RICE UNIVERSITY

A METHOD FOR MEASURING THE CONDUCTIVITY OF HIGH
RESISTIVITY SEMICONDUCTOR MATERIALS

by

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ABSTRACT

A simple method of measuring the conductivity of semiconductor materials is studied both analytically and experimentally. The basic method utilizes two pairs of coils, operating as transformers, and excited with a source of sinusoidal voltage. The two primary coils are connected in series, so that they have the same current flowing through them; the secondary coils are connected in series, in such a manner that the induced voltages tend to cancel. An approximate null in the secondary circuit is produced by positioning of the coils. A sample (usually small) of semiconductor material is placed between one pair of coils. A voltage proportional to the bulk conductivity of the sample is induced in the secondary winding for that pair, disturbing the voltage balance in the secondary circuit. Because this voltage is 90 degrees out of phase with the original voltage, it may be detected by methods that take account of the phase relations. In the preliminary experimental studies reported in this thesis, the out-of-phase voltage component was determined by simple trigonometric relations. A practical laboratory instrument would use a phase sensitive detector excited in such a manner that it responds to the desired component.

The experimental study verified the essential correctness of the theoretical analysis and demonstrated the potential usefulness of a practical instrument based upon the principles utilized. Certain refinements and simplifications of design for a practical instrument are indicated.
1. **INTRODUCTION**

1-1. **Description.**

During the past several years, interest in the measurement of semiconductor resistivity has been increasing, and several methods are in use. Among these are the four-point probe method\(^1\)-\(^2\), a method using microwave techniques\(^6\)-\(^10\), the Q-meter method\(^3\)-\(^4\) at the megacycle frequencies, and the method of strong electric field\(^11\)-\(^12\).

The measurement of semiconductors with the microwave techniques has been of particular interest. These methods have both advantages and disadvantages. The measuring procedure is not simple; and they impose limitations on the size, the thickness of the samples, and the region of low and high resistivity at gigacycles. The four-point probe method essentially has limitations because of contact potentials and surface effects. An important measurement in the manufacture of semiconductor materials is the volume resistivity of the material. At present, this is measured by an ohmic contact method, but the difficulty in this method is encountered in maintaining the correct probe shape and spacing, the degree of accuracy being low over the high resistivity range.

Volume resistivity (volume conductivity) may be measured precisely with sensitive equipment by the method described in this project. The equipment contains an impedance bridge made from coaxial coil systems in the primary and secondary circuits. This new method is entirely different from the methods mentioned above; also it has essentially no limitations on size and thickness of samples, since the conduction voltage due to the conductivity of a sample is exactly proportional to the volume conductivity in the sample \((V_o = k\sigma)\) when the semiconductor
material to be measured is placed between a coaxial coil pair in the equipment.

In the low and high resistivity range of semiconductor material, a high degree of accuracy is obtained by this method. The measurement is simple and rapid. This method represents a new approach to the measurement of volume resistivity of semiconductor, as compared with conventional methods. The advantages in this method are as follows:

(1) A high degree of accuracy is obtained in the measurement of semiconductor material from low resistivity to high resistivity.

(2) There are essentially no limitations on size and shape of the sample to be tested.

(3) The measurements are made directly, rapidly, and simply, as compared with other methods.

(4) Sheet resistivities of diffused layer may be measured by this method.
2. PRINCIPLES AND TECHNIQUES FOR MEASURING THE VOLUME CONDUCTIVITY OF SEMICONDUCTORS.

2-1. Configuration of the Electrical Circuits.

The circuit configuration consists of a pair of primary coils, often called the transmitter coils, and a pair of secondary coils, called the receiver coils. The circuit of this instrument is essentially a mutual impedance bridge circuit as shown in Fig. 1.

The primary circuit, consisting of two series-connected primary coils, is energized by an oscillator; and the second circuit, consisting of two bucking series-connected secondary coils, is connected to an amplifier system having a voltmeter at its output. We suppose the currents and voltages vary sinusoidally with time. The current in either primary coil sets up a magnetic field which induces a voltage, referred to as the mutual inductance voltage, in the corresponding receiver coil. When a sample of semiconductor material is placed in the region between one of the primary coils and its secondary coil, the flux field for that pair is modified to induce a new component of voltage in the secondary coil. The analysis of Section 2-3 indicates that for small conductivity of the sample (high resistivity) there is a component of voltage $V_\sigma$ proportional in magnitude to the product of the conductivity of the sample and the number of ampere turns in the primary coil. This voltage $V_\sigma$ is $90^\circ$ out of phase with the mutual voltage $V_m$ induced whether or not a sample is present.

The behavior of this instrument depends on the frequency applied by the oscillator. The mutual inductance voltage, for a given current, increases with frequency. On the other hand, for a fixed driving
voltage, the primary current decreases with increasing frequency. When the sample of semiconductor material is placed between one pair of coils, a voltage proportional to the conductivity is induced in the secondary coil of this pair, changing the balance condition.

This voltage, for sinusoidal operation, is very nearly 90° out of phase with the mutual inductance voltage. This voltage is determined from the voltage $V_m$ (mutual inductance voltage in the balanced condition) and the voltage $V_s$ produced when the sample is placed in position. The voltage $V_\sigma$ due to the conductivity may be determined by the relationship

$$V_\sigma^2 = V_s^2 - V_m^2$$

It is necessary to maintain good balance -- that is small $V_m$ -- so that the amplifier-detector will not be overloaded and so that it is not necessary to take small differences of large quantities when determining $V_\sigma$. If an amplifier is used, its gain must be known in determining the voltage $V_m$ and $V_s$. The conductivity $\sigma$ may be determined from the relation

$$\sigma = K V_\sigma$$

$$= K \sqrt{V_s^2 - V_m^2}$$

where $K$ = proportionality constant called the instrument constant.

The instrument constant may be expressed in the form

$$K = \frac{\mu_0 s^2 \text{eff} I_p \pi}{D}$$

where $f$ = frequency in cycle/sec.

$\mu$ = magnetic permeability of free space ($4\pi \times 10^{-7}$ henry/meter)

$Seff$ = effective area of coil in square meters

$I_p$ = current in the primary circuit in amperes
D = coil space of two coaxial coils in meters

\( V_\sigma \) = conduction voltage in volts

The factors determining this constant are discussed further in the next section.

A direct measurement of \( V_\sigma \) may be made with a phase-sensitive detector-amplifier arranged to respond to the quadrature component of \( V_s \). This method was not used in this investigation of the feasibility of the basic method, although it should be used in a practical instrument. The conduction voltage \( V_\sigma \), as discussed above, is exactly proportional to the volume conductivity. Samples having a wide range of conductivity above \( 10^{-4} \) mho/cm may easily be determined by direct measurement. Samples having a conductivity below \( 10^{-4} \) mho/cm may be determined by refund measuring techniques.

