RICE UNIVERSITY

AN EXPERIMENTAL INVESTIGATION OF
WATER WAVE INERTIAL FORCES ON
LARGE DIAMETER PILES

by

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ABSTRACT

An experimental investigation was carried out in the Rice University wave tank facility with the object of determining the magnitude and variation in time of inertial forces due to wave action on large diameter pilings of circular cross-section. Mass coefficients were computed for various wave periods and wave heights using two different wave theories and comparisons are made between the results. The measured wave force and predicted wave force for a second order Stokes wave approximation are both plotted versus time. All of the waves studied were "deep-water" waves.

A discussion of the deviations of the experimental results from theory is included.

A detailed description is provided of the equipment in the Rice University wavetank facility, including the "L"-shaped wavetank. The equipment performance is discussed.
The writer wishes to express his gratitude to Dr. Murphy H. Thibodeaux, Associate Professor of Civil Engineering, Rice University, and Dr. Herbert K. Beckmann, Professor of Mechanical Engineering, Rice University, who suggested the thesis topic and provided guidance during its development.

In particular, the author wishes to thank the personnel of the Rice Mechanical Engineering Shop, who were a very great aid in building the equipment for the wavetank.
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1. Wave Profile Program
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INTRODUCTION

The purpose of this investigation was to determine experimentally the magnitudes and the variations in time of inertial forces due to periodic deep water waves acting on surface-penetrating, large-diameter piles.

Lamb gives the theoretical value of the mass coefficient (or inertial coefficient) as 2 for a non-viscous fluid accelerating perpendicular to the axis of a totally immersed infinite circular cylinder. (1). The mass coefficient in such a case is independent of fluid velocity and pile size.

However, at least two organizations involved in offshore structural design use mass coefficients of 1.5 for predicting wave inertial forces on circular piles with diameters small with respect to wave height. (See page 86 in "Discussion") This would seem to indicate that use of the theoretical value of 2 for the mass coefficient is open to question. It is known that the actual oscillating flow around piles due to waves is not accurately described by potential theory. Viscous effects, such as the presence of a boundary layer and boundary layer separation accompanied by vortex formation, as well as general turbulence, act to alter the water particle motions around piles from the motion patterns predicted by potential flow. In addition, surface effects alter the fluid flow around piles from that of potential flow.
Little data has been published to date on measured mass coefficients for large diameter circular piles. Therefore, since large piles are coming into increasingly common use for offshore structures, an experimental investigation of large diameter circular piles was made in the Rice University wavetank facility.

The circular pile used for the tests had a very large diameter to wave height ratio; this ratio varied from approximately 2.8 to 3.5. Water particle motions and behavior were predicted by two different solutions of the wave equation and the associated boundary value problem. The total inertial force and the location of its resultant were predicted on the basis of the two wave solutions utilized. A comparison is made between the experimentally measured total wave force and the theoretically predicted values of the inertial force. A comparison is also made between the theoretically predicted and the observed locations of the resultant forces on the test pile.
DESCRIPTION OF WAVETANK FACILITY

The series of tests described in this thesis were carried out in the Rice University wavetank facility in the Ryon Civil Engineering Laboratory.

Wavetank:

The test tank is an "L"-shaped monolithic reinforced concrete structure one hundred feet long, eight feet wide, and seven feet deep. (See Figure 1) The right angle extension "L" is seventeen feet long and has the same width and depth as the main portion of the tank. The tank cross-section is constant within ± 1.0 inch width, and the depth is constant within ± 0.25 inch. The bottom surface of the tank is smooth concrete; the walls are roughened slightly by protruding sand grains in the cement finishing course.

The tank is provided with a drain valve, filling spout, and overflow pipe for maintaining constant water surface elevation. The overflow level of the tank can be varied as desired. This equipment is located in the end of the "L" section of the tank.

Wavemaker:

Essentially long-crested, two-dimensional wave trains are generated at one end of the tank by a rigid-flap type wavemaker.
Figure 1

General Plan View of Wavetank
The wavemaker utilizes a hydraulic power system and a mechanical control apparatus. (See Figures 2 & 3) The hydraulic fluid is delivered by a ten-horsepower, electric motor driven, positive displacement pump unit operating at a pressure of approximately 800 psig and producing a flow rate of up to eighteen gallons per minute.

A trunnion-mounted, double-end, double-acting hydraulic cylinder actuates the rigid flap. The hydraulic cylinder has an 18.0 inch stroke, 0.625 inch pushrod diameter, and an 1.00 inch piston diameter. The net piston area is 0.479 square inches; 0.0021 gallons are required per one-inch piston displacement.

Because it was necessary to choose a small-displacement hydraulic cylinder for the hydraulic power system in order to make the hydraulic piston quick to react and sensitive to input commands, the hydraulic piston operates close to its load capacity for the 800 psig pump operating pressure.

The flow to the hydraulic cylinder is metered through a special four-way valve "Orbitrol" unit adapted from a commercially available tugboat power-steering system. The valve is controlled by rotation of its input shaft. A given rotation of the input shaft of the valve from its neutral position produces a proportional flow of hydraulic fluid to the active side of the hydraulic cylinder. When the input shaft is in a neutral
Figure 2

Schematic Diagram of Wavemaker

- Overload Clutch
- Drive Chain
- Drive Sprocket
- Electric Motor with 10:1 Reduction
- Interchangeable Gears for varying input speed
- 3:1 Reduction Gears
- Variable Stroke Crank
- Slotted Connecting Rod
- Power Steering Control Valve
- Control Valve Drive Sprocket
- Rack Gear
- Push Rod
- Return Line
- Hydraulic Pump
- Hydraulic Fluid Reservoir
- Hydraulic Cylinder
- Mounting Hinge for Flap
- Flap

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Figure 3. Photo of wavemaker control unit
position, the flow from the pump unit bypasses the hydraulic cylinder.

Lag times between input commands and response of the hydraulic piston are relatively small because of the high pump delivery rate. The time lag is dependent upon the magnitude of the input shaft rotation with respect to the neutral position of the valve; i.e., the amount of fluid which has to be displaced by the pump to rotate the valve to its new neutral position.

The control system for the hydraulic unit is sufficiently flexible so that, for suitable input motions applied to the control valve, an irregular sea state can be produced by the wavemaker. However, this was not within the scope of this investigation.

For the purposes of the research described in this thesis, it was desired to produce a regular sea state in the wavetank. Thus the input shaft of the control valve was radially oscillated with a sinusoidal input, resulting in the corresponding output motion of the hydraulic cylinder being approximately a sinusoidal motion with a slight time phase shift from the input.

The period and amplitude of the input motion are independently variable, but cannot be changed while the input device is in operation. The power to the input unit is provided
by a 0.25 horsepower electric motor (rated full load speed 1725 rpm) through a 10.0 : 1 reduction gearbox and a chain-driven overload clutch. The use of the overload clutch is necessary for the protection of the control valve, which is easily damaged by the application of excessive torque. The variation in wave periods is obtained by use of several combinations of gears which can be mounted on the driving and driven shafts of the input apparatus. In this way the period of the input motion can be varied without changing the drive motor speed. Gear ratios of 60 : 60 and 49 : 71 were used in the tests described in this thesis.

The input device shaft rotation is subjected to a further 3.0 : 1 reduction in a gearbox before it is applied to a variable-stroke crank rigidly mounted on the drive shaft. The essentially constant shaft speed motion of the variable-stroke crank is used to drive a scotch crank by means of a connecting pin working in an elongated slot perpendicular to the crank axis. The result is the production of a rectilinear sinusoidal motion of the scotch crank along its axis. The control valve is provided with a pinion gear which is driven directly by a rack gear mounted on the midsection of the scotch crank. Thus the control valve is operated with a sinusoidal input motion.

The rigid flap is composed of two 4' x 8' sheets of marine plywood mounted on a heavy-duty galvanized steel frame built of 3" x 1.5" channels and 2" x 2" angles. (See Figure 4.) The flap
FLAP IN POSITION
(SIDE VIEW)

REAR VIEW of FLAP

Scale: 2' = 1"
was hinged 37.25 inches above the bottom of the tank for the series of tests discussed in this thesis. However, the location of the hinge points and, therefore, the amount of immersion of the flap for a given water depth can be varied. This is done by means of a slideable cam-locking device enclosed in a slotted steel section which is rigidly mounted to the walls of the tank. The cam-locking device supports the fixed hinge mounts for the flap at the desired level. The hydraulic piston is connected to the flap by a mounting bracket located 2.5 inches above the top of the flap generating surface.

Since the hinges of the flap are positioned 23.0 inches from the end wall of the tank, a column of water is present behind the flap. At the beginning of each test, the nodal points of the flap motion are adjusted so that the flap is vertical when in a nodal position. Thus this column of water (in still water conditions) is 96 inches wide, 24.5 inches thick, and of depth equal to the water depth minus 30.5 inches. Water flow communication between the region behind the flap and the main portion of the wavetank is minimized by a removable barrier of large concrete blocks positioned behind and under the flap as shown in Figure 4. The body of water behind the flap is thus essentially isolated from the rest of the tank for short period oscillations of the flap.

The water behind the flap exhibits various modes of oscillation.
lation whenever waves are being generated in the tank. It was observed during this investigation that the entrapped water column was apparently in resonance for an exciting force period of approximately one second.

The water oscillations behind the flap attained larger magnitudes than those in the wavetrain concurrently produced. It was therefore necessary to locate a splash shield at the back of the tank to prevent the water column behind the flap from overtopping the tank walls.

**Turning Vanes:**

A novel approach to the problem of reducing wave reflections from the end of the wavetank was utilized at the Rice facility. As has already been mentioned, the tank was designed with an unusual "L" shape. At the far end of the wavetank from the wavemaker an array of turning vanes is located at the junction of the two legs of the "L". (See Figures 2 and 5.) The primary function of the turning vanes is to cause the waves propagating down the length of the tank to change direction upon entering the vane array and enter the transversely oriented section of the tank.

The vanes were fabricated from 0.10 inch thick galvanized steel sheets 8.0 feet high bent to the desired curvatures and spliced where necessary with lapped and bolted connections. The vanes are spaced at the following radii from
Figure 5. General view of turning vanes from main portion of tank
the inside corner of the "L": 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 5.0, 6.0, 7.0, and 8.0 feet. The current number and spacing of the vanes was selected after observation of the performance of a lesser number of similar vanes indicated that more vanes would more effectively turn the waves into the short leg of the "L". The vanes are held in position by rigid upper and lower frames. (See Figures 5 and 6) The vanes themselves are somewhat flexible; some of the vanes with larger radii may be observed to flex as much as 0.5 inch when large waves pass through the turning vane array.

