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THE USE OF COMPUTER-PREDICTED PERFORMANCE IN THE DESIGN OF A
THREE-PORT H-PLANE WAVEGUIDE CIRCULATOR WITH CENTRAL FERRITE
POST

by

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ABSTRACT

THE USE OF COMPUTER-PREDICTED PERFORMANCE IN THE DESIGN OF A THREE-PORT H-PLANE WAVEGUIDE CIRCULATOR WITH CENTRAL FERRITE POST

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An approximate evaluation procedure to predict the performance of a circulator is justified. The scattering matrix description of a circulator is developed. Based on the theory by Davies, the scattering matrix elements are developed from a field theory analysis of the junction. A computer program which incorporates this theory is described. For an input of ferrite parameters and internal biasing magnetic field, a simulated swept-frequency graph of circulator performance is obtained. A three port H-plane waveguide junction circulator with a central ferrite post is tested in the laboratory, agreeing well with the predicted performance. Possibilities for further work are stated.
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I. Introduction

The m-port microwave circulator is a device having wide application over the whole microwave spectrum. For example, it is useful as a duplexer, which permits the use of a single antenna for both transmitting and receiving a signal, an electronic switch, or an isolator in radar systems operating in the frequency range of from approximately 500 MHz to 35 GHz. The circulator may be built as either a stripline junction or a waveguide junction device. Additionally, if the waveguide junction device is desired, it may have either an H-plane or E-plane configuration.

Whether stripline or waveguide, the operation of the device is described schematically in fig. 1-1.

![Fig. 1-1 m-Port Circulator](image)

The direction of circulation is indicated by the arrow, in this instance counter-clockwise. For an input at port one, output is obtained only at port two. Or, for the general case, an input at port \( k \) results in an output at port \( k+1 \) and at no other port. The particular characteristics of a circulator are achieved using some configuration of appropriate materials in the junction to make it nonreciprocal. The nonreciprocity may be obtained with a gyrotropic material such as a ferrite, or in principle, even a plasma. Typically,
this configuration consists of a ferrite material, of optimum geometry, biased by a static magnetic field. The direction of circulation may be reversed by reversing the field.

Thus, the design of a circulator consists of choosing the proper ferrite, magnetic field, and geometry that results in the required performance in the desired frequency range. Due to the lack of a well developed circulator theory, the design procedure has usually been one of experimentation in the laboratory, i.e., a certain combination will be deduced by intuition or ingenuity, and tested. If the results are encouraging, perturbations of the design are tried and retested, the process continuing until satisfaction is achieved. If the initial trial is unsatisfactory, the design is discarded, and a new starting point is sought.

Obviously, this process can be tedious and time consuming, as well as expensive. It is the purpose of this discussion to demonstrate an approximate evaluation procedure based on the theory developed by Davies. A computer program has been written which, when given the electromagnetic properties of the ferrite, its physical dimensions, and the internal biasing magnetic field, will evaluate the ideal performance of the device. Only if the performance meets certain criteria, is it presented as a graphical output. Thus, the time consuming task of parameter variation has been shifted from the laboratory to the high-speed computer. Hundreds of configurations per hour may be tested and only promising ones experimentally evaluated.

As a first measure of the computer evaluation process, a three port, H-plane, waveguide junction circulator with a central ferrite post has been built and tested, and compared with the predicted performance.
II. Theory

2.1 Scattering Matrix Description of a Circulator

The circulator performance may be completely described in terms of its scattering matrix. (2) The scattering matrix, \([S]\), of a device is defined by the relationship

\[ b = [S]a \]

where \(b\) and \(a\) are column vectors with elements \(b_1\) and \(a_1\), respectively. \(a_1\) represents the incident transverse electric field at the terminal plane of the 1st port, and \(b_1\) represents the reflected or outgoing wave. The terminal planes, \(t_1\), are chosen to give a convenient phase factor. (fig. 2.1-1). Since this choice is arbitrary, we choose the terminal planes far enough away from the junction so that we may neglect evanescent waveguide modes.

\[
\begin{align*}
\text{Incident and Reflected Waves} \\
\text{Fig. 2.1-1}
\end{align*}
\]
The most general scattering matrix of a three port, symmetrical, non-reciprocal junction is given by:

\[
[S] = \begin{bmatrix}
S_{11} & S_{12} & S_{13} \\
S_{21} & S_{22} & S_{23} \\
S_{31} & S_{32} & S_{33}
\end{bmatrix}
\]  (2.1-1)

If the junction is perfectly matched, then \(S_{11}=S_{22}=S_{33}=0\).

\[
[S] = \begin{bmatrix}
0 & S_{12} & S_{13} \\
S_{21} & 0 & S_{23} \\
S_{31} & S_{32} & 0
\end{bmatrix}
\]  (2.1-2)

If the junction is assumed to be lossless, then conservation of energy requires that the scattering matrix be unitary, i.e., \([S]^t[S]^*=[I]\) where \([I]\) is the unit matrix, \([S]^t\) the transpose, and \([S]^*\) the complex conjugate of \([S]\). Expanding the unitary expression shows that this condition is equivalent to requiring:

a) The sum of the product of the elements of any column of the scattering matrix with the complex conjugate elements of this same column is unity, i.e.,

\[
S_{21}S_{21}^*+S_{31}S_{31}^*=1 \quad (2.1-3)
\]
\[
S_{12}S_{12}^*+S_{32}S_{32}^*=1 \quad (2.1-4)
\]
\[
S_{13}S_{13}^*+S_{33}S_{33}^*=1 \quad (2.1-5)
\]

b) The sum of the product of the elements of any column of the scattering matrix with the complex conjugate of the elements of any other column is zero, i.e.,

\[
S_{31}S_{32}^*=0 \quad (2.1-6)
\]
\[
S_{12}S_{13}^*=0 \quad (2.1-7)
\]
\[
S_{23}S_{21}^*=0 \quad (2.1-8)
\]
From (6), assume $S_{31} \neq 0$ which implies $S_{32} = 0$. Thus in (4), $|S_{12}| = 1$, which means from (7) that $S_{13} = 0$. Equation (5) implies that $|S_{23}| = 1$. This results in demanding in (8) that $S_{21} = 0$, which gives, finally, from (3) that $|S_{31}| = 1$.