The mutual inductance voltage is an important factor in this instrument. The characteristics of this voltage may be seen clearly by comparison of experimental values and theoretical calculation.

Since the two coaxial coil pairs are located at such a distance that these coil pairs do not affect each other, the mutual inductances may be determined for each pair independently. By Neuman's formula, mutual inductance \( M \) is given by

\[
M = \frac{M}{4 \pi} \int_0^{2\pi} \frac{2 \int_0^{2\pi} \frac{\partial \cos \phi \, d\phi}{\sqrt{d^2 + a_1^2 + a_2^2 - 2a_1a_2 \cos \phi}}}
\]
where

\( a_1, a_2 = \) the radii of the loops
\( d = \) the distance between the planes of the coils
\( r = \) the distance between current elements
\( r_1 = \) the projection of this distance

then

\[
 r = \sqrt{d^2 + r_1^2}, \quad r_1^2 = a_1^2 + a_2^2 - 2a_1a_2 \cos \varphi 
\]

\( ds, ds_2 = a_1a_2 \varphi \, d\Psi \, \cos \varphi \)

The integration with respect to \( \varphi \) is given by

\[
 M = \frac{\mathcal{M} a_1 a_2}{2} \int_0^{2\pi} \cos \varphi \, d\varphi 
\]

putting \( \varphi = 2\alpha \), \( r \) is given by

\[
 \gamma = \sqrt{d^2 + \alpha_1^2 + \alpha_2^2 + 2 \alpha_1 \alpha_2 - 4 \alpha_1 \alpha_2 \cos \alpha} = \sqrt{d^2 + (\alpha_1 + \alpha_2)^2} \sqrt{1 - \kappa^2 \cos^2 \alpha}
\]

where

\[
 \kappa = \frac{2}{\sqrt{d^2 + (\alpha_1 + \alpha_2)^2}}
\]

Consequently, as \( \kappa < 1 \) always,

\[
 \frac{\alpha_1 \alpha_2}{\gamma} = \frac{\kappa \sqrt{\alpha_1 \alpha_2}}{2 \sqrt{1 - \kappa^2 \cos^2 \alpha}}
\]

Hence

\[
 M = \mathcal{M} \sqrt{\alpha_1 \alpha_2} \frac{\kappa}{2} \int_0^{\pi} \frac{\cos^2 \alpha \, d\alpha}{\sqrt{1 - \kappa^2 \cos^2 \alpha}}
\]

But

\[
 K \cos 2\alpha = K (2 \cos^2 \alpha - 1) = \frac{2}{K} (K^2 \cos^2 \alpha - 1) + \frac{2}{K} - K
\]

\[
 \frac{1}{\sqrt{1 - \kappa^2 \cos^2 \alpha}} = \frac{2}{K} \frac{1}{\sqrt{1 - \kappa^2 \cos^2 \alpha}} - \frac{2}{K} \sqrt{1 - \kappa^2 \cos^2 \alpha}
\]

Furthermore

\[
 \frac{1}{2} \int_0^{\pi} \sqrt{1 - \kappa^2 \cos^2 \alpha} \, d\alpha = \int_0^{\pi} \sqrt{1 - \kappa^2 \cos^2 \alpha} \, d\alpha = \int_0^{\pi} \sqrt{1 - \kappa^2 \sin^2 \alpha} \, d\alpha
\]
\[ M = \mu \sqrt{\alpha \beta} \, g(K) \]

\[ g(K) = \left( \frac{2}{K} - K \right) K(K) - \frac{2}{K} E(K) \]

\[ K(K) = \int_0^{\frac{\pi}{2}} \frac{d\varphi}{\sqrt{1 - K^2 \sin^2 \varphi}} \]

\[ E(K) = \int_0^{\frac{\pi}{2}} \sqrt{1 - K^2 \sin^2 \varphi} \, d\varphi \]

\[ K = \frac{(\alpha + \alpha_x)^2}{(\alpha + \alpha_x)^2 + d^2} \]
$K(k)$ is called the complete elliptic integral of the first kind.

$E(k)$ is called the complete elliptic integral of the second kind.

In order to demonstrate the dependence of instrument operation on frequency, curves of primary current and secondary voltage for given primary voltage are shown in Fig. 2. Because of the high resistivity or low conductivity, the primary current changes by a negligible amount when the sample is inserted. Therefore, no correction is made for current changes.
<table>
<thead>
<tr>
<th>F. Kc/Sec.</th>
<th>Et. Volts</th>
<th>Er. Millivolts</th>
<th>Ir Milliamps</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>18 = 6.36</td>
<td>2.8</td>
<td>86</td>
</tr>
<tr>
<td>150</td>
<td>18</td>
<td>4.7</td>
<td>58</td>
</tr>
<tr>
<td>200</td>
<td>18</td>
<td>7.58</td>
<td>44</td>
</tr>
<tr>
<td>250</td>
<td>18</td>
<td>12.6</td>
<td>34</td>
</tr>
<tr>
<td>300</td>
<td>18</td>
<td>17.8</td>
<td>28</td>
</tr>
<tr>
<td>350</td>
<td>18</td>
<td>26.2</td>
<td>24</td>
</tr>
<tr>
<td>400</td>
<td>18</td>
<td>36.5</td>
<td>21</td>
</tr>
<tr>
<td>450</td>
<td>18</td>
<td>52.0</td>
<td>18</td>
</tr>
<tr>
<td>500</td>
<td>18</td>
<td>66.5</td>
<td>16</td>
</tr>
<tr>
<td>550</td>
<td>18</td>
<td>75.0</td>
<td>14</td>
</tr>
<tr>
<td>600</td>
<td>18</td>
<td>80.2</td>
<td>13</td>
</tr>
<tr>
<td>650</td>
<td>18</td>
<td>83.1</td>
<td>12</td>
</tr>
</tbody>
</table>
2-2. **Configuration of the Measuring Instrument.**

In mechanical design, the following points must be taken into account in the design of the instrument. All the material in the supporting structure of the instrument must be insulators with very small dissipation factors over the frequency range of interest. It is especially necessary in the technical mechanical design to consider the magnetic shielding and coaxial coil systems, since this instrument is a sensitive electronic device. Input wave forms in such a sensitive electronic device are often altered by stray fields resulting from extraneous fields and leakage flux of neighboring components, such as a transformer, a choke, or a transmitter-receiver operating near by. The proper positioning of the components is necessary to minimize interaction between the two coil systems. Since the magnitude of the stray fields is generally very small, these affects may be removed by applying good shielding. The shielding efficiency should be tested over the surrounding equipment with a current probe or magnetometer. Thus the shielding percentage represents the reduction of the voltage induced by stray fields. A further consideration in the mechanical design of an experimental instrument is the provision for precision adjustment of the position of various points of the instrument.

