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A JUMP-SEARCH PROCEDURE FOR SEQUENTIAL DECODING SYSTEMS

by

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ABSTRACT

A JUMP-SEARCH PROCEDURE FOR SEQUENTIAL DECODING SYSTEMS

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The introduction of the concept of sequential decoding by Wozencraft has opened up the possibility of more efficient decoding search procedures than those available with block code..

The purpose of this study is to try to improve the decoding scheme proposed by Pfeiffer and Lin. Because of the group property of the code, we do not have to search every branch when the tentatively decoding path has violated the threshold function. If the errors occur at 2nd, 3rd branches, we connect the 1st and 4th branches. By jumping over the errors, a correct path can be obtained.

This article describes the use of the jumping method, according to the group property of the code word, in order to minimize the decoding operation.

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CHAPTER I. INTRODUCTION

In recent years, interest in the study of sequential decoding has been increasing. Many of the well known results have been published in papers by Wozencraft, Fano and others.

The process of generating the convolutional tree codes from a generator sequence is described in "Sequential Decoding" by Wozencraft and Reiffen and a paper of Lin and Lyne. The "Threshold Test" is described in " A Sequential Decoding System Utilizing Distance Properties of Convolutional Tree Code" by Pfeiffer and Lin.

In this work, a new sequential type decoding scheme for binary channels has been proposed. The algebraic properties of the codes have been utilized in the decoding scheme. The threshold function is based on the distance property of codes. It seems that this decoding scheme is more efficient than the others.

CHAPTER II. DESCRIPTION OF BASIC SYSTEM

2-1. Convolutional Tree Codes

In this thesis we consider only the binary convolutional tree code with a single-generator sequence. For simplicity, each message digit will encode into $\alpha=3$ channel digits. The code constraint length n corresponds to message sequence of length ν digits. The ratio $R=\nu/n=1/\alpha$ is called the transmission rate. The distance between two sequence is the Hamming distance, which is determined by counting the number of places at which the sequence differ.

As an example, consider two sequences $\underset{\sim}{U}$ and $\underset{\sim}{V}$. Let

$$\underset{\sim}{U} = 001011101$$

$$\underset{\sim}{V} = 101011001$$

$$\underset{\sim}{U} \oplus \underset{\sim}{V} = 100000100$$

where the basic operation $\oplus$ for combining two sequences is modulo-2 addition of corresponding elements in the sequences to be combined.

Let $d(\underset{\sim}{U},\underset{\sim}{V})=|\underset{\sim}{U}\oplus\underset{\sim}{V}|$ be the distance between two sequences $\underset{\sim}{U}$ and $\underset{\sim}{V}$ (in the example above, $d(\underset{\sim}{U},\underset{\sim}{V})=2$)

The process of generating tree convolutional codes from a generator sequence is described by Wozencraft and Reiffen. Here we will give an example:

Let the generator sequence $\mathbf{g}$ = 111,010,001,000,.... for which α =3 and ν =3, so that n=9 and R=1/3.

The tree will be like that shown in Fig.2-1. Let $\mathbf{m}$ be the message sequence; if $\mathbf{m}$ = 1010...., then the encoded sequence is $\mathbf{S}$ = 111,010,110,010,......

Convolutional codes of the kind considered here have some important properties, as described by Lin. Since some of these are utilized in the decoding scheme, we shall simply state them as follows:

1. The code produced is a group code: if two paths of the same length in the tree extending through branches of the same order "i" are combined by the operation, the resulting sequence corresponding to another path in the tree of the same length and extending through branches of order "i".

2. The codes possess a fundamental distance property which provides the error-correcting capability of the code. According to this property, the distance between the upper half and the lower half of any k-unit in the tree depends only upon the length k and does not depend upon the node from which the k-unit stems. Hence we can introduce a distance function d(.) for k-unit which depends only upon k. Thus d(k) is the distance between halves of any k-unit in the code tree.