Thus it is discovered that $|S_{12}| = |S_{23}| = |S_{31}| = 1$ and $S_{32} = S_{13} = S_{21} = 0$. The scattering matrix now becomes

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 \\ 0 & 0 & S_{23} \\ S_{31} & 0 & 0 \end{bmatrix}$$

or

$$[S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

(2.1-9)

for proper choice of the terminal planes. For circulation in the opposite direction

$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

(2.1-10)

This corresponds to an ideal circulator, circulating as shown in fig. 2.1-1. Equation (2.1-10) shows that there is perfect transmission from port one to port two, from two to three, from three to one, and there is zero transmission in any other direction. Thus, we may state the theorem: any lossless, matched, non-reciprocal, three-port network is a perfect circulator. This result was first given by Carlin. (3)

Now equation (2.1-10) gives the scattering matrix for a perfect circulator. But the physical device can be far from perfect in many ways. If the junction is not perfectly matched, then $S_{11}$, $S_{22}$, and $S_{33}$ are not zero. The junction
can be lossy, i.e., the ferrite can dissipate energy. If there is not perfect transmission in the circulation direction, then $|S_{13}|$, $|S_{21}|$, $|S_{32}|$ are not equal to unity. If perfect isolation does not occur, then $S_{31}$, $S_{12}$, and $S_{23}$ are not equal to zero.

The approximation of the physical device to the ideal model is measured by the departure of the elements of the scattering matrix from the ideal values. There are three performance indices, expressible in terms of the scattering matrix elements, which, when observed, indicate the utility of the device. These are: isolation, insertion loss due only to scattering and not to dissipation, and reflection coefficient.

To illustrate, consider the scattering matrix of equation (2.1-1) with an input at port one only. We may write

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} a_1 \\ 0 \\ 0 \end{bmatrix}$$

or

$$b_1 = S_{11}a_1$$
$$b_2 = S_{21}a_1$$
$$b_3 = S_{31}a_1.$$ 

Isolation is expressed by the ratio of the output at port three to the input at port one, i.e.,

$$b_3 = S_{31}; \quad \text{Define } S_{31} = \beta \frac{a_1}{a_1}$$

Insertion loss is given by the ratio of the output at port two to the input at port one, i.e.,
\[ b_2 = S_{21}; \text{ Define } S_{21} = \gamma \]
\[ \frac{b_1}{a_1} \]

The reflection coefficient is given by the ratio of the output of port one to the input of port one, i.e.,

\[ b_1 = S_{11}; \text{ Define } S_{11} = \alpha \]
\[ \frac{a_1}{\alpha} \]

Because the junction is symmetric, similar expressions arise from inputs into port two only, and port three only, yielding:

\[ S_{11} = S_{22} = S_{33} = \alpha \]
\[ S_{12} = S_{23} = S_{31} = \beta \]
\[ S_{13} = S_{21} = S_{32} = \gamma \]

Equation 2.1-1 now becomes:

\[
\begin{bmatrix}
    b_1 \\
    b_2 \\
    b_3
\end{bmatrix} =
\begin{bmatrix}
    \alpha & \beta & \gamma \\
    \gamma & \alpha & \beta \\
    \beta & \gamma & \alpha
\end{bmatrix}
\begin{bmatrix}
    a_1 \\
    a_2 \\
    a_3
\end{bmatrix}
\]

where an input may be incident at any of the three ports.

Thus, if the scattering matrix elements can be computed in some manner for the proposed physical device, it is possible to predict its performance as a practical and not an ideal circulator.
2.2 Development of the Scattering Matrix Elements by Field Theory Analysis of the Junction

We make the following assumptions:

1. A junction of \( m \) identical rectangular waveguides equally spaced around the junction comprises the device.

2. Only the \( \text{TE}_{10} \) mode, with the electric field in the direction of the symmetry axis is allowed to propagate in the waveguide.

3. The gyromagnetic material (ferrite) is placed at the center of the junction on the axis of symmetry and is uniformly magnetized.

