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TURBO-ALTERNATOR MODELING AND CONTROL

by

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Dedicated to my
Father and Mother
and
Brothers and Sisters
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This thesis is concerned with the problem of designing a constant coefficient feedback controller that will allow arbitrary pole placement of a single synchronous generator connected through a reactance to an infinite bus.

The nonlinear system model is derived and linearized. The controller regulates voltage, frequency and real power outputs of the synchronous machine. It is in cascade with the plant and uses voltage, frequency and real power deviations to generate the control signals. It has the following features, which others do not possess:

1) It is designed for arbitrary pole placement of the closed loop system.
2) It is a multi-input, multi-output controller and
3) It permits an algorithmic approach which can be completely computerized.
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Symbols

$\phi_{fd}(t)$ ... Field flux linkages

$\phi_{d}(t)$ ... direct axis flux linkages

$\phi_{q}(t)$ ... quadrature axis flux linkages

$L_{ffd}$ ... field self inductance

$L_{afd}$ ... Mutual inductance between stator and field

$L_{d}$ ... direct axis self inductance

$L_{q}$ ... quadrature axis self inductance

$i_{fd}(t)$ ... field current

$i_{d}(t)$ ... direct axis current

$i_{q}(t)$ ... quadrature axis current

$V_{fd}(t)$ ... field voltage

$V_{d}(t)$ ... direct axis generator terminal voltage

$V_{q}(t)$ ... quadrature axis generator terminal voltage
\[ R_{fd} \ldots \text{Resistance of field winding} \]

\[ \omega(t) \ldots \text{actual frequency} \]

\[ \omega_0 \ldots \text{synchronous frequency} \]

\[ V_{t}(t) \ldots \text{generator terminal voltage} \]

\[ V_B \ldots \text{Bus voltage} \]

\[ X_e \ldots \text{external reactance} \]

\[ \delta(t) \ldots \text{generator torque angle} \]

\[ T_m(t) \ldots \text{mechanical torque} \]

\[ T_{v}(t) \ldots \text{valve opening of the turbine} \]

\[ T_e(t) \ldots \text{electrical torque} \]

\[ T_t \ldots \text{turbine time constant} \]

\[ T_g \ldots \text{governor time constant} \]

\[ T_E \ldots \text{exciter time constant} \]

\[ T_A \ldots \text{standard voltage regulator time constant} \]
\[ T_R \] ... standard voltage regulator time constant

\[ K_t \] ... Turbine gain

\[ K_g \] ... governor gain

\[ K_E \] ... exciter gain

\[ K_A \] ... standard voltage regulator gain

\[ K_F \] ... standard voltage regulator gain

\[ V_R(t) \] ... standard voltage regulator state variable

\[ X_R(t) \] ... standard voltage regulator state variable

\[ U_1(t) \] ... input to the governor

\[ U_2(t) \] ... input to the standard voltage regulator

\[ P(t) \] ... real power

\[ Q(t) \] ... reactive power

\[ f(V_{rd}) \] ... saturation function

\[ R \] ... frequency feedback gain
\( M \) ... inertia constant

\( D \) ... rotor damping coefficient

\[ L_d' = L_d - \frac{L_{afd}^2}{L_{ffd}} \] is called direct axis transient inductance

\( X_d \) ... direct axis reactance

\( X_q \) ... quadrature axis reactance

\( X_d' \) ... direct axis transient reactance

\( \delta \) ... steady state Torque angle

\( T_m \) ... steady state mechanical torque

\( V_V \) ... steady state valve opening

\( \Phi_{fd} \) ... steady state field flux

\( V_{fd} \) ... steady state field voltage

\( V_R \) ... steady state standard voltage regulators state variable

\( \overline{X}_R \) ... steady state standard voltage regulators state variable

\( \overline{V}_t \) ... steady state generator terminal voltage
\( \mathbf{P} \) ... steady state generator real power

\( \mathbf{Q} \) ... steady state generator reactive power
1.1 Introduction

In a power system, the synchronous machine has to be controlled about certain desirable operating points. This is usually achieved by designing a feedback control system. Design of such controllers has been considered by many authors, however the design procedures in the past have been based essentially on frequency domain techniques and trial and error computer simulation. The object of this thesis is to apply some recently developed methods of designing dynamic controllers, using the powerful analytical approach of modern control theory.

The system considered is a single synchronous machine connected to a large system through a reactance and the object is to design a controller which uses frequency, terminal voltage and real power deviations to regulate frequency, terminal voltage and real power output.

1.2 Background

A conventional method of controlling frequency and voltage is shown schematically in Fig. 1. It consists principally in feeding back a frequency deviation signal to the governor and a terminal voltage deviation signal to a voltage regulator.
FIGURE 1

BLOCK DIAGRAM OF THE CONVENTIONAL METHOD USING STANDARD VOLTAGE REGULATOR AND FREQUENCY DEVIATION FEEDBACK
In 1944 Concordia [1] presented a method for predetermining the optimum regulator and excitation-system characteristics to give maximum gain in stability for a given synchronous machine.

In 1967 Schleif, Martin and Angell [2], suggested a special control (Fig. 2) derived from local frequency to improve system damping. It consisted of differentiating the local frequency and using the phase advance of the first derivative to offset inherent lags in the governor and turbine.

Byerly, Skooglund and Keay [3,4] suggested an additional feedback signal derived from real power superposed on the voltage regulator to achieve improved damping (Fig. 3).

Schleif [5,6] introduced a supplementary controller (Fig. 4), which in essence was driven by an error signal derived from frequency deviations, the output of which was used as an additional driving signal for the voltage regulator.

Concordia and DeMello [7] examined a wide range of system and machine parameters and arrived at a set of guide rules and recommendations for a transfer function characteristic for a machine speed derived signal superposed on the voltage regulator reference (Fig. 4). The above methods do not exploit the systematic analytical tools of modern control theory.

Recently in [8] it was shown that a single input, multi output, linear time-invariant system can be optimally controlled by feeding the system outputs back through a dynamic controller of appropriate order. The results were extended in [9] to multi-input,
Figure 2: Block diagram of the conventional method using an auxiliary signal derived from frequency deviation as suggested by Schleif in 1967.
Figure 3  Block diagram of the conventional method using an auxiliary signal derived from real power as suggested by Byerly, Skooklund and Keay
FIGURE 4  BLOCK DIAGRAM OF THE CONVENTIONAL METHOD USING AN AUXILIARY SIGNAL DERIVED FROM FREQUENCY DEVIATIONS AS SUGGESTED BY SCHLEIF IN 1968 & 1969 & CONCORDIA IN 1969
multi-output, linear, time-invariant systems.

In [10J the problem of designing a compensator using output feedback only to obtain arbitrary pole placement in the system consisting of the plant and compensator in cascade was considered. Such a design procedure has been applied in this thesis, to the case of a single synchronous machine connected through a reactance to a large system.

1.3 Outline of Thesis

This thesis consists of first modeling a single synchronous machine connected through a reactance to a large system in the form:

\[ x(t) = f_1(x(t), u(t)) \quad (1.3.1) \]
\[ V_t^2(t) = f_2(x(t)) \quad (1.3.2) \]
\[ P(t) = f_3(x(t)) \quad (1.3.3) \]
\[ Q(t) = f_4(x(t)) \quad (1.3.4) \]

where \( x(t) \) is an \( n \)-vector, the plant state and \( u(t) \) is an \( r \)-vector, the plant input.

It turns out that \( Q \) is fixed by \( V_t \) and \( P \). Thus the steady state values \( \bar{x} \) and \( \bar{u} \) are solved for different real power levels \( P \), for \( V_t = 1 \) p.u. and \( \omega_0 = 377 \) radians per second. Linearization of
equations (1.3.1) - (1.3.3) about $\bar{x}, \bar{u}$ yields the linearized perturbation model:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1.3.5)$$
$$y(t) = Cx(t) \quad (1.3.6)$$

where

- $A$ is an $n \times n$ constant matrix, the transition matrix
- $B$ is an $n \times r$ constant matrix, the input matrix
- $C$ is an $m \times n$ constant matrix, the output matrix

Using the linearized model and the results of [10] a compensator using output feedback (frequency, terminal voltage and real power) is designed to regulate the outputs. A comparison of this controller with the conventional voltage regulator and with the method suggested by DeMello and Concordia is also made.
CHAPTER 2

2.1 The System

The system (Fig. 5) considered here, is a synchronous machine connected through a reactance to a large system. The governor, turbine and exciter are treated as integral parts of this system.

2.2 Derivation of the System Equations

The state space approach has been chosen for the mathematical description of the model, because it lends itself readily to systematic analytical methods. The mathematical equations consist of the machine equations, power transfer equations relating the mechanical input power and the electrical output power, and the equations for the governor, turbine and exciter.

Assumptions

The following assumptions are made:

1. The stator windings are distributed to produce a sinusoidal magnetomotive force along the air gap.
2. Machine saturation is neglected.
3. Stator slots produce negligible variations in the rotor inductances.
4. Hysteresis is negligible.
FIGURE 5
BLOCK DIAGRAM OF THE OPEN LOOP SYSTEM
The armature voltages induced by the rate of change of flux linkages during rotor oscillation about synchronous speed are negligible compared to the voltages generated by the flux rotating at the synchronous speed.

The armature and external resistances are negligible.

The damper windings are also neglected.