The turning vanes prove quite effective in their intended purpose of turning the impingeing waves. In addition, they provide an added benefit by contributing to the turbulent dissipation of waves passing through the vanes. The waves entering the vane array are uniform, two-dimensional waves traveling with constant phase speed. As the waves enter between the different vanes, they are subdivided into separate waves which have to travel paths with lengths directly proportional to the path radii. Thus a phase shift results between the subdivided waves as they emerge from the vanes into the short leg of the "L". Spilling occurs at the lips of the vanes as the oscillating fluid attempts to equalize its surface elevation with the fluid between the adjacent vanes. As a result, very large eddies or vortices up to one foot in diameter form at both the leading and,

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Plan of Lower Turning Vane Mounting Frame & Turning Vanes

Figure 6

Inside corner of 7" in wavetank
particularly, the trailing edges of the vanes. (See Figure 7.)
The formation of these vortices persists for significant periods of time (frequently five minutes or more) after cessation of wave generation. In addition to the phase shift and turbulence induced by the vanes, further disruption of the uniform wavetrains entering the transverse section of the tank is produced by the tendency of a certain portion of the waves to bypass the vane array between the radius point of the array and the first vane. (See Figure 8.)

Screens:

Additional wave absorption is provided by a series of ten closely packed 4.0' x 8.0' screens supported on tubular perimeter frames located in the short leg of the "L". (See Figures 2 and 9.) The screening is chain-link fencing with a 2.25 inch square mesh. The screens are suspended from the sides of the tank so that they are on a 1 : 2.0 slope from the horizontal and positioned with approximately 3 inches of the screen protruding above the still water surface.

Beach:

A perforated beach was installed at the extreme end of the short leg of the "L" to further assist in wave absorption. (See Figures 2 and 9.) This beach is composed of a flat 8' x 8' sheet of 0.5 inch plywood randomly perforated over the entire beach.
Figure 7. Vortices at the trailing edge of the turning vanes

Figure 8. A bypassing wave combined with waves passing through turning vanes
Figure 9. View of the turning vanes, screens, and beach (looking in the direction of initial wave travel)

Figure 10. Turbulence produced by the perforated beach
surface with 0.625 inch and 1.25 inch holes on approximately 4 inch centers. The beach is supported at inclinations which can be varied as desired by adjustment of the submerged wooden beach support frame; beach inclinations vary from 1:8 to 1:12 were used in most cases discussed in this thesis. Generally only the lower half of the beach is submerged. Graphs presented in the Beach Erosion Board Technical Memo No. 11: Reflection of Solitary Waves, November, 1949, indicate that the absorption efficiency of the beach is a function of the beach slope and the hole size and spacing. Figure 10 shows some of the turbulence induced by the beach and the absorber screens.

Any reflections from the permeable beach are required to pass through the absorber screens and the vane array before they re-enter the main portion of the tank, thereby causing additional disruption and damping of the reflected waves.

Recorder:

The output of all electronic measuring equipment used for test measurements in the wavetank was recorded with a Honeywell Model 1506 Visicorder Oscillograph. The galvanometers in the oscillograph are Honeywell Series "M" subminiature optical galvanometers (type M40-350A) with a nominal undamped natural frequency of 40 cycles per second and a required external damping resistance of 350 ohms. The current sensitivity (+ 5%) is 4.10 microamps per inch of galvanometer deflection. The records of
galvanometer deflections are preserved on photosensitive paper which was driven past the galvanometer light beams at a constant speed, resulting in traces being left on the paper where it was exposed to the light beams.

Wave Gauge:

A special electronic instrument constructed by a local manufacturer was used to measure the water surface profile as a function of time. This wave gauge operates by measuring (by means of a bridge circuit operating out of balance) the variations in electrical current flowing through the water between two parallel platinum wires immersed vertically in the wave-tank. A constant electrical potential is applied between the two wires; as the water level changes, the area and, therefore, the resistance of the conducting path also changes. Figure 11 shows the circuit diagram for the wave gauge. A 15 inch scale, rack and pinion drive, and a friction lock are incorporated in the mount for the probe holding the two platinum wires. (See Figure 12) A hook gauge is located on the side of the probe to ensure that the zero point immersion of the gauge will always be the same.

Test Frame and Force Measuring Unit:

The frame which supports the test pile, wave gauge probe, and the equipment for measuring the loads applied to the pile
Figure 12. The wave gauge probe and its mount
was built of light gauge 6.0 inch high-strength steel I-beams. The frame is a 30.0" x 110.5"
rectangle with two cross-braces located with their centerlines 6.5 inches to either side of mid-
span of the frame. (See Figure 13.)

The test frame is supported on the heads of the rails provided along the sides of the tank for tow-testing. The elevation of each corner of the test frame can be varied by means of leveling screws located there. The leveling screws are the actual supporting members of the frame. The frame is held in position on the rails mainly by its dead weight, but the lateral positioning screws which are used to center the frame over the tank also provide clamping action when tightened against the web of the rails.

It was necessary for the purposes of this investigation to measure the forces acting on the test pile in the direction of wave propagation and the resultant moments due to those forces. To accomplish that end, the following system was used. The pile was hung by means of two hinges in series from an upright structural member vertically fixed to the test frame. (See Figures 14 and 15.) The two hinges in series, the equivalent of a roller support, comprise the upper hinge unit of the force measuring device. The actual hinges are 0.020 inch thick, 2.00 inch wide, and 0.50 inch long beams of thin stainless steel shim stock sheeting. The upper hinges do, of course, have some finite stiffness, but this was taken into account by calibrating the
Figure 13

Plan View of the Test Frame
Figure 14. View of the top hinge in position on test frame
force measuring unit. The upper hinge provides some small torsional stiffness for the pile and effectively resists any tendency for motion of the pile in the plane transverse to the axis of the wavetank.

The principal load upon the hinges in the top hinge unit is axial tension because the hinges are required to support the dead load weight of the pile unit; thus buckling of the thin stainless steel sheeting in the upper hinges due to vertical wave force components acting on the pile is not a possibility.

A pair of load cells are located horizontally in the plane of the tank axis 4.00 feet apart on the vertical upright supporting the pile. Both load cells are Statham Instruments temperature-compensated, solid-state load cells; the top load cell has a capacity of ± 10 pounds axial load, and the bottom load cell has a capacity of ± 20 pounds axial load. The load cells are connected to the pile by lateral hinges which are stiff only in an axial direction. (See Figures 15 and 16.) The lateral hinges consist of two 0.041 inch diameter music wire columns 0.05 inch long separated by a rigid intermediate section 1.00 inches long. The hinges are able to withstand without buckling small axial misalignments in addition to the maximum allowable 20 pound axial compressive loads applied to the load cells.
Figure 15
Top Hinge and Lateral Hinge Construction

Top Hinge

Lateral Hinge
Figure 16. View of the top load cell and lateral hinge in position.

Figure 17. View of the torsion-resisting hinges in position.
If the pile is assumed to be supported from its upper end on rollers (i.e., if the top hinge unit has zero stiffness), then the only restraint on the pile in the vertical plane containing the tank axis is due to forces applied axially through the load cells. Thus the net horizontal force on the test pile in the direction of the tank axis is the algebraic sum of the forces measured by the two load cells. The magnitude of the resultant moment vector acting horizontally and transverse to the axis of the tank can then be obtained by statics. The actual behavior of the force measuring system closely approaches the idealized case described above.

The torsional rigidity of the pile is provided primarily by two hinges similar to the lateral hinges used to join the pile to the load cells. (See Figure 17.) These torsion resisting hinges are mounted horizontally transverse to the tank axis and fixed to a cross-member of the test frame. The torsion resisting hinges function as a pair of rollers (with some resistance due to beam action in the wire columns of the hinges) to constrain the motion of the test pile to the vertical plane containing the tank axis.

Figure 18 shows the load cell circuit diagram. The circuit employs essentially the same components as recommended by the load cell manufacturer, but no calibrating sub-circuit is used. The input voltage is provided by two 1.35 volt mercury
LOAD CELL CIRCUIT
batteries in series. Mercury batteries are utilized because of their very constant electrical potential during use. The output of the separate load cells is recorded by means of the Honeywell Model 1508 Visicorder previously described. The galvanometer type is the same as for the wave gauge: Honeywell type M40-35QA.

The load cells are quite linear and respond rapidly; calibrations for the separate cells show the cell repeatability is excellent.

The vertical upright support for the pile and the pile itself were both plumbed before installation in order to eliminate any preloading of the strain gauges.

The force measuring unit was quite stiff; the natural period of vibration of the unit in air was approximately 0.08 seconds.

Test Pile:

The pile itself is a smooth-surfaced aluminum pipe of 7.00 inch outer diameter and 0.0625 inch wall thickness. The lower end of the pile is capped with a flat, smooth plate perpendicular to the pile axis. Round-head machine screws attach this cap plate to the pile, and the screw heads protrude out from the body of the pile 0.08 inch. The effect of the protruding screw heads on the force measurements was quite small, since they are near the bottom of the pile and also near the bottom of the tank, where the deep water waves generated for the tests did not cause
appreciable water particle motions. There are some small leaks around the cap so that the pile gradually fills with water, but the leaks are too small to permit variations in the amount of water contained inside the pile during wave generation.

The length of the portion of the pile of 7.00 inch diameter is 5.942 feet. Above the 7.00 inch diameter section of the pile is a 0.50 inch aluminum plate to which a 4.0" x 4.0" x 53.0" piece of square steel structural tubing is attached by means of clip angles. This 4.0" x 4.0" steel tubing serves as an axial extension of the pile. (See Figure 19.)

The whole pile is supported from its top by the top hinge unit. The lateral hinge for the upper load cell is attached to the pile 4.00 inches below the top of the pile. The lateral hinge for the lower load cell is located 4.00 feet below the top load cell, i.e., 1.00 inch above the 0.50 inch plate at the transition in pile section. Only the 7.00 inch aluminum pipe portion of the pile is immersed in the water for tests.

