

RICE UNIVERSITY

MIXING MODEL OF A REAL PLATE
FOR VAPOR-LIQUID CONTACTING PROCESSES

BY

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Thesis Director's signature

A handwritten signature in cursive script, written over a horizontal line. The signature appears to be "J. H. Alford".

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LIST OF SYMBOLS

A	Parameter defined by equation (8)
B	Parameter defined by equation (9)
C	Constant
C_1	Constant
C_2	Constant
E_L	Liquid point efficiency, percent
E_V	Vapor point efficiency, percent
E_{ML}	Murphree liquid plate efficiency, percent
E_{MV}	Murphree vapor plate efficiency, percent
k	Slope of the vapor-liquid equilibrium curve of y vs. x
L	Liquid flow rate, lb.-mole per unit time
M	Total number of plates
N	Total number of contacting stages per plate
V	Vapor flow rate, lb.-mole per unit time
v	Vapor flow rate per stage
w	Outlet weir height, in.
x	Liquid composition, mole fraction
x_{n-1}	Composition of the liquid entering the n^{th} stage, mole fraction
x_n	Composition of the liquid leaving the n^{th} stage, mole fraction
x_n^*	Composition of the liquid in equilibrium with the vapor leaving the n^{th} stage
X_{m-1}	Composition of the liquid entering the m^{th} plate
X_m	Composition of the liquid leaving the m^{th} plate
X_m^*	Composition of the liquid in equilibrium with the vapor leaving the m^{th} plate
y	Vapor composition, mole fraction

y_n	Composition of the vapor leaving the n^{th} stage, mole fraction
y_{n+1}	Composition of the vapor entering the n^{th} stage, mole fraction
y_n^*	Composition of the vapor in equilibrium with the vapor leaving the n^{th} stage, mole fraction
Y_m	Composition of the vapor leaving the m^{th} plate, mole fraction
Y_{m+1}	Composition of the vapor entering the m^{th} plate, mole fraction
Y_m^*	Composition of the vapor in equilibrium with the liquid leaving the m^{th} plate, mole fraction
α	Fraction of the liquid leaving the n^{th} stage that is in equilibrium with the vapor leaving the n^{th} stage
λ	Parameter LN/kV
μ	Liquid viscosity (lb)/(ft)(hr)

ABSTRACT

Ernesto J. Requena

A mixing model for a real plate is presented that relates liquid point efficiency and liquid plate efficiency. The model was tested with available data in the literature and it was found that the main variables that affect liquid point efficiency are liquid rate, vapor rate per stage, weir height and liquid viscosity. Liquid point efficiency increases with increasing vapor rate and weir height and decreases with increasing liquid rate and liquid viscosity. The effect of plate layout on liquid point efficiency is not as marked as the effects of weir height and liquid viscosity.

INTRODUCTION

It is the purpose of this work to present a mathematical model for liquid efficiencies in vapor-liquid contacting plates. The model is based on the idea that the vapor flows perpendicular to the direction of liquid flow and the contact between the two phases occurs in successive stages thus allowing the formulation of a finite difference equation to describe the mass transfer between the liquid and vapor streams. The solution of this difference equation relates the efficiency of the contacting stage to the efficiency of the over-all plate.

The study of the variables that determine the plate efficiency, and the prediction of such efficiencies from these variables, is of utmost importance to design engineers. Processes, such as absorption and distillation, that are based on the mass transfer between vapor and liquid are probably the most widely used methods in industry for separation and purification of different components.

The mathematical theory for the calculation of rectifying columns is based on the concept of the ideal or perfect plate. In a perfect plate the vapor and liquid leaving are in equilibrium and thus the separation obtained in each plate can be obtained from the equilibrium relations of the system. Several methods^{9,11,12,15,17,18,22} based on the concept of the perfect plate, have been proposed since the idea was first introduced by Sorel²¹. These methods vary in the degree of complexity and the most recent ones have taken advantage of the use of digital computers to reduce the calculation time^{23,25}.

The condition of equilibrium is rarely met in practice, and it becomes necessary to determine the number of real

plates that is equivalent to one perfect plate. The relation between the number of real and perfect plates is determined by the efficiency of the real plate.

Efficiencies are defined in three different ways:

(1) over-all column efficiency; (2) Murphree plate efficiency; and (3) point efficiency.

The concept of over-all column efficiency is the simplest. It is defined as the ratio of the number of theoretical plates to the number of actual plates.

Murphree plate efficiency is the efficiency determined for a single plate, and it may be defined in terms of the vapor phase or the liquid phase. In terms of the vapor phase it is defined by the following equation:

$$E_{MV} = \frac{Y_m - Y_{m+1}}{Y_m^* - Y_{m+1}}$$

where E_{MV} is Murphree vapor plate efficiency; Y_{m+1} is the composition of the vapor entering the m^{th} plate; Y_m is the composition of the vapor leaving the m^{th} plate; and Y_m^* is the composition of the vapor in equilibrium with the liquid leaving the m^{th} plate.

In terms of the liquid phase Murphree plate efficiency is defined by:

$$E_{ML} = \frac{X_{m-1} - X_m}{X_{m-1} - X_m^*}$$

where E_{ML} is the Murphree liquid plate efficiency; X_{m-1} is the composition of the liquid entering the m^{th} plate; X_m is the composition of the liquid leaving the m^{th} plate; and X_m^* is the composition of the liquid in equilibrium with the vapor leaving the m^{th} plate.

The definition of point efficiency is based on the same concept as the plate efficiency but it is applied to one point rather than the whole tray. It can also be defined in terms of the vapor or the liquid phase and the definitions are the same as the plate efficiency except that they refer to a point in the plate.

The variables that affect the efficiency of real plates may be grouped into three categories: operating variables, plate-design variables, and system variables. Operating variables include gas rate, liquid rate, temperature, and pressure. Plate-design variables include plate dimension and spacing, bubble cap design, outlet weir height, etc. System variables depend on the system and include relative volatility, surface tension and gravity, viscosity and diffusivity of the gas and liquid phases.

The effect of no one of these variables can be singled out since change in one variable such as temperature or pressure will produce change in other variables such as gas and liquid viscosities. Due to this interrelated effect, it has not been possible to correlate the value of efficiency to these variables in one equation. Most of the studies performed in efficiencies have tried to determine the effect of a few of these variables holding the rest constant.

The effect of operating and system variables have been studied by Langdon and Keyes¹⁰ on bubble cap trays using the isopropyl-alcohol-water system. Efficiencies in distillation have been studied by Nord¹³ for the benzene-toluene-xylene system and by Shilling and Watson²⁰ for the methanol-water system. Walter and Sherwood²⁴ studied the efficiencies obtained in absorbers for different absorption systems. Sherwood and Jenny¹⁹, Rhodes¹⁶, and Coulburn⁶ have studied

the effect of entrainment on plate efficiency. Plate efficiencies in perforated trays have been studied by Hellums⁸ using the n-octane-toluene system.

Many correlating formulas have been proposed to predict efficiencies in distillation and absorption towers. O'Connell¹⁴ developed a correlation of over-all column efficiency as a function of feed viscosity, and relative volatilities of the key components for fractionation columns and as a function of Henry's law constant, system pressure and viscosity of absorbent oil for absorbers. Chaiyavech and Van Winkle⁵ presented an empirical correlation to predict plate efficiencies in small distillation columns based on system properties. English and Van Winkle⁷ developed a correlation that takes into account operating and system properties.

One of the most complete studies of plate efficiencies was performed by the A.I.Ch.E. Research Committee^{1,2,3}. In this study the effect of operating, design and system variables were determined for different systems and tray-designs; a method for estimating plate efficiencies, based on the two film resistance theories, has been presented in the Bubble-Tray Design Manual⁴.

MIXING MODEL FOR EFFICIENCIES IN REAL PLATES

An equilibrium stage is based on the assumption that the vapor and liquid leaving the stage are in equilibrium with each other. The equilibrium constant of the system being considered gives then a direct relationship in going from one stage to the next. This idealized equilibrium stage is seldom met in practice and the departure from equilibrium may be considered as a measurement of the inefficiency of the real plate.

In a real plate the liquid flows across the plate and gets in contact with successive portions of the vapor from the plate below as it travels from one stage to the next. However, only a fraction of the liquid is in direct contact with the vapor, and thus the mass transfer between the vapor and liquid phases is dependent on both diffusional and mixing effects.

In order to simplify the model, consider a single row of caps (or holes in the case of a perforated plate), perpendicular to the flow of liquid as a stage, and consider a distillation or absorption column with M plates and N stages per plate, as shown in Figure 1.

All the liquid flowing across the plate and only a portion of the vapor from the plate below will pass through one stage. If it is assumed that a fraction, α , of the liquid is in equilibrium with the vapor leaving the stage and $(1 - \alpha)$ is unchanged in composition, then α can be defined as the stage efficiency, since α represents the fraction of the liquid that is in equilibrium with the vapor.

From the definition of α the following relationship is obtained for the n^{th} stage for the component under consideration:

$$L \cdot x_n = \alpha L x_n^* + (1-\alpha) L x_{n-1}$$

where L = total liquid rate, moles per unit time
 x_{n-1} = mole fraction of the component in the liquid leaving the $(n-1)^{\text{th}}$ stage
 x_n = mole fraction of the component in the liquid leaving the n^{th} stage
 x_n^* = mole fraction of the component in the liquid in equilibrium with the vapor leaving the n^{th} stage.

Rearranging:

$$\alpha = \frac{x_{n-1} - x_n}{x_{n-1} - x_n^*} = E_L \quad (1)$$

which is the definition of Murphree point efficiency.

The vapor from the plate below, $(m+1)^{\text{th}}$ plate, contacts all stages in parallel, thus the vapor per stage, v , is the total vapor flow, V , divided by the number of stages, N . The liquid flowing across the plate contacts all stages in series, thus the liquid flow per stage is equal to liquid flow, L .

A material balance, at the m^{th} plate, around the n^{th} stage gives:

$$y_n = \frac{L}{v} (x_{n-1} - x_n) + Y_{m+1} \quad (2)$$

where y_n = mole fraction of the component in the vapor leaving the n^{th} stage at the m^{th} plate

v = vapor flow per stage, moles per unit time
 Y_{m+1} = mole fraction of the component in the total vapor leaving the $(m+1)^{\text{th}}$ tray.

If it is considered that the equilibrium constant, k , is either constant across the plate or an average value is used, then:

$$y_n = kx_n^* , \text{ by definition.} \quad (3)$$

Substituting in equation (2), and solving for x_n^* gives:

$$x_n^* = \frac{L}{kv} (x_{n-1} - x_n) + \frac{Y_{m+1}}{k} \quad (4)$$

Eliminating x_n^* between equations (1) and (4) and defining

$$\lambda = \frac{L}{kv} = \frac{LN}{kV} \quad (5)$$

it is obtained

$$x_n - \left[\frac{(1 + \lambda E_L) - E_L}{1 + \lambda E_L} \right] x_{n-1} - \frac{E_L Y_{m+1}}{k(1 + \lambda E_L)} = 0 \quad (6)$$

If the assumption is made that the stage efficiency E_L , is constant across the plate and the vapor from the plate is well mixed, equation (6) is a finite difference equation with constant coefficient that can be solved by the rules of finite differences. The assumption of constant stage efficiency is probably very good since it depends on the dynamics of the stages, which are essentially the same for each stage. The assumption of mixed vapor entering the plate is good for small columns or for columns having large enough plate spacing to allow the vapor to mix.

Subject to these conditions, equation (6) may be written as:

$$x_n - Ax_{n-1} - B = 0 \quad (7)$$

where A and B are constant and are defined by:

$$A = \frac{1 + \lambda E_L - E_L}{1 + \lambda E_L} = 1 - \frac{E_L}{1 + \lambda E_L} \quad (8)$$

$$B = \frac{E_L Y_{m+1}}{k(1 + \lambda E_L)} \quad (9)$$

Equation (7) is solved in the Appendix. Its solution is:

$$x_n = \left(X_{m-1} - \frac{B}{1-A} \right) A^n + \frac{B}{1-A} \quad (10)$$

but $\frac{B}{1-A} = \frac{Y_{m+1}}{k}$

then

$$x_n = \left(X_{m-1} - \frac{Y_{m+1}}{k} \right) A^n + \frac{Y_{m+1}}{k}$$

or $x_n = X_{m-1} A^n + \frac{Y_{m+1}}{k} (1 - A^n)$ (11)

The plate efficiency based on overall concentration is:

$$E_{ML} = \frac{X_{m-1} - X_m}{X_{m-1} - X_m^*} \quad (12)$$

where X_{m-1} = mole fraction of the component in the total liquid from the (m-1)th tray
 X_m = mole fraction of the component in the total liquid from the mth tray
 X_m^* = mole fraction of the component in the liquid in equilibrium with the total vapor leaving the mth tray.

