KLYSTRON-FREQUENCY STABILIZATION
FOR A PARAMAGNETIC-RESONANCE
SPECTROMETER

by

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ABSTRACT

The requirements for the frequency stability of a microwave oscillator for use in a paramagnetic-resonance spectrometer are discussed. A design for a frequency-stabilization system, which is based on developments by R. V. Pound, is presented. Tests show that the system has a short-time stability of better than one part in $10^8$ and a long-time stability of one part in $10^7$. These results are compared with the required frequency stability for paramagnetic-resonance experiments.
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INTRODUCTION

Since its inception approximately fifteen years ago, paramagnetic-resonance spectroscopy in solids has grown to be an important field of research. This field of study yields information about the magnetic properties of matter, the chemical bonding in solids, the interactions between the spins and the lattice of a solid, and many other phenomena. Experimental and theoretical details have been described extensively in the literature.¹,²

Experimentally the resonance condition is obtained in a sample by the application of an r-f magnetic field and an adjustable d-c magnetic field which is perpendicular to the r-f field. For best results the r-f frequency is in the microwave part of the spectrum. The requirement for frequency stability in the microwave signal exceeds the performance of available oscillators. It is possible, however, to increase the frequency stability of a klystron oscillator, through the use of auxiliary equipment, to such an extent that its use as a source of microwave radiation in a paramagnetic-resonance experiment is possible.

The need for frequency stability arises from the desire to obtain a sensitive experimental apparatus. It is the purpose of this thesis to consider some of the limitations on sensitivity and to discuss in detail the elimination of one of these limitations, namely, frequency instability. Several methods for the frequency stabilization of a klystron will be examined. Design considerations for one method are presented
along with a system which meets these requirements. The performance of the finished system has been tested, and the results are evaluated by comparing them with the requirements for a sensitive paramagnetic-resonance spectrometer.
SPECTROMETER NOISE SOURCES

Quite obviously the usefulness of any paramagnetic-resonance spectrometer depends upon its sensitivity. The problems involved in reaching the maximum possible sensitivity have been considered in detail by Feher. Most spectrometer noise falls into one of the following five classes.

Regardless of the detection system used the spectrometer will have a source resistance, $R$, at an absolute temperature, $T$. The available noise power, $P_N$, is due to thermal agitation, and is termed Johnson noise. Its magnitude is

$$P_N = k T \Delta \nu,$$

and the resulting open circuit rms noise voltage is

$$V_{N\text{rms}} = \sqrt{4k T R \Delta \nu},$$

where $k$ is Boltzmann's constant and $\Delta \nu$ is the bandwidth of the detection system. The value of $R$ depends upon the type of spectrometer, and it will be discussed more fully in a following section.

Very little data is available on noise resulting from the microwave oscillators used. The electron beam which passes through the $r$-$f$ gap of a klystron resonator contains a continuous spectrum of noise currents arising from the shot effect, the thermal velocity fluctuations due to the hot cathode, and from partition effects. For those frequencies at which the resonator has an appreciable shunt resistance, output voltages appear. Thus the noise is in the form of amplitude modulation which is difficult to eliminate.
Noise originating in the detection system presents perhaps the most serious limitation to sensitivity in present spectrometer systems. Of the several detection schemes in use it will be worthwhile to comment only on the noise in a crystal detector as straight crystal detection is to be used in this laboratory. For a crystal diode operating in the linear region of its characteristic, the noise power is given by 6, 7

\[ P_N = \left( \frac{\gamma P_{rf}}{\nu} + 1 \right) kT\Delta \nu, \]  

(3)

where \( \nu \) is the frequency about which the detection system bandwidth, \( \Delta \nu \), is centered and \( \gamma \) is a constant dependent upon the type of crystal.

In practice the frequency of interest is the frequency of a modulated magnetic field which is superimposed on the d-c magnetic field. Thus, for low-frequency modulation fields the noise from this source is high.

The detection systems in use are amplitude sensitive; however, if the klystron frequency differs from the resonant frequency of the sample cavity the klystron frequency modulation will become a noise source. In order to see the effects of frequency instability reference must be made to the reflection coefficient, \( \Gamma \), from the cavity with a sample inside. This is given in Appendix D as

\[ \Gamma = \Gamma_o + \frac{2\beta Q_o f_m}{(1 + \beta)^2} \left( \chi'' + j \chi' \right) - \frac{4\beta \delta_o}{(1 + \beta)^2} (1 - jQ_o), \]  

(4)

where:

\[ \Gamma_o = \frac{1 - \beta}{1 + \beta}, \]

\[ \delta_o = \frac{\omega - \omega_o}{\omega}, \]
\( \beta = \) cavity coupling coefficient

\( \omega = \) frequency of incident r-f

\( \omega_0 = \) cavity resonant frequency

\( Q_0 = \) unloaded \( Q \) of cavity.

\( f_m = \) magnetic filling factor

\( \chi' = \) real part of magnetic susceptibility of sample

\( \chi'' = \) imaginary part of magnetic susceptibility of sample.

In order to detect changes in \( \chi' \) or \( \chi'' \) through changes in \( \Gamma \), as in the reflection spectrometer to be described, all terms in equation (4) except those proportional to \( \chi' \) or \( \chi'' \) must be kept reasonably constant.

In addition equation (4) is valid only when \( \delta_0 \) is small; otherwise other effects of frequency modulation must be included. Since the klystron oscillators in use are rather unstable in frequency some means of maintaining their frequencies within very narrow limits must be devised. Such frequency control is obtained through the use of the klystron-frequency stabilizer described in this thesis.

The last important source of noise is cavity noise. It may be seen from equation (4) that for a constant microwave frequency any change in the resonant frequency of the cavity will result in a noise signal. The changes in resonant frequency are brought about by the modulated external magnetic field which sets up eddy currents in the cavity walls. These eddy currents interact with the d-c component of the field, resulting in vibration of the cavity walls and a change of the resonant frequency at the rate of the modulation frequency. Unfortunately, this noise is phase
coherent with a resonance signal and is not rejected by the narrow-band detection system. Cavity noise can be largely eliminated if the klystron frequency is maintained nearly identical to the cavity resonant frequency, thus keeping $\delta_o$ in equation (4) as small as possible.
Causes of Frequency Instability

In order to design an effective stabilization system one should become familiar with the causes of frequency instability. There are five major causes of frequency instability in a reflex-klystron oscillator.\textsuperscript{8,9} Many of the terms used in the following discussion are explained in Appendix A which reviews the principles of operation for this type of oscillator.

Since the frequency of a klystron is largely controlled by its resonant cavity, any change in the resonant frequency of the cavity will result in a frequency change. The chief causes of a change in the resonant frequency are changes in the size of the cavity due to thermal expansion or contraction and due to mechanical vibration. In addition, oscillator cavities are designed to produce high fields in a narrow gap region which results in high current densities and low values of $Q$. As a consequence the frequency stability is degraded.

A klystron oscillates at a frequency at which the total susceptance of the oscillator self-admittance and load admittance is zero so that the largest r-f voltage will be present.\textsuperscript{10} Susceptance introduced into the oscillator tuned circuit from the load generally produces a change in frequency of the oscillator so that the total admittance is real at the new frequency of oscillation. Such an effect is called pulling.
The oscillator is said to be pushed in frequency when variations in power supply voltages cause frequency changes. For typical klystrons the electronic tuning rate is such that a change of one volt in reflector potential will cause a frequency change of one megacycle.

Two lesser causes of frequency instability are noise in the electron beam, which has been mentioned earlier, and the presence of stray magnetic fields.

