RICE UNIVERSITY

DEFORMATION AND MOTION OF A GAS-LIQUID INTERFACE IN A CYLINDRICAL CAPILLARY

by

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ABSTRACT

The motion of a gas-liquid interface in a cylindrical capillary was studied.

Stream function and vorticity equations were set up to describe a two dimensional flow in the liquid phase.

Only momentum transfer is of interest; no heat transfer and mass transfer are involved. The driving force for flow is the pressure in gas phase; the surface tension is assumed negligible.

Numerical solutions were obtained for a few time steps. The deformation of interface and the pattern of stream lines have been plotted.

NOMENCLATURE

ρ	Density of liquid
μ	Viscosity of liquid
ϕ'	Vorticity
ϕ	Dimensionless vorticity
Ψ'	Stream function
Ψ	Dimensionless stream function
r_1	Radius of capillary
t	time
T	Dimensionless time
v_r	Radial component of velocity
U	Dimensionless radial component of velocity
v_z	Axial component of velocity
V	Dimensionless axial component of velocity
p	Pressure
P	Dimensionless pressure
p_∞	Pressure at time approaching infinity (in gas phase)
p_o	Pressure at z approaching infinity
ν	Kinematic viscosity

TABLE OF CONTENTS

I. INTRODUCTION

The purpose of this thesis is to make a theoretical study of flow of liquid out of small cylindrical capillary with a moving liquid-gas interface as its boundary.

Cylindrical coordinate was adopted. The mathematical model is symmetrical about the z axis, so only two dimensions are concerned in this case. Because of the small diameter of the capillary, the flow is free from turbulence.

Two second order partial differential equations were set up to describe the flow in liquid phase. One is parabolic, and the other is an elliptic. Since the interface boundary is moving, approximate boundary conditions were used at this boundary.

In order to simplify the problem, mass transfer and heat transfer across the boundary were ignored. The pressure in gas phase is an arbitrary exponential function of time. The force of a surface tension is negligible as compared to the pressure variation.

Numerical solutions have been obtained for a few time levels. Stream function pattern, velocity distribution and the variation of the interface has been plotted for those time levels.

This thesis is a preliminary work for a theoretical study of bubble growing out of a capillary. Actually there are heat transfer and mass transfer involved in the bubble growing process, if the difference of temperature is not too large. The influence of temperature on the physical constants will be small. The momentum transfer still can be solved alone without the coupling of heat transfer.

II. MATHEMATIC MODEL AND SOME MAJOR ASSUMPTIONS

In this thesis, only momentum transfer in liquid phase is of interest. The driving force for flow is the pressure variation in vapor phase. The solution is started from an arbitrary curved interface. For time greater than zero, pressure begins to increase; the interface is pushed upward, and the liquid begins to flow out of the capillary into an infinite liquid pool above the surface. Since it is impossible to carry out numerical solution for an infinite space, the boundary is cut off at a certain distance from the mouth of the capillary. The model is shown in Fig. 1.

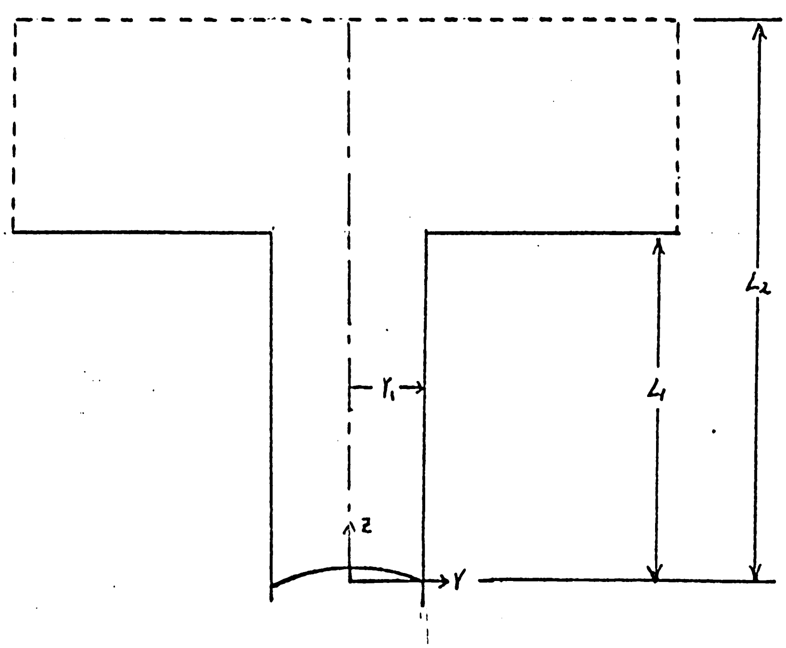


FIG. 1. The Model

In order to set up transport equation for this model, the following major assumptions have been made:

1. The model is symmetrical with respect to θ, i.e., only axial and radial variations are involved.
2. Density and viscosity are constant throughout the domain.
3. The diameter of the capillary is small enough, so the flow is laminar in this region.
4. The fluid is Newtonian.
5. The fluid behaves as a continuum in the small capillary.
6. There is no slip at the wall.

III. TRANSPORT EQUATIONS

With the previous assumptions Navier-Stokes equation can be applied to this model. Because of symmetry, θ component of the equation of motion can be dropped, and all the terms including v_θ and the partial derivatives with respect to θ are canceled. The simplified transport equations can be written [1]:

Equation of Continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{\partial v_z}{\partial z} = 0 \tag{1}$$

Equation of Motion:

r component:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{\partial^2 v_r}{\partial z^2}\right] \tag{2}$$

z component:

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{\partial^2 v_z}{\partial z^2}\right] . \tag{3}$$

Because these equations are non-linear, it is too difficult to solve these three equations simultaneously. This difficulty can be reduced by introducing stream function Ψ'.

Let

$$v_r = \frac{1}{r}\frac{\partial \Psi'}{\partial z} \qquad v_z = -\frac{1}{r}\frac{\partial \Psi'}{\partial r} . \tag{4}$$

Substitute equation (4) into (1), (2), and (3), equation of continuity is satisfied automatically, cancel the pressure term between equation (2) and (3), then a fourth order partial differential equation is obtained [1]

$$\frac{\partial^2}{\partial t}(E^2\Psi') - \frac{1}{r}\frac{\partial(\Psi',E^2\Psi')}{\partial(r,z)} - \frac{2}{r^2}\frac{\partial \Psi'}{\partial z} E^2\Psi' = \nu E^4\Psi' \tag{5}$$

where

$$E^2\Psi' = \frac{\partial^2\Psi'}{\partial r^2} - \frac{1}{r}\frac{\partial \Psi'}{\partial r} + \frac{\partial^2\Psi'}{\partial z^2}$$

$$\frac{\partial(\Psi',E^2\Psi')}{\partial(r,z)} = \begin{vmatrix} \partial\Psi'/\partial r & \partial\Psi'/\partial z \\ \partial E^2\Psi'/\partial r & \partial E^2\Psi'/\partial z \end{vmatrix} .$$

Equation (5) can be reduced to two second order differential equations by introducting vorticity ϕ', by definition

$$\begin{aligned} \phi' &= (\vec{\nabla} \times \vec{v})_\theta \\ &= \frac{\partial u_r}{\partial z} - \frac{\partial v_z}{\partial r} . \end{aligned} \tag{6}$$

Insert relation (4) into (6),

$$\begin{aligned} \phi' &= \frac{1}{r}\left[\frac{\partial^2\Psi'}{\partial r^2} - \frac{1}{r}\frac{\partial \Psi'}{\partial r} + \frac{\partial^2\Psi'}{\partial z^2}\right] \\ &= \frac{1}{r}E^2\Psi' . \end{aligned} \tag{7}$$

Let $E^2\Psi' = r\phi'$, substitute it into equation (5) then expanded and rearranged.

$$r^2 \frac{\partial\phi'}{\partial t} = \frac{\mu}{\rho} r^2\left(\frac{\partial^2\phi'}{\partial r^2} + \frac{\partial^2\phi'}{\partial z^2}\right) + r\left(\frac{\mu}{\rho} - \frac{\partial\Psi'}{\partial z}\right)\frac{\partial\phi'}{\partial r}$$

$$+ \; r\,\frac{\partial\Psi'}{\partial r}\,\frac{\partial\phi'}{\partial z} - \left(\frac{\mu}{\rho} - \frac{\partial\Psi'}{\partial z}\right)\phi' \; . \qquad (8)$$

Thus a fourth order partial differential equation can be reduced into two second order differential equation, equation (7) is elliptic and equation (8) is parabolic. The solution can be obtained by solving these two equations simultaneously.

