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The Application of Shallow Shells in Structural Floor Systems

by

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ABSTRACT

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The purpose of this thesis is to demonstrate the application of shallow concrete shells for structural floor systems. The shell is first placed in its historical context. The writer then proposes the application of shells for floors through rationale based on three points:

(1) historical precedent;
(2) structural capability;
(3) aesthetic qualities.

The application having been proposed, the writer then proceeds to an analysis of the problem - from the simplest stages to the more complex applications. The specific criteria which are the basis of the application of shallow shells for floors follow the analysis. The last part is a demonstration of several prototype systems of shells for floors.
CHAPTER 1

Historical Review and Preliminary Analysis
Thin shells have, in recent years, come into wide use as a means of architectural and structural expression. Their first application in construction, however, dates back to the turn of the twentieth century, and many of the principles of shell structures were recognized and developed by the Romans as early as 100 B.C. through arch and vault construction.

The arch, or in its more complex form, the vault, is one of two structural forms that has been primarily used by man in enclosing space - the other being the beam. While the beam is a prehistoric form, the archetype being Stonehenge, more than 3,000 years of recorded history passed before the arch form was evolved.

It was the Romans, then, who first exploited the arch to its fullest significance. Their development of arch and vault construction in cut stone, brick, and concrete was perhaps Rome's greatest contribution to the evolution of the building art.

One thing which made this possible was the discovery of the pozzuolana earths (after the town Pozzuoli), which form a hydraulic cement when mixed with slaked lime. This gave the Romans a material almost as strong as modern concrete.

The geometric forms of shell concrete frequently bear an architectural resemblance in silhouette to these earlier structures, particularly in the spaces they create in internal volume. An examination of the use of reinforced concrete as a plastic material for the enclosure of space, however, will show the difference of these outwardly similar forms. The architecture of mass is essen-
ially the architecture of equilibrium - an equilibrium of compression, with such varying expressions as the massive Roman arch and the soaring Gothic vault. The introduction of materials in which their sense of their tensile quality is allowed to be fully expressed has created a new aesthetic. Reinforced concrete has given this new aesthetic a new dimension.4

A comparison of some structures of similar span, of old and new origin, is of value in contrasting the structural capabilities of reinforced concrete and the concrete and stone of previous generations.

The pantheon of Rome, built in various stages and completed in A.D. 124, has an internal span of 142 feet. The dome is of travertine, tufa, brick and pumice and is 4 feet thick at the thinnest point.5 By contrast, the auditorium at the Massachusetts Institute of Technology by Eero Saarinen spans 155 feet, with a maximum thickness of 20 inches near the edge to a minimum of 3.5 inches at the crest.6 (See illustrations, p.3 )

Another and perhaps more striking example is a comparison of the sixteen century dome of St. Peter's Cathedral in Rome with a modern dome in Jena, Germany. Both domes have a span of 132 feet. The modern dome, however, is considerably flatter; its rise-of-arch-to-span proportion is shallower than that of the dome of St. Peter's. The latter, which is partially constructed as a double shell, has an average thickness of about 10 feet and a total dead weight of about 11,000 short tons compared to the reinforced concrete dome
with a shell thickness of only 2.36 inches and a total dead weight of 364 short tons.\(^7\) (See illustrations, p. 5).

Following the time of Rome and the medieval and Renaissance periods there was little development of the vault and dome as a structural system and only in the 20th century has the curved form returned as an architectural expression.

Today, working chiefly in reinforced concrete, architects are moving toward more open and flexible designs. Having purposely abandoned the simple beam, they are forced back to methods devised by earlier builders for spanning large open areas. The striking shell structures that are now appearing all over the world bear a fundamental relation to the cathedrals of the Middle Ages. Vaulted roofs, after an almost total eclipse, are back in the mainstream of architecture.\(^8\)

So it seems that the history of architecture, as also expressed in the opinion of Mr. Serge Chermayeff of Harvard University, begins to take on a "cyclic" form - the primitive post and beam to the daring domes of the Renaissance and the imposing stone vaults of the Gothic; back to the rectilinear forms, and finally to the present portion of history which concludes the second cycle, a period in which curvature once again emerges as an architectural form.\(^9\)

Historically, the mathematics involved in designing concrete shells had origin in 1828 when the work of G. Lame and E. Clapeyron was published containing the 'membrane analogy' by
ST. PETER'S CATHEDRAL, ROME

SCHOTT WORKSHOP, JENA, GERMANY
which a shell was considered capable of resisting external loads by direct stress unaccompanied by any bending. Not until many years later, in 1892 when the use of reinforced concrete was rapidly developing, were further contributions made to shell theory. The first expressions with concrete shells were made in Germany and France in 1910.\(^\text{10}\)

In 1924 the form of Carl Zeiss, optician, made some studies on a small barrel vault. It is interesting to note that a differential equation that applied to one of their lenses also applied to cylindrical shells. Following their studies the famous Zeiss-Dywidag patents were issued\(^\text{11}\). After experiments, the first sizeable shell dome was constructed at Jena, Germany in 1925, spanning 82 feet with the concrete no more than 2 3/8 inches thick.\(^\text{12}\)

In the early 1930's the firm of Roberts and Schaefer Co. acquired United States rights under the Zeiss-Dywidag patents. The first structure built here was a doubly curved shell 9/16 inches thick with a span of 21 feet. It was built as a test exhibit at Chicago's Century of Progress Exposition in 1932.\(^\text{13}\)

Other U.S. examples of note include the Hayden Planetarium at the American Museum of Natural History in New York City. Built in 1933 this structure was the first concrete shell dome to be built in the United States.\(^\text{14}\) More recently there is the Air terminal building at St. Louis Airport, (1954) and the Civic Auditorium at M.I.T. (1955), which was mentioned earlier.
But the greatest advances in shell technology have not been made in the United States. This has been brought about for many reasons: an unfavorable economic situation for one and perhaps just as importantly, the gap between engineers and architects working together to mutually solve the complex problems involved in shell building. The three most notable shell designers have been Eduardo Torroja, Spain; Felix Candela, Mexico; and Pier Luigi Nervi, Italy.

It is at this point then that we should consider the shell more specifically in its relation to other structures. In his publication "Schalen und Rippenjüppeln" which appeared in 1928, Dischinger gave the classic definition of a shell: "By shells we mean structures formed by singly or doubly curved surfaces, the thickness of which is slight in comparison with the superficial area."

