

# THREE PUBLIC LECTURES ON SCIENTIFIC SUBJECTS<sup>1</sup>

## I

### A MATHEMATICAL APPROACH TO ETHICS<sup>2</sup>

**I**N January, 1932, when I had the privilege of lecturing at the Rice Institute, my topic was that of quantitative aesthetics. In the first of the lectures which I am giving now, I would like to call your attention to similar possibilities in the field of ethics. It is an especial pleasure for me to be with you again; and I can only hope that you will find your confidence justified, at least to the extent that the three very diverse subjects considered will be novel and interesting. Only in the last of these will any mathematical technique be involved.

Since the time of the German philosopher, Immanuel Kant, it has been clear that, for certain purposes, philosophic thought may be treated separately in its logical, aesthetic, and ethical aspects, concerned respectively with the true, the beautiful, and the good.

In the last century logic has developed into an independent discipline—the edifice of syllogistic thought—of which all of mathematics appears as the grandiose superstructure.

The concept of “aesthetic measure” which I laid before

<sup>1</sup>Delivered at the Rice Institute, March 6, 7, and 8, 1940, by George David Birkhoff, Ph.D., Sc.D., LL.D., Perkins Professor of Mathematics at Harvard University.

<sup>2</sup>Many of the ideas presented in this lecture were first given by the author in a lecture entitled “A Program for Quantitative Ethics,” delivered at the University of Washington under the auspices of the Walker-Ames Foundation, July 19, 1939.

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you in 1932<sup>3</sup> made possible a more or less mathematical treatment of aesthetics giving promise of taking the subject of analytical aesthetics out of the domain of philosophic speculation into the region of common sense thought. The question thus presents itself almost irresistibly to the mind: Is not a similar treatment of analytic ethics possible? My aim here is to show that such a program seems to be feasible.

To most mathematicians the tendency towards increasing mathematization in these three fundamental aspects of philosophic thought—logic, aesthetics, and ethics—is only what was to be expected; for they are likely to agree with the dictum of the great French philosopher and mathematician, René Descartes, *omnia apud me mathematica fiunt*—with me everything turns into mathematics!

Even in early Greek times the philosopher Pythagoras tried to bring mathematical order into the ethical field by asserting that justice is represented by a square number. This must be looked upon as a mystical conjecture of real importance for ethics. Similarly Plato and Aristotle were always desirous of showing the close relationship of the good and the beautiful, if not their essential identity; and they regarded the beautiful as characterized by unity in variety. Thus, there has always been observable in ethics, as well as in aesthetics, a tendency towards quantitative formulation. The supreme goal of the *summum bonum* or highest good, adopted by the Greeks, is suggestive of this; and the modern utilitarian principle of “the greatest good of the greatest number” reveals still more clearly the same tendency.

A very interesting analogy between aesthetics and ethics is the following. Individuals of so-called artistic tempera-

<sup>3</sup>See my lectures, “A Mathematical Theory of Aesthetics and its Application to Poetry and Music,” delivered at the Rice Institute in January, 1932, published in the *Rice Institute Pamphlet*, Vol. XIX, No. 3, July, 1932; and also my book, *Aesthetic Measure* (Cambridge: Harvard University Press, 1933).

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ment often look upon their personal experiences as a succession of aesthetic adventures from which they try to extract the greatest possible enjoyment. Similarly, persons of predominantly moralistic type strive for a maximum of moral satisfaction by making in their daily lives such ethical decisions as will best promote the material and spiritual well-being of their fellows.

Just as the analysis of experience from the aesthetic point of view yields the concept of "aesthetic measure"—the ratio of aesthetic reward to effort of attention—as basic in the evaluation of aesthetic pleasure, so the consideration of experience in its ethical aspects leads to an analogous concept of "ethical measure"—the amount of moral satisfaction based on good accomplished.