Figure 20 shows a general view of the test frame. Figure 21 shows a view of the tank looking from the wave generator towards the test frame and turning vane array.
Figure 19

Side View of the Force-Measuring Unit

\( \frac{1}{8} \) Scale
Figure 20. Photo view of the test frame and pile in position
Figure 21. View down the length of the wavetank from the flap
THEORY

The accurate prediction of wave forces on rigid pilings requires a knowledge of the water particle kinematic behavior in the undulating fluid surrounding the piling.

The determination of water particle behavior during wave motion may be approached in two ways: by actual measurements of the particle motions, or by solution of the wave equation with its associated boundary conditions. The latter approach for the two dimensional case was used for the purposes of this investigation.

The solution of the water wave boundary value problem requires that certain parameters and functions be known: the water depth, density, the acceleration of gravity, and the surface profile of the water at a point as a function of time. The wave period is assumed deducible from the water surface profile. The wave equation is, of course, assumed valid.

There are several suitable references in which various solutions of the water wave equation are given; Wiegel (3), Kinsman (4), and Stoker (5) are just a few of many. The two solutions utilized in this thesis are due to Airy and Stokes. The Stokes wave solution may be extended to different orders; however, only the second order Stokes wave solution will be used in this investigation.

The method of wave force prediction described by Morison, O'Brien, Johnson, and Schaff (6) is based upon the assumption
Figure 22  The Coordinate System (at time t=0)

Positive Force in direction of wave propagation

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that the force exerted on an object immersed in a fluid through which surface waves are propagating is composed of two additive components: a drag force, and an inertial force.

The drag force is due to two separate components: form drag, and skin drag, but the two effects are combined in this treatment.

The drag force in a given direction is equal to the dynamic pressure of the oscillating flow (due to the component of the flow velocity in the given direction), multiplied by both the frontal area of the body and by an empirically obtained constant called the drag coefficient. Thus the drag force on a pile may be expressed as:

\[ F_D = \int C_D \cdot \rho \frac{u \cdot |u|}{2} \cdot dA_f \]  

where \( F_D \) = drag force, \( A_f \) = frontal area of the pile, \( C_D \) = drag coefficient, \( \rho \) = mass density of the fluid around the pile, and \( u \) = component of the fluid velocity in the direction of \( F_D \). The magnitude of the drag force is seen to be dependent on the square of the horizontal water particle velocity. The direction of the drag force vector is the same as that of the horizontal water particle velocity vector.

Henceforth, only horizontal forces will be considered. The pile will be assumed to be vertical and to penetrate the water surface. The pile is taken as a circular cylinder of constant
The drag coefficient is a function of body shape, Reynolds number, body roughness, and fluid turbulence. An experimentally derived plot of drag coefficient, \( C_D \), versus Reynolds number, \( R \), for the case of steady flow around a circular cylinder is given in Schlichting (7). The Reynolds number is taken with the pile diameter as the reference dimension; thus:

\[
R = \frac{u \cdot D}{\nu},
\]

where \( D \) = outer diameter of the pile and \( \nu \) = kinematic viscosity of the fluid around the pile.

If the flow around the pile were steady, a numerical integration of the drag force increments down the length of the pile would be possible. However, since the pile is in an oscillating flow, the relation of the drag coefficient to the Reynolds number is unknown. Therefore this investigation will deal only with an average drag coefficient for the total immersed length of the pile. The average drag coefficient is obtained by means of a modification of equation (1):

\[
\overline{C_D} = \frac{2 \cdot F_D}{\rho \cdot u \cdot |u| \cdot d A_p},
\]

where \( F_D \) in this case is the measured drag force at either a crest or a trough of a wave and \( \overline{C_D} = \) the average drag coefficient.
If there is any general similarity between the relationship of the drag coefficient to Reynolds number for the case of steady flow and the $C_D - R$ relationship for unsteady flow, then it is to be expected that $C_D$ will not remain constant throughout an entire wave cycle. For the purposes of this investigation, it will be assumed that the $C_D$ measured for a given wave crest or trough remains constant in the vicinity of the point at which it was evaluated. This will later be shown to be only a small source of possible error in predicting the maximum total force on the pile. The magnitude of the drag force on the pile is found to be quite small in relation to the magnitude of the inertial force.

The inertial force on a body immersed in an undulating fluid is due to the fluid particle accelerations around the body. The inertial force in a given direction is equal to the fluid particle acceleration in that direction in the undisturbed flow field, multiplied by both the mass displaced by the body and a mass coefficient. The mass coefficient takes into account the fact that the virtual mass encountered by the accelerating fluid is greater than the fluid mass displacement of the column.

Thus the expression for the inertial wave force acting on the pile may be taken as:

$$ F_I = \int C_M \cdot A \cdot \rho \cdot a \cdot d\ell \ , \quad (h) $$

where $F_I =$ the inertial force, $C_M =$ the mass coefficient, $\rho =$
mass density of the fluid around the pile, \( A = \) cross-sectional area of the pile, \( a = \) undisturbed water particle acceleration, and \( l = \) the immersed length of the pile.

It is assumed that the mass coefficient of the pile is constant down the length of the pile. Theoretically it has been shown that the mass coefficient is equal to 2 for infinitely long circular columns. (1), (8).

The measured value of the average mass coefficient for the immersed portion of the pile may be determined when the drag force is zero (i.e., at the wave nodes; assuming the wave behaves according to the Airy solution, or, in general, when the horizontal component of the water particle velocity is zero) if the water particle accelerations are known. Thus:

\[
C_M = \frac{F_I}{\int A \cdot \rho \cdot a \cdot dl},
\]

(5)

where \( C_M \) = the average mass coefficient for the immersed length of the pile. According to theory, the mass coefficient is not dependent upon the Reynolds number of the fluid flow around the pile.

Let a "large pile" be considered to be a pile which has a diameter in excess of \( 20\% \) of the wave height for a given wave train. A pile may be "large" for a normal sea state, but for its design wave, the same pile may be "small".

Beckmann (9) has shown that for an Airy or sinusoidal wave
in deep water and $C_D = 0.67$, the maximum wave force on a pile with diameter in excess of approximately 20% of the wave height occurs at the wave nodes. This implies that the maximum wave force acting on a large pile will be an inertial force. If $C_D = 1.20$, then Beckmann's criterion requires that a pile have a diameter in excess of 38% of the wave height in order to be considered "large". The test pile was exposed to waves whose heights were such that the pile was "large" in all cases.

The formulas for the drag force and inertial force exerted on a pile by a wavetrain will be developed from the following water wave equation solutions: (1) Airy wave, deep water approximation, and (2) second-order Stokes wave, deep water approximation.

The use of the deep water approximations for water particle behavior simplifies the calculations and does not significantly affect the accuracy of the predictions, since the water depths were at least 50% of the wavelengths for the tests described herein. Considering the magnitudes of the irregularities in the wave profiles, it seems certain that the error introduced in the computations due to the use of the deep water approximations will be an order of magnitude smaller than the other errors in the analysis.

For both the Airy wave solutions and second-order Stokes wave solutions, it will be assumed that the waves propagate in
the tank with their deep water phase speed, $c_o$. The speed of wave propagation is given by:

$$ c = \frac{g \cdot T}{2\pi} \tanh \left( \frac{2\pi d}{L} \right) = \frac{L}{T} \quad , \quad (6) $$

and

$$ c_o = \frac{g \cdot T}{2\pi} \quad , \quad (7) $$

where $c$ = wave phase speed in any water depth, $d$, $c_o$ = deep water wave phase speed, $L$ = wave length, and $T$ = wave period. For a water depth $d = 0.5 \cdot L$, this deep water assumption for $c$ is approximately $0.4\%$ too high. (See Wiegel, p. 522)

**Airy Wave, Deep Water Approximation:**

The Airy wave solution is a linear, sinusoidal wave which is assumed irrotational and of infinitesimal amplitude. The water particle motions describe closed elliptical paths in the vertical plane extending in the direction of wave propagation. The size of the ellipses decreases approximately exponentially with water depth for waves in deep water. In addition, the eccentricity of the ellipses decreases with increasing water depth.

Therefore, in deep water, the water particle orbits are essentially closed circles with magnitudes which decrease exponentially with water depth below the surface.

The water surface profile is assumed to be given by:

$$ h = \frac{H}{2} \cdot \cos 2\pi \left( \frac{k}{L} - \frac{t}{T} \right) \quad , \quad (8) $$
where \( H \) = double amplitude wave height, \( T \) = wave period, \( x \) = distance from wave crest to centerline of the pile when \( t = 0 \) (positive in direction of wave propagation), \( t \) = time (beginning at \( t = 0 \) and \( x = 0 \) when crest passes the centerline of the pile), and \( h \) = instantaneous water surface elevation above the still water level.

The horizontal water particle velocity, \( u \), for an Airy wave in deep water is:

\[
u = \frac{2\pi y}{T} e^{\frac{2\pi y}{L}} \cos \left( \frac{x}{L} - \frac{t}{T} \right), \tag{9}\]

where \( y \) = water depth below the still water surface.

The effects of large water particle orbit diameters with respect to wave lengths are neglected in the case of horizontal water particle velocities, so that the water particle motions at the location of the pile centerline are assumed all in phase. Substituting from equation (9) into the equation for the drag force, equation (1), the following result is obtained after integration of equation (1):

\[
F_D = C_D \left[ \frac{H^2 \cdot D \cdot \gamma}{16} \cos \left( \frac{2\pi t}{T} \right) \cdot \left| \cos \left( \frac{2\pi t}{T} \right) \right| \cdot e^{\frac{h_{\text{sg}}}{{\frac{L}{T}}}} \right]_{y=0}^{y=-\frac{L_p}{T}} \tag{10}
\]

where \( \gamma \) = weight density of water and \( L_p \) = immersed length of the pile below the still water level. The effect of variation in water level due to passage of the wavetrain is neglected.
With the same assumptions about water particle orbit sizes being small with respect to the wavelength as for the drag force calculations, it is possible to predict the inertial force on the pile. With \( a = \frac{D(u)}{D(t)} = \) the total derivative of the horizontal water particle velocity with respect to time, it can be said that:

\[
a = \frac{2\pi^2 H}{T^3} \cdot \sin 2\pi \left( \frac{X}{L} - \frac{t}{T} \right) \cdot e^{\frac{2\pi y}{L}}.
\] (11)

Substituting equation (11) into equation (4) and performing the required integration, equation (12) is obtained:

\[
P_I = - \frac{C_M \gamma \cdot A \cdot H}{2} \cdot \sin \left( \frac{2\pi t}{T} \right) \cdot e^{\frac{2\pi y}{L}} \bigg|_{y=0}^{y=-\frac{h}{h'}}.
\] (12)

where \( h \) is found from equation (9) and \( A \) = cross-sectional area of the pile. Again, the effect of variation in water surface elevation is neglected. This introduces no error into the computation of the mass coefficient, since \( h = 0 \) at the quarter points of an Airy wave.