From this two fundamental features of shell geometry emerge: a shell is not flat, but curved, and it is thin. In statics, therefore, a shell is referred to as a surface structure, as opposed to a beam, which is called a linear member (an element in skeleton construction). One way of classifying structure then, is by dividing them generally into two categories - surface and linear. (See illustrations, p. 8).

It is important here to distinguish the 'shell' as a curved surface structure from the surface structure vaults of the past, which were mentioned earlier. The thin sections achieved today are
LINEAR STRUCTURE

Two Dimensions Small In Relation To Third.

EXAMPLE: Skeleton Steel High Rise

SURFACE STRUCTURE

Two Dimensions Large In Relation To Third.

EXAMPLE: Shell
not only the result of new systems of calculation, which in contrast to the empirical methods of the past offer a relatively exact reflection of the pattern of stresses, but of the use of appropriate materials. As previously pointed out, only when the construction consists of a material which withstands compression and tension can it work like a shell. Medieval vaults are made of stone and can, therefore only resist compression.\textsuperscript{18} The linkage of several vaulted shells allows the same classic solutions that were built in stone in antiquity, yet with more freedom and possibilities than stone affords.\textsuperscript{19}

A surface structure must exemplify four characteristics in order to work like a shell:

(a) a singly or doubly curved form;
(b) a thickness which is small in relation to its superficial area;
(c) a material resistant to compression and tension;
(d) proper support; it will thus develop, at most, a small amount of bending in a limited portion of the shell.\textsuperscript{20}

The material most commonly used in shell construction is reinforced concrete but shells can also be built in timber, steel, aluminum, etc.

There are many other ways of classifying structures and a look at some of these might help to clarify the nature of various structural systems and particularly the shell in relation to other systems.
One of these is a division of structures into categories of very flexible and stiff. (See illustrations, p.11). A very flexible structure is incapable of resisting compressive forces acting upon it (e.g. cable, membrane). A stiff structure is required to withstand a certain amount of compressive forces in addition to tensile and shearing stresses (e.g. column, beam, shell, arch).

Another more general classification than the previous two would be according to how many different sources of structural strength are present. (See illustrations, p.12). These include:

1. tensile and compressive strength in x - direction
2. tensile and compressive strength in y - direction
3. in-surface shearing resistance in x - direction
4. in-surface shearing resistance in y - direction
5. transverse shearing strength in x - direction
6. transverse shearing strength in y - direction
7. bending resistance in x - direction
8. bending resistance in y - direction
9. torsion resistance around x - axis
10. torsion resistance around y - axis

From these general classifications of structures we can proceed to a breakdown of categories within the family of shells. There are many ways to classify shells but to the architect a classification based on outward form would be most pertinent. From this standpoint, every possible shell form falls into two groups: doubly curved shells and singly curved shells. Doubly curved shells
VERY FLEXIBLE SYSTEM

VERY STIFF SYSTEM
Tensile and Compressive Strength and In-Surface Shearing Resistance

Transverse Shearing Strength

Bending and Torsion Resistance
may then be further broken down into three groups:

(1) doubly curved with principal curvatures in same direction;
(2) doubly curved with principal curvatures in opposite directions;
(3) doubly curved with principal curvatures in same and opposite directions. 21 (See illustrations, p. 14).

By virtue of the curvature of shells, the internal forces are transmitted chiefly by the direct stresses of tension, compression and shear (all in the plane of the surface), rather than by bending as in the case of flat plates. Such stresses, characteristic of shells, are called membrane stresses. Such membrane action makes shells extremely efficient because the transmission of force by direct stress is more efficient than by bending. 22 Curvature, as well as continuity, is necessary to create a uniform three-dimensional stress distribution.

Thinness is a requirement, as unnecessary thickness increases the dead weight load and may introduce undesirable bending action. Thin shells often require stiffening ribs to prevent lateral buckling caused by compressive stresses within the plane of the shell.

Shell structures have achieved extraordinary practical importance in the last decades because the characteristic interplay of forces in spacial surface structures result in a considerable savings in building costs. However, the laws governing this interplay of forces cannot be explained by the elementary equations of statics. For this reason the practical application
FOUR TYPES OF SHELLS

SINGLY CURVED

DOUBLY CURVED, PRINCIPAL CURVES IN SAME DIRECTION

DOUBLY CURVED, PRINCIPAL CURVES IN OPPOSITE DIRECTIONS

DOUBLY CURVED, PRINCIPAL CURVES IN SAME AND OPPOSITE DIRECTIONS
of shells became possible only after their special theory had been developed. However, the various aspects of shell design and analysis will not be covered in this thesis. Two excellent resource books for those further interested in this aspect of shells are *Elementary Statics of Shells* by Alf Pflüger and *Thin Concrete Shells* by A. M. Haas.

The structural solutions with shells have, to date, been limited primarily to roofs. So much effort has been put into refining the shell as a roof structure that the possibilities of shells applied to flooring systems have gone virtually unexplored.

There are several reasons to account for this:

1. Historically, because of its obvious efficiency for long spans, man has employed the dome and vault as a solution to the problem of spanning large areas.

2. Shells are still in an early stage of development and are not fully understood. Naturally then, architects and engineers are hesitant to explore areas of potential other than roof construction.

3. Shells, to be efficient in long spans, need a significantly large rise in relation to the span. It is not immediately apparent that when the span is short, as in floors, the ratio of rise to span can be markedly reduced and still maintain low stresses.

4. A shell of only 2 or 3 inches thick will span a hundred feet. A span to thickness ratio of this magnitude cannot be maintained in an application to floors,
for a minimum thickness of at least 1 1/2 inches is required in some cases just to cover the reinforcing. (However, this does not necessarily mean such structures are uneconomical.)

(5) There are obvious problems apparent in the application of shells for floors and people fail to look beyond these to inherent advantages.

Consideration of shallow shells for floors may be correlated with an existing floor system - the flat slab. Just as there is a membrane action that develops in the shell, there is also a membrane action that develops in flat slabs. (See illustrations, p. 17). This action is described extensively in the book *Plastic and Elastic Design of Slabs and Plates* by R. H. Wood.

With this understanding, it becomes apparent that shallow shells are at least feasible as a structural system for floors, assuming they will meet requirements of vertical and wind loads. At a later point in this paper it will be shown that shells are not only feasible as a flooring system but have certain structural advantages over other systems.