**Methods of Frequency Stabilization**

There are three general methods for frequency stabilization of klystron oscillators. The degree of stabilization obtained by means of a given method is sometimes described by specifying a stabilization factor, \( S \), which is the ratio of the frequency change without stabilization to the frequency change with stabilization. More often the stability is specified by the ratio of the center frequency to the rms frequency deviation so that a 10 kMc oscillator stable to one part in \( 10^6 \) would exhibit an rms frequency deviation of 10 kc. In addition stability figures are quoted in two categories, determined by the period of the frequency variation components. Short-time stability applies to variations with periods shorter than a few seconds. Long-time stability covers periods up to several hours.

The first and most obvious method of frequency stabilization is to attempt to eliminate the five causes of instability listed above. The klystron cavity may be insulated from temperature changes and
vibration. Placement of a suitable amount of isolation between the klystron and its load will reduce the pulling effect. Finally, extremely stable power supplies may be used. A short time stability of one part in $10^5$ may be obtained by this means.

Frequency stability may be improved by increasing the effective Q of the klystron resonant cavity with an external high Q cavity. This so-called "dynamic" stabilization is difficult to use in practice mainly because of the limited frequency range over which it is effective and the elaborate tune-up procedure involved. Reports on this method indicate a short-time stability of better than one part in $10^6$ is possible.\textsuperscript{12,13}

The last method of frequency stabilization, termed electronic stabilization, is by far the most successful of the three. Several different types of electronic systems are in use; however, certain characteristics are common to all. In a typical system such as that shown in figure 1, a portion of the microwave power from the oscillator is diverted to a detection system which utilizes a frequency standard in order to derive an electrical output which indicates frequency drift of the oscillator. This signal is returned to the klystron through a control device which applies the proper corrections to the klystron.

Of the six different classes of electronic stabilization systems perhaps the most widely used is the d-c system developed by Pound.\textsuperscript{15,16,17} This method has been chosen for use in the spectrometer for this laboratory because of its simplicity and because of its excellent performance. A detailed discussion of this method appears in a following section.
FIGURE 1
 BLOCK DIAGRAM OF A TYPICAL
 ELECTRONIC STABILIZATION SYSTEM
The Pound a-c system is an improvement over the d-c system.\textsuperscript{15, 16, 17} It utilizes a high Q cavity in a bridge circuit that provides an intermediate frequency signal for the control system. By detecting the amplitude and phase of this signal a control amplifier may correct the klystron frequency by changing the reflector potential. This system is capable of attaining a short-time stabilization of one part in $10^8$.

Several variations and improvements on the two Pound systems have been reported. The Pound d-c discriminator has been used with a servomechanism to control a klystron to one part in $5 \times 10^5$.\textsuperscript{18} The usefulness of the Pound a-c system has been improved without deterring its effectiveness.\textsuperscript{19} The improved Pound a-c system and a servomechanism have been combined to give control that is better than one part in $10^8$.\textsuperscript{20}

Since a waveguide cell filled with a suitable gas will show a sharp stable absorption line at a certain frequency, it can be used in place of a resonant cavity to control the frequency of an oscillator.\textsuperscript{21, 22} Results indicate that stabilities of one part in $5 \times 10^{13}$ are not impossible.\textsuperscript{23}

The remaining two systems use a special transmission type cavity to form a discriminator.\textsuperscript{24} In one case frequency modulation of the klystron produces an amplitude-modulated output from the detection unit which allows control by a phase sensitive detector and a d-c amplifier. The method of resonant-circuit sensing is similar to
the frequency-modulation method except in this case the resonant
frequency of the cavity is modulated. Both these approaches
provide improved long-time stability; however, they are limited by
the inability to respond to frequency changes faster than the modulation
frequency.
Description of Operation

The Pound d-c stabilization method has been chosen for use in the paramagnetic-resonance spectrometer for this laboratory. This system has the dual advantages of simplicity and high stability.

The detection system in this case is a microwave discriminator whose output voltage indicates the magnitude and direction of a drift in frequency away from the resonant frequency of a high Q cavity. The control device is a direct-coupled amplifier, and it applies the proper frequency correction by means of the reflector-voltage power supply. Thus the system attempts to maintain the klystron frequency coincident with the resonant frequency of the cavity.

One form of the Pound microwave discriminator is shown in figure 2. The components used are a stable, high Q, resonant cavity and a microwave magic tee, the characteristics of which are described in Appendix B. Microwave power from the klystron which enters H divides equally to arms 1 and 2; the power into arm 2 is absorbed in the matched load $Y_0$. The power in arm 1 enters $H'$ and divides equally to arms $1'$ and $2'$. The reflected waves from the cavity arm may be described in one of three ways depending on their frequency.
FIGURE 2
THE POUND MICROWAVE DISCRIMINATOR

FIGURE 3
OSCILLOSCOPE PHOTOGRAPH OF A TYPICAL DISCRIMINATOR CURVE
(1) At frequencies far off resonance the cavity acts as a short and gives a reflected wave reversed $180^\circ$ in phase.

(2) At resonance the admittance of the cavity consists of a small conductance and zero susceptance giving a reflected wave with no change in phase.

Since one arm is of length $X$ and the other is $X + \frac{\lambda}{8}$, where $\lambda$ is the wavelength at the resonant frequency, returning waves for case (2) will be $90^\circ$ out of phase and equal in magnitude. The result is approximately the same for case (1). It is pointed out in Appendix B that when power enters both side arms it is delivered by the E' and H' arms depending on the relative magnitude and phase of the inputs, being equal when the inputs are equal and $90^\circ$ out of phase. Thus, for both case (1) and case (2) the returning power divides equally between arms E' and H'. The power which enters H' again divides equally to arms E and H. Hence the difference between the output of diode B and half the output of diode A is zero at resonance and far off resonance.

(3) At frequencies near resonance the admittance of the cavity is complex having capacitive susceptance at high frequencies and inductive susceptance at low frequencies. The phase shift is no longer zero and its sign differs for the two conditions. Thus, the division of power at the tee will not be equal so that one diode output will increase while the other will decrease, the relation
being dependent upon whether the incident r-f frequency is above or below resonance.

If the diode outputs are subtracted, as described earlier, the familiar discriminator characteristic shown in figure 3 results. In this graph of d-c voltage versus frequency there is zero output at cavity resonance, and for frequencies above and below resonance the output is negative and positive respectively. One may then use this device as a detector for klystron frequency-modulation.

Theory of Discriminator

In the case of the discriminator described in figure 2 the power incident upon the crystal in arm 4' is given by equation (B-8) in Appendix B as

\[ P_4 = 4P_0 g_3 g_4 \left| \frac{(Y_1 - Y_2)}{(1 + Y_1 Y_4)(1 + Y_2 Y_3) + (1 + Y_1 Y_3)(1 + Y_2 Y_4)} \right|^2 \]

The detectors are assumed to be matched, and their admittances in terms of \( Y_0 \) are

\[ Y_3 = Y_4 = 1. \]  

Arm 1' is terminated in a short giving

\[ Y_1 = -j. \]
The cavity has an admittance given by

\[ Y_2 = \frac{\delta_o}{\delta_1} + j \frac{2\Delta \nu}{\delta_1}, \quad (8) \]

where:

\[ \delta_o = \frac{1}{Q_o} \quad \delta_1 = \frac{1}{Q_{ext}} \quad \Delta \nu = \frac{\nu - \nu_o}{\nu_o} \]

\[ Q_o = \text{unloaded } Q \text{ of cavity} \]
\[ Q_{ext} = Q \text{ of matched external circuit}. \]

\[ \nu_o = \text{cavity resonant frequency} \]
\[ \nu = \text{oscillator frequency}. \]