It is desirable to know where the predicted resultant inertial force will act during the instant when the wave node passes the centerline of the pile, as this is when the Airy wave solution predicts the maximum wave force. This predicted location of the resultant inertial force may be compared with the measured location of the resultant. The location of the predicted inertial force:

\[-44-\]
tial force with respect to the instantaneous water surface is found by dividing the analytically predicted moment on the pile about the water surface by the predicted force on the pile. The predicted moment is obtained from:

\[ M_I = \int C_M \cdot \rho \cdot A \cdot a \cdot y \cdot dl, \quad (13) \]

where \( M_I \) = the predicted moment due to the inertial force.

With substitution from equation (11), it is found that:

\[ M_I = \frac{-C_M \cdot \gamma \cdot A \cdot H \cdot L}{4} \cdot \sin \left( \frac{2\pi t}{T} \right) \cdot \left[ e^{\frac{2\pi y}{L}} \cdot \left( \frac{2\pi y}{L} - 1 \right) \right] \bigg|_{y=0}^{y=-\frac{L}{p}}. \quad (14) \]

Thus \( d_{RA} \), the location of the resultant Airy inertial force with respect to the free water surface is obtained by dividing equation (14) by equation (12) to get:

\[ d_{RA} = \frac{L}{2\pi} \left[ \frac{K \cdot p \cdot e^{-K \cdot p \cdot L}}{1 - e^{-K \cdot p \cdot L}} - 1 \right] \cdot (K = \text{wave number.}) \quad (15) \]

Although the effects of varying pile length exposed to wave action are neglected in equations (10), (12), and (14), it is necessary to take the instantaneous wave height, \( h \), into account when considering the location of the measured resultant wave force with respect to the water surface. If the upper hinge system on the pile support is assumed frictionless, then the measured value of \( d_{RA} \) at the wave quarter points where \( h = 0 \)
according to Airy theory is:

\[ d_{RM} = \left( \frac{4h'}{F} \right) - d' \]  

where \( F_L \) = the measured force in the lower load cell, \( F \) = the total resultant horizontal force on the pile, and \( d' \) = the distance from the upper load cell hinge to the still water surface.

**Second-Order Stokes Wave, Deep Water Approximation:**

The Stokes wave solutions are valid for irrotational waves of finite amplitude. The water particle motions are unclosed ellipses in the vertical plane extending in the direction of wave propagation. In deep water, the water particle orbits are not circular.

The second-order Stokes wave in deep water exhibits a water surface profile described by the following equation:

\[ h = \frac{H}{2} \cdot \cos 2\pi \left( \frac{x}{L} - \frac{t}{T} \right) + \frac{H^3}{4L} \cdot \cos 4\pi \left( \frac{x}{L} - \frac{t}{T} \right), \]  

where the same symbols are used as for the Airy wave solution. The Stokes wave exhibits longer troughs, shorter crests, and less trough depression below than crest elevation above the still water level. The surface profile exhibited by the second-order Stokes wave describes more nearly the actual surface configuration measured for the test wavetrains than does the Airy wave solution. The wave propagation speed is the same for the second-order Stokes wave as for the Airy wave. (See equations
The horizontal water particle velocity is given by:

\[
u = \left(\frac{\pi H}{T}\right) \cdot e^{K \cdot y} \cdot \cos (K \cdot x - \sigma \cdot t)
+ \frac{3\pi K \cdot H^2}{T} \cdot e^{2K \cdot (y-\delta)} \cdot \cos 2(K \cdot x - \sigma \cdot t), \quad (18)
\]

where \(K = 2\pi/L\) = the wave number, \(d = \) water depth measured from still water level, and \(\sigma = 2\pi/T = \) the wave frequency. However, since the tests were conducted in a closed tank, any mass transport in the direction of the wave propagation due to the wave action was necessarily counteracted by an equal, but opposite, mass transport distributed across the entire tank width and depth in a presumably uniform manner. This mass transport velocity is neglected in this analysis, since the contribution of the reverse mass transport velocity in the fluid to the drag force is rather small for the cases experimentally investigated. The error due to neglecting this mass counterflow effect is further diminished in importance when compared to the total force on the pile.

As for the Airy wave, an average drag coefficient is assumed for the entire length of the pile. The water particle motions at the centerline of the pile are considered to all be in phase.

Substituting equation (18) into equation (1) for drag force and neglecting terms containing \(\delta = \frac{H}{L}\) to order higher than the first (permissible for low wave steepnesses), the following is obtained:
\[ F_D = \frac{\gamma H^2 D}{16} \left[ e^{2K \cdot y} \cdot \cos(\omega t) \cdot \cos(\omega t) + \frac{4nH}{L} \cdot e^{K(3y-2d)} \cdot (\cos(\omega t) \cdot \cos(3\omega t) + |\cos(\omega t)| \cdot \cos(3\omega t)) \right]_{y=h}^{y=-L_p} \]

It is assumed that the water particle accelerations at the pile centerline are all in phase and that the mass coefficient for the pile is constant. The approximation due to assuming \( a = \frac{D(u)}{D(t)} \cdot \frac{\partial u}{\partial y} \) does not induce significant error into the computations.

Investigation of the additional term due to inclusion of \( u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y} \) in the expression for acceleration reveals that the convective acceleration term is more than one order of magnitude smaller (for the cases studied) than the coefficient of \( \sin 2(K \cdot x - \omega t) \), which is itself a small correction term. Thus,

\[
a = \frac{\gamma H}{L} \left[ e^{K \cdot y} \cdot \sin(K \cdot x - \omega t) \cdot \left\{ 1 - \frac{6nH}{L^3} \cdot e^{2K(y-d)} \right\} + \frac{12nH}{L} \cdot e^{K(y-d)} \cdot \sin 2(K \cdot x - \omega t) \right]
\]

is the total fluid particle horizontal acceleration.

From equation (20) and equation (14), the following is obtained after integration:

\[
F_I = \frac{c_m \cdot \gamma \cdot A \cdot H}{2} \left[ \sin(\omega t) \cdot e^{K \cdot y} \cdot K \cdot H \cdot e^{2K(y-d)} \cdot \sin(2\omega t) - \frac{K^2 \cdot H^3}{2} \cdot e^{K(3y-2d)} \cdot \sin(\omega t) \right]_{y=h}^{y=-L_p}
\]
Using Stokes wave solutions for the case of a "large" pile, it is evident that the maximum wave force on the pile occurs with some phase shift from the quarter points of the wave. This is true even if drag forces are neglected. To find the time during a wave cycle when the inertial force is maximal, equation (21) may be differentiated once with respect to time and the derivative set equal to zero. However, use of this approach is both long and involved; therefore, the phase shift is determined from a plot of the predicted Stokes wave inertial force versus time.

The theoretical moment arm for the resultant inertial force on the pile may be obtained in the same manner as for the Airy wave. It is found that, with the assumption that \( \alpha = \frac{\partial u}{\partial t} \), the \( d_{Rs} \) for the Stokes wave at both the quarter points and at the times when the maximum inertial forces are predicted by second-order Stokes theory are almost the same as for the Airy wave. Equation (22) gives the expression for \( d_{Rs} \), the location below the still water surface of the resultant Stokes wave inertial force. The second-order and third-order terms are neglected in obtaining the Stokes wave \( d_{Rs} \), but this produces only a slight error in the computation.

\[
d_{Rs} = \left[ \frac{h \cdot e^{K \cdot h} + \ell \cdot e^{-K \cdot \ell}}{e^{K \cdot h} - e^{-K \cdot \ell}} \right] - \frac{1}{K}.
\]  

(22)
TABLE OF SYMBOLS

A = cross-sectional area of pile
a = undisturbed horizontal fluid particle acceleration in an undulating fluid
\( A_F \) = frontal area of pile
\( c \) = speed of propagation (phase speed) of a wavetrain in any water depth
\( c_o \) = deep-water phase speed of a wavetrain
\( C_M \) = mass coefficient for a cylindrical pile of circular cross-section
\( \overline{C_M} \) = average mass coefficient for the immersed length of the pile
\( C_D \) = drag coefficient for cylindrical pile of circular cross-section
\( \overline{C_D} \) = average mass coefficient for the total immersed length of the pile
\( D \) = pile diameter
\( d' \) = distance from upper load cell to still water surface
\( d_{RA} \) = predicted location of resultant Airy inertial force with respect to the instantaneous water surface
\( d_{RM} \) = measured location of the resultant inertial force with respect to the still water surface
\( d_{RS} \) = predicted location of the resultant Stokes inertial force with respect to the still water surface
\( \delta \) = wave steepness
\( \frac{D(u)}{D(t)} \) = total or substantial derivative with respect to time
\( d \) = still water depth

\( e \) = exponential \( e \) of natural logarithm base

\( F \) = measured resultant horizontal force on pile

\( F_I \) = horizontal inertial force exerted on the pile by the undulating fluid

\( F_D \) = horizontal drag force exerted on pile by the undulating fluid

\( F_L \) = measured force in lower load cell

\( F_U \) = measured force in upper load cell

\( g \) = gravitational constant \((32.16 \text{ ft/sec}^2)\)

\( \gamma \) = weight density of fluid around the pile

\( H \) = double amplitude wave height

\( h \) = instantaneous undisturbed water surface elevation at the position of the pile centerline

\( K = \frac{2\pi}{L} \) = the wave number

\( L \) = the wave length

\( \ell \) = immersed length of the pipe

\( \ell_p \) = immersed length of the pile below still water level

\( R \) = Reynolds number

\( \rho \) = mass density of fluid around the pile

\( \sigma = \text{wave frequency} = \frac{2\pi}{T} \)

\( T \) = wave period

\( t \) = time elapsed since passage of a wave crest by the centerline of the pile

\( u \) = undisturbed horizontal fluid particle velocity component

\( v \) = undisturbed vertical component of fluid particle velocity
v = kinematic viscosity of the fluid around the pile

x = distance along tank axis from wave crest to centerline of pile when t = 0 (positive in direction of wave propagation)

y = distance measured vertically from the still water surface (positive y above still water surface)
TEST PROGRAM

Wavegauge Calibration:

The output signal of the wave gauge varies nonlinearly with the water surface elevation. In addition, the magnitude of the wave gauge output signal is also dependent upon the conductivity of the water between the wires on the probe. The nonlinearity and conductivity effects were taken into account by running static calibrations of the wave gauge both preceding and following every series of tests.