Equation (4), (7), (8), and (3) can be changed into dimensionless form by introducing following dimensionless group:

$$\Psi = \frac{\Psi'}{\Delta p\left(\frac{r_1^{\,4}}{4\mu L_2}\right)}$$

$$\phi = \frac{\phi'}{\Delta p\left(\frac{r_1}{4\mu L_2}\right)}$$

$$x = \frac{r}{r_1}$$

$$y = \frac{z}{L_2}$$

$$T = \frac{t}{\rho\,\frac{r_1^{\,2}}{\mu}}$$

$$A = \frac{\Delta p r_1^{\,4}\rho}{4\mu^2 L_2^{\,2}}$$

$$B = \frac{r_1}{L_2}$$

$$U = \frac{v_r}{\Delta p \left(\frac{r_1^2}{4\mu L_2}\right)}$$

$$V = \frac{v_z}{\Delta p \left(\frac{r_1^2}{4\mu L_2}\right)}$$

$$\Delta p = p_\infty - p_o$$

$$P = \frac{p - p_o}{\Delta p}$$

The dimensionless equations are:

$$V = -\frac{1}{x}\frac{\partial \Psi}{\partial x} \qquad U = \frac{B}{x}\frac{\partial \Psi}{\partial y} \tag{9}$$

$$x\phi = B^2 \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial x^2} - \frac{1}{x}\frac{\partial \Psi}{\partial x} \tag{10}$$

$$x^2 \frac{\partial \phi}{\partial T} = x^2\left(\frac{\partial^2 \phi}{\partial x^2} + B^2 \frac{\partial^2 \phi}{\partial y^2}\right) + \left(1 - A \frac{\partial \Psi}{\partial y}\right) \cdot x \frac{\partial \phi}{\partial x}$$

$$+ A x \frac{\partial \Psi}{\partial x}\frac{\partial \phi}{\partial y} - \left(1 - A \frac{\partial \Psi}{\partial y}\right) \phi \tag{11}$$

$$A \frac{\partial p}{\partial y} = \frac{1}{x}\frac{\partial}{\partial x}\left(x \frac{\partial}{\partial x}\right) + B^2 \frac{\partial^2 V}{\partial y^2} - \frac{\partial V}{\partial T} - \frac{A}{B} U \frac{\partial V}{\partial x}$$

$$- AV \frac{\partial V}{\partial y} \tag{12}$$

IV. DISCRETE FORM OF DIFFERENTIAL EQUATION AND THEIR BOUNDARY CONDITIONS

Central difference form is used to evaluate all the first and second derivative terms.

F: any dependent variable

For equal lattice space:

$$\frac{\partial F}{\partial x} = \frac{F_{i+1,j} - F_{i-1,j}}{2\Delta x} \qquad \frac{\partial F}{\partial y} = \frac{F_{i,j+1} - F_{i,j-1}}{2\Delta y}$$

$$\frac{\partial^2 F}{\partial x^2} = \frac{F_{i+1,j} - 2F_{i,j} + F_{i-1,j}}{\Delta x^2} \qquad \frac{\partial^2 F}{\partial y^2} = \frac{F_{i,j+1} - 2F_{i,j} + F_{i,j-1}}{\Delta y^2} \tag{13}$$

For the lattice points near the interface, discrete forms of unequal lattice space are used: See Fig. 2

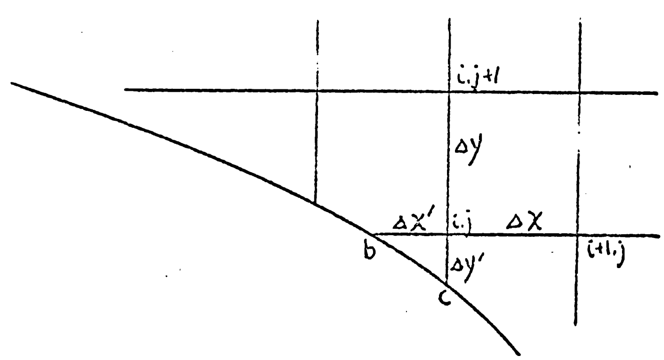


FIG. 2. Discrete forms at the interface boundary

$$\left(\frac{\partial F}{\partial x}\right)_{i,j} = \frac{\Delta x'}{\Delta x + \Delta x'}\left(\frac{F_{i+1,j} - F_{i,j}}{\Delta x}\right) - \frac{\Delta x}{\Delta x' + \Delta x}\left(\frac{F_b - F_{i,j}}{\Delta x'}\right)$$

$$\left(\frac{\partial F}{\partial y}\right)_{i,j} = \frac{\Delta y'}{\Delta y + \Delta y'}\left(\frac{F_{i,j+1} - F_{i,j}}{\Delta y}\right) - \frac{\Delta y}{\Delta y + \Delta y'}\left(\frac{F_e - F_{i,j}}{\Delta y'}\right)$$

$$\left(\frac{\partial^2 F}{\partial x^2}\right)_{i,j} = \frac{2}{\Delta x + \Delta x'}\left(\frac{F_{i+1,j} - F_{i,j}}{\Delta x} + \frac{F_b - F_{i,j}}{\Delta x'}\right)$$

$$\left(\frac{\partial^2 F}{\partial y^2}\right)_{i,j} = \frac{2}{\Delta y + \Delta y'}\left(\frac{F_{i,j+1} - F_{i,j}}{\Delta y} + \frac{F_e - F_{i,j}}{\Delta y'}\right) \qquad (14)$$

A. The discrete form for equation (10):

$$\left(\frac{2}{\Delta x^2} + \frac{2B^2}{\Delta y^2}\right)\Psi_{i,j} = -(i-1)\,\Delta x \phi_{i,j} + \frac{B^2}{\Delta y^2}\left(\Psi_{i,j+1} + \Psi_{i,j-1}\right)$$

$$+\left(\frac{1}{\Delta x^2} - \frac{1}{2(i-1)\Delta x^2}\right)\Psi_{i+1,j} + \left(\frac{1}{\Delta x^2} + \frac{1}{2(i-1)\Delta x^2}\right)\Psi_{i-1,j} \qquad (15)$$

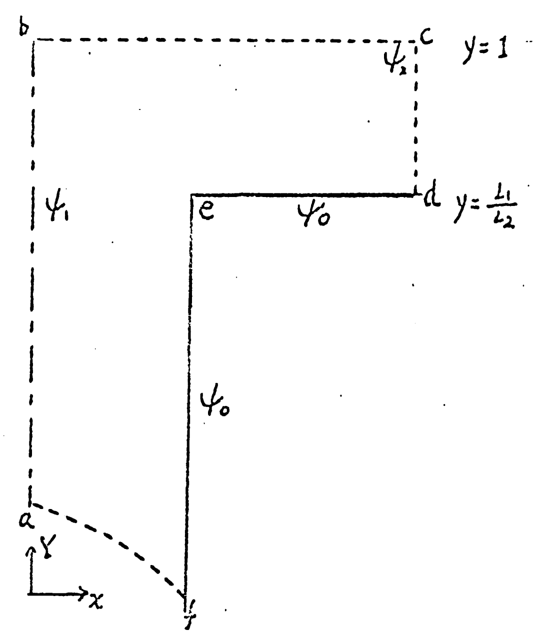


FIG. 3. The Boundary Conditions

The boundary conditions are specified in the following ways:

. Boundary along the wall: As shown from f to e and e to d in Fig. 3.

By the assumption of no slip at wall V, U are zero along the wall. From equation (9)

$$\frac{\partial \Psi}{\partial x} = 0 \qquad \frac{\partial \Psi}{\partial y} = 0 \ .$$

So the stream function must be constant along the wall, denoted by Ψ_o . Since only the distribution of stream function is significant, this constant can be assigned an arbitrary value. For convenience, let $\Psi_o = 0$.

b. Gas-liquid interface: from f to a in Fig. 3.