4. The ferrite post completely spans the height of the cavity.

5. The junction is lossless.

The computation of the scattering matrix of the \( m \)-port junction is achieved by solving for its eigenvalues, from which the matrix elements may be expressed. Since the junction is symmetrical, symmetry properties demand that the scattering matrix have eigenvectors \( \mathbf{v}_0, \mathbf{v}_1, \ldots, \mathbf{v}_{m-1} \), where \( \mathbf{v}_j \) is a column vector with elements

\[
\mathbf{v}_j = \begin{bmatrix}
\exp(2\pi i j/m) \\
\exp(4\pi i j/m) \\
\exp(6\pi i j/m) \\
\vdots \\
1
\end{bmatrix}
\]

with corresponding eigenvalues \( \lambda_0, \lambda_1, \ldots, \lambda_{m-1} \), or

\[
[S]\mathbf{v}_j = \lambda_j \mathbf{v}_j
\]
where

\[
[S] = \begin{bmatrix}
S_0 & S_1 & S_2 & \cdots & S_{m-1} \\
S_{m-1} & S_0 & S_1 & \cdots & S_{m-2} \\
& \cdots & \cdots & \cdots & \cdots \\
S_1 & S_2 & S_3 & \cdots & S_0 \\
\end{bmatrix}
\]

and \( S_j = \frac{1}{m} \sum_{k=0}^{m-1} \lambda_k \exp(-2\pi ijk/m). \) (2.2-1)

The fact that the junction is lossless and \([S]\) is unitary, implies that its eigenvalues must have unit magnitude. Thus the behavior of the nonreciprocal \(m\)-port symmetrical junction is described by the arguments of the \(m\) eigenvalues.

The analysis of the electromagnetic fields in the device may be accomplished with reference to the geometry of figure 2.2-1.
Since the TE₁₀ waveguide mode excites the junction fields, and there are no Z dependent boundary conditions, solutions are desired with the electric field purely in the Z direction, and magnetic field purely in the x-y plane. Thus these fields are Z-independent and are taken to have time dependence exp(ıwt).

Solving the wave equation in the ferrite, \( \nabla^2 E_z + \gamma^2 E_z = 0 \),
yields:

\[
E_z = \sum_{n=-\infty}^{\infty} A_n J_n(\gamma r) e^{-in\phi} \quad (2.2-2)
\]

From which:

\[
H_\phi = -\frac{i}{\omega(\mu^2-k^2)} \sum_{n=-\infty}^{\infty} \{ \mu \gamma A_n J_n(\gamma r) + \kappa A_n n J_n(\gamma r) \} e^{-in\phi} \quad (2.2-3)
\]

\[
H_r = \frac{1}{\omega(\mu^2-k^2)} \sum_{n=-\infty}^{\infty} \{ \gamma \mu A_n J_n(\gamma r) + \frac{\mu^2}{\kappa} A_n n J_n(\gamma r) \} e^{-in\phi} \quad (2.2-4)
\]

\[
E_r = E_\phi = H_z = 0 \text{, and where } \gamma^2 = \frac{\omega^2 \epsilon (\mu^2-k^2)}{\mu} ,
\]

\( \epsilon \) is the ferrite permittivity, and

\[
\begin{pmatrix}
\mu & -i\kappa \\
 i\kappa & \mu
\end{pmatrix}
\]
is the ferrite permeability tensor in the x-y plane. (5)

Similarly, for cavity fields outside the ferrite (r>R)

\[
E_z = \sum_{n=-\infty}^{\infty} \{ B_n J_n(\gamma_o r) + C_n Y_n(\gamma_o r) \} e^{-in\phi} \quad (2.2-5)
\]
\[ H_\phi = -i\sqrt{\varepsilon_0/\mu_0} \sum_{n=-\infty}^{\infty} \{B_n J_n^2(\gamma_0 r) + C_n Y_n^2(\gamma_0 r)\}e^{-in\phi} \quad (2.2-6) \]

\[ H_R = \frac{1}{\omega \mu_0 r} \sum_{n=-\infty}^{\infty} n\{B_n J_n(\gamma_0 r) + C_n Y_n(\gamma_0 r)\}e^{-in\phi} \quad (2.2-7) \]

and \( E_T = E_\phi = H_Z = 0; \gamma_0^2 = \omega^2\varepsilon_0\mu_0 \).

The condition that tangential \( E \) and \( H \) must be continuous at the ferrite-air boundary, i.e., at \( r=R \), yields, for each mode

\[ E_z : A_n J_n (\gamma R) = B_n J_n (\gamma_0 R) + C_n Y_n (\gamma_0 R) \quad (2.2-8) \]

and for \( H_\phi \):

\[ \frac{\varepsilon \gamma_0}{\varepsilon_0 Y} = \{J_n^2(\gamma R) + \frac{\kappa n J_n}{\gamma R J_n(\gamma R)}\}A_n = B_n J_n^2(\gamma_0 R) + C_n Y_n^2(\gamma_0 R) \quad (2.2-9) \]

which gives

\[
\begin{align*}
C_n &= -J_n(\gamma_0 R) \left[ \frac{J_n(\gamma R)}{\gamma R J_n(\gamma R)} + \frac{\kappa n}{\mu(\gamma R)^2} \right] \frac{\varepsilon}{\varepsilon_0} - \frac{J_n^2(\gamma_0 R)}{\varepsilon_0 Y_0 R J_n(\gamma_0 R)} \\
B_n &= Y_n(\gamma_0 R) \left[ \frac{J_n(\gamma R)}{\gamma R J_n(\gamma R)} + \frac{\kappa n}{\mu(\gamma R)^2} \right] \frac{\varepsilon}{\varepsilon_0} - \frac{Y_n^2(\gamma_0 R)}{\varepsilon_0 Y_0 R Y_n(\gamma_0 R)}
\end{align*}
\]  

(2.2-10)

If the m-port junction is excited with a particular eigenvector of the scattering matrix, the electric field of the eigensolution differs only by a phase factor for each port. Because of this fact, and using superposition of the eigensolutions, it is sufficient to discuss the fields in one waveguide together with the area of the cylindrical cavity between it and the junction axis.

