RICE UNIVERSITY

AN INVESTIGATION OF THE RISE TIMES OF OUTPUT PULSES OF SEMICONDUCTOR NUCLEAR RADIATION DETECTORS

by

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ABSTRACT

This paper is a report of the investigations carried out to determine whether or not the rise time of the output pulses of semiconductor nuclear particle detectors is influenced by the type of ionizing particle striking the detector.

Charge collection rise times are calculated in the customary fashion, neglecting interaction between the lines of charge produced by the ionizing particle. A model of the detector is developed on which rise time calculations may be carried out taking into account charge interaction. The calculations are carried out for several different types of ionizing particles. Comparing the calculated rise times shows no difference between the rise time calculated taking into account charge interaction and those neglecting charge interaction, to within the accuracy of the method of calculation.

The design of a wide-band amplifier is discussed, and the performance characteristics of the system used to observe detector charge collection rise times are presented.

Resistivity and junction capacitances of several detectors were measured. In only one of the available detectors could the charge collection time be observed. Using an eight m.e.v. alpha source, the charge collection time of the detector was measured and found to be several times longer than was predicted. No other types of particle sources were readily available, so it was not possible to determine if this effect varied with the type of ionizing particle striking the detector. This effect (i.e. an apparent lengthening of the charge collection time) has, however, been observed by others, who have determined experimentally that the effect does not depend on the particle type.
The conclusion then, of this paper is that there is a definite lengthening of the charge collection time over that calculated by usual methods. This effect does not vary with different types of ionizing particles. No explanation for this effect is offered.
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An Investigation of the Rise Times of Output Pulses Of Semiconductor Nuclear Radiation Detectors

Introduction

It has been suggested that the rise time of the output pulse of a semiconductor nuclear particle detector might be affected by the specific ionization of the incident particle, thus making it possible to determine the type of ionizing particle (i.e., alpha, proton, etc.,) striking the detector by measuring the rise time of the pulse produced by the particle. It is the purpose of the work reported in this paper to determine by both theoretical and experimental investigation whether or not this suggestion is correct.

First we consider the mechanism by which a particle produces a pulse when it strikes a detector. A brief qualitative explanation of this mechanism is as follows: When an ionizing particle strikes a detector, it loses energy through collisions with the atoms of the semiconductor. These collisions occur mainly with the electrons in the outer shells of the atoms, only rarely with a nucleus. In silicon, the particle loses 3.6 electron-volts of energy per collision and liberates an electron in the process, creating a free hole-electron pair. These holes and electrons are swept across the depletion region of the detector by the field produced by the external bias and the internal field, and their motion produces the output pulse. Since the depletion region constitutes the active volume of the detector, we will concern ourselves only with this region for the present. Other factors of detector construction influencing the pulse rise time will be considered briefly in Section 3-2.

The number of collisions the particle undergoes per unit length of travel, hence the energy loss per unit length and the number of hole-electron pairs created per unit length, is a function of both the ionizing particle and the detector material. The energy loss per unit length, denoted as \( -\frac{dE}{dx} \), may be determined conveniently by either of two ways: In many references\(^1\), there are curves giving the range (i.e., the distance of penetration) of particles of different energies
in various materials. Dividing the particle energy by the range gives the average \(-\frac{dE}{dx}\). Then dividing \(-\frac{dE}{dx}\) by the energy loss per collision (3.6 eV in silicon) gives the number of hole-electron pairs per unit length (specific ionization) created by the particle. Formulae also exist\(^2\) for calculation of \(-\frac{dE}{dx}\) as a function of particle velocity.

The hole-electron pairs created by the particle initially form a cylinder of charge with a diameter on the order of one micron\(^3\). For the purposes of this paper, little error will be introduced, if instead of a cylinder of charge, it is assumed that two lines of charge are produced, with their initial separation approximately equal to one micron. The charge per unit length of these lines of charge is, of course, determined by the specific ionization of the ionization particle.

In the usual calculation of pulse shapes, it is assumed that diffusion and the drift field move the charges apart in a relatively short time to the point where the interaction between them is negligible, and their motions are governed solely by the drift field in the depletion region. In order to test the validity of this assumption, expressions will be developed for the transit times of electrons and holes in the detector depletion layer; in one case, interaction between the two lines of charge is neglected, and in the other case it is taken into account.
I. Theoretical Rise Time Calculations

1-1. Depletion Region Characteristics.