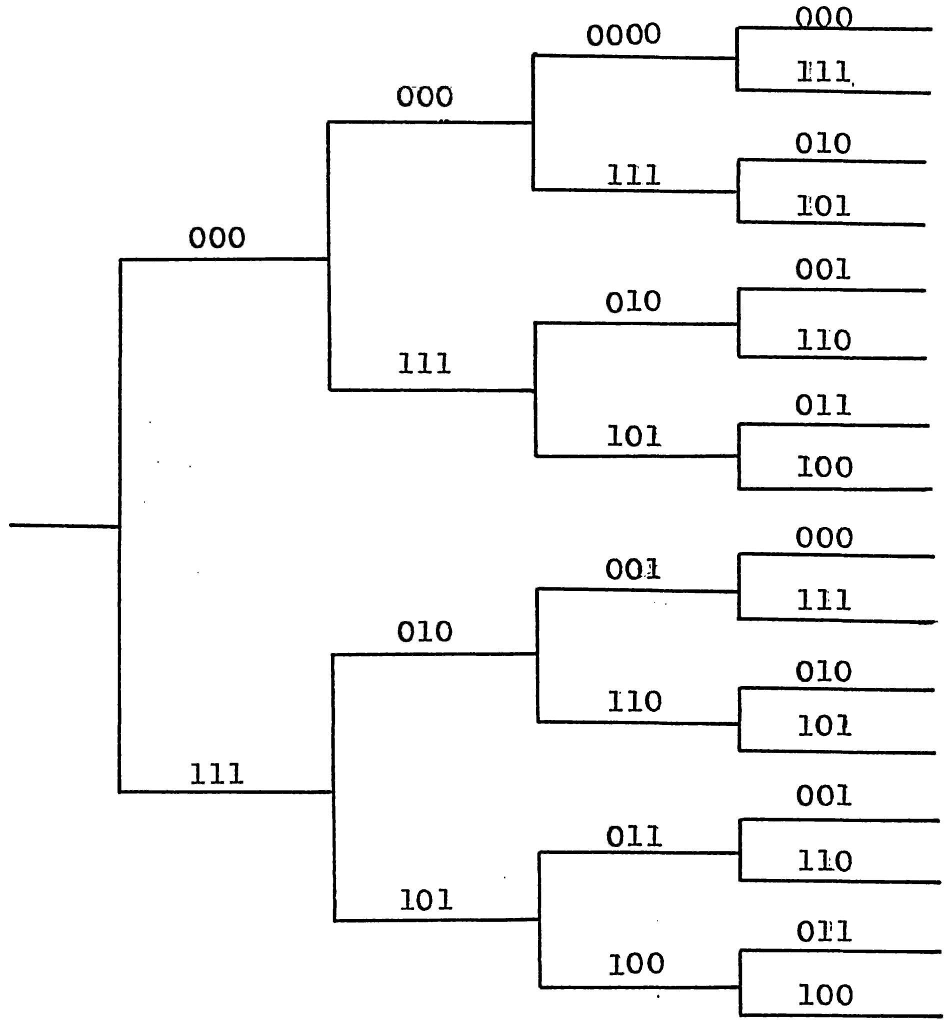


Fig. 2-1 g=111, 010, 001, 000, ------

3. If a code tree is entered at any node, the paths stemming from that node form a tree. If any two sequences of message digits agree in the last ν-1 places, the trees stemming from the nodes corresponding to the last digit in the two cases are identical.

2-2. Minimum Distance Decoding

We shall assume that the decoder stores $n = \alpha\nu$ channel output digits and compares this sequence of digits with possible paths in the code tree. In the process of decoding, a tentative choice of a path means a tentative decoding of the channel output digits in the decoder. If there were no noise in the transmission channel to perturb the transmitted signal, the sequence in the decoder would correspond exactly with a path in the tree, and this path could be identified with certainty. In the presence of noise,however, the received sequence may correspond exactly with no path in the tree. In this case, a choice must be made as to which path is most probable. When a choice has been made as to the most probable path, the decoder shifts out α channel digits and pyints out the corresponding decoded message digit. When perturbations due to noise are infrequent, the search for a path proceeds by a "jumping process", with a minimum number of decoding operations. Distance properties of the code provide very sensitive tests for an error in the choice of path, which amounts to providing tests for an error in decoding.

2-3. The Threshold Test

Once an incorrect choice of a branch is made, the subsequent branches must be incorrect. As successive branches are chosen, the fundamental distance property of the code drives the selected (but incorrect) path away from the correct path. The distance increases with the number of branches beyond the node of separation, that is, beyond the node at which the first incorrect choice is made. For k such branches, the distance between the correct path and the incorrect one must be at least d(k). To see this, we may argue as follows. Let $\underline{U}$ be the correctly transmitted sequence, $\underline{W}$ the tentatively decoded sequence corresponding to the path chosen, and $\underline{V}$ the sequence received at the channel output. We suppose that the first $\mathcal{J}-k$ branches in the decoder are correct and the last k are incorrect. The distance between $\underline{U}$ and $\underline{W}$ will then be due to the distance in the last k branches. In fact, $\underline{U}$ and $\underline{W}$ will lie in opposite (upper and lower) halves of the k-unit stemming from the node of separation. We must therefore have

$$d(\underline{U},\underline{W}) \geq d(k)$$

We may utilize this property to obtain an effective test for detecting the fact of an incorrect choice before the first incorrect branch is shifted out of the decoder.