The importance of the shallowness of the shell should be reemphasized here. The shell must be shallow in order to maintain a minimum overall thickness of floor construction (including fill and wearing surface). This in turn affects the floor to floor height. If, because the shells are not shallow enough, the floor to floor height must be increased unwarrantedly, then
COMPRESSIVE MEMBRANE STRESSES

TENSILE MEMBRANE STRESSES
the economic feasibility of using them will be reduced.

Certain types of shells are more capable of withstanding floor loads than others. Of these, doubly curved shells are best since they have more sources of structural strength than singly curved shells such as barrel vaults.

Hyperbolic paraboloids are one type of doubly curved shell which warrant study in their possible application to floors. However, this writer has decided to approach the problem from a different direction for several reasons:

(1) Hypars have a tendency to develop bending moments and often develop large bending moments if boundary stiffeners are not rigidly supported;

(2) Colin Faber, in his book, *Candela: The Shell Builder*, suggests that umbrella type hypars must have a large rise in order to act effectively. Further research indicates this statement cannot be fully confirmed, but this illustration does at least cast doubt on the economic feasibility of umbrella hypars for floors because of the problem with rise-of-arch-to-span ratio. This would be a good area for investigation at some future time.

Here we should point out some differences between synclastic (all principal curves up or down) shells and saddle shells (principal curves alternate - both positive and negative). An example of a saddle shell is the hyperbolic paraboloid, which was just dis-
cussed. Synclastic shells are supported by shear stresses on boundary arches. Saddle shells, on the other hand, are supported by shear stresses in end stiffeners. Uneven stresses will develop in hyperbolic paraboloids if the boundary stiffeners are not rigidly supported. In this case some of the stiffeners' weight hangs from the shell, and the two halves of the paraboloid behave very much like cantilevered beams.  

Among the types of shells which do show definite promise are paraboloids of revolution and elliptical domes. Both are synclastic shells with thickened edges and the bending disturbances at the boundaries penetrate only a short distance into such shells. The rigidity is exceptionally high, even with very shallow shells. Preliminary analysis by this writer and Dr. Nat Krahl of Rice University indicates that paraboloids up to a 40 feet span with as little as 5% rise will easily stand conventional office floor loads. Also indications are that with a span of less than 10 feet the rise can be reduced to 4% or less.

The application of concrete shells for floors has an ancient as well as a modern forerunner in the State of Jalisco and its principal City, Guadalajara, in Mexico.

This construction system, known in Guadalajara as "techo de bóveda," is essentially a structural system whereby a roof or floor is constructed of brick multiple barrel vaults, almost flat (3.6% rise), which are supported on a framework of steel, or sometimes concrete. The system is apparently unique to this portion of Central Mexico.  

(See illustrations, p. 20).
Ancient techo de bóveda construction system
This system will be described and illustrated in more detail in Chapter 2 of this thesis. Let us suffice to say here that if floors can be built of brick vaults as strong as these are known to be then the same system could be built much stronger and with a thinner section in reinforced concrete.

Felix Candela of Mexico built two schools in 1953-54 which were multi-story structures employing prismatic or folded slabs for floors. (See illustration, p.22). In the book *Candela: The Shell Builder* the author suggests that this may have been the first time that shells were used for floors. The author goes on to say that Candela considers shells for floors a "hopeless path" of investigation.28 This writer does not consider folded slabs or plates as true shells because they react in a different manner structurally. In this light, however, other structures have since been built using folded plates for floors, among them is the Arts and Architecture Library at Washington University in St. Louis, Missouri.

To further illustrate the potential of shells for floors - when the structural capabilities of the ancient "techo de bóveda" system became known through load tests and experimental models, it was decided that this system would be used as a *structural design determinant in all the buildings* on the new campus of the Universidad Autonoma de Guadalajara.29 This project, under the guidance of Mr. Harry Ransom, architectural professor at Rice University, will be detailed and illustrated in Chapter 2.
Convent School, Guerrero, D.F., Felix Candela, Engineer.

Built 1953, Prismatic Slab Construction.
CHAPTER 2

Simplest Application of Cast-In-Place
The simplest application of concrete floors is a cast-in-place method with no consideration for mechanical systems, plumbing, or interior partitions. These considerations would be applicable primarily to building types such as warehouses and parking garages. The extension of the system into precasting and other building types involves problems that will be discussed in Chapter 3.

There are many methods of structuring a floor system. Four obvious methods will be illustrated. They are:

(1) barrel shells and a one-way beam system (this is simply an application in reinforced concrete of the "techo de bóveda" system used in Guadalajara);

(2) double curvature shells and a one-way beam system;

(3) double curvature shells and a two-way beam system (this is an application in concrete of the system employed by Mr. Harry Ransom for the Universidad Autónoma de Guadalajara);

(4) double curvature (parabolic) shells resting directly on columns.

An understanding of the "techo de bóveda" system is important for the problems encountered here are similar to any application with concrete shells.

This system is a combination of two separate structures;
first the brick vaults, which support the floor and the loads imposed on the floor, and which carry these loads to the supporting structure; and second, the supporting framework of steel or concrete beams, girders, and columns which directly support the vaults.30

The illustrations on the following page show the various stages of development of vaulted construction in Central Mexico culminating with the "techo de boveda" system.

The supporting beams are placed 32 - 51 inches center-to-center, and this distance becomes the span of the barrel vault. The bricks comprising the barrel vaults are laid without framework, each successive row of bricks in itself becoming an arch rib spanning between steel beams. The vaults themselves are almost flat, the rise being only 1.5 - 3 inches (3-8 cm.).31

A level surface above the vault is achieved by filling with a light-weight concrete made from hydrated lime and "Jal", which is a local pumice sand and gravel. This fill acts to distribute evenly over the vaults the concentrated floor loads. The floor live load used locally for design of dwellings is 30.8 lbs/ft.$^2$ (150 Kg/m$^2$) and 71.8 lb/ft$^2$ (350 kg/m$^2$) for office buildings.32 A wearing surface of burnt clay brick or tile is usually placed on top of the concrete fill.