In calculating the effective area of coil, it is useful to modify the conventional formula for calculating the effective area. Since the measurement of flux density with a fluxmeter or ballistic galvanometer depends on the effective area of the coil, precision calculation of effective area of coil is very important. A coil constant is the product of the number of turns \( N \) by its effective area \( S_{eff} \), so that
in a round coil with an evenly distributed winding, the area may be
determined by the modified formula

\[ NS_{\text{eff}} = \int_0^r S(r) dN = \int_{r_1}^{r_2} \pi r^2 \frac{N}{r_1 + r_2} dr \]

\[ S_{\text{eff}} = \frac{\pi}{3} (r_e^3 + r_e r_i + r_i^2) \]

where \( r_e \) is the external radius of the coil, and
\( r_i \) is its internal radius of the coil.

The calculation of effective area of coil in the coaxial coil system
may be determined by the following:

\[ S_{\text{eff}} = (S_1 - S_e) + (S_2 - S_e) + (S_3 - S_e) + \cdots (S_n - S_{n-1}) \]

\[ S_e = \pi r_e^2 \] initial radius

\[ S_i = \frac{\pi}{3} (r_i^2 + r_e r_i + r_i^2) \] internal radius

\[ S_2 = \frac{\pi}{3} (r_2^2 + r_e r_i + r_i^2) \] internal radius

\[ S_3 = \frac{\pi}{3} (r_3^2 + r_e r_i + r_i^2) \] internal radius

\[ S_n = \frac{\pi}{3} (r_n^2 + r_e r_i + r_i^2) \] internal radius

\[ S_{\text{eff}} = \frac{\pi}{3} \left\{ (r_e^2 + r_i^2 + r_e^2) - 3r_i^2 \right\} + \left\{ (r_e^2 + r_i^2 + r_e^2) - (r_e^2 + r_i^2 + r_e^2) \right\} \]

\[ + \left\{ (r_3^2 + r_i^2 + r_i^2) - (r_3^2 + r_i^2 + r_i^2) \right\} \]

\[ + \left\{ (r_n^2 + r_i^2 + r_i^2) - (r_n^2 + r_i^2 + r_i^2) \right\} \]
Let \( r_i = (r_0 + d) \)
\( r_i = (r_i + d) = r_0 + 2d \)
\( r_3 = (r_2 + d) = r_0 + 3d \)
\( \cdots \)
\( r_n = (r_{n-1} + d) = r_0 + nd \)

\( r_i = \) initial radius
\( d = \) diameter of coil
\( n = \) number of turns of coil

\[
S_{\text{eff}} = \frac{\pi}{3} \left( (r_0 + nd)^2 + (r_0 + nd)(3r_0 + n-1d) + (r_0 + n-1d)^2 \right) - r_0 \pi
\]
\[= \frac{\pi}{3} \left( 3r_0^2 + (nd - n-1d)(3r_0 + n-1d) + (n-1d)^2 \right) - r_0 \pi \]

\[
S_{\text{eff}} = \frac{\pi}{3} \left( 3r_0^2 + (nd - n-1d)(3r_0 + n-1d) + (n-1d)^2 \right) - r_0 \pi
\]
The coil used in the experimental instrument had the following dimensions:

\[
d = 0.017 \text{ inches} \\
r_o = 0.25 \text{ inches} \\
n = 68 \text{ turns} \\
n-1 = 67 \text{ turns}
\]

The effective area is thus

\[
S_{\text{eff}} = \frac{\pi}{3} \left[ 3 \times 0.25^2 + (68 \times 0.017 + 67 \times 0.017)(3 \times 0.25 + 68 \times 0.017) \\
+ (67 \times 0.017)^2 \right] - 0.25^2 \times 3.14
\]

\[
= 5.9362525 \text{ inch}^2 \\
= 5.9362525 \times 6.4516 \text{ cm}^2 \\
= 38.2979799 \text{ cm}^2 \\
= 3.8298 \times 10^{-4} \text{ m}^2
\]

\[
S_{\text{eff}} = 3.8298 \times 10^{-4} \text{ m}^2
\]
In considering the electrical properties of materials, an examination of the properties of several possible materials showed that polyethylene and polystyrene had the most desirable properties. The most important characteristics for purposes of the investigation are shown in the following table.

<table>
<thead>
<tr>
<th>Name of Materials</th>
<th>Composition</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polyethylene</td>
<td>Dielectric strength [VPM-60 cps]</td>
<td>400-500</td>
</tr>
<tr>
<td></td>
<td>Dielectric constant</td>
<td></td>
</tr>
<tr>
<td></td>
<td>i) 60 cps at 23° C</td>
<td>2.28-2.32</td>
</tr>
<tr>
<td></td>
<td>ii) 10⁶ cps at 23° C</td>
<td>2.28-2.32</td>
</tr>
<tr>
<td></td>
<td>Dissipation factor (tan φ)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>i) 60 cps at 23° C</td>
<td>1 - 5 x 10⁻⁴</td>
</tr>
<tr>
<td></td>
<td>ii) 10⁶ cps at 23° C</td>
<td>1 - 5 x 10⁻⁴</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>Dielectric constant</td>
<td></td>
</tr>
<tr>
<td></td>
<td>i) 60 cps at 25° C</td>
<td>2.45-2.50</td>
</tr>
<tr>
<td></td>
<td>ii) 10⁶ cps at 25° C</td>
<td>2.50-2.60</td>
</tr>
<tr>
<td></td>
<td>Dissipation factor</td>
<td></td>
</tr>
<tr>
<td></td>
<td>i) 60 cps at 25° C</td>
<td>10-54 x 10⁻⁴</td>
</tr>
<tr>
<td></td>
<td>ii) 10⁶ cps at 25° C</td>
<td>10-54 x 10⁻⁴</td>
</tr>
<tr>
<td>Polytetrafluoroethylene</td>
<td>Dielectric strength [VPM-60 cps]</td>
<td>400-500</td>
</tr>
<tr>
<td></td>
<td>Dielectric constant</td>
<td></td>
</tr>
<tr>
<td></td>
<td>i) 60 cps at 25° C</td>
<td>2 - 2.2</td>
</tr>
<tr>
<td></td>
<td>ii) 10⁶ cps at 25° C</td>
<td>2-2.2</td>
</tr>
<tr>
<td></td>
<td>Dissipation factor</td>
<td></td>
</tr>
<tr>
<td></td>
<td>i) 60 cps at 25° C</td>
<td>2 x 10⁻⁴ - 3 x 10⁻⁴</td>
</tr>
<tr>
<td></td>
<td>ii) 10⁶ cps at 25° C</td>
<td>2 x 10⁻⁴ - 3 x 10⁻⁴</td>
</tr>
</tbody>
</table>
These characteristic curves of the electrical properties are shown by Figs. 3 and 4. These materials, as shown in the figure, are but little affected by the frequency below several megacycles. Thus we can say that the dielectric constants and dissipations of the materials used in the experimental instrument are constant at all frequencies of interest below one megacycle.
The fundamental equation to be considered in measuring the conductivity of semiconductors may be derived from Maxwell's equation as follows:

\[ \nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \quad \nabla \times \vec{D} = \rho \quad (1) \]

\[ \nabla \times \vec{H} - \frac{\partial \vec{D}}{\partial t} = \vec{J} \quad \nabla \times \vec{B} = 0 \quad (2) \]

where

\[ \vec{B} = \mu \vec{H} \]

\[ \vec{D} = \varepsilon \vec{E} \]

\[ \vec{J} = \sigma \vec{E} \quad (3) \]

An exact analysis of the voltages induced in the secondary coil when the sample is placed between the primary and the secondary coils is a very difficult problem. In order to understand the behavior, we consider a related case whose solution has been carried out \[ \] . We suppose the coils are immersed in a homogeneous medium which has a small conductivity \( \sigma \) as well as permeability \( \mu \) and permittivity \( \varepsilon \). The analysis shows that the voltage induced in the secondary consists of a term 90° out of phase with the primary coil current plus a term proportional to the conductivity which is in phase with the primary coil current. It is this second component which is used to determine the conductivity. Physical reasoning and experimental evidence indicate that quite similar behavior is obtained in the case of a small sample injected into the space between a pair of coils. Various geometrical factors are modified, but the general nature of the solution should be unchanged.
Considering the condition of the time dependence and substituting the Eq. 3 into Eq. 1 and Eq. 2, we have
\[ \nabla \times \vec{E} - i \omega \mu \vec{H} = 0 \quad \text{where} \quad \nabla \times \vec{E} = \rho / \epsilon \] (4)
\[ \nabla \times \vec{H} - (\sigma - i \omega \epsilon)\vec{E} = 0 \quad \text{where} \quad \nabla \times \vec{H} = 0 \] (5)
We may assume that there is no buildup of a charge anywhere as the current loop formed between two coil systems, since the conductivity medium is the same along all parts of the current loop and the lines of current flow are very close. Then accordingly
\[ \nabla \times \vec{E} = 0 \] (6)
to conditions such that \( \nabla \times \vec{H} = 0 \), \( \nabla \times \vec{E} = 0 \), it may be written in the form
\[ \vec{H} = \nabla \times \vec{A} \] (7)
\[ \vec{E} = i \omega \mu \vec{A} \] (8)
Substituting these equations into Eq. 4, we have
\[ \nabla \times \nabla \times \vec{A} - (\sigma - i \omega \epsilon) i \omega \mu \vec{A} = 0 \] (9)
This Eq. 9 may be reduced in the simple form since \( \nabla \times \vec{A} = 0 \) according to vector identity.
\[ \nabla \left( \nabla \cdot \vec{A} \right) - \nabla^2 \vec{A} - (\sigma - i \omega \epsilon) i \omega \mu \vec{A} = 0 \] (10)
where
\[ \nabla^2 = i \omega \mu (\sigma - i \omega \epsilon) \]
\[ = i \omega \mu \sigma + \omega^2 \mu \epsilon \] (11)
\( \mu = 4\pi \times 10^{-7} \) henries/meter, the magnetic permeability of free space.
\( \epsilon = 8.854 \times 10^{-12} \) farads/meter, the permittivity of free space.
The second term of Eq. 11 can be neglected since it is of small value.
\[ \sigma^2 = i \omega \mu \sigma \]
Hence

$$\frac{\partial^2 \mathbf{A}}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \mathbf{A}}{\partial \phi} + \frac{\partial^2 \mathbf{A}}{\partial Z^2} + \Phi^2 \mathbf{A} = 0$$  \hspace{1cm} (13)$$

Since the surrounding medium is homogeneous and all the sources of vector potential are included in the current density vector \( \mathbf{J}_s \), the particular solution of this equation may be given by the following form [Reference 39-41,]

$$\mathbf{A} (\rho, Z, \phi) = \frac{1}{4\pi} \int_{\mathcal{U}} \frac{J_s (\rho', Z', \phi')}{R} e^{-\alpha R} d\mathcal{U}$$  \hspace{1cm} (14)$$

\( \mathbf{A} \) = vector potential at a point \( p(\rho, Z, \phi) \)

\( J_s (\rho', Z', \phi') \) = the source current density at the point \( (\rho', Z', \phi') \)

\( R \) = the distance from the current density component to the point \( p \)

\( e^{-\alpha R} \) = the phase shift and attenuation in the electromagnetic wave when it travels from the source to the point \( p \)

\( P(\rho, Z, \phi) \) = arbitrary point with cylindrical coordinates between two coil pairs

\( \alpha \) = propagation constant \( (\alpha^2 = i\omega \mu \sigma \text{ when displacement current can be neglected}) \)

Since the source current \( J_s \) and vector potential \( \mathbf{A} \) have the single components under the condition of cylindrical symmetry, \( J_s \) and \( \mathbf{A} \) are functions of \( R \) and \( Z \). Thus the vector potential may be expressed
in the following form

\[ \mathbf{A}(r, \varphi) = \frac{1}{4\pi} \int \mathbf{J}_S(r', \varphi') \cos (\varphi - \varphi') \frac{e^{-\alpha R}}{R} d\mu \]  

(15)

In fact, the current source consists of the transmitter coil and it is the circular current in the coaxial coils with \( N \) turns. Expanding the exponential factor and denominator in powers of a \( \cos (\varphi - \varphi') \), we have

\[ \mathbf{A} = \frac{II(\pi a^3)}{4\pi} \frac{\rho}{R^3} (1 + \alpha R) e^{-\alpha R} \]  

(16)

If \( A \) is recognized as the vector potential due to a magnetic dipole moment \( M \), then \( M = \tau I (\pi a^3) \)

This eq. 16 may be found by the Laplace transform directly from electromagnetic wave eq. 13, and also found by integral equation \( 2 \).