**Nonlinear System Equations**

With the usual representation of the synchronous machine in terms of its direct- and quadrature-axis windings, and with the standard sign convention [12, 13, 14], the equations expressing the relationships between machine fluxes and currents can be expressed by

\[ \Phi_{fd}(t) = L_{ffd} i_{fd}(t) - L_{afd} i_d(t) \]  
\[ \Phi_d(t) = L_{afd} i_{fd}(t) - L_d i_d(t) \]  
\[ \Phi_q(t) = -L_q i_q(t) \]

and the equations relating machine voltages and currents are

\[ V_{fd}(t) = \frac{d}{dt} \Phi_{fd}(t) + R_{fd} i_{fd}(t) \]  
\[ V_d(t) = -\omega \Phi_q(t) \]
\[ V_q(t) = \omega(t) \phi_d(t) \quad (2.2.2c) \]

Assumption 5 implies that the terminal voltage can be considered as if in sinusoidal steady state and the relation between the terminal voltage d-q components and the bus voltage is

\[ V_d(t) = V_B \sin \delta(t) - x_e i_q(t) \quad (2.2.3a) \]
\[ V_q(t) = V_B \cos \delta(t) + x_e i_d(t) \quad (2.2.3b) \]

Using equations (2.2.1a) through (2.2.3b) one easily obtains:

\[ \phi_{fd}(t) = -\frac{R_{fd}}{L_{ffd}} \frac{L_d}{L_d} \phi_{fd}(t) + \frac{R_{fd}}{L_{ffd}} \frac{L_{afd}}{L_d} \phi_d(t) + V_{fd}(t) \quad (2.2.4a) \]

\[ \phi_d(t) = \frac{x_e}{x_e + x_d} \frac{L_{afd}}{L_{ffd}} \phi_{fd}(t) + \frac{V_B \cos \delta(t)}{x_e + x_d} \frac{L_d}{(x_e + x_d)} \quad (2.2.4b) \]

\[ \phi_q(t) = -\frac{V_B \sin \delta(t)}{x_e + x_q} \frac{L_g}{(x_e + x_q)} \quad (2.2.4c) \]

The constraints on the change of rotor position, due to the machine inertia and the system damping is expressed as
\[ M \ddot{\delta}(t) + D \dot{\delta}(t) = T_m - T_e \quad (2.2.5) \]

where \( \dot{\delta}(t) = \omega(t) - \omega_0 \quad (2.2.6) \)

and the electrical torque is

\[ T_e(t) = i_q(t) \phi_d(t) - i_d(t) \phi_q(t) \quad (2.2.7) \]

which, with equations (2.2.1a), (2.2.1b), (2.2.1c), (2.2.4b) and (2.2.4c)

can be expressed as

\[ T_e(t) = \frac{V_B L_{afd}}{L_{ffd}(x_e + x_d)} \phi_{fd} \sin \delta(t) + \frac{V_B^2 (x_d' - x_q)}{2 (x_e + x_d')(x_e + x_q)} \sin 2 \delta(t) \quad (2.2.8) \]

**Governor, Turbine and Standard Voltage Regulator**

The governor and turbine (Fig. 6) are in cascade. It is assumed that the governor controls the steam valve position of the turbine and the operation of the governor is such that the steam admitted to the turbine is a linear function of the speed of the turbine throughout the governor's working range. If constant steam pressure is assumed, the necessity of including any boiler representation is eliminated. This as a reasonable assumption for small variations in load and frequency and for the study of rapid electro-
FIGURE 6  BLOCK DIAGRAM OF THE GOVERNOR AND TURBINE IN CASCADE
mechanical transients. The differential equations describing the turbine and governor are

\[ T_m(t) = - \frac{T_m(t)}{t} + \frac{K_t}{T_t} T_v(t) \quad (2.2.9) \]

\[ T_v(t) = - \frac{T_v(t)}{T_g} + \frac{K_E}{T_g} U_1(t) \quad (2.2.10) \]

The standard voltage regulator [11] used (Fig. 7) is described by the following differential equations.

\[ \dot{V}_{fd}(t) = - \frac{f(V_{fd}) + K_E}{T_E} V_{fd}(t) + \frac{1}{T_E} V_R(t) \quad (2.2.11) \]

\[ \dot{V}_R(t) = - \frac{1}{T_A} V_R(t) - \frac{K_A}{T_A} X_F(t) + \frac{K_A}{T_A} U_2(t) \quad (2.2.12) \]

\[ \dot{X}_F(t) = - \frac{K_F K_R + K_F f(V_{fd})}{T_E T_F} V_{fd}(t) + \frac{K_F}{T_E T_F} V_R(t) - \frac{1}{T_F} X_F(t) \quad (2.2.13) \]
Figure 7: Block Diagram of a Standard Voltage Regulator.

SAT. FUNCTION
\[ SE = f(V_{fd}) \]

\[ \frac{1}{K_{E} + T_{ES}} \]

\[ T_{AS} + 1 \]

\[ SKF \]

\[ T_{FS} + 1 \]

\[ X_{F} \]

\[ U_{2} \]

\[ V_{fd} \]
2.3 Terminal Voltage, Real and Reactive Power in terms of Field Flux, Torque Angle and Frequency

The terminal voltage of the synchronous machine in terms of its d-q components can be expressed as

\[ v_t(t)^2 = v_d(t)^2 + v_q(t)^2 \]  \hspace{1cm} (2.3.1)

and with equations (2.2.2b), (2.2.2c), (2.2.4a) and (2.2.4b) one obtains

\[ v_t(t)^2 = \left[ \frac{x_e \omega(t) L_{afd}}{L_{ffd}(x_e + x_d')} \right]^2 \phi_{fd}(t)^2 + \left[ \frac{v_B x_d'}{x_e + x_d} \right]^2 \cos^2 \delta(t) \]

\[ + \frac{2 x_e \omega(t) L_{afd} x_d'}{L_{ffd}(x_e + x_d')^2} \phi_{fd}(t) \sin \delta(t) \]

\[ + \left[ \frac{v_B x_q}{x_e + x_q} \right]^2 \sin^2 \delta(t) \]  \hspace{1cm} (2.3.2)

The real power delivered to the infinite bus is

\[ P(t) = i_d(t) v_d(t) + i_q(t) v_q(t) \]  \hspace{1cm} (2.3.3)
With equations (2.2.1a), (2.2.1b), (2.2.1c), (2.2.2a), (2.2.2b), (2.2.2c), (2.2.4b) and (2.2.4c) one obtains

\[ P(t) = \frac{\omega(t) v_B L_{afd}}{L_{fd}(x + x_d^{'})} \phi_{fd}(t) \ \sin \delta(t) \]

\[ + \frac{v_B^2 (x_d^{'1} - x_q)}{2(x_e + x_d^{'1})(x_e + x_q)} \ \sin 2 \delta(t) \]

\[ (2.3.4) \]

The reactive power is expressed by

\[ Q(t) = i_d(t) v_q(t) - i_q(t) v_d(t) \]

\[ (2.3.5) \]
With equations (2.2.1a), (2.2.1b), (2.2.1c), (2.2.2a), (2.2.2b), (2.2.2c), (2.2.4b) and (2.2.4c) one obtains

\[ Q(t) = \frac{L_{afd}(X_d - X_e)}{L_{ffd}(X_d^1 + X_e^1)^2} \cdot V_B \cdot \omega (t) \cdot \phi_{fd}(t) \cdot \cos \delta(t) \]

\[ + \frac{L_{afd}^2 X_e}{L_{ffd}^2(X_e + X_d^1)^2} \cdot \omega^2(t) \cdot \phi_{fd}^2(t) \]

\[ - \left[ \frac{X_d^1}{(X_e + X_d^1)^2} + \frac{X_q^1}{(X_e + X_q)^2} \right] \cdot \frac{V_B^2}{2} \]

\[ + \left[ \frac{X_q^1}{(X_e + X_q)^2} - \frac{X_d^1}{(X_e + X_d^1)^2} \right] \cdot \frac{V_B^2}{2} \cdot \cos 2 \delta(t) \]

(2.3.6)

2.4 Summary of power plant equations

Using the equations of section 2.2 and section 2.3 one can summarize the power plant equations (Fig. 5)

\[ \dot{\delta}(t) = \omega(t) - \omega_0 \]
\[
\dot{\omega}(t) = -\frac{D}{M}(\omega - \omega_0) - \frac{V_B L_{afd}}{M L_{ffd}(x_e + x'_d)} \phi_{fd}(t) \sin \theta(t)
\]

\[
- \frac{v_B^2 (x'_d - x_q)}{2M \omega(t)(x_e + x'_d)(x_e + x_q)} \sin 2 \theta(t) + \frac{T_m(t)}{M}
\]

\[
T_m(t) = - \frac{T_m(t)}{T_t} + \frac{K_t}{T_t} T_v(t)
\]

\[
T_v(t) = - \frac{T_v(t)}{T_g} - \frac{1}{T_{gR}} (\omega(t) - \omega_0) + \frac{K_e}{T_g} u_1(t)
\]

\[
\phi_{fd}(t) = - \frac{R_{fd}(x_e + x_d)}{L_{ffd}(x_e + x'_d)} \phi_{fd}(t) + \frac{V_B L_{afd} R_{fd}}{L_{ffd}(x_e + x'_d)} \cos \theta(t) + v_{fd}(t)
\]

\[
v_{fd}(t) = - \frac{f(V_{fd}) + K_e}{T_E} v_{fd}(t) + \frac{V_R(t)}{T_E}
\]

\[
v_{R}(t) = - \frac{1}{T_A} v_{R}(t) - \frac{K_A}{T_A} x_p(t) + \frac{K_A}{T_A} u_2(t)
\]

\[
x_p(t) = - \frac{(K_{FE} + K_F f(V_{fd}))}{T_{EF} T_F} v_{fd}(t) + \frac{K_F}{T_E T_F} v_{R}(t) - \frac{1}{T_F} x_p(t)
\]
\[ v^2_t(t) = \left( \frac{x_e \omega(t) L_{af}}{L_{rfd}(X_e + X_d')} \right)^2 \phi_{fd}^2(t) + \left( \frac{V_B X_d'}{X_e + X_d} \right)^2 \cos^2 \delta(t) \]

\[ + \frac{2X_e \omega(t) L_{af} X_d V_B}{L_{rfd}(X_e + X_d')^2} \phi_{fd}(t) \cos \delta(t) \]

\[ + \left[ \frac{V_B X_q}{X_e + X_q} \right]^2 \sin^2 \delta(t) \]

\[ P(t) = \frac{\omega(t) V_B L_{af}}{L_{rfd}(X_e + X_d')} \phi_{fd}(t) \sin \delta(t) + \frac{V_B^2(X_d' - X_q)}{2(X_e + X_d')(X_e + X_q)} \sin 2 \delta(t) \]

\[ Q(t) = \frac{L_{af}(X_d' - X_e) V_B}{L_{rfd}(X_d' + X_e)^2} \omega(t) \phi_{fd}(t) \cos \delta(t) \]

\[ + \frac{L_{af} X_e}{L_{rfd}(X_e + X_d')^2} \cos^2(t) \phi_{fd}^2(t) - \left[ \frac{X_d'}{(X_e + X_d')^2} + \frac{X_q}{(X_e + X_q)^2} \right] \frac{V_B^2}{2} \]

\[ + \left[ \frac{X_q}{(X_e + X_q)^2} - \frac{X_d'}{(X_e + X_d')^2} \right] \frac{V_B^2}{2} \cos 2 \delta(t) \]