The substitution of (6), (7), and (8) in (5) gives

\[ \frac{P_4}{P_o} = 4 \cdot 1 \cdot 1 \left[ \frac{-j - \frac{\delta_o}{\delta_1} - j \frac{2\Delta \nu}{\delta_1}}{(1 - j) \left(1 + \frac{\delta_o}{\delta_1} + j \frac{2\Delta \nu}{\delta_1}\right) + (1 - j) \left(1 + \frac{\delta_o}{\delta_1} + j \frac{2\Delta \nu}{\delta_1}\right)} \right]^2 \]

\[ = 4 \left[ \frac{-j - \frac{\delta_o}{\delta_1} - j \frac{2\Delta \nu}{\delta_1}}{\frac{2}{(1 + \frac{\delta_o}{\delta_1} + j \frac{2\Delta \nu}{\delta_1})} \left(1 - j\right)} \right]^2. \quad (9) \]

Multiply the numerator and the denominator of (9) by \( \frac{\delta_1}{\delta_o} \) and substitute

\[ \beta = \frac{\delta_o}{\delta_1} = \frac{Q_{ext}}{Q_o} \quad \text{and} \quad \alpha = \frac{2\Delta \nu}{\delta_o}, \quad (10) \]
so that (9) becomes

\[
\frac{P_4}{P_o} = 4 \left[ \frac{-j\beta - 1 - j\alpha}{2(\beta + 1 + j\alpha)(1 - j)} \right]^2
\]

\[
= 1/2 \left[ \frac{1 + (\beta + \alpha)^2}{(\beta + 1)^2 + \alpha^2} \right].
\]  

Similarly the power reflected to arm 3, half of which reaches crystal B, is given by

\[
\frac{P_3}{P_o} = 1/2 \left[ \frac{1 + (\beta - \alpha)^2}{(\beta + 1)^2 + \alpha^2} \right].
\]  

If the detectors are operated in the square law region their outputs will be proportional to the incident power. The discriminator characteristic is formed by subtracting the output of crystal B from half the output of crystal A which gives

\[
\frac{P_4 - P_3}{P_o} = \frac{\beta \alpha}{(\beta + 1)^2 + \alpha^2}.
\]  

It should be noted that this result is positive for \( \nu > \nu_o \), zero for \( \nu = \nu_o \), and negative for \( \nu < \nu_o \) giving the desired discriminator characteristic.

It is informative to examine the slope and points of inflection of the discriminator curve. The slope at any point is
\[
\frac{d}{d\alpha} \left( \frac{P_4 - P_3}{P_o} \right) = \frac{[ (\beta + 1)^2 + a^2]\beta - \beta a (2a)}{[ (\beta + 1)^2 + a^2]^2}. \tag{14}
\]

At resonance the slope reduces to

\[
\left[ \frac{d}{d\alpha} \left( \frac{P_4 - P_3}{P_o} \right) \right] \bigg|_{\alpha=0} = \frac{\beta}{(\beta + 1)^2} = S. \tag{15}
\]

If the detectors have a sensitivity of \(b\) volts per incident watt, the rate of change of output voltage with frequency is (using the definition of \(a\) from (10))

\[
\frac{dV}{dv} = \frac{2b \beta Q_o P_o}{(\beta + 1)^2 \nu_o}. \tag{16}
\]

The quantity \(\beta\) is known as the cavity coupling coefficient and has an optimum value for this application. Since

\[
\frac{dS}{d\beta} = \frac{(\beta + 1)^2 - \beta}{(\beta + 1)^4} - \frac{2(\beta + 1)}{(\beta + 1)^4} \]

\[
= \frac{(\beta + 1) - 2\beta}{(\beta + 1)^3} \]

\[
= \frac{1 - \beta}{(\beta + 1)^3}. \tag{17}
\]
Then for a maximum discriminator slope, $\beta$ should be unity, and the cavity $Q$ should be high. The inflection points may be found from (14). The condition is

$$\left[(\beta + 1)^2 + \alpha^2\right] \beta - 2 \alpha^2 \beta = 0,$$

so

$$\alpha = \pm (\beta + 1).$$

Recalling the definitions in equation (10), the frequency values are

$$\frac{2Q_o (\nu - \nu_o)}{\nu_o} = \pm \left(\frac{Q_o}{Q_{ext}} + 1\right)$$

$$\nu = \pm \frac{\nu_o}{2} \left(\frac{1}{Q_{ext}} + \frac{1}{Q_o}\right) + \nu_o$$

$$= \nu_o \pm \frac{\nu_o}{2Q_L},$$

where $Q_L$ is loaded $Q$ of the cavity. Thus, for the higher $Q$ cavities there is a smaller range of frequencies over which stabilization is effective.

**Performance of System**

The Pound d-c stabilization system has been reported to
achieve a short-time stability of one part in $10^8$. No reports on long-time stability have been made except that frequency deviations are less than one part in $5 \times 10^5$ over a period of several hours.

There are three major difficulties which limit the effectiveness of the Pound D-C system. Since the system is error-dependent for its correction signal it is subject to drift errors over long periods of time. Any steady drifts in reflector supply voltage or cavity resonant frequency would result in errors of this sort. It has been pointed out earlier that microwave crystals are subject to excessive noise voltages in the low frequency region. Since the correction amplifier response goes to d-c, this source of error can become serious. The need for a direct-coupled amplifier in this system presents the common problem of errors due to drift in the d-c level of the amplifier output. The large electronic tuning rate of klystrons requires that this drift be kept very low.
SPECTROMETER STABILIZATION SYSTEM

Description of Spectrometer

In order to obtain a clearer picture of klystron stabilization and how it relates to a paramagnetic-resonance experiment one should be familiar with the spectrometer as a whole. Figure 4 shows a block diagram of the spectrometer constructed in this laboratory. The microwave power from the klystron is divided at a directional coupler, part of the power going to the frequency measurement apparatus and the rest being divided at a magic tee. Half the power is sent to the sample cavity and half to the microwave discriminator. A circulator isolates the input power to the cavity from the reflected power, half of which goes to the detection system; while the other half forms a second input to the discriminator. Provision is made for sweeping the large magnetic field so that the resonance condition is established at some time during the sweep. A small modulated field is superimposed upon the larger field so that in passing through a resonance the reflected power from the cavity will be modulated with an amplitude proportional to the derivative of the resonance absorption. The detection system is narrow-banded about the modulation frequency to prevent noise at other frequencies from interfering with the resonance signal. A phase-sensitive detector then
FIGURE 4

BLOCK DIAGRAM OF PARAMAGNETIC-RESONANCE SPECTROMETER
provides a d-c output which is proportional to the derivative of
the absorption signal.

The Pound d-c stabilization method has been chosen in
this case because it provides a degree of stability which is
adequate for paramagnetic-resonance experiments. In addition
the design problems involved are less difficult than in other
methods. The sample cavity is used as the stabilizer cavity in
this system so that there will be no difference between the micro-
wave frequency and the resonant frequency of the cavity. Thus,
the term containing $\delta_o$ in equation (4) is zero giving the dual
advantage of no signal resulting from modulation field inter-
actions or drift in the resonant frequency of the cavity because
of temperature changes.

**Discriminator**

The discriminator used in this system is slightly
different from the one described in the previous sections in that
the detectors are on the E and H arms while the cavity and
oscillator connections are on the symmetrical arms. The
operation is exactly the same provided the phase shifters are
adjusted so that incoming waves to the symmetrical arms are
exactly $90^\circ$ out of phase at resonance.