When potential \( A \) is located between the transmitter coil and the receiver coil, \( E = i\omega \mu A \). Thus the induced voltage \( V \) in the receiver coil

\[ \mathbf{V} = 2\pi \omega R E(a, d) = 2\pi \omega R i\omega \mu \mathbf{A}(a, d) \]

\[ \mathbf{V} = \frac{II(\pi a^3)}{4\pi} \frac{A2\pi i\omega R u}{d^3} e^{-\alpha d} \]

\[ \mathbf{V} = i \frac{2i\omega R u}{4\pi} \frac{(1 + \alpha d)}{d^3} e^{-\alpha d} \]

for \( \rho = a, d = R \)

\[ \mathbf{V} = i \frac{2S_i\omega R u}{4\pi} \frac{(1 + \alpha d)}{d^3} e^{-\alpha d} \]

for \( S_i = \pi a^2 = \pi a^2 R = S \)

\[ \mathbf{V} = \frac{i MS^2 \omega R}{4\pi d^3} \left( 1 - \frac{(\alpha d)^2}{2} + \frac{(\alpha d)^3}{3} - \frac{(\alpha d)^4}{8} + \cdots + \frac{(\alpha d)^n (\alpha d)^{n-1}}{n!} \right) \]  

(17)

Expanding with respect to \( \alpha d \) we have

To study the solution, we consider the first two terms of the expansion, neglecting higher order terms.
\[ \vec{\nabla} = j \frac{M^2 \omega I}{2 \pi d^3} \left( 1 - \frac{(d \cdot d)^2}{2} \right) \]
\[ = j \frac{M^2 \omega I}{2 \pi d^3} \left[ \left( 1 - \frac{\omega^2 \mu_0 d^2}{2} \right) - j \frac{\omega \mu_0 d^2}{2} \right] \]
\[ = \frac{M^2 \omega^2 I}{4 \pi d} \sigma + j \left( 1 - \frac{\omega^2 \mu_0 d^2}{2} \right) \frac{M^2 \omega I}{2 \pi d^3} \]

For \( \sigma = 0 \)

\[ \vec{\nabla} = \nabla \sigma = j \left( 1 - \frac{\omega^2 \mu_0 d^2}{2} \right) \frac{M^2 \omega I}{2 \pi d^3} \]

The term \( \nabla \sigma \) must be

\[ \nabla \sigma = \frac{M^2 \omega^2 I}{4 \pi d} \sigma \]

which is 90° out of phase with \( \nabla I \).

The second term in parenthesis \( \frac{\omega^2 \mu_0 d^2}{2} \) is usually neglected as very small compared to unity.

In order to take account of the fact that the sample occupies only a small part of the region surrounding the coils, we must use a sample factor. We thus suppose

\[ \nabla \sigma = K_s \frac{M^2 \omega^2 I}{4 \pi d} \sigma \]

If we assume the field is constant throughout the sample, then the sample factor must vary directly with sample volume \( v \). This has been verified experimentally

\[ K_s = K_s' v \]

Substituting in the above equation and solving for \( \sigma \) we obtain

\[ \sigma = \frac{d}{f^2 I \sqrt{K_s'a}} = K_r \nabla \sigma \]
where

\[ a = \mu^2 \pi s^2 \]

\[ v = \text{sample volume} \]

\[ d = \text{separator of coils} \]

\[ f = \text{frequency of the sinusoidal variations} \]

\[ I = \text{primary coil current} \]

\[ K_s' = \text{sample factor which depends upon the shape of the sample.} \]

\[ K_s' \] must be determined experimentally in the absence of a detailed analysis.

In Eq. 18, if \( \sigma = 0 \), \( \alpha = \omega \mu \sigma \),

\[ \nabla_m = i' \frac{2 s^4 I \omega \mu}{4 \pi d^3} \]  \hspace{1cm} (19)

This equation 19 represents the mutual-inductance voltage induced in the receiver coil by direct induction from the current in the transmitter coil. This voltage has a phase angle 90° relative to the transmitter current. When \( \sigma \) increases, the voltage \( V_x \), the component 90° out of phase with \( I \), will decrease while the voltage \( V_r \) in the real part to have component in phase with \( I \) will begin to grow. This behavior can be seen more clearly by separating the two parts, expanding in power of \( \alpha d \), and writing the results as follows:

\[ \nabla_k = K \sigma (1 - \frac{2}{3} \frac{d}{\beta}) \]  \hspace{1cm} (20)

\[ \nabla_x = i' K \sigma \frac{\beta^3}{d^4} (1 - \frac{2}{3} \frac{d^3}{\beta^4}) \]  \hspace{1cm} (21)

where \[ K = \frac{\omega^2 \mu^2 s^2}{4 \pi} \cdot \frac{d}{\beta} \] , \( \beta = \sqrt{\frac{2}{\omega \mu \sigma}} \)  \hspace{1cm} (22)
$\beta$ is sometimes represented in skin depth, giving the order of magnitude of the penetration depth of the electromagnetic field into a conductor. $K$ is also represented as the instrument constant, depended on frequency applied under the condition of constant spacing between two coil pairs. When the conductivity $\sigma$ increases, the terms below second term in Eq. 20, real part, and Eq. 21, imaginary part, can be neglected. Then the first term will remain in each equation, namely, the terms of the conductive voltage and mutual inductance voltage may remain.

From Eq. 20 and 21

$$V = V_r + V_x = V_x + V_m$$

$$= K\sigma + \frac{MWSz}{4\pi d^2}$$

Equation (23)

Since the conductivity is proportional to only conduction voltage, Eq. 23 may be in the form $V = K\sigma$ (24)

If we consider the geometric factor, the Eq. 24 may be given the form

$$V = KG\sigma$$

Equation (25)

Where $G = \text{geometric factor}$

$$= \int\int g \, d\phi \, d\tau$$

Equation (26)

It is very convenient to write instrument constant in the reversed form, such that $K_r = 1/K$ (27)

For example
\[ G = K_r \cdot V_o \quad \text{mho/metre} \]

Where \[ K_r = \frac{D}{f^2 \cdot \pi \cdot a} \] \[ a = \mu \pi S^2 = 7.27715 \times 10^{-9} \]

\[ G = 9.95 \div 1 \quad \text{According to experimental results} \]

- \( I = \text{Current} \quad \text{--- Ampere} \)
- \( D = \text{Coil Spacing} \quad \text{--- meter} \)
- \( V_o = \text{Conduction Voltage} \quad \text{--- Volts} \)
- \( f = \text{Frequency} \quad \text{--- cycle/sec} \)