The equilibrium relation is again expressed as

$$X_m^* = \frac{Y_m}{k} \quad (13)$$

The concentration Y_m is the average concentration of the mixed vapor leaving the m^{th} tray, so:

$$Y_m = \frac{\sum_{n=1}^N vY_n}{V} \quad (14)$$

By definition $V = Nv$ and since v is the same for every stage

$$Y_m = \frac{1}{N} \sum_{n=1}^N Y_n \quad (15)$$

Substituting (2) in (15) it is obtained

$$Y_m = \frac{1}{N} \sum_{n=1}^N \left[\frac{L}{V} (x_{n-1} - x_n) + Y_{m+1} \right] \quad (16)$$

Since Y_{m+1} is constant, and thus independent of the value of n it is obtained

$$Y_m = \frac{L}{V} \sum_{n=1}^N (x_{n-1} - x_n) + Y_{m+1} \quad (17)$$

The value $x_{n-1} - x_n$ is equal to the change in liquid concentration across the n^{th} stage. The sum of these changes across the plate is to the total change $X_{m-1} - X_m$. Therefore equation (17) becomes:

$$Y_m = \frac{L}{V} (X_{m-1} - X_m) + Y_{m+1} \quad (18)$$

Substituting (18) into (13) gives

$$X_m^* = \frac{\lambda}{N} (X_{m-1} - X_m) + \frac{Y_{m+1}}{k} \quad (19)$$

Substituting (19) into (12) and simplifying gives:

$$E_{ML} = \frac{X_{m-1} - X_m}{\left(1 - \frac{\lambda}{N}\right)X_{m-1} - \frac{Y_{m+1}}{k} + \frac{\lambda}{N} X_m} \quad (20)$$

Since X_m is the liquid concentration leaving the m^{th} plate, which is the concentration leaving the N^{th} stage, it is obtained from equation (11), by setting $n = N$

$$X_m = x_N = X_{m-1} A^N + \frac{Y_{m+1}}{k} (1 - A^N) \quad (21)$$

Substituting (21) into (20) gives:

$$E_{ML} = \frac{X_{m-1} - X_{m-1} A^N - \left(\frac{Y_{m+1}}{k}\right)(1-A^N)}{\left(1 - \frac{\lambda}{N}\right)X_{m-1} - \frac{Y_{m+1}}{k} + \frac{X_{m-1} A^N \lambda}{N} + \frac{\lambda Y_{m+1} (1-A^N)}{Nk}} \quad (22)$$

which can be simplified to:

$$E_{ML} = \frac{1 - A^N}{1 - \frac{\lambda}{N} (1-A^N)} \quad (23)$$

where (5) $\lambda = \frac{LN}{kV}$

and (8) $A = 1 - \frac{E_L}{1 + \lambda EL}$

Equation (23) relates the plate efficiency E_{ML} to the stage efficiency, E_L , and the conditions at the plate given by λ . For $N = 1$ the plate efficiency is equal to the point efficiency. As N approaches infinite the limit is given by the expression:

$$\lim_{N \rightarrow \infty} E_{ML} = \frac{1 - e^{-\frac{kV}{L}}}{1 - \frac{L}{kV} (1 - e^{-\frac{kV}{L}})}$$

The over-all plate efficiency as given by equation (12) and calculated by equation (23) permits to proceed from plate to plate. A material balance around the m^{th} plate gives:

$$LX_{m-1} + VY_{m+1} = LX_m + VY_m$$

or
$$Y_m = \frac{L}{V} (X_{m-1} - X_m) + Y_{m+1}$$

since
$$E_{ML} = \frac{X_{m-1} - X_m}{X_{m-1} - X_m^*}$$

and
$$X_m^* = \frac{Y_m}{k}$$

it is obtained

$$(1 - E_{ML} + \frac{L}{kV} E_{ML})X_{m-1} = (1 + \frac{L}{kV} E_{ML})X_m - \frac{E_{ML}}{k} Y_{m+1} \quad (24)$$

Equation (24) permits calculation of X_{m-1} from X_m , the plate efficiency E_{ML} , the liquid rate, L , and the vapor rate, V . Since k is a function of temperature, an average value must be used for the plate. The plate efficiency, E_{ML} , can be obtained from the stage efficiency by equation (23) or from separate correlations.

Values of E_{ML} were calculated by equation (23) for different values of E_L , N and L/Vk . These results are presented on tabular form in Tables 1 through Table 10 for values of E_L of 0.1 to 0.9, N of 2.0 to 99.0 and L/kV of 0.01 to 100.0. Values of E_{ML} greater than 1.0 were obtained at high enough values of E_L and N for all the L/kV ratios used. The effect of plate sizing on plate efficiency is presented in Figure 2 for $L/kV = 1.0$.

It is believed by the author that equation (23) represents a more basic approach to the relationship between point and plate efficiency and is simpler than other proposed models. It allows for individual differences in plate efficiency for each component and gives a physical model that can be easily visualized. The stage or point efficiency as given by α , will depend on flow geometry, viscosity and to a minor degree on component diffusivities.

One application of this model to industrial uses will be in scaling pilot plant studies, which are usually done in small diameter towers and with small numbers of stages, to actual plant size scale in which larger towers and flow rates are used. The stage efficiency can be obtained from the overall plate efficiencies obtained in pilot plant studies, and the plate efficiency for plant scale operations can in turn be calculated from this stage efficiency.

RESULTS

The study of the factors that affect liquid point efficiency was based on the results obtained by the A.I.Ch.E. Distillation Research Program^{1,2,3}. The reported values of plate efficiencies were converted to point efficiencies by the following equations that were obtained from equations (8) and (23)

$$E_L = \frac{1 - A}{1 - \lambda(1-A)}$$

and

$$A = \left(1 - \frac{E_{ML}}{1 + \frac{\lambda}{N} E_{ML}} \right)^{1/N}$$

1. Effect of operating and design variables.

The effect of operating and design variables on bubble cap trays were studied at the University of Delaware¹ for several different tray designs and over a wide range of operating variables. The results reported in this work are based on the data obtained for the absorption of ammonia from air into water and for the desorption of oxygen from oxygen rich water by air.

Due to the high solubility of ammonia in water the ammonia-air-water system has essentially no liquid phase mass transfer resistance. The mass transfer resistance in the oxygen-air-water system is all in the liquid phase due to small solubility of oxygen in water. Experiments were done at atmospheric pressure for both systems and at average temperatures of 20°C for the ammonia-air-water system and at 25°C for the oxygen-air-water system. For both systems a 2 ft. diameter, single tray column was used. Gas velocity was varied from 0.5 ft/sec to 8.0 ft/sec,

and liquid rates varying from 10 gpm to 66 gpm were studied. Twelve different tray designs were used in these studies. Table 11 summarizes the details for tray designs 1 and 2. For details on the other tray designs the reader is referred to the University of Delaware Report ¹.

The effect of gas and liquid rates on liquid point efficiency are presented in Figures 3 to 7 for the ammonia-air-water system, and in Figures 8, 9, and 10 for the oxygen-air-water system. Liquid point efficiencies are presented as a function of gas rate for liquid rates of 5.7, 21.6 and 37.5 gpm per foot of average column width. For both systems efficiencies increased for increasing gas rates and decreased with increasing liquid rates. The increase in efficiency is more pronounced at low gas and liquid rates.

Figures 11 and 12 show the effect of outlet weir height on liquid point efficiency. Efficiencies were plotted against LN/V with outlet weir height as a parameter. For both systems higher efficiencies were obtained for higher weir height; however, for the oxygen-air-water system no difference in the value of efficiency could be observed by increasing the weir height from 3 in. to 5 in.

Liquid hold-up has quite an appreciable effect on efficiency as shown in Figure 13. Efficiencies increase quite rapidly as the liquid hold-up decreases. This result is in agreement with the effect of liquid rate discussed previously since liquid hold-up is a function of liquid rate and weir height.

Figure 14 shows the effect of slot area distribution on efficiency. Tray designs 1 and 2 have the same slot spacing, area per tray and spacing, but since the bubble caps are of different sizes the slot distribution varied with the two trays. Tray design 1 has more uniformly distributed slots. The efficiencies obtained for tray design 1 are higher than for tray design 2, indicating that efficiency is slightly improved by uniform distribution of the slot area.

All the results of the ammonia-air-water system for all the tray designs studied are presented in Figure 15. Liquid point efficiency is related to LN/V and weir height by a general curve. These results verify the validity of the mixing model since the number of contacting stages varied from one to five for the different tray designs.

2. Effect of system variables.

The effect of physical properties of the system were studied at North Carolina State College³ and the University of Michigan² using towers with identical tray layout. The towers used were rectangular and $1\frac{1}{2}$ in. diameter caps were used. The caps were arranged on a $2\frac{1}{2}$ in. center to center square pitch with three rows of three caps each. Five trays were used.

The results presented on this work are based on the study with the carbon-dioxide-cyclohexanol system performed at the University of Michigan².

From the results obtained it was found that liquid viscosity is the main variable that affects efficiency. Figure 16 shows the results obtained for 2 in. weir height

and $2\frac{1}{2}$ in. splash baffle height. Efficiency decreases considerably as the liquid viscosity increases. When liquid viscosities were increased from 58 lb/(ft)(hr) to 237 lb/(ft)(hr), the values of liquid point efficiencies were reduced by nearly fifteen percent units. Figure 17 shows the results obtained for weir height of $3\frac{1}{2}$ in. and 4 in. splash baffle height.

SUMMARY AND DISCUSSION

A mixing model of a real plate has been presented. This model relates liquid point efficiency to liquid plate efficiency by means of the number of vapor-liquid contacting stages, N , and the parameter, L/kV .

The model was tested by all available complete data in the literature. The validity of the model is supported by the fact that similar trends in the effect of the different variables were obtained for all systems studied. The data in the literature is not complete enough to obtain over-all correlations but are sufficient to indicate magnitude of effect on liquid point efficiency.

The main variables that affect liquid point efficiency are liquid rate, gas rate per stage, weir height, and liquid viscosity. As shown in the mixing model plate, efficiency is a function of point efficiency, number of contacting stages and the parameter L/kV .

The data studied indicates that the liquid rate, L , and the vapor rate per stage, V/N , can be grouped by the single function (LN/V) , indicating that the effect of liquid rate on point efficiency is compensated by the effect of vapor rate per stage. The effect of plate layout is not as marked as the effect of weir height and liquid viscosity.

From the results presented in Figures 11 and 12 it can be seen that the liquid point efficiencies for the oxygen-air-water system are higher than for the ammonia-air-water system. This discrepancy between these two systems can be explained by the fact that the absorption

of ammonia by water is limited both by the resistance to diffusion and by the finite velocity of the reaction between ammonia and water. The absorption of oxygen by water is pure physical absorption since this gas is inert to water.

In the absorption of ammonia by water as water is first brought into contact with the ammonia-rich air, the ammonia will dissolve and react with the water at the interphase. The product, ammonium hydroxide, will then diffuse toward the main body of the liquid. The liquid near the interface will soon be depleted of water molecules which will begin to diffuse from the main body of the liquid to the interface. The rapid removal of water molecules from near the interface makes necessary for the incoming ammonia molecules to diffuse through part of the liquid to meet the molecules of water diffusing in the opposite direction. The system reaches equilibrium when equimolar counter diffusion between ammonia and water molecules is attained; this process requires a short, but definite amount of time, during which the absorption rate falls considerably.

CONCLUSIONS

1. A mixing model for a real plate has been developed that relates liquid point efficiency to liquid plate efficiency.
2. The results obtained in this study indicates that the model is a valid one.
3. The use of point efficiency presents a more fundamental basis for correlation with operating, design and system variables than does plate efficiency. In the mixing model presented the effect of the number of contacting stages on plate efficiency is taken into account.
4. The most important variables that affect liquid point efficiency are liquid rate, vapor rate per stage, weir height and liquid viscosity. The value of the liquid point efficiency increases with increasing gas rate and weir height and decreases with increasing liquid rate and liquid viscosity.
5. The effect of liquid rate and gas rate per stage can be described by the effect of the single parameter, LN/V .
6. The effect of plate layout is not as marked as the effect of weir height and liquid viscosity.
7. Available data in the literature is not complete enough for overall correlations, but is sufficient to indicate the magnitude of effects on point efficiency.

