A RESONANT CAVITY FOR A MICROWAVE MASER

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INTRODUCTION

1.1 Introduction

Seeking to enlarge very small signals of electromagnetic energy can prove frustrating to the engineer if his signals are accompanied by random energy fluctuations. These random, unwanted fluctuations are termed noise. An ideal amplifier would enlarge a signal and add no noise to the output; amplifiers which approach this ideal are termed low noise amplifiers. In recent years two approaches toward low noise amplification have become interesting: the parametric amplifier and the maser amplifier. The maser amplifier utilizes several branches of technology which are new to the field of electrical engineering and is of interest from the point of view of expanding the skills of the designer. It was for the reasons of low noise amplification and expanding the skills of the designers that a project of maser construction was initiated in the department of electrical engineering at Rice University.

A maser is an amplifier utilizing emission of photons of electromagnetic energy to achieve power gain. Coined by Gordon and Townes, who built the first maser, the word maser is an anacronym for Microwave Amplification by Stimulated Emission of Radiation. As the result of activity in the field of this type of amplifier, masering has come to apply to all types of stimulated emission amplification.
regardless of the frequency range of the device. At the time of this writing a continuous wave ruby light amplifier has been announced using the principles of maser amplification. Photons emitted by molecular or atomic energy level transitions are the basis for maser action.

It is known that matter exists only in certain allowed energy states or energy levels. The most common example for this is the hydrogen atom. The electron circling the nucleus is allowed only certain distinct orbits, and with each orbit there is associated a unique energy state for the atom. A system may make a transition from one energy level to another. This transition is accompanied by the absorption or radiation of a photon of electromagnetic energy, absorption if the system energy increases, radiation if the system energy decreases. This situation results from the principles of conservation of energy and momentum. The frequency of the absorbed or admitted photon of radiation is given by Planck's Law

\[ \Delta E = hf. \]  

The type of transition which is interesting for maser work is the stimulated transition, i.e., the transitions caused by electromagnetic radiation which is applied externally to the system. This radiation, applied at a frequency determined by Planck's Law, induces the system to make a transition. Upward transitions, absorbing energy,
represent a loss to the applied radiation; downward transitions, emitting energy, represent an amplification. Contrasted with the stimulated transition is the spontaneous transition. Spontaneous transitions occur randomly with provocation only from energy sources within the system. Associated with each of these transitions is a probability of its occurrence. This is just the probability that a system will have a transition from energy state $i$ to energy state $j$. For a stimulated transition the probability of an upward or downward transition is the same.

1.2 A Qualitative Description of Maser Action

With these ideas in mind the concepts of maser action may be introduced. Let us assume energy levels $E_1$, $E_2$ with a number of occupied states $n_1$, $n_2$ respectively. In this case we will consider $E_2 > E_1$ and the thermal equilibrium population of state given by Boltzmann's distribution law:

$$n_2 = n_1 e^{-\frac{(E_2 - E_1)}{kT}}. \quad (2)$$

Where $k =$ Boltzmann's constant the $T$ is the temperature in degrees Kelvin. If electromagnetic radiation is present at the frequency given by (1), then the power absorbed due to upward transitions is given by

$$P_A = W_{12} n_1 hf_{12} \quad (3)$$
where $W_{12}$ is the probability of a stimulated transition from state 1 to state 2. The power emitted is given by

$$P_E = W_{21} n_2 h f_{12}. \quad (4)$$

But $W_{12} = W_{21}$ hence the net power absorbed is

$$P_{\text{net}} = W_{12} (n_1 - n_2) h f_{12}. \quad (5)$$

It is clearly seen that if $n_2$ is greater than $n_1$ the net power absorbed is actually power emitted. For this to occur a condition of non-thermal equilibrium must exist. Considerations which lead to coherent absorption for $n_1 > n_2$ suggest that when $n_2 > n_1$ coherent amplification will result.

There are various methods for creating the required non-thermal equilibrium state. However, there exists a tendency for any system to fall back to the thermal equilibrium state in time. The time associated with the system returning to equilibrium is called the relaxation time.

The system of energy states required may be bound states of molecules or spin states in a magnetic field. The first maser used bound states of molecules incorporating an ammonia beam.\(^2\) An electric field focuser was used to separate the high energy molecules from the low. Then electromagnetic radiation of 23.870 Gc was amplified when the high state to low state transitions were stimulated.

A system of energy levels of larger interest is the spin-states of a paramagnetic salt in the presence of an
external magnetic field. A scheme for continuous wave maser operation utilizing this system was proposed in 1956 by Bloembergen. ³

This idea involved a system with three energy levels. Radiation of one frequency was used to saturate two levels, i.e., to create equal population in each of the levels. If the frequency is chosen properly, a population inversion is achieved between the third level and one of the saturated levels. This type of action allows a continuous amplification of the signal frequency.

A material commonly used for this scheme is a ruby crystal, which consists of trivalent chromium ions in a sapphire (Al₂O₃) lattice. In the presence of an external magnetic field certain spin states of the ruby are allowed to exist in four separate energy levels. ⁴ The separation of these levels is dependent upon the strength of the magnetic field and the orientation of the crystal axis of symmetry. A sketch is shown, fig. 1, of the energy levels for a given angle and variable field strength. ⁵

Although four levels are present in ruby, only three are needed.
\[ \theta = 50^\circ \]

Figure 1. Zeeman splitting of Cr\(^{+++}\) ions in an Al\(_2\)O\(_3\) lattice. \(\theta\) measured between the \(H_0\)-field axis and the crystal c-axis.
2.1 The Three Level Continuous Wave Maser

This section is devoted to a development of the theory of a simple three level maser scheme primarily to develop the requirements for a maser cavity. Initially the maser to be constructed was planned to be a narrow band amplifier, similar to the original masers built. This involves a sample which is fairly small in volume compared to the total volume of the cavity. In this event the characteristics of the cavity with sample may be treated as a perturbation from the fields in the vacant cavity.

The amplification process is accomplished in the following manner. Let us assume three levels \( E_3 > E_2 > E_1 \) with populations \( n_3 < n_2 < n_1 \) related to each other by Boltzmann's thermal equilibrium distribution law (equation 2). A plot of energy versus population of states will have the appearance of a bar graph looking somewhat like figure 2a.

We apply radiation of frequency

\[
f_{13} = \frac{E_3 - E_1}{h}
\]

and stimulate transitions from level 1 to level 3. If the applied power is great enough, then a state may be reached where
Figure 2a. Schematic showing energy level population at thermal equilibrium (Boltzmann's distribution).