The solution for the fields in the waveguide is given by:
\[ E_z = \sum_{p=-1}^{\infty} D_p \sin[(1 + y/a)p\pi] e^{i\beta_p x} \quad (2.2-11) \]

\[ H_x = \frac{i\pi}{2\omega \mu_0 a} \sum_{p=-1}^{\infty} p D_p \cos[(1 + y/a)p\pi] e^{i\beta_p x} \quad (2.2-12) \]

\[ H_y = \frac{1}{\omega \mu_0} \sum_{p=-1}^{\infty} D_p \beta_p \sin[(1 + y/a)p\pi] e^{i\beta_p x} \quad (2.2-13) \]

where \( \beta_1 = [\omega^2 \mu_0 \varepsilon_0 - (\pi/2a)^2]^{1/2} = 2\pi/\lambda_g \)
\( \beta_{-1} = -\beta_1 \)

and \( \beta_p = 1[(p\pi/2a)^2 - \omega^2 \mu_0 \varepsilon_0]^{1/2} \) for \( p \geq 2 \).

The above expressions detail that the only propagating modes are the incident and reflected \( \text{TE}_{10} \) modes. The \( \text{TE}_{p0} \) modes, for \( p \geq 2 \), are the evanescent reflected modes. Thus \( D_1 \) and \( D_{-1} \) are the amplitudes of the incoming and outgoing propagating waves, and with the above fields excited by the \( j \)th eigensolution, the corresponding eigenvalue is \( \lambda_j = -D_{-1}/D_1 \).

The next step is to match the tangential field components of the cavity modes to the waveguide modes along the arc of the junction of radius \( a/\sin(\pi/m) \). This is done by equating \( E_z \) of (2.2-5) and (2.2-11), \( H_\phi \) to \( (H_y \cos \phi - H_z \sin \phi) \) and integrating from \( -\pi/m \) to \( \pi/m \), incorporating the orthogonality properties of \( \exp(\text{in}\phi) \). The result, for any \( n \), of the infinite set is a system in the infinity of unknown waveguide mode amplitudes, \( D_p \), which for the \( j \)th eigensolution must be satisfied for \( n = mq+j, \ q = 0,1,2,... \). Hence, for each
eigensolution, only the ratio $D_{-1}/D_1$ is required from the infinite system.

Defining

$$G_p = D_p \sin(p\pi/2) + i \cos(p\pi/2),$$

the system becomes:

$$\sum_{p=-1}^{\infty} A_p(q) G_p = 0 \quad q = 0, \pm 1, \pm 2, \ldots \quad (2.2-14)$$

where for $p$ odd

$$A_p(q) = \left[ \frac{\pi}{2} \exp[i\beta \rho \cos \phi] \cos[(m q + j) \phi] \right] \cdot \sin(\pi/m)$$

$$\cdot \left\{ \frac{i \beta \rho \lambda}{2\pi} \cos^2 \frac{p\pi \sin \phi}{2 \sin(\pi/m)} - \frac{p \lambda}{4a} \sin \phi \sin \frac{p\pi \sin \phi}{2 \sin(\pi/m)} \right\}$$

$$- \sum_{m}^{m} m q + j \cos \frac{p\pi \sin \phi}{2 \sin(\pi/m)} \right\} d\phi \quad (2.2-15)$$

and for $p$ even

$$A_p(q) = \left[ \frac{\pi}{2} \exp[i\beta \rho \cos \phi] \sin[(m q + j) \phi] \right] \cdot \sin(\pi/m)$$
\[
\left\{ \frac{ip\lambda}{2\pi} \cos \phi \sin \left[ \frac{p\pi \sin \phi}{2 \sin (\pi/m)} \right] + \frac{p\lambda}{4a} \sin \phi \cos \left[ \frac{p\pi \sin \phi}{2 \sin (\pi/m)} \right] \right\}
- S_{mq+j}^m \sin \left[ \frac{p\pi \sin \phi}{2 \sin (\pi/m)} \right] d\phi
\]

(2.2-16)

where

\[
S_n^m = \frac{J_n[-\gamma/a]}{\sin(\pi/m)} + \frac{C_n Y_n[-\gamma/a]}{B_n \sin(\pi/m)}
\]

\[
= \frac{J_n[-\gamma/a]}{\sin(\pi/m)} + \frac{C_n Y_n[-\gamma/a]}{B_n \sin(\pi/m)}
\]

for \( n = mq+j \). The solution to the infinite system can be approximated by taking successively more terms from eq. (2.2-14). Taking the leading \( 2r+1 \) terms for \( q = 0, \pm 1, \ldots, \pm r \), the solution is

\[
\begin{vmatrix}
A_1(r) & A_2(r) & \cdots & A_{2r+1}(r) \\
A_1(r-1) & A_2(r-1) & \cdots & A_{2r+1}(r-1) \\
& & \ddots & \vdots \\
& & & A_1(-r) & A_2(-r) & \cdots & A_{2r+1}(-r)
\end{vmatrix}
\]

\[
\frac{G-1}{G_1} = \lambda j
\]

\[
= \begin{vmatrix}
A_1(r) & A_2(r) & \cdots & A_{2r+1}(r) \\
A_1(r-1) & A_2(r-1) & \cdots & A_{2r+1}(r-1) \\
& & \ddots & \vdots \\
& & & A_1(-r) & A_2(-r) & \cdots & A_{2r+1}(-r)
\end{vmatrix}
\]
The first approximation may be taken as the solution, i.e.,
\[ \lambda_j = -\frac{A_1(q)}{A_1^*(q)} \]
where \( q \) is chosen to minimize \( |mq+\lambda_j| \), and the lowest possible order, \( n \), of cylindrical modes, is taken. This approximation is equivalent to neglecting the evanescent waveguide modes and considering only the dominant propagating modes.