Unfortunately the physical layout of the spectrometer is
such that the microwave power reaching one discriminator arm
travels over a long length of wave guide while the length for the
remaining arm is much shorter. This condition can lead to a serious error as can be seen in the following example. Suppose that the cavity resonant frequency changes due to a temperature change so that the klystron frequency, of guide wavelength $l$, will tend to follow it resulting in a new guide wavelength $l + \Delta l$. If the path lengths differ in the two arms by $n$ wavelengths then an extra phase shift of $n \Delta l$ will exist. Thus the condition of exactly $90^\circ$ phase difference between incident waves will be violated, and the klystron will seek some new frequency, other than the resonant frequency of the cavity, at which this condition is satisfied.

Figure 3 shows a typical discriminator characteristic obtained with this apparatus. The cavity used in the discriminator is described in Appendix C. The cavity has a $Q_0$ of approximately 10,000, is unity coupled, and operates, in this case, at an incident power level of three milliwatts. The slope of one volt per megacycle is in agreement with the slope predicted by equation (16) when the value of $b$ is one volt per megacycle (1N23C crystal).

**Requirements for Stabilizer**

In order to estimate the frequency stability required for paramagnetic resonance experiments one must know just how small the noise due to frequency instability must be. Among the noise sources investigated earlier, the only one which cannot be
significant. Considering only the signal due to frequency
modulation we have

\[ \Gamma \simeq \Gamma_0 - \delta_0 + j Q_0 \delta_0 \]

\[ = \Gamma_0 \left[ 1 - \frac{\delta_0}{\Gamma_0} + j \frac{Q_0 \delta_0}{\Gamma_0} \right], \quad (24) \]

so

\[ |\Gamma|^2 = \Gamma_0^2 \left[ 1 - \frac{2 \delta_0}{\Gamma_0} + \frac{\delta_0^2}{\Gamma_0^2} + \frac{Q_0^2 \delta_0^2}{\Gamma_0^2} \right]. \quad (25) \]

Then since \( \delta_0 \) is considered small

\[ |\Gamma| = \Gamma_0 \left[ 1 - \frac{2 \delta_0}{\Gamma_0} + \frac{Q_0^2 \delta_0^2}{\Gamma_0^2} \right]^{1/2}, \]

\[ \simeq \Gamma_0 \left[ 1 - \frac{\delta_0}{\Gamma_0} + \frac{Q_0^2 \delta_0^2}{2 \Gamma_0^2} + \ldots \right] \]

\[ = \Gamma_0 - \delta_0 + O \left( \frac{Q_0^2 \delta_0^2}{2 \Gamma_0^2} \right), \quad (26) \]

where the symbol \( O \) indicates that the term is only an order
of magnitude. But \( \Gamma_0 \) is a constant, and \( \delta_0 \) is very small so
that finally the noise voltage appearing at the detector due to
frequency modulation only is
The matched detector will present a terminating impedance of $R_o$ to the transmission line. The open circuit noise voltage across the detector would be given by

$$V_{\text{rms}} = \sqrt{4kTR_o \Delta \nu},$$

(28)

where $\Delta \nu$ is the bandwidth over which this voltage is being observed, $k$ is Boltzmann's constant, and $T$ is the absolute temperature. In addition the transmission line itself adds another resistance, $R_o$, at the same temperature. The resulting noise voltage at the detector is then

$$V_{\text{rms}} = \sqrt{2kTR_o \Delta \nu}.$$  

(29)

The rms value of $\delta_o$ which contributes an rms detector voltage equivalent to that from thermal noise is found by equating (29) and (27). Thus

$$V_{\text{rms}} = \sqrt{2kTR_o \Delta \nu} = \sqrt{2R_oP_o} \delta_{\text{rms}},$$

(30)

and

$$\delta_{\text{rms}} = \sqrt{\frac{kT\Delta \nu}{P_o}}.$$  

(31)

Table 1 gives the resulting values of $\delta_{\text{rms}}$ and $\Delta \nu_{\text{rms}}$, the rms frequency deviation, for the three detection system band-
widths used in the spectrometer. The conditions used are

TABLE 1

<table>
<thead>
<tr>
<th>Detection System</th>
<th>Maximum Allowable Frequency Deviation</th>
</tr>
</thead>
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<tr>
<td>Time Const. Sec.</td>
<td>Bandwith cps</td>
</tr>
<tr>
<td>0.01</td>
<td>31.8</td>
</tr>
<tr>
<td>0.15</td>
<td>2.12</td>
</tr>
<tr>
<td>3.0</td>
<td>0.106</td>
</tr>
</tbody>
</table>

Short-Time Stability Requirements
Dictated by Detection System Noise

$P_0 = 1.4$ milliwatts, $\nu_0 = 9.25$ kMc, and $T = 300^\circ$K. It should be remembered that in a practical spectrometer there will be noise sources many times larger than thermal noise. The figures given in Table 1 may be considered to be exceptionally rigorous.

Amplifier Requirements

The requirements for an amplifier to be used in the Pound d-c stabilization system are similar to those for any amplifier to be used in a feedback control system.

The frequency response needed corresponds to the range of frequencies over which it is desired to eliminate frequency
modulation in the klystron output. In this case the elimination of slow drifts in frequency is important along with the reduction of noise at the field modulation frequencies used. Thus the amplifier must have a response from d-c to over 10 kc. The upper half power point for a feedback amplifier is

\[ f_{2f} = f_2 (1 - A_o \beta_f) , \]  

(32)

where \( f_2 \) is the upper half power point without feedback, \( A_o \) is the amplifier gain without feedback and \( \beta_f \) is the fraction of output voltage fed back to the input. \( \beta_f \) is negative in the case of negative feedback. With the Pound system a typical value of \( \beta_f \) is -0.1 so that if \( A_o \) is large the value of \( f_2 \) need not be very high.

The gain of a feedback amplifier is given by

\[ A_f = \frac{A_o}{1 - A_o \beta_f} , \]  

(33)

where the terms are defined as in equation (32). For perfect stabilizer performance the value of \( A_o \) needed would be infinite, which would give a closed loop gain of

\[ A_f = \frac{1}{-\beta_f} . \]  

(34)

Obviously this goal is unattainable; however, it would be wise to have the open loop gain as high as possible. This is especially
important at d-c where frequency deviations are large due to drift effects.

In order for a feedback amplifier to function properly careful attention must be paid to the problem of stability. The factor $A_o \beta_f$ in equation (33) is in general a complex number which is frequency dependent. Nyquist has shown that a plot of this function in the complex plane must not enclose the point $1 + j0$ if the system is to be stable. An alternate approach described by Bode relates the rate of amplifier attenuation to phase shift so that the open loop characteristics of an amplifier will predict its closed loop performance. The approach by Bode is to be used in the following work.

As mentioned before, the electronic tuning rate of a klystron is quite high, so that any changes in reflector potential caused by a drift in the d-c output of the amplifier will be serious. Ordinarily d-c amplifiers are poor in this respect. Noise in the output of the amplifier results in the same effect. It should be noted that an increase in the gain of the amplifier or the application of more feedback will not eliminate these sources of frequency modulation. Hence this application calls for a low-noise, drift-free amplifier.

Amplifier Design

One form of amplifier which will meet the above requirements is a chopper-stabilized d-c amplifier combined with
an a-c coupled amplifier. A partial schematic diagram of such a system appears in figure 5. In this amplifier the input signal is filtered in a low pass filter and then converted into an a-c signal (a carrier modulated by the input signal) which is amplified in an a-c coupled amplifier. The amplified signal is rectified by a second converter and then filtered in a low pass filter to produce the output. The other half of the system is a conventional a-c coupled amplifier for higher frequency signals. The two amplifiers in parallel provide the desired response from d-c to high frequencies.