The correct units are listed in Appendix I.
2.5 Conversion to the Per-Unit System

The machine parameters are almost universally in per-unit. Thus it seems worthwhile to express the previously derived equations in per-unit quantities. With the choice of the base quantities as in Appendix I, the equations of section 2.4 can be written as:

\[ \delta(t) = \omega(t) - \omega_0 \]  
\[ \dot{\omega}(t) = -\frac{D}{M} (\omega(t) - \omega_0) - \frac{V_B L_{afd} \phi_{fd}(t) \sin \delta(t)}{M L_{fd}(X_e + X_d)} \]  
\[ -\frac{V_B^2 \omega_0 (X_d' - X_q)}{2M \omega(t)(X_e + X_d')(X_e + X_q)} \sin 2 \delta(t) + \frac{T_m(t)}{M} \]  
\[ \dot{T}_m(t) = -\frac{T_m(t)}{T_t} + \frac{K_t}{T_t} T_v(t) \]  
\[ \dot{T}_v(t) = -\frac{T_v(t)}{T_g} - \frac{1}{T_{gR}} (\omega(t) - \omega_0) + \frac{K_e}{T_{ge}} U(t) \]
\[ \dot{\phi}_{fd}(t) = -\omega_0 \frac{R_{fd}}{L_{ffd}} (x_e + x_d') \phi_{fd}(t) + \frac{\omega_0 V_{B} L_{afd} R_{fd}}{L_{ffd}} \cos \delta(t) \]

\[ + \omega_0 V_{fd}(t) \quad (2.5.5) \]

\[ \dot{V}_{fd}(t) = -\frac{(f(V_{fd}) + K_E)}{T_E} V_{fd}(t) + \frac{V_{R}(t)}{T_E} \quad (2.5.6) \]

\[ \dot{V}_{R}(t) = -\frac{V_{R}(t)}{T_A} - \frac{K_A}{T_A} x_F(t) + \frac{K_A}{T_A} U_2(t) \quad (2.5.7) \]

\[ \dot{x}_F(t) = -\frac{(K_E K_F + K_F f(V_{fd}))}{T_E T_F} V_{fd}(t) + \frac{K_F}{T_E T_F} V_{R}(t) - \frac{x_F(t)}{T_F} \quad (2.5.8) \]

\[ V_{t}^2(t) = \left[ \frac{x_e \omega(t) L_{afd}}{\omega_0 L_{ffd} (x_e + x_d')} \right]^2 \phi_{fd}(t) + \left[ \frac{V_{B} X_d'}{x_e + x_d'} \right]^2 \cos^2 \delta(t) \]

\[ + \frac{2 x_e \omega(t) L_{afd} x_d' V_B}{\omega_0 L_{ffd} (x_e + x_d')^2} \phi_{fd}(t) \cos \delta(t) \]

\[ + \left[ \frac{V_B X_q}{x_e + x_q} \right]^2 \sin^2 \delta(t) \quad (2.5.9) \]
\[ P(t) = \frac{\omega(t) V_B L_{afd}}{\omega_0 I_{rfd}(X_e + X_d)} \phi_{fd}(t) \sin \delta(t) \]

\[ + \frac{V_B^2(x_d' - x_q)}{2(x_e + x_d')(X_e + X_d)} \sin 2 \delta(t) \]

\[ Q(t) = \frac{L_{afd}^2 x_e \omega^2(t) \phi_{fd}^2(t)}{L_{rfd}^2 (X_d' + X_e)^2 \omega_0^2} + \left[ \frac{x_q}{(X_e + X_d)^2} - \frac{x_d'}{(X_e + x_d')^2} \right] V_B^2 \cos 2 \delta(t) \]

\[ + \frac{L_{afd} (x_d' - x_e) V_B \omega(t)}{L_{rfd} (X_d' + X_e)^2 \omega_0} \phi_{fd}(t) \cos \delta(t) \]

\[ - \left[ \frac{x_d'}{(X_e + x_d')^2} + \frac{x_q}{(X_e + X_d)^2} \right] \frac{V_B^2}{2} \]

(2.5.10)

(2.5.11)

2.6 Evaluation of the Steady State operation points

The steady state operating points \( \delta, \omega, \overline{T_m}, \overline{T_v}, \phi_{fd} \), \( \overline{V}_{fd}, \overline{X}_F, \overline{U}_1 \) and \( \overline{U}_2 \) depend on the terminal voltage, the real power level and the synchronous frequency. Setting the derivatives of the dynamical equations equal to zero yields eight algebraic equations, which together with the equations for the terminal voltage and real...
power can be solved to give the steady state operating points.

The program to evaluate these points is in Appendix II.

2.7 Linearized Perturbation Model

If a Taylor-series expansion is formed about any operating point for the equations 2.5.1 through 2.5.10, the resulting linear perturbation relationships of the first order terms are expressed as follows:

\[ X(t) = A X(t) + B U(t) \]
\[ Y(t) = C X(t) \]

where

\[
X(t) = \begin{bmatrix}
\Delta \delta(t) \\
\Delta \omega(t) \\
\Delta T_m(t) \\
\Delta T_v(t) \\
\Delta \phi_{fd}(t) \\
\Delta V_{fd}(t) \\
\Delta V_R(t) \\
\Delta X_F(t)
\end{bmatrix}
\]

\[
Y(t) = \begin{bmatrix}
\Delta \omega(t) \\
\Delta V(t) \\
\Delta P(t)
\end{bmatrix}
\]

\[
U(t) = \begin{bmatrix}
\Delta U_1(t) \\
\Delta U_2(t)
\end{bmatrix}
\]

The coefficients of the A, B and C matrices are in Appendix III. The corresponding A, B and C matrices for real power levels ranging from 0.5 to 1.4 per unit are evaluated and shown in Appendix III.
CHAPTER 3

Design and Performance of the Controller

In this chapter the results of [10] are applied to the problem of controller design for the system under consideration.

3.1 Background and Notation

Consider the controllable, observable, linear, time-invariant, real system

\[ X(t) = A X(t) + B U(t) \]
\[ Y(t) = C X(t) \]

where

- \( X(t) \) is an \( n \) - vector, the plant state
- \( U(t) \) is an \( r \) - vector, the control vector
- \( Y(t) \) is an \( m \) - vector of measurable outputs
- \( A \) is an \( n \times n \) constant matrix, the transition matrix
- \( B \) is an \( n \times r \) constant matrix, the input matrix and
  \[ \text{rank} (B) = r \]
- \( C \) is an \( m \times n \) constant matrix, the output matrix and
  \[ \text{rank} (C) = m \]

Let \( A \) be cyclic (This is no restriction, because if \( A \) is not cyclic, then there exists an \( r \times m \) matrix \( Q \) such that \( (A+BQC) \) is cyclic [10]).
Define \( P_0 \) and \( P_c \) to be the smallest integers such that

\[
\text{rank } \begin{bmatrix} B, AB, A^2B, \ldots, A^{P_c}B \end{bmatrix} = n
\]

\[
\text{rank } \begin{bmatrix} C^T, A^TC^T, \ldots, (A^T)^{P_c}C^T \end{bmatrix} = n
\]

Then the order of the compensator to achieve arbitrary pole-placement of the closed loop system consisting of the plant and compensator in cascade is

\[
P = \min (P_0, P_c)
\]

and there exists an \((r+p) \times (m+p)\) matrix \( K \)

\[
K = \begin{bmatrix}
    K_1 (r \times m) & K_2 (r \times p) \\
    \hline
    K_3 (p \times m) & K_4 (p \times p)
\end{bmatrix}
\]

such that the poles \( \Lambda = \{\lambda_i\} \) \( i = 1, 2, \ldots, n+p \) of the closed loop system

\[
\dot{\bar{X}}(t) = \bar{A} \bar{X}(t)
\]

can be placed arbitrarily.
Here:

\[ \bar{X}(t) = \frac{X(t)}{\dot{X}(t)} \quad \text{and} \quad \hat{x}(t) = \begin{bmatrix} \hat{x}_1(t) \\ \hat{x}_2(t) \\ \vdots \\ \hat{x}_p(t) \end{bmatrix} = \text{The state vector of the controller} \]

\[ \bar{A} = \hat{A} + B \hat{K} \]

\[ \hat{A} = \begin{bmatrix} A \ (n \times n) & 0 \\ \hline \hline K_3 \ C & K_4 \\ (p \times m) \times (m \times n) & (p \times p) \end{bmatrix} \]

\[ \hat{B} = \begin{bmatrix} B \\ \hline 0 \\ \hline (p \times r) \end{bmatrix} \]

and

\[ \hat{K} = \begin{bmatrix} K_1 \ C & K_2 \\ (r \times m) \times (m \times n) & (r \times p) \end{bmatrix} \]
Furthermore, any controllable, observable system may be made controllable (observable) from a single input (output) by incorporating output feedback. In other words, given a controllable, observable system \((A, B, C)\) there is a matrix \(Q\), a vector \(b\) in Range \((B)\) and a vector \(c\) in Range \((C^T)\) such that \((A + BQC, b, c^T)\) is controllable and observable. If \(A\) is cyclic, then \(Q = 0\) and the plant is controllable (observable) from a linear combination of the inputs (outputs). Once \(Q\) is known, \(K\) can be computed.