If \( \theta_j \) is defined by \( \lambda_j = \exp(i\theta_j) \), the approximation becomes
\[ \tan(\theta_j + \pi) = \frac{\text{Im}[A_1(q)]}{\text{Re}[A_1(q)]} = \frac{D_n^m + E_n^m(C_n/B_n)}{F_n^m + G_n^m(C_n/B_n)} \]
where \( C_n/B_n \) is given in eq. (2.2-10) and \( D_n^m, E_n^m, F_n^m, \) and \( G_n^m \) are integral functions of only the number of ports, \( m \), the free space wave length, \( \lambda_0 \), and the guide halfwidth, \( a \). This form is most convenient for computer analysis. For a given value of \( m \) and \( n \), and type of waveguide, these integrals may be computed once, and then stored for later use as known constants. Now change the index for \( \lambda_j \). Let \( \lambda_j = \lambda_k \) and substitute into equation (2.2-1) to obtain the scattering matrix elements from the known eigenvalues. This expression becomes:

\[ S_{11} = S_{22} = S_{33} = \alpha = \frac{\lambda_0 + \lambda_1 + \lambda_2}{3} \]
\[ S_{12} = S_{23} = S_{31} = \beta = \frac{\lambda_0 \lambda_1 e^{\frac{2\pi}{3}} + \lambda_2 e^{-\frac{2\pi}{3}}}{3} \]
\[ S_{13} = S_{32} = S_{21} = \gamma = \frac{\lambda_0 \lambda_1 e^{-\frac{2\pi}{3}} + e^{\frac{2\pi}{3}}}{3} \]

Hence, the computation of the elements of the scattering matrix
finally is accomplished by the evaluation of the function $C_n/B_n$, which requires only the values of the ferrite radius and dielectric constant, and the value of the biasing magnetic field.

2.3 Approximation of the Internal Biasing Field of the Ferrite

Because of uncompensated surface poles on a material specimen immersed in a magnetic field, the internal field in the specimen is not the same as the applied field. This phenomenon is expressed by means of the demagnetization factor, which is dependent upon the sample geometry. For the ferrite samples of cylindrical shape used in the circulator, the internal field, for an applied field in the same direction as the cylinder's long axis, is approximated by

$$H_0 = H_A - N_z 4\pi M_S$$

where $H_A$ is the applied field, $N_z$ is the demagnetizing factor, and $M_S$ is the saturation magnetization of the sample. The above expression would be exact only for specimens with an ellipsoidal shape. Hence, to achieve saturation magnetization of a sample, where the internal field is just equal to zero, requires an applied field equal to $N_z 4\pi M_S$ oe.
III. Computer Prediction of Performance

The computer program based on the foregoing theory incorporates several important assumptions:

a) The ferrite post is saturated and uniformly magnetized. For lower values of magnetic field, the phenomenon of low field loss occurs due to individual domain rotation resonance absorption which disappears when the ferrite is saturated. (7)

b) The theoretical values of the susceptibility tensor components are calculated from the Polder-Hogan equations. (8),(9) These equations have proven to be adequate for the prediction of component values not known experimentally. They are of special value in this work since they include the parameters which we wish to adjust over the frequency range of circulator performance. These parameters are: $4\pi M_s$, the saturation magnetization; $\varepsilon_r$, the dielectric constant; $H_0$, the internal biasing field; and $\Delta H$, the ferrite line width. Ferrite line-width may be defined as the difference between the two magnetic field values, one on either side of resonance, where the imaginary part of the diagonal component, $\chi''$, of the susceptibility tensor is half its value at resonance.

c) There is no $Z$ dependence of electromagnetic fields in the junction. If this were not so, the field equations would become inseparable and make a solution much more difficult.

The predicted performance of the circulator is determined by the computation of the scattering matrix at small intervals over the waveguide bandwidth. We selected the frequency band 7-13.0 GHz and standard waveguide for convenience. By appropriate scaling, other frequency ranges can be investigated. The scattering matrix elements are obtained through the calculation of the function $C_n/B_n$ given by equation (2.2-10). For each frequency point, internal biasing magnetic field, and the desired ferrite parameters, the elements of the susceptibility tensor are calculated and
substituted into $C_n/P_n$, which is then evaluated.

The computer program output is presented in the form of a simulated swept frequency graph of reflection coefficient, isolation, and insertion loss. This corresponds to a plot of the output from each port in terms of power loss relative to the input power. Figure 3.1 is a typical output graph. In section 2.1 we defined $\alpha, \beta,$ and $\gamma$. Each may be expressed as a product of magnitude and phase factor:

$$\alpha = A \exp(j\phi_A)$$

$$\beta = B \exp(j\phi_B)$$

$$\gamma = G \exp(j\phi_G)$$

Thus, for the output curves, $A$ is used for the reflection coefficient, $B$ is used for isolation, and $G$ is used for insertion loss.