TABLE 1

OVERALL PLATE EFFICIENCY

$$L/Vk = 0.01$$

$E_L \backslash N$	2.0000	5.0000	10.0000	20.0000	40.0000	70.0000	99.0000
.01	.0199	.0490	.0956	.1820	.3310	.5052	.6305
.10	.1900	.4095	.6516	.8807	.9922	1.0090	1.0100
.20	.3600	.6727	.8951	.9958	1.0098	1.0100	1.0101
.30	.5100	.8334	.9774	1.0087	1.0100	1.0101	1.0101
.40	.6402	.9255	1.0021	1.0100	1.0101	1.0101	1.0101
.50	.7506	.9741	1.0085	1.0100	1.0101	1.0101	1.0101
.60	.8412	.9971	1.0098	1.0101	1.0101	1.0101	1.0101
.70	.9123	1.0064	1.0100	1.0101	1.0101	1.0101	1.0101
.80	.9640	1.0094	1.0100	1.0101	1.0101	1.0101	1.0101
.90	.9963	1.0100	1.0101	1.0101	1.0101	1.0101	1.0101

TABLE 2

OVERALL PLATE EFFICIENCY

$$L/Vk = 0.10$$

$E_L \backslash N$	2.0000	5.0000	10.0000	20.0000	40.0000	70.0000	99.0000
.01	.0198	.0490	.0956	.1821	.3311	.5061	.6331
.10	.1900	.4098	.6546	.8986	1.0477	1.0934	1.1036
.20	.3601	.6761	.9152	1.0548	1.1000	1.1083	1.1098
.30	.5107	.8452	1.0222	1.0917	1.1076	1.1101	1.1105
.40	.6423	.9508	1.0685	1.1030	1.1095	1.1105	1.1108
.50	.7555	1.0159	1.0897	1.1071	1.1102	1.1107	1.1109
.60	.8512	1.0555	1.0998	1.1089	1.1105	1.1108	1.1109
.70	.9301	1.0792	1.1049	1.1098	1.1107	1.1109	1.1109
.80	.9934	1.0932	1.1076	1.1103	1.1108	1.1109	1.1109
.90	1.0420	1.1014	1.1090	1.1105	1.1109	1.1109	1.1110

TABLE 3

OVERALL PLATE EFFICIENCY

$$L/Vk = 0.50$$

$E_L \backslash N$	2.0000	5.0000	10.0000	20.0000	40.0000	70.0000	99.0000
.01	.0198	.0490	.0956	.1821	.3316	.5090	.6410
.10	.1900	.4109	.6638	.9444	1.1804	1.3128	1.3705
.20	.3606	.6864	.9658	1.1958	1.3462	1.4189	1.4486
.30	.5130	.8766	1.1286	1.3049	1.4088	1.4566	1.4758
.40	.6486	1.0127	1.2282	1.3650	1.4415	1.4759	1.4896
.50	.7692	1.1137	1.2947	1.4028	1.4615	1.4876	1.4979
.60	.8764	1.1909	1.3420	1.4288	1.4750	1.4954	1.5035
.70	.9717	1.2515	1.3772	1.4476	1.4848	1.5011	1.5075
.80	1.0566	1.3001	1.4045	1.4620	1.4921	1.5053	1.5105
.90	1.1323	1.3398	1.4261	1.4732	1.4978	1.5086	1.5128

TABLE 4

OVERALL PLATE EFFICIENCY

$$L/Vk = 0.75$$

$E_L \backslash N$	2.0000	5.0000	10.0000	20.0000	40.0000	70.0000	99.0000
.01	.0199	.0490	.0956	.1821	.3318	.5102	.6442
.10	.1900	.4115	.6674	.9595	1.2188	1.3739	1.4445
.20	.3609	.6904	.9823	1.2363	1.4144	1.5057	1.5442
.30	.5140	.8873	1.1600	1.3642	1.4928	1.5547	1.5802
.40	.6511	1.0321	1.2733	1.4375	1.5349	1.5803	1.5988
.50	.7741	1.1425	1.3515	1.4850	1.5612	1.5961	1.6101
.60	.8847	1.2292	1.4086	1.5182	1.5791	1.6067	1.6177
.70	.9844	1.2989	1.4522	1.5427	1.5922	1.6143	1.6232
.80	1.0746	1.3561	1.4864	1.5616	1.6021	1.6201	1.6273
.90	1.1563	1.4039	1.5141	1.5766	1.6099	1.6247	1.6305

TABLE 5

OVERALL PLATE EFFICIENCY

$$L/Vk = 1.00$$

$E_L \backslash N$	2.0000	5.0000	10.0000	20.0000	40.0000	70.0000	99.0000
.01	.0199	.0490	.0956	.1821	.3320	.5113	.6466
.10	.1900	.4119	.6701	.9699	1.2436	1.4121	1.4904
.20	.3611	.6935	.9935	1.2624	1.4569	1.5589	1.6026
.30	.5147	.8949	1.1806	1.4013	1.5443	1.6145	1.6436
.40	.6530	1.0452	1.3020	1.4824	1.5919	1.6438	1.6650
.50	.7777	1.1613	1.3871	1.5355	1.6219	1.6618	1.6780
.60	.8906	1.2537	1.4500	1.5729	1.6424	1.6741	1.6868
.70	.9930	1.3288	1.4984	1.6007	1.6574	1.6829	1.6931
.80	1.0864	1.3911	1.5368	1.6222	1.6688	1.6896	1.6979
.90	1.1717	1.4435	1.5679	1.6393	1.6778	1.6949	1.7016

TABLE 6

OVERALL PLATE EFFICIENCY

$$L/Vk = 1.25$$

$E_L \backslash N$	2.0000	5.0000	10.0000	20.0000	40.0000	70.0000	99.0000
.01	.0198	.0490	.0956	.1822	.3322	.5121	.6485
.10	.1900	.4123	.6723	.9775	1.2609	1.4381	1.5213
.20	.3612	.6958	1.0017	1.2805	1.4856	1.5947	1.6416
.30	.5154	.9004	1.1950	1.4267	1.5790	1.6545	1.6859
.40	.6545	1.0545	1.3219	1.5128	1.6301	1.6861	1.7089
.50	.7804	1.1746	1.4115	1.5694	1.6623	1.7056	1.7230
.60	.8949	1.2707	1.4781	1.6096	1.6845	1.7188	1.7326
.70	.9992	1.3494	1.5296	1.6395	1.7007	1.7284	1.7395
.80	1.0947	1.4149	1.5705	1.6627	1.7131	1.7357	1.7447
.90	1.1824	1.4703	1.6039	1.6811	1.7228	1.7414	1.7487

TABLE 7

OVERALL PLATE EFFICIENCY

$$L/Vk = 2.0$$

$E_L \backslash N$	2.0000	5.0000	10.0000	20.0000	40.0000	70.0000	99.0000
.01	.0198	.0490	.0956	.1822	.3326	.5139	.6524
.10	.1901	.4131	.6766	.9915	1.2909	1.4823	1.5733
.20	.3617	.7005	1.0166	1.3119	1.5342	1.6542	1.7063
.30	.5168	.9109	1.2204	1.4698	1.6369	1.7207	1.7557
.40	.6575	1.0715	1.3561	1.5638	1.6935	1.7559	1.7816
.50	.7857	1.1980	1.4529	1.6262	1.7294	1.7778	1.7974
.60	.9029	1.3002	1.5255	1.6706	1.7542	1.7926	1.8081
.70	1.0104	1.3846	1.5819	1.7038	1.7723	1.8034	1.8159
.80	1.1094	1.4553	1.6270	1.7296	1.7861	1.8116	1.8217
.90	1.2009	1.5156	1.6639	1.7502	1.7971	1.8180	1.8263

TABLE 8

OVERALL PLATE EFFICIENCY

$$L/Vk = 5.0$$

$E_L \backslash N$	2.0000	5.0000	10.0000	20.0000	40.0000	70.0000	99.0000
.01	.0199	.0490	.0956	.1823	.3334	.5169	.6582
.10	.1902	.4146	.6831	1.0099	1.3273	1.5335	1.6325
.20	.3624	.7074	1.0361	1.3497	1.5902	1.7219	1.7794
.30	.5191	.9250	1.2516	1.5201	1.7027	1.7948	1.8335
.40	.6620	1.0931	1.3969	1.6224	1.7650	1.8339	1.8624
.50	.7931	1.2268	1.5014	1.6909	1.8046	1.8586	1.8799
.60	.9136	1.3358	1.5803	1.7397	1.8321	1.8752	1.8923
.70	1.0248	1.4262	1.6418	1.7764	1.8524	1.8870	1.9003
.80	1.1278	1.5025	1.6912	1.8050	1.8678	1.8958	1.9075
.90	1.2234	1.5677	1.7318	1.8279	1.8798	1.9034	1.9120

TABLE 9

OVERALL PLATE EFFICIENCY

$$L/Vk = 10$$

$E_L \backslash N$	2.0000	5.0000	10.0000	20.0000	40.0000	70.0000	99.0000
.01	.0199	.0490	.0956	.1824	.3339	.5184	.6609
.10	.1903	.4154	.6860	1.0173	1.3409	1.5521	1.6550
.20	.3629	.7105	1.0439	1.3638	1.6106	1.7456	1.8050
.30	.5202	.9308	1.2636	1.5386	1.7263	1.8222	1.8620
.40	.6641	1.1016	1.4120	1.6437	1.7902	1.8604	1.8900
.50	.7962	1.2379	1.5194	1.7139	1.8313	1.8876	1.9079
.60	.9180	1.3492	1.6003	1.7644	1.8602	1.9028	1.9225
.70	1.0306	1.4417	1.6636	1.8025	1.8813	1.9156	1.9299
.80	1.1350	1.5199	1.7145	1.8319	1.8971	1.9260	1.9373
.90	1.2321	1.5869	1.7564	1.8553	1.9087	1.9338	1.9410

TABLE 10

DESIGN DETAIL

	Tray Design 1	Tray Design 2
<u>Bubble Caps</u>		
Diameter (O.D.), in.	1.5	3.0
Height, in.	1.5	2.625
Metal thickness, in.	1/16	1/16
Number per tray	41	17
Number of cap rows	5	3.5
Distance between centerline of last row and outlet weir, in.	4.00	1.38
Cap layout	On 2.5-in. square inline spacing	On 4.5-in. triangular spacing
<u>Cap Slots</u>		
Height, in.	0.75	0.75
Width, in.	0.125	0.125
Number per cap	18	38
Number per tray	738	646
Area per cap, sq. ft.	0.0117	0.0247
Area per tray, sq. ft.	0.480	0.421
<u>Cap Rises</u>		
Diameter (I.D.), in.	0.938	1.843
Height above tray, in.	1.172	1.610
Area per tray, sq. ft.	0.197	0.315
Weir length, in.	15.25	15.25
Bubbling Area, sq. ft.	2.57	2.57
Distance from inlet downpipe to outlet weir, in.	15.60	15.60