The simple ethical formula evidently suggested is:

$$M \text{ (ethical measure)} = G \text{ (total good achieved)}.$$

From this point of view the ethically-minded person<sup>4</sup> endeavors always to select that one of the possible courses of action which *maximizes* the ethical measure  $G$ , just as the aesthetically-minded person continually compares aesthetic objects and prefers those which maximize the aesthetic measure  $O/C$ <sup>5</sup>. The utilitarian calculus of Jeremy Bentham represents a suggestive semi-philosophical attempt in the same direction.<sup>6</sup>

Let us consider a little more in detail this general parallelism between the aesthetic and ethical domains. In order to do this the use of parallel columns is convenient.

<sup>4</sup>Or corporate body or state.

<sup>5</sup> $O$  = order,  $C$  = complexity.

<sup>6</sup>In this connection, Mr. P. A. Samuelson of the Society of Fellows of Harvard University calls my attention to F. Y. Southworth's very interesting volume on *Mathematical Psychics* (1881).

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### *Aesthetics*

Some of the principal aesthetic 'factors' are: (+, of positive type) repetition, similarity, contrast, balance, sequence, centers of interest or repose; (-, of negative type) complexity, ambiguity, undue repetition, unnecessary imperfection. These factors enter into the terms *O* and *C* of the aesthetic formula,

$$M = O/C$$

The factors involved in the order *O* may be divided into formal and connotative elements of order, while the complexity *C* is formal. Only the formal type of elements in *O* admits of quantitative treatment.

In aesthetics, objects of a definite class are to be compared in regard to their relative aesthetic measures *M*. Such classes are of extraordinary variety. The theory of aesthetic measure is best exemplified by certain simple formal visual

### *Ethics*

Some of the principal ethical 'factors' are: (+, of positive type) material good, sensuous enjoyment, happiness, intellectual and spiritual achievement; (-, of negative type) material waste and destruction, pain, sorrow, intellectual and spiritual deterioration. These enter into the term *G* of the ethical formula,

$$M = G$$

The factors involved in the good, *G*, may be divided into the material and the immaterial elements of the good. Only the material type of elements admits of quantitative treatment by the formula.

In ethics, each single definite problem is to be considered by itself, and the possible solutions are compared as to their ethical measures, *M*. These problems are also of extraordinary variety. The main interest in ethics is provided

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and auditory fields, provided by art rather than by nature.

Artists, connoisseurs, and critics of all kinds are considered to be especially competent judges in their special aesthetic fields. But the aggregate opinion of ordinary lay observers plays a vital rôle.

Aesthetic tastes vary from one individual to another, and are relative to the period and culture concerned. Nevertheless there is a certain grand parallelism to be discerned, due to the presence of certain absolute elements of order, as, for instance, rhythm in music. Cultivated human beings are generally able to understand and appreciate aesthetic objects of all kinds and periods.

Finally, the main phases in the history of aesthetic ideas and literary criticism of special artistic forms can be concisely interpreted by use of the concept of aesthetic measure.

by problems arising in practice rather than by artificial problems.

Religious leaders, statesmen, judges, and the socially elect are regarded as the best judges in their several ethical fields. But the general intuitive opinion of mankind often has decisive weight.

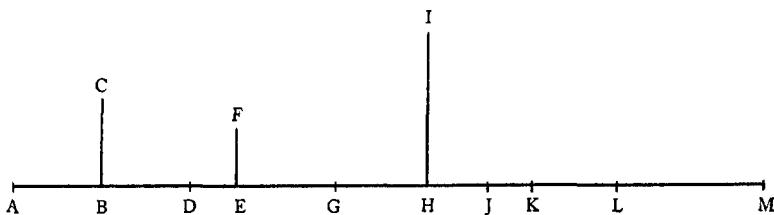
Ethical values and ideals vary in a similar manner. Nevertheless, there are always to be found certain absolute elements of the good as, for instance, bravery and loyalty in their socially validated forms. Careful study of the development of such specific forms serves to explain them acceptably to men everywhere as varied manifestations of these absolute elements of the good.

Similarly, the main phases in the history of ethical ideas and of their many special social manifestations admit of concise interpretation through the concept of ethical measure.

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Having called attention to this significant general parallelism between aesthetics and ethics, I propose to consider some specific problems which illustrate how the concept of ethical measure can be used. I make no apologies for the simple character of the ideas involved, since it is inevitable that the initial results obtained be rudimentary. As possibly suggestive in this connection, it may be recalled that the first classification of matter as solid, liquid, or gaseous provided a crude trifurcation of nature, which ultimately led to the mathematical theories of elasticity and hydrodynamics.