Figure 2b. Schematic showing energy level population with radiation $f_{13}$ applied strongly enough to saturate levels 1 and 3.
When this state is reached the graph takes on the appearance of figure 2b. As can easily be seen, a population inversion exists between levels $2$ and $1$ and a signal of frequency

$$f_{21} = \frac{E_2 - E_1}{\hbar}$$

may be amplified.

The radiation creating the inversion is called the **pump**; while the radiation amplified is called the **signal**. Locating the ruby crystal in a microwave cavity terminating a transmission line is one way to place the system of energy levels in the vicinity of the required electromagnetic radiation. The cavity containing the ruby must be fed by transmission lines for both the pump and signal energies and be located in a constant magnetic field. The cavity need not terminate the line for the signal frequency—in this case the amplifier is termed a transmission cavity maser. The cavity terminating the transmission line was chosen for work here at Rice.

A more detailed analysis follows to give further insight into maser action. The development here follows that outlined by A. E. Siegman of Stanford University in unpublished course notes. It is seen by those acquainted with the field of masers and electron paramagnetic resonance studies that this development is similar to other
developments in the literature. The primary fruits of this labor will be requirements of signal frequency and signal level.

Let us consider energy levels and populations as before denoting \( f_{31} = f_p \) as the pump frequency and \( f_{32} = f_s \) as the signal frequency. The remaining transition will be denoted by \( f_1 = f_{21} \) or the idle frequency. We note that

\[
f_s + f_1 = f_p.
\]

The complete rate equations for the three levels may be written as follows: \(^7\)

\[
\frac{dn_1}{dt} = -w_{12}n_1 - w_{13}n_3 + w_{21}n_2 + w_{31}n_3 + W_{12}(n_2 - n_3) + W_{13}(n_3 - n_1)
\]

\[
\frac{dn_2}{dt} = -w_{21}n_2 - w_{32}n_2 + w_{12}n_1 + w_{32}n_3 + W_{12}(n_1 - n_2) + W_{23}(n_3 - n_2)
\]

\[
\frac{dn_3}{dt} = -w_{31}n_3 - w_{32}n_3 + w_{13}n_1 + w_{23}n_2 + W_{13}(n_1 - n_3) + W_{23}(n_2 - n_3)
\]

where the \( w_{ij} \) are the transition probabilities induced on the spin system by its coupling to the lattice and the \( W_{ij} \) are the stimulated transition probabilities. With a few assumptions, primarily that \( hf/kT \) is much less than 1, and some algebraic manipulation these equations may be rewritten.

\[
\frac{dn_1}{dt} = \frac{\Delta n_{12} - \Delta N_{12}}{2T_1(12)} - \frac{\Delta n_{13} - \Delta N_{13}}{2T_1(13)} - W_p \Delta n_{13}
\]
\[
\frac{dn_2}{dt} = - \frac{\Delta n_{21} - \Delta N_{21}}{2T_1(12)} - \frac{\Delta n_{23} - \Delta N_{23}}{2T_1(23)} - w_s \Delta n_{23}
\]

\[
\frac{dn_3}{dt} = - \frac{\Delta n_{31} - \Delta N_{31}}{2T_1(13)} - \frac{\Delta n_{32} - \Delta N_{32}}{2T_1(23)} - w_p \Delta n_{31} - w_s \Delta n_{32}
\]

Where

\[
\Delta n_{ij} = - \Delta n_{ji} = n_i - n_j
\]

\[
\Delta N_{ij} = - \Delta N_{ji} = N_i - N_j
\]

and \(T_1(ij)\) is the spin-lattice relaxation time for transition \(ij\). The lower case letters denote the population of the levels when thermal equilibrium does not exist; the upper case letters denote population under thermal equilibrium conditions.

The three equations (7) above are set equal to zero because the steady state, with both pump and signal applied, of the inverted population difference \(\Delta n_{32}\) is of interest. The general solution to the above equations is somewhat complicated and so some simplifying assumptions must be used. If the pumping signal is strong enough the pump transition will be saturated and hence \(\Delta n_{13}\) will be zero. Also it is clear that

\[
\Delta n_{32} = -\Delta n_{23} = \Delta n_{21}
\]

at saturation. Taking advantage of these simplifications leads to the following solution for \(\Delta n_{32}\):
\[ \Delta n_{32} = \frac{\Delta N_{12}}{T_1(12)} - \frac{\Delta N_{23}}{T_1(23)} \]

\[ \frac{1}{T_1(12) + 1/T_1(23) + 2W_s} \]

(8)

We note that the thermal equilibrium populations and the upwards and downwards relaxation rates between any two levels must obey the Boltzmann relation

\[ \frac{N_i}{N_j} = \frac{w_{ji}}{w_{ij}} = e^{-hf_{ij}/kT}. \]

(9)

Using the assumption that \( hf_{ij}/kT \) is much less than 1, we can write

\[ N_{ij} = (hf_{ij}/kT) (N/\ell). \]

where now \( \ell \) is the number of levels and \( N \) is the total population of all of the levels. In this case

\[ N = N_1 + N_2 + N_3 = n_1 + n_2 + n_3. \]

Using these results we can rewrite equation (8) as follows:

\[ \Delta n_{32} = -\Delta n_{21} = \frac{hf_{12}}{kT} \frac{N}{3} \left[ \frac{f_{12}/T_1(12) - f_{32}/T_1(23)}{1/T_1(12) + 1/T_1(23) + 2W_s} \right]. \]

(10)

If \( W_s \) is small compared to the other terms in the denominator, it may be neglected. It is clearly seen that an inversion will occur in either the 3-2 transition or the 2-1 transition depending upon which term in the numerator is larger. The condition for inversion can be seen to be
\[
\frac{f_1}{f_s} = \frac{f_p}{f_s} - 1 > \frac{T_l(\text{idle})}{T_l(\text{signal})}.
\]

In practice the values of the \( T_l \)'s are not well known;\(^8\) this leads to the assumption that they are of the same order of magnitude. This requires that the ratio \( f_p/f_s > 2 \) for maser action. More sophisticated analysis together with more energy levels leads to other conclusions. Masers have recently been operated with \( f_p/f_s < 1 \).

If the \( W_s \) term in equation (8) is not small compared with its companion terms in the denominator, this will of course have an effect on the maser performance. The emitted power will begin to decrease as the signal strength increases, as may be seen from a consideration of equations (5) and (8). This effect is known as signal saturation.