The structure of the compensator is shown in Fig. 8. Here the \(K_{ij}^{th}\) element corresponds to the gain between the \(j^{th}\) output and the \(i^{th}\) input, where \(i = 1, 2, \ldots, r, r + 1, \ldots r + p\) and \(j = 1, 2, \ldots m, m + 1, \ldots m + p\).

The computer program used to compute the \(K\) matrix was written by F.M. Brasch. Given \(A, B\) and \(C\) and \(\Lambda\), where

\[
\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_n + p\}
\]

subject to the constraint that \(\lambda_i\) with \(\text{Im} \lambda_i \neq 0\) appears as one of a complex conjugate pair, the program computes the \(K\) matrix to meet the specifications.
FIGURE 8 BLOCK DIAGRAM ILLUSTRATING HOW THE CONTROLLER (REF. 10) IS CONNECTED TO THE PLANT
The background and notation has been stated in section 3.1 and the data for the synchronous machine, turbine, governor and standard voltage regulator along with the A, B, and C matrices for $P = 0.5$ to $P = 1.4$ is listed in Appendix IV.

It is true that one can place the poles of the closed loop system arbitrarily [10], but the question is: where should one place them? This of course depends on the customer, but it is reasonable to assume that one would demand stable operation for the range of expected power levels, and for that matter, not even allow the poles to come anywhere close to the imaginary axis for the range of expected power levels.

It would be very expensive and unreasonable to design different compensators for different power levels. It is desirable to build a compensator that will allow a good response over the range of expected power levels.

Unfortunately one cannot exactly tell in which direction the poles will move when the machine operates at a power level other than the one for which it was designed. In fact it is feasible and possible for the closed loop system to be unstable at power levels other than for which the compensator is designed. Thus the procedure followed is to design a compensator for a certain power level and then test if the performance is considered good for all other expected power levels.
Example 1

The compensator has been designed for $P = 1.0$.

The open loop poles of the system under consideration at $P = 1.0$ are

$$A_{OL} = \{-18.83, -4.33 + j 1.418, -2.57, -0.411 + j 8.41, -0.288, -0.205\}$$

The closed loop poles are chosen only slightly different from the open loop poles, with the intention of keeping the sensitivity low.

$$A_{CL} = \{-18.83, -4.33 + j 1.418, -2.57, -2.0, -1.6, -1.3, -1.0, -0.7, -0.6 + j 7.0\}$$

The compensator (Fig. 9) uses deviations ($\Delta \omega, \Delta V_t, \Delta P$) from a desired level of operation ($\omega_o, V_{to}, P_o$) to generate control signals $\Delta U_1(t)$ and $\Delta U_2(t)$ which are superposed on $U_1$ (ref.) and $U_2$ (ref.) with the object of maintaining the desired level of operation ($\omega_o, V_{to}, P_o$).

It turns out that $A$ is cyclic and thus $Q = (0)_{2 \times 3}$.

Here $n = 8$, $r = 2$, $m = 3$ and $p = 3$ and the computed $K$ matrix is

$$K = \begin{bmatrix}
0.2863416 & 0.00 & -0.3182513 & 10.00 & 0.00 & 0.00 \\
0.0014317 & 0.00 & -0.0015912 & 0.050 & 0.00 & 0.00 \\
0.128005 & 0.00 & 0.1242625 & 0.00 & 1.00 & 0.00 \\
0.181389 & 0.00 & -0.826475 & 0.00 & 0.00 & 1.00 \\
0.000603 & -0.329497 & 7.020192 & 0.00 & 5.895031 & -6.48084
\end{bmatrix}$$

Diagrams 1 and 2 display the pole movement of the closed loop system for $P = 0.5 - 0.9$ and $P = 1.1 - 1.4$. 
FIGURE 9
BLOCK DIAGRAM OF THE PLANT AND COMPENSATOR IN CASCADE
\[ \text{Re}(\lambda) = \text{REAL PART OF } \lambda \]
\[ \text{Im}(\lambda) = \text{IMAGINARY PART OF } \lambda \]

Diagram 2: Pole movement of the closed loop system from \( P=1.1 \) to \( P=1.4 \) (example 1)
Example 2

The compensator has been designed for \( P = 1.0 \)

The open loop poles of the system under consideration

at \( P = 1.0 \) are

\[
\Lambda_{OL} = \{ -18.83, -4.33 + j 1.418, -2.57, -0.411 + j 8.41, \\
-0.288, -0.205 \} 
\]

The closed loop poles are chosen much further into the left half plane than in example 1.

\[
\Lambda_{CL} = \{ -18.83, -4.33 + j 1.418, -2.57, -4.0, -3.5, -3.0, \\
-2.0, -1.5, -1.6 + j 7.0 \} 
\]

Again A is cyclic, \( Q \equiv (0)^2 x 3 \), \( n = 8 \), \( r = 2 \), \( m = 3 \) and \( p = 3 \).

The computed K matrix is

\[
K = \begin{bmatrix}
0.0026508 & 0.00 & 0.4134051 & 0.00 & 0.00 \\
0.000132 & 0.00 & 0.0020672 & 0.00 & 0.00 \\
0.2638776 & 0.00 & -0.2357313 & 0.00 & 1.00 \\
-2.432512 & 0.00 & -0.1087801 & 0.00 & 0.00 \\
22.596810 & -11.483120 & 17.704630 & 0.00 & -80.12486 \\
\end{bmatrix}
\]

Diagrams 3 and 4 display the pole movement of the closed loop system

for \( P = 0.5 - 0.9 \) and \( P = 1.1 - 1.4 \).
\( \text{Re} (\lambda) = \text{REAL PART OF } \lambda \)

\( \text{Im} (\lambda) = \text{IMAGINARY PART OF } \lambda \)

Diagram 3. Pole movement of the closed loop system from \( p=0.5 \) to \( p=0.9 \) (example 2)
Discussion

It was found that a choice of the closed loop poles very far from the open loop poles was associated with high compensator gains, which in turn cause a greater sensitivity of the poles to changes in power levels. Comparing the gains, one can see that the gains in example 2 are higher than those of example 1. The closed loop system of example 1 is less sensitive than that of example 2, but the Poles \((I, I')\) of example 1 move dangerously close to the imaginary axis \((-0.0794 + j0.78023\) for \(P = 0.5\). Obviously one should strive to keep the sensitivity low, but it is far more important to make sure that all the poles are sufficiently far away from the imaginary axis for the range of expected power levels. Thus the design in example 2 is considered better than example 1.

3.3 Comparison of different methods

At any given oscillation frequency about synchronous frequency, braking torques are developed in-phase with the machine rotor angle and in phase with the machine rotor speed. The former are termed synchronizing torques and the latter damping torques. Stability can be endangered by a lack of either or both synchronizing and damping torques. The traditional stability criterion with which industry is most acquainted concerns the tests for positive synchronizing torques which will determine whether or not forces will be set up to restore
the rotor angle of the machine following an arbitrarily small displacement of this angle.

By means of small perturbation analysis, the terminal voltage deviations $\Delta V_t$ are related to changes in angle $\Delta \theta$ by the proportionality factor $C_{21}$ (Appendix III), which can have either sign and considerable range in magnitude depending on the impedances and operating conditions.

Under normal conditions (i.e. $C_{21} > 0$) the voltage feedback helps in producing damping torques, but in the case when the external reactance is very high, $C_{21}$ can be negative and in such cases, voltage feedback produces negative damping torques. This is well explained in [7]. These negative damping torques can lead to instability.

This section is essentially devoted to the comparison of the pole movement of

a) The method using standard voltage regulators, without any supplementary control (Fig. 1)

b) The method proposed by Demello and Concordia (Ref. 7, Fig. 4)

and c) The method using the results of [10]

in the case of a synchronous machine connected to an infinite bus through a very high external reactance.