*Problem I.* A bus driver regularly takes passengers from the starting point  $A$  to their destinations along the main road from  $A$  to  $M$  and along certain side roads on one side of the main road. The majority of the passengers live along the main road, and the side roads are short. The driver wishes to be as accommodating as possible and to give all the passengers equal consideration. In what order should he take the passengers to their destinations?



His decision is always to deliver the passengers in the natural order of going from  $A$  to  $M$  along the main road. To justify this decision he might argue as follows:

Suppose first that all the passengers on some trip wished to alight at points on the main route, as not infrequently was the case. If he took them to their destinations in other than the natural order, the series of passengers (as a series)

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would be less quickly delivered one by one than otherwise, i.e., the first passenger would alight later, the second passenger also, etc. Since all the passengers are to be treated as well as possible, this would be regarded as extremely objectionable by them. But in the event that some of the passengers wish to alight along the short side roads, the additional times required are very small and in the driver's judgment do not need to be considered. Hence he finds that the delivery of passengers should always be as stated.

Let us attempt to formalize this simple reasoning. The underlying good here,  $G$ , may be regarded here as negative ( $-$ ), if we reckon upon the unrealizable good of immediate delivery of the passengers as the neutral point (0) from which the reckoning starts. Thus we write

$$G = -(\text{sum of all the trip-durations for the passengers}).$$

The possible solutions to be considered are the various ways of taking the passengers to their destinations.

The two basic assumptions of the driver are almost but not entirely in agreement with this definition of  $G$ ; they are: (1) the individual trip-durations along the main road are to be diminished as far as possible; (2) the trip-durations along the side roads need not be considered. On this basis his decision is obviously as stated and in general will maximize the good,  $G$ , as just defined.

However, there might occasionally arise situations in which this solution was not actually the best one by the formula written above. Suppose, for example, there were six passengers, one to be delivered at  $C$ , and five at  $D$ , with equal distances  $AB$ ,  $BC$ , and  $BD$  (see the figure above). Clearly the 'best' solution in this exceptional case would be to deliver the five passengers at  $D$  along the main road, and then to return along the main road and deliver the remaining

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passenger at  $C$ . In fact, if the driver follows his general rule, we have

$$G = -22a,$$

where  $a$  stands for the time required for the bus to go over any one of the equal distances, while if the driver were to deliver his passengers in the reverse order, we would have

$$G = -14a,$$

so that  $8a$  units of time would be thereby saved to the passengers.

Nevertheless, the driver decides to deliver the passengers in the usual way. In doing so he goes directly against a perfectly natural postulate referred to above, namely, that if he can shorten (or in no case lengthen) the trip-durations of the successive passengers, he certainly should do so.

Obviously the naturalness and uniformity of the solution adopted by him operates as an important factor in its favor. For the rule of procedure chosen by the driver is readily understood by the passengers and any modification of it in the direction of increased complication might lead to dissatisfaction, especially because the time-schedules of the trip would become even more unpredictable.

*Thus we are led to realize that there are instances in which the simplicity and elegance of the solution of an ethical problem must itself be regarded as one of the imponderable elements of the good which enter into  $G$ .*

There is a kind of counterpart to this phenomenon in the aesthetic field: Apparently the intuitive aesthetic judgment tends through an inner necessity to prefer formally simple elements of order in the aesthetic object.

It would be easy to propose other allied problems in this field of the ethics of procedure. In fact, the last two prob-



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lems here considered ( $V$  and  $VI$ ), dealing briefly with the proportionate representation of the states in the House of Representatives and with the question of preferential balloting, are both clearly of the procedural type. All such problems are characterized by the fact that they involve the determination of the most equitable procedure by an agent to whom the choice of procedure is personally indifferent, while the good,  $G$ , in question is a material good.