The allowable loads and practices used by the local people are based primarily on accumulated experiences. However, load tests on experimental vaults similar to those of local construction demonstrated a capability of sustaining a superimposed uni-
Primitive Terrado Construction by Indians

Modification of Terrado Construction by Spaniards

"Techo de bóveda"
formly distributed load of 388 lb/ft. Based on maximum live loads in current use this is a safety factor of greater than 5.4. 33

These examples illustrate the necessity for at least 4 basic elements in shell floor construction:

(1) shell;
(2) secondary system of columns, or columns and beams;
(3) fill to distribute loads;
(4) wearing surface.
Test Barrel Vaults, Guadalajara

It is important here to understand the need for containing the horizontal thrust of the vault coming into the beam. As shown in the illustration above, tie rods have been added to counteract the force of the vault and thus equilize the forces acting at the beam. This will be discussed further at a later point in the thesis.
Type 1: Barrel Shells - One-Way Beam System

Plan 1/8" = 1' 0"

Section A 1/8" = 1' 0"
Type 1

Section B \(1" = 1' 0"\)

Section C \(1" = 1' 0"\)
BARREL SHELLS AND ONE-WAY BEAM SYSTEM:

Barrell Shell, 6" Rise in 8' - 0" Span

Span: 8' - 0"  
Live Load 100 psf  
Dead Loads:  
1" Wearing Surface 13  
Foam Concrete, 4" avg.  
at 60 pcf (1"-7") 20  
2 1/2" Conc. Shell 31  
Total: 164 psf

Horizontal Thrust $= \frac{WL^2}{8h} = \frac{164 \times 8.00^2 \times 12}{8 \times 6} = 2,620$ lbs./ft.

Compressive Stress $= \frac{H}{A} = \frac{2,620}{12 \times 2.5} = 87$ psi

Barrel Shell, 3" Rise in 8' - 0" Span

Span: 8' - 0"  
Live Load 100 psf  
Dead Loads:  
1" Wearing Surface 13  
Foam Concrete, 2 1/2" avg.  
at 60 pcf (1"-4") 12  
2 1/2" Conc. Shell 31  
Total: 156 psf

Horizontal Thrust $= \frac{156 \times 8.00^2 \times 12}{8 \times 3} = 4,990$ lbs./ft.

Compressive Stress $= \frac{4,990}{12 \times 2.5} = 166$ psi

Barrel Shell, 6" Rise in 24' - 0" Span

Span: 24' - 0"  
Live Load 100 psf  
Dead Loads:  
1" Wearing Surface 13  
Foam Conc., 5" avg.  
at 60 pcf (2" - 8") 25  
2 1/2" Conc. Shell 31  
Total: 169 psf

Horizontal Thrust $= \frac{169 \times 24.00^2 \times 12}{8 \times 6} = 24,300$ lbs./ft.

Compressive Stress $= \frac{24,300}{12 \times 2.5} = 810$ psi
It is apparent from the calculations on the previous page that for barrel vaults the span may be much greater than 8 ft. (166 psi compressive stress with 3" rise) and still be within the allowable limit for compressive stresses (810 psi with 6" within 24').

A comparison with flat concrete slab construction under the same conditions will be advantageous in clarifying the advantages and disadvantages of barrel vault structures.

THE CRSI DESIGN HANDBOOK (Revised 1959) gives comparative information in the table: "Solid Concrete Slabs - End Spans (approximately 2/3% Reinf.)."

An 8'-0" span with 156 psf floor load requires a 4" slab thickness. A 4" slab may be used with an 8' span with up to 192 psf (total live and dead load). For the same span the barrel shell as presented in the calculations has a 3 1/2" structural thickness.*

A 24' 0" span with 169 psf floor load requires a 9 1/2" slab. The live load is 100 psf and a 9 1/2" slab will safely support a 106 psf live load. The same span with a barrel shell requires only a 3 1/2" thickness.

*3 1/2" thickness - 1" conc. wearing surface + 2 1/2" shell thickness: minimum - 2" to 2 1/2" required for cast-in-place shells including conc. cover over reinf. (3/4" min.), ACI BUILDING CODE (318 - 63).
We may deduce from this that barrel shells have relatively no economic structural advantages over flat slabs with short spans but have definite advantages with longer spans. For shorter spans there would be some advantage with precasting. It would be more economical because it makes unnecessary the use of formwork and shoring, which is very expensive.
Type 2: Double Curvature Shells - One-Way Beam System

Plan 1/8" = 1' 0"

Section A 1/8" = 1' 0"
Type 2

Section B 1" = 1' 0"

Section C 1" = 1' 0"
The third method is an outgrowth of the system employed by Mr. Harry Ransom of Rice University for the new campus of the Autonomous University of Guadalajara. That method, in turn, came about as a result of studies made on the "techo de bóveda" system. The analysis made there for the need of a two-way system is also of importance to an understanding of some of the problems encountered in shell floor systems.

In the architectural programming for all the proposed new buildings for the Universidad Autonoma de Guadalajara it became evident that a system of architectural flexibility must be an integral part of the structural skeleton. It is imperative that, with relative ease, walls may be relocated, modules added, spaces rearranged, versatility. The barrel brick vault system satisfied these requirements to a workable degree. But it was recognized as basically a one-way growth system with the opportunity to expand in only one direction. For example, partitions can be only reasonably located at the supporting beams, framing in the same direction. A two-way directional system was therefore desired so that expandibility could occur in either of two directions.34

The brick vaults are doubly curved - built without the use of formwork-spanning 5' - 2 3/8", rising 3.9". These vaults are in turn supported upon an aggregate network of reinforced concrete beams and columns. The underside of the brick vaults are exposed to take advantage of their rich color and textured surface.35
Construction of test vaults two-way grid system, to be employed in the buildings for the new campus of the Universidad Autónomos de Guadalajara.
Test vault construction, bottom photograph indicates quality of underside of finished vaults.
Type 3: Double Curvature Shells - Two-Way Beam System

Plan 1/8" = 1' 0"

Section A 1/8" = 1' 0"
Type 3

--- 1" CONC. WEARING SURFACE ---

--- FOAM CONC. FILL ---

--- SHELL REINF. ---

--- BEAM REINF. 1½" CLEAR CONC. COVER ALL AROUND ---

Section B 1" = 1' 0"

--- COLUMN REINF. ---

Section C 1" = 1' 0"
Type 4: Double Curvature Shells - resting directly on Columns

Plan 1/8" = 1' 0"

Section A 1/8" = 1' 0"
Type 4

Section B 1" = 1' 0"

Section C 1" = 1' 0"
CHAPTER 3

Integration of More Complex Variables—Precasting, Mechanical, Lighting Interior Partitions
An extension of the application of shells for floors to include more complicated building types (i.e., addition of mechanical systems, interior walls, and the possibility of precasting) involves a number of problems. It is impossible to consider all the problems but the writer will illustrate some of the more important ones and include some alternative solutions to these.