For \( 300 \text{ Kc} \) \[ K_r = 1.2313 \times 10^5 \quad \text{at} \quad D = 5 \text{cm} \]

\[ G = 1.2313 \times 10^5 \cdot \frac{2 \times 10^4}{1.2} \times 1 \]

\[ G = 2.4626 \text{ mho/m} \]
3. EXPERIMENTAL CONSIDERATION

The characteristics of main parameters \((V_m, V_\sigma, K \text{ and } M)\) in the theory of this measuring system may be seen clearly by the comparison with the experimental results of these parameters. These characteristics may, however, be determined by considering the following relations:

1. The characteristics of mutual inductance voltage \(V_m\) and frequencies (Fig. 5, 6)

2. The characteristics of instrument constant \(K\) and frequencies (Fig. 7)

3. The characteristics of conduction voltage \(V_\sigma\) and frequencies (Fig. 8, 9)

The mutual inductance voltage \(V_m\) may be represented in the following form, as discussed in the previous section.

\[
\sqrt{V_m} = M \omega i
\]

\[
M = \frac{M}{4\pi} \int_0^{2\pi} \int_0^{2\pi} \frac{V_{\text{m}} \cos \phi \, d\phi}{\sqrt{K^2 + K^2 \sin^2 \phi - 2\pi K \cos \phi}}
\]

\[
M = M r \varphi(k)
\]

where

\[
\varphi(k) = \left(\frac{2}{k} - k\right)K(k) - \frac{2}{k}E(k)
\]

\[
K(k) = \int_0^{2\pi} \frac{d\phi}{\sqrt{1 - k^2 \sin^2 \phi}}
\]

\[
E(k) = \int_0^{2\pi} \frac{d\phi}{\sqrt{1 - k^2 \sin^2 \phi}}
\]

\[
\varphi^2 = \frac{4Y^2}{4r^2 + D^2}
\]

where \(r\) = radius of coaxial coil system and \(D\) = distance between its coil.

The mutual inductance has been calculated numerically with results given in the following table (Table 1). The mutual inductance voltage \(V_m\) is a function of distance \(D\) and radius \(\sigma\) under the constant
frequency. Since the radius is essentially a constant in the instrument, the mutual inductance voltage \( V_m \) is a function of the distant \( D \) between coil pair in the primary and secondary circuits. This relation is represented by Fig. 5. If the distance \( D \) is fixed as a constant, the mutual inductance voltage \( V_m \) is proportional to the product of frequency and current in the primary circuit, this current being 90° out of phase with \( V_m \). This relation has been represented by Fig. 6 and Table 2. The curves of Fig. 5 and Fig. 6 will aid in designing a coaxial coil system.

It is convenient to write instrument constant \( K \) as the reversed form \( 1/K \) in calculating a conductivity. From previous sections, we have

\[
\nu r = K \sigma \quad \text{or} \quad \sigma = \frac{1}{K} \nu r = K_r \nu r
\]

If the sample fills the whole space

\[
K_r = \frac{1}{K} = \frac{D}{f \mu_0 S^2 \pi} = \frac{D}{f^{2} \pi a}
\]

where \( a = \text{constant} = \mu_0 S^2 \pi = 7.2767 \times 10^{-5} \),

\( S = \text{effective area of coaxial coil system.} \)

Since the current \( I_c \) depends on frequency, if the frequency and distance \( D \) is fixed the instrument constant \( K \) is constant. For example, if \( D = 5 \text{ cm} \), \( f = 300 \text{ KC} \), then \( K_r = 1.2313 \times 10^5 \), namely

\[
K_r = \frac{\frac{f \times 10^{-2}}{(3 \times 10^5)^2 \times 62 \times 10^{-3} \times 7.2767 \times 10^{-7}}} = 1.2313 \times 10^5 \text{ volts-ohm meter}^3
\]

This relationship between \( K_r \) and \( f \) are represented in Fig. 7 and Table 3.
The equation above for $K_r$ is valid only if the sample fills the whole space. In the case of small samples, such as those used in these experiments, a factor depending upon shape and volume must be included. Table 4 gives experimental evidence of the proportionality of $V_\sigma$ to volume. To determine the sample factor analytically, one might utilize the method of Doll, with suitable changes in the region of integration, to obtain a good approximation to the required factor. In practice, one would calibrate the instrument by using sample of known dimensions and conductivity.

The conductivity is proportional to conduction voltage, the conduction voltage being proportional to the frequencies under the constant distance between coil pair in the primary and secondary circuits, so that the characteristics of these relations are represented by the Fig. 8, 9 and Table 5.

The conduction voltage is directly proportional to the frequencies as shown in Fig. In the characteristics between conductivities and frequencies, the curve of high resistivity (200 cm) of semiconductor material is increased rapidly by frequency, the curve of very high resistivity (26,000 cm) of semiconductor material being increased slowly by the same frequency ranges.

The Table 4 gives the ratio of conductivity and the ratio of volume for semiconductor materials; this table show that conduction voltage is proportional exactly to volume conductivity of semiconductor.
4. CONCLUSIONS

It has been demonstrated both mathematically and experimentally that the mutual inductance voltage $V_m$ is independent of the conductance voltage $V_\sigma$, and that these two are $90^\circ$ out of phase. The magnitude of the conductance voltage $V_\sigma$ is directly proportional to the conductivity and the square of the frequency $F$. The mutual inductance voltage varies directly as square of the frequency.

The mathematical determination was based on the assumption of a very large sample. The experimental results show that the same mathematical form holds for small samples. The evidence is summarized in Figs. 5 through 8 and in Tables 1 through 4.

This system of measurement puts essentially no limitations on the size and shape of the sample of semiconductor material. Also the system measures conductivities over a wide range of values. This method of measurement may be used to determine the sheet resistivity of a diffused layer in a semiconductor sample. The method may be realized by a simple, practical instrument which is easy to use in the laboratory.