Our second problem is intended to present a very different type of ethical situation which is of high significance, and which involves both material and immaterial elements of the good. Though presented in a specialized form, I believe that the problem selected embodies a situation characteristic of critical moments in the lives of many human beings—moments when the choice must be made between material good with attendant failure in loyalty, on the one hand, or the sacrifice of this material gain with preservation of loyalty, on the other. As has been indicated previously, a complete quantitative treatment cannot be hoped for in such a problem.

*Problem II.* One or the other of two friends of long standing,  $A$  and  $B$ , is to be advanced to an opening in the organization in which they hold positions of the same rank.  $A$  happens to learn that the actual selection will hinge upon the judgment of a certain person  $L$  belonging to the same organization. Ought  $A$  to pass this information on to  $B$ ?

The answer of course is that in the circumstances stated  $A$  ought to inform his friend  $B$ .

$A$ 's reasons for this decision might be formalized as follows: The material goods  $g_A$  and  $g_B$  which will accrue to him or to his friend through such an advancement are the same:  $g_A = g_B = g$ . If  $A$  informs  $B$ , the immaterial good of

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his friendship,  $f$ , with  $A$  is retained. Therefore, we have simply

$$M = g.$$

On the other hand, if  $A$  does not tell  $B$  what he has learned, the friendship between them is destroyed, even if  $B$  never learns of  $A$ 's unfriendly act; and so we have

$$M = g - f.$$

Since  $g$  exceeds  $g - f$ ,  $A$  ought to tell  $B$ , although he realizes that by doing so he gives up a definite personal advantage. In the above reckoning the unfavorable effect upon  $A$ 's character of not informing  $B$  is intentionally disregarded although it might really be the most important consideration of all.

$A$ 's decision to pass on the information to  $B$  is here assumed to be made on the utilitarian basis. On a hedonistic basis,  $A$  might conclude that if he fails to inform  $B$ , then

$$M = g - f,$$

since he will be certain to win  $L$ 's special favor, whereas, in the contrary case,

$$M = \frac{g}{2},$$

inasmuch as he would then only have an equal chance with  $B$ . In this event, he would have to balance the prospect of material advancement against his friendship with  $B$ .

Again, according to the extent that  $A$  believes himself inferior to  $B$ , he will feel that his chances are lessened by telling  $B$ . If  $A$  is a loyal friend, however, he will not be moved from his decision by such thoughts.

The basic hypothesis has been made here that the information about  $L$  is of legitimate practical advantage to  $A$

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and  $B$ . It is also assumed that the friendship between  $A$  and  $B$  is a sincere one, founded upon mutual esteem. For clearly if there were no real friendship,  $A$  would not be under any obligation to inform  $B$ , any more than he would consider it an obligation on  $B$ 's part to tell him. Of course if  $A$  believes that  $B$  would not tell him if circumstances were reversed, or that  $B$  would employ unfair or unscrupulous tactics to gain  $L$ 's favor, the bond of friendship between them is already weak; and so the situation would not be the one envisaged in the problem under consideration.

A somewhat similar type of problem, also not infrequently exemplified in human experience, is the following:

*Problem III.* Two men,  $A$  and  $B$ , among six,  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$ ,  $F$  in control of a certain business, have orally agreed to exchange all relevant information before entering into any arrangement with the others.  $A$  and  $B$  do this in order to protect their interests in the business.  $A$  is approached confidentially by  $C$ ,  $D$ ,  $E$ , and  $F$ , and asked if he will concur in a vote giving him important special privileges which are to be withheld from  $B$ . Actually  $A$  does not feel he is entitled to these special privileges any more than  $B$  is. How ought  $A$  to act?

The ethical course for  $A$  to follow is clearly to refuse to connive with  $C$ ,  $D$ ,  $E$ , and  $F$ . He should further inform  $C$ ,  $D$ ,  $E$ , and  $F$  that in his opinion to do otherwise would not be fair to  $B$ .

If  $A$  acts in this manner we may write

$$M = 0,$$

meaning thereby that the *status quo ante* is not altered. If  $A$  consents to their proposal, we may write for  $A$

$$M = g_A - f - e_B,$$

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meaning that  $A$  gains the privileges mentioned ( $g_A$ ), loses  $B$ 's friendship ( $f$ ), and possibly incurs  $B$ 's positive enmity ( $e_B$ ), fraught with danger to him—for instance, the enmity of  $B$  might lead to his loss of a valuable reputation for business integrity. Here we find in  $G$  two elements of mainly material nature ( $g_A, e_B$ ) and one of immaterial nature ( $f$ ).