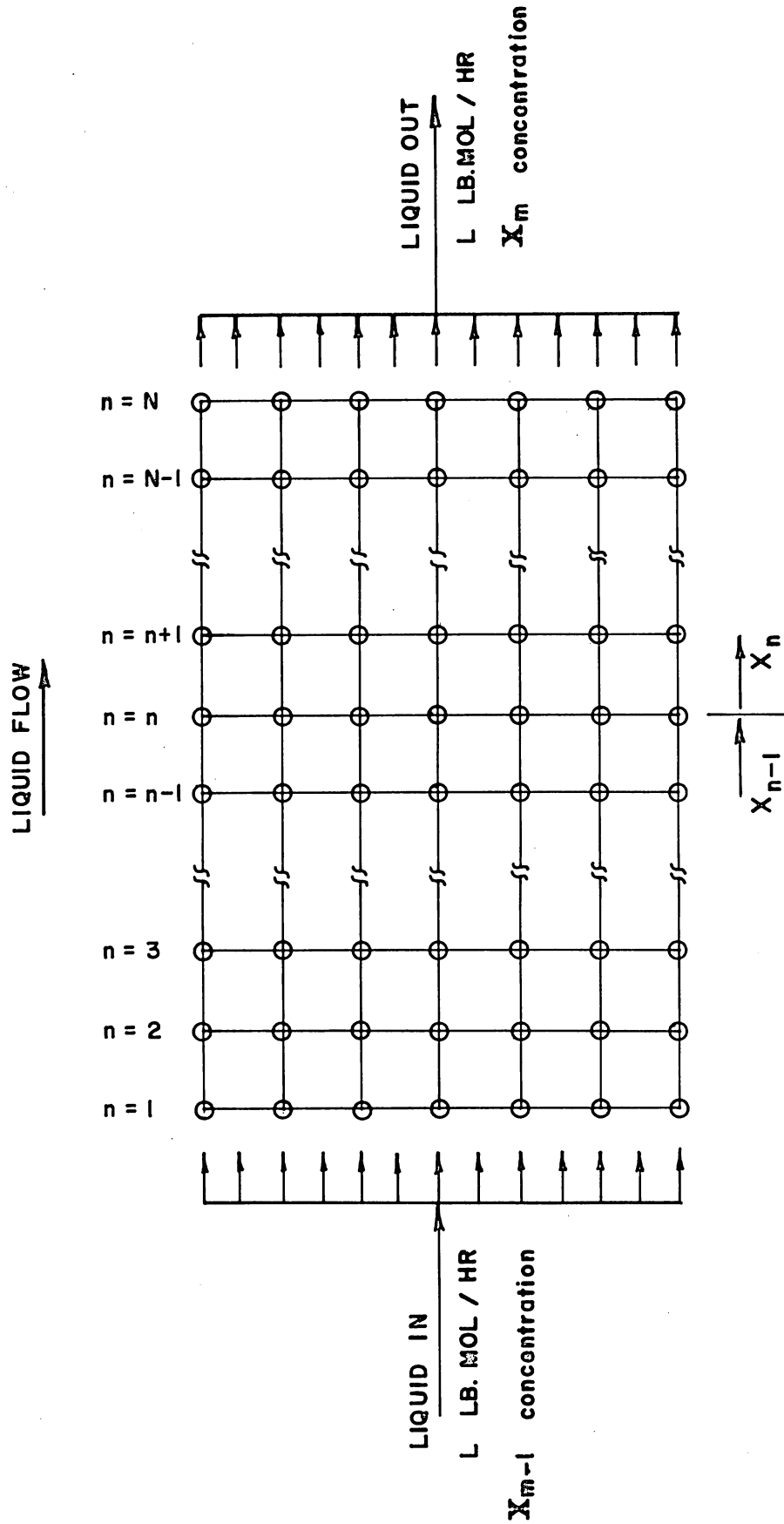


FIGURE 1
Simplified Model of Plate M

FIGURE 2

The Effect of Plate Sizing on
Plate Efficiency

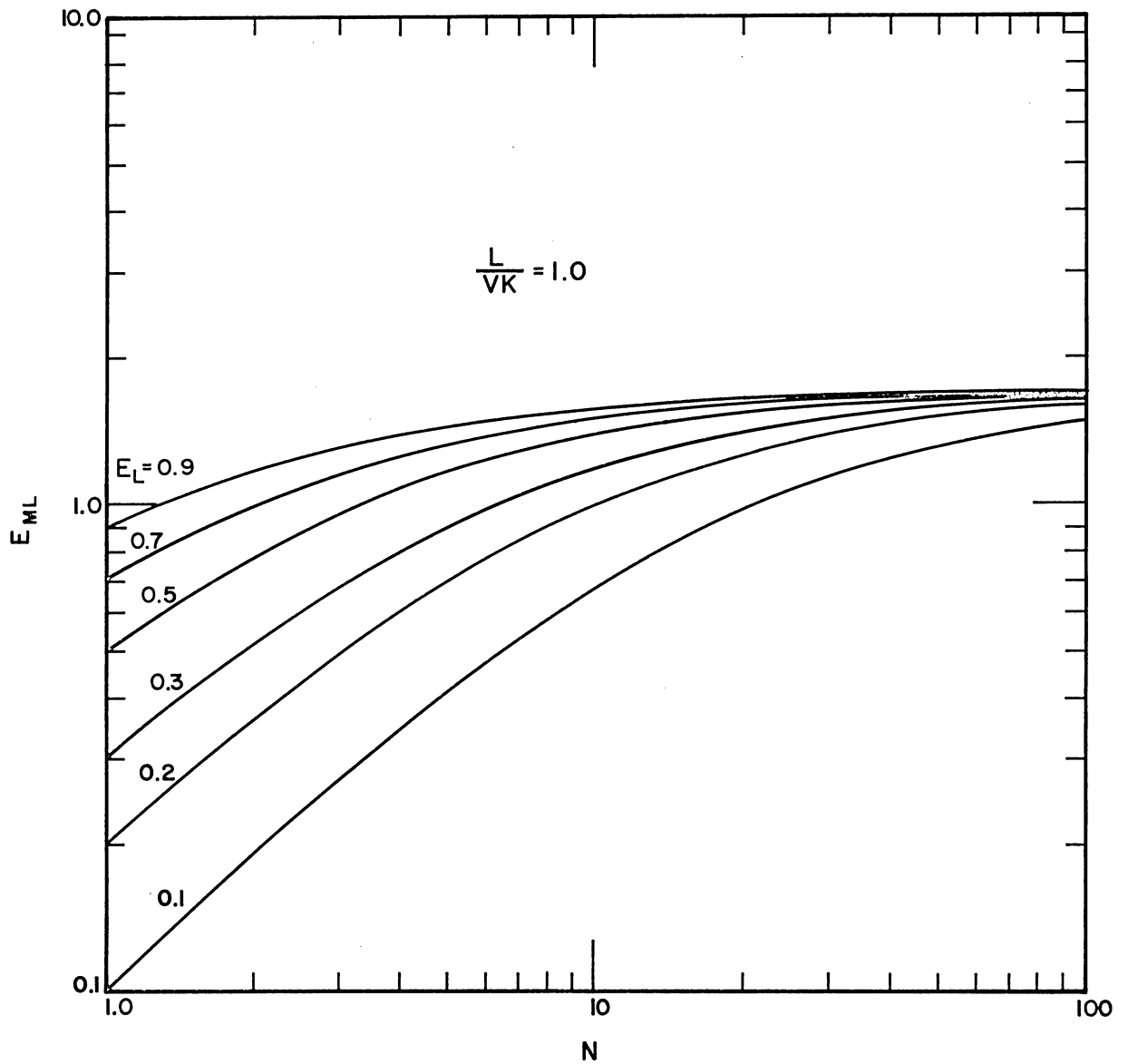


FIGURE 3

Effect of Gas and Liquid Rates
on Efficiency

Ammonia-Air-Water System
at 20°C and 1 Atm

Tray Design 1

Outlet Weir Height = 1 in.

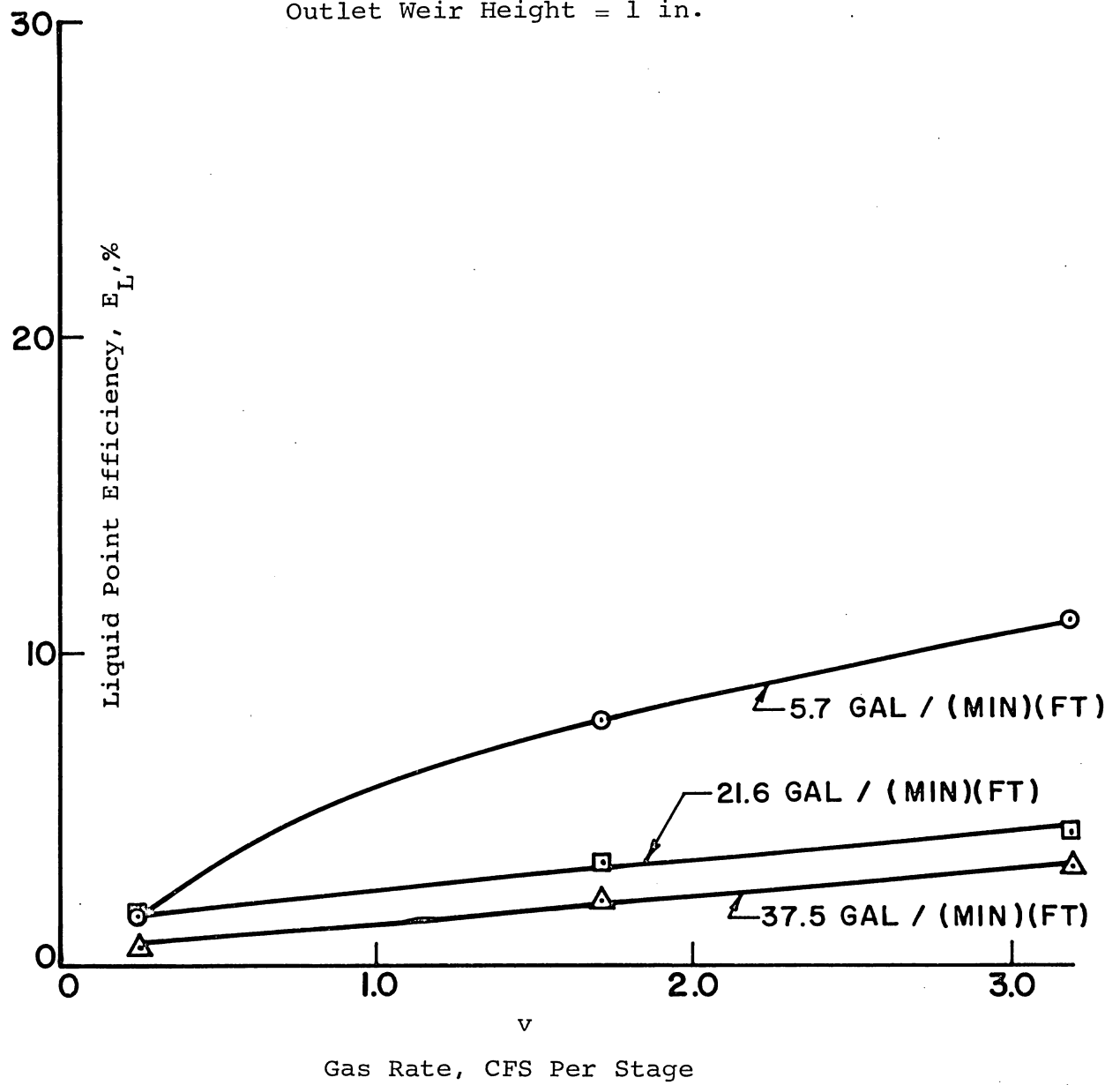


FIGURE 4

Effect of Gas and Liquid Rates
on Efficiency

Ammonia-Air-Water System
at 20°C and 1 Atm

Tray Design 1

Outlet Weir Height = 3 in.

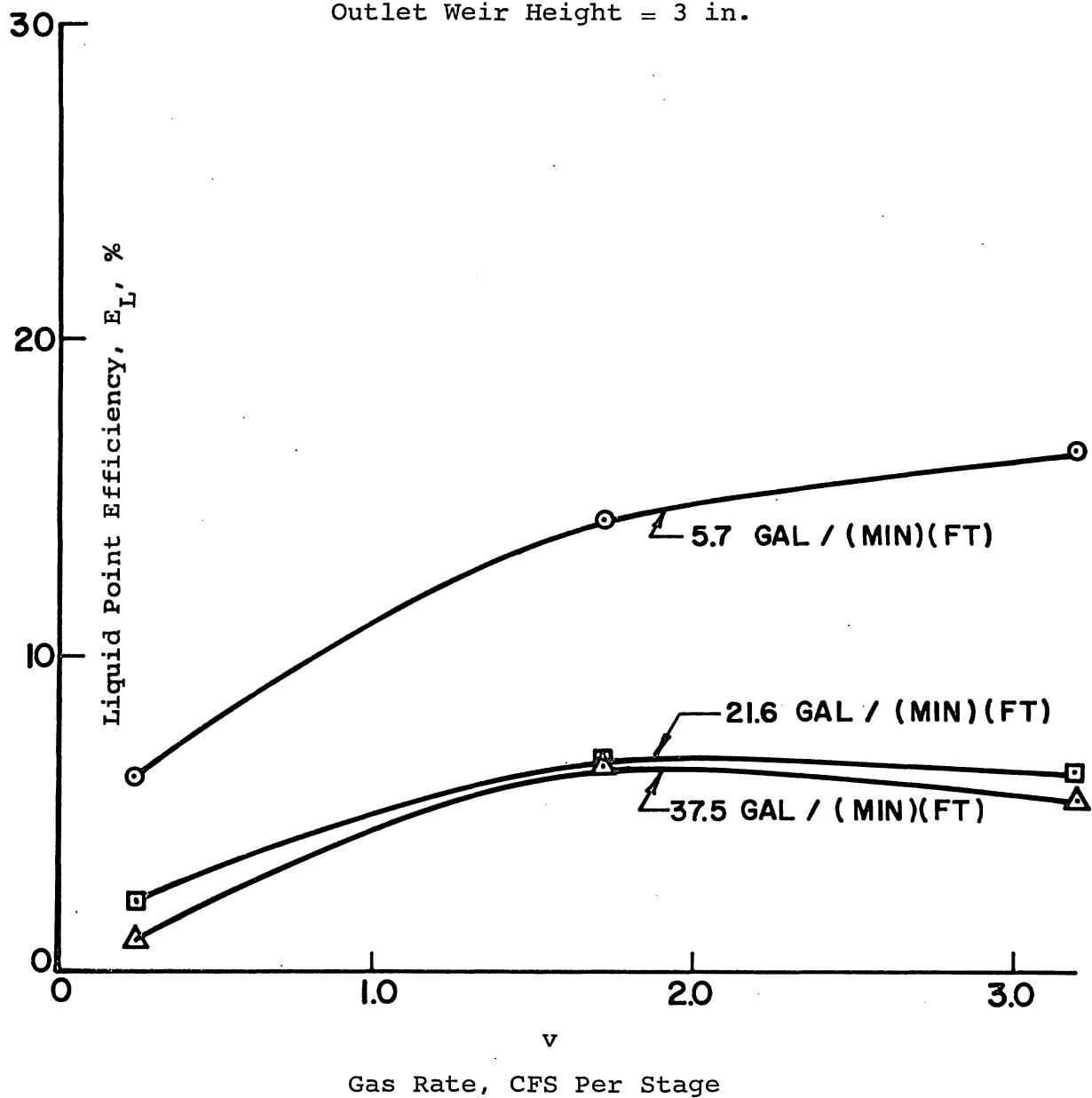


FIGURE 5

Effect of Gas and Liquid Rates
on Efficiency

Ammonia-Air-Water System
at 20°C and 1 Atm

Tray Design 1

Outlet Weir Height = 5 in.

