we calculate \(k'\) at 2.8 MeV, where the energy correction is \(a\). Then denoting the unsaturated constant by \(k\), we know

\[
E(k', f) = E(k, f) (1+a)
\]

\[
\gamma_0 c^2 \left( \sqrt{1 + \frac{2k'^{1/2}}{m_0 c^2}} - 1 \right) = \gamma_0 c^2 \left( \sqrt{1 + \frac{2k^{1/2}}{m_0 c^2}} - 1 \right) (1+a)
\]

\[
k = \frac{k'}{(1+a)^2} + a \frac{\gamma_0 c^2}{f^2} \left( \sqrt{1 + \frac{2k'^{1/2}}{m_0 c^2}} - 1 \right)
\]

The correction expression for \(k\) and the energy correction curve were programmed for an IBM 1620 computer so the energies corresponding to a series of frequencies could be quickly calculated when the frequency of the \(^{13}\text{C}(\alpha,n)^{16}\) resonance was known.
I wish to thank Dr. J. R. Risser for suggesting this experiment, and for his advice and encouragement throughout the work on it. I also wish to thank Dr. T. A. Belote for help in taking part of the data. I am grateful to Texas A. & M. College for the use of their computing facilities. Finally, I wish to acknowledge the help and encouragement given me by my wife, Martha.
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