For each new time level, the interface is moving up to a new position, the shape of curve is changed and so does the velocity distribution.

The difficulty is that it can not be known exactly, so it depends on trial and error procedure. The detail of this procedure will be described in part V.

In order to facilitate of guessing work, a set of curves has been adopted, the algebra equation for this set of curves is represented by

$$y = h(1 - x^n)$$

for n greater than one. The parameters h denotes the height of the interface on the central line, n is proportional to the slope of the curve at the wall.

The velocity distribution on the curve boundary can be approximated by assuming that in a short time step the interface is growing in normal direction.

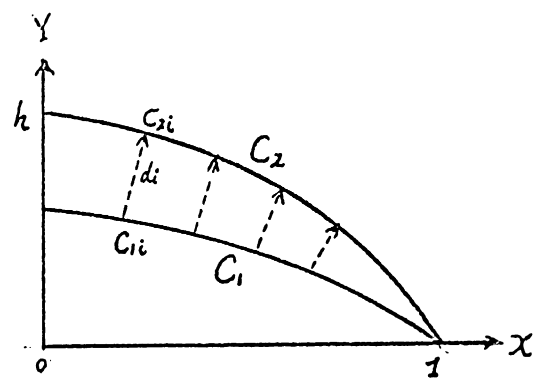


FIG. 4. Interface boundary

In Fig. 4, C_1 denotes interface of previous time level, C_2 is an assumed interface for new time level, d_i is the normal distance between two corresponding points on C_2 and C_1 so the average velocity V_{ai} must equal to $d_i/\Delta t$ then

$$V_{2i} = 2\,V_{ai} - V_{1i}$$

V_{2i} is the flow velocity at point C_{2i}, V_{1i} is the velocity at point C_{1i}.

The velocity distribution on the curve can be set by the above method.

The stream function value for each point on the curve can be obtained by numerical integration of equation (9) from wall to the center. Trapezoidal Rule was used in this calculation.

c. Central line: (From a to b in Fig. 3).

Because of symmetry, the velocity in radial direction should be zero.

By equation (9)

$$\frac{\partial \Psi}{\partial y} = 0$$

this means that stream function is constant along this line, let it be denoted by Ψ_1. This value is the same as that for point a. It can be obtained by numerical integration along the curved boundary as stated in part b.

d. Boundary from b to c in Fig. 3.

This is the boundary cut off at a certain distance from the mouth of the tube, the stream function value distribution along this line is approximated by an exponential function

$$\Psi = A' e^{-B' x^C} .$$

While A' and B' can be fixed by the value of stream function at point b and c.

$$A = \Psi_1$$

$$B = \frac{1}{x_c^{\,C}} \ln \frac{\Psi_1}{\Psi_2}$$

Ψ_2 is determined by trial, it is a value between Ψ_1 and Ψ_o. C is the parameter left for adjusting the distribution curve.

e. Boundary from c to d in Fig. 3.

The stream function along this line is also approximated by an exponential function.

$$\Psi = A'' e^{(y - y_1)^{B''}} + C'$$

A'' and C' is fixed by the value of stream function at point c and d.

$$A'' = (\Psi_2 - \Psi_o)/(e^{(y_2 - y_1)^{B''}} - 1 .)$$

$$C' = \Psi_o - A'$$

B'' is left for adjusting.

B. The discrete form for equation (11)

$$\left(2K^2 + K^2\Delta x^2/\Delta T + G + 2K^2B^2\Delta x^2/\Delta y^2\right) \phi_{i,j}$$

$$= K^2\Delta x^2/\Delta T \,\phi_{i,j} + (K^2 + GK/2)\,\phi_{i+1,j} + (K^2 - GK/2)\,\phi_{i-1,j}$$

$$+ (K^2\Delta x^2B^2/\Delta y^2 + AK\,(\Psi_{i+1,j} - \Psi_{i-1,j})/(4\Delta y))\,\phi_{i,j+1}$$

$$+ (K^2\Delta x^2B^2/\Delta y^2 - AK\,(\Psi_{i+1,j} - \Psi_{i-1,j})/(4\Delta y))\,\phi_{i,j-1}$$

where

$$K = i - 1$$

$$G = 1 - A\,\frac{\Psi_{i,j+1} - \Psi_{i,j-1}}{2\Delta y}$$

a. Boundary at interfacial: From f to a in Fig. 4.

Calculated from equation (10) from an assumed distribution of Ψ .

b. Central line: From a to b in Fig. 3.

$$\phi' = \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r}$$

equal to zero along the central line, so $\frac{\partial v_r}{\partial z}$ vanished.

$\frac{\partial v_z}{\partial r}$ also vanished because of symmetry.

$$\phi' = 0 \qquad \text{that is} \qquad \phi = 0$$

c. Boundary from b to c, and c to d.

ϕ can be calculated by equation (10) from a known distribution of Ψ .

d. Boundary from d to e:

Because Ψ equal to Ψ_o along the wall, $\frac{\partial^2\Psi}{\partial x^2}$ and $\frac{\partial\Psi}{\partial x}$ can be dropped out from equation (10),

$$\phi = \frac{1}{x}\left(B^2 \frac{\partial^2\Psi}{\partial y^2}\right) . \qquad (17)$$

e. Boundary from e to f:

Ψ equal to Ψ_o along this line, so $\frac{\partial^2\Psi}{\partial y^2}$ is equal to zero by equation (9)

$$\frac{\partial\Psi}{\partial x} = 0$$

then equation (10) becomes:

$$\phi = \frac{1}{x} \frac{\partial^2 \Psi}{\partial x^2} \qquad (18)$$

The second order partial derivatives in equation (17), and (18) can be estimated by using the value of Ψ on the boundary and three internal points near the boundary.

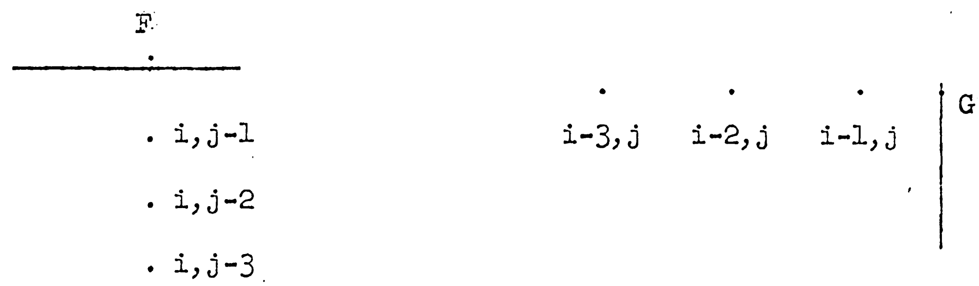


FIG. 5. Calculate second partial derivative at the boundary

$$\left(\frac{\partial^2 \Psi}{\partial y^2}\right)_F = \frac{2\Psi_F - 5\Psi_{i,j-1} + 4\Psi_{i,j-2} - \Psi_{i,j-3}}{\Delta y^2}$$

$$\left(\frac{\partial^2 \Psi}{\partial x^2}\right)_G = \frac{2\Psi_G - 5\Psi_{i-1,j} + 4\Psi_{i-2,j} - \Psi_{i-3,j}}{\Delta x^2}$$

The above formula is obtained by truncate first four terms of Newton's backward formula [2] .