2.2 The Concept of Negative Q

In this section a simplified equivalent circuit will be derived for the cavity maser. The major approximation binding this equivalent circuit is that the maser action be relatively weak compared with the magnetic resonance line width. The results of this section are intended to be used as guideposts to understanding of maser operation in circuit terms and are not intended to reflect capabilities of maser operation in view of advances in the state of the art. The analysis is fairly indicative of the
planned Rice maser in its initial operating state.

Consider a resonant cavity containing a maser crystal and tuned to the signal frequency as shown in figure 3a. Only the signal guide to the cavity is shown, since for the moment we are not interested in the pump circuit. An equivalent circuit for the cavity is shown in figure 3b for the cavity in the frequency range of maser action. The ideal transformer represents the coupling from the line to the cavity. By transforming the impedance through the transformer the equivalent circuit of figure 3c is obtained.

From a consideration of transformers it is known that

\[ Z_p = n^2 Z_s \]  

(12)

for an ideal transformer. \( Z_p \) is the impedance looking in to the primary, \( Z_s \) is the secondary or load impedance, and \( n \) is the turns ratio of the transformer. If the factor \( n \) is variable, the impedance presented to the line by the cavity will vary. A means of changing the coupling will then change the load characteristics.

For circuits such as this it is convenient to talk in terms of a circuit quality factor, termed \( Q \), defined in the following manner:\(^{10}\)

\[ Q = \frac{\omega_0 \text{ maximum energy stored in network}}{\text{average power dissipated}} \]  

(13)
Figure 3a. Schematic representation of a reflection cavity maser.

![Schematic representation of a reflection cavity maser](image)

Figure 3b. An approximate equivalent circuit for microwave maser cavity, frequencies near the maser operating frequency.

![Equivalent circuit](image)

Figure 3c. Transmission line terminated at sending end by matched impedance; terminated at receiving end by

\[ Z_p = -R_m + j\omega L_0 + \frac{1}{j\omega C_0}. \]

**FIGURE 3.** Maser equivalent circuits.

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In the case of the maser in its operating state the crystal is emitting power which by convention is termed negative power. Using this concept we will define the "magnetic Q" of the maser cavity to be

\[ Q_m = \frac{\omega_0 W_s}{-P_m}. \]  (14)

where \( W_s \) represents the stored energy of the cavity and \(-P_m\) is the power emitted by the spin system. The cavity can be represented by the tuned circuit of figure 3b, if the resonant frequency and the Q are the same for both.

2.3 Calculation of Magnetic Q

To calculate the magnetic Q we will need to find the power emitted from the maser crystal and the energy stored in the cavity. It can be shown that the power emitted from the spin system may be calculated by

\[ P_m = \left(\frac{1}{2\pi}\right) \frac{h f I}{\Delta f_L} \int_{V_{x'tal}} \gamma^2 \left[ \vec{H}^* \cdot \vec{H} \right] \, dV. \]  (15)

where

- \( h \) = Planck's constant in joule-seconds
- \( f \) = the signal frequency in cycles per second
- \( I \) = inversion ratio \( \Delta f_p / 2f_s - 1 \)
- \( \Delta N \) = thermal equilibrium population difference per cm\(^3\)
- \( \gamma = \frac{g \beta \nu}{h} \) in meters per ampere
- \( g \) = the spectroscopic splitting factor = 2 for ruby
\[ \beta = \text{the Bohr magneton} = 9.27 \times 10^{-25} \text{ amp meter}^2 \]
\[ \mu_0 = \text{the permeability of free space} = 4 \pi \times 10^{-7} \text{ henries per meter} \]
\[ \hbar = h/2\pi \text{ in joule-seconds} \]
\[ \Delta f_L = \text{the magnetic frequency line width in sec}^{-1} \]
\[ \hat{H} = \text{peak r-f magnetic intensity in amperes/meter} \]
\[ \mathcal{T}^{ik} = \text{the dimensionless probability tensor}. \]

The term
\[ \left[ \hat{H}^k \cdot \mathcal{T}^{ik} \cdot \hat{H} \right] \tag{16} \]

arises from the derivation of the stimulated transition probability \( W_{ij} \) which has a factor

\[ | \langle i | \hat{H}^k \cdot \hat{S} | j \rangle |^2. \tag{17} \]

This term is a quantum mechanical expression in which

\( \langle i | \) and \( | j \rangle \) are the bra and ket vectors for the quantum eigenstates associated with levels \( i \) and \( j \) and \( \hat{S} \) is the effective spin vector considered as a quantum mechanical operator. The quantity \( \langle i | \hat{H}^k \cdot \hat{S} | j \rangle \) is called the transition-probability matrix element, since it is the matrix element of the operator \( \hat{H}^k \cdot \hat{S} \) between the states \( \langle i | \) and \( | j \rangle \).

It is often convenient to rewrite equation (17) in the form of equation (16) where the quantity \( \mathcal{T}^{ik} \) is a three by three tensor also called a dyadic product.

This tensor has the form
£T* = 
\begin{bmatrix}
\sigma_x \sigma_x^* & \sigma_y \sigma_y^* & \sigma_z \sigma_z^* \\
\sigma_x^* & \sigma_y^* & \sigma_z^* \\
\sigma_x & \sigma_y & \sigma_z
\end{bmatrix}
\quad (18)

Essentially \( \sigma^* \) is a normalized form of the imaginary susceptibility tensor \( \times \).

The total stored energy is given by

\[ W_s = \frac{1}{2} \mu \int_{\text{cavity}} \hat{\mathbf{H}}^* \cdot \hat{\mathbf{H}} \, dV, \quad (19) \]

because the electric and magnetic fields are in time quadrature. Combining equations (14), (15), and (19), we obtain

\[ \frac{1}{Q_m} = \gamma_0 \Delta N \frac{I}{\Delta f_L} \frac{\int_{\text{x'tal}} \left[ \hat{\mathbf{H}}^* \cdot \mathbf{\sigma} \hat{\mathbf{C}}^* \cdot \hat{\mathbf{H}} \right] dV}{\sqrt{\int_{\text{cavity}} \left[ \hat{\mathbf{H}}^* \cdot \hat{\mathbf{H}} \right] dV}} \quad (20) \]

We can simplify the writing of equation (20) somewhat by using the concept of the filling factor \( \eta \) defined by:

\[ \eta = \frac{\int_{\text{x'tal}} \left[ \hat{\mathbf{H}}^* \cdot \mathbf{\sigma} \hat{\mathbf{C}}^* \cdot \hat{\mathbf{H}} \right] dV}{\frac{1}{2} \int_{\text{cavity}} \left[ \hat{\mathbf{H}}^* \cdot \hat{\mathbf{H}} \right] dV} \quad (21) \]

where \( \mathcal{U}^2 \) denotes the effective value of \( \mathbf{\sigma} \hat{\mathbf{C}}^* \). For an order of magnitude the filling factor may be taken as the ratio of the volume of the crystal to the volume of the cavity. This approximation is made on the assumption that the magnetic field is approximately the same in the crystal as it is in the cavity with the crystal absent. The magnetic field must also be oriented properly to
produce maximum value of $\bar{\gamma}_l^2$ everywhere. Filling the cavity with the maser crystal does not guarantee unity filling factor, however.