A fundamental property of distance functions (the triangle inequality) enables us to assert

$$d(\underset{\sim}{U},\underset{\sim}{V}) + d(\underset{\sim}{V},\underset{\sim}{W}) \geq d(\underset{\sim}{U},\underset{\sim}{W})$$

$$d(\underset{\sim}{V},\underset{\sim}{W}) \geq d(\underset{\sim}{U},\underset{\sim}{W}) - d(\underset{\sim}{U},\underset{\sim}{V})$$

Utilizing the separation property derived above, we have the relation

$$d(\underset{\sim}{V},\underset{\sim}{W}) \geq d(k) - d(\underset{\sim}{U},\underset{\sim}{V})$$

The distance between the received sequence $\underset{\sim}{V}$ and the tentatively decoded sequence $\underset{\sim}{W}$ is at least as great as the distance $d(k)$, reduced by the amount of the distance between the received sequence $\underset{\sim}{V}$ and the correct (transmitted) sequence $\underset{\sim}{U}$. Note that $d(k)$ is a known property of the code and $d(\underset{\sim}{V},\underset{\sim}{W})$ is an observable quantity in the decoder.

To simplify the statement of the test, we introduce a threshold function $T(.)$ derived from the distance function $d(..)$ as follows:

Definition: For each integer k, the value $T(k)$ of the threshold function $T(.)$ is that integer satisfying the relationship $T(k) < d(k)/2 \leq T(k) + 1$

If $k = \nu$, we must have

$$d(\underset{\sim}{V},\underset{\sim}{W}) \geq d(\nu) - T(\nu) \quad \text{and} \quad d(\nu) > 2T(\nu)$$

so that

$$d(\underset{\sim}{V},\underset{\sim}{W}) > T(\nu)$$

Then we have

$$d(\mathbf{U},\mathbf{W}) \geq \left| d(\mathbf{V},\mathbf{W}) - d(\mathbf{U},\mathbf{V}) \right|$$

The conditions $d(\mathbf{V},\mathbf{W}) > T(\nu)$ and $d(\mathbf{U},\mathbf{V}) \leq T(\nu)$ thus imply $d(\mathbf{U},\mathbf{W}) > 0$, which means that at least one branch is incorrect.

Suppose the distance between the transmitted sequence $\mathbf{U}$ and received sequence $\mathbf{V}$ at length ν segments is less than the error-correcting capacity $T(\nu)$ of the code, then:

1. If the distance $d(\mathbf{V},\mathbf{W})$ between the received sequence $\mathbf{V}$ and the tentatively decoded sequence $\mathbf{W}$ in the decoder is greater than $T(\nu)$, there must be at least one incorrect segment in $\mathbf{W}$.

2. If the first branch in the decoder is incorrect, then $d(\mathbf{W},\mathbf{V})$ exceeds $T(\nu)$ for each possible sequence $\mathbf{W}$.

This threshold test provides a highly reliable basic test for decoding errors within the error-correcting capabilities of the code. But if the number of channel errors exceeds this level the test may or may not be reliable.

2-4. Some Distance Property Used in Decoding

Convolutional codes of the kind considered here have some important properties, as described in 2-1. Since some of these are utilized in the decoding scheme, we shall simply state them as follows:

1. The code produced is a group code.

2. The codes possess a fundamental distance property which provides the error-correcting capacity of the code.

Let $T(\nu)$ be the error-correcting capacity of the codeword, $\mathbf{U}$ the correctly transmitted sequence, $\mathbf{W}$ the tentatively decoded sequence corresponding to the path chosen, and $\mathbf{V}$ the sequence received at the channel output. Suppose decoding is correct up to the sequence $\mathbf{V}$ in the decoder. To decode correctly the sequence in the decoder, we must determine whether the correct decoded path $\mathbf{W}$ lies in the upper or lower half of the code tree.

<u>Theorem 2-4-1:</u>

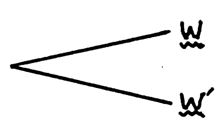
Suppose $d(\mathbf{U},\mathbf{V}) \leq T(\nu)$. If there is a $\mathbf{W}$ which satisfies the following condition

$$|\mathbf{W} \oplus \mathbf{V}| \leq T(\nu)$$

then all $\mathbf{W}'$ belonging to the other half tree must satisfy

$$|\mathbf{W}' \oplus \mathbf{V}| \geq T(\nu) + 1$$

Proof:



$$|\mathbf{t}| = |\mathbf{W} \oplus \mathbf{V}|$$

where $\underset{\sim}{t}$ is the difference between received sequence $\underset{\sim}{V}$ and tentatively decoded sequence $\underset{\sim}{W}$.