C. The discrete form for equation (12):

For $i > 1$

$$4(P_{i,j+1,\ n+1} - P_{i,j,\ n+1}) = \left[\Delta y/\Delta x^2 + \Delta y/2\Delta x(1/(i-1)\Delta x - \frac{A}{B} U_{i,j})\right] V_{i+1,\ j,\ n}$$

$$- (2\Delta y/\Delta x^2 + 2B^2/\Delta y + \Delta y/\Delta T)\ V_{i,\ j,\ n}$$

$$+ (\Delta y/\Delta x^2 - \Delta y/2\Delta x\ (1/(i-1)\Delta x - A/B\ U_{i,j}))\ V_{i-1,j,n}$$

$$+\left(\frac{B^2}{\Delta y} - A\ \frac{V_{i,j}}{2}\right) V_{i,j+1} + \left(\frac{B^2}{\Delta y} + A\ \frac{V_{i,j}}{2}\right) V_{i,j-1,n}$$

$$+ \ \Delta y\ \frac{V_{i,j,n+1}}{\Delta T} \qquad (20)$$

For $i = 1$

$$4\left(P_{i,j+1} - P_{i,j}\right) = \left(2\Delta y/\Delta x^2\right)V_{2,j} - \left(2\Delta y/\Delta x^2 + 2B^2/\Delta y + \Delta y/\Delta T\right) V_{1,j}$$

$$+ \left(B^2/\Delta y - A\ V_{1,j}/2\right) V_{1,j+1} + \left(B^2/\Delta y + A\ V_{1,j}/2\right) V_{1,j-1}$$

$$+ \ \frac{V_{1,j}\ \Delta y}{\Delta T} \qquad (21)$$

These equations are used for check. The pressure at the interface should be almost the same value as in the gas phase, if the surface tension force is negligible small as compared to the pressure.

The pressure at top boundary from b to c (Fig. 3) is almost con-constant, if the distance from c to d is large enough, the equal pressure line will be flat near the center, so that the pressure at the top boundary can be set as a constant. For convenience, let it be zero. The interface pressure can be obtained by numerical integration of equation (20), and (21).

V. PROCEDURE OF SOLVING THE PROBLEM

1. Assign the boundary conditions for stream function, then assume a stream function distribution within the domain.

2. The boundary condition of vorticity equation is calculated as described in III B, then assumed a vorticity distribution within the domain.

3. Equation (16) is iterated by Gauss-Seidel method to obtain a new set of vorticity distribution.

4. Solve equation (15) by Gauss-Seidel iteration and get a new set distribution of stream function.

5. Repeat step 2, 3, and 4 until it gets converged.

6. Using the final converged stream function calculate velocity distribution by numerical differentiation.

7. Numerical integration equation (20) to get the pressure at the interface. If it is not the same pressure as in the vapor phase or is not equal everywhere, then another set of boundary condition is assumed and repeat the whole procedure again.

8. The parameters for setting the boundary conditions as stated in part III A are h, n, for the interface, C for the boundary b to c (Fig. 3), B' for the boundary c to d.

VI. DISCUSSION OF RESULT

In this thesis, the solution for several time steps have been solved by using IBM 7040 digital computer.

The parameters using for this calculation are as follows:

$A = 0.1 \qquad B = 0.025$

$\Delta T = 1.0$

$\Delta x = 0.1$

$\Delta y = 0.0333$

The function of pressure variation with respect to time is an exponential function.

$$p = 1 - e^{-0.5T}$$

A. Stream line pattern:

As presented in Figs. 6, 11, 12, and 13 for each time step, the stream line stretch outward near the interface. That indicates the interface will grow in the normal direction. Just a little distance from the interface the stream lines become vertical, this is the same situation as the flow of fluid in a cylindrical tube. They become curved again near the mouth of the tube, because of sudden enlargement.

B. Velocity distribution:

As shown in Fig. 7, the vertical component of velocity decrease a little bit from the interface, then holding constant through the most part of the tube and decrease again when approaching the mouth of the capillary. The distribution of velocity along the radial direction is parabola in the tube, it will become smaller and flatter after flow out of the tube.

The radial component of velocity will be seen in Fig. 8, it is very small at interface and it will decrease to zero near the middle part of the tube then increasing again near the mouth of the tube.

The variation of central line velocity with respect to time can be seen in Fig. 10. Because of pressure increasing, the flow is accelerated.

C. Pressure distribution:

The pressure drop is almost a straight line along the y direction within the tube (Fig. 9). The drop will decrease after the flow comes out of the tube.

D. The deformation and position of gas-liquid interface can be seen in Fig. 15. The interface is going up and becomes sharper, that is because the velocity on the central line is higher than that near the wall, the central part of the interface is moving faster than that near the wall.

The results are all presented in dimensionless form, in order to get some phsyical picture, an example is given as follows:

Let:

Y_1 = 0.001" L_2 = 0.04"

Take water for example:

$$\rho = 62.4 \text{ lb/ft}^3 \qquad \mu = 6.721 \times 10^{-4} \text{ lb/ft-sec}$$

$$A = \frac{\Delta p \ \rho r_1^{\ 4}}{4\mu^2 L_2^{\ 2}} = 0.1 \qquad B = \frac{r_1}{L_2} = 0.025$$

$$\Delta p = 0.126 \text{ psi}$$

$$\frac{\Delta p r_1^{\ 2}}{4\mu L_2} = 0.66 \text{ ft/sec}$$

if

$$V = 0.4$$

$$v_z = 0.4 \times 0.66 \text{ ft/sec} = 3.2 \text{ in/sec}$$

VII. SUGGESTIONS FOR FURTHER INVESTIGATION

1. The calculation is not successful when the interface comes near the mouth of capillary, that is because the top boundary is too near to the mouth. If the study of interface growing out of the capillary is of interest, the top boundary should be moved far away from the mouth. In order to save storage position and computing time, an exponential increasing of lattice space is suggested.

2. The situation of momentum transfer in the infinite pool is very similar to the case of the circular jet. The top boundary conditions can be approximated by the circular jet problem. For example, in this thesis, the top boundary conditions for the first time step is approximated by the equation

$$\Psi = Ae^{-B\, x^{C}}$$

where

$$A = \Psi_1 \qquad B = \frac{1}{x_c^{2.7}} \ln \frac{\Psi_1}{\Psi_2} \qquad C = 2.7$$

the distribution of stream function along this boundary for the circular jet is [3]

$$\Psi = \Psi_1 - \frac{(y - y_1)}{A} \frac{1}{\left(\frac{y - y_1}{Bx}\right)^2 + 0.25}$$

$$y = 0.1666$$

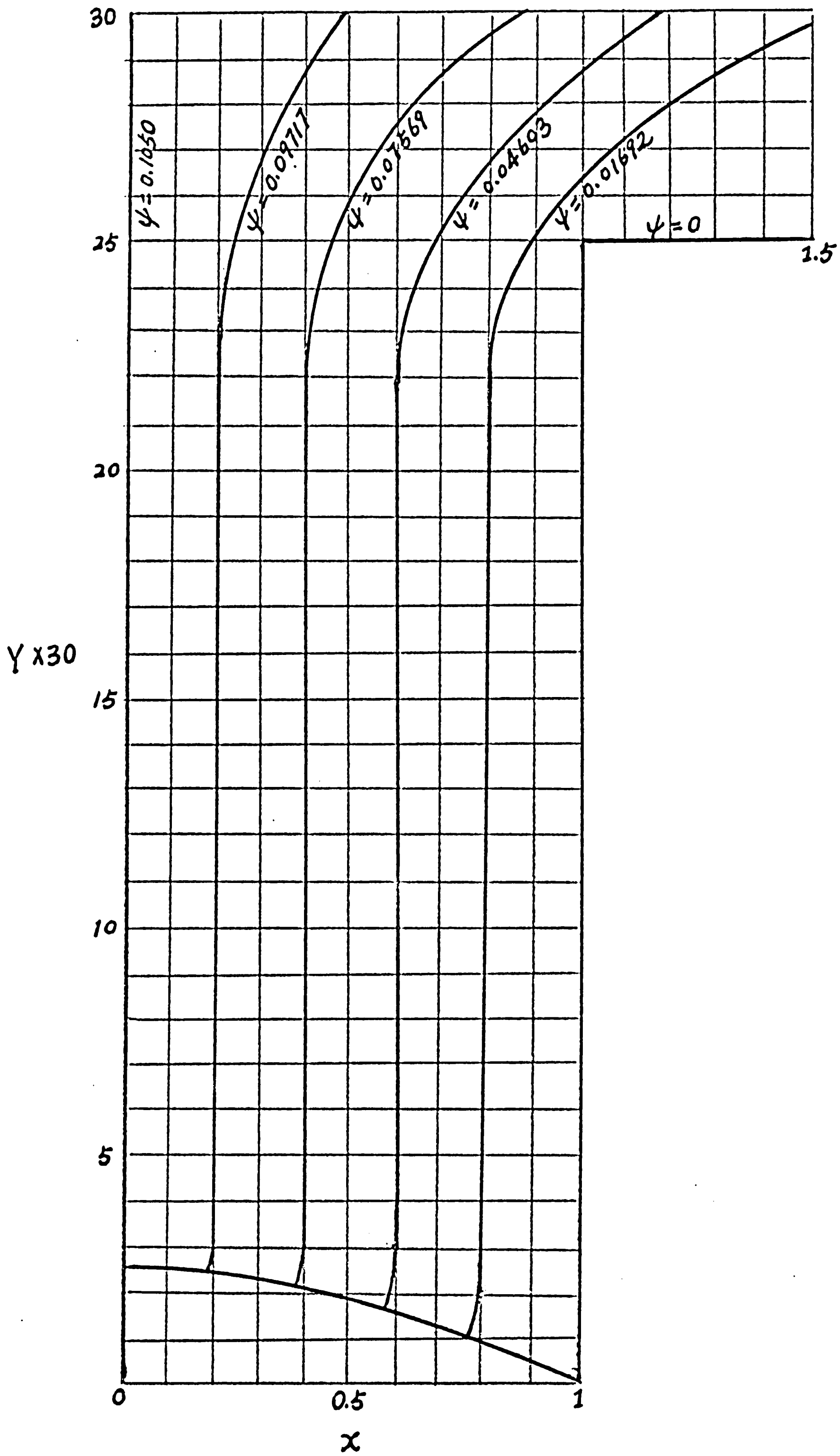
Both distributions are shown in Fig. 14, it is very similar. So this boundary condition can be approximated by the soltution of circular jet problem.