The representation of the voltage regulator exciter system in each case is that suggested by Demello and Concordia [7].
The data for the synchronous machine, turbine and governor is listed in Appendix V.

a) Pole Movement in conventional case

Diagram 5 displays the movement of the poles of the system (Fig. 4) for power levels between \( P = 0.5 \) to \( P = 1.4 \). Poles \( (I', I'') \) lead to instability at about \( P = 1.15 \). An enlarged version of Poles \( (I', I'') \) is shown in diagram 6. (See table 1)

b) Pole Movement using the method proposed by Demello and Concordia

Demello and Concordia \( [7] \) recommended two transfer functions (Fig. 4):

\[
G_1(s) = \frac{K S (1 + S/8 + S^2/64)}{(1 + T S) (1 + S/20 + S^2/400)}
\]

and

\[
G_2(s) = \frac{K S(1 + S/8)^2}{(1 + T S)(1 + S/20)^2}
\]

where the recommended range for \( K/T \) and \( T \) are:

\[
10 \leq K/T \leq 40, \quad 2 < T < 4
\]

The values chosen for \( K \) and \( T \) are

\( K = 60 \) and \( T = 3 \).
Re(\(\lambda\)) = REAL PART OF \(\lambda\)
Im(\(\lambda\)) = IMAGINARY PART OF \(\lambda\)

Diagram 5 Pole Movement
Using the Standard Voltage
Regulator Without Any Supplementary Control (\(X_e = 0.70\))
Re($\lambda$) = REAL PART OF $\lambda$
Im($\lambda$) = IMAGINARY PART OF $\lambda$

Diagram 6: Enlarged version of poles ($I'$, $I''$) from Diagram 5
Diagrams 7 and 8 display the movement of the poles of the closed loop system for $G_1(S)$ and $G_2(S)$ respectively for power levels ranging from $P = 0.5$ to $P = 1.4$. Poles $(VI, VI')$ move very rapidly into the right half plane and cause instability between $P = 0.9$ and $P = 1.0$. An enlarged version of Poles $(VI, VI')$ is shown in diagram 9 (see table 2 and 3).
Re(\lambda) = \text{REAL PART OF } \lambda \\
\text{Im}(\lambda) = \text{IMAGINARY PART OF } \lambda

Diagram 7: Pole movement using the standard voltage regulator, with a supplementary control as suggested by DeMello and Concordia (G_1(S))
Diagram 8: Pole Movement Using the Standard Voltage Regulator, with a Supplementary Control as Suggested by DeMello and Concordia ($G_2(s)$)
\[
P = 0.5 \quad -0.0198 + j \ 4.378 \\
-0.0198 - j \ 4.378
\]
\[
P = 0.6 \quad -0.0429 + j \ 4.596 \\
-0.0429 - j \ 4.596
\]
\[
P = 0.7 \quad -0.0585 + j \ 4.799 \\
-0.0585 - j \ 4.799
\]
\[
P = 0.8 \quad -0.0655 + j \ 4.964 \\
-0.0655 - j \ 4.964
\]
\[
P = 0.9 \quad -0.0632 + j \ 5.066 \\
-0.0632 - j \ 5.066
\]
\[
P = 1.0 \quad -0.0511 + j \ 5.073 \\
-0.0511 - j \ 5.073
\]
\[
P = 1.1 \quad -0.0256 + j \ 4.939 \\
-0.0256 - j \ 4.939
\]
\[
P = 1.2 \quad +0.0277 + j \ 4.567 \\
+0.0277 - j \ 4.567
\]
\[
P = 1.3 \quad +0.1815 + j \ 3.638 \\
+0.1815 - j \ 3.638
\]
\[
P = 1.4 \quad +0.681 + j \ 0.669 \\
+0.681 - j \ 0.669
\]

Table 1: Movement of Poles \((I^I, I^{"I})\) in the case of the method using standard voltage regulators, without any supplementary control.
Table 2: Movement of Poles (VI, VI") in the method suggested by Demello and Concordia using $G_1(S)$.

<table>
<thead>
<tr>
<th>$P$</th>
<th>Pole VI'</th>
<th>Pole VI&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>$-3.015 + j, 66.090$</td>
<td>$-3.015 - j, 66.090$</td>
</tr>
<tr>
<td>0.6</td>
<td>$-1.976 + j, 65.093$</td>
<td>$-1.967 - j, 65.093$</td>
</tr>
<tr>
<td>0.7</td>
<td>$-1.135 + j, 64.261$</td>
<td>$-1.135 - j, 64.261$</td>
</tr>
<tr>
<td>0.8</td>
<td>$-0.498 + j, 63.616$</td>
<td>$-0.498 - j, 63.616$</td>
</tr>
<tr>
<td>0.9</td>
<td>$-0.063 + j, 63.163$</td>
<td>$-0.063 - j, 63.163$</td>
</tr>
<tr>
<td>1.0</td>
<td>$+0.171 + j, 62.905$</td>
<td>$+0.171 - j, 62.905$</td>
</tr>
<tr>
<td>1.1</td>
<td>$+0.196 + j, 62.850$</td>
<td>$+0.196 - j, 62.850$</td>
</tr>
<tr>
<td>1.2</td>
<td>$-0.023 + j, 63.033$</td>
<td>$-0.023 - j, 63.033$</td>
</tr>
<tr>
<td>1.3</td>
<td>$-0.635 + j, 63.597$</td>
<td>$-0.635 - j, 63.597$</td>
</tr>
<tr>
<td>1.4</td>
<td>$-2.125 + j, 65.965$</td>
<td>$-2.125 - j, 65.965$</td>
</tr>
<tr>
<td>Table 3: Movement of Poles (VI', VI'') in the method suggested by Demello and Concordia, using $G_2(S)$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>$P = 0.5$</td>
<td>$-3.018 + j 66.091$</td>
<td>$-3.018 - j 66.091$</td>
</tr>
<tr>
<td>$P = 0.6$</td>
<td>$-1.979 + j 65.094$</td>
<td>$-1.979 - j 65.094$</td>
</tr>
<tr>
<td>$P = 0.7$</td>
<td>$-1.139 + j 64.262$</td>
<td>$-1.139 - j 64.262$</td>
</tr>
<tr>
<td>$P = 0.8$</td>
<td>$-0.502 + j 63.617$</td>
<td>$-0.502 - j 63.617$</td>
</tr>
<tr>
<td>$P = 0.9$</td>
<td>$-0.067 + j 63.165$</td>
<td>$-0.067 - j 63.165$</td>
</tr>
<tr>
<td>$P = 1.0$</td>
<td>$+0.166 + j 62.907$</td>
<td>$+0.166 - j 62.907$</td>
</tr>
<tr>
<td>$P = 1.1$</td>
<td>$+0.192 + j 62.851$</td>
<td>$+0.192 - j 62.851$</td>
</tr>
<tr>
<td>$P = 1.2$</td>
<td>$-0.027 + j 63.034$</td>
<td>$-0.027 - j 63.034$</td>
</tr>
<tr>
<td>$P = 1.3$</td>
<td>$-0.639 + j 63.598$</td>
<td>$-0.639 - j 63.598$</td>
</tr>
<tr>
<td>$P = 1.4$</td>
<td>$-2.128 + j 65.966$</td>
<td>$-2.128 - j 65.966$</td>
</tr>
</tbody>
</table>
c) **Pole Movement using the results of [10]**

The pertinent results of [10] are stated in section 3.1. The compensator has been designed for the system (Fig. 5) operating at \( P = 1.4 \).

The open loop poles of the system at \( P = 1.4 \) are

\[
\Lambda_{OL} = \{-20.00, -5.623 \pm j 2.782, -1.328, -0.753, 3.997\}
\]

The closed loop poles are chosen at:

\[
\Lambda_{CL} = \{-24.00 + j 7.00, -3.00 \pm j 1.00, -12.00, -6.00, -4.00, -0.20\}
\]

Again A is cyclic and thus \( Q = (0)^2 \times 3 \)’. Here \( n = 6, r = 2, m = 3 \) and \( p = 2 \) and the computed \( K \) matrix is

\[
K = \begin{bmatrix}
0.3611051 & 0.0000000 & -1.1874320 & 10.0000000 & 0.0000000 \\
0.0361105 & 0.0000000 & -0.1187432 & 1.0000000 & 0.0000000 \\
-1.3638032 & 0.0000000 & 3.8499140 & 0.0000000 & 1.0000000 \\
64.7268200 & -3.3819230 & -181.3613000 & 0.0000000 & -46.8686000
\end{bmatrix}
\]

Diagram 10 displays the pole movement of the closed loop system for \( P = 0.5 \) to \( P = 1.3 \).

**Discussion**

The voltage feedback to the voltage regulator exciter system used in this section causes "relatively high frequency" oscillations (see Diagram 5, Poles III' III") which are very well damped and are thus no cause for concern. In this case poles (I', I") are the ones
that cause instability. While the method suggested by Demello and Concordia does help in removing this problem by moving the troublesome poles further into the left half plane (see Diagrams 7, 8 and 9; Poles (II, II')), unfortunately it reduces the damping of the high frequency poles, which then are the cause of instability (see Diagrams 7, 8 and 9; Poles (VI, VI')).

The method suggested in [10] is definitely a systematic way of computing feedback gains for arbitrary pole placement of the closed loop system for a given power level. However it is difficult to predict the behaviour of the closed loop system for other power levels, particularly when one or more of the open loop poles are in the right half plane, as is the case in this example. For this example, it was difficult to find a closed loop pole location (or for which to design the compensator, such that the closed loop system remain sufficiently stable over the expected range of power levels. This indicates that a systematic approach is desireable to the problem of specifying closed loop pole locations. Although the work of Demello and Concordia is a systematic approach to the control of damping torques, it should be noted that instabilities can arise due to changes in other parts of the system and an overemphasis on any one particular aspect of the problem with the neglect of another part can produce unsatisfactory results. Stability must be viewed as a system property and improving the "damping" in one part of a system doesn't necessarily make the entire system more stable.
Summary and Conclusions

This thesis has been concerned with the problem of designing a constant coefficient feedback controller, that would allow arbitrary pole placement of a single synchronous machine connected through a reactance to an infinite bus.

The approach used was first to derive the nonlinear system model. This model has two independent inputs (input to the governor and input to the voltage regulator) and three independent outputs (frequency, terminal voltage and real power). The steady state values of the state variables are solved for at different real power levels $P$, for $V_t = 1$ p.u. and $\omega_0 = 377$ radians per second. Linearization of the model about the steady state values yields the linearized perturbation model for which the controller is designed.

It is considered unreasonable and expensive to design different controllers for different power levels. It is desirable rather to design a controller that will allow a sufficiently stable response over the expected power levels.

Computer simulations revealed that in some cases the closed loop system becomes unstable for power levels other than that for which the controller is designed. It was found that a choice of the closed loop poles very far away from the open loop poles incorporated high controller gains, which in turn can cause greater sensitivity of the poles to changes in power levels. Thus the controller was designed for a given real power level and then tested to see if the closed loop system remained sufficiently stable for all expected power levels.
The controller regulates voltage, frequency and power outputs of a synchronous machine. It is in cascade with the plant and uses voltage, frequency and real power deviations to generate the control signals. It has the following features, which other methods do no possess:

1) It is designed for arbitrary pole placement of the closed loop system.
2) It is a multi-input, multi-output controller and
3) It permits an algorithmic approach which can be completely computerized.