Using this notation the expression for magnetic $Q$ becomes

$$\frac{1}{Q_m} = \frac{\gamma^2 \hbar}{\gamma \omega} \frac{I \Delta N \bar{\gamma}^2 \gamma}{\Delta f_L} \approx 10^{-18} \frac{hf N I \bar{\gamma}^2 \gamma}{kT I \Delta f_L}$$  \hspace{1cm} (22)$$

where $l$ represents the number of levels in the spin system and $N$ is the total population density of all the levels.

Some figures for determining an order of magnitude of $Q_m$ are as follows for a ruby cavity maser of the type planned.

Material: pink ruby 0.1% chromium concentration

$N = 5 \times 10^{19}$ spins/cm$^3$

$l = 4$

$f = 10$ Gc/sec

$T = 4.2 \, ^\circ$K

$I = \frac{1}{2}$

$\gamma = 0.1$

$\gamma^2 = 100$ Mc/sec

$\Delta f_L = 100$ Mc/sec

Substituting in equation (22) we obtain

$$Q_m \approx 10$$

Siegman's calculation$^{13}$ gives a $Q_m = 100$ for a somewhat similar maser.
2.4 Gain of the One Port Cavity Maser

One way of understanding the operation of a cavity maser is to consider it as a transmission line terminated in a negative resistance. Qualitatively speaking the approach comes from consideration of the incident and reflected waves on a transmission line. If the unmatched line termination is negative real, the reflected wave has a larger amplitude than the incident wave. In this respect a negative resistor and a power source are similar.

A quantity often used in transmission line calculations is the reflection coefficient given by

\[ \rho = \frac{Z_L - Z_0}{Z_L + Z_0} \]  

(23)

where \( Z_L \) represents the load impedance, and \( Z_0 \), the characteristic impedance of the transmission line. As it is basically defined \(|\rho|\) represents the fraction of the incident wave amplitude reflected at the load terminals. In the case of the cavity maser, let \( Z_0 = R_e \) and \( Z_L = -R_m' + j\omega L_0' + 1/j\omega C_0' \). The primes refer to the cavity equivalent circuit referred to the primary side of the ideal transformer representing the coupling iris. The sending end of the line is terminated with a matched impedance. A crude picture of the arrangement is shown in figure 3c.

A commonly accepted way of writing the impedance of a tuned circuit for frequencies in the vicinity of the
resonant frequency is

\[ Z_L = -R_m + 2j \omega_0 L_0 \delta, \]  

(24)

where \( \delta = \frac{\omega - \omega_0}{\omega_0} \), the frequency deviation. Using the above expression in equation (23) the equation for the voltage gain of the maser is

\[ g(\omega) = \left| \frac{R_0 + R_m - 2j \omega_0 L_0 \delta}{R_0 - R_m + 2j \omega_0 L_0 \delta} \right|. \]  

(25)

Defining \( Q_e \), the external \( Q \), to be

\[ Q_e = \frac{\omega_0 L_0}{R_e}, \]  

(26)

this reduces the maser gain expression to the following form:

\[ g(\omega) = \frac{1 - 2j \frac{\omega_0 L_0}{R_e} \delta}{1 + 2j \frac{\omega_0 L_0}{R_e} \delta}. \]  

(27)

Equation (27) may be simplified by noting that

\[ \frac{2\omega_0^2 L_0 \delta}{R_e + R_m} \ll 1; \]  

as follows:

\[ g(\omega) = \frac{1 - 2j \frac{\omega_0 L_0}{R_e} \delta}{1 + 4Q_t^2 \delta^2} \]  

(28)

where

\[ \frac{1}{Q_t} = \frac{1}{Q_e} - \frac{1}{Q_m}. \]  

(29)
The effects of cavity losses are such as to put a resistance, $R_q$, in series with the negative resistance, $R_{m}$, of the maser material in the equivalent circuit for the maser cavity. In this event we can define an equivalent magnetic 

$$ Q = Q_m' $$

by

$$ Q_m' = \frac{\omega_0 L_0}{R_m - R_o} \quad (30) $$

It is easy to see that since the unloaded $Q = Q_o$ of the cavity is given by

$$ Q_o = \frac{\omega_0 L_0}{R_o} \quad (31) $$

the effective magnetic $Q = Q_m'$ is related to $Q_o$ and $Q_m$ by

$$ \frac{1}{Q_m'} = \frac{1}{Q_m} - \frac{1}{Q_o} \quad (32) $$

By merely replacing $Q_m$ by $Q_m'$ in the above equation we can account for the cavity losses. If the losses caused by the coupling iris are considered as series losses in the transformer representation, then the effect of these losses may be handled in the same manner.

The loaded $Q$ of a microwave cavity may be made quite high (on the order of 1000 at least) so that the maser gain will be determined by $Q_m$ largely. This is clearly seen in the light of the order of magnitude calculations made previously.
Figure 4. Maser voltage gain $|g(\omega_0)|$ and 3 db bandwidth as a function of $1/Q_t = 1/Q_e - 1/Q_m$, with $Q_m = 50$, and $f_o = 10$ Gc.
3.1 Design Considerations

A right circular cylindrical cavity was chosen primarily for ease of construction. By a consideration of various field configurations, frequency ranges, possible tuning mechanisms and the like, a cavity resonant in the TE_{111} mode at X band and the TE_{013} mode at K band was picked. Although the Q of the cavity at either frequency was not critical, both presented the possibility of high Q. One problem to be faced was that of cavity tuning. The TE_{013} mode displays circumferential currents on the end plates, and hence current does not cross directly to the side walls of the cavity from the end plate. On the other hand, the TE_{111} mode has currents which do cross from the side walls to the end plates. A tuning mechanism using a moving end plate was desired for mechanical simplicity. This problem was solved by machining a half-wave length shorted transmission line in parallel electrically with the moving end plate and wall connection.