For a given ferrite saturation magnetization, linewidth and dielectric constant the program input allows variation of the sample radius and the internal biasing magnetic field. A performance graph is printed for each value of the ferrite parameters, for each value of the internal field. Thus, each data input card requires the following information:

1. Ferrite $4\pi M_s$
2. Ferrite $\Delta H$
3. Ferrite dielectric constant
4. Final ferrite radius
5. Increment in ferrite radius
6. Initial ferrite radius
7. Waveguide half width
8. Final internal magnetic field
9. Increment in magnetic field
10. Initial magnetic field
11. Final frequency value
Typical Computer Predicted Performance

Fig. 3.1
12. Frequency increment
13. Initial frequency
14. Performance test parameter value, or alpha-test value.

The machine calculation of the circulator performance is achieved in one or two seconds at the most, for each set of data values. However, the print-out requires 10 seconds for each graph. Hence it is convenient to have a performance criterion which would evaluate each circulator and print out graphs only for those which are promising. This is achieved by the alpha test, referred to in the data input, as follows: From the well-known theorem by Carlin, any lossless, perfectly matched, nonreciprocal three-port microwave junction is a perfect three-port circulator. With the assumption of a lossless junction, the value of the reflection coefficient becomes the measure of the performance of the circulator. The alpha test is such that output graphs are printed only if the magnitude of the reflection coefficient falls below a certain value somewhere in the frequency range. If one is testing a configuration about which nothing is known, a relatively large value of reflection coefficient may be used. As one becomes more selective, output graphs may be restricted to very good values of reflection coefficient, for example, 0.2 or less.
IV. Experimental

4.1 Procedure

A three port, H-plane waveguide junction with a threaded, removable plug (see fig. 4.1-1 photograph) was fabricated for test purposes. The removable plug facilitated the change of ferrite samples. Swept frequency measurements of isolation, reflection coefficient, and insertion loss for several ferrite samples were made. The same basic microwave circuit and equipment, with slight variations, were used for each measurement.

Figure 4.1-2 is a block diagram of the experimental setup for the isolation measurement. It will serve to discuss the basic experimental apparatus. The signal source used delivered a levelled output over the range 7-12.5 GHz. Output frequency could be swept at variable rates over all, or part of this range. A low pass filter with cutoff at 16 GHz was used on the output for harmonic suppression. Next, a 20-dB directional coupler sampled the output, and a hybrid-tee divided this sample between the levelling loop input and a power meter for system monitoring. The levelling loop consisted of a broad band travelling-wave tube amplifier to amplify the low level r-f sample, which was then detected and fed to the levelling input of the swept source. An attenuator on the amplifier output allowed regulation of the feedback amplitude to insure a levelled source output over the desired frequency range.

The biasing magnetic field for the circulator was obtained from a large permanent magnet with an adjustable air gap. The field of the magnet for a fixed air gap was adjustable by use of rotating shunt rings. The pole faces were large (5 cm. diameter) and flat, so that the circulator could be securely clamped by them to fashion a good magnetic circuit.

For the appropriate test, the output and isolated arms of the circulator were terminated in 10-dB directional couplers, which in turn had the through arm terminated in a
Experimental 3-Port, Symmetrical H-Plane Waveguide Junction

Fig. 4.1-1
Fig. 4.1-2
Experimental Setup for
Isolation Measurement
matched waveguide termination, and a crystal detector on the sampling arm. This particular configuration was used in order that the arms of the circulator be matched over the widest frequency range possible. Direct termination of the arms with a crystal detector would have presented a poor match over a wide band. The two crystal detectors were obtained as a matched pair, so that their frequency response would cancel out in the ratio of their output. This output from each detector was fed to the ratio meter, which produced a d.c. voltage proportional to the ratio of the two signals in decibels. This signal was fed to the vertical axis input of the x-y recorder, which received its horizontal input from the swept source. The swept source also provided frequency calibration markers for the horizontal trace.

Modifications to the above described setup are shown in figs. 4.1-3 and 4.1-4 for measurement of reflection coefficient and insertion loss, respectively.

In the experimental procedure, to produce a desired value of internal biasing magnetic field in the sample, an applied field in excess of that value was necessary, the different amount depending upon the demagnetization factor and $\chi' T M_S$ of the sample. Fig. 4.1-5 shows a curve of the appropriate demagnetization factor as a function of the ratio of sample length to diameter. (12)

Four samples of ferrite were used in the experimental circulator. The materials were chosen from a readily available commercial source, with a moderate range in values of magnetic and electrical properties, and physical dimensions. (13) Relatively high values of dielectric constant were desired, since it was predicted that, for operation at a given center frequency, the higher the dielectric constant, the smaller the sample radius. Table 4.1-1 summarizes the sample properties, while Table 4.1-2 gives the demagnetizing factors.
Experimental Setup for Reflection Coefficient Measurement
Experimental Setup for Insertion Loss Measurement
Demagnetizing Factor for Rod Magnetized Parallel to the Long Axis

$K = \frac{\text{length}}{\text{diameter}}$

Demagnetizing Factor, $N_z$

Fig. 4.1-5
<table>
<thead>
<tr>
<th>SAMPLE</th>
<th>MATERIAL</th>
<th>$\Delta H$, o.e.</th>
<th>$4\pi M_s$, g</th>
<th>$\epsilon_r$</th>
<th>R, cm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TT 2-130</td>
<td>300</td>
<td>1000</td>
<td>12.0</td>
<td>0.350</td>
</tr>
<tr>
<td>2</td>
<td>TT 1-109</td>
<td>155</td>
<td>12.50</td>
<td>11.5</td>
<td>0.350</td>
</tr>
<tr>
<td>3</td>
<td>G-1002</td>
<td>120</td>
<td>1000</td>
<td>15.5</td>
<td>0.300</td>
</tr>
<tr>
<td>4</td>
<td>G-1002</td>
<td>120</td>
<td>1000</td>
<td>15.5</td>
<td>0.250</td>
</tr>
</tbody>
</table>