There are three methods of construction with reinforced concrete: cast-in-place, precasting, and a combination of both. Cast-in-place has been considered in Chapter 2 and the more complex problem of precasting will be reviewed here. There have been considerable advances in precast shell technology in Eastern Europe in recent years but Pier Luigi Nervi of Italy is probably the notable designer of precast concrete shells. In Structures by Nervi, he states, "besides its technical advantages, prefabrication possesses interesting aesthetic-architectural characteristics deriving from the inherent lightness of prefabricated structures and from the speed of construction typical of mass-produced elements. Moreover, prefabrication allows the use of elements of complicated shape without construction difficulties or expensive formwork, since each form is used to pour a large number of elements."
Along with the advantages are these problems:

(1) jointage of precast units;
(2) proper size and weight of units;
(3) breaking off of thin shell edges;
(4) precasting of uniform thickness with very small tolerances.

The problem of jointage includes not only that of joining the various shell units together but shells to columns, shells to beams, and beams to columns - according to the system being employed. The jointage must be firm in order to allow for continuity throughout the structure.

One method used by Nervi for the structural connection between elements is to weld the reinforcing bars and pour the joints with high strength concrete.37

Problems of jointage are further complicated by the small tolerances required in making the units. Tolerances are affected by the preciseness of the design, the precision of the forms, the skill of the laborers, the materials used and the weather. This writer constructed some models of doubly curved precast shells. The purposes for the models were:

(1) to study the visual effects and possibilities which would not be apparent from drawings;
(2) to study problems of jointage and edge conditions of precasting;
(3) to perceive a better realization of the scale involved.

Many of the problems encountered in making these models of plaster
will also be encountered in construction of full size concrete units.

Unlike the hyperbolic paraboloid which can be generated with straight line forms, the paraboloid of revolution which was used for the models presented problems in proper control of the curvature with small tolerances. This problem, however, could be overcome with proper equipment in a laboratory or a manufacturing plant.

The material first used for the form was potters clay. However, unless a properly controlled humidity was maintained, the clay mould would shrink. The form was then cast permanently in plaster at $1/4'' = 1' 0''$ and shell units were in turn cast from this. At this point the following problems arose:

(1) what reinforcing material to use to maintain strength and yet be flexible enough to assume the contours of the form;

(2) how to maintain uniform thickness at a small scale;

(3) how to assure uniform, slow drying of plaster to prevent shrinkage and warpage;

(4) breaking of edges.

Several reinforcing materials were tried. The most reliable proved to be nylon mesh, which maintained reinforcing strength yet easily assumed the curvature of the shell. The other three problems mentioned above were never completely solved.
Another consideration is that of size and weight. The weight and dimensions must be carefully considered in relation to the mechanical equipment available to lift them into position. Usually a complete scaffolding must be designed and horizontal and vertical transportation equipment provided for. The efficiency of this handling system may be responsible for the success or failure of the project. 38

The next few pages are photographs of a model illustrating prefabricated umbrellas 40 feet square and 2 inches thick except at the valleys where the thickness is increased to 4 inches. The total weight of such a unit is 46,000 lbs. Conventional equipment could easily move and lift such a shell. The main problem would be to prevent breaking of the edges in handling. Some other examples of precast units and their weights are:

(1) post-tensioned bridge girder (940 lb/ft x 95' = 89,400 lbs.);
(2) pretensioned roof beam (520 lb/ft. x 66' = 34,400 lbs.). 39
The following five pages present alternative solutions to the problems of precasting.
ALTERNATIVE 1

SECTION

GIRDER — COLUMN

ONE-WAY BEAM SYSTEM

PLAN
ALTERNATIVE 2

ALTERNATIVE 3
ALTERNATIVE 4

SECTION

PLAN
ALTERNATIVE 5

SECTION

PLAN
ALTERNATIVE 6

SECTION

COLUMN

FILL

WEARING SURFACE

PRECAST UMBRELLA SHELL
A second major problem is how to integrate the mechanical system into the shell flooring system. Some aspects of this problem are:

1. Whether to expose the duct system below the shell or place it in the fill above (basic assumption - shell is to be exposed from below and not covered with suspended ceiling);

2. if the answer to question (1) is to expose the ducts, the question is how can this be done properly from both an architectural and mechanical standpoint;

3. if the answer to question (1) is to put the ductwork in the fill, there are several needs to be met:
   a. how to keep the size of the ductwork small enough to make putting it in the fill feasible,
   b. how to allow for concentrated loads which are directly above ducts,
   c. where to place ducts (at valleys or at peak),
   d. how to distribute air out into the room.

A good example of the first aspect, exposing the ducts, is the Skidmore, Owings & Merrill - Great Southern Life Building, Houston, Texas. Here the ducts run between girders of a one-way beam system.

A similar solution would prove interesting for shell floors. The idea of expressing both the structure and mechanical as a pure
expression seems justified. Care would have to be taken not to "clutter" the ceiling and destroy the appearance of the shell forms.

Ductwork should be placed at low point of shell because:

(1) this allows for greatest flexibility in duct size;
(2) decreases risk with concentrated loads above ducts;
(3) easiest to dissipate air into room from this point.

(See illustration, p. 58).
MECHANICAL CONSIDERATIONS

LOCATIONS OF DUCTS

SPACE REQUIRED FOR DUCTS

METHOD OF DISTRIBUTION OF AIR
How much space do the mechanical ducts require? The answer to this question is dependent on several things:

(1) the amount of air needed in the room;
(2) the distance from central unit;
(3) the type of system, conventional or high-speed.

The amount of space required for ducts is critical to the design, for a matter of inches saved in depth could mean great savings in overall cost of materials. (See illustration, p.58).

How is the air dissipated into a room? The question involves not how is it to be distributed down into a room but whether down through the ceiling or up through the floor.