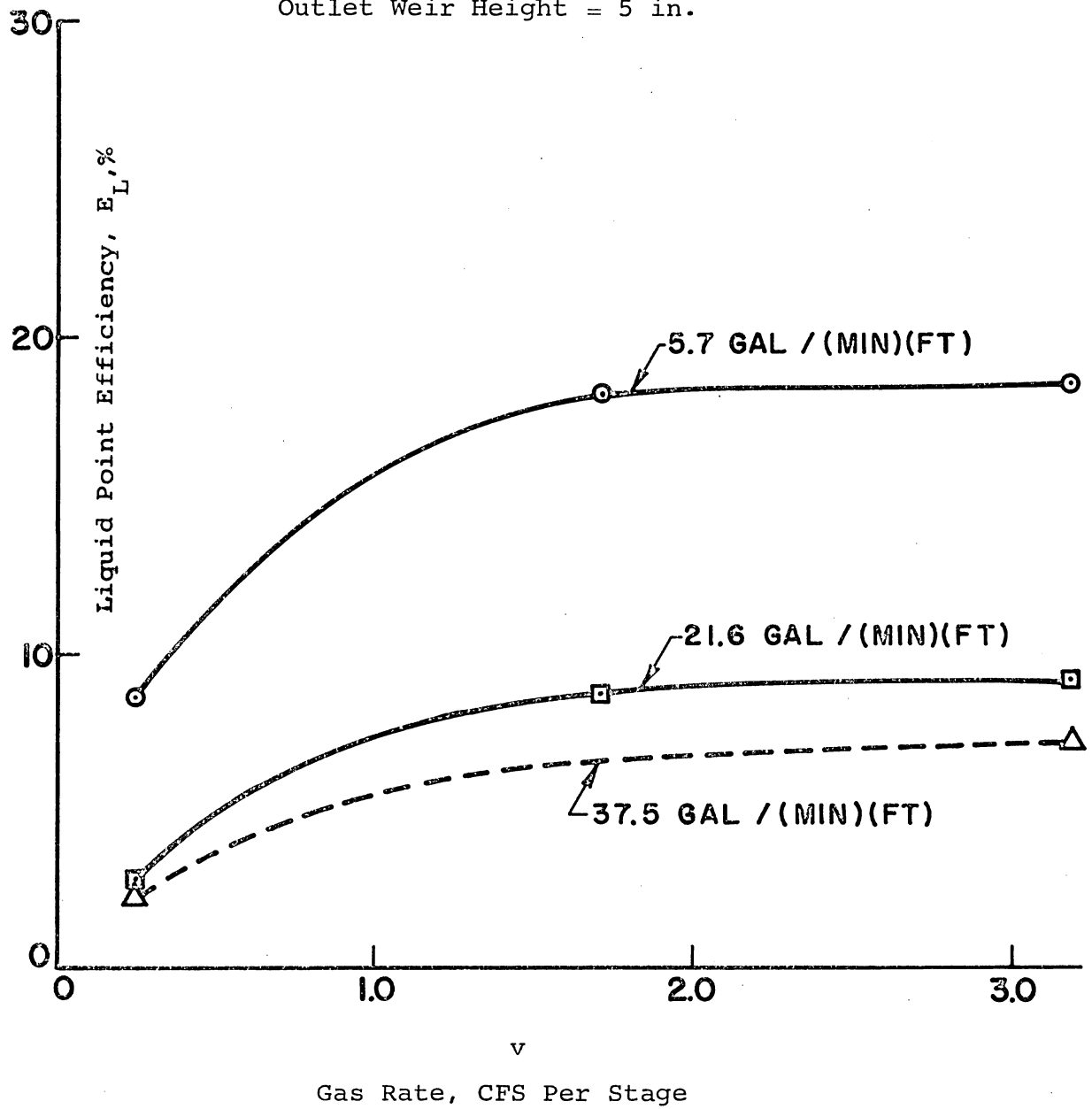


FIGURE 6

Effect of Gas and Liquid Rates
on Efficiency

Ammonia-Air-Water System
at 20°C and 1 Atm

Tray Design 2

Outlet Weir Height = 1 in.

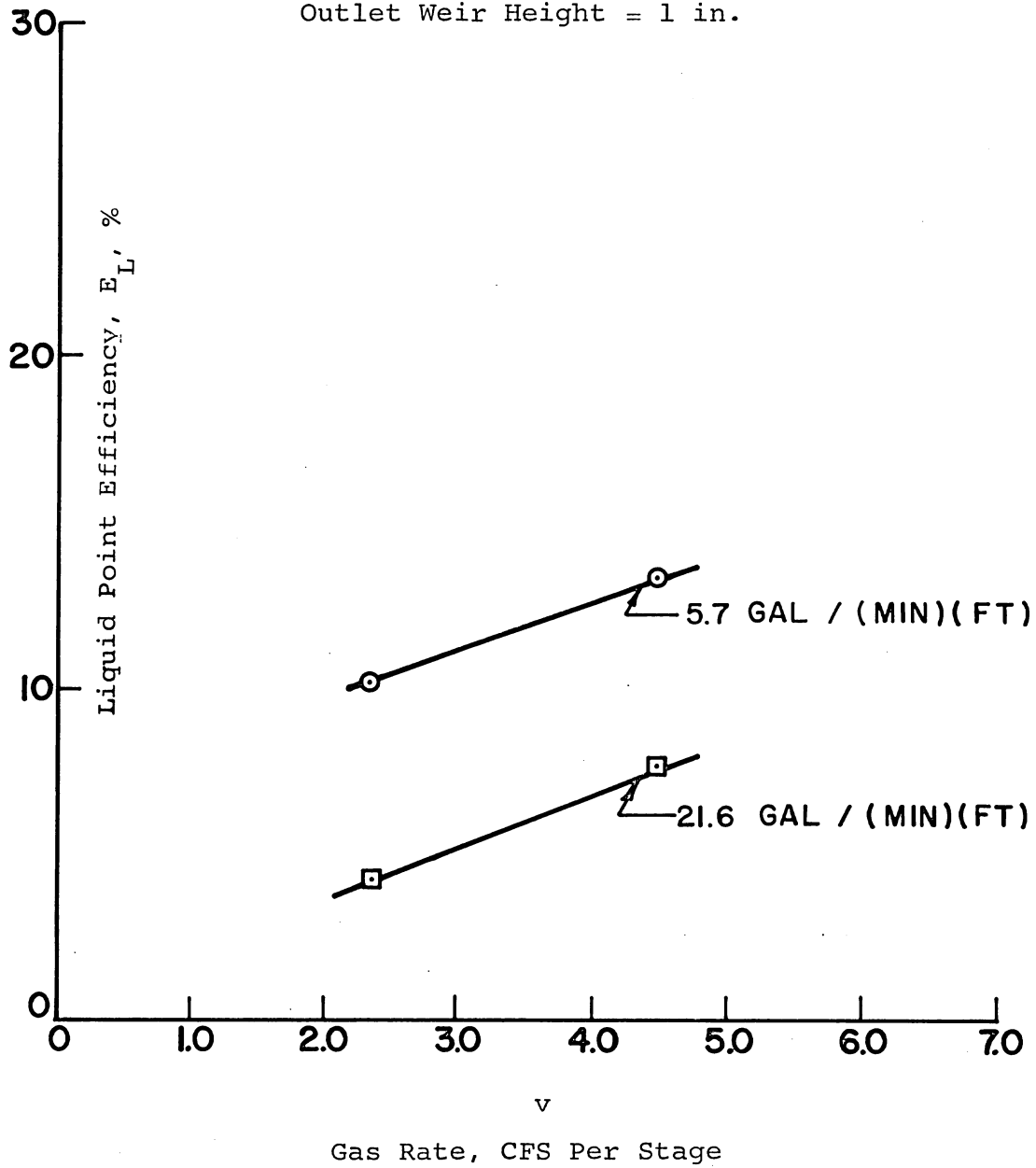


FIGURE 7

Effect of Gas and Liquid Rates
on Efficiency

Ammonia-Air-Water System
at 20°C and 1 Atm

Tray Design 1

Outlet Weir Height = 3 in.

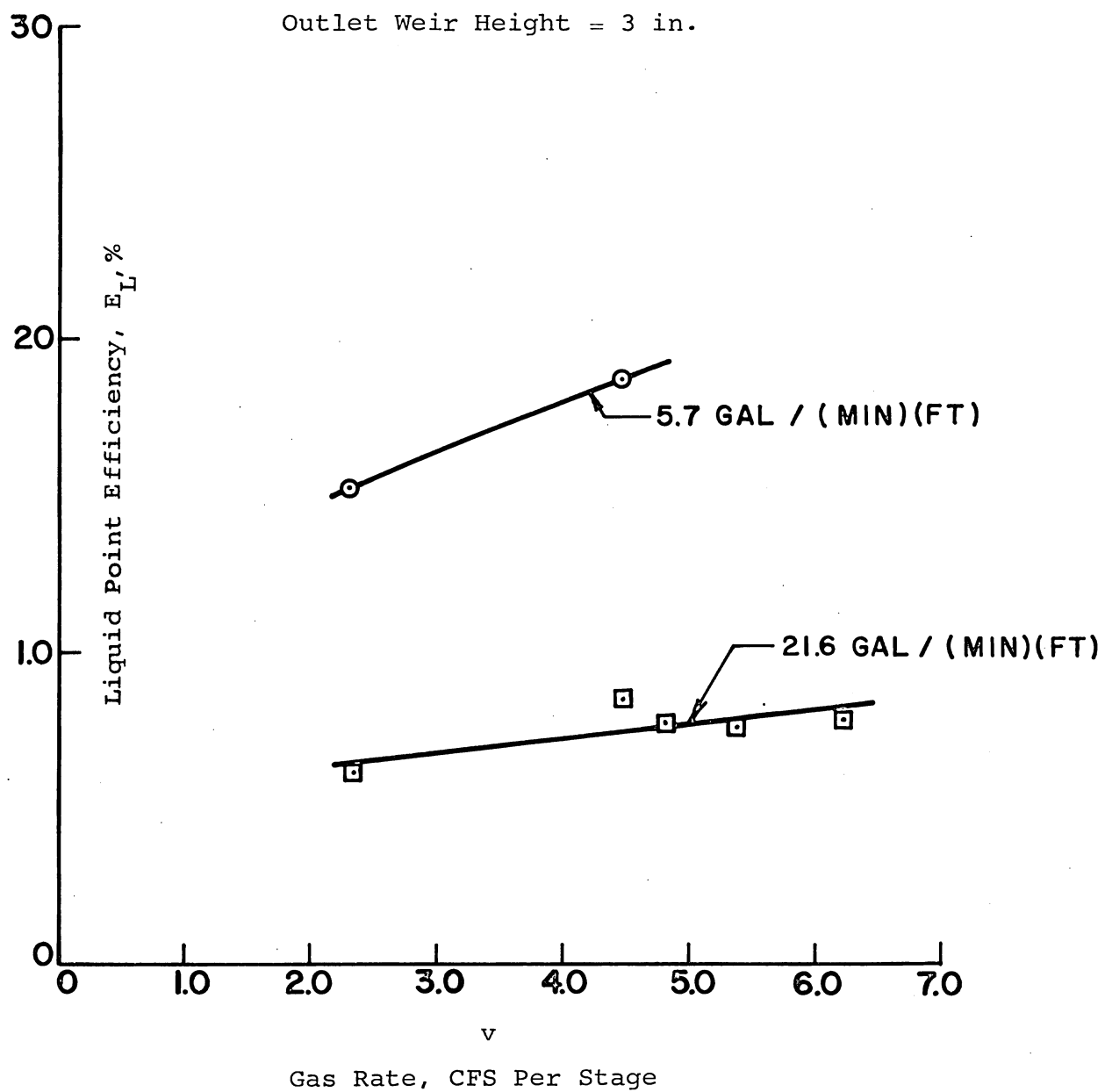


FIGURE 8

Effect of Gas and Liquid Rates
on Efficiency

Oxygen-Air-Water System
at 25°C and 1 Atm

Tray Design 1

Outlet Weir Height = 1 in.