In order to develop the expressions for hole and electron transit times, it is necessary to consider the characteristics of the detector depletion region. The usual semiconductor nuclear particle detector consists of a step junction, one side of which is lightly doped, the other side is heavily doped. The junction is reverse biased. If we assume for the purposes of this paper that the n-side of the detector is much more heavily doped than the p-side, then the following expressions hold for the depletion layer width, the electrostatic potential, and the electric field in the depletion region:

\[
(1-1) \quad x_n - x_p = -x_p (1 + M_A/M_D)^{\frac{1}{2}} - x_p, \text{ since } M_A \ll M_D
\]

\[
(1-2) \quad \psi = - \left(\frac{qM_A}{e} \right) (x_p x - x^2/2)
\]

\[
E = 2(-V + \frac{kT}{q} \ln \frac{M_A M_D}{n_i^2}) \left(1 - x/x_p\right)
\]

(1-3) \quad E \approx -(2V/x_p)(1 - x/x_p) \text{ for large reverse bias.}

In the above expressions, \(x_n - x_p\) is the depletion layer width, \(\psi\) is the electrostatic potential, \(E\) is the electric field, \(M_A\) and \(M_D\) are the acceptor and donor densities respectively, \(x\) is the position in the depletion layer, and \(V\) is the bias voltage, negative for reverse bias. \(\epsilon = \epsilon_0 \epsilon_r = 1.06 \times 10^{-12}\) farad/cm. for silicon. Throughout this paper the rationalized mks system of units of units is used, with the exception of the units for length. The unit of length used here is the centimeter.

Note that equation (1-1) shows that almost all of the depletion region lies in the p-side of the junction. Figure 1 shows the assumed detector configuration. The contribution to the output pulse from particles passing from \(x = 0\) to \(x = -x_n\) is negligible.
Under the previous assumptions concerning detector doping, we also have
\[ (1-4) \quad M_A^0 = \frac{1}{(q \rho_h \mu_h)}, \]
where \( \rho_h \) is the resistivity of the p-type silicon, and for large reverse bias \( (-V \approx 5 \text{ volts}) \),
\[ (1-5) \quad V = \frac{q/2 \varepsilon M_A x_p^2}{A_p}. \]

1-2. Hole and Electron Position and Velocity, Neglecting Charge Interaction

The expressions for the hole and electron velocities, neglecting charge interaction, are as follows:

\[ (1-6) \quad \nu_e = -\mu_E^r \frac{x_e}{x_p} = \left( \frac{2 \mu_n V x_p}{x_e} \right) \left( 1 - \frac{x_e}{x_p} \right) = 2 \mu_n V (x_p - x_e)/x_p^2 \]

\[ (1-7) \quad \nu_h = \mu_E^r \frac{x_h}{x_p} = -\left( \frac{2 \mu_n V x_p}{x_h} \right) \left( 1 - \frac{x_h}{x_p} \right) = 2 \mu_n V (x_h - x_p)/x_p^2 \]

Have \( \nu_e \) is the electron velocity, \( \nu_h \) is the hole velocity, \( \mu_E^r \) and \( \mu_E^r \) are the hole and electron mobilities, respectively. \( V \) is again negative for reverse bias. \( x_e \) and \( x_h \) are the electron and hole positions, respectively, and \( x_p \) is approximately the depletion layer width. Now

\[ \nu_e = \frac{dx_e}{dt} = -\mu_E^r V, \]

\[ \frac{dx_e}{dt} = \frac{x_p^2}{2 \mu_n V (x_p - x_e)} - \frac{x_e^2}{2 \mu_n V (x_e - x_p)} \]

\[ t_e = \int_0^t \frac{dx_e}{x_e} \left( \frac{x_p^2}{2 \mu_n V (x_p - x_e)} - \frac{x_e^2}{2 \mu_n V (x_e - x_p)} \right) = -\left( \frac{x_e^2}{2 \mu_n V} \right) \int_0^t \frac{dx_e}{(x_e - x_p)} \]

\[ (1-8) \quad t_e = \frac{(x_e^2/2 \mu_n V)}{(x_e - x_p)} \ln \frac{(x_p - x_e)}{(x_p - x_o)}. \]

Similarly, it is found that

\[ (1-9) \quad t_h = \frac{(x_h^2/2 \mu_n V)}{(x_h - x_p)} \ln \frac{(x_p - x_o)}{(x_p - x_h)}. \]

In the above equations, \( x_o \) is the initial position of either a hole or an electron, \( t_e \) is the time it takes an electron to travel from \( x_o \) to \( x_e \), and \( t_h \) is the time it takes a hole to travel from \( x_o \) to \( x_h \). Equation (1-8) is valid only until the electron reaches the edge of the depletion layer; i.e., until \( x_e = 0 \). We may solve equations (1-8) and (1-9) explicitly for \( x_e \) and \( x_h \) and obtain.
The above expressions, since they do not take into consideration charge interaction, are valid regardless of the path along which the ionising particle passes through the detector.

1-3. Development of the Model for Charge Interaction Calculations

In the discussion that follows, as shown in Figure 2, the normal force is defined as the component of force acting normal to the two lines of charge when \( x_2 = x_1 + s \), and \( d \) is allowed to vary. The tangential force is the component of force acting parallel to the lines of charge when \( d \) is held constant and \( x_1 \) and \( x_2 \) are allowed to vary. In order to develop a model for further calculations, it is necessary to compare the magnitudes of these two forces. Figure 2 explains the notation used in the following equations.

We assume that the specific ionization of the particle producing the line charges is constant; further, that the lines of charge are constrained so that the forces between them do not change the charge distribution. The charge per unit length, which is the specific ionization of the particle multiplied by the electronic charge (-q for holes, -q for electrons), is denoted by \( n \).