The two preceding problems have been taken from the field of social ethics. It is of interest that similar problems can be drawn from the field of international ethics. In the problem about to be stated there is no intention to parallel closely any actual problem. The intention is rather to suggest that there may exist somewhat analogous problems which admit of clarification when approached from the point of view of ethical measure.

*Problem IV.* As the result of a war,  $B$  has lost a colony  $C$  to the nation  $A$ . This colony  $C$  has subsequently been given nearly complete independence by  $A$ . This action leaves  $C$  well satisfied with her status and favorably disposed towards  $A$ . However,  $B$  has an economic need for her former colony  $C$ , by reason of lack of raw materials which  $C$  had formerly supplied; and for this and other more political reasons,  $B$  demands the cession of  $C$  back to her by  $A$ . How is  $A$  to reply to the demand?

A reasonable analysis on  $A$ 's part might be the following:  $A$  concludes that to return the colony  $C$  would not only be objectionable to  $C$  but extremely detrimental to  $A$ 's international standing and prestige as a concession under duress. Furthermore,  $A$  feels that if she did agree to  $B$ 's demand, other similar demands reinforced by further military threats would soon follow. Thus  $A$  (and  $C$ ) might write in the event of return of  $C$  to  $B$

$$M = g_B - h_{A,C}$$

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(where  $g_B$  = material good to  $B$ ;  $h_{A,C}$  = material and immaterial harm to  $A$  and  $C$ ) and, in the contrary case,

$$M = 0,$$

since there is no reason to believe that the prospect of ultimate war is effectively lessened. Hence  $A$  and  $C$  would have to balance  $B$ 's good against their own harm; and so  $A$  would almost certainly refuse to cede  $C$  back to  $B$ .

From  $B$ 's standpoint, however, the analysis in the case of cession would more correctly be

$$M = g_B + p,$$

(where  $p$  = good of peace), since  $B$  would not admit that  $A$  or  $C$  would suffer much economic loss thereby, nor that there could be enduring peace without cession; and  $B$ 's analysis in the contrary case would be

$$M = 0.$$

Thus the balance in favor of cession is  $g_B + p$  in  $B$ 's estimate, and at least  $h_{A,C} - g_B$  against cession in  $A$  and  $C$ 's estimate. Thus there is a very serious conflict of ethical judgment. Such a situation naturally raises the question of possible compromise.

*In this and similar cases of apparent conflict in ethical judgments the thorough exploration of all possibilities of compromise is absolutely essential.*

The following is a suggestion of a possibility of such a compromise in this particular case:  $A$  notifies  $B$  and  $C$  that in recognition of  $B$ 's economic needs and of her claims, she will henceforth not accept from her colony  $C$  any more favorable trade status than  $C$  accords to  $B$ .

There is then the possibility that despite  $A$ 's refusal of  $B$ 's demand for the return of  $C$  to her,  $B$  can recover a sub-

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stantial portion of her former trade with *C*. Thus one might write, in behalf of all the three parties *A*, *B*, and *C*, a formula such as

$$M = \frac{1}{2}g_B - \frac{1}{4}g_A + \frac{1}{2}p,$$

in the event of such a compromise, as against  $M=0$  if the *status quo ante* is preserved. The sole loss for *A* would be loss of an estimated quarter of *B*'s trade with her ( $\frac{1}{4}g_A$ ); *B* would recover an estimated half of her former trade with *C* ( $\frac{1}{2}g_B$ ) without loss to *C*; and it might be that the resultant improvement in the friendliness of relations between *A* and *B* would increase the likelihood of a permanent peace and so slow down the expensive armament race between *A* and *B* ( $\frac{1}{2}p$ ).