A possible extension of this work is the study of where to place the closed loop poles, such that the closed loop system is in some sense a minimally sensitive one. Another useful extension would be the multi-machine case.
Appendix I

Per Unit Bases

Base $V_t$ and $V_B$ = Base $V_{fd}$ = Base $U_2$ = Base $V_R$ = Base $X_F$ = Rated terminal voltage (Line to Line) in Kilovolts = $V_o$

Base $P$ = Rated MVA = $P_o$

Base $\omega$ = Synchronous frequency = $\omega_o = 377$ (Rad/Sec)

Base Torque = Base $P$/Base $\omega$ = $P_o/\omega_o = T_o$

Base $\phi_{fd} = Base V_{fd}/Base \omega = V_o/\omega_o$

Base Reactance = Base Resistance = $V_o^2$ = $V_o^2/P_o$

Base $L = Base \text{ Reactance}/Base \text{ frequency} = V_o^2/\omega_o P_o$

Base $D = Base M = Base \text{ Torque}$

Base $T_v = \text{Fraction of valve opening that yields Base torque at Base frequency} = T_v o$

Base $U_1 = \text{Position of speedchanger which delivers rated Power at rated frequency}$

Base $t = 1 \text{(rad)}/Base \omega = 1/377$ seconds
The units to be used in equations of section 2.4 are:

\( \theta \) (Radians)
\( \omega \) and \( \omega_0 \) in (rad/sec)
\( V_t, V_B, V_{fd}, U_2, V_R, X_F \) (per unit)
\( T_m \) (per unit)
\( \phi_{fd} \) (per unit)
\( X_e, X_d, X_d', X_q, R_{fd}, L_{afd}, L_{ffd} \) (per unit)
\( D \) (per unit torque/rad/sec)
\( M \) (per unit torque/rad/sec\(^2\))
\( T_v \) (per unit)
\( P \) (per unit)

All time constants are in seconds
Appendix II

Purpose and Description of the Program

From equation (2.5.10) it follows for steady state conditions that

\[
F = \frac{V_B^2 (X_d' - X_q)}{2(X_e + X_d')(X_e + X_q)} \sin 2 \delta
\]

\[
\overline{\Phi}_{fd} = \frac{V_B L_{afd} \sin \overline{\delta}}{L_{fd}(X_e + X_d')}
\]  \hspace{1cm} (II.1)

Then with equation (2.5.9) one obtains:

\[
A_1 \sin^2 \overline{\delta} - 2A_2 \sin \overline{\delta} \cos \overline{\delta} - 4A_3 \sin^2 \overline{\delta} \cos^2 \overline{\delta} - A_4 = 0 \hspace{1cm} (II.2)
\]

where

\[
A_1 = \left[ \frac{V_t^2}{V_B^2} - \frac{V_B^2 X_q^2}{(X_e + X_q)^2} \right]
\]

\[
A_2 = \frac{P X_e X_q}{X_e + X_q}
\]
The purpose of the program is to solve equation (II.2) for given values of $A_1$, $A_2$, $A_3$, and $A_4$.

Iteration procedure

Equation (II.2) can be solved as follows:

Let $S_N$ be a nominal choice for which

$$F(S_N) = A_1 \sin^2 S_N - 2A_2 \sin S_N \cos S - 4A_3 \sin^2 S_N \cos^2 S_N - A_4 \neq 0$$

Let $S_{New} = S_N + h$ such that $F(S_{New}) = 0$.

Neglecting higher order terms

$$F(S_{New}) = F(S_N + h) = F(S_N) + h F'(S_N)$$

or

$$h = -\frac{F(S_N)}{F'(S_N)}$$
Thus

\[ \delta_{\text{New}} = \delta_{N} - \frac{F(\delta_{N})}{F'(\delta_{N})} \]

Iterate till

\[ \left| \frac{\delta_{\text{New}} - \delta_{N}}{\delta_{\text{New}}} \right| \leq \varepsilon \]

Description of Variables

\begin{align*}
VT & = \text{Terminal Voltage} \\
VB & = \text{Bus voltage} \\
XQ & = \text{Quadrature Axis reactance} \\
XE & = \text{External reactance} \\
XD1 & = \text{Direct Axis transient reactance} \\
P & = \text{Real power} \\
A1, A2, A3 & \text{ and } A4 = \text{coefficients of equation (II.2)} \\
FX & = F(\delta_{N}) \\
DFX & = F'(\delta_{N}) \\
X & = \text{Torque angle in radians} \\
XN & = X - \frac{FX}{DFX} = \text{New choice of Torque angle in radians} \\
XY & = \text{Torque Angle in degrees} \\
XPF & = \text{Auxiliary variable for storage of } X \\
\end{align*}

Input Quantities read in

\[ VT, VB, XQ, XE, XD1 \]

Output Quantities printed out

\[ VT, VB, XQ, XE, XD1, P, A1, A2, A3, A4, X, XY \]
BEGIN

FILE IN CARD 0(2,10);
FILE OUT PRINT 15(2,15);
INTEGER JOB, IVP, I;
REAL X, XP, VT, VB, XQ, XE, XD1, A1, A3, P, A2, A4, S, C, F, X, X5, SS, CC, DFX,
XN, XY;
FORMAT FL10(8R10.5);

FL10(F12.2,4F10.4,F10.6,X5,F10.6),
"X(DEGREES)"/),
"XN1="X6.3/),
FL29(I5);

LIST LIST1(JOB);
LIST LIST2(VT,VB,XQ,XE,XD1);
LIST LIST3(P,A1,A2,A3,A4,X,XY);
LABFL L13,L11,L17,FINISH;
READ(CARD,FL29,LIST1)(FINISH);
COMMENT MAKE INITIAL CHOICE OF Y;
X1=35;
COMMENT INITIALIZE XP;
XP=.0J
IVP=1;
DO BEGIN
COMMENT READ AND PRINT VT, VB, XQ, XE, XD1;
READ(CARD,FL10,LIST2)(FINISH);
WRITE(PRINT(PAGE1));
WRITE(PRINT,FL25,LIST2);
COMMENT EVALUATE A1 AND A3;
A1:=VTxVT-(VBxVB)(x0xQ/(xExQ));
A3:=(25xVBxVBxXQ1xXQ1(1/(xExQ)(xExQ))-1/(xExXQ1xXQ1));
COMMENT INITIALIZE P;
P:=40;
WRITE(PRINT,FL15);
I:=1;
Dr BEGIN
P:=P+1;
COMMENT EVALUATE A2 AND A4;
A2:=PxExXQ/(xExQ);
A4:=PxExVB;
A4:=A4xA4;
COMMENT REPLACE SIN(X) AND COS(X) BY S AND C RESPECTIVELY;
L13: S:=SIN(X);
C:=COS(X);
COMMENT EVALUATE THE FUNCTION FX;
FX:=A1xSxS-2xA2xSxC-A3xSxSxCxC-A4;
XX:=2xX;
SS:=SIN(XX);
CC:=COS(XX);
COMMENT EVALUATE THE DERIVATIVE DFX;
DFX:=A1xSS-2xA2xC-4xA3xSxCC;
COMMENT OBTAIN NEW CHOICE OF Y(I.E. XN);
XN:=X-FX/DFX;
IF ABS((X-XN)/XN) LEQ .00001 THEN GO TO L11;
X:=XN;
GO TO L13;}
L11: X := XN
COMMENT CONVERT X IN RADIANS TO XY IN DEGREES;
XY := X * 180 / 3.14159273
IF P NEQ 5 THEN GO TO L17;
XPF := X;
L17: WRITE(PRINT, FL14, LIST3);
END UNTIL (I := (I + 1) GTR 10);
X := XPF;
END UNTIL (IVP := (IVP + 1) GTR JOB);
FINIS: END.
BEGIN

**COMMENT** 

REAL VB, XD, XO1, XO, XE, RFD, LFD, LAFD, KE, KEG1
REAL X5, TF, C3, D*M, IT, TR
REAL KS, TF, KF, TA, KA, C16

**COMMENT**

IN THE PREVIOUS PROGRAM THE STEADY STATE TORQUE ANGLE WAS COMPUTED FOR DIFFERENT POWER LEVELS AND FOR DIFFERENT EXTERNAL REACTANCES.

THIS PROGRAM COMPUTES THE A, B AND C MATRICES FOR DIFFERENT POWER LEVELS AND FOR XE = 0.2 AND XE = 0.7.