One possibility is a 'mushroom' type rising up out of the floor diffusing cool air into the upper portion of the space. (See illustration, p.60). They should be placed where they do not interfere with room functions. They might also serve as an ordering device for the rooms and might be integrated with the lighting.40

If the air is dissipated out through the ceiling there are several ways in which this could be done. Page 60 illustrates the use of paraboloids of revolution resting on a one or two-way grid work of beams. Return could be carried out through the corridor to a return air plenum.
"MUSHROOM" DIFFUSION

DIFFUSION THROUGH CEILING
Lighting

A third major area of consideration is artificial lighting. The governing light source for past structures was of course daylight and, in a sense, this has continued to present day. Since "structure" has developed using natural lighting as its criterion of form, when new structures (i.e. shells) have been developed, the models and perspectives have been illustrated as for use in natural light. The artificial lighting is only afterward added to the structure.41

This procedure is illustrated by the early development of shell constructing where the basic simplicity of the structure so appealed to architects that the needs of other services were neglected; the result was shell structure designed for natural daylighting in which the artificial lighting and other services were hung below the shell, often producing a confusing and untidy appearance.42

This trend is changing along with the development of shells and architects are realizing the possibilities with artificial lighting of shells. The aesthetic possibilities of artificial lighting are more limited in thin shell application than in conventional structure, but, at the same time, the possibilities that are present suggest some exciting solutions. The matter of daylighting will be considered in Chapter 4.

Some of the problems involved are:

(1) how to best accentuate and express the exposed structure
while giving proper light to the room;

(2) how to integrate the lighting fixtures so that they do not interfere visually with the structure. The more technical problems involved with illumination level, lighting materials, etc. will not be considered in this thesis.

The six methods of distributing light are illustrated graphically on page 63. 43

The ones that can probably be most usefully employed in shell lighting are upward and downward diffused and upward concentrating. Light reflected from the curved surface of the shell can serve to accentuate the strong form of the structure.
SIX METHODS OF LIGHTING

- **DOWNWARD**
  - CONCENTRATING

- **DOWNWARD**
  - DIFFUSING

- **UPWARD**
  - CONCENTRATING

- **UPWARD**
  - DIFFUSING

- **MULTI-DIRECTIONAL**
  - CONCENTRATING

- **MULTI-DIRECTIONAL**
  - DIFFUSING
The type of lighting shown at the top of page 65 is known as cove lighting. If possible it is best to use the entire shell as a reflector for cove lighting. This preserves and emphasizes the purity, and simplicity of the structure. Boyd Anderson says "treating the shell as a reflector is better appreciated with each application." In the bottom figure the lighting units are simply suspended from the ceiling. To achieve the proper lighting quality the rules of thumb shown in figures should be followed with regard to distance of light source from shell.

Rotterdam, Holland's train stations, illustrate a successful relationship between the artificial sources and the structural shell. While a high proportion of light is directed downward toward the platform edge, some light is permitted upward to give a gentle light over the whole curve of the roof. This both emphasizes the character of the roof form and adds a sense of protection from the canopy to travelers at night.

There are many psychological effects that the quality of light may have upon a space. Light can give the shell a 'hardness' or 'softness' depending upon how it is used and as well has an effect upon the sense of scale of the space. Scale will be discussed further in Chapter 4. The effects of natural light will not be considered here.
Cove Lighting

Top Lighting

Suspended Lighting
Another problem of significance but which will not be considered here is the method of handling plumbing. Consideration must be given to incorporating hot and cold water supply lines, waste and vent stacks into a service core.

A fourth major area of consideration is integration of interior partitioning with the shell flooring system.

The problems involved are:

(1) how to solve aesthetic problem of connection of rectilinear wall plane with curved surface of shells;
(2) the technical problem of coordinating walls with a gridwork of beams, if such are used;
(3) retaining sense of curved ceiling while incorporating wall planes.

The solid wall should be detached from the shell surface if at all possible. This will allow the shell to float in space in its own articulate form, free from any superficial support. The void between the top of the wall can be filled with glass or clear plastic panel. Page 67 will illustrate this situation.

It is not always known exactly where interior walls will need to be placed in a building, particularly a speculative structure. For this reason a grid system on a small module has advantages over larger modules in some cases.
WALLS CARRIED SOLID TO CEILING

SOLID WALL BROKEN - SPACE FLOWS THROUGH
CHAPTER 4

Consideration of Architectural Aesthetics
Chapter 4 deals with some of the considerations of architectural expression involved. The problems are:

(1) how to best express the structural system of shells on both interior and exterior;
(2) how to exploit the shells for sun control;
(3) how to express the light, airy quality of shells;
(4) how to terminate the shell at ground and roof;
(5) scale.

Robert Maillart achieved with his bridge designs a distinctly graceful quality peculiar to the forms he used. The bridges purely express the manner in which the forces act upon the structures. (See illustration, p. 69).

Similarly it is important that shell designs convey a sense of lightness and airiness expressive of their true nature. Felix Candela has achieved this with many of his shell structures, illustrated on page 70.

Pages 71 and 72 illustrate the manner in which the shell might be handled at the exterior, both in expressing the structural system and exploiting the shell for sun control.

Another major problem is how to express the termination of the shell at ground and roof. For purposes of drainage and insulation the roof should probably be flat although it would have a pleasing effect to express the shells on the roof.

The fifth major concern is scale. The module or size of the shells bears a distinct relationship to the scale of the building.
Salginatobel bridge, designed by Robert Maillart. Maillart's bridges purely expresses the manner in which the forces act upon the structures.
Church of our Miraculous Lady, Felix Candela, 1953

Indicative of light-lacy quality of much of Candela's work.
The rhythm set up in a shell flooring system should carry through to the exterior. It should be readily apparent that shells form a significant portion of the structural system.

HORIZONTAL ACCENTUATION

HORIZONTAL AND VERTICAL ACCENTUATION
RELATIONSHIP OF SHELL AT EXTERIOR WALL

Cantilevered Shell and Fill

Cantilevered Shell

Cantilevered Beam and Shell
From Architectural Scale we read "...when there is a structural skeleton on columns and beams, it supplies a large module running throughout the design. Where the structural units are shells, vaults, or folded concrete planes, they may easily provide a unit that is an eighth or a quarter of a small building. And where a single shell or vault embraces the whole building, all of its divisions will necessarily be related to the structural span. Can there be any doubt that structural behavior and systems of construction determine orders of physical dimensions that are, as a rule, the natural bases for a building's visual scale?"

The discipline of planning is as exacting as that of construction. For instance, where a work space is custom-fitted, as in the modern kitchen or bathroom, the accommodations of people's bodies and bodily movements fills the room with dimensions that - within an inch or two - are determined by use. The more intimately and carefully a building is designed for the activities of its occupants, the more these measurements determine its minimum.