The effects of external fields and conducting material may be minimized by the use of suitable mechanical design and electromagnetic shielding. The use of a phase sensitive detector to read the changes in the conductive voltage $V_\sigma$ directly should further minimize such effects.
The author wishes to thank Dr. Thomas A. Rabson, who suggested the basic idea and actively supervised and supported the work described in the paper. Thanks are also due Dr. Paul E. Pfeiffer, who made several suggestions regarding the experimental work and the mathematical analysis. Mr. William R. Peters gave considerable help in several matters of instrumentation.
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44. J. C. Brice and P. Moore, Contactless Resistivity Meter for Semiconductors


**MUTUAL INDUCTANCE VOLTAGES AT CONSTANT FREQUENCY**

**TABLE 1**

<table>
<thead>
<tr>
<th>R</th>
<th>D</th>
<th>M</th>
<th>V_{m1}</th>
<th>V_{m2}</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>$2.97 \times 10^{-6}$</td>
<td>$307.7 \times 10^{-5}$</td>
<td>240</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>$1.58 \times 10^{-6}$</td>
<td>$163.7 \times 10^{-5}$</td>
<td>136</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>$9.44 \times 10^{-6}$</td>
<td>$97.8 \times 10^{-5}$</td>
<td>80</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>$5.97 \times 10^{-7}$</td>
<td>$61.9 \times 10^{-5}$</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>$3.95 \times 10^{-7}$</td>
<td>$40.9 \times 10^{-5}$</td>
<td>34</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>$2.7 \times 10^{-7}$</td>
<td>$28.0 \times 10^{-5}$</td>
<td>26</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>$1.92 \times 10^{-7}$</td>
<td>$19.9 \times 10^{-5}$</td>
<td>21</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>$1.40 \times 10^{-7}$</td>
<td>$14.5 \times 10^{-5}$</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>$1.05 \times 10^{-7}$</td>
<td>$10.9 \times 10^{-5}$</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>$8.03 \times 10^{-8}$</td>
<td>$8.3 \times 10^{-5}$</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>$6.26 \times 10^{-8}$</td>
<td>$6.5 \times 10^{-5}$</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>$4.97 \times 10^{-8}$</td>
<td>$5.2 \times 10^{-5}$</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
<td>$3.99 \times 10^{-8}$</td>
<td>$4.1 \times 10^{-5}$</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
<td>$3.26 \times 10^{-8}$</td>
<td>$3.4 \times 10^{-5}$</td>
<td>10</td>
</tr>
</tbody>
</table>

**NOTE:** $V_{m1}$ is theoretical values

$V_{m2}$ is measuring values
### MUTUAL INDUCTANCE VOLTAGE VARIATION WITH FREQUENCY

#### TABLE 2

<table>
<thead>
<tr>
<th>D</th>
<th>R</th>
<th>F</th>
<th>M</th>
<th>$V_{m1}$</th>
<th>$V_{m2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3</td>
<td>100</td>
<td>$3.95 \times 10^{-9}$</td>
<td>$21.33 \times 10^{-5}V$</td>
<td>$20 \times 10^{-5}V$</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>150</td>
<td>$3.95 \times 10^{-9}$</td>
<td>$29.02 \times 10^{-5}V$</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>200</td>
<td>&quot;</td>
<td>$34.73 \times 10^{-5}V$</td>
<td>$34 \times 10^{-5}V$</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>250</td>
<td>&quot;</td>
<td>$38.07 \times 10^{-5}V$</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>300</td>
<td>&quot;</td>
<td>$40.93 \times 10^{-5}V$</td>
<td>$43 \times 10^{-5}V$</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>350</td>
<td>&quot;</td>
<td>$43.41 \times 10^{-5}V$</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>400</td>
<td>&quot;</td>
<td>$44.64 \times 10^{-5}V$</td>
<td>$46 \times 10^{-5}V$</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>450</td>
<td>&quot;</td>
<td>$45.00 \times 10^{-5}V$</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>500</td>
<td>&quot;</td>
<td>$45.20 \times 10^{-5}V$</td>
<td>$47 \times 5 \times 10^{-5}V$</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>550</td>
<td>&quot;</td>
<td>$45.2 \times 10^{-5}V$</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>600</td>
<td>&quot;</td>
<td>$45.2 \times 10^{-5}V$</td>
<td>$48 \times 10^{-5}V$</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>650</td>
<td>&quot;</td>
<td>$46.3 \times 10^{-5}V$</td>
<td></td>
</tr>
</tbody>
</table>

$R$ is radius of the coaxial coil  
$V_{m1}$ is theoretical values  
$V_{m2}$ is experimental values
**VARIATION OF INSTRUMENT CONSTANT WITH FREQUENCY**

**TABLE 3**

<table>
<thead>
<tr>
<th>$F$</th>
<th>$D$</th>
<th>$I_t$</th>
<th>$K_r = \frac{1}{K}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>6</td>
<td>86</td>
<td>$9.5872 \times 10^5$</td>
</tr>
<tr>
<td>150</td>
<td>6</td>
<td>78</td>
<td>$4.6978 \times 10^5$</td>
</tr>
<tr>
<td>200</td>
<td>6</td>
<td>70</td>
<td>$2.9446 \times 10^5$</td>
</tr>
<tr>
<td>250</td>
<td>6</td>
<td>63</td>
<td>$2.0939 \times 10^5$</td>
</tr>
<tr>
<td>300</td>
<td>6</td>
<td>55</td>
<td>$1.6656 \times 10^5$</td>
</tr>
<tr>
<td>350</td>
<td>6</td>
<td>50</td>
<td>$1.3461 \times 10^5$</td>
</tr>
<tr>
<td>400</td>
<td>6</td>
<td>45</td>
<td>$1.1451 \times 10^5$</td>
</tr>
<tr>
<td>450</td>
<td>6</td>
<td>40</td>
<td>$1.0179 \times 10^5$</td>
</tr>
<tr>
<td>500</td>
<td>6</td>
<td>36</td>
<td>$9.161 \times 10^4$</td>
</tr>
<tr>
<td>550</td>
<td>6</td>
<td>33</td>
<td>$8.2594 \times 10^4$</td>
</tr>
<tr>
<td>600</td>
<td>6</td>
<td>30</td>
<td>$7.6342 \times 10^4$</td>
</tr>
<tr>
<td>650</td>
<td>6</td>
<td>28</td>
<td>$7.0754 \times 10^4$</td>
</tr>
</tbody>
</table>
TABLE 4