THE DATA OF THE PLANT IS AS FOLLOWS:

VTR = RATED TERMINAL VOLTAGE = 1 [PER UNIT]

VB = BUS VOLTAGE = 0.95 [PER UNIT]

P = RATED POWER

D = ROTOR DAMPING COEFFICIENT = 1/377 [PER UNIT TORQUE/RAD/SEC]

M = INERTIA CONSTANT = 0.0277 [PER UNIT TORQUE/RAD/SEC SQUARED]

R = FREQUENCY FEEDBACK GAIN = -15.00

XD = DIRECT AXIS REACTANCE = 1.383 [PER UNIT]

XO = QUADRATURE AXIS REACTANCE = 1.32 [PER UNIT]

XO1 = DIRECT AXIS TRANSIENT REACTANCE = 0.135 [PER UNIT]

XE = EXTERNAL REACTANCE = 0.2 OR 0.7 [PER UNIT]

LFD = FIELD SELF INDUCTANCE = 1242.0 [PER UNIT]

LAFD = MUTUAL INDUCTANCE BETWEEN STATOR AND FIELD = 39.5 [PER UNIT]

RFD = FIELD RESISTANCE = 0.3462 [PER UNIT]
TF = EXITING TIME CONSTANT = 0.50 OR 0.05 [SEC]
TA = STANDARD VOLTAGE REGULATOR TIME CONSTANT = 0.1 [SEC]
TF = STANDARD VOLTAGE REGULATOR TIME CONSTANT = 1.0 [SEC]
TG = GOVERNOR TIME CONSTANT = 0.20 [SEC]
TT = TURBINE TIME CONSTANT 0.25 [SEC]
KE = EXITING GAIN = -0.05 [PER UNIT]
KS = SATURATION FUNCTION OF EXITING = 0.4
KEG = EXITING GAIN = 25.00 [PER UNIT]
KA = STANDARD VOLTAGE REGULATOR GAIN = 50.00 [PER UNIT]
KF = STANDARD VOLTAGE REGULATOR GAIN = 0.01 [PER UNIT]
KG = GOVERNOR GAIN = 1.00 [PER UNIT]
KT = TURBINE GAIN = 1.00 [PER UNIT]
XIRAD[1] = STEADY STATE TORQUE ANGLE AT POWER LEVEL 1.
X5 = STEADY STATE FIELD FLUX
C3 = AUXILIARY VARIABLE
C13 = AUXILIARY VARIABLE
C16 = AUXILIARY VARIABLE
BASE V1 = VO = 13.8 [KV]
BASE P = PO = 129 [MVA]
BASE D = BASE M = BASE TORQUE
BASE X = BASE R = (VO)*(VO)/PO = 1.477 [OHMS]
BASE OMEGA = 377 [RAD/SEC]
BASE TORQUE = BASE POWER / BASE OMEGA = 129 [MVA]/377 [RAD/SEC] = 0.342
BASE L = BASE X / BASE OMEGA = 0.0392 [HZ]
BASE FIELD FLUX = BASE V / BASE OMEGA = 13.8 * 1000/377 = 36.6 [V.S.]
COMMENT $$$$$$$$$$$$$$$ DECLARATION OF FILES $$$$$$$$$$$$$$$$
FILE IN FIN1 (2, 10)
FILE OUT FOUT2 152(2,17);  
COMMENT $$$$$$$$$$$$$$$$$$$$$$$$$$$ DECLARATION OF FORMATS $$$$$$$$$$$$$$$$
FORMAT IN FMT (R5.2/10(XA,RA,2)/10R8,6); 

COMMENT $$$$$$$$$$$$$$$$$$$$$$$$$$$ DECLARATION OF LABELS $$$$$$$$$$$$$$$$$$$$
LABEL TOP,ENDGO,DETOUR1,DETOUR2,DETOUR3; 
COMMENT $$$$$$$$$$$$$$$$$$$$$$$$$$$ DECLARATION OF REAL ARRAYS $$$$$$$$$$$$$$$$
REAL ARRAY P[0:91,X1RADI0:9]; 
REAL ARRAY A[0:8*0:8], C[0:3*0:8]; 
REAL ARRAY B[0:8*0:2]; 

COMMENT **************** MACHINE PARAMETERS ******************;

| VB<0,95| 
| D+1/377; |
| M+0,0277; |
| R+15,00; |
| X0+1,383; |
| X0+1,32; |
| X01+0,135; |
| X01+0,135; |
| RDF+0,3462; |
| LFD+1.242,01 |
| LAF+39,5; |
| TE<0,05; |
| TA<0,05; |
| TF+1,0; |
| TG<0,20; |
| T+0,25; |
| KE<0,05; |
| KEG<25,00; |
| KA<50,00; |
| KF<0,01; |
| KS<0,4; |
COMPUTATION OF THE A MATRIX

BEGIN

IF XE=0.2 THEN B[7,2]=KA/TA ELSE IF XE=0.7 THEN B[6,2]=KEG/TE;

READ(XE,FMT,VE) FOR I=1 STEP 1 UNTIL 9 DO X1RAD[I]=EDF/0.1;

FOR I=0 STEP 1 UNTIL 9 DO

C3+(VB*LAFD/(X*LFFD*(XE+X1)));
C13*XE*LAFD*VD*(X*LFFD*(XE+X1)*X*(XE+XQ))/((VB*LAFD*XD1*VE*(X+X1))*(X+XQ)));
C16*VB*LAFD/(LFFD*(XE+X1));

FOR I=0 STEP 1 UNTIL 9 DO BEGIN

TE=0.5;

C1+[C1+1];
C2+[C2+1]+(VB/XE+XQ)*(VB/XE+XQ)*X*(XQ-XQ-X1*XD1)*SIN(X1RAD[I])X*COS(X1RAD[I]);
C3+[C3+1]+C16*X5*COS(X1RAD[I])+(VB*VB*(X+X1)*(X+XQ));
C1+[C1+1]*X5*COS(X1RAD[I])+(VB*VB*(X+X1)*(X+XQ));
C3+[C3+1]+C16*X5*COS(X1RAD[I])+(VB*VB*(X+X1)*(X+XQ));

END

END.
COMMENT $\text{EVALUATION OF THE A MATRIX}$

\begin{verbatim}
A[1,2]=1.00;
A[2,1]=C3*X5*COS(X1RAD[I])+(V8*V8*(X0-YD))/
(M*(X0+X1)*(X0+X2))\times\cos(2\times X1RAD[I]);
A[2,2]=D/M+(V8*V8*(X0-YD)\times\sin(2\times X1RAD[I])/
(2\times 377\times M\times(X0+X1)\times(X0+X2));
A[2,3]=1/4;
A[2,5]=C3\times\sin(X1RAD[I]);
A[3,3]=1/TT;
A[3,4]=1/TT;
A[4,4]=1/TT;
A[5,1]=(377\times V8\times LAFD\times RFD*(LFFD\times(X0+X1))\times\sin(X1RAD[I]);
A[5,5]=(377\times RFD\times(X0+X1)/(LFFD\times(X0+X1));
A[5,6]=377;
IF XE=0.7 THEN GO TO DETOUR1;
A[6,6]=-(XE*KE/TE);
A[6,7]=1/TE;
A[7,6]=1/TA;
A[7,8]=K/TA;
A[8,6]=-(KE*KE+KE*KS)/(TE*TF);
A[8,7]=KF/(TE*TF);
A[8,8]=1/TF;
DETOUR1: IF XE=0.2 THEN GO TO DETOUR2;
TE=0.05;
A[6,6]=1/TE;
COMMENT $\text{PRINT OUT THE A,R,C MATRICES}$
WRITE(FOUT2,\"P=",F5,\"X5","XE="\",\"F5,\"X5P[i]=\"XE\")
WRITE(FOUT2[DBL]);
WRITE(FOUT2,"A MATRIX\")
\end{verbatim}
FOR K = 1 STEP 1 UNTIL 6 DO
   FOR L = 1 STEP 1 UNTIL 6 DO A(K,L);
WRITE(FOUT2, "B MATRIX");
WRITE(FOUT2, <BF13,5/>);

FOR K = 1 STEP 1 UNTIL 6 DO
   FOR L = 1 STEP 1 UNTIL 2 DO B(K,L);
WRITE(FOUT2, "C MATRIX");
WRITE(FOUT2, <BF13,5/>);

FOR J = 1 STEP 1 UNTIL 3 DO
   FOR L = 1 STEP 1 UNTIL 6 DO C(J,L);
WRITE(FOUT2, PAGE);
   IF YE = 0.7 THEN GO TO DETOUR3;
   DETOUR2: WRITE(FOUT2, "P="EF5.2X5,"XE="EF5.2X5,"P1,"X E");
WRITE(FOUT2, "A MATRIX");
WRITE(FOUT2, <BF13,5/>);

FOR K = 1 STEP 1 UNTIL 6 DO
   FOR L = 1 STEP 1 UNTIL 8 DO A(K,L);
WRITE(FOUT2(DBL));
WRITE(FOUT2,"B MATRIX");
WRITE(FOUT2,<2F13,5/>);
FOR K+1 STEP 1 UNTIL A DO
FOR L+1 STEP 1 UNTIL 2 DO B[K,L];
WRITE(FOUT2(DBL));
WRITE(FOUT2,"C MATRIX");
WRITE(FOUT2,<8F13,5/>);
FOR J+1 STEP 1 UNTIL 3 DO
FOR L+1 STEP 1 UNTIL A DO C[J,L];
WRITE(FOUT2(PAGE));
DETOUR3:END;
GO TO TOP;
EOFGO:END,
Appendix III

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & a_{23} & 0 & a_{25} & 0 & 0 & 0 \\
0 & 0 & a_{33} & a_{34} & 0 & 0 & 0 & 0 \\
a_{42} & 0 & a_{44} & 0 & 0 & 0 & 0 & 0 \\
a_{51} & 0 & 0 & 0 & 0 & a_{55} & a_{56} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & a_{66} & a_{67} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{77} \\
0 & 0 & 0 & 0 & 0 & 0 & a_{86} & a_{87} & a_{88}
\end{bmatrix}
\]

\[
a_{21} = - \left[ \frac{V_B L \bar{a}_{fd} \bar{\phi}_{fd} \cos \bar{\theta}}{M \bar{L}_{ffd} (\bar{X}_e + \bar{X}_d)} + \frac{v_B^2 (\bar{X}_d^i - \bar{X}_e) \cos 2\bar{\theta}}{M (\bar{X}_e + \bar{X}_d^i) (\bar{X}_e + \bar{X}_q)} \right]
\]

\[
a_{22} = - \frac{D}{M} + \frac{v_B^2 (\bar{X}_d^i - \bar{X}_q) \sin 2\bar{\theta}}{2 \omega_0 M (\bar{X}_e + \bar{X}_d^i) (\bar{X}_e + \bar{X}_q)}
\]

\[
a_{23} = \frac{1}{M}
\]

\[
a_{25} = - \frac{v_B L \bar{a}_{fd} \sin \bar{\theta}}{M \bar{L}_{ffd} (\bar{X}_e + \bar{X}_d)}
\]
\[ a_{33} = - \frac{1}{T_t} \]