$$|\underset{\sim}{t}'| = |\underset{\sim}{W}' \oplus \underset{\sim}{V}|$$

where $\underset{\sim}{t}'$ is the distance between received sequence $\underset{\sim}{V}$ and tentatively decoded sequence $\underset{\sim}{W}'$.

$$|\underset{\sim}{t}| + |\underset{\sim}{t}'| \geq |\underset{\sim}{t} \oplus \underset{\sim}{t}'| = |\underset{\sim}{W} \oplus \underset{\sim}{V} \oplus \underset{\sim}{W}' \oplus \underset{\sim}{V}|$$

$$= |\underset{\sim}{W} \oplus \underset{\sim}{W}'| \geq d(\nu)$$

where ν is the length of the segments of the sequence.

$$|\underset{\sim}{t}| \leq T(\nu)$$

$$2T(\nu) < d(\nu) \leq 2T(\nu) + 2$$

$$2T(\nu) + 1 \leq d(\nu) \leq 2T(\nu) + 2$$

$$|\underset{\sim}{t}'| \geq 2T(\nu) + 1 - T(\nu) = T(\nu) + 1$$

$$|\underset{\sim}{t}'| \geq T(\nu) + 1$$

$$|\underset{\sim}{W}' \oplus \underset{\sim}{V}| \geq T(\nu) + 1$$

It is proved that all $\underset{\sim}{W}'$ belonging to the other half tree must be

$$|\underset{\sim}{W}' \oplus \underset{\sim}{V}| \geq T(\nu) + 1$$

So if there is a $\underset{\sim}{W}$ satisfying the condition

$$|\underset{\sim}{W} \oplus \underset{\sim}{V}| \leq T(\nu)$$

Then we know the first branch of this $\underset{\sim}{W}$ is a correct decoded segment, we can shift out this branch from the decoder.

Theorem 2-4-2:

If $\underset{\sim}{W}$ satisfies the following condition

$$|\underset{\sim}{W} \oplus \underset{\sim}{V}| = T(\nu) - A$$

where A is an integer, then we know the first A/r + 1 branches are correct. We can shift the first $(A/r + 1)$ branches out from the decoder.

where $$r = \frac{d(1)-1}{2} \qquad \text{when } d(1) \text{ is odd}$$

$$r = \frac{d(1)}{2} \qquad \text{when } d(1) \text{ is even}$$

Proof:

In the branch-by-branch search, a distance of at most r per segment is allowed.

If $|\mathbf{W} \oplus \mathbf{V}| = T(\nu) - A$ and A/r segments are shifted out, branch-by-branch choice of the A/r new segments must guarantee $|\mathbf{W} \oplus \mathbf{V}| \leq T(\nu)$. At least one more branch may be selected before the threshold could be violated.

CHAPTER III. ANALYSIS OF THE SEARCH PROCEDURE

3-1. Some Further Properties of the Convolutional Tree Code

In this section, we shall examine the structure of the convolutional tree code and find some properties which will be useful in decoding.

Group Property Let $\underset{\sim}{g}=g(1),g(2),g(3),\ldots.g(\nu)$ be the generator sequence. Consider the transmitting rate $R_t=1/\alpha$ (i.e., each message digit will be encoded into α channel input digits). If the generator segments satisfy $g(i)\neq g(j)$ $i,j\leq\alpha$ and no two segments are complements of each other, the last column of the initial α-unit would contain all 2^{α} possible segments of length α, with no segment repeated. The segments will appear in a particular order depending on the choice of $g(1)$, $g(2),\ldots g(\alpha)$. This set of all possible segments forms a group under addition term by term modulo-2 (operation $\oplus$). When the permutation order is preserved, we refer to the set as the fundamental group.

The last column of any α-unit is a permutation of the fundamental group, obtained by adding a fixed segment to each member of the fundamental group. This means that each possible segment of α digits appears one and only one time in the last column. The last column of any $(\alpha+1)$-unit consists of two groups, each being a permutation of the fundamental group. In general, the last column of any $(\alpha+m)$-unit will consist of 2^m such permutations of the fundamental group. Thus, each possible segment will appear exactly 2^m times in the last column of the $(\alpha+m)$-unit.

To illustrate the group property, consider the case $\alpha=3$ with generator sequence 111,010,001,011,.... The tree will be that shown in Fig.3-1. All possible segments are in the third column and the upper half of the fourth column. The result of adding g(4) to the various elements of the upper half of the fourth column is to give all possible segments (in a different order if g(4) is not all zeroes) in the lower half of the fourth column.

From the development of the tree code shown in Fig.3-1, if any two of the first α segments are equal or are complements of each other, then the condition that the 2^α segments are different will be destroyed.