Table 4.1-1

Ferrite Sample Properties
<table>
<thead>
<tr>
<th>SAMPLE</th>
<th>COMPOSITION</th>
<th>LENGTH(_{in})</th>
<th>DIAMETER(_{in})</th>
<th>K</th>
<th>(N_z)</th>
<th>(N_z 4\pi M_s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Nickel Ferrite</td>
<td>0.398</td>
<td>0.275</td>
<td>1.445</td>
<td>0.200</td>
<td>200</td>
</tr>
<tr>
<td>2</td>
<td>Magnesium Ferrite</td>
<td>0.398</td>
<td>0.275</td>
<td>1.445</td>
<td>0.200</td>
<td>250</td>
</tr>
<tr>
<td>3</td>
<td>Gadolinium-YIG</td>
<td>0.398</td>
<td>0.236</td>
<td>1.688</td>
<td>0.170</td>
<td>170</td>
</tr>
<tr>
<td>4</td>
<td>Gadolinium-YIG</td>
<td>0.398</td>
<td>0.197</td>
<td>2.020</td>
<td>0.140</td>
<td>140</td>
</tr>
</tbody>
</table>

Table 4.1-2
Ferrite Sample Demagnetization Factors
4.2 Experimental Results

The experimental results are shown in figures 4.2-1 through 4.2-8. For each sample, at a specified value of internal biasing field ($H_o=200$ oe.), the reflection coefficient, isolation, and insertion loss as a function of frequency are given (figs. 4.2-1 through 4.2-4). The continuous curves are the measured results, and the broken curves are the computer predicted results.

The results for sample one are shown in figure 4.2-1. The measured isolation is shifted 5-7% from the predicted value, but the form of the curve is generally agreeable with predictions. The reflection coefficient shows a deviation from predictions between 9 and 9.5 GHz, and is also shifted in frequency. The insertion loss curve shows excellent agreement above 10.5 GHz, but relatively poor agreement otherwise.

Samples two and three gave the curves of figures 4.2-2 and 4.2-3, respectively. For sample two, the isolation center frequency agrees to within one or two percent, while for sample three, it agrees exactly. The isolation curve for sample three is quite close to predictions above 12 dB, while usually the measured isolation is less than the predicted values for all samples. The reflection coefficient and insertion loss show the deviation from predicted values between 9 and 9.5 GHz, as observed for sample one.

Figure 4.2-4 shows the results for sample four. The generally good agreement between the measured and predicted results for all three parameters is apparent. The insertion loss agreement is excellent over the whole range of measurement. The isolation center frequency is shifted slightly by about 2%, while the reflection coefficient is moderately greater over most of the range.

Of particular importance in the design of narrowband circulators is the center frequency, $f_0$, of circulator operation. $f_0$ is usually taken as the frequency of maximum iso-
CIRCULATOR PERFORMANCE, SAMPLE I.
FIG. 4.2-1
CIRCULATOR PERFORMANCE, SAMPLE 2

FIG. 4-2-2
CIRCULATOR PERFORMANCE, SAMPLE 3

FIG. 4.2-3
CIRCULATOR PERFORMANCE, SAMPLE 4

FIG. 4.2-4
lation. For a ferrite sample of given properties and radius, the center frequency is a function of the internal biasing magnetic field, $H_0$. Figures 4.2-5 through 4.2-8 are plots of center frequency vs. $H_0$ for the four experimental samples. The open circles are the range of measured values, while the triangles are predicted values. The bars represent the predicted variation of center frequency due to the variation of the sample dielectric constant over the manufacturer's stated tolerances. The higher frequency bar represents the minimum dielectric constant, and the lower frequency bar, the maximum dielectric constant. The dielectric constant for the predicted points marked by triangles was taken to be the nominal value given by the manufacturer's specifications.

Sample number one is the only sample whose measured results vary widely from the predicted ones. For samples two and three, the agreement is excellent for field values below 800 oe. There is a relatively constant difference between the actual and predicted values for sample four of about 1.4% for all values of $H_0$, where $H_0$ is the internal biasing field estimated from equation (2.3-1), and is assumed to be uniform. It is interesting to note that, for samples two, three, and four, all but two measured points fell within the range allowed by the tolerance of the dielectric constants. As a practical matter, it is frequently desired to operate the circulator with as low a value of biasing field as possible, so that small permanent magnets may be used on the circulator package. Thus, it is encouraging that the experimental results agree well with predictions at low field values.

From the computer predictions and the measured results, one might consider the circulator in terms of a dielectric loaded resonant cavity. As the radius of the ferrite increases, the center frequency $f_0$, decreases, like the decrease in a cavity resonant frequency as the effective cavity size increases. Similarly, an increase in ferrite dielectric constant with the radius held constant causes $f_0$ to decrease.
Center Frequency vs. Internal Field, Sample 1

Fig. 4.2-5
$4\pi M_s = 1250$
$\Delta H = 155$
$\varepsilon_r = 11.5 \pm 5$

Center Frequency vs. Internal Field, Sample 2

$\Delta H = 0.700 \text{ cm}$

Fig. 4.2-6
Center Frequency vs. Internal Field, Sample 3

Fig. 4.2-7

- $\mu_0M_0 = 1000$
- $\Delta H = 120$
- $\varepsilon_r = 15.5 \pm 5\%$

D = 0.600 cm.
$4\pi M_s = 1000$
$\Delta H = 120$
$\varepsilon_r = 15.5 \pm 5\%$

$D = 0.500 \text{cm.}$

Center Frequency vs. Internal Field, Sample 4

Fig. 4.2-8
The analogous situation for a resonant cavity is that its resonant frequency is inversely proportional to the square root of the dielectric constant, giving the same result. The effect of the internal biasing field on $f_0$ is seen in figs. 4.2-5 through 4.2-8, while the effect of dielectric constant, ferrite radius, and saturation magnetization on $f_0$ can be seen in figs. 4.2-9, 10, and 11.