REFERENCES

[1] Transport Phenomena   Bird, Steward Lightfoot

[2] Numerical Mathematical Analysis   Scarborough

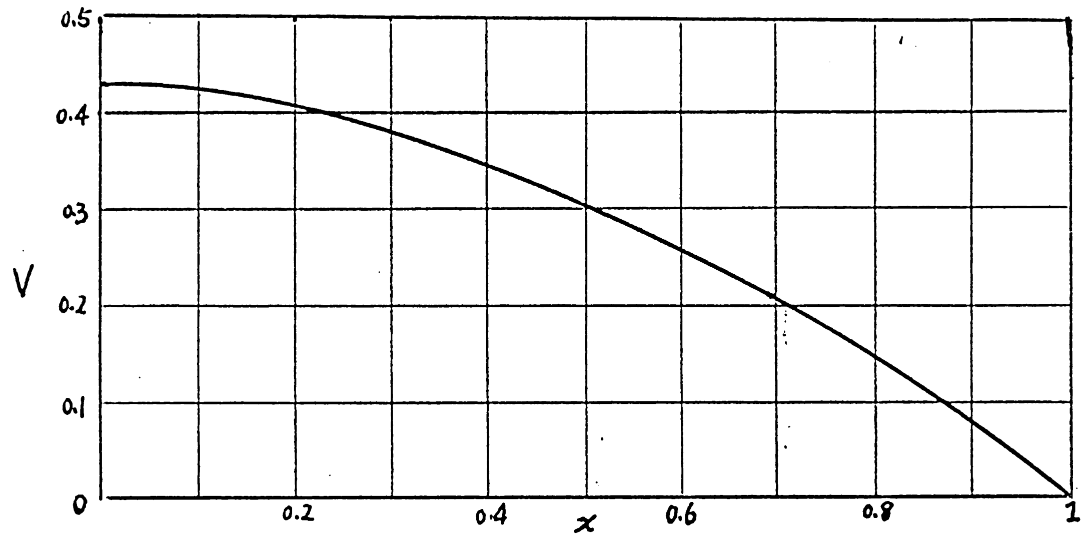
[3] Boundary Layer Theory   Schlichting



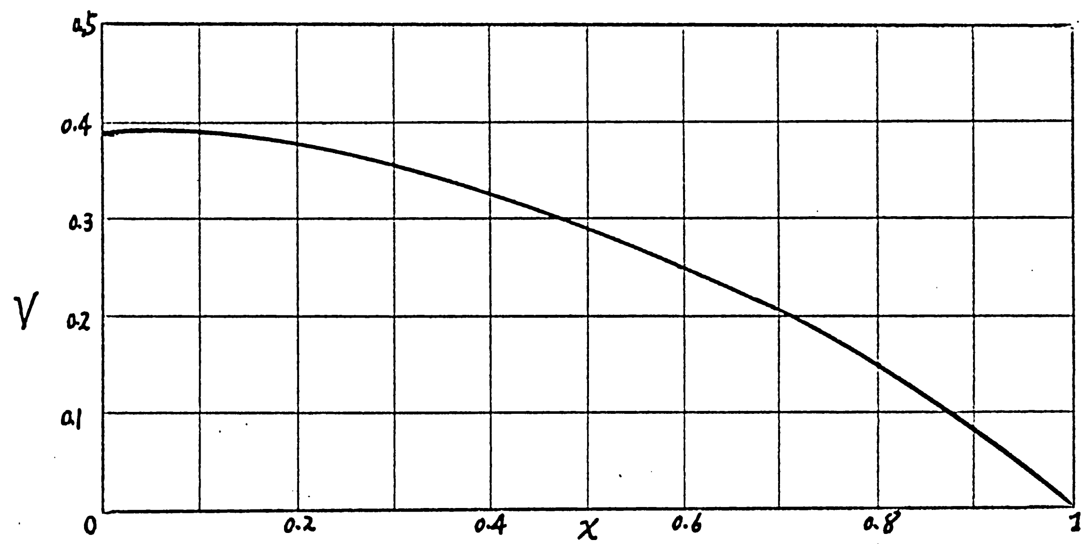
T = 1

P = 0.3935

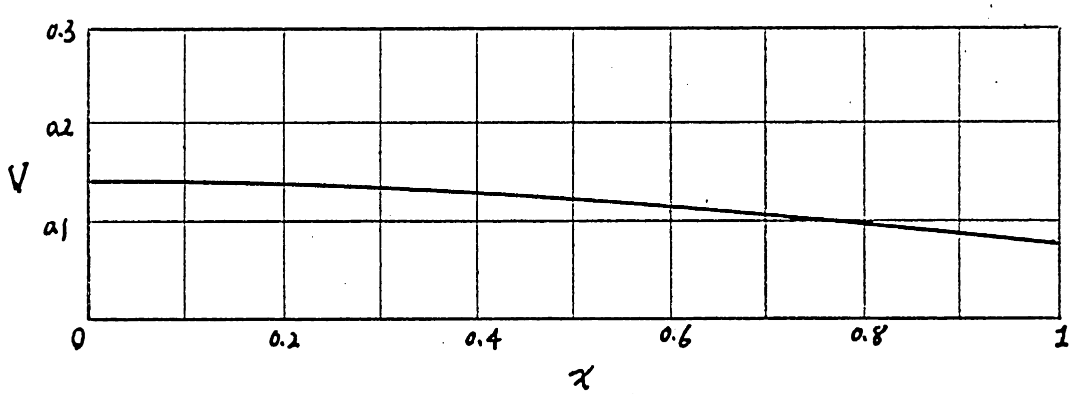
FIGURE 6. First time step stream line pattern