<u>The Relation Between Intermediate Segments</u> The structure of the convolutional tree code is entirely determined by the ν th column of the initial tree. Let the elements be H(0),H(1),....H(n). The order number n is in the range $0 \leq n \leq 2^\nu - 1$. The last column of the initial tree will be repeated after ν segments from the initial node. The above relation can be shown in Fig.3-2, for $\nu=3$.

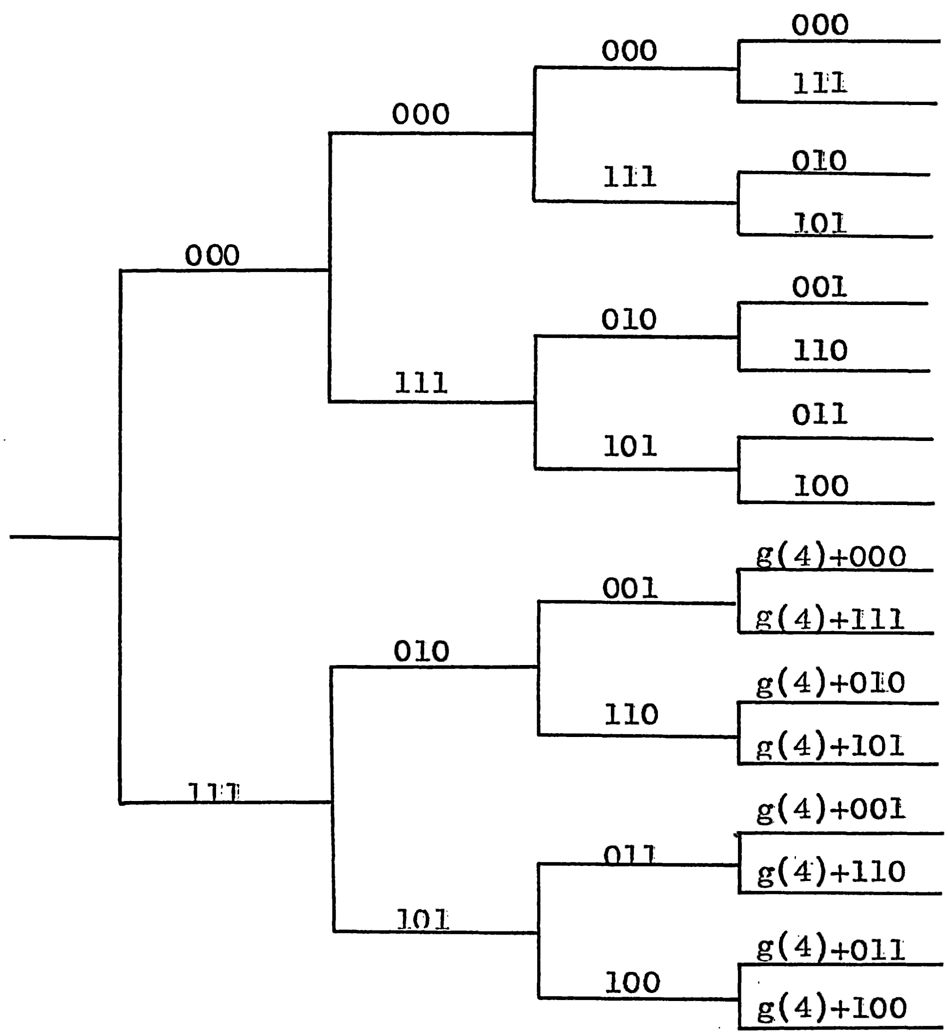


Fig. 3-1 g=111, 010, 001, 011, -------

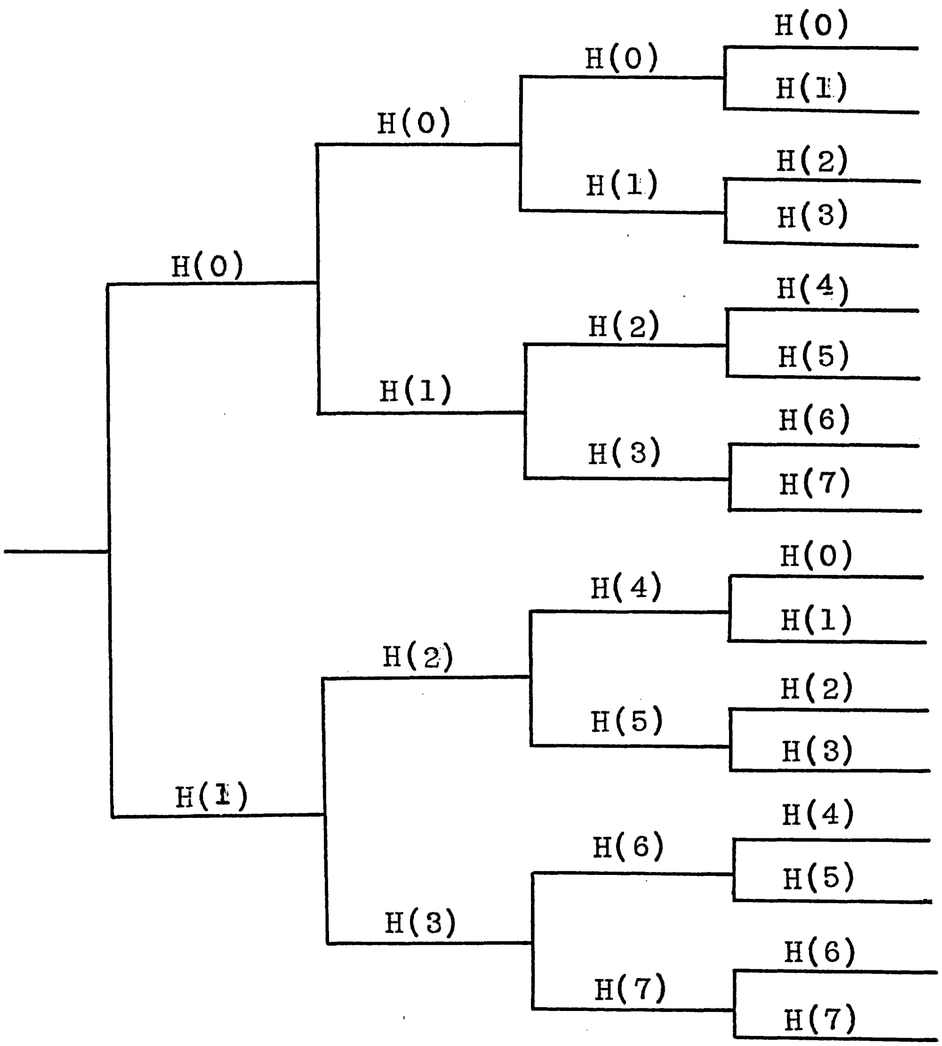


Fig. 3-2 For $\nu = 3$

In the initial tree, if we know that a segment in the Nth column is H(n), where $N \leq \nu$, then the segment preceeding H(n) in the (N-1)th column will be H(n/2) if n is an even number, or $H(\frac{n-1}{2})$ if n is an odd number. Consider an α-unit stemming from a branch H(k). The uppermost element of the last column of the α-unit is H(j), where j is the remainder of $\frac{k.2^{\alpha}}{2^{\nu}}$; i.e., j is $k.2^{\alpha}$ (mod. 2^{ν}). Any other element in the last column of the α-unit is determined by adding H(j) to the proper element in the fundamental group. Thus the last column in the α-unit is determined by the uppermost element and the fundamental group (in the initial order). The relation is shown in Fig.3-3.

If we know a segment in the Nth column is H(j+a) and the uppermost element of that column is H(j), then the element preceeding H(j+a) in the (N-1)th column will be H(l) where l is the remainder of $\frac{k.2^{\alpha}+ a}{2.2^{\nu}}$, when numerator is even, or l is the remainder of $\frac{k.2^{\alpha}+a-1}{2.2^{\nu}}$, when numerator is odd.

The above relations between intermediate segments can be summarized as follows:

1. When the uppermost element in the last column of any α-unit is known, every element is known.

2. When the last column in the α-unit is known, the complete α-unit is known.

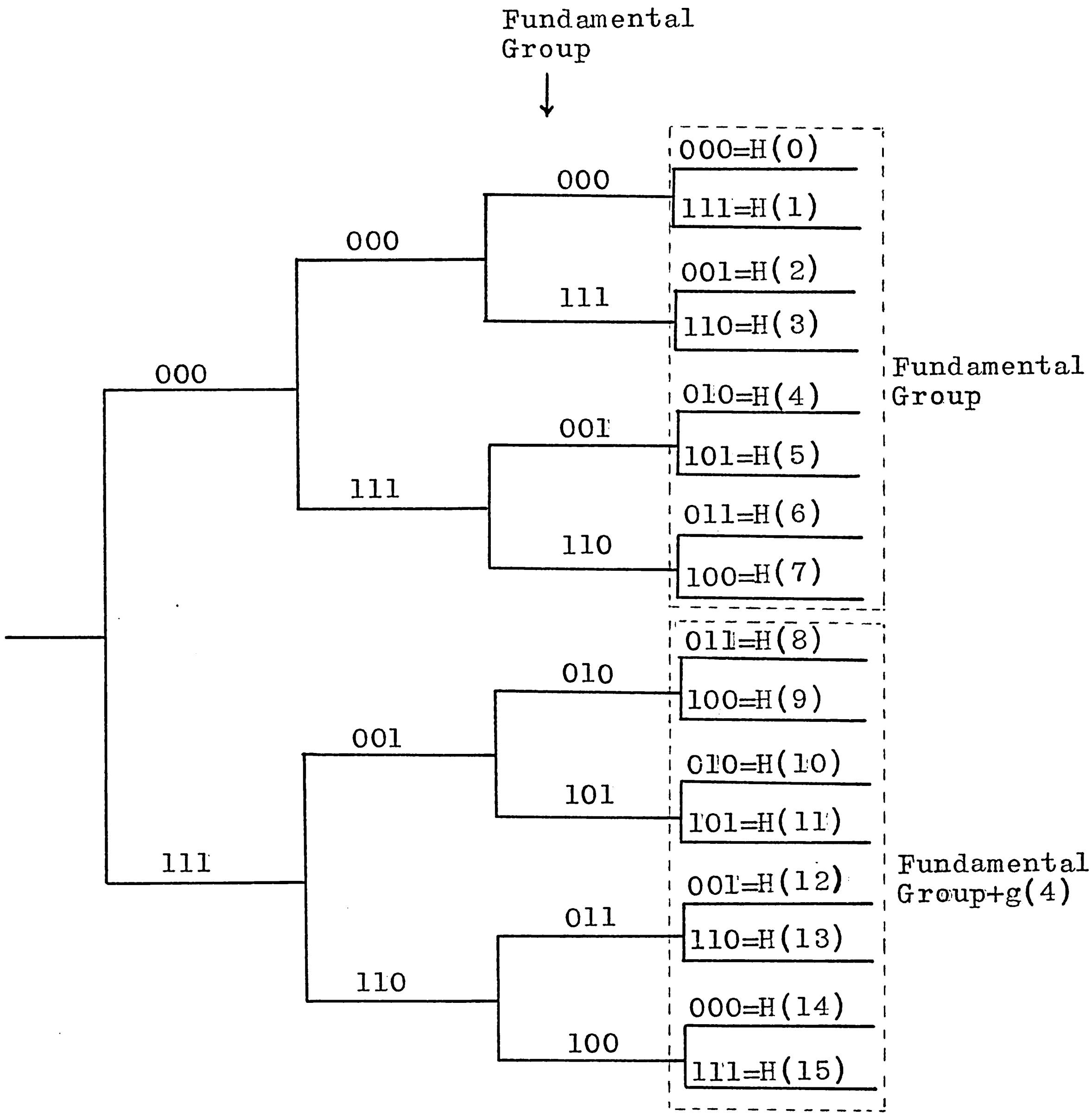


Fig. 3-3 $\alpha=3$, $\nu=4$, $\underset{\sim}{g}$=111, 010, 001, 011,------

3. When the branch from which any α-unit is known, the uppermost element in the last column is determined.

These facts imply that we need to know only branches of order $1, 1+\alpha, 1+2\alpha, \ldots\ldots$ to determine completely any path in the code tree.

3-2. Basic Concepts of Jump-Search Procedure

Because of the properties of the convolutional tree code, we can exploit the following theorem and the jump-search procedure in decoding.

Jump-Search Procedure The properties developed in the last section show that it is necessary to determine only the segments at $k=1, 1+\alpha, 1+2\alpha, \ldots$ in order to determine a path in the tree. We call a search for the correct segments of these orders the jump-search procedure. We make use of this procedure and the following theorem in decoding.

Theorem 3-2-1 Let $a+b=\nu$ and let $\underset{\sim}{W}_A$ be the first a segments of $\underset{\sim}{W}$ and $\underset{\sim}{W}_B$ be the last b segments of $\underset{\sim}{W}$ in the decoder, with similar notation for other sequences. As before, let $r = \frac{d(1)}{2}$ for d(1) even and $\frac{d(1)-1}{2}$ for d(1) odd.

If (i) $|\underset{\sim}{e}| \leq T(\nu)$

(ii) for some $b < \nu$, $rb \leq T(\nu) - |\underset{\sim}{t}_A|$

Then the first branch of $\underset{\sim}{W}$ is correct.

Proof:

$\because |\underset{\sim}{t}_B| \leq rb$

$$T(\nu) \geq rb + |\underset{\sim}{t}_A| \geq |\underset{\sim}{t}_B| + |\underset{\sim}{t}_A| = |\underset{\sim}{t}| = d(\underset{\sim}{W}, \underset{\sim}{Y})$$

Therefore it implies the first branch is correct.

3-3. Decoding Procedure

Suppose that the decoder stores $n=\alpha\nu$ channel output digits and operates to compare this sequence of digits with possible paths in the code tree. An attempt is made to identify the corresponding path traced in the code tree by the encoder, under the control of the original message. In the process of decoding, a tentative choice of a path means a tentative decoding of the channel output digits in the decoder.