In order to examine stability a Bode plot, gain versus frequency diagram, for this system is shown in figure 6. Bode has shown that, to a good approximation, the amplifier phase shift in radians at a given frequency, for linear circuit elements, is

$$\phi = \frac{\pi a}{12},$$

(35)

where a is the rate of attenuation in decibels per octave. Thus, an attenuation rate of greater than 12 decibels per octave combined with a gain greater than unity produces oscillations at the corresponding frequency while smaller rates produce no oscillation. Figure 6 shows no rates of attenuation greater than 6 decibels per octave; therefore, the overall amplifier will be stable. Part of the response is
FIGURE 5
PARTIAL SCHEMATIC OF A CHOPPER STABILIZED D-C AMPLIFIER

FIGURE 6
BODE PLOT OF A TYPICAL CHOPPER STABILIZED D-C AMPLIFIER
determined by the d-c amplifier input and output time constants, which are specified empirically by

\[ R_1 C_1 \approx \frac{1.6}{f_c} \]  

(36)

\[ R_2 C_2 \approx \frac{6.4 A_2}{f_c A_1} \]  

(37)

where:

- \( A_1 \) = gain of the a-c coupled amplifier
- \( A_2 \) = d-c gain of the chopper amplifier
- \( f_c \) = chopper frequency.

The time constant \( R_2 C_2 \) establishes the break frequency, \( f_w \), (see figure 6) while \( R_1 C_1 \) determines \( f_x \) and \( f_y \), two octaves above \( f_x \). The input filter \( R_1 C_1 \) is essential in that it eliminates ambiguous modulation components. The break frequency \( f_z \) is determined by a lag circuit in the a-c amplifier. This results in the attenuation rate of -6 decibels per octave provided the original upper half power point is above \( f_o \).

A complete schematic diagram of the stabilization amplifier is shown in figure 7; figure 8 shows a photograph of the completed unit. R-1, R-2, and R-3 form a balancing network for the matched crystal detectors of the discriminator.
FIGURE 7
SCHEMATIC DIAGRAM OF KLYSTRON FREQUENCY STABILIZER
R-2 is adjusted for zero input to the amplifier when the klystron frequency coincides with the resonant frequency of the cavity. The signal is fed to parallel d-c and a-c amplifiers. R-4 and C-1 form an input low-pass filter while R-5 prevents the discharge of C-1 should the chopper, K-1, drift momentarily into make-before-break operation. A high quality capacitor is used for C-2 in order to prevent any voltage due to grid current in V-1 from being modulated along with the signal. V-1 is a low noise input stage, and V-2 provides two high gain stages which give a total amplifier a-c gain of 97 decibels. R-14 and C-8 eliminate chopper transients from the output by rolling off the high frequency response. V-3 is a cathode follower with a transformer (designed for this system by the Electrodynamic Instrument Corporation) as its cathode load. The transformer provides isolation for the amplifier from reflector potentials which may reach 1500 volts. A center tapped secondary makes full wave demodulation by K-2 possible. R-20 and C-13 form the output filter with the voltage across C-13 placed in series with the reflector supply. The overall gain of the amplifier is about 87 decibels at d-c. V-3 also supplies a signal to a peak-responding voltmeter which may be used to monitor the correction signal. V-4 is a low-noise first stage for the a-c amplifier, and V-5 is a high gain second stage which gives a
total a-c gain of 66 decibels. V-6 is a cathode follower to provide a low output impedance. C-23 isolates the system from the high reflector voltage. The correction signal is developed across the high output impedance of the klystron power supply. Pentodes are used to obtain high frequency response, and the use of d-c coupling where possible results in a good low frequency response. R-33 and C-21 form a lag circuit with a step of greater than 66 decibels beginning at a frequency of 300 cycles. Thus, a value of $\beta_f$ of 0.02 or greater will give the proper frequency response for the system. Large amounts of decoupling have been used to discourage low frequency oscillation. An adjustable 60 cycle modulation voltage may be placed on the reflector using S-1B and T-1; this feature assists in the adjustment of the stabilizer. S-3 removes the low impedance a-c amplifier from the klystron so that it may be modulated externally.

**Operating Procedure**

An effort has been made to keep the procedure for locking the klystron to the cavity frequency as simple as possible. Only a d-c coupled dual trace oscilloscope is needed so that one may monitor the discriminator characteristic and the cavity mode simultaneously. The klystron may be modulated externally with the power-supply sawtooth output or internally at 60 cycles.
With the discriminator-adjust control at its middle position the discriminator may be adjusted to give a zero d-c level at resonance using the phase shifters and the detector-mount shorting plugs. When internal modulation is used a turn of S-1 to the "on" position will lock the klystron.

A serious difficulty in the use of this stabilization system which has not been mentioned up to now is the pulling of the klystron by the high Q cavity. In some cases it will be impossible to lock the klystron to the cavity. This difficulty is discussed in detail by Pound $^{31, 32}$ and others. $^{33}$ Fortunately the precaution to be taken is a simple one, being the isolation of the oscillator by a suitable amount of attenuation. The degree of padding needed depends upon the situation and may be determined from the references cited.
Feedback Factor Determination

The degree of stability to be expected in any situation depends on the amount of feedback applied. The feedback factor, $\beta_f$, may be determined if both the slope of the discriminator characteristic, $S$, and the klystron electronic tuning rate, $R$, are known by

$$\beta_f = \frac{S}{R}. \quad (38)$$

Near the center of a mode the klystron electronic tuning rate is approximately constant so that a graph of frequency versus reflector voltage is a straight line whose slope is the desired rate. It should be pointed out that the electronic tuning rate is different for different modes. The discriminator slope may be determined from an oscilloscope photograph such as the one shown in figure 9. The frequency scale may be calibrated by making a double exposure which shows the absorption dip of a tunable cavity wavemeter at different points on the cavity mode. In any measurement of slope, equation (16) shows that $\beta$, $Q_o$, and $P_o$ must be specified. In some cases the value of $\beta_f$ may be complex and a function of frequency; however, this will not be investigated.
FIGURE 8
PHOTOGRAPH OF THE KLYSTRON FREQUENCY STABILIZER

FIGURE 9
OSCILLOSCOPE PHOTOGRAPH INDICATING SLOPE MEASUREMENT TECHNIQUE
Stabilizer Performance

The microwave discriminator provides an excellent means of detecting frequency modulation in the klystron output which makes it suitable for measuring short-time frequency stability. The signal from the discriminator will be a complex waveform which is not at all periodic. One way to investigate the stability is to perform a harmonic-wave analysis on the signal. Figure 10 shows the needed equipment, which includes a very-low-noise a-c amplifier and a frequency-selective voltmeter. The Hewlett Packard Model 302A harmonic wave analyzer has a selectivity curve which is symmetrical and has a slope of 24 decibels per octave being 12 decibels down at 10 cycles off the center frequency. The voltmeter reads an average rms value of the input so that if the discriminator slope in volts per megacycle is known a graph of the average rms frequency deviation versus component frequency may be plotted. The values given in Table 1 of allowable rms frequency deviation $\Delta \nu_{\text{rms}}$ are calculated for a detection system with a symmetrical bandpass having a slope of 6 decibels per octave and being 3 decibels down at half the bandwidth off the center frequency. In order to compare the measured results with the calculated requirements, it will be assumed that the measurements give readings equivalent to those made by a detection-system
FIGURE 10
BLOCK DIAGRAM OF SHORT-TIME STABILITY TEST EQUIPMENT

FIGURE 11
BLOCK DIAGRAM OF LONG-TIME STABILITY TEST EQUIPMENT
selectivity of 6 cycles per second bandwidth. Since the rms
noise voltage is proportional the square root of the bandwidth
the remaining calculations require no such approximations.