At interface



In the middle of the capillary



At y = 0.924

FIGURE 7. Velocity Distribution

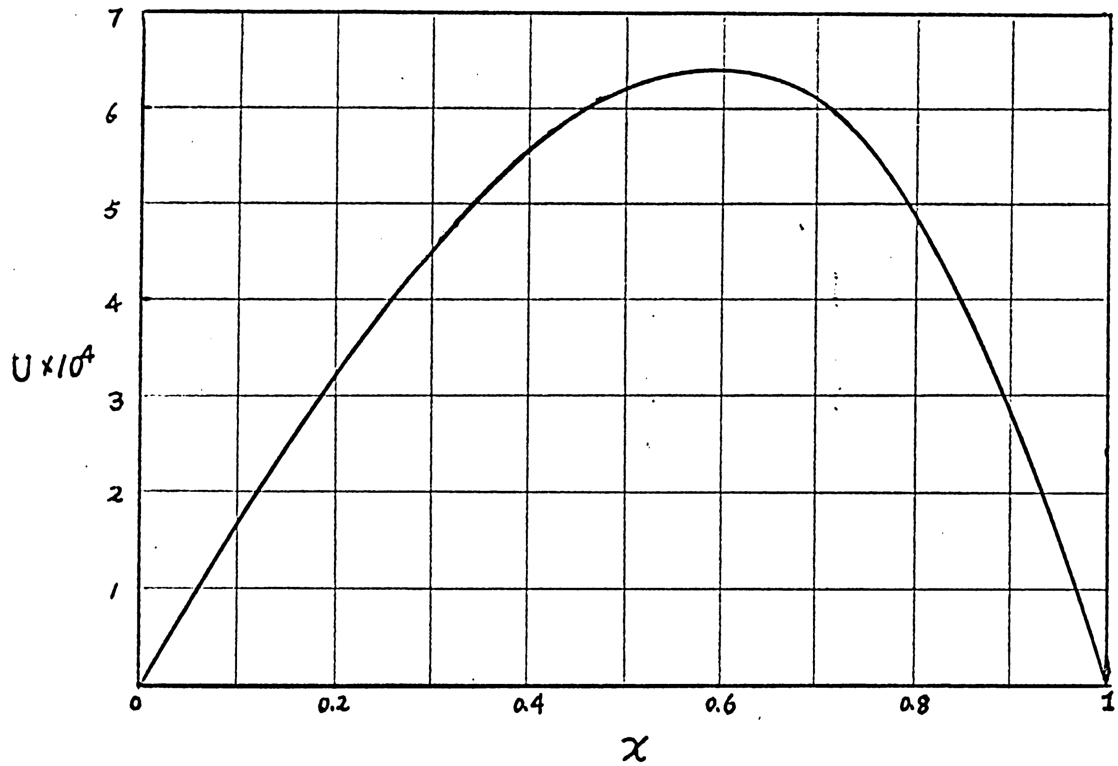


FIGURE 8. Radial velocity distribution at interface

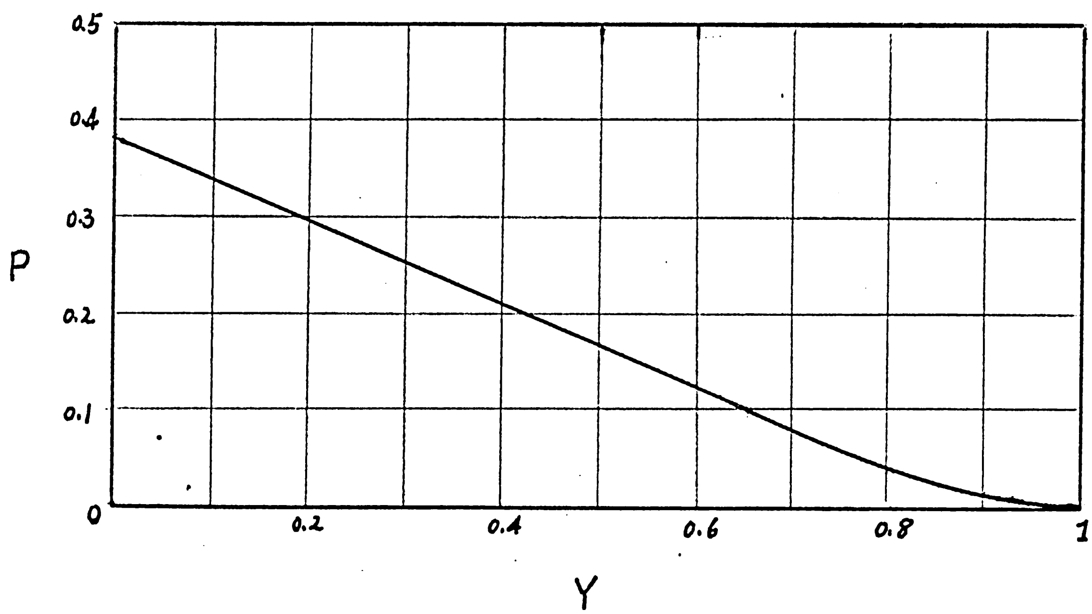


FIGURE 9. Pressure drop along the central line