Problems are also caused by the extraneous modes which would exist in the K band tuning range. Since a number of these modes required the end plate currents to cross directly to the side wall, it was felt that these extraneous modes would be eliminated by the moving joint between the side wall and the end plate. The crystal is placed on the tuning
plunger with a low dielectric spacer to place the ruby in the optimum field configuration.

3.2 Field Configurations

The theory of resonant cavities involves the solution to Maxwell's equations subject to the boundary conditions on the cavity walls.\textsuperscript{15, 16, 17} The cavity walls are generally assumed to be perfect conductors requiring the tangential component of the electric field and the normal component of the magnetic field to be zero at the walls. The electric and magnetic fields must satisfy the vector wave equations as follows:

\[
\nabla^2 \bar{E} = \mu \frac{\partial^2 \bar{E}}{\partial t^2}
\]

\[
\nabla^2 \bar{H} = \mu \frac{\partial^2 \bar{H}}{\partial t^2} .
\] (33)

It must be remembered that the Maxwell's equations relate the $\bar{E}$ and $\bar{H}$ fields. In the case of sinusoidally varying fields of the form

\[
\bar{E}(t) = \bar{E}_{\text{max}} \text{Re}(e^{j\omega t}),
\] (34)

usually written

\[
\bar{E}(t) = \bar{E} e^{j\omega t},
\] (35)

with the real part understood, the vector wave equations above reduce to solving the following equations:
\[ \nabla^2 E = -\omega^2 \mu \varepsilon E \]
\[ \nabla^2 H = -\omega^2 \varepsilon \mu H . \]  

(36)

For these equations two types of solutions are encountered. In one case the electric field along the axis of cylindrical symmetry (z-axis) is zero; in the other the magnetic field is zero along the z-axis. The former solutions are termed transverse electric solutions, the latter transverse magnetic. The transverse electric solutions are of the most interest in this particular case and the equations for the fields in terms of cylindrical coordinates are given below.

\[ H_z = H_0 j_n \left( \frac{S_{nm} r}{D} \right) \cos \theta \cos \left( \frac{p_n z}{L} \right) \]

\[ H_r = \frac{p_n D^2 H_0}{S_{nm} L} \frac{\partial j_n \left( \frac{S_{nm} r}{D} \right) \cos \theta \cos \left( \frac{p_n z}{L} \right)}{\partial r} \]

\[ H_\theta = -\frac{p_n D^2 H_0}{S_{nm} L} \frac{n}{r} j_n \left( \frac{S_{nm} r}{D} \right) \sin \theta \cos \left( \frac{p_n z}{L} \right) \]

\[ E_r = \frac{j \omega D^2 H_0}{S_{nm}^2} \frac{n}{r} j_n \left( \frac{S_{nm} r}{D} \right) \sin \theta \sin \left( \frac{p_n z}{L} \right) \]

\[ E_\theta = \frac{j \omega D^2 H_0}{S_{nm}^2} \frac{\partial j_n \left( \frac{S_{nm} r}{D} \right) \cot \theta}{\partial r} \sin \theta \sin \left( \frac{p_n z}{L} \right) \]

\[ E_z = 0 \]

(37)

where \( H_0 \) is the maximum amplitude of the time varying magnetic field intensity expressed in the appropriate units; \( D \) is the diameter of the cylinder; \( L \) is the height of the cylinder; \( S_{nm} \) is the \( m \)th zero of \( \frac{d j_n (r)}{dr} \), with \( r \)
representing the radial dimension. The letters m, n, p are
the mode numbers; these are the eigenstate numbers arising
from the application of the boundary conditions. The in-
tegral mode numbers determine the nomenclature for a par-
ticular solution or mode. These are $\text{TE}_{nmp}$ for transverse
electric modes and $\text{TM}_{nmp}$ for transverse magnetic modes.

3.3 The Mode Lattice

As a consequence of the separation of variable method
solution of the wave equation, the mode numbers, dimensions
of the cavity, and frequency are related in the following
way:

$$\omega^2 \varepsilon = \left( \frac{S_{nm}}{D} \right)^2 + \left( \frac{p \eta}{L} \right)^2. \quad (38)$$

From this equation the resonant frequency of any mode, $f_{nmp}$,
is readily determined.

$$f_{nmp} = \frac{1}{2 \pi \sqrt{\varepsilon}} \left[ \left( \frac{S_{nm}}{D} \right)^2 + \left( \frac{p \eta}{L} \right)^2 \right]^{\frac{1}{2}}. \quad (39)$$

To non-dimensionalize the equation a few manipulations
result with the following expression:

$$\left( \frac{D}{\lambda} \right)^2 = \frac{1}{4 \pi^2} \left[ S_{nm}^2 + (p \eta)^2 \left( \frac{D}{L} \right)^2 \right]. \quad (40)$$

Now the equation is independent of the medium filling the
cavity, since
\[ \lambda = \frac{c}{f_{\text{mp}}} , \]

where
\[ c = \sqrt{\frac{\mu}{\varepsilon}} \]

is the velocity of electromagnetic propagation in the medium. As outlined in an article by Bracewell, this equation may be represented in nomograph form. The conditions for the various modes are indicated by a lattice of points in between two parallel scales of \( D/\lambda \) and \( D/L \). A nomograph for right circular cylindrical cavities is shown in figure 5.

The use of this nomograph is straightforward as shown in the following examples. Suppose the dimensions of a cavity and its resonant frequency are known along with the medium filling the cavity. The proper points are picked on the two scales; the line connecting the two points passes through the point designating the mode of operation. By choosing a mode of operation and frequency, the appropriate dimensions may be found in the reverse fashion. This type of chart is useful in the design of a dual-mode, double-frequency cavity. One can easily see the interfering modes affecting the pump frequency mode for a given dimensional choice satisfying the signal frequency mode qualifications. For a tunable cavity the mode points inside the trapezoid formed by connecting the extreme points on the frequency scale to the extreme points on the length scale represent extraneous mode possibilities. All of these interior points except the desired mode denote modes to be suppressed.
Figure 5. The mode lattice showing the first 260 modes for a right circular cylindrical cavity.
3.4 Dimensional Choice

The limiting factors on the choice of cavity diameter are the inner diameter of the dewar system and the size of the connecting waveguide. The maser is to operate with an X band signal and a K band pump. For the pump source a 2K33B klystron is considered; this tube has a tunable range of 22 Gc to 26 Gc. A suitable circulator for the signal line has a pass band of 8.2 Gc to 9.6 Gc; this fixes the signal frequency range. The ratio of these two ranges is about 2.6 to 1. By trial and error the requirements are worked out using the mode lattice. The results for this cavity are a diameter of 1.35 inches operating in the TE\textsubscript{111} mode for the X band signal and the TE\textsubscript{013} mode for the K band pump. The 8.2 Gc to 9.6 Gc signal tuning range leads to a change in length from 0.921 inches to 0.716 inches and results in a 22 Gc to 27.1 Gc range for the pump frequencies.