The question of compromise is extremely important in many ethical problems. Is it reasonable to suppose that such compromises can generally be reached? In this connection I recall a conversation with Dean Roscoe Pound and Count Korzybski some years ago. Count Korzybski had expressed the opinion that many conflicts of points of view had their origin mainly in misunderstandings as to the meaning of terms, so that the conflict would disappear as soon as these meanings were agreed upon. I replied that in many disputes the situation resembled rather that arising between two boys contending for a single piece of pie; and Dean Pound was inclined to agree with me. In the tragic condition of the world today, the suggestion might be made that if the division of the single piece of pie into two equal pieces were made (a reasonable compromise), both boys could be induced to accept their portion!

Our last two problems, like the first, fall in the field of ethical procedure, and are interesting in showing that a certain amount of technical mathematical consideration may

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be necessary. The first of these is that of the equitable apportionment of Representatives in our Congress to the several states in accordance with the constitutional provision to that effect:

*Problem V.*<sup>7</sup> The set of states is to elect biennially an assigned total number of Representatives to Congress, at least one from each state; and, in accordance with the Constitution of the United States, the numbers of Representatives allotted to the several states are to be as nearly proportional as possible to the populations of the respective states. What is the best method of the apportionment of Representatives to the various states?

It is first of all to be observed that various intuitively reasonable postulates may be formulated, such as, for instance, the two following: Of two unequal states  $A$  and  $B$ , the one with greater population should have at least as many Representatives as the other; every state should receive at least the integral part of the exact (fractional) number which it is ideally entitled to.

These two postulates were satisfied by the former Vinton method of apportionment which may be described as follows. The theoretical size of a congressional district (i.e., its population) is first calculated, and on this basis the exact number of Representatives (not in general an integer) for each state is determined. Each state then receives at least as many Representatives as the integral parts of these numbers; and one additional Representative is assigned to the states in the order of decreasing fractional parts of these numbers until the required number of congressional Representatives is reached. Thus, if there were three states with the calculated numbers 3.72, 2.41, 1.87, with a correspond-

<sup>7</sup>For the mathematical conclusions accepted below without analysis see E. V. Huntington's article "On the Method of Equal Proportions," *Trans. Amer. Math. Soc.*, pp., 85-110, vol. 30, (1930).

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ing total of eight Representatives, there would first be assigned respectively, 3, 2, and 1 Representatives, leaving two still to be assigned. These would then go to the third and first states with the larger fractional parts, (.87 and .72 respectively) so that the final assignments by the Vinton method are 4, 2, and 2 respectively.

It is significant that this very simple method was accepted until it led to the "Alabama paradox," exemplified when an increase in the total number of Representatives from 299 to 300 led to an actual decrease in Alabama's quota of Representatives! Here there was violated another very simple and natural postulate: If the assigned total of Representatives for all the states is increased, the assignment to each of them should certainly not be diminished. This outcome was manifestly unreasonable from the political point of view, and led to the use of Willcox's so-called "method of major fractions" instead of the Vinton method.

Willcox's method is very simple in statement and is at once seen to avoid the "Alabama paradox." It may be formulated as follows: Determine as before the number of Representatives for each state, and assign to each state the next lesser integral number if the fractional part is less than one-half, and the next greater if it exceeds one-half (i.e., is a *major fraction*). Then, at least if this rule yields the desired total number of Representatives, the assignment will be that designated by the Willcox method. If it does not do so, increase or decrease the (theoretical) size of a "congressional district" until the correct total number is secured. This allocation of Representatives will yield the proper result, according to the Willcox method.

More recently, Huntington has proposed his method of equal apportionment which not only avoids the Alabama



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paradox but has two additional formal advantages over the Willcox method: (1) it *automatically* assigns at least one Representative to every state, in accordance with the constitutional requirement, as the Willcox method may fail to do; (2) the Huntington method makes no distinction between the mathematically dual notions of "congressional district" and "individual share" while the Willcox method arbitrarily prefers the latter.\*

Thus here again considerations of simplicity and elegance enter in determining the relative merits of the two methods of apportionment.