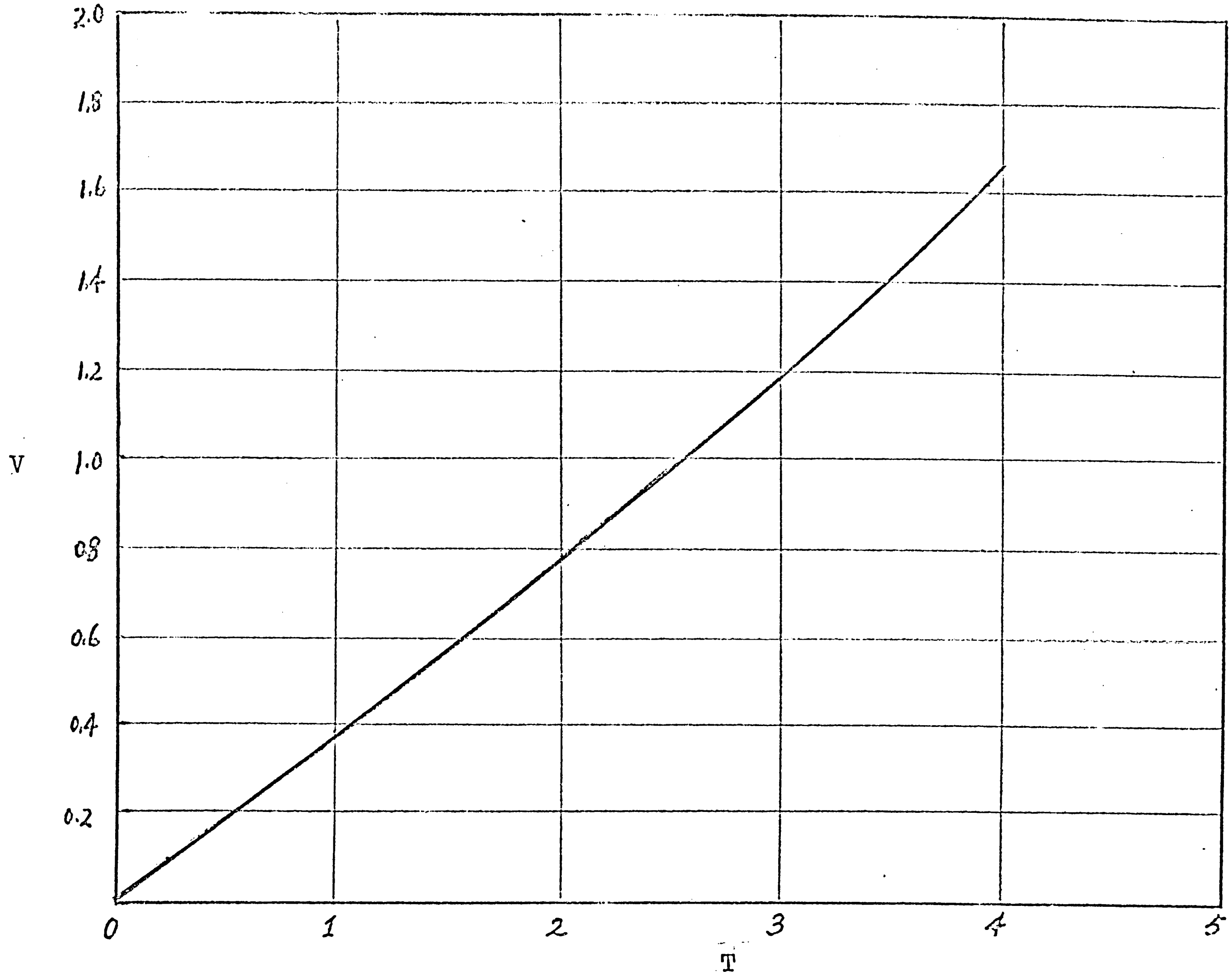


FIGURE 10. The Variation of Central Line Velocity With Respect to Time

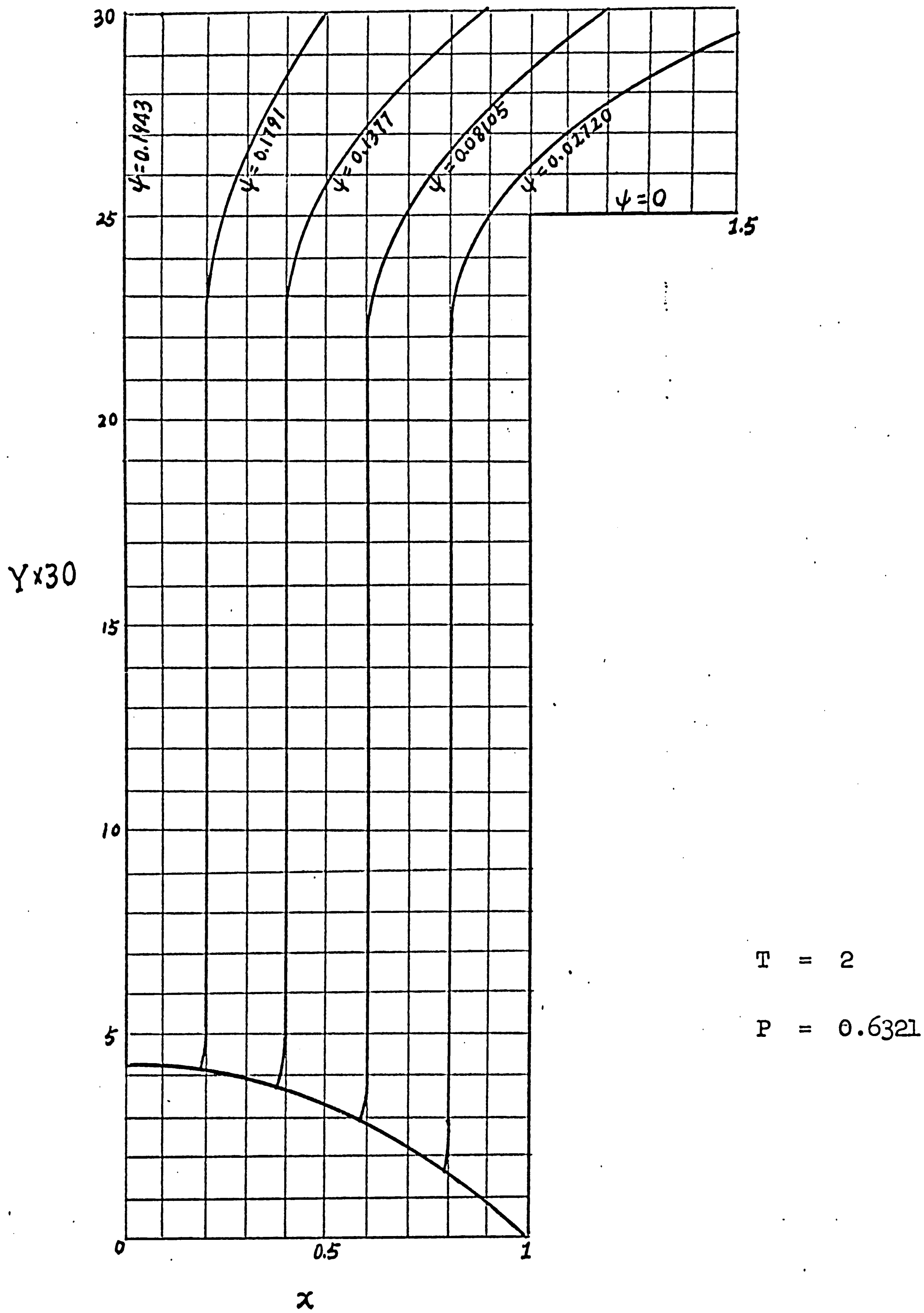


FIGURE 11. Second time step stream line pattern

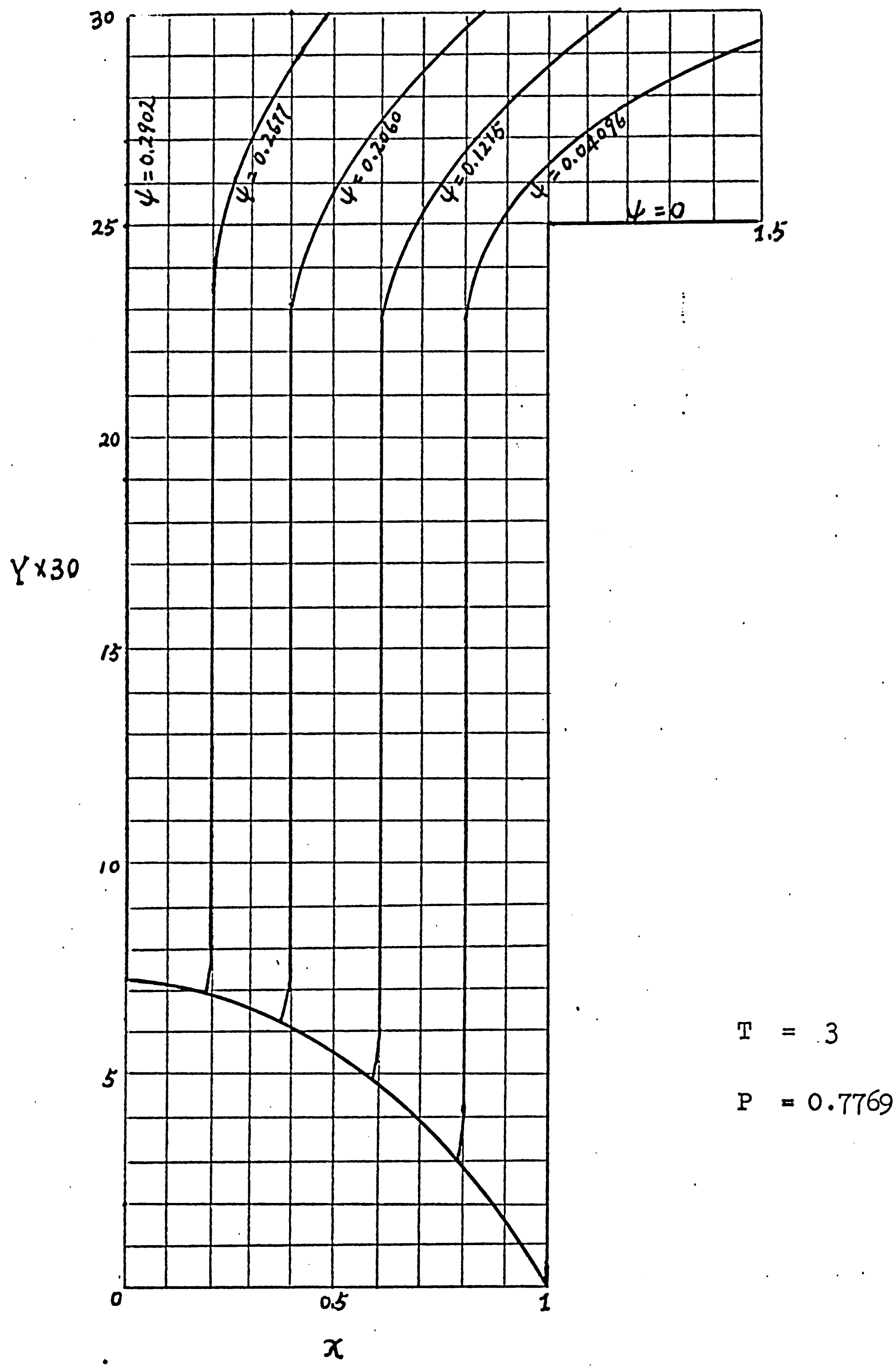
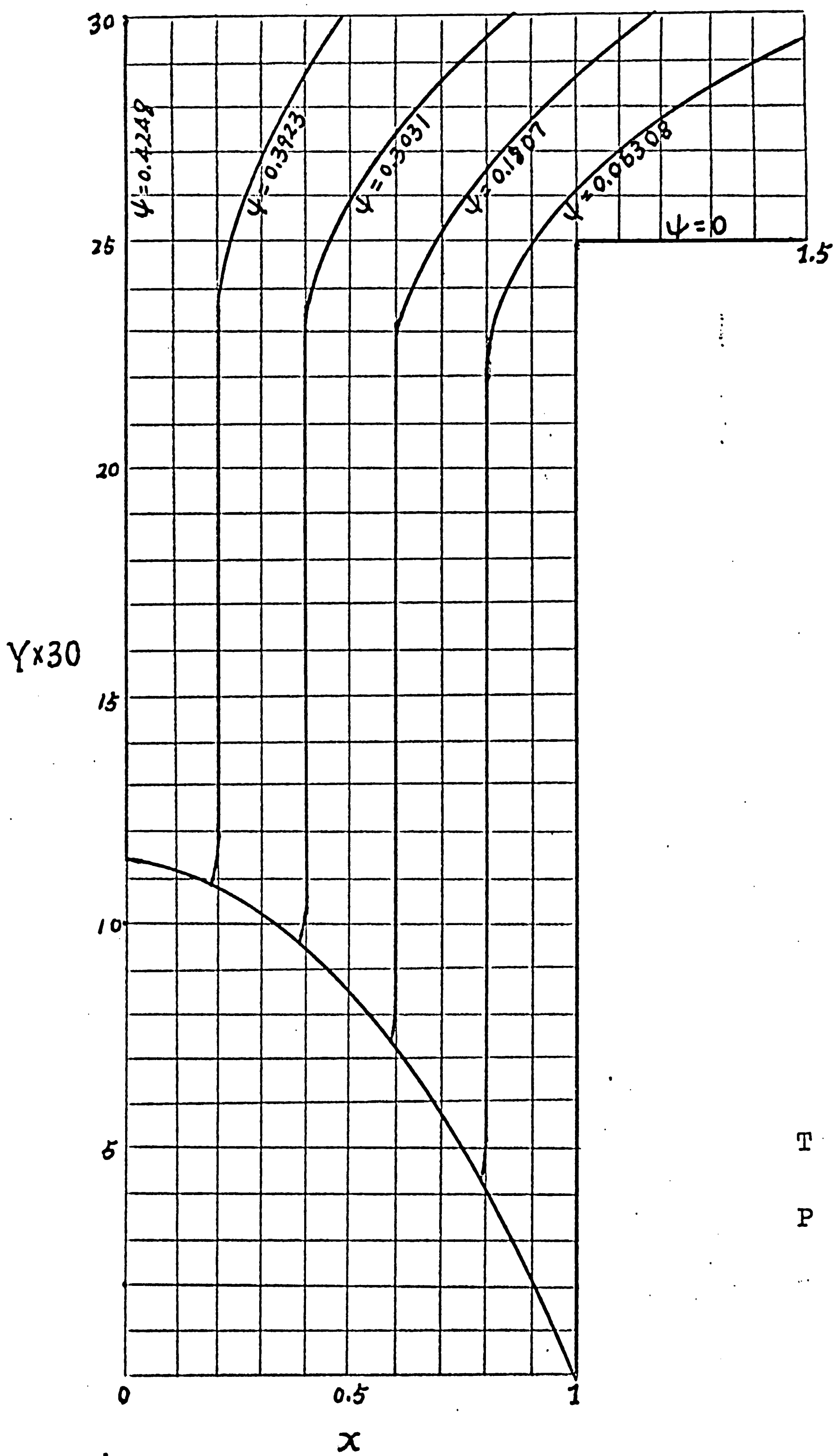