The measurement of long-time stability has proved very
difficult. The methods suggested by Pound\textsuperscript{34} and others\textsuperscript{35}
require a second, stabilized, oscillator, which is unavailable
in this laboratory. If it is assumed that there is no change in
the resonant frequency of the cavity, the stability can be
measured by simply recording the drifts in klystron frequency.
However, obtaining a cavity with a resonant frequency stable to
one part in \(10^8\) is virtually impossible. The cavity described in
appendix C has a temperature coefficient of frequency of 200 kc
per degree centigrade so that its temperature must be kept
constant to a few parts in \(10^3\). An ice bath has proved helpful,
though not sufficient, and a thermistor has been used to record
temperature changes. The cavity temperature coefficient of
frequency is much lower at liquid helium temperature; however,
when operation at this temperature was attempted the changing
dielectric constant inside the cavity due to the evaporation of
helium partially filling the cavity caused large drifts in the
resonant frequency.

The measurement of frequencies in this range to accuracies
of one part in \(10^8\) is also difficult. The apparatus used in this
case is shown in figure 11. The transfer oscillator generates a stable signal, adjustable in frequency from 100 to 220 Mc, which is continuously monitored to better than one part in $10^7$ accuracy by the frequency counter. Harmonics of the transfer oscillator are then compared in a mixer with the frequency to be measured, and the difference frequency is observable on an oscilloscope or at a video output terminal. In order to make continuous frequency measurements a frequency controller was designed to keep a constant difference frequency between the transfer oscillator harmonic and the microwave signal. A schematic diagram of this device appears in figure 12. V-1 is a high gain amplifier for the 50 kc difference signal. V-3 is a Schmitt trigger which gives one positive pulse out for each cycle input. V-4 clamps the pulse to ground. V-5 is a cathode coupled multivibrator which gives output pulses of constant width and height, one for each trigger signal. R-21, R-22, and R-23 form a voltage divider before the pulses go to a cathode follower, V-6, and are again clamped to ground by CR-1 and R-28. R-29 and C-15 form a low pass filter to provide a d-c output voltage proportional to the video input frequency. BT-1 provides a reverse bias so that the input voltage to the chopper stabilized d-c amplifier may be set to zero for a 50 kc beat frequency. Thus, by tuning the 540A oscillator to the low side
FIGURE 12
SCHEMATIC DIAGRAM OF TRANSFER OSCILATOR
FREQUENCY CONTROLLER
of zero beat a null will appear on the correction voltage meter indicating that S-1 may be placed in the "on" position locking the oscillator to the microwave signal. The beat frequency has an rms deviation of less than 100 cycles so that the accuracy of frequency measurements are limited only by the electronic counter accuracy.

The subject of transient response for a servomechanism of this type is lengthy and will not be considered in detail. A qualitative knowledge of the system's behavior under disturbances might be useful, however. A d-c step voltage applied at the input of the stabilizer amplifier will create the disturbance needed. Transient response following the step function may be observed by an oscilloscope photograph of the noise signal present in the discriminator. This test enables the closed loop performance to be compared with what would be expected from the open loop performance.
RESULTS

Amplifier Performance

The open loop frequency response of the stabilization amplifier is shown in figure 13. In addition the amplifier phase shift is shown as a function of frequency. The similarity between figure 13 and figure 6 shows that the desired results have been achieved. In this case $f_w \cong 0.1$ cps, $f_x \cong 1.0$ cps, $f_z \cong 350$ cps, and $f_o \cong 800$ kc. The maximum rates of attenuation are no more than 6 decibels per octave, and the corresponding phase shifts approach $90^\circ$ as predicted by equation (35). The results indicate that the amplifier cannot oscillate at any frequency without the introduction of an additional phase shift externally. Also, the upper half power point of 500 cps indicates that the minimum value of $\beta_f$ which may be used is 0.01.

Feedback Factor

The way in which the feedback factor varies with incident power to the discriminator cavity is shown in figure 14. It may be seen that $\beta_f$ is a linear function of cavity power, which is to be expected. One should always keep in mind that the numerical results are valid only for the test conditions indicated. This set of conditions will be carried through the remaining investigations for comparison purposes. In some paramagnetic
FIGURE 13
OPEN LOOP FREQUENCY RESPONSE AND PHASE SHIFT FOR STABILIZER AMPLIFIER
resonance experiments saturation effects dictate the use of a very low cavity power. The results indicate that a cavity power as low as 40 microwatts may be used under these conditions before $\beta_f$ becomes too small.

Stabilizer Performance

Figure 15 shows the short-time stability results for several operating points for the system. The harmonic wave analysis indicates that the low-frequency modulation components are predominant with the power-line frequency and its harmonics having the greatest contribution. The high-frequency components are relatively low in magnitude as would be expected since there is no power supply ripple at those frequencies. The two curves for the 3-3/4 mode have the same general form with the lowest frequency modulation present when $\beta_f$ is large. The single curve for the 5-3/4 mode shows a great difference in shape for a $\beta_f$ value in the same range. These results are to be expected since a large value of $A_0/\beta_f$ gives the best stabilization factor, and the noise spectrum in a higher order mode is both greater and of a different form than that of lower modes. The magnitude of frequency deviations which occur at the field modulation frequencies are given in Table 2. These are the modulation components of major interest since they contribute noise which is detected. It has been pointed out earlier that these measurements...
FIGURE 15
GRAPH OF AVERAGE R-M-S FREQUENCY DEVIATION VERSUS COMPONENT FREQUENCY

COMPONENT FREQUENCY - CYCLES PER SECOND

AVERAGE R-M-S FREQUENCY DEVIATION

TEST CONDITIONS:

<table>
<thead>
<tr>
<th>MODE</th>
<th>E₀</th>
<th>E₀</th>
<th>CAVITY POWER</th>
<th>FEEDBACK</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>300V</td>
<td>410V</td>
<td>1.4 MW</td>
<td>0.111</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>445 μW</td>
<td>0.034</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>140V 82 μW</td>
<td>0.048</td>
</tr>
</tbody>
</table>
TABLE 2

<table>
<thead>
<tr>
<th>Klystron Mode</th>
<th>$\beta_f$</th>
<th>$\Delta \nu_{rms}$ Measured - cps</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_b$ Volts</td>
<td>$E_r$ Volts</td>
<td>No.</td>
</tr>
<tr>
<td>300</td>
<td>410</td>
<td>3-3/4</td>
</tr>
<tr>
<td>300</td>
<td>410</td>
<td>3-3/4</td>
</tr>
<tr>
<td>300</td>
<td>410</td>
<td>3-3/4</td>
</tr>
<tr>
<td>300</td>
<td>140</td>
<td>5-3/4</td>
</tr>
</tbody>
</table>

Short-Time Stability Results
at the Modulation Frequencies
are taken with a selectivity different from that of the detection system.

In order to compare the results with the allowable frequency
deviations, corrections have been made to obtain the data given
in Table 3. In no case has the observed short-time stability
been poorer than needed. The expected stability of one part

TABLE 3

<table>
<thead>
<tr>
<th>Detection System</th>
<th>$\Delta \nu_{rms}$ max. rms cps</th>
<th>$\Delta \nu_{rms}$ Corrected - cps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Const. Sec.</td>
<td>Bandwith cps</td>
<td>$\nu_c = 100\text{~cps}$</td>
</tr>
<tr>
<td>.01</td>
<td>31.8</td>
<td>90</td>
</tr>
<tr>
<td>.15</td>
<td>2.12</td>
<td>23</td>
</tr>
<tr>
<td>3.0</td>
<td>.106</td>
<td>5.2</td>
</tr>
</tbody>
</table>

* Mode: $E_b = 300\text{~V}$, $E_r = 410\text{~V}$, $\beta_f = 0.111$

Short-Time Stability Results
Corrected to the Detection System Bandwidth
in $10^8$ has been exceeded and it undoubtedly would approach one part in $10^9$ with larger values of feedback.