In Willcox's method, from our standpoint of "ethical" measure, it is not hard to show that there is an underlying ethical measure for any two states

$$M = - \left| \frac{C_1}{P_1} - \frac{C_2}{P_2} \right|$$

where  $P_1$  and  $P_2$  are the populations of the two states, and  $C_1$  and  $C_2$  are the proposed numbers of congressional seats for these states. In the unnamed dual method we have similarly

$$M = - \left| \frac{P_1}{C_1} - \frac{P_2}{C_2} \right|.$$

The corresponding choice of  $M$  for Huntington's method of equal apportionment is

$$M = - \left| \log \frac{P_1}{C_1} - \log \frac{P_2}{C_2} \right|.$$

\*According to Huntington (Congressional Record, April 28, 1941) the test of equal proportions is to be stated as follows: "A proposed transfer of a seat from one state to another state should be made when and only when the percentage inequality between the congressional districts in the two states is reduced by the transfer." The words "congressional districts" may be replaced by "individual shares" if one so desires, since Huntington's method of apportionment is self-dual.

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All three methods fulfill the following further important and natural requirement: The assignment should be made in such wise that it cannot be improved between any pair of states by transferring a Representative from one state to the other.

The second postulate mentioned above as satisfied by the Vinton method is not satisfied by any of these three methods.

*It seems to be abundantly borne out in this and other examples that apparently justifiable postulates are often mutually contradictory, so that a choice has to be made between them.*

Two further remarks of general import may well be made. If the populations of the states were to remain nearly fixed for long periods of time, injustices might be regularly inflicted upon certain of them, so that some further modification of the method of apportionment mentioned might become desirable. In the second place, from the standpoint of ethical measure, it would be of interest to investigate other possibilities. For example it might be our goal to minimize the largest injustice of underrepresentation for any state, then the next largest injustice for some other underrepresented state, etc., and we might ask what would then be the corresponding method of apportionment. Here we might adopt the specific measure of injustice suggested by the value of  $M$  given above for the method of equal apportionment of Huntington.

We turn now to a very brief consideration of our final problem of preferential balloting:

*Problem VI.* A group is required to elect one of a number of candidates to an office on the basis of the relative preferences of the group. How should the successful candidate be determined from the ordering of names on the various ballots?

Here, too, various natural postulates obviously apply, such,

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for example, as the following: If one of the candidates,  $A$ , is named in higher position than another,  $B$ , throughout (i.e.,  $A$  has more first positions, more first and second positions combined, etc.) then  $A$  is to be chosen in preference to  $B$ . By the aid of this principle alone the choice is generally narrowed down to a few individuals.

The *simplest* rule of procedure perhaps is to rate the first position as 1, the second as 2, etc., and to add the ratings for each candidate. The candidate with the least total is then taken to be elected<sup>8</sup>. In this case we may write for each candidate

$$M = k - (\text{sum of positions obtained}),$$

where  $k$  is the number of members in the voting group. Thus, if one of the  $k$  candidates receives all the first choices, we have  $M = 0$ , the ideal case; otherwise  $M$  is negative and is smallest for the least sum of indices of position.

Another possible definition of  $M$  would be

$$M = 1 - \sqrt[k]{\text{product of positions obtained}}.$$

These two methods would be related much as the arithmetic and geometric means are. However the first (usual) method has the advantage of being simpler to apply in practice.

It is clear that a politically-minded person could manipulate his ballot in favor of special candidates by not indicating his real preferences. However, such action would constitute a serious misuse of the preferential form of balloting.

This problem of preferential balloting has not been treated as thoroughly as it should be.

*In my judgment it would be a very constructive program for analytic ethics, to catalogue systematically various significant*

<sup>8</sup>The case of a tie must be otherwise decided of course.

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*problems in the three fields of procedural ethics, social ethics, and international ethics, and to classify the main types of solutions on the basis of the formula for ethical measure.*

This has been attempted to a very rudimentary extent in the six special problems above.

More specifically, social customs and systems of law and of religion contain a vast mass of ethical data, embodying the accepted ethical solutions of innumerable practical problems of analytic ethics; and the inductive method can generally be applied to treat new problems when they arise. In so far as these solutions are not purely empirical, they could be codified by means of the ethical formula. Such a codification would list and classify the very extensive variety of ethical intuitions (postulates), in part the cause of, and in part the result of, specific social interactions. There is little doubt of the basic rôle which the sentiments of love, goodwill, loyalty, and other feelings of sensuous, aesthetic, or intellectual type play in such intuitions. These provide a substratum of absolute elements, of which the specific manifestation depends on the particular culture and period concerned.