The hook gauge on the side of the wave gauge probe was used to ensure that the immersion of the probe in still water was always the same before initiation of testing.

Static calibrations were performed by changing the probe immersion in known increments, usually 0.50 inch, and then noting the corresponding deflection of the wave gauge galvanometer. Figure 23 gives a typical wave gauge calibration curve.

Force Measuring Unit Calibration:

It was necessary to calibrate the force measuring system in order to take support hinge bending moments into account. The calibrations were static load tests carried out when the wavetank was dry. Both positive and negative loads were applied in the direction of the wavetank axis at various distances below the load cells. The nominal forces in the load
cells \(F_U\) and \(F_L\) were computed from statics by assuming the supporting hinges had zero stiffness. The computed nominal forces were then plotted against the corresponding load cell galvanometer deflections to form calibration curves. Figures 24 and 25 give the calibration curves for the upper (10 lb) and the lower (20 lb) load cells, respectively.

Wave Parameters:

Waves of two different periods were generated for this investigation: (1) approximately 1.0 second waves, and (2) approximately 1.5 second waves. Waves of longer periods were not generated, since, for such waves, the fluid particle motions would deviate excessively from those of the deep water wave assumptions.

For each wave period, several series of test runs were made at different wave amplitudes. In general, \(\delta = \frac{H}{L}\), the wave steepness, was between 0.016 and 0.045; for these steepnesses, the waves more nearly satisfied the Airy and second-order Stokes solutions than if the waves were steeper. The results of a few of the test runs displaying the best waveforms and regularities were selected for analysis in this investigation.

Data Reduction:

Figure 26 shows a tracing of a segment of a typical oscillograph record. The sign convention is indicated on the tracing.
Figure 2.5
Calibration Curve for Load Cell

Calculated Load in Cells (lb.)

Deflection (inches)

Load Cell

-14 -12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12

-1 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1.0 1.2

L.R.R.
5/15/66
Typical Oscillograph Record

actual scale

2.0 sec  1.0 sec  1.0 sec  1.0 sec

zero line  Compression in Cell  Tension in Cell  zero line

Wave Gauge

crest  trough  crest

Lower Load Cell

zero line

Upper Load Cell

zero line

increasing time

-L.R.R.
5-7-60
(See Figure 22 for force sign convention.) Transformation of the load cell and wave gauge readings recorded on the oscillograph record to the actual forces and surface elevations was done by means of a scale and the calibration curves. (See Figures 23-25.)

The measurement of the wave periods was performed according to the arbitrary assumption that the crest of a wave occurs for $t = 0$ and $t = T$. This was consistent with the sign convention of the mathematical wave models shown in Figure 22. Thus the time intervals between neighboring relative maximum water surface elevations at the centerline of the test pile correspond to the wave periods for the one wave portions of the wavetrain between the crests. The time of occurrence of a wave trough was taken to be the time elapsed between the initial crest of a wave and the time when the lowest water surface elevation was obtained during that one wave cycle. The quarter points of the wave cycles were then taken as the quarter points of the wave periods measured according to the preceding assumptions. The wave nodes were assumed to occur when the surface elevation at the pile was that of the still water level.

According to both Airy and Stokes theory, the wave force at the quarter points of a wave is purely an inertial force. The force exerted by a given wave must therefore be measured at
these points in order to evaluate the mass coefficients. In addition, the crest height and trough height must be measured for each wave to be investigated. The crest height, \( H_c \), of a wave is assumed to be the water surface elevation above still water level at \( t = 0 \) for that wave; the trough height, \( H_T \), is the water surface elevation at the trough. The total wave height is the sum of the crest height and the trough height:

\[ H = H_c + H_T. \]

**Computation Procedures:**

Some of the more routine calculations for this investigation were performed by an IBM 1620 computer at Rice University. The programs for the computer computations are included in the appendices at the end of this thesis. The changes in notation and symbols necessitated by the use of the Fortran I programming language are shown in Appendix V.

Appendix I contains a computer program by which the Airy wave and second order Stokes wave water surface profiles are calculated with respect to time for a certain number of discrete points in time during the wave cycle. The calculated heights are given both as the predicted height in inches and as a dimensionless height equal to the computed height divided by the single amplitude height of the wave. The program requires that the wave period, crest height, and trough height be provided as input data. The theoretical wave profiles ob-
tained from the results of the computer program in Appendix I were plotted versus time with the measured wave profiles superposed on the plot in order to ascertain the accuracy of the two mathematical models of the undulating fluid system. (See Figures 27 and 28.)

Appendix II contains a computer program by means of which calculations are made for a table of values of certain Airy wave parameters as a function of wave period and water depth. The parameters obtained are for drag force, inertial force, Reynolds number, and wave number for wave periods varying from 0.900 seconds to 1.580 seconds in 0.005 second increments. Separate tables are compiled for each of the two water depths at which the tests were run. The constant $C_1$ multiplied by both the drag coefficient and the wave height in inches gives the predicted drag force at the crests and trough of the wave. The constant $C_2$ multiplied by the mass coefficient and the wave height in inches gives the predicted inertial force at the quarter points of the wave. The Reynolds number is obtained by multiplying the appropriate coefficient in the table by the wave height in inches. The values for the inertial force coefficients obtained with the computer program were used for the determination of the Airy theory mass coefficients. Linear interpolations for periods not integral multiples of 0.005 second were made between the appropriate values of the parameters in
the tables generated by the computer. No input data are required for the use of the computer program in Appendix II.

The computer program in Appendix III is used to calculate the locations of the resultant inertial wave forces for both wave solutions and the predicted phase shift away from the wave cycle quarter points of the maximum inertial force for the Stokes wave. In addition, the program provides coefficients for the determining of the mass coefficient and maximum inertial force for the Stokes wave. The required input data for the program are the wave period, immersed pile length, and wave height. The term labeled P SHIFT in the computer output is the phase angle in degrees at which the maximum inertial wave force occurs preceding the first wave quarter point. The value of the phase shift is found by calculating the Stokes inertial force on the pile for incrementally varying lead times from the wave quarter point and determining when the force is a maximum. Since the inertial force versus time record is anti-symmetric, the phase shift angle is also the angle by which the maximum wave force lags the wave three-quarter point. The coefficient FFI multiplied by the Stokes mass coefficient gives the predicted value of the maximum wave inertial force. The Stokes wave mass coefficient is obtained by dividing the measured wave force at the wave quarter point and three-quarter point by the coefficient F/CM. The term Hl is the predicted Stokes wave height in inches.
for the maximum inertial wave force, and WL is the wave length. RDSQ is the location of the Stokes resultant inertial force below the still water surface.

In Appendix IV is the computer program which may be used to calculate the theoretical values of Airy and Stokes inertial forces, Airy and Stokes drag forces, and the location of resultant inertial force from Stokes theory for the waves. The calculations are made at intervals of 0.01 T during the wave cycle, beginning at t = 0. The required input data for the computer are the wave period, pile length, and wave height, as well as the Airy and the Stokes measured mass coefficients. C4 is the Airy wave inertial force, C7 is the Airy wave drag force with an assumed drag coefficient of C_D = 1.2, and Q is the dimensionless time, t/T, at which the values are computed. FS is the computed Stokes inertial wave force, RDS is the location of the resultant FS below the still water surface in feet, and FD is the Stokes wave drag force with an assumed C_D = 1.2. C_D = 1.2 is assumed since it is roughly the drag coefficient in steady flow for the same Reynolds number as exhibited by the wave-pile system. (7)
RESULTS

The results of computations made for this investigation are presented in four separate tables, one table for each wave-train considered. The water depth for DATA SETS II and III was 5.528 feet; for DATA SETS VI and VII, the water depth was 6.300 feet.

In the data presented for each wave, the third row \((t = 0.5 \cdot T)\) gives both the measured value of the water surface elevation at the trough of the wave and the time of occurrence of the wave trough. For DATA SET VII, the starting time for waves 1 and 2 was assumed to be at the wave trough, rather than at the crest.

A rough picture of the measured waveforms is provided by the included values of wave height versus time.

Three significant figure precision was estimated for the values of the measured forces, wave heights, and computed mass coefficients given in the tables. Computations were made using four digit values for the data, and results are given in four digit form. The computations of the mass coefficients and resultant force locations were made using the tables created by the computer programs in Appendices II and III. The calculations were performed according to the formulas in the "Theory" section of this thesis.
The mean of the computed Airy wave mass coefficients is $C_{Ma} = 2.03$; the mean of the computed Stokes wave mass coefficients is $C_{Ms} = 2.04$. The root-mean-square deviation from the mean of the Stokes mass coefficient is 0.08.

Figures 27 and 28 show dimensionless plots of the measured wave profile of wave 1, DATA SET II, with superposed plots of the corresponding Airy wave approximation (Figure 27) and the second order Stokes wave approximation (Figure 28). The axis of the Stokes wave profile is shifted upwards slightly to achieve a better approximation to the measured surface profile of the wave. The theoretically predicted profiles were plotted by means of values provided by results of the computer program in Appendix I.

Figure 29 shows a plot of the measured values of wave force and the predicted values of the second order Stokes wave inertial force versus time.
Figure 27  DIMENSIONLESS PLOT
WAVE PROFILE
DATA Set II, Wave #1

AMPLITUDE vs TIME
Measured Profile
& Airy Approximation

$T = 0.995$ seconds
$H = 2.498$ inches
Water Depth $d = 5.522$ feet

$L.R.R.$
4-20-66
### DATA SET II

Water depth $d = 5.522$ ft

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<th>NO. OF WAVE, T(SEC), H(IN)</th>
<th>TIME FROM ZERO (SEC)</th>
<th>TIME/T</th>
<th>MEASURED FORCE (LB)</th>
<th>MEASURED SURFACE ELEV. (IN)</th>
<th>AIRY MASS COEFF.</th>
<th>STOKES MASS COEFF.</th>
<th>MEASURED RESULTANT LOCATION/L</th>
<th>AIRY RESULTANT LOCATION/L</th>
<th>STOKES RESULTANT LOCATION/L</th>
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<td>TIME/T</td>
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<td>STOKES MASS COEFF.</td>
<td>MEASURED RESULTANT LOCATION/L</td>
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Water depth \( d = 5.522 \) ft
**DATA SET VI**

Water depth d = 6.300 ft

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DISCUSSION

Equipment Characteristics:

The performance of the wave maker was satisfactory for the series of tests performed for this investigation. There were slight variations in the waveform, symmetry, wave heights, and periods in any given set of consecutive waves in a wavetrain. For these reasons, the waves generated did not conform precisely to the relatively accurate Stokes mathematical model of the waves. However, these variations were not judged to be of sufficient magnitude as to impair the reliability of the mathematical model of the undulating fluid. These variations were in part due to both performance characteristics of the wave-maker-wavetank system and the inaccuracy of the mathematical model of the waves.