As can be seen from the mode lattice of figure 5, a large number of extraneous modes are present in the pump frequency range. A large number of these modes, especially the TE\textsubscript{113} mode, should be suppressed because of the poor connection between the moving end-plate and side-wall at the K band frequencies. Other degenerate mode possibilities could be damped by preferentially exciting the TE\textsubscript{013} mode over the undesired mode. Since sidewall coupling was not considered because of space limitations in the dewar, proper placement of the K band coupling iris in the cavity top was
given priority over placing X band iris. The tangential component of the electric field must be zero at the walls of the cavity, hence magnetic coupling must be used for iris coupling through an end plate. The main idea is to position the iris in the location that the magnetic field in the cavity is maximal. The waveguide is oriented so that the field lines of the waveguide are parallel to the field lines of the cavity mode.

In the case of the TE_{0lp} modes the magnetic field at the end plates is all radial and satisfies the equation

$$H_r \left|_{z = 0} \right. = H_{max} \frac{d}{dr} J_0 \left( S_{0l} \frac{r}{D} \right).$$

The field has a maximum at

$$S_{0l} \frac{r}{D} = 1.841,$$

or for $$S_{0l} = 3.832, D = 1.350$$ inches,

$$r = 0.324$$ inches.

The distance $$r$$ above is the location of the center of the K band coupling iris. For the TE_{111} mode the magnetic field at the end plate is composed of a radial and angular term. A little investigation similar to the above shows the maximum magnetic intensity to be at the center of the plate or $$r = 0$$.

Placing the two waveguides with the long sides
perpendicular gives the best location for the coupling. The center of the K band guide is located for optimum coupling, thus by necessity the center of the X band guide cannot be at the center of the cavity. This location of the guides also sets up the required orthogonality of the fields and facilitates attachment of the external microwave plumbing. The resulting arrangement is shown in figures 6 and 7. The iris sizes were determined by scaling published results from other cavities and then adjusted by experiment.

3.5 Cavity $Q_0$

A formula for determining the $Q_0$ of a cylindrical cavity is presented by Wilson, Schramm, and Kinzer in an article concerned with resonant cavity design for radar testing. Since the value of $Q_0$ is not too critical it seems justifiable to use these results to predict values for $Q_0$ of the cavity designed. Prior to the use of these formulas a calculation was made for Wetsel's design and compared with his results for theoretical $Q_0$. 19 The following is the formula for $Q_0$ of a right circular cylindrical cavity:

$$Q_0 = \frac{\lambda S_{nm}}{2 \pi \delta} \left[ 1 + k^2 R^2 \right]^{3/2} \frac{1 - \left( \frac{n}{S_{nm}} \right)^2}{1 + k^2 R^3 + k^2 (1-R) R^2 \left( \frac{S_{nm}}{S_{nm}} \right)^2} \quad (42)$$

where

- $R = D/L$
- $k = \pi \gamma / 2 S_{nm}$

$S_{nm}$, $m$, $n$, $p$, $D$, and $L$ are all as previously defined; $\lambda$ is
the wave length of the electromagnetic wave, \( \delta \) is the skin depth defined in the normal fashion. For silver

\[
\delta = 0.0642 \left( \frac{f}{s} \right)^{\frac{1}{2}} \text{meters}.^2
\]

Using the cavity dimensions as determined, \( Q_0 \) was calculated for a center frequency in each band. The cavity was considered to be silver plated so that the skin depth for silver was used. The following are the results of the calculations. For the TE\(_{\text{l1l}}\) mode at 8.95 Gc

\[
Q_0 = 20,600.
\]

For the TE\(_{\text{013}}\) mode at 23.97 Gc

\[
Q_0 = 48,600.
\]

3.6 Tuning Plunger

As mentioned previously the cavity was to be tuned by a moving end wall. This necessitates a sliding joint. Since a close fit is not desirable at the extremely low temperature that would be encountered, a means for allowing current passage from the side-walls to the end-plate was needed. To provide this means a tuning plunger was designed using the concepts behind choke-flange coupling for waveguides. In other words, a half wave length shorted transmission line was to be placed in parallel electrically with the joint between the side-wall and end-plate of the cavity. This procedure provides a low impedance path for current travel.
Reference to the literature showed the important consideration for the design of choke flanges to be a half wave length transmission line in two quarter wave length sections. This places an open circuit in parallel with the mechanical break between the two surfaces in the "waveguide" transmission line. A design would be similar to that shown in figure 8. This places a short circuit between the cavity wall and the surface of the tuning plunger at the frequency for which the choke is a half wave length long.

For the TE_{11} mode in cylindrical waveguide the cut-off wave length is

$$\lambda_c = \frac{2 \pi D}{1.841}.$$  

Hence, since the guide wave length

$$\lambda_g = \frac{\lambda}{\left[1 - \left(\frac{\lambda_c}{\lambda}\right)^2\right]^{1/2}},$$  \hspace{1cm} (43)

where \(\lambda\) is the free space wave length,

$$\lambda_0 = 1.65 \text{ inches}.$$  

Then

$$\frac{\lambda_g}{4} = 0.414 \text{ inches}.$$  

The width of the slot was determined by what seemed standard practice on waveguide choke flanges as no guideposts for these choices were found in the literature.
NOTE:
1. All dimensions are in inches.
2. Finish inside of barrel & end plate.
3. Shirt Teflon for a finish possible.
4. Coat blade rod hole with Teflon film not more than 0.001" thick.
5. Cavity material: Yellow Brass.

FIGURE 1
Dual Frequency Cavity for Rice Maser
July 17, 1961  F.E. Emery
Figure 8. Cross section of cavity wall and tuning plunger showing method of choke groove shaping.
A means for positioning the tuning mechanism in the cavity while the cavity is in the dewar system is desired. Since an accurate knowledge of the length of the cavity is required and a controlled drive mechanism is desirable, a micrometer head drive was chosen. It is hoped that the two other rods shown in figure 7 will help to keep the tuning plunger straight in the cylinder. To date no attempt has been made to check the success of this tuning mechanism at low temperatures.

The result of placing the cavity in a bath of liquid helium should have little effect on the resonant frequency of the cavity, other than the change in the dimensions of the cavity, since the relative dielectric constant of liquid helium is 1.048 at 4.19°K. The tuning plunger movement can counteract the dimension change caused by cooling.