The kitchen and bath are one example. On a building scale, the function of an auditorium would, of course, require a larger minimum dimension than that of a school room. The auditorium space would also dictate a larger shell module (perhaps even a single shell spanning the entire area) than the school - not only because of reasons of structural efficiency but because the size of the space would visually warrant a larger scale module.
SHELL AS SMALL BASIC UNIT

SHELL AS LARGE BASIC UNIT
This is related to unit size. The synthesis of physical dimensions often finds its expression in a bay unit. In a medieval hall, for instance, the synthesizing bay unit may coordinate a useful plan area, a handsome visual division, and a convenient structural unit for stone vaulting and large window openings. In a modern office building, the bay may be even more comprehensive. It may provide a suitable unit of steel skeleton construction (or shell construction) and a useful area of rental space; and it may also allow the space to be divided in many ways by a modular partition system and furnish an elaborate system of lighting and air-conditioning.49

The bay unit might be much larger for a warehouse or a garage, than for an office building. There are two readily apparent reasons for this; one is functional, the other is related to human scale. The office building has a smaller module to which man may more readily relate.
LARGE MODULE-LARGE SCALE
(Warehouse, Department Store Prototype)

SMALL MODULE-SMALL SCALE
(Office, Apartment, Hotel Prototype)
CHAPTER 5

Limiting Factors and Considerations for Future Investigation
This thesis has been primarily an architectural approach to the application of shells for floors. There are two basic aspects of the problem which have been mentioned but should here be stressed as an area for additional research:

(1) the effects of concentrated loads upon the fill and, consequently the shell;

(2) the appropriate materials and proper design of the fill and wearing surface.

The intent here is only to define the problems, not to solve them. Solving these problems is a matter for further engineering research. It can be assumed, however, from the experience of "techo de bóveda" that some combination of shell, fill and wearing surface will withstand conventional floor loads. The fill material used in Guadalajara is a lightweight concrete made from hydrated lime and "jal", which is a local pumice sand and gravel. From test results its compressive strength against cracking was 50 lb/in\(^2\) and 61.9 lb/in\(^2\) against rupture. Its volumetric weight at 10 days was 58.5 lb/ft\(^3\). Its average thickness was 2.8 inches. The wearing surface of burnt clay brick was .4 inches in thickness. These details are obviously good for ordinary office floor loads where severe concentrations are not present, and hence these strengths, unit weights, and thicknesses become a starting point for design or additional research. Studies aimed at optimum combinations of materials could proceed from this point.
The strength of the concrete and the depth are not arrived at analytically but are based primarily on accumulated experience. This fill is only structural in that it sustains the load from the floor above and distributes it across the vault. The vaults, just as shells, will not withstand large concentrated loads because of their thinness. The "jal" is used not only for its strength but because of its light weight. This writer has suggested throughout this text the use of foam concrete, an aerated concrete which has a range of strength and unit weight. Other materials which might be considered for this fill are inert earth materials and synthetics such as foamed plastic. The material must be stable enough to support evenly the wearing surface above or else cracking will occur if, for instance, concrete is used for the wearing surface. This is the extent to which these problems will be considered here.
CHAPTER 6

Review and Synthesis
The shell was described in Chapter 1 in its historical context and in relation to other structural systems. The proposition was then posed that shallow shells are applicable for flooring systems in multi-story buildings with a preliminary basis presented for this proposition.

Chapters 2 - 4 were an analysis of the proposition. In Chapter 2 was demonstrated the simplest application - cast-in-place shells of several types with structural analysis. In Chapter 3 were enumerated the several problems and alternatives involved with extending the proposition to additional building types and precasting. In Chapter 4 were presented the problems and alternatives of architectural expression involved.

From the investigation and analysis on the previous chapters we may draw several general conclusions:

(1) shells are structurally feasible for floor systems and, in fact, have some advantages over other systems;

(2) precasting and integration of mechanical systems present problems not ordinarily experienced with other systems but, conversely, potential with mechanical and lighting not available with other systems;

(3) shells for floors have unique potential for architectural and structural expression from both interior and exterior.

There are, however, some limiting factors which must be pointed out in regard to this study:

(1) only simple guideline structural analyses have been made.
A complete structural analysis would have to be made on a system before detailed dimensions and reinforcement could be determined;

(2) There are, as has been said, some particular structural problems which should be investigated further - i.e. the effects of concentrated loads upon the fill and shell, the optimum design of fill and wearing surface;

(3) The important consideration of economy has not been dealt with by this writer. A complete cost analysis of a system will naturally follow as a sequel in any further investigation of shells for floors. Systems used are not always the most inexpensive but, if they are not, there must be other advantages.

The general conclusions above have basis in specific criteria which reinforce the thesis that shallow shells are applicable in flooring systems:

I. Historical precedent

A. Singly curved, "flat" vaults were used in Mexico for many centuries. This has evolved into the "Techo de boveda" system.

B. Doubly curved "flat" vaults are being used in construction for new campus of Autonomous University of Guadalajara.

C. Use of folded plates in multi-story construction by Candela and others is analogous in its similarity.
II. Structural properties

A. Structural analysts have shown that it is possible for membrane action to develop even within the shallow depth of flat slab construction, and hence very shallow shells, carrying load entirely through membrane action, should be possible.

B. Shallow shells of very thin sections and relatively long span are capable of carrying normal and above normal floor loads.

C. Thinness of shell is an economical factor with certain spans.

D. Shells are the most efficient type of structure available and hence should be efficient used as shallow shells for floors.

III. Architectural and mechanical potential

A. There are a variety of ways in which to handle mechanical, lighting and interior walls.

B. There is a unique aesthetic quality of shell forms.

There will, in Chapter 7 be demonstrated three prototype shell flooring systems, based upon the above criteria and their relationship to the material presented in the previous chapters. I again point out that this thesis, as an investigation into this area, covers only limited aspects of the possibilities of shells for floors and further investigation along these avenues of thought is encouraged.
CHAPTER 7
Demonstration
Prototype I is the result of an initial three week investigation of shells for floors in the fall of 1966. An evaluation follows the demonstration of this prototype.
A speculative office building for

Houston, Texas
FRAMING PLAN  \( \frac{1}{2}" = 1 '0" \)
89

BEAM-GIRDER CONNECTION 34'-10"

EXTERIOR WALL @ COLUMN 34'-1"
ELEVATION DETAIL
NORTHWEST CORNER

$\frac{1}{8}" = 1.0"$
In an evaluation of the design several points were brought out:

(1) The distribution of the mechanical through the 'cap' as shown in the isometric, presents problems related to combining the mechanical outlet and point of structural bearing into one unit.