Lag times between input commands and response of the hydraulic piston and flap were relatively small because of the high pump delivery rate. The time lags were dependent upon the magnitude of the input shaft rotation with respect to the neutral position of the control valve; i.e., the amount of fluid which had to be displaced by the pump to rotate the valve to its new equilibrium position. Thus the time lag for the response of the piston was greatest near the nodal points of the wave where the scotch crank was required to move fastest. At other times, near the wave crests or troughs, the response of the hydraulic piston was rapid, so that the control valve closed suddenly,
thereby causing slight impulsive motions or jerks of the flap. These jerky motions produced small amplitude, sharp-crested waves of periods much shorter than the period of the primary wave motion. The waves produced by the jerks propagated at a rate slower than that of the primary wave motion, since the speed of wave propagation is directly proportional to the wave period (Airy and second order Stokes theory). Test measurements were always discontinued before the short waves produced by the jerky flap motions reached the test cylinder.

When the operation of the wavemaker was initiated, it was observed on the wave record that very low waves of long period reached the pile before the apparent front of the wavetrain consisting of waves with the primary frequency of oscillation, since the longer waves propagated faster. The apparent periods of these initial long waves reaching the wave gauge probe decreased as the primary wavefront approached the probe. After the primary wavefront reached the probe, the presence of the low waves was not directly observable in the wave record.

The forces exerted on the flap by the water column behind it were superposed upon the forces exerted by the water in the main portion of the tank. Since the hydraulic piston was apparently working rather close to its load capacity with the 800 psig pump operating pressure, the flap response to input commands was affected to some extent by the forces exerted by
the water column behind the flap.

The blocks installed below the flap did not completely block the passage between the fluid bodies on opposite sides of the flap. (See Figure 4.) For this reason, some flow could have taken place through the gap, thereby producing some irregularities of the waveform.

An additional aberration of the wavemaker was the tendency of the nodal points of the flap motion to drift (i.e., translate) with time. The direction and speed of nodal point drift were observed to be functions of both the period and magnitude of the primary flap oscillation. The drift was observed even when there was no water in the tank. The drifting was basically the result of an internal leak within the control valve. Generally the drift averaged no more than 2% of the piston stroke per stroke. For the series of tests treated in this thesis, the drift was always in the direction of wave propagation and averaged less than 2% of the stroke length per stroke.

Although the wavemaker system did display certain irregular characteristics, the various perturbations of the flap motion did not result in excessive deviations of the wave surface profiles from the desired behavior; that is to say, the wave-trains produced were essentially uniform.

The use of the turning vanes and the "L" shaped tank was highly effective in enhancing the wave damping characteristics of the wavetank. The vortices and other disturbances produced
in the waves incident on the vanes greatly reduced the reflections from the end of the tank. It seems probable that the use of more vanes in the array would increase the effectiveness of the system in dissipating waves. The screens and perforated beach also contributed considerably to the damping capability of the tank. The effectiveness of the wave damping system can probably be increased by "tuneing" the system to the most effective configuration for the frequency of waves being generated.

The heights of the reflected waves were generally reduced considerably from the heights before absorption. No measurements were made of reflected wave heights, but a conservative estimate based upon visual observation is that the damping system decreased the amplitude of reflected waves to 20% of the undamped amplitude.

When the wavetank was calming after cessation of wave generation, the last wave action of any significance to be visible in the tank was the propagation of long, low waves back and forth down the tank. These long waves exhibited wavefronts which were not perpendicular to the wavetank axis, due to the effect produced when the waves passed through the turning vanes. The lower effectiveness of the wave absorption system in the case of long waves indicated that the long waves were present in the wavetank and were propagating back and forth in the tank during testing.
The behavior of the wave gauge was quite repeatable. However, the zero position of the galvanometer recording the wave gauge output (i.e., the wave gauge zero point itself) was prone to drift perhaps 0.02 inches during a 20 minute period. This drift never injected significant error into the wave measurements, however, since the test durations were usually less than 45 seconds long and the galvanometer was rezeroed immediately preceding every test.

It was necessary to assume the dynamic response of the wave gauge was identical to its statical response, since there was no suitable means available for calibrating the instrument dynamically. (Kinsman, pp. 430-433). For the low frequencies of waves measured by the wave gauge, the dynamic response of the electronic equipment was assumed instantaneous.

A small capillary rise of the water surface was present around the two wave gauge sensor wires. When the water surface was rising, this capillary fringe tended to be smaller; conversely, when the water surface was falling, the capillary fringe height was larger. The effect of this appeared to be small near the crests and troughs of the wave, but elsewhere during the wave cycle some error was due to this capillary rise. Visual observations of the capillary effect indicated that the maximum magnitude of the error was definitely less than ± 0.10 inch. Wetting of the wires reduced the capillary effect to a certain extent.
An investigation of the wave gauge records revealed that the capillary fringe resisted moving up the previously unwetted sections of the platinum wires for the first waves from a wave-train reaching the probe. The third wave crest of any appreciable size was almost invariably the highest crest in the whole wavetrain, so the severe capillary fringe drag which occurred on the unwetted wires was unobserved after the passage of the third wave crest.

The performance of the test frame and force-measuring device supporting the test pile was satisfactory for the series of tests performed for this thesis. The response of the system to the calibration static load tests was highly repeatable; variance in response to the static load tests was ascribed to pulley friction in the load applying device. The dynamic response of the force-measuring unit was assumed identical with the static response of the unit, since no suitable means for a dynamic calibration was available. The force-measuring unit was very stiff; when large impulsive loads were applied to the unit in order to determine its natural frequency of vibration in air, the vibrations of the system were undetectible by the unaided eye.

Some wave reflections from the test pile were observed. These reflections were quite small for the waves of low steepness utilized in the tests. In addition, since the reflections from the pile were propagating radially outwards, their height
decreased as their distance from the pile increased. Small waves were also reflected from the probe holding the two sensor wires for the wave gauge. These effects can easily be seen in Figure 30. These reflections from the pile were not abnormal phenomena, but they did affect the wave gauge reading to a slight extent. The waves in Figures 30 through 33 were steeper than those used in the test series, but the periods in all cases shown were about 1.5 seconds.

**Accuracy of Measurements:**

The following errors give the estimated limits upon the precision of the measurements made for this investigation: (1) wave periods were known within $\pm 2\%$, (2) wave heights were known within perhaps $\pm 4\%$ at the wave gauge probe, and (3) wave forces were known within about $\pm 2\%$. The scatter of the wave force measurements was due to round-off error and inaccuracies induced by the use of the calibration curves for the force-measuring unit. Thus the wave force measurement error probably obeyed the normal distribution laws. The scatter of the wave height and period measurements was primarily due to reflections from the test pile and absorber battery. Therefore the wave height and period scatter did not obey the normal distribution laws.

**Waveforms:**

All of the waves examined for this investigation displayed wave profiles that were approximately trochoidal in character.
Figure 30. Wave action around the test pile, rear view. Note the reflections from the test pile.

Figure 31. Closeup of wave action around the test pile at a wave crest, rear view.
Figure 32. Closeup of wave action around the test pile

Figure 33. Closeup of wave action around the test pile
For this reason, the surface was better described by the second-order Stokes theory than by the Airy theory.

The data given for each test wave considered include the initial wave crest height, the trough height and time of occurrence, and the wave heights for $t = 0.25\cdot T$ and $t = 0.75\cdot T$. This information may be used to give some idea of the regularity of the test wave form.

The waves measured for the tests exhibited some slight irregularities of various kinds, but these irregularities were not highly significant in their effect on the experimental accuracy. The apparent periods of consecutive waves in the individual wave-trains considered in this thesis varied by a maximum of 3.7% from each other; in most cases, the variation was 2% or less. The variations in apparent periods could be ascribed to several possible causes, most of which undoubtedly occurred simultaneously.

Irregular flap response to wavemaker control signals probably occurred as a result of the oscillations of the water column behind the flap. This behavior would have resulted in the production of waves of different periods. Such waves would propagate at different rates. For such behavior, the waves would not behave as discrete units; rather, some modification of waveforms would result from interaction between adjacent waves.

The use of apparent wave periods measured from the wave record, rather than some average wave period computed for an entire wave-train, is a controversial point. Little definite infor-
mation was available for rectifying the question of what the wave period should be taken to be. Accordingly, the period was assumed to be the apparent measured period. It was assumed that interactions between adjacent waves with slightly differing periods would not produce intensive modifications in the wave behavior in the distance between the flap and the test pile. The distance the wave propagated before reaching the test pile was approximately 10 wavelengths for the 1 second wave and 5 wavelengths for the 1.5 second wave. The interactions between adjacent waves of slightly different periods could also account for the variations in the wave crest and trough heights measured.

The occurrence of the long waves propagating in the tank concurrent with the primary wavetrain contributed somewhat to shifts in the apparent wave periods measured. In addition, some portion of the primary wavetrain was reflected back into the tank, though at considerably reduced amplitude, thereby contributing further to apparent period variations. Reflections from the test pile also produced aberrations of the water surface from that of the wave if the pile were not present. However, the reflections from the pile should have been rather regular in character, so that they would be expected to occur at approximately the same phase angle with respect to the primary wave motion at all times. The presence of the pile also acted to constrict the flow between the pile and the tank wall; this effect was not taken into account theoretically, but pre-
sumably was quite small, since the wave gauge sensors were located 27.4 inches (nearly 4 pile diameters) laterally from the axis of the test pile. This, again, should have been a consistent effect, occurring with the same phase relationship to the primary motion for all waves of similar periods.

Some small error was induced into the measurement of wave periods by visually locating the extreme of the variations in the water surface level on the wave gauge record. In addition, there was a negligible error produced by the failure of the recorder device to move the photosensitive paper at a constant speed past the galvanometer beams.