Volume Conductivities*

<table>
<thead>
<tr>
<th>No.</th>
<th>Grams</th>
<th>Volts = V17</th>
<th>Ratio of volume</th>
<th>Ratio of Conductivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.682</td>
<td>$4.89 \times 10^{-4}$</td>
<td>$\frac{2}{2'} = \frac{1.682}{1.010}$</td>
<td>$\frac{2}{2'} = \frac{4.89 \times 10^{-4}}{2.945 \times 10^{-4}}$</td>
</tr>
<tr>
<td>2'</td>
<td>1.010</td>
<td>$2.945 \times 10^{-4}$</td>
<td>$\frac{7}{7'} = \frac{1.810}{1.656}$</td>
<td>$\frac{7}{7'} = \frac{6.10 \times 10^{-4}}{5.55 \times 10^{-4}}$</td>
</tr>
<tr>
<td>7</td>
<td>1.810</td>
<td>$6.10 \times 10^{-4}$</td>
<td>$\frac{10}{10'} = \frac{1.107}{0.879}$</td>
<td>$\frac{10}{10'} = \frac{7.58 \times 10^{-5}}{6.19 \times 10^{-5}}$</td>
</tr>
<tr>
<td>7'</td>
<td>1.656</td>
<td>$5.55 \times 10^{-4}$</td>
<td>$\frac{10}{10'} = \frac{1.107}{0.879}$</td>
<td>$\frac{10}{10'} = \frac{7.58 \times 10^{-5}}{6.19 \times 10^{-5}}$</td>
</tr>
<tr>
<td>10</td>
<td>0.879</td>
<td>$6.19 \times 10^{-5}$</td>
<td>$\frac{10}{10'} = \frac{1.107}{0.879}$</td>
<td>$\frac{10}{10'} = \frac{7.58 \times 10^{-5}}{6.19 \times 10^{-5}}$</td>
</tr>
<tr>
<td>9</td>
<td>2.049</td>
<td>$1.805$</td>
<td>$\frac{10}{10'} = \frac{2.049}{35.455}$</td>
<td>$\frac{9}{9'} = \frac{1.805}{31.355}$</td>
</tr>
<tr>
<td>9'</td>
<td>35.455</td>
<td>31.355</td>
<td>$= 17.32$</td>
<td>$= 17.42$</td>
</tr>
<tr>
<td>5</td>
<td>2.049</td>
<td>$1.90 \times 10^{-4}$</td>
<td>$\frac{5}{5'} = \frac{2.049}{2.049}$</td>
<td>$\frac{5}{5'} = \frac{1.90 \times 10^{-4}}{39.8 \times 10^{-4}}$</td>
</tr>
<tr>
<td>5'</td>
<td>2.049</td>
<td>$39.8 \times 10^{-4}$</td>
<td>$= 1$</td>
<td>$= 30.42$</td>
</tr>
</tbody>
</table>

*Notes:

1. Each pair of samples except 5 and 5' are of silicon having the same resistivity
2. Sample 5 is silicon before diffusion; sample 5' is silicon diffused with gold.
VARIATION OF CONDUCTIVITY WITH FREQUENCY

<table>
<thead>
<tr>
<th>No.</th>
<th>$f$</th>
<th>$V_\sigma$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>100</td>
<td>$4.30 \times 10^{-5}$ v.</td>
<td>32.11 mho/m</td>
</tr>
<tr>
<td>8</td>
<td>150</td>
<td>$1.123 \times 10^{-5}$ v.</td>
<td>37.40 mho/m</td>
</tr>
<tr>
<td>8</td>
<td>200</td>
<td>$1.837 \times 10^{-5}$ v.</td>
<td>40.05 mho/m</td>
</tr>
<tr>
<td>8</td>
<td>250</td>
<td>$2.699 \times 10^{-5}$ v.</td>
<td>41.36 mho/m</td>
</tr>
<tr>
<td>8</td>
<td>300</td>
<td>$3.417 \times 10^{-5}$ v.</td>
<td>42.10 mho/m</td>
</tr>
<tr>
<td>8</td>
<td>350</td>
<td>$4.192 \times 10^{-5}$ v.</td>
<td>44.35 mho/m</td>
</tr>
<tr>
<td>8</td>
<td>400</td>
<td>$5.089 \times 10^{-5}$ v.</td>
<td>46.25 mho/m</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>$2.150 \times 10^{-5}$ v.</td>
<td>16.05 mho/m</td>
</tr>
<tr>
<td>10</td>
<td>150</td>
<td>$5.260 \times 10^{-5}$ v.</td>
<td>17.30 mho/m</td>
</tr>
<tr>
<td>10</td>
<td>200</td>
<td>$8.020 \times 10^{-5}$ v.</td>
<td>18.00 mho/m</td>
</tr>
<tr>
<td>10</td>
<td>250</td>
<td>$1.130 \times 10^{-4}$ v.</td>
<td>18.48 mho/m</td>
</tr>
<tr>
<td>10</td>
<td>300</td>
<td>$1.442 \times 10^{-4}$ v.</td>
<td>19.00 mho/m</td>
</tr>
<tr>
<td>10</td>
<td>350</td>
<td>$1.762 \times 10^{-4}$ v.</td>
<td>19.51 mho/m</td>
</tr>
<tr>
<td>10</td>
<td>400</td>
<td>$2.08 \times 10^{-4}$ v.</td>
<td>19.98 mho/m</td>
</tr>
</tbody>
</table>

No. is sample number.
A = Coaxial Coil system in the primary circuits
A' = Coaxial Coil system in the secondary circuits
B = Holder for a sample
C = Slider adjusting a distance between two coils
E = Shielding coils
D = Sets for measuring the geometric factors
Fig. 9. The relationships of conductivities as the function of frequencies between the resistivity and very high resistivity of semiconductor materials. 
$\sigma$ is conductivity of semiconductor 
No. 8 is high resistivity of semiconductor (200 cm) 
No. 10 is very high resistivity of semiconductor (26000 cm)
Fig. 8. Relationships between the high resistivity and very high resistivity of conduction voltages as the function of frequencies.
Fig. 7. The characteristics of instrumental constant as the frequencies for the constant coil spacing \( D (K_r = \frac{K}{K}) \) is the reversed instrumental constant. \( f \) is kilocycles.
Fig. 6. The characteristics of mutual inductance voltage having a function of the frequencies at constant distance $D$. $V_m$ is mutual inductance voltage, $f$ is frequency in kilocycle. * is Theoretical Curve * is experimental curve
Fig. 5. The characteristics of mutual inductance voltage having a function of the coil spacing, $D$. $V_m$ is a mutual inductance voltage, $D$ coil spacing between coaxial coil system at constant frequency.

- Theoretical curve
- Experimental curve
1. Polytetrafluoroethylene
2. Polyethylene
3. Polystyrene
4. Test temperature = 20° - 27° C.
5. Tan $\delta$ is dissipation factor
6. Reference (41)
Fig. 2. Characteristics of the circuits in the phase sensitive device. $E_r$ is output voltage in the secondary coil system. $I_t$ is input current in the primary circuits at input voltage $E_t = \text{constant}$.
O = Oscillator
A = Amplifier
S = Sample of Semiconductor Material
D = Phase Sensitive Detector
$L_1$ & $L_1'$ are coaxial coils in the primary circuits
$L_2$ & $L_2'$ are coaxial coils in the secondary circuits
MODEL B
(For Light Sample)