The relative dielectric constant of ruby is quite high, \( \varepsilon_r \approx 12 \), so that a large ruby will affect the resonant frequency a great deal. However, if the ratio of the volume of the ruby to the volume of the cavity is kept small (a requirement so that the fields may be fairly accurately predicted) then a small effect on the cavity frequency is expected. In this case the effect of the ruby can be treated as a perturbation and it is felt that the tuning range of the cavity will be able to handle the change in resonant frequency.
Figure 10

Photograph of cavity and tuning mechanism.
CAVITY TESTING

4.1 Theory

Two methods were used for testing the characteristics of the cavity— one as outlined by Ginzton and another outlined by Lebowitz. An excellent summarization of Ginzton's method is contained in Wetsel's thesis and it is felt that little would be accomplished by a second summary. The Lebowitz method is a fairly accurate means of quickly determining the coupling coefficient and Q of a cavity. It is a good method to use while adjusting the size of the coupling irises, or the coupling coefficient.

An approximate equivalent circuit of a cavity terminating the end of a transmission line is shown in figure 11. This circuit is postulated for the cavity in the vicinity of the resonant mode of interest and is made with the following assumptions in mind. First, the line loss is negligible; second, the resistive loss in the coupling mechanism is very small; and third, the line and the generator are matched. At points x, y the impedance looking towards the cavity is

\[ Z = \frac{n^2}{1/R_c + j(\omega C - 1/\omega L)} \]

if xy is an integral number of half wave lengths from the cavity. If the line is assumed lossless then the impedance presented an odd number of quarter wave lengths from
Figure 11. Equivalent circuit for cavity terminating waveguide, assuming matched source impedance. $L$ is shunt inductance, $C$ is shunt capacitance, $R$ is shunt resistance, $n$ is the ideal transformer turns ratio between guide and the cavity.
the cavity is

\[
\frac{Z}{A} = \frac{Z_0^2}{Z} = \frac{Z_0^2/n^2}{1/R_0 + j(\omega C - 1/\omega L)}.
\]  

(45)

At resonance, since \( \omega C = 1/\omega L \),

\[
\frac{Z}{A} = \frac{Z_0^2}{R_0 n^2}
\]

(46)

and the voltage at an odd number of quarter wave lengths away from the load is given by

\[
E_{\frac{A}{4}} = \frac{(Z_0^2/R_0 n^2)(E)}{Z_0 + Z_0 Z_0^2/R_0 n^2}
\]

(47)

For frequencies far off resonance, the magnitude of the load impedance is approximately zero. Far off the resonant frequency the voltage at odd quarter wave lengths from the load is just the source voltage, \( E \). Consider the ratio

\[
R = E/E_{\frac{A}{4}} = \frac{R_0 n^2}{Z_0} + 1.
\]

(48)

For the equivalent circuit presented the coupling coefficient, \( \beta \), between the line and the cavity is defined as follows:

\[
\beta = \frac{n^2 R_0}{Z_0}.
\]

(49)

Hence

\[
R = \beta + 1 \quad \text{or} \quad \beta = R - 1.
\]
The loaded Q of the cavity may be measured at integral half wave lengths from the load, since the impedance seen there is the same as the load impedance. The assumption of lossless lines is necessary here. The loaded Q may be determined for a tuned circuit from the relation

$$Q_L = \frac{f_0}{f_2 - f_1},$$

where \(f_2 > f_0 > f_1\); \(f_2\) and \(f_1\) are the half-power frequencies. If the cavity is some distance from the probe, then error is introduced because of the changing frequencies and hence changing wave lengths. Under these conditions a correction term must be added to the cavity impedance when the frequency is off resonance. However, Lebowitz shows that for a fairly high Q and reasonably high coupling the correction is negligible if the point of measurement and the cavity are fairly close together. The Q measured is the loaded Q and is related to the Q of the cavity by

$$Q_0 = Q_L (1 + \beta).$$

The equipment necessary to measure Q by this method is similar to that required by the standing wave ratio measurements. A typical laboratory setup is shown in figure 12. (Photograph of apparatus figure 13.) The mode of the klystron is displayed on the screen of the oscilloscope in the usual fashion and the cavity is detuned completely out of the operating mode of the klystron. The probe in the
For Lebowitz method

For SWR method

Figure 12

Schematic showing equipment for measuring $Q_o$. 
Figure 13

Photograph of laboratory equipment used in measuring cavity Q and coupling coefficient.
Photograph of klystron mode with cavity detuned from the mode.

Crystal output with probe at odd number of quarter wave lengths from the cavity.

Figure 14.

Photograph of klystron mode with cavity tuned to resonate at the frequency which the klystron had a maximum output.
Klystron mode with cavity detuned from the mode.

Crystal output with probe at an integral number of half wave lengths from the cavity

Figure 15

Klystron mode with cavity tuned to resonate at frequency which klystron has a maximum output.
slotted section is then placed at a maximum in the standing wave pattern on the line. This places the probe at an odd number of quarter wave lengths from the load for the frequency of maximum mode power. The position point of maximum height is noted along with the distance to the baseline, A, as shown in the diagram. Taking the detector characteristic into account, the distance A represents the voltage on the line away from the resonant frequency, or $E$ in the terminology expressed above. The cavity is then tuned to resonate at the frequency in the mode which gave the maximum height. The distance, B, from the vertex of the dip to the baseline represents the voltage on the line at resonance, or $E_{1/4}$ in the nomenclature above. Again the detector characteristic must be accounted for. The ratio of A to B is just $R$. Hence

$$\beta = \frac{A}{B} - 1.$$  

The probe is then moved a quarter of a wavelength to the position where the signal from the detector looks somewhat like a resonance curve. The scope pattern should be somewhat symmetrical at the points on the line half wave lengths from the cavity. The measurement of the bandwidth corresponding to the half-power bandwidth is accomplished by the use of a wavemeter. The wavemeter presents a dip in the display on the oscilloscope at its tuned frequency. However, the wavemeter dip is sometimes broad and difficult
to position, leading to only a close approximation of the frequency.

4.2 Results

The $Q_0$ of the cavity was measured using both techniques mentioned at various stages of final finishing of the cavity. The primary reason for doing so was to satisfy the curiosity of the author as to what effect the various finishing procedures would have on the value of cavity $Q$. The most complete set of measurements was made on the X band mode because of equipment difficulties at the K band frequencies. The $Q_0$ was measured first on the cavity immediately following assembly, prior to the polishing phase. The interior surface had several scratches of about 0.00025 inch depth. The interior surface was then polished in the normal fashion, ending with a final polish with 0.3 micron particles of aluminum oxide suspended in distilled water. The curves surfaces were done by hand and the flat surfaces done on polishing machines. The $Q_0$ was measured immediately after polishing.