<u>Decoding Strategy</u> The decoding strategy is to determine the half tree such that it will have at least one $\underset{\sim}{W}$ satisfying $|\underset{\sim}{t}| \leq T(\nu)$. Then if the number of channel errors $|\underset{\sim}{e}|$ does not exceed the error-correcting capacity $T(\nu)$ of the code, we are assured by theorem 2-4-1, the $\underset{\sim}{W}$ and $\underset{\sim}{U}$ belong to the same half tree. This means that the first segment of $\underset{\sim}{W}$ and $\underset{\sim}{U}$ are the same. Decoding of the first segment of $\underset{\sim}{W}$ is therefore correct. Usually it is not necessary to determine the whole path $\underset{\sim}{W}$ in decoding. If a partial path $\underset{\sim}{W}$ satisfies both theorems 2-4-2 and 3-2-1, the first $(A/r+1)$ segments of $\underset{\sim}{W}$ and $\underset{\sim}{U}$ are the same. Decoding of the first $(A/r+1)$ segments of $\underset{\sim}{W}$ is correct.

<u>A Search Procedure</u> The problem is to try to determine a partial path or whole path which satisfies the threshold test. Assuming that the decoder stores

ν segments, we can start our decoding according to the following steps:

1. We enter into the tree and choose the segments at $1, 1+\alpha, 1+2\alpha, \ldots, 1+(n-1)\alpha, 1+n\alpha$, until the condition $\nu - n\alpha \leq b_{max}$ is satisfied. (where $b_{max} = \frac{T(\nu)}{r}$)

2. If the last $b = \nu - n\alpha$ segments satisfy the condition $b \leq \frac{T(\nu) - |\underset{\sim}{t}_A|}{r}$, we find $A = T(\nu) - |\underset{\sim}{t}_A| - rb$ and the first $A/r + 1$ segments can then be shifted out from the decoder with $(A/r + 1)$ new segments shifted in. The decoding can then be continued, starting at the last $(1+n\alpha)$th segment following the above procedure. If the condition $b \leq \frac{T(\nu) - |\underset{\sim}{t}_A|}{r}$ is not satisfied, the selection procedure should be continued until the proper n is found.

3. If a partial path can not be determined using the above procedures, we can further start our jump at the $(n\alpha - m)$th segment, where $m = 1, 2, 3, \ldots\ldots$. Beginning with $m=1$, successive value of m could be tried until a partial path is found. The procedures (1) and (2) can then be followed in decoding. For $m = \alpha$, we should jump $(\alpha+1)$ segments so that we can begin our further jump at the $(n\alpha+1)$th segment.

CHAPTER IV. GENERAL DISCUSSION AND CONCLUSION

The development of sequential decoding is due largely to the need for minimizing the number of decoding operations. In the searching process of this scheme, the decoder always attempts to determine a path or a partial path in the tree which differs from the received sequence in less than $T(\lambda)$ places, where $T(\lambda)$ is the error correcting capacity of the tree code.

In this decoding scheme, all operations such as modulo-2 addition and register shifting can be made by a general purpose computer. We can store the last column of the initial truncated tree in the computer and decode very rapidly. Thus, the method is quite well suited for programming on a general purpose computer.

We have shown that it is not necessary to determine the whole path for our decoding. Under no-noise or low-noise conditions, it is only necessary to choose the corresponding segments and to generate segments between them in determining a unique path. By this process, the purpose of correct decoding can be achieved. Using jump-search procedure developed in this work, for α-unit, $\alpha+1$ operations consisting of one choice, $(\alpha-1)$ generating and one comparison will be needed, whereas branch-by-branch search needs 2α operations consisting of α choices and α comparisons. It appears that fewer operations are needed using jump-search procedure than using branch-by-branch search for same α-unit. If the burst errors occur at 2nd, 3rd branches, we choose the 1st and 4th branches and generate the branches between them. By jumping over the errors, a correct path can be obtained. So under burst-error condition, this decoding scheme gives fewer decoding operations.

If $|\underset{\sim}{e}| > T(\nu)$, we can use the same procedure as in the case $|\underset{\sim}{e}| \leq T(\nu)$ except changing the threshold function from $T(\nu)$ to $T(\nu) + n$, where n is a positive integer. Since the search procedure is not as complete as that used by Ong [5] or Lin and Lyne [3,4], the probability of error can be expected to be somewhat higher, although many error patherns with weight $|\underset{\sim}{e}| > T(\nu)$ will be corrected. Because the search procedure is quite definite, the maximum number of operations for any message digit is kept reasonably small.

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