\[ a_{34} = \frac{K_t}{T_t} \]

\[ a_{42} = - \frac{1}{T_g R} \]

\[ a_{44} = - \frac{1}{T_g} \]

\[ a_{51} = - \frac{\omega_o V_B L_{afd} R_{fd} \sin \delta}{L_{ffd} (X_e + X_d)} \]

\[ a_{55} = - \frac{\omega_o R_{fd} (X_e + X_d)}{L_{ffd} (X_e + X_d)} \]

\[ a_{56} = \omega_o \]

\[ a_{66} = - \frac{(f(V_{fd}) + K_E)}{T_E} \]

\[ a_{67} = \frac{1}{T_E} \]

\[ a_{77} = - \frac{1}{T_A} \]

\[ a_{78} = - \frac{K_A}{T_A} \]
\[ a_{86} = - \frac{(K_F K_E + K_F f(V_{fd}))}{T_E T_F} \]
\[ a_{87} = \frac{K_F}{T_E T_F} \]
\[ a_{88} = - \frac{1}{T_F} \]

\[
B = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
b_{41} & 0 \\
0 & 0 \\
0 & 0 \\
0 & b_{72} \\
0 & 0 \\
\end{bmatrix}
\]

\[ b_{41} = \frac{K_g}{T_g} \]
\[ b_{72} = \frac{K_A}{T_A} \]

\[
c = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
c_{21} & c_{22} & 0 & 0 & c_{25} & 0 & 0 & 0 \\
c_{31} & c_{32} & 0 & 0 & c_{35} & 0 & 0 & 0 \\
\end{bmatrix}
\]
$$c_{21} = \frac{v_B^2 (x_q - x_d)^2 \sin \theta \cos \theta}{(x_e + x_q)^2 \bar{V}_t} - \frac{v_B x_e L_{afd} x_d^\prime \bar{\Phi}_{fd} \sin \theta}{L_{ffd} (x_e + x_d')^2 \bar{V}_t}$$

$$c_{22} = \frac{x_e^2 L_{afd} \bar{\phi}_{fd}}{\bar{V}_t L_{ffd}^2 (x_e + x_d')^2 \omega_o} + \frac{x_e L_{afd} x_d^\prime v_B \bar{\Phi}_{fd} \cos \theta}{\bar{V}_t L_{ffd} (x_e + x_d')^2 \omega_o}$$

$$c_{25} = \frac{x_e^2 L_{afd} \bar{\phi}_{fd}}{\bar{V}_t L_{ffd}^2 (x_e + x_d')^2} + \frac{x_e L_{afd} x_d^\prime v_B \cos \theta}{\bar{V}_t L_{ffd} (x_e + x_d')^2}$$

$$c_{31} = \frac{v_B L_{afd} \bar{\phi}_{fd} \cos \theta}{L_{ffd} (x_e + x_d')} + \frac{v_B^2 (x_d' - x_q) \cos 2 \theta}{(x_e + x_d')(x_e + x_q)}$$

$$c_{32} = \frac{v_B L_{afd} \sin \theta \bar{\phi}_{fd}}{L_{ffd} (x_e + x_d') \omega_o}$$

$$c_{35} = \frac{v_B L_{afd} \sin \theta}{L_{ffd} (x_e + x_d')}$$
Appendix IV

Plant Data

\[ X_d = 1.3830 \]
\[ X_d' = 0.1350 \]
\[ R_{fd} = 0.3462 \]
\[ L_{ffd} = 1242.0000 \]
\[ L_{afd} = 39.5000 \]
\[ X_q = 1.3200 \]
\[ V_B = 0.95 \]
\[ K_E = -0.05 \]
\[ T_E = 0.50 \]
\[ D = 1/377 \]
\[ M = 0.0277 \]
\[ T_t = 0.25 \]
\[ T_g = 0.20 \]
\[ K_t = 1.00 \]
\[ K_g = 1.00 \]
\[ R = -15.00 \]
\[ X_e = 0.2 \]
\[ f(V_{fd}) = 0.4 \]
\[ T_F = 1.0 \]
\[ K_F = 0.01 \]
\[ T_A = 0.05 \]
\[ K_A = 50.00 \]
B Matrix for $P = 0.5$ to $P = 1.4$

$X_e = 0.20$

\[
B = \begin{bmatrix}
0.000 & 0.000 \\
0.000 & 0.000 \\
0.000 & 0.000 \\
5.000 & 0.000 \\
0.000 & 0.000 \\
0.000 & 0.000 \\
0.000 & 1000.000 \\
0.000 & 0.000
\end{bmatrix}
\]
\[ P = 0.50 \quad XE = 0.20 \]

**A Matrix**

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<th>0.00000</th>
<th>1.00000</th>
<th>0.00000</th>
<th>0.00000</th>
<th>0.00000</th>
<th>0.00000</th>
<th>0.00000</th>
<th>0.00000</th>
</tr>
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<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
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<td>-0.17286</td>
<td>36.10108</td>
<td>0.00000</td>
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<td>0.00000</td>
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</tr>
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</tr>
<tr>
<td>-0.00000</td>
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\[ P = 0.60 \quad x_\varepsilon = 0.20 \]
\[ P = 0.70 \quad \chi^2 = 0.20 \]

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\[ P = 0.90 \quad \chi^2 = v, 20 \]

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\[ P = 1.00 \quad V = 0.20 \]

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\[ P = 1.20 \quad \chi = 0.20 \]

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**C Matrix**

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\[ P = 1.40 \quad XE = 0.20 \]

### A_\text{MATRIX}

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Appendix V

Plant Data

\[ X_d = 1.3830 \]
\[ X'_d = 0.1350 \]
\[ R_{fd} = 0.3462 \]
\[ L_{ffd} = 1242.0000 \]
\[ L_{afd} = 39.5000 \]
\[ X_q = 1.3200 \]
\[ V_B = 0.9500 \]
\[ K_E = 25.00 \]
\[ T_E = 0.0500 \]
\[ D = 1/377 \]
\[ M = 0.0277 \]
\[ T_t = 0.1500 \]
\[ T_g = 0.2000 \]
\[ K_t = 1.0000 \]
\[ K_g = 1.0000 \]
\[ R = -15.0000 \]
\[ X_E = 0.7 \]

Note: The equations of the coefficients of the \( A(6 \times 6) \) and \( B(6 \times 2) \) matrices as found in Appendix III change only very slightly. The changes are
\[ a_{66} = -\frac{1}{T_{E}}, \quad b_{62} = \frac{K_{E}}{T_{E}} \]

The equations of the coefficients of the C(3 x 6) remain unchanged.

**B Matrix for \( P = 0.5 \) to \( P = 1.4 \)**

\[ X_{e} = 0.70 \]

\[
\begin{bmatrix}
0.000 & 0.000 \\
0.000 & 0.000 \\
0.000 & 0.000 \\
5.000 & 0.000 \\
0.000 & 0.000 \\
0.000 & 500.000 \\
\end{bmatrix}
\]
\begin{tabular}{cccccccc}
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 & A Matrix & & & & & & \\
\hline
 & $0.00000$ & $1.00000$ & $0.00000$ & $0.00000$ & $0.00000$ & $0.00000$ & \\
-30.04424 & -0.12317 & 36.10108 & $0.00000$ & -1.10433 & $0.00000$ & \\
0.00000 & $0.00000$ & -4.00000 & 4.00000 & $0.00000$ & $0.00000$ & \\
0.00000 & 0.33333 & $0.00000$ & -5.00000 & $0.00000$ & $0.00000$ & \\
-3.99270 & $0.00000$ & $0.00000$ & $0.00000$ & -0.26215 & 377.00000 & \\
0.00000 & $0.00000$ & $0.00000$ & $0.00000$ & $0.00000$ & -20.00000 & \\
\hline
\end{tabular}

\begin{tabular}{cccccccc}
\hline
 & C Matrix & & & & & & \\
\hline
 & $0.00000$ & $1.00000$ & $0.00000$ & $0.00000$ & $0.00000$ & $0.00000$ & \\
0.07188 & $0.00175$ & $0.00000$ & $0.00000$ & 0.02278 & $0.00000$ & \\
0.83223 & $0.00235$ & $0.00000$ & $0.00000$ & 0.03059 & $0.00000$ & \\
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\[ \begin{array}{cccc}
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0.00000 & 1.00000 & 0.00000 & 0.00000 \\
0.00000 & 0.00000 & 1.00000 & 0.00000 \\
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\end{array} \]
\[ P = 0.80 \quad \chi^2 = 0.70 \]

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\[
P = 0.90 \quad \nu = 0.70
\]

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0.0000 & 0.3333 & 0.0000 & -5.0000 & 0.0000 & 0.0000 \\
-4.63631 & 0.0000 & 0.0000 & 0.0000 & -0.26215 & 377.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & -20.0000
\end{array}
\]

**C Matrix**

\[
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\[ P = 1.20 \quad x_0 = 0.70 \]

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\[ P = 1.30 \quad X = 0.70 \]

A Matrix

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\text{A Matrix} & \begin{bmatrix}
0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
12.50298 & -0.06930 & 36.10109 & 0.0000 & -1.12758 & 0.0000 \\
0.0000 & 0.0000 & -4.0000 & 4.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.33333 & 0.0000 & -5.0000 & 0.0000 & 0.0000 \\
-4.07659 & 0.0000 & 0.0000 & 0.0000 & -0.26215 & 377.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & -20.0000 \\
\end{bmatrix}
\end{array}

\begin{array}{ccccccc}
\text{C Matrix} & \begin{bmatrix}
0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
-0.29336 & 0.00224 & 0.0000 & 0.0000 & 0.02351 & 0.0000 \\
-0.34633 & 0.00298 & 0.0000 & 0.0000 & 0.03123 & 0.0000 \\
\end{bmatrix}
\end{array}
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