Most of the deviant behavior of the test waves appeared to be due to the long waves and other reflected waves superposed on the primary wavetrain. The presence of the long waves seems to offer the most likely explanation of the variances in the measured wave heights and periods. The dissymmetry of the waveforms also appears to be attributable to waves frequencies other than that of the primary wavetrain. The capillary fringe behavior around the wave gauge sensor wires also was responsible for apparent wave dissymmetries and possibly also apparent variations in the measured wave periods.

Unfortunately, a Fourier analysis was not made of the measured wavetrains, so that the periods and amplitudes of the spurious waves were not known. The long waves may have been higher harmonics of the primary wave frequency. The long waves
were propagating in both directions in the tank, but it is unknown whether or not standing waves were produced. The long wave fronts were not normal to the tank axis.

Figure 27 shows a dimensionless plot of one of the test wave profiles versus a cosine or Airy wave profile; Figure 28 shows a dimensionless plot of the same wave profile versus that of a second order Stokes wave. The second order Stokes wave profile fits the wave in Figures 27 and 28 fairly well, particularly during the first half of the wave cycle, when the surface profile actually measured is almost identical to that of the Stokes model. The agreement between the second order Stokes profile and that of the measured profile is not quite as good for the second half of the wave, particularly from \( t = 0.85T \) until \( t = T \).

The superposition of the wave surface profiles was achieved in Figure 28 by shifting the effective still water surface of the second order Stokes wave upward an amount \( 0.035H = 0.087 \) inches. The upward shift was made with the assumption that a long, low amplitude wave with positive height (at the same time as the Stokes wave was passing the pile) was propagating past the pile.

Wave Forces:

The magnitudes of the wave forces acting on the test pile were dependent upon the water particle motions; in particular, the water particle acceleration was important. The water par-
ticle motions were deduced from the water surface profile record, since prediction of the water motions was a boundary value problem. Hence, uncertainty or inaccuracy in water particle motion prediction implied uncertainty in wave force prediction.

The maximum wave forces on the test pile were almost entirely inertial forces, as expected for "large" piles. Figure 29 shows a plot of measured wave force and predicted second-order Stokes wave inertial force versus time. The predicted Airy wave inertial force would be sinusoidal and would display almost the same maximum amplitude as the Stokes wave inertial force. The mass coefficient was the mean of the two determined according to the test procedure outlined in this thesis. The second-order Stokes wave inertial force more nearly approximated the wave force measured than did the Airy inertial force. This was expected, since the Stokes wave profile also more closely matched the measured profile than the Airy wave.

Some phase shift is apparent between the two curves shown in Figure 29. The reason for this phase shift is not known; the wave gauge sensors were definitely located during the tests in a plane transverse to the wavetank axis and containing the axis of the test pile. Perhaps part of the phase shift was due to the spurious long wave fronts not being perpendicular to the wavetank axis, resulting in the wave gauge experiencing a water surface profile record different from that seen by the
test pile. However, this was a small effect and could not have accounted for all of the phase shift.

In addition to the inertial force, some small drag force acted on the test pile near the wave crests and troughs. However, it was impossible to make an accurate estimate of the drag force (and, therefore, also the drag coefficient) because of the phase shift between the actual and the predicted forces.

The behavior of the predicted inertial force and the measured force were, for the wave in Figure 29, basically the same except for the phase shift. The measured force was fairly constant near the wave quarter points for the wave of Figure 29; thus, in spite of the phase shift, only a small error was produced in the calculations of the mass coefficients by either of the two wave theories used.

For the other waves considered for this investigation, the phase shifts of the maximum measured wave forces from the predicted maximum inertial forces were of varying magnitudes, but were such that the measured wave force peaks led the theoretically predicted wave force peaks. The presence of this phase shift was not an important consideration except to the slight extent to which it affected the computation of the mass coefficient.

Most of the scattering in the measured values for the maximum wave forces appeared to be due to irregularities in the wave-train which caused dissymmetric waves and therefore dissymmetric
water particle accelerations. For example, most of the waves in
DATA SET III were distorted from the Stokes profile so that the
troughs occurred prior to the midpoint of the wave. For most
of these waves, the mass coefficient (a fairly accurate indi-
cator of the magnitude of the local relative maximum measured
wave force) was somewhat larger than the mass coefficient ob-
tained for the quarter point following the wave trough. The
likely cause of this, as mentioned before, was the presence of
the long waves superposed on the primary wavetrain.

Other factors which acted to cause variance between the
wave forces measured for the two quarter points of the indi-
vidual waves were differences in wetted pile length at the
quarter points and the presence of turbulence around the pile.
The effect of the former was essentially negligible, while the
effect of the latter is unknown. Another possible effect which
could have contributed to the variance of the two maximum wave
forces measured during the wave cycles was the occurrence of
surface-induced wave forces. (9). The surface-induced forces
are more severe for steep waves; the test waves were not steep.
The surface-induced forces theoretically are zero at the crests,
trough, and quarter points of an Airy wave.

Some assumptions were made in the theoretical treatment of
the water particle motion, but the magnitude of error induced
in the wave force predictions due to these assumptions was small
compared to possible errors from other sources. The wavelength
was taken to be the deep-water wavelength. For the waves with periods of approximately one second, the error due to this assumption was strictly negligible; in the case of the waves with periods of approximately 1.5 seconds, the deep water wavelength was approximately 0.2% longer than the actual wavelength according to both wave theories used for this investigation. This deep-water approximation introduced very small errors into the computation of the water particle accelerations, velocities, and wave surface profiles for the approximately 1.5 second waves. The wavetrains were assumed to have been generated irrotationally and the wave equations were assumed to be valid. The force expected on the unsupported end of the test pile was assumed negligible; this was an insignificant source of error.

The value of the mass coefficient given for a cylinder with a circular cross-section in Lamb (1) is 2. Lamb's value of $C_m = 2$ was obtained by considering one-dimensional pile motion through an infinite non-viscous fluid. Potential flow was used to describe the flow field around the cylinder. The fluid in which the test pile was immersed was viscous, so that there was some boundary layer around the test pile. It is not known whether vortices or other turbulent effects modified the flow patterns around the test pile significantly from the idealized potential flow case. Certainly there was some likelihood of flow modification due to turbulence.
Waves were reflected from the test pile, so that additional forces, unaccounted for in a totally submerged pile, were acting on the pile.

The mean mass coefficient computed for the second order Stokes model of the test waves was 2.04; for the Airy wave model, the mean mass coefficient was 2.03. The root-mean-square of the deviation of the measured mass coefficients for the Stokes wave model was 0.08. The corresponding value obtained for the Airy wave model was virtually the same as for the Stokes wave model. Thus the scatter for the measurements of the mass coefficient was relatively small, indicating that the effects of wave dissymmetries and wave period estimates were not highly deleterious sources of error. Because of the undefined nature of the long waves propagating in the tank, no corrections were made for the presence of the long waves. For this reason, the normal distribution laws do not pertain to the error distribution of the computed mass coefficients. The scatter of the measurements for the mass coefficient was quite small compared to the results obtained by some investigators for small piles in the ocean. (Wiegel, p. 261). However, the ocean is far from being a carefully controlled laboratory.

The location of the resultant maximum predicted inertial wave force and the wave force measured at the same time were
quite close. The measured location of the resultant force was
deep than predicted. For the waves of DATA SET III, the re-
sultant force at the first quarter point of the waves acted
farther below the still water surface than the resultant at
the three-quarter point of the waves. This effect seemingly
was the result of the long waves propagating in the tank.

Since the test waves were deep water waves according to
the criteria that, for \( d/L > 0.5 \), the water is deep, while the
long waves were intermediate water depth waves with \( 0.005 <
\frac{d}{L} < 0.5 \), a slightly different type of force distribution
occurred for the long waves. (Kinsman, p. 132). In addition,
the decay of the long wave fluid particle accelerations with
depth was less rapid than with the primary waves, so that the
resultant inertial force therefore acted farther below the
water surface, for the cases measured, than if only the pri-
mary wave were propagating in the tank. This was the probable
cause for the discrepancy between the predicted location and
the measured location of the resultant inertial force. It
therefore appears reasonable to assume the water particle ac-
celerations for the primary wavetrain were distributed as the
second-order Stokes theory predicted.
CONCLUSIONS

1. The wave force on circular piles with diameters large with respect to incident wave height is primarily an inertial force.

2. The mean mass coefficient obtained from the test series was $C_w = 2.0\hat{4}$. This value was obtained with the use of a relatively accurate second order Stokes solution of the wave equation. The root-mean-square deviation from the mean mass coefficient was 0.08.

3. The resultant force acted on the test pile slightly farther below the free water surface than predicted by theory. This effect is thought to be due to the presence of small perturbations due to long waves in the primary wavetrain.

4. The measured total wave force exhibited magnitudes and timewise behavior similar to the predicted inertial wave force, but the measured total wave force occurred before the predicted time, due to a phase shift between the measured and the predicted force behavior.

5. The "L"-shaped wavetank performed very efficiently in reducing reflections of the waves from the end wall of the test tank.
BIBLIOGRAPHY


APPENDIX I

Calculation of Airy wave and second order Stokes wave profiles

1 FORMAT (3F7.4)
2 FORMAT(34H TIME HA A HS S)
3 FORMAT(6F13.6)
4 READ 1, T, HC, HT
8 PUNCH1, T, HC, HT
9 PUNCH2
P=3.14159
G=32.16
H=(HC+HT)/12.
WN=4.*P*P/G/T/T
D05I=0.40
V=I
Q=V*.05*P
HA=H/2.*COS(Q)
A=COS(Q)
HS=HA+WN*H*H/8.*COS(2.*Q)
S=A+WN*H/4.*COS(2.*Q)
O=T*.025*V
HA=HA*12.
HS=HS*12.
5 PUNCH3, Q, O, HA, A, HS, S
GOT04
6 STOP
7 END
APPENDIX II

Calculation of parameters for Airy wave.