Another useful service of such a program might be to treat the extremely interesting history of ethical ideas by use of the same ethical formula. Thus the early Greek conception of ethical behavior as directed towards the attainment of the *summum bonum* is evidently in consonance with the ethical formula. The customary threefold division of ethical theories into those of hedonistic (or egoistic) type, of utilitarian (or universalistic) type, and of altruistic type is immediately explained in the same way; for if  $G_I$  denotes the good of an individual and  $G_F$  the good of his fellows, then the three types of ethical theory correspond to the respective formulas:  $M = G_I$ ,  $M = G_I + G_F$ ,  $M = G_F$  respectively. It is hardly necessary to say that in promulgating a theory which is

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(supposedly) of the first or last non-utilitarian type, it is frequently necessary to rob Peter to pay Paul!

Many ethical theorists have tended to take the good and the pleasurable as synonymous; thus, according to Bentham, pleasures differ quantitatively but not qualitatively. From our point of view, this is a necessary assumption if all the constituents of the good in  $G$  are looked upon as comparable quantities, as required by the ethical formula.

Some have regarded the striving for perfection as supremely important, thereby emphasizing the achievement of potential good as the final goal; this reaches far into the domain of the *qualitative* application of the ethical formula.

Still others, like Kant, insist upon the dominating rôle of the sense of duty as the "categorical imperative." This validates the innumerable ethical intuitions on which concrete decisions concerning the immaterial good must always depend. Through the sense of duty we feel that it is possible to distinguish clearly between right and wrong, independently of our particular backgrounds, although careful analysis reveals that this independence is by no means complete. In fact the formalization of such intuitions, combined with the use of the general ethical formula, leads to the analytic solution of ethical problems by means of reasoning—a point of view going back to Socrates, Plato, and Aristotle.

There is a further reason why the systematic codification of ethical notions might be of genuine service. Ordinary language provides a vast storehouse of convenient symbols, which (as has been recently emphasized) often bring together under one name a number of quite different entities. For example, we speak of "fatigue" with a good deal of certainty. But what is fatigue? There are specific conditions of fatigue of the muscles, of special nerves, etc; but what

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have they in common? Careful experimentation, systematic analysis, and detailed classification are necessary for the proper elucidation of this question. This has indeed been accomplished recently by Professor L. J. Henderson and his colleagues in the Harvard Fatigue Laboratory; and the final upshot is that "fatigue" designates not one thing but many, grouped together largely because of intellectual convenience. Of course the notion of fatigue has no immediate ethical import.

Now many of the terms used constantly in ethical discussions have an even less definite meaning, and frequently provide a convenient emotional support for ethical or unethical action. Certain of these general terms, such as "wisdom" and "justice" seem to be mainly constructive in their effect, but others, like "racial superiority," for example, are positively destructive and dangerous, unless their various meanings have been made very specific. For instance, in speaking of racial superiority, which of the qualities listed below do we regard as really characteristic? Physical prowess and beauty; racial purity; descent from divine ancestors; intellectual capacity and achievement; aesthetic sensibility and artistic creativeness; unselfish idealism; unlimited devotion to the state; economic efficiency; military might; high potentiality of further development? Evidently there are many consistent points of view as to what constitutes "racial superiority"; and so wherever the idea is used it needs to be properly defined and accurately applied in the selected sense.

Our thought here is akin to that of Korzybski, that when human beings realize that certain important general terms have a variety of distinct meanings, the effect of this realization is definitely prophylactic against misunderstanding, prejudice, and intolerance. If the clarification of such im-

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portant but multiple-valued ideas is not effected soon, the present tragic confusion among men may end in the destruction of civilization.

Such is the general program for ethics to which I desire to direct attention. It is analogous in many respects to that which I have attempted to carry out provisionally in aesthetics. The program involves the introduction of elementary quantitative ideas based on a simple formula for "ethical measure" in order to clarify and codify the vast ethical domain. Conceivably such a program might perform the same kind of useful service for ethics as elementary logic performs for mathematics, and grammar for language.