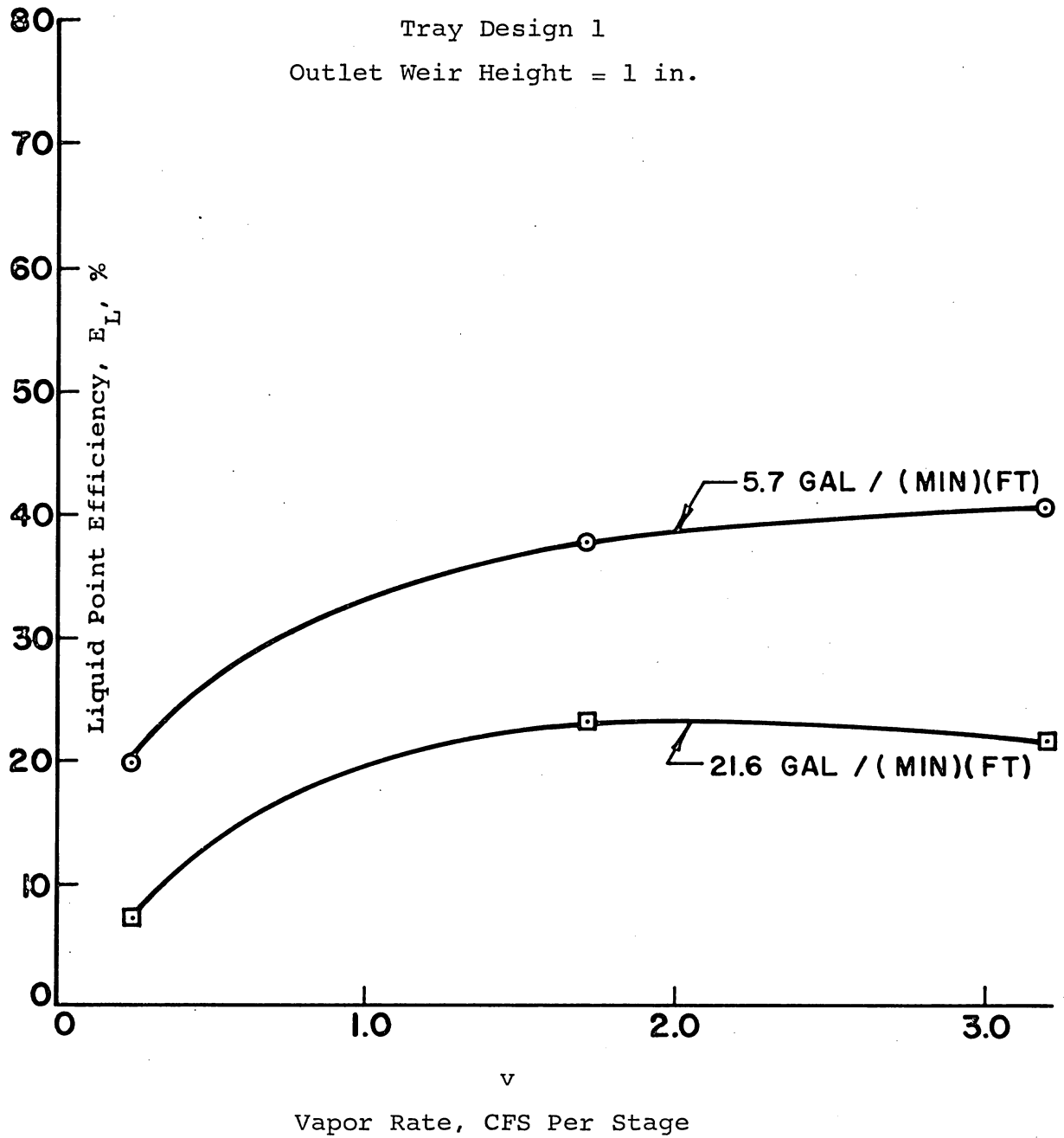


FIGURE 9

Effect of Gas and Liquid Rates
on Efficiency

Oxygen-Air-Water System
at 25°C and 1 Atm

Tray Design 1

Outlet Weir Height = 3 in.

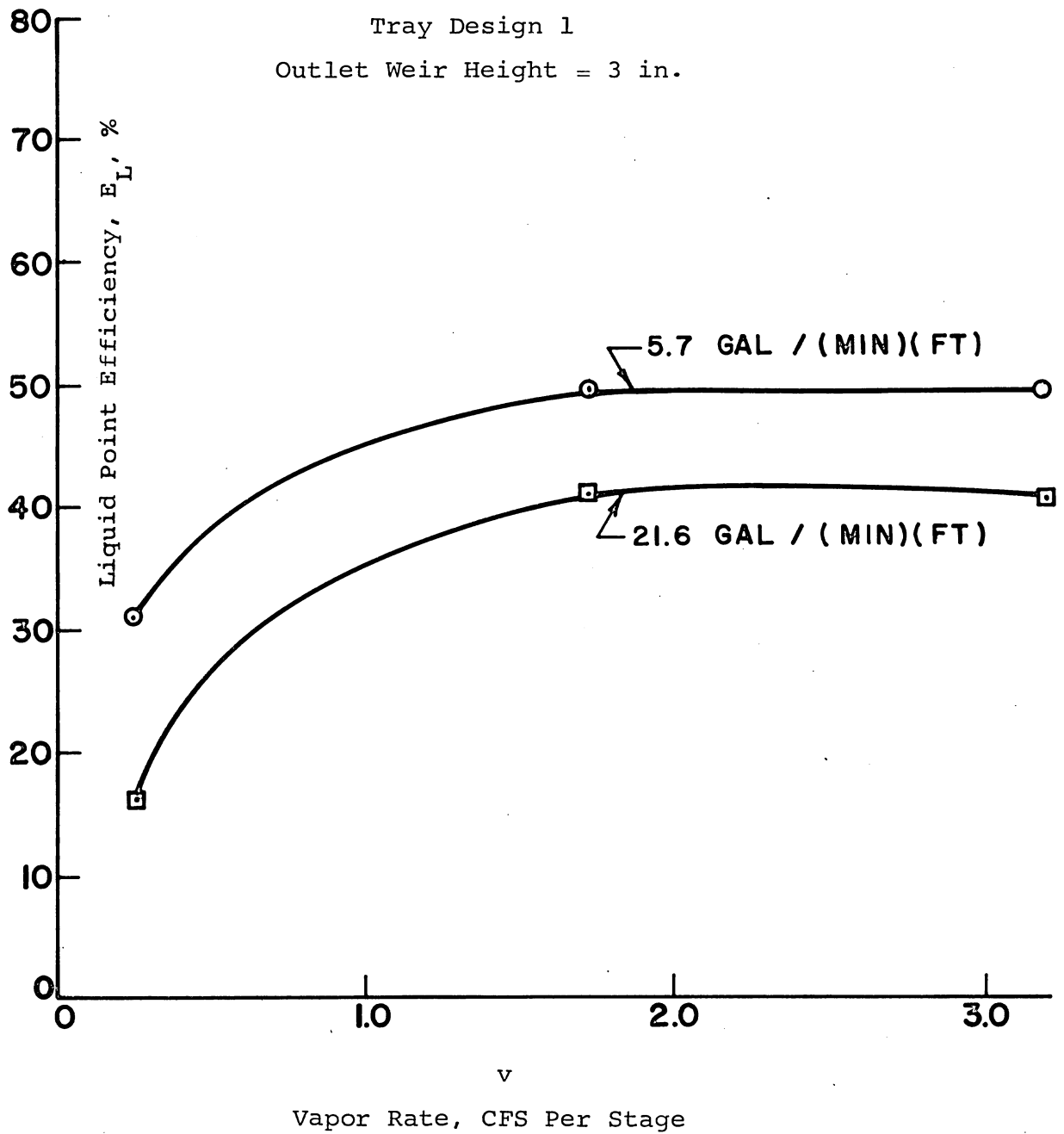


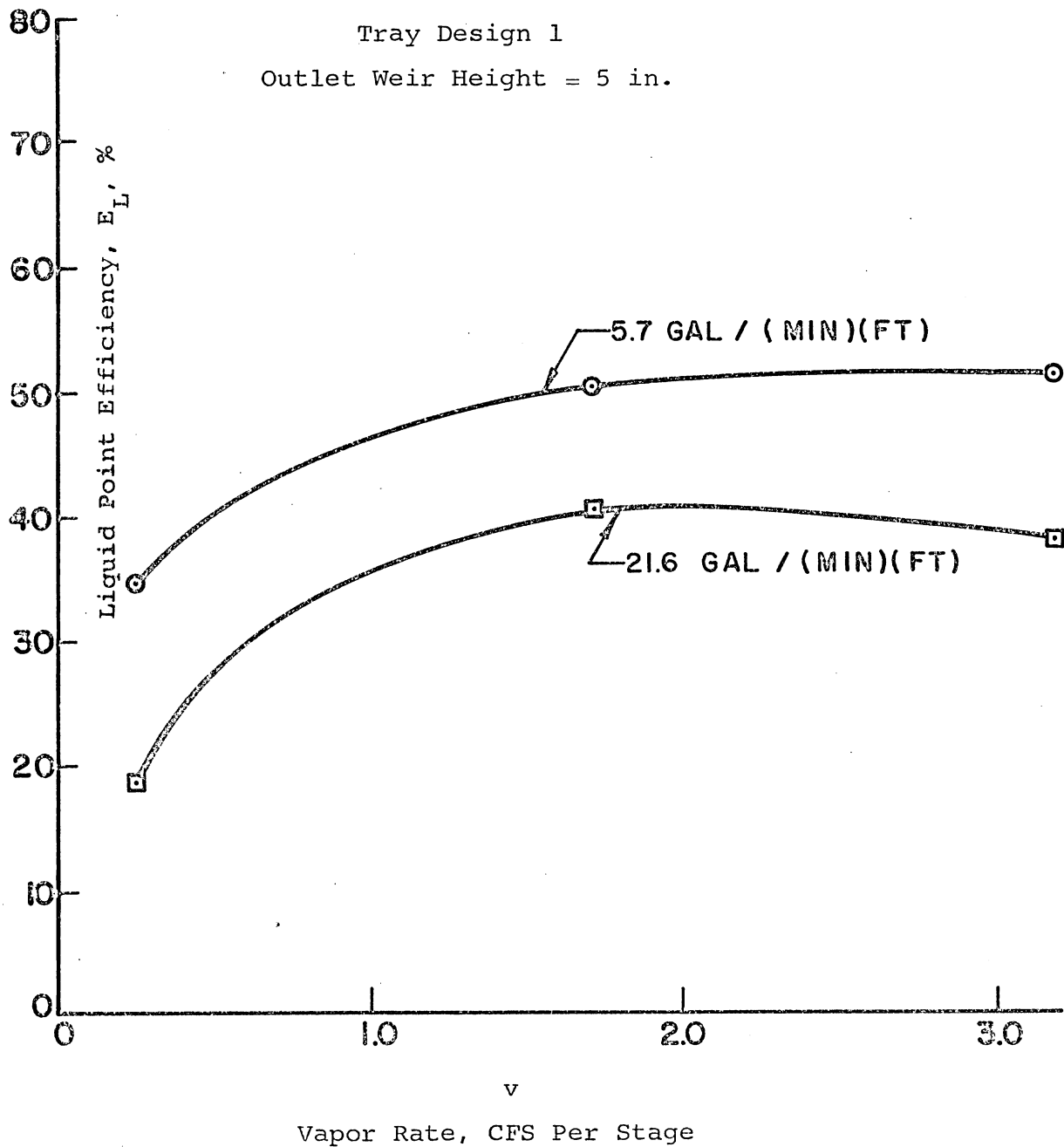
FIGURE 10

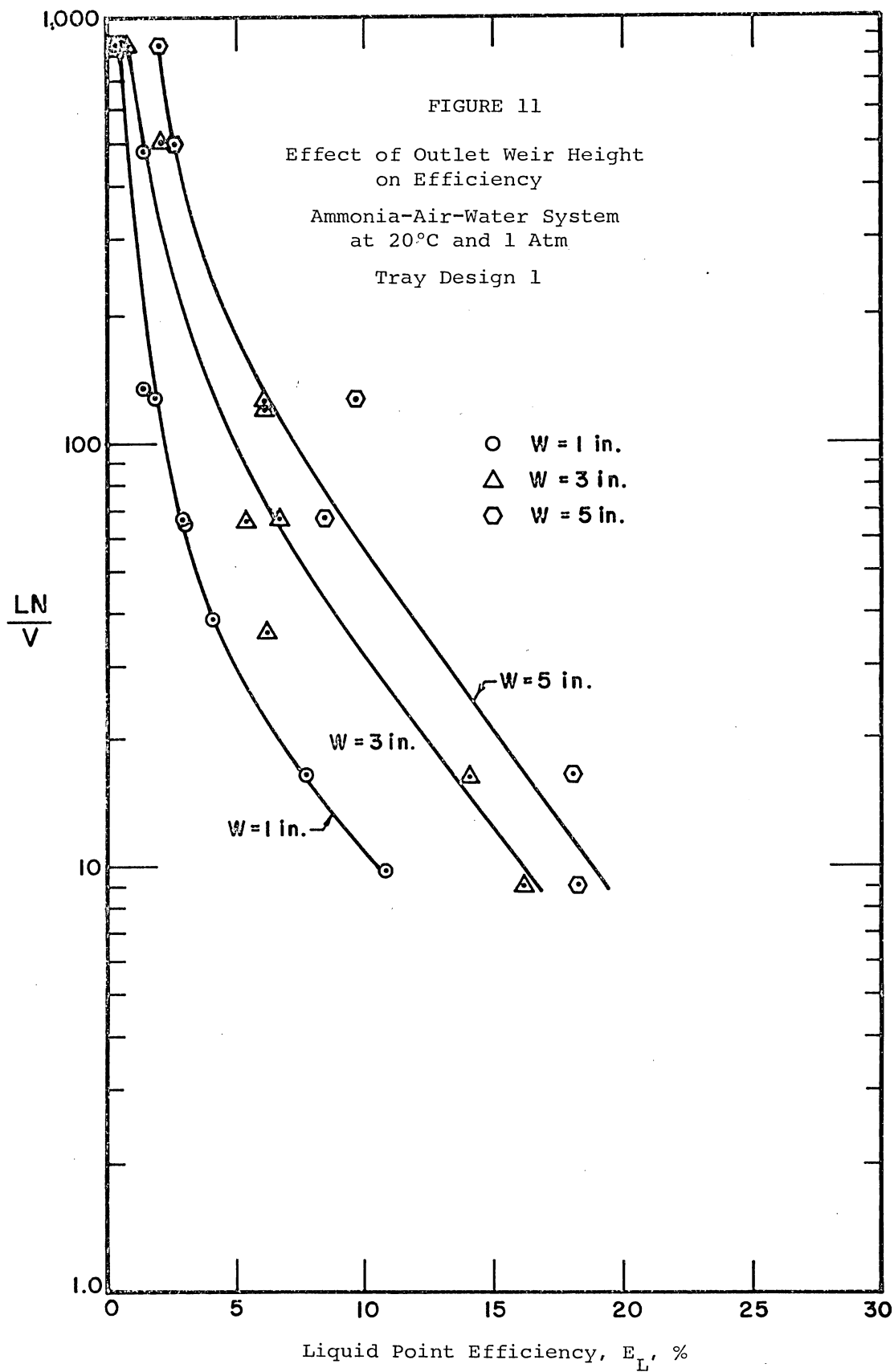
Effect of Gas and Liquid Rates
on Efficiency