(2) It has been suggested that there is a certain aesthetic and perhaps structural impurity in employing doubly-curved shells with a one-way beam system.

(3) Round ducts are better suited for these circumstances than rectangular ducts.

(4) The sun-screen solution at the exterior walls on the east and west is not well resolved.
Prototype II.

Plan 1/8" = 1' 0"

Section A 1/8" = 1' 0"
Prototype II.

Section B. 1" = 1' 0"

Section C 1" = 1' 0"
Scale Model, Prototype II.
Scale Model, Prototype II.
Prototype III.

Plan 1/8" = 1' 0"

Section A 1/8" = 1' 0"
Doubly curved shell supported directly by columns

Paraboloid of Revolution, 18" Rise in 24' - 0" Square Bay

Live Load
Dead Loads: 1" Wearing Surface 100 psf
Foam Concrete, 2" - 20", avg. 11" 13
at 60pcf
2 1/2" Conc. Shell 55
Total: 31

Total Load in one panel = 199 x 24 x 24 = 114,700 lbs.

Vertical Component of Thrust = \( \frac{114,700}{4} \) = 28,700 lbs. = 28.7\( k \) = F_y

\[ y = Kx^2 \]
\[ 9 = K \times 144 \times 144 \]
\[ K = \frac{9}{144 \times 144} = 0.000434 = 4.34 \times 10^{-4} \]

\[ y = 4.34 \times 10^{-4} x^2 \]
\[ \frac{dy}{dx} = 8.68 \times 10^{-4} x \]

\[ \frac{dy}{dx} \bigg|_{x=144} = 8.68 \times 10^{-4} \times 144 = 0.1250 = \frac{F_y}{F_x} \]

\[ F_x = \frac{F_y}{0.1250} = \frac{28,700}{0.1250} = 229,000 \text{ lbs.} = 229k \]

\[ F = \sqrt{229^2 + 28.7^2} = 231k \]

Compressive Stress = \( \frac{231,000}{48 \times 4} \) = 1,202 psi
DOUBLY CURVED SHELL SUPPORTED DIRECTLY BY COLUMNS (cont'd.)

Paraboloid of Revolution, 24" Rise in 24' - 0" Square Bay

Live Load: 100 psf
Dead Loads: 13
- 1" Wearing Surface
- Foam Concrete, 2" - 26", avg. 14" at 60 pcf
- 2 1/2" Conc. Shell 31

Total: 214 psf

Total Load in One Panel = 214 x 24 x 24 = 123,200 lbs. = 123.2k

\[ F_y = \frac{123,200}{4} = 30,800 \text{ lbs.} = 30.8k \]

\[ y = K x^2 \quad 12 = K x 144 x 144 \quad K = 0.000579 = 5.79 \times 10^{-4} \]

\[ y = 5.79 \times 10^{-4} x^2 \]

\[ \frac{dy}{dx} = 11.58 \times 10^{-4} x \]

\[ \frac{dy}{dx} x = 144 = 11.58 \times 10^{-4} x 144 = 0.1668 = \frac{F_y}{F_x} \]

\[ F_x = \frac{30,800}{0.1668} = 184,700 \text{ lbs.} = 184.7k \]

\[ F = \sqrt{184.7^2 + 30.8^2} = 187.4k \]

Compressive Stress = \[\frac{187,400}{48 \times 4}\] = 975 psi
At exterior column (not corner column)

If we arbitrarily limit the working shearing stress to about 200 psi, we shall then require

Effective Column Area = \[bd = \frac{V}{V} = \frac{169,600}{200} = 848 \text{ in.}^2\]

E.G., \(b=30", d = 30", bd = 30 \times 30" = 900 \text{ in.}^2\)
Tie Size

\[ \text{Reg'd. A} = \frac{T}{F_{all}} = \frac{185 \text{k}}{20 \text{ksi}} = 9.25 \text{ in.}^2 \]

6 - #11 bars
(int. grade billet steel)

Alternatively

\[ \text{Reg'd. A} = \frac{185 \text{k}}{110 \text{ksi}} = 1.68 \text{ in.}^2 \]

16-3/8" high-strength bars
(ult. strength = 200 ksi)

Alternatively

\[ \text{Reg'd. A} = \frac{185 \text{k}}{87 \text{ksi}} = 2.13 \text{ in.}^2 \]

2-1 1/2" Regular Stress Steel bars
(ult. strength = 145 ksi)

2 - 1 1/2" Regular Stress steel bars selected - fewer bars,
easier to handle for erection, maneuvering of A/c ducts.
Prototype III.

Section B. 1" = 1' 0"
Prototype III.

Section C.  1" = 1' 0"
Cutaway Exploded Isometric Column-Shell Connection

.1" = 1' 0"
Prototype III.

Precast Shell Connection  \(3'' = 1' 0''\)
Prototype III.

Mechanical Detail  $1" = 1' 0"$
Prototype III.

Free-standing Panel Room Diffuser

Free-standing Cylindrical Room Diffuser


17. Ibid., p. 19.

18. Ibid., p. 19.
19 Eduardo Torroja, Philosophy of Structure, (University of California, Berkeley) p. 175.


21 Joedicke, op. cit., p. 20.


23 Pfliüger, op. cit., p. 2.


26 Salvadori and Heller, op. cit., p. 342 - 344.


28 Faber, op. cit., p. 43.

29 Krah1, Ransom, op. cit., p. 51.

30 Ibid., p. 19.

31 Ibid., p. 20.

32 Ibid., p. 20.

33 Ibid., p. 23.

34 Ibid., p. 51.


37 Ibid., p. 64.

38 Ibid., p. 65.


40 Buckley, op. cit., p. 22.


42 Ibid., p. 199.


44 Robinson, op. cit., p. 25.

45 Ibid.

46 Phillips, op. cit., p. 20.


48 Ibid., p. 20.

49 Ibid., p. 27.

50 Krahl and Ransom, op. cit., p. 20.

51 Ibid., p. 41.

52 Ibid., p. 39.

Buckley, William, "Thin Concrete Shells," Rice Institute, 1958, A paper submitted to the Department of Architecture in partial fulfillment of the requirements for the degree of Master in Architecture.


Joedicke, Shell Architecture, (Reinhold Corp. N.Y.), 1963.


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