The cavity was then silver plated with a thickness of approximately 0.002 inch of silver by a commercial plating shop using a cyanide process. The silver plate was then finish polished by hand with 0.3 micron aluminum oxide. $Q$ was measured again. The results of these measurements are listed in table I. The coupling factor was being
### TABLE OF \( Q \) MEASUREMENTS AND COUPLING

**Resonances in the \( \text{TE}_{111} \) Mode**

<table>
<thead>
<tr>
<th>State of Cavity Finish</th>
<th>( f_0 (\text{Gc}) )</th>
<th>( \phi )</th>
<th>( Q_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fine lathe finish</td>
<td>9.393</td>
<td>1.04</td>
<td>2160</td>
</tr>
<tr>
<td>Polished Brass</td>
<td>8.500</td>
<td>0.74</td>
<td>2430</td>
</tr>
<tr>
<td>Silver Plated</td>
<td>8.875</td>
<td>1.14</td>
<td>3350</td>
</tr>
</tbody>
</table>

**Resonances in the \( \text{TE}_{013} \) Mode**

| Polished Brass         | 23.84                | 0.55   | 1010   |

**TABLE I**
Determination of Cavity Q
VSWR versus FREQUENCY

Cavity after Silver Plating and Polishing

\[ f_0 = 8.875 \text{ Gc} \]
\[ f = 2.65 \text{ Mc} \]
\[ Q = 3350 \]
experimentally changed over this period of time and is entered in the table also.

It will be noted that there is very little difference between the Q obtained by finely machining the brass and that obtained after polishing the cavity. Note also that all values fall short of the predicted Q.

The reason that the measured values of Q are low compared with the theoretical values is felt to lie with the design of the tuning mechanism. Wetsel reports a marked reduction of Q with a change in end-plate design to reduce the degenerate TM_{111} mode in his TE_{011} cavity. The results of Wetsel's design show his measured Q to be approximately one-third of the predicted Q.

The accuracy of these Q measurements is low. The frequency calibration leaves much to be desired. In all cases frequency was determined by use of a cavity wavemeter which has 0.01 per cent accuracy. However, it is doubtful whether the Q's are accurate to more than one part in a thousand and possibly less. This accuracy is adequate for a maser cavity because of the broad requirement that the Q_0 be high enough to neglect cavity losses when concerned with maser action.

The characteristics of the cavity at the K band frequencies were also measured. It was found that many modes were not damped by the moving end wall connection; see table II. The degenerate TM_{113} mode and the TE_{022}, TM_{122} modes are especially troublesome. With the coupling iris located
Figure 17

DETERMINATION OF CAVITY Q

VSWR versus FREQUENCY

Cavity after silver plating and polishing

\[ f_0 = 23.960 \text{ Gc} \]

\[ \Delta f = 34 \text{ Mc} \]

\[ Q = 705 \]
List of Discernable Modes Resonant to 23.76 Gc Within the Range of the Cavity Tuning Mechanism (Polished Brass Finish)

<table>
<thead>
<tr>
<th>Micrometer Setting</th>
<th>Mode</th>
<th>$\beta$</th>
<th>Approximate $Q_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>449</td>
<td>TE$_{122}$</td>
<td>0.07</td>
<td>-----</td>
</tr>
<tr>
<td>415</td>
<td>TM$_{022}$</td>
<td>0.09</td>
<td>-----</td>
</tr>
<tr>
<td>404</td>
<td>TE$_{113}$</td>
<td>0.03</td>
<td>-----</td>
</tr>
<tr>
<td>398</td>
<td>TM$_{013}$</td>
<td>0.41</td>
<td>1000</td>
</tr>
<tr>
<td>366</td>
<td>TE$_{222}$</td>
<td>0.14</td>
<td>-----</td>
</tr>
<tr>
<td>336</td>
<td>TE$_{013}$</td>
<td>1.12</td>
<td>1480</td>
</tr>
<tr>
<td></td>
<td>TM$_{113}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>302</td>
<td>TM$_{122}$</td>
<td>0.09</td>
<td>-----</td>
</tr>
<tr>
<td></td>
<td>TE$_{022}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>287</td>
<td>TE$_{022}$</td>
<td>0.07</td>
<td>-----</td>
</tr>
<tr>
<td></td>
<td>TM$_{122}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>240</td>
<td>TE$_{313}$</td>
<td>0.17</td>
<td>-----</td>
</tr>
<tr>
<td>214</td>
<td>TM$_{213}$</td>
<td>0.07</td>
<td>-----</td>
</tr>
<tr>
<td>184</td>
<td>TE$_{413}$</td>
<td>0.03</td>
<td>-----</td>
</tr>
<tr>
<td>154</td>
<td>TE$_{114}$</td>
<td>0.07</td>
<td>-----</td>
</tr>
<tr>
<td>107</td>
<td>TM$_{014}$</td>
<td>0.10</td>
<td>-----</td>
</tr>
<tr>
<td>58</td>
<td>TM$_{114}$</td>
<td>0.91</td>
<td>1180</td>
</tr>
<tr>
<td></td>
<td>TE$_{014}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE II**
at the top of the cavity no preferential coupling can exist for the $TE_{013}$, $TM_{113}$ degeneracy. The resonant frequency for the two modes does not seem to exist at the same frequency. The curve plotted from standing-wave data is quite unsymmetrical. Upon the introduction of end plate irregularity---in the form of an aluminum foil sheet inserted between the top and the cylinder---a marked separation of the two modes was observed. Both modes appear in the same klystron mode however.

The introduction of the ruby crystal into the cavity should effect a mode separation between these two modes so that good pumping can be obtained. If not, the pump mode may be rechosen and the coupling scheme redesigned easily. The modes which are available in the tuning range of the cavity for a single frequency are listed in table II.

The Q of the cavity for the K band mode is much lower than the calculated value. It is felt that there are two primary reasons for this---first, the interaction between the $TE_{013}$ and $TM_{113}$ modes; secondly, the losses caused by the large gap between the end wall and side wall because of the tuning mechanism. This gap is probably an almost open circuit for the K band radiation. Cavity masers have been designed with little regard to resonance at the pump frequencies. It is felt that these difficulties may be overcome in actual maser operation.
ACKNOWLEDGMENTS

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