From Figure 2 it may be seen that

\[ r = d / \sin \alpha, \quad dx_e = (d / \sin^2 \alpha) \, dx. \]

To compute the normal force, \( f_N \), exerted on a small portion of the line of holes by the line of electrons (or vice versa, interchanging \( x_e \) and \( x_h \)), let \( x_1 = 0, x_2 = s \). Then

\[ f_N = \left[ \frac{\pi}{2} \sin \alpha \, dx_e / x^2 \right. - \left. \frac{\pi}{4} \sin \alpha \, dx_h / (\gamma \sigma \, x^2) \right] \sin \alpha \, dx \]

\[ f_H = \left[ \frac{\pi}{2} \sin \alpha \, dx_h / (\gamma \sigma \, x^2) \right] \sin \alpha \, dx_h. \]

\[ f_E = \left[ \frac{\pi}{2} \sin \alpha \, dx_e / (\gamma \sigma \, x^2) \right] \sin \alpha \, dx_e. \]

\[ f_N = \frac{(s - x_h)}{\sqrt{d^2 + (s - x_h)^2}} + \frac{x_h}{\sqrt{d^2 + x_h^2}}. \]
The total normal force between the two lines of charge, $F_N$, is given by

$$F_N = \sum_{0}^{s} F_N \, dx_n = \left[ \frac{n^2}{(2\pi d)} \right] \left[ \sqrt{\frac{s^2}{d^2} + 1} - 1 \right].$$

It should be noted again that the above expressions for $F_N$ and $F_T$ are valid only if the two lines are directly opposite each other and allowed to move only in the normal direction; i.e., only the distance $d$ may vary. In a similar fashion, expressions for $F_T$ (analogous to $F_N$ but tangential instead of normal forces) may be derived, holding $d$ constant and allowing motion in only the tangential direction.

We find that

$$F_{Te} = \left[ \frac{n^2 \, dx}{(4\pi \varepsilon)} \right] \left[ \frac{1}{\sqrt{(x_1 + s - x_2)^2 + d^2}} - \frac{1}{\sqrt{(x_1 - x_2)^2 + d^2}} \right]$$

$$F_{Th} = \left[ \frac{n^2 \, dx}{(4\pi \varepsilon)} \right] \left[ \frac{1}{\sqrt{(x_2^2 - x_1)^2 + d^2}} - \frac{1}{\sqrt{(x_1^2 - x_2)^2 + d^2}} \right]$$

where $F_{Te}$ is the force exerted on a small part of the line of electrons by the line of holes; $F_{Th}$ is the force exerted on a small part of the line of holes by the line of electrons.

$F_T$, the total tangential force between the lines, may be derived from either $F_{Te}$ or $F_{Th}$. To simplify computation, a coordinate system is chosen such that $x_1$ is zero. Then

$$F_T = \sum_{0}^{s} F_{Th} \, dx_n = n^2/(4\pi \varepsilon) \log \left\{ \frac{[(s - x_2) + \sqrt{(s - x_2)^2 + d^2}]^2}{[(2s - x_2) + \sqrt{(2s - x_2)^2 + d^2]}} \right\}$$

We now need to compare $F_N$ and $F_T$. In all cases of interest, we have initially $s \gg d$. As mentioned before, $d$ is approximately one micron initially. If we consider a particle with a range of 0.1 mm, then ($s/d$) = 100. In this case, $F_N = (50 \, n^2)/(\pi \varepsilon)$ initially. From the above formula for $F_T$, it is seen that $F_T = 0$ initially. $F_T$ reaches a maximum when only 40% of the lines of charge are overlapping, as shown in Figure 3. At this point $F_T = (2.05 \, n^2)/(\pi \varepsilon)$. The value of $F_T$ will be still less in an actual detector. Consider a particle entering the detector of Figure 1 from the left (p) side, perpendicular to the depletion layer edges. The field due to the applied bias will immediately begin to remove holes from
the depletion region, and the charge per unit length of the line of electrons will be diminished, since the field due to the bias increase from left to right, moving the electrons at the right faster than those on the left. A more realistic estimate of $F_{\text{max}}$ in a detector is probably on the order of $n^2/(\pi \varepsilon)$. Considering again the normal force between the two lines, if we let $d$ increase until $d = s/2$, $F_N = 0.618 \frac{n^2}{(\pi \varepsilon)}$.

It appears from the above calculations that if there is a significant lengthening due to charge interaction of the rise time of the output pulse, then it may be calculated from a detector configuration as shown in Figure 4. It is admitted that this configuration is highly impractical, in that it would be nearly impossible to position an actual detector and source so that the ionizing particles passed through the depletion layer parallel to its edges. However, if the effect we are interested in does, in fact, exist, then it should show up in calculations made on the model.
Hole and Electron Velocity and Position, Considering Charge Interaction

We now return to the equation derived previously for $f_N$. Near the center of the line of holes, i.e., for $x_n \leq s/2$, we may make the approximation

$$f_N = \frac{n^2 dx_n}{4\pi d} \left[ \frac{2x_n}{\sqrt{d^2 + x_n^2}} \right],$$

which, for $d < s/2$ may be further simplified to

$$f_N = \frac{n^2 dx_n}{(2\pi d)}.$$

Finally, if we let $dx_n$