The long-time stability of the system has been measured many times, and a typical result is shown in figure 16. The graph shows only the amount of drift in frequency away from the initial value and does not show the deviation from the resonant frequency of the cavity. Since the cavity is unity-coupled the d-c level of the output from the detector crystal gives a rough indication of how close the klystron frequency is to resonance. Qualitative measurements by this method indicate that the long-time deviation is considerably less than 1 kc. Thus, figure 16 in reality indicates a drift in cavity resonant frequency which could be brought about by a temperature change of 0.04 °C over a period of 4-1/2 hours. Calculations attempting to correlate these drifts with temperature measurements have been unsuccessful, largely because of the large size and high heat capacity of the cavity. Nevertheless, one may draw some conclusions by assuming that the rate of drift in resonant frequency due to temperature changes is constant. From this viewpoint the average rms frequency deviation is considerably less than 1 kc. Hence, it may be concluded that the long-time stability for this system is of the order of one part in $10^7$. 
Figure 17 shows the transient response for the system with the same operating conditions used in the long-time stability measurements. The response is an exponential whose time constant is about 50 microseconds. This indicates a good high frequency response for the system which would be expected with $\beta_f = 0.11$. The fact that there is no overshoot indicates that the open loop attenuation characteristic of the amplifier has the proper form for this application. The frequency step applied to the system is considerably greater than 100 kc so that one may conclude that disturbances from shock or line voltage surges will not offer any problems.
FIGURE 17
OSCILLOSCOPE PHOTOGRAPH OF STABILIZER TRANSIENT RESPONSE FOLLOWING A STEP FUNCTION DISTURBANCE
CONCLUSIONS

It has been shown that this klystron-stabilization system meets the criteria for use in a paramagnetic-resonance spectrometer. If necessary a better stabilization system could probably be designed without changing from the Pound d-c system. A self-contained stable microwave source utilizing a commercially available cavity and using a small physical layout would eliminate several design problems. A Chopper stabilized operational amplifier\(^37, 38\) used with a specially designed reflector power supply would complete the system. One advantage of the present system which has not been pointed out is that it operates externally to the klystron power supply, and thus it may be used with any existing unstabilized power supply.

The measurements of long-time stability obtained are not accurate and might well be improved upon. The obvious problem is the stabilization of the resonant frequency of the cavity to one part in \(10^8\). Although operation of the cavity at liquid helium temperature was tried without success, and no further attempt was made to correct the difficulties encountered, one should be able to use a helium bath to obtain the stability needed. Other methods for the measurement of long-time stability which need an identically stabilized oscillator have
been cited earlier. One advantage of this approach is that ultra-stable cavities are not required.

Another point worth noting is that the stabilized oscillator frequency is so sensitive to small changes in cavity dimensions that it might be used to measure these changes. One might use this property to measure temperature changes or expansion coefficients to a high degree of accuracy, by measuring the beat frequency between two oscillators.
APPENDIX A

Reflex Klystron Operation

Figure 18 shows a schematic diagram of a reflex klystron and the supply voltages needed for its operation. The elements that compose the tube are the cathode, a focusing electrode at cathode potential, a resonator which also serves as an anode, and a reflector that is at a negative potential with respect to the cathode. An electron beam passes through the resonator gap and out toward the reflector. The electrons are turned back by the negative reflector and pass through the gap a second time to the anode which gives rise to the name of reflex klystron.

When the klystron is oscillating an alternating electric field is present in the gap space. As electrons pass through the gap, they are either accelerated or decelerated by the field. Because of this variation in exit velocity the electrons have different transit times. Thus, the electrons group together in bunches as they return to the gap. If the bunches pass through the gap at a time in the gap voltage cycle such that the electrons are slowed down then energy will be delivered to the resonator and oscillation will be sustained. Strongest oscillations will occur when the transit time, \( T \), is

\[
T = (n + 3/4) \tau , \tag{A-1}
\]

where \( n \) is an integer and \( \tau \) is the period of oscillation of the r-f voltage. The quantity \( (n + 3/4) \) designates the mode of oscillation.
FIGURE 18
SCHEMATIC DIAGRAM OF A TYPICAL REFLEX KLYSTRON
The frequency of oscillation may be changed by changing the cavity resonant frequency, $\nu_o$, or by changing the reflector voltage so that a phase shift, $\phi$, is introduced in $T$. This effect is more precisely given by\(^3\)

$$\tan \phi = 2Q \frac{(\nu_o - \nu)}{\nu_o},$$  \hspace{1cm} (A-2)

where $\nu$ is the oscillation frequency and $Q$ is the resonator $Q$.

Since $\phi$ is related to the reflector voltage, there is a means for electronically tuning the klystron frequency over a small range. This characteristic makes electronic frequency stabilization systems feasible.
APPENDIX B

Microwave Magic Tee

A microwave magic tee is a circuit consisting of a waveguide with two other waveguides connected perpendicularly to it, one in the broad wall and the other in the narrow wall, at a common point. Figure 19 shows that a magic tee is simply a combination of an E-plane tee and an H-plane tee having common side arms 1 and 2 which are often called the symmetrical arms.

To understand the special properties of the magic tee it is necessary to realize the difference in coupling between E-plane tees and H-plane tees. In figure 20A and B are shown, respectively, the field configurations in the region of the junction produced by a wave traveling into the side arm of each. Lines are drawn, in figure 20A, to represent the wave front and the circles with crosses represent the E-vector which points into the paper. Note that waves traveling outward from the junction have the same phase at points equidistant from the junction. In figure 20B, which shows the E-plane tee, the E-vector is in the plane of the paper and is represented by lines with arrows. The electric field fringes at the junction and excites waves in the arms having opposite phases at points equidistant from the junction.

In the composite junction note that the E-vector in the E-plane arm is normal to the E-vector in the H-plane arm so that there can be no direct transmission between the two. When waves of equal magnitude and phase enter the E and H arms, the outputs cancel in one of the side
Figure 19
Oblique view of a magic tee

Figure 20
Coupling in a simple T-junction
(A) H-plane tee; (B) E-plane tee
arms and add in the other. By the reciprocity principle energy applied at either arm 1 or arm 2 will divide equally between the E and H arms transmitting nothing to the opposite arm. When the entering waves are not of the same phase or amplitude the resulting fields are given by the same convention and are vector sums and differences. Hence, the division of power is dependent upon the phase of the entering waves, and the magic tee may be used as a phase sensing device.