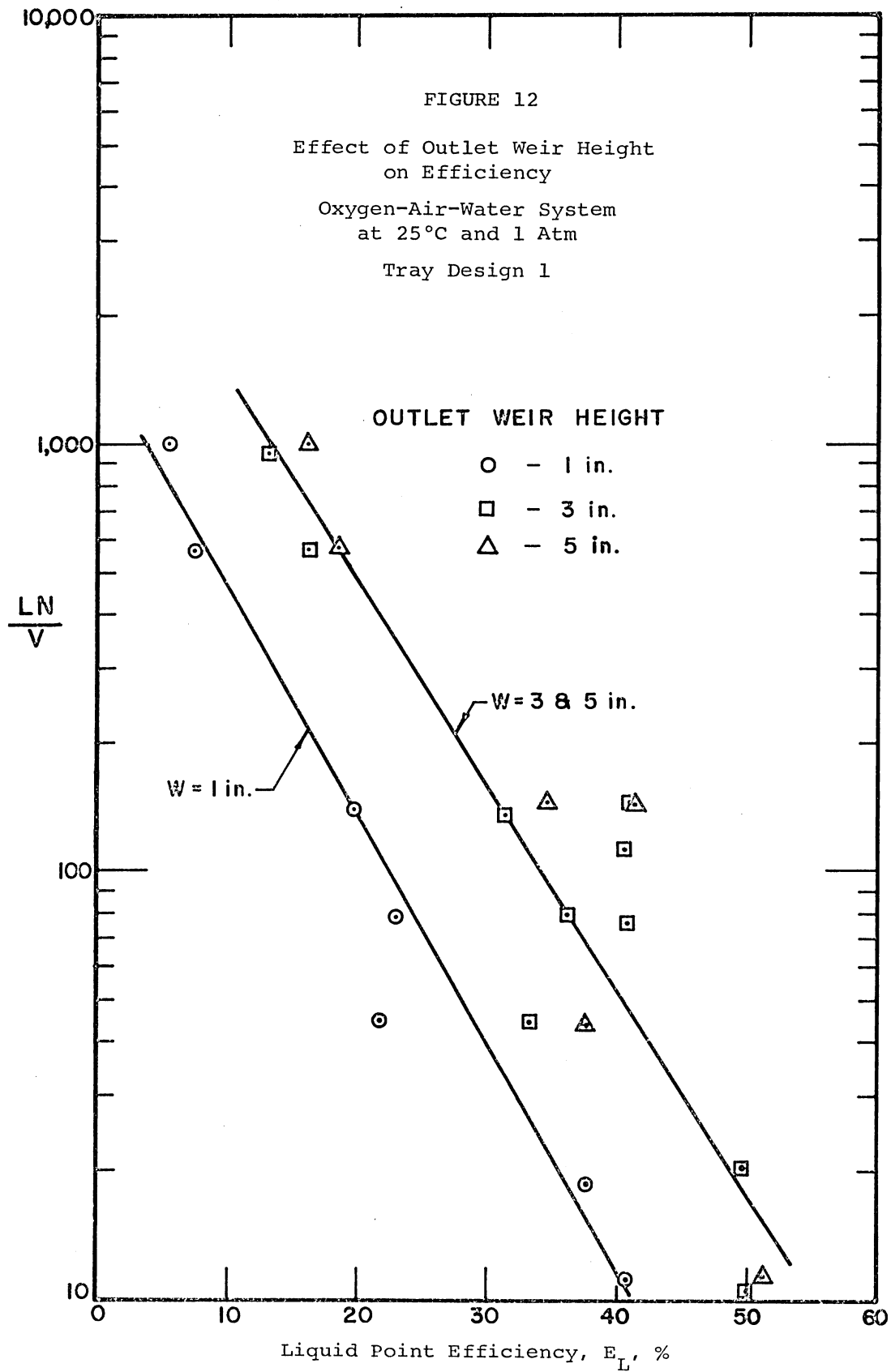
Oxygen-Air-Water System
at 25°C and 1 Atm

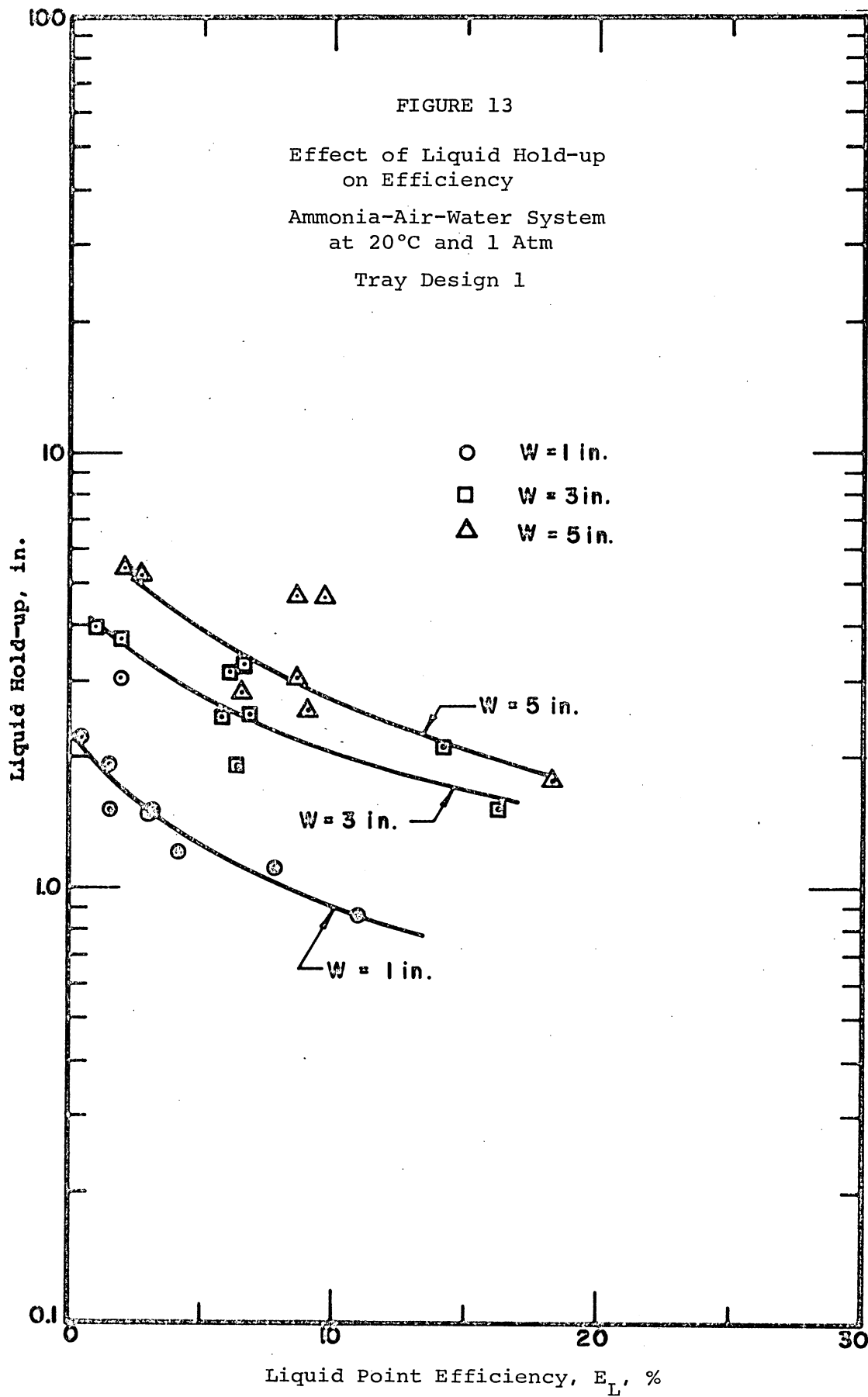
Tray Design 1

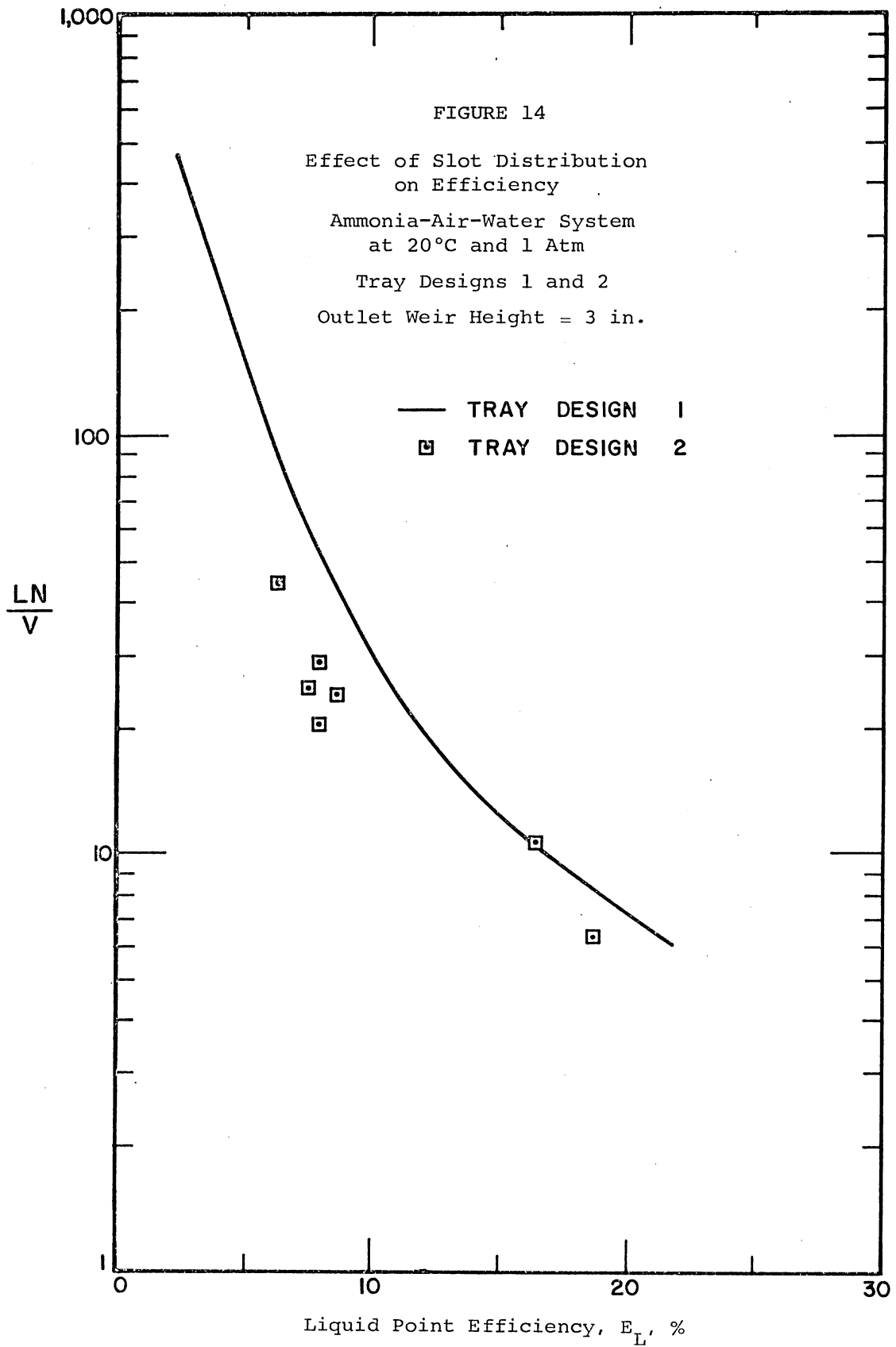
Outlet Weir Height = 5 in.