1 FORMAT (F10.4*, F10.8*, F10.6*, F10.0*, F10.6*)

2 FORMAT (49H PERIOD DRAG MASS REYNOLDS WAVE NO.)

3 FORMAT (1H )

4 PUNCH 2

5 PUNCH 3

6 PUNCH 3

P=3.14159
D=7.12
A=P/4.*D*D
G=32.16
DNS=1.934*32.16
DO 10 I=0,1
V=I
PL=-4.75-V*.778
DO 10 J=0,136
Z=J
T=.9+Z*.005
WN=4.*P*P/G/T/T
C1=DNS*D/16.*(1.-EXP(2.*WN*PL))
C1=C1/144.
C2=-DNS*A/2.*(1.-EXP(WN*PL))
C2=C2/12.
R=14700.*/T
WN=WN/12.
10 PUNCH 1, T,C1,C2,R,WN

8 STOP

9 FND
APPENDIX III
Calculation of parameters for second order Stokes wave

1  FORMAT(3F6.3)
2  FORMAT(F6.3,2F7.3,F9.4,F8.5,F8.4,F10.5)
3  FORMAT(3H T,5X,2HPL,6X,1HH,5X,2HRD,7X,23HRD/WL WL P SHIFT)
14 FORMAT(2F11.4,F8.5,F10.7,F9.5)
15 FORMAT(7X,19HFFI RD STOKES HS,6X,JHF/CM,7X,4HRDSQ)
18 FORMAT(1X)

4 READ1*T*PL*H
   H = H/12.
   G = 32.15
   P = 3.14159
   DN = 1.934*G
   A = 49./144.*P/4.*
   WL = G*T*T/(2.*P)
   WN = 4.*P/P/(G*T*T)
   RD = (WN*PL*EXP(-WN*PL)/(1.-EXP(-WN*PL)))-1./WN
   UZ = RD/WL
   VC = 1.*
   DO 40 M = 1, 360
   YR = M-1
   YR = YR*P/600.*
   PS3 = COS(P/2.-YR)
   HS = H/2.*PS3+WN*H*H/8.*{2.*PS3*PS3-1.}
   BG = HS*EXP(WN*HS)+PL*EXP(-WN*PL)
   BH = EXP(2.*WN*HS)-EXP(-2.*WN*PL)
   BI = EXP(WN*HS)-EXP(-WN*PL)
   SN1 = SQRT{1.-PS3*PS3}
   SN2 = 2.*SN1*PS3
   UB = -DN*A*H/2.*
   U1 = BI*SN1
   U7 = 3.*WN*H/EXP(2.*WN*(PL+.772))
   U1 = SN2*U7*BH
   U12 = WN*WN*H*H/2.*EXP(2.*WN*(PL+.772))*SN1
APPENDIX III, continued

U3=U2*(EXP(3.*WN*HS)-EXP(-3.*WN*PL))
FFI=UB*(U1+U-U3)
IF(ABS(FFI)-ARS(VC)) 41*41*40
40 VC=FFI
41 YR=YR*180./P
   RDS=(BG/BI)-1./WN
   HS=-WN*H*H/8.
   DU=EXP(WN*HS)-EXP(-WN*PL)
   DV=EXP(3.*WN*HS)-EXP(-3.*WN*PL)
   FC=UB*(DU-WN*WN*H*H/2*EXP(2.*WN*(PL+772)))*DV
   RDS0 = (HS*EXP(WN*HS)+PL*EXP(-WN*PL))/DU-1./WN
   HS=12.*HS
7 PUNCH 3
8 PUNCH 2, T*PL*H*RD*UZ*WL*YR
16 PUNCH 15
12 PUNCH 14, FFI*RDS*HS*FC*RDSQ
17 PUNCH 18
   GOTO 4
5 STOP
6 END
APPENDIX IV

Calculation of wave inertial force and location of resultant force

1 FORMAT (3F6.3)
3 FORMAT (6F9.6)
6 FORMAT (3X,1HT,8X,1HH,7X,2HPL,7X,2HCM,6X,3HCM1)
9 FORMAT (3X,1H9,8X,2HC4,7X,2HC7,7X,3HRDC,6X,2HF5,6X,3HFD)
12 FORMAT (1X)
28 FORMAT (2F7.4)
10 READ 1*T,PL,H
6 READ 28, CM,CM1

P=3.14159
D=7/12.
A=P/4.*D*D
G=32.16
DN=1.934*G
H=H/12.

11 PUNCH 6
5 PUNCH 3, T,H,PL,CM,CM1

WN=4.*P*P/G/T/T
C=-DN*A*H/2.*(1.-EXP(-WN*PL))
S=2.*P
B=1.2*H*H*D*DN/16.*(1.-EXP(-2.*WN*PL))
DO41=1.101
V=1-1
Q=V*0.01
C4=CM*C*SIN(S*Q)
C7=B*COS(S*Q)*ABS(COS(S*Q))
HI=H*COS(S*Q)/2.
HS=HI+WN*H*H/8.*COS(2.*S*Q)
X=1.2*DN*H*H*D/16.

X1=COS(S*Q)*ABS(COS(S*Q))*(EXP(2.*WN*HS)-EXP(-2.*WN*PL))
X2=WN*H*0.2*EXP(-2.*WN*(PL+7721))*(EXP(3.*WN*HS)-EXP(-3.*WN*PL))
FD=X*(X1+X2)*(COS(S*Q)*ABS(COS(2.*S*Q))+ABS(COS(S*Q))*COS(2.*S*Q))
UB=-DN*A*H/2.
APPENDIX IV, continued

\[ U_1 = \sin(S \cdot Q) \cdot (\exp(WN \cdot HS) - \exp(-WN \cdot PL)) \]

\[ U = 3 \cdot WN \cdot \frac{1}{\exp(2 \cdot WN \cdot (PL + 0.772))} \cdot \sin(2 \cdot S \cdot Q) \]

\[ U_2 = U \cdot (\exp(2 \cdot WN \cdot HS) - \exp(-2 \cdot WN \cdot PL)) \]

\[ U_3 = WN \cdot WN \cdot H \cdot H / 2 \cdot \exp(2 \cdot WN \cdot (PL + 0.772)) \]

\[ U_4 = U_3 \cdot (\exp(3 \cdot WN \cdot HS) - \exp(-3 \cdot WN \cdot PL)) \cdot \sin(S \cdot Q) \]

\[ FS = CM1 \cdot UB \cdot (U_1 + U_2 - U_4) \]

\[ RDS = (HS \cdot \exp(WN \cdot HS) + PL \cdot \exp(-WN \cdot PL)) / U_2 \cdot U_1 / WN \]

14 PUNCH 9
7 PUNCH 3, Q*C4*C7*RDS*FS*FD
4 CONTINUE

15 PUNCH 12
16 PUNCH 12
17 PUNCH 12
GOTO 10
8 STOP
FND
APPENDIX V

Some of the Fortran computer language symbols are redefined for the different computer programs in Appendices I through IV.

<table>
<thead>
<tr>
<th>Appendix</th>
<th>Fortran Symbol</th>
<th>Equivalent Symbol</th>
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<tbody>
<tr>
<td>I</td>
<td>A</td>
<td>2·(h_{\text{airy}}/H)</td>
</tr>
<tr>
<td></td>
<td>HC</td>
<td>measured crest height of wave</td>
</tr>
<tr>
<td></td>
<td>HT</td>
<td>Measured trough height of wave</td>
</tr>
<tr>
<td></td>
<td>WN</td>
<td>K</td>
</tr>
<tr>
<td></td>
<td>O</td>
<td>t</td>
</tr>
<tr>
<td></td>
<td>HA</td>
<td>predicted Airy wave height</td>
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<tr>
<td></td>
<td>HS</td>
<td>predicted Stokes wave height</td>
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<td>S</td>
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</tr>
<tr>
<td></td>
<td>Q</td>
<td>(ct)</td>
</tr>
<tr>
<td>II</td>
<td>D</td>
<td>pile diameter in feet</td>
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<tr>
<td></td>
<td>DNS</td>
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<td></td>
<td>PL</td>
<td>(l_p)</td>
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<td></td>
<td>WN</td>
<td>K</td>
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<tr>
<td></td>
<td>Cl</td>
<td>(F_D/(C_D'H)) for Airy wave</td>
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<tr>
<td></td>
<td>C2</td>
<td>(F_I/(C_M'H)) for Airy wave</td>
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<tr>
<td>III</td>
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<tr>
<td></td>
<td>WL</td>
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<td>Appendix</td>
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<td></td>
<td>RD</td>
<td>(d_{RA})</td>
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<tr>
<td></td>
<td>UZ</td>
<td>(d_{RA}/L)</td>
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<td></td>
<td>HS</td>
<td>(h_{\text{stokes}}) at the wave quarter point</td>
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<tr>
<td></td>
<td>FFTI</td>
<td>maximum (F_{I}/C_M) for Stokes wave</td>
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<tr>
<td></td>
<td>RDS</td>
<td>(d_{Rs}) when (F_I) is maximum</td>
</tr>
<tr>
<td></td>
<td>YR</td>
<td>angle in degrees by which maximum (F_I) leads first quarter point</td>
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<td></td>
<td>RDSQ</td>
<td>(d_{Rs}) at (t = 0.25\cdot T)</td>
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<td>FC</td>
<td>(F_{I}/C_M) for the Stokes wave at the quarter points</td>
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<td>IV</td>
<td>D</td>
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</tr>
<tr>
<td></td>
<td>DN</td>
<td>(\gamma)</td>
</tr>
<tr>
<td></td>
<td>WN</td>
<td>K</td>
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<tr>
<td></td>
<td>PL</td>
<td>(l_p)</td>
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<td></td>
<td>CM</td>
<td>mean of 2 computed mass coefficients for Airy model of wave</td>
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<tr>
<td></td>
<td>CML</td>
<td>mean of 2 computed mass coefficients for Stokes model of wave</td>
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<tr>
<td></td>
<td>Q</td>
<td>(t/T)</td>
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<tr>
<td></td>
<td>C4</td>
<td>(F_I) for Airy wave</td>
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<tr>
<td></td>
<td>C7</td>
<td>(F_D) for Airy wave ((C_D = 1.2))</td>
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<td>Appendix</td>
<td>Fortran Symbol</td>
<td>Equivalent Symbol</td>
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</tr>
<tr>
<td>IV</td>
<td>HI</td>
<td>$h_{airy}$</td>
</tr>
<tr>
<td></td>
<td>HS</td>
<td>$h_{stokes}$</td>
</tr>
<tr>
<td></td>
<td>FD</td>
<td>$F_D$ for Stokes wave ($c_D = 1.2$)</td>
</tr>
<tr>
<td></td>
<td>FS</td>
<td>$F_I$ for Stokes wave</td>
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<tr>
<td>RDS</td>
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<td>$d_{Rs}$</td>
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