The performance of a magic tee may be described more rigorously in terms of voltages and currents in an equivalent circuit. For any linear passive four terminal pair network the currents and voltages are related in general by the four simultaneous linear equations

\[ i_1 = y_{11} e_1 + y_{12} e_2 + y_{13} e_3 + y_{14} e_4 \]
\[ i_2 = y_{21} e_1 + y_{22} e_2 + y_{23} e_3 + y_{24} e_4 \]  
\[ i_3 = y_{31} e_1 + y_{32} e_2 + y_{33} e_3 + y_{34} e_4 \]
\[ i_4 = y_{41} e_1 + y_{42} e_2 + y_{43} e_3 + y_{44} e_4 \]  

where the coefficients depend on the properties of the network. Using the conventional choice of planes of reference for the junctions and the fact that there is no direct coupling between opposite arms gives

\[ Y_{11} = y_{22} = y_{33} = y_{44} = y_{12} = y_{34} = 0. \]  

Thus equations (B-1) reduce to

\[ i_1 = y_{13} e_3 + y_{14} e_4 \]
\[ i_2 = y_{23} e_3 + y_{24} e_4 \]  
\[ i_3 = y_{13} e_1 + y_{23} e_2 \]
\[ i_4 = y_{14} e_1 + y_{24} e_2 \]  

-49-
For a network containing no power dissipating elements the coefficients, $Y_{nm}$, are imaginary. If matched loads, $Y_o$, are connected to arms 1 and 2 and a current is induced in arm 3, the symmetry condition for the H arm and the conservation of power give

$$\frac{i_3^2}{Y_o} = e_1^2 Y_o + e_2^2 Y_o.$$  \hspace{1cm} (B-4)

Using the fact that $e_1 = e_2$ and that $Y_{nm}$ is imaginary we have

$$i_3 = j \frac{\sqrt{2}}{2} Y_o (e_1 + e_2).$$  \hspace{1cm} (B-5)

Thus

$$Y_{13} = Y_{23} = j \frac{\sqrt{2}}{2} Y_o.$$  \hspace{1cm} (B-6)

The remaining parameters may be similarly derived giving the set of transformation equations

$$i_1 = j Y_o \frac{\sqrt{2}}{2} (e_3 + e_4)$$

$$i_2 = j Y_o \frac{\sqrt{2}}{2} (e_3 - e_4)$$

$$i_3 = j Y_o \frac{\sqrt{2}}{2} (e_1 + e_2)$$  \hspace{1cm} (B-7)

$$i_4 = j Y_o \frac{\sqrt{2}}{2} (e_1 - e_2).$$

If a generator having an available power $P_o$ and an internal admittance $Y_3$ is connected to arm 3, the power delivered to a load of admittance $Y_4$ on arm (4) may be shown to be
\[ P_4 = 4P_o g_3 g_4 \frac{Y_1 - Y_2}{(1 + Y_2 Y_3)(1 + Y_1 Y_4) + (1 + Y_1 Y_3)(1 + Y_2 Y_4)}, \quad \text{(B-8)} \]

where \( g_3 \) and \( g_4 \) are the conductance parts of \( Y_3 \) and \( Y_4 \), \( Y_1 \) and \( Y_2 \) are the load admittances connected to arms 1 and 2, and all admittances are expressed in units of \( Y_o \).

The power delivered to the adjacent arm 1 is

\[ P_1 = 8P_o g_1 g_3 \frac{(1 + Y_2 Y_4)}{(1 + Y_2 Y_3)(1 + Y_1 Y_4) + (1 + Y_1 Y_3)(1 + Y_2 Y_4)}, \quad \text{(B-9)} \]

Equations (12) and (13) give the power relations for any opposite or adjacent arms respectively by permutation of subscripts.
APPENDIX C

Stabilizer Cavity

The cavity used in the microwave discriminator circuit is a cylindrical type operating in the $\text{TE}_{011}$ mode. The $\text{TE}_{011}$ mode was chosen because of its high $Q$ relative to other modes. Construction details of the cavity are shown in figure 21. The material used is yellow brass. The inner walls of the cavity have been polished and silver evaporated onto them to form a layer approximately a skin depth in thickness. The iris used gives a coupling coefficient, $\beta$, of approximately unity without the aid of tuning screws. The completed cavity has a $Q_0$ of about 10,000 and a resonant frequency of about 9.25 kMc. The temperature coefficient of frequency for the cavity is about 200 kc per degree centigrade.
SECTION A-A
CAVITY ENDPLATE

FIGURE 21
MICROWAVE DISCRIMINATOR CAVITY

FIGURE 22
EQUIVALENT CIRCUIT FOR A CAVITY INDUCTIVELY COUPLED TO A WAVEGUIDE
APPENDIX D
Cavity Reflection Coefficient

In a paramagnetic-resonance experiment the paramagnetic sample is placed in a microwave cavity. Resonances in the sample are observed by detecting the changes which result in the cavity reflection coefficient. The theoretical derivation of the reflection coefficient would require a great deal of space and only some essential steps will be described here.

A paramagnetic sample has, in general, complex magnetic and electric susceptibilities given by

\[
\chi_m = \chi_m' + j \chi_m'' \quad \text{(D-1)}
\]

\[
\chi_e = \chi_e' + j \chi_e''
\]

Experimental technique is such that changes in \(\chi_m\) may be detected while changes in \(\chi_e\) have a small effect and may be neglected.

Figure 22 shows one form of equivalent circuit for a cavity coupled to a waveguide. The cavity reflection coefficient, \(\Gamma\), for this inductively coupled model, assuming that \(\omega L_1 \ll Z_o\), is

\[
\Gamma = |r| = \frac{1 - \frac{\omega^2 M^2}{Z_o R}}{1 + \frac{\omega^2 M^2}{Z_o R}} + j Q_o \left( \frac{\omega^2 - \omega_c^2}{\omega^2} \right)
\]

\[
\Gamma = \frac{1 - \frac{\omega^2 M^2}{Z_o R}}{1 + \frac{\omega^2 M^2}{Z_o R}} + j Q_o \left( \frac{\omega^2 - \omega_c^2}{\omega^2} \right)
\]

(D-2)
where:

\( \omega_c = \) cavity resonant frequency with sample

\( \omega = \) incident microwave frequency

\( R = \) cavity equivalent resistance with sample

\( Z_o = \) characteristic impedance of waveguide.

Here it is necessary to define three quantities:

\[
\beta = \frac{\omega^2 M^2}{R_o Z_o} \quad (D-3)
\]

\[
\delta = \frac{\omega - \omega_c}{\omega} \quad (D-4)
\]

\[
\epsilon = \frac{Q_o - Q}{Q} = \frac{R - R_o}{R_o} \quad (D-5)
\]

where:

\( \omega_o = \) cavity resonant frequency without sample

\( R_o = \) cavity equivalent resistance without sample

\( Q = \) cavity Q without sample

\( Q_o = \) cavity Q with sample.

For the case where \( \delta \) and \( \epsilon \) are small, equation (D-2) may be expanded neglecting all terms in \( \delta \) and \( \epsilon \) of second order or greater giving

\[
\Gamma \approx \frac{1 - \beta}{1 + \beta} \left[ 1 + \frac{2\beta}{1 - \beta^2} \left( \epsilon - 2\delta + 2j\delta Q_o \right) \right]. \quad (D-6)
\]
The values for $R$, $L$, and $C$ for the cavity equivalent circuit contain perturbation terms due to the presence of a paramagnetic sample. It can be shown that

$$R \approx R_0 \left(1 + \chi_m'' f_m Q_o\right), \quad \text{(D-7)}$$

and

$$\omega_c \approx \omega_o \left(1 - \frac{\chi_m f_m}{2}\right), \quad \text{(D-8)}$$

When equations (D-7) and (D-8) are substituted into (D-4) and (D-5), equation (D-6) reduces to

$$\Gamma = \Gamma_o + \frac{2 \beta Q_o f_m}{(1+\beta)^2} \left(\chi_m'' + j \chi_m'\right) - \frac{4 \beta \delta_o}{(1+\beta)^2} (1 - jQ_o), \quad \text{(D-9)}$$

where:

$$\Gamma_o = \frac{1 - \beta}{1 + \beta}$$

$$\delta_o = \frac{\omega - \omega_o}{\omega}$$

$$f_m = \text{magnetic filling factor for sample.}$$

This is the desired reflection coefficient.
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