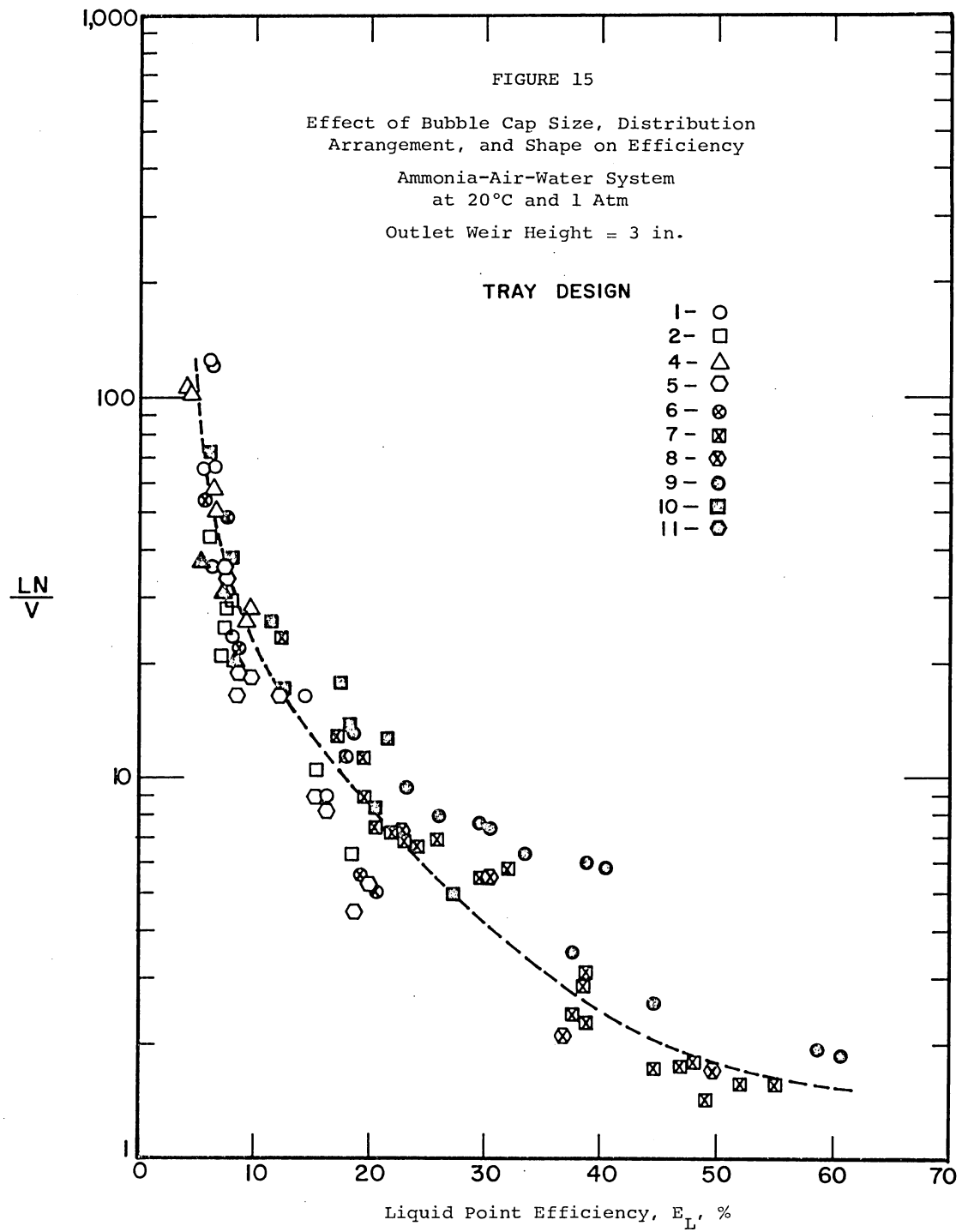


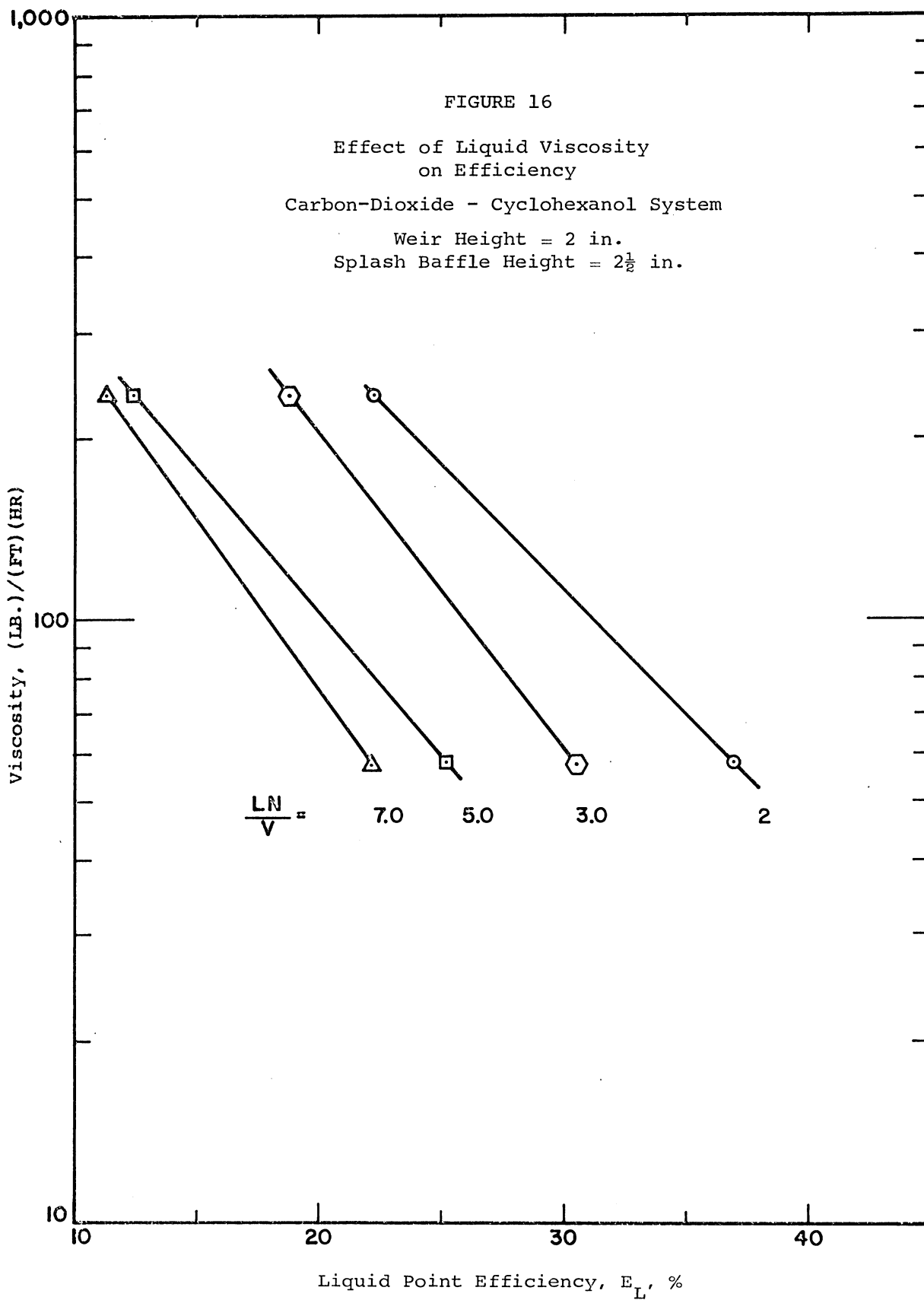


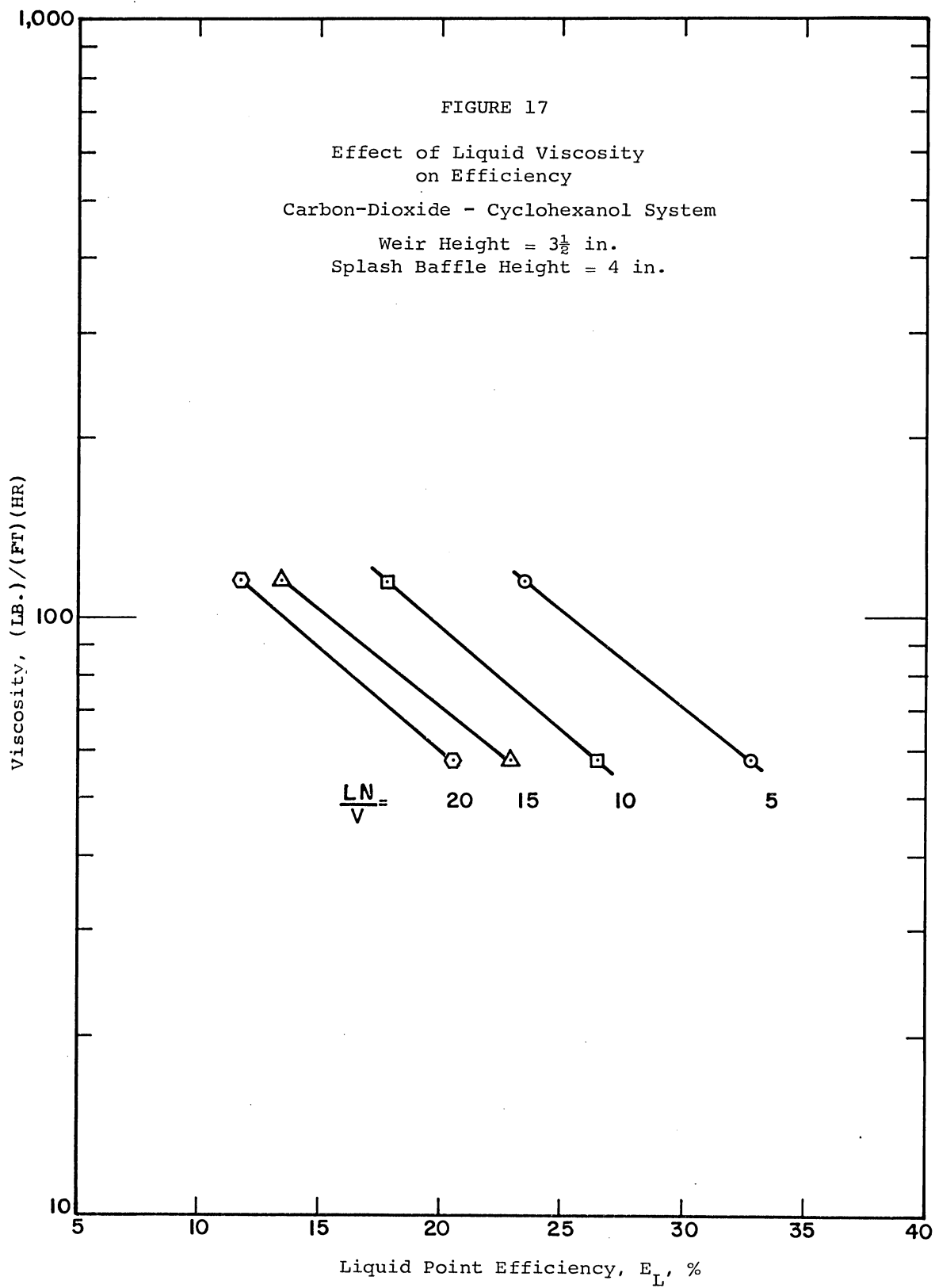












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APPENDIX

Solution of the difference equation

$$x_n - Ax_{n-1} - B = 0 \quad (1)$$

The reduced equation is

$$x_n - Ax_{n-1} = 0 \quad (2)$$

Assume a solution of the form

$$x_n = CB^n \quad (3)$$

Substituting (3) into the reduced equation (2) gives

$$CB^n - ACB^{n-1} = 0 \quad (4)$$

which reduces to

$$B = A \quad (5)$$

Thus the general solution of the reduced equation is:

$$x_n = CA^n \quad (6)$$

Assume that a particular solution of the complete equation is

$$x_n = C_1A^n + D \quad (7)$$

Substituting (7) into (1) gives

$$C_1A^n + D - AC_1A^{n-1} - AD - B = 0 \quad (8)$$

which reduces to

$$D - AD - B = 0 \quad (9)$$

Solving for D yields

$$D = \frac{B}{1-A} \quad (10)$$

Thus the general solution to the complete equation is

$$x_n = CA^n + C_1 A^n + \frac{B}{1-A} \quad (11)$$

This may be written as

$$x_n = C_2 A^n + \frac{B}{1-A} \quad (12)$$

The boundary conditions are

when $N = 0$ $x_n = X_{m-1}$

where X_{m-1} is the composition of the entering liquid.

Substituting this boundary condition in (12) gives

$$X_{m-1} = C_2 + \frac{B}{1-A} \quad (13)$$

or

$$C_2 = X_{m-1} - \frac{B}{1-A} \quad (14)$$

Thus the solution to the complete equation is

$$x_n = \left(X_{m-1} - \frac{B}{1-A}\right) A^n + \frac{B}{1-A} \quad (15)$$