FIGURE 12. Third time step stream line pattern



T = 4

P = 0.8647

FIGURE 13. Fourth time step stream function pattern

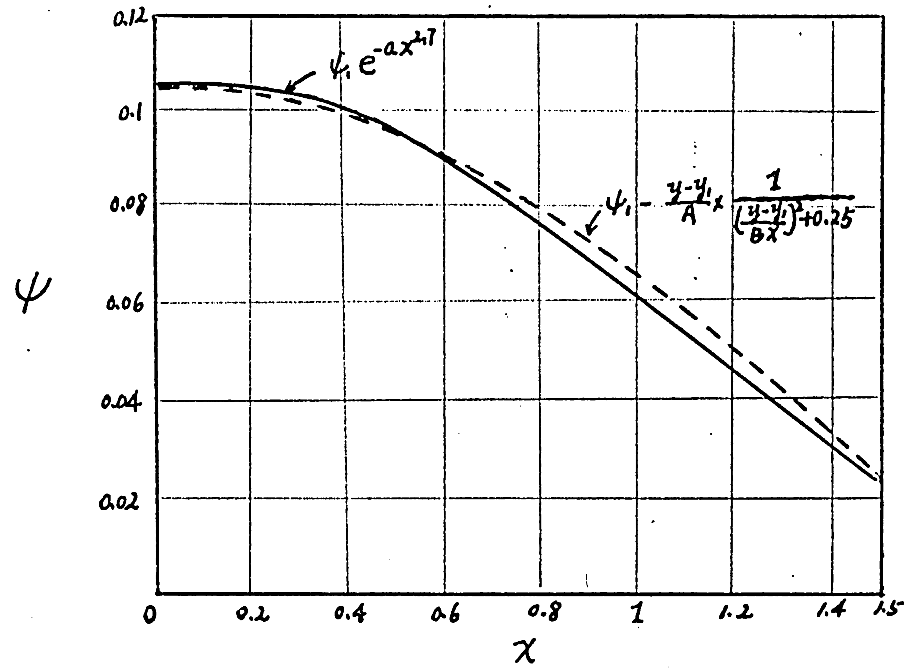


FIGURE 14. Boundary condition at the top boundary

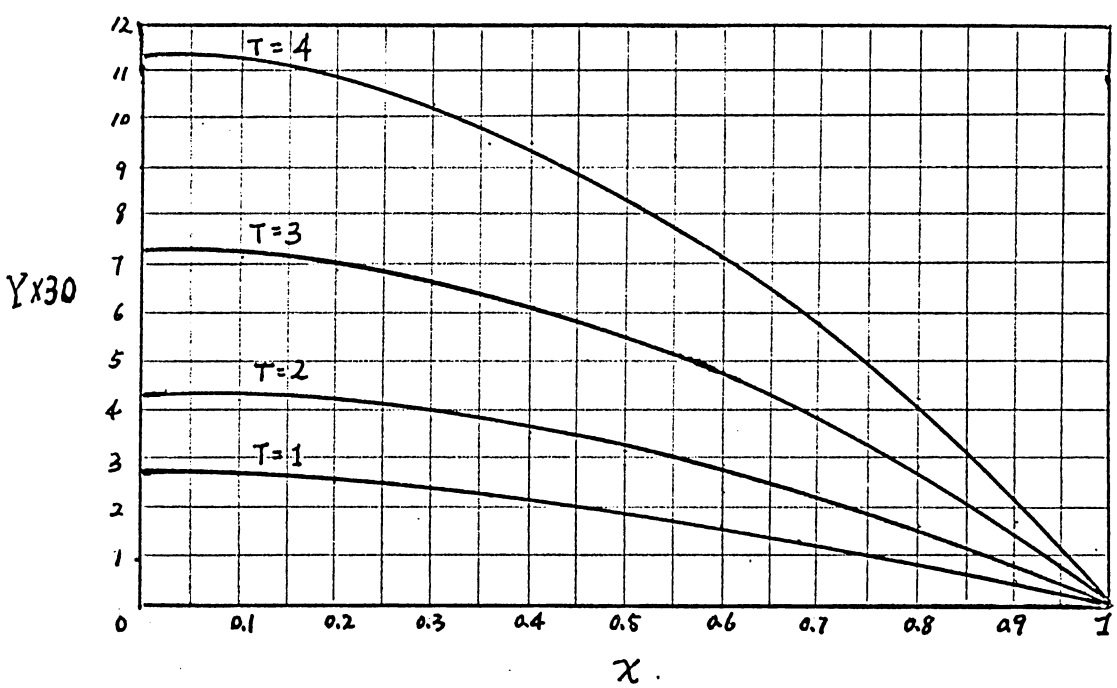


FIGURE 15. Deformation of liquid-gas interface