4.3 Experimental Errors

The most consistent and the largest disagreement between the predicted and measured performance of the circulator occurred for sample number one. The other three samples showed good and sometimes excellent agreement. The correlation between the predicted and measured results should be judged relative to the fact that in the past, no prediction of circulator performance was possible at all.

The disparity is perhaps best shown in fig. 4.2-5, the graph of center frequency against internal field. The difference between the maximum allowed by the dielectric constant variation and the measured results ranges from about $1.2\%$ to $9\%$. If the curves for isolation, reflection coefficient, and insertion loss were shifted lower by this amount, they would be in fairly good agreement with the computer predicted curves. As a check on the experimental procedures, measurements on sample one were interspersed between measurements of other samples, with always the same disparity. To investigate the possibility of an error in the ordering or delivery of the samples, another specimen of the same material was fabricated and tested, yielding no better results. A slight improvement in agreement was achieved by noticing that the Landé $g$ factor of the material was $2.6$ instead of the nominal value of $2.0$ which was used in the program. While not an explicit parameter in the program, $g$ appears in the gyromagnetic ratio, which becomes the proportionality constant
$\varepsilon_r$ vs. $f_0$, GHz

$H_0 = 200$, $\Delta H = 120$, $4\pi M_s = 1000$

Center Frequency vs. Dielectric Constant
(Predicted)
Fig. 4.2-9
Ferrite G1002

Center Frequency vs. Ferrite Radius
(Measured)
Fig. 4.2-10
$\epsilon = 15.5, \Delta H = 120$

Center Frequency vs. $4\pi N_s$
(Predicted)
Fig. 4.2-11
between the resonance frequency of the material and the internal biasing magnetic field. At this time there is no adequate explanation for the sensitivity of the procedure to this particular ferrite sample.

There are small sources of error always present when swept source measurements with waveguide systems are done. For example, the slight day-to-day variations in the tightness of waveguide joints will be observed to have an effect over the broad frequency range of the measurements. Sources such as these are tried to be compensated for by including as much as possible of the system external to the circulator in the calibration procedures, so that the circulator performance is measured independent of the system frequency response. The calibration of the instruments will sometimes become invalid during the period of a measurement, requiring that the collected data be rejected. Hopefully, all such events were noted, and the data not incorporated in the results.

The computer program assumed that a uniform magnetic field was applied to the ferrite sample, which was saturated. With the broad magnet pole-pieces, and the clamping of the junction between them, this condition was probably fairly well met. The applied field was measured at the center of the junction with a Hall-probe calibrated to \( \pm 0.25\% \) error.

The initial experimental measurements exhibited sharp spikes in the swept frequency curves at several points. These spikes were attributed to the presence of a small air gap between the ferrite cylinder and the top of the junction cavity. A small square of metal foil, slightly larger than the sample diameter, was fastened to the bottom of the removable plug to insure direct metal-to-ferrite contact. The correct foil thickness, a few thousandths of an inch, was experimentally determined. Upon addition of the foil, the spikes disappeared, and the center frequency of circulation was very much improved. This problem and solution are well known in the empirical design and production of these devices.
A physical device cannot, of course, be lossless, although sometimes it can come quite close. Besides the finite conductivity of the waveguide, there were small scratches and irregularities in the junction, resulting from much cleaning and replacing of ferrite samples, which could contribute to losses. The ferrite can contribute to losses in two ways. The ideal material is assumed to be a nonconductor. But the presence of an electric field on the real ferrite causes currents which result in heating losses. There are magnetic losses also, due to absorption by the material when the biasing magnetic field is large enough, or the operating microwave frequency nears the resonance frequency. The presence of the removable plug in the cavity also contributes to the overall losses. Hence, the assumption needed in the analysis that the junction is lossless clearly is a small source of difference between any predicted and measured performance.
V. Conclusions

The experimental results show that for the simple three-port H-plane waveguide junction circulator with a central ferrite post, the theory is adequate to allow confidence in the computer predicted performance of the device, and to allow the use of this computer method for the design of a circulator to meet modest requirements.

The bandwidth of a circulator may be defined as the ratio of the frequency band for which the isolation is greater than a specified value, to the frequency of maximum isolation. For isolation greater than 10db, the measured and predicted bandwidths are in adequate agreement. While the theory suffices for the simple device investigated, of more practical interest is a circulator which would have much greater isolation over a very wide bandwidth, say 10-20% compared with the 2-3% obtained. Having been experimentally verified, this theory and procedure can be used as a starting point for a more exhaustive and exact analysis for the development of better circulators. Instead of only a ferrite post, various combinations of ferrites, dielectrics, and conducting pins in concentric cylinders or tubes may produce wider band three and four port devices.
VI. References


7. Soohoo, op. cit., p. 87.


13. Trans-Tech Inc., 12 Meem Avenue, Gaithersburg, Maryland.
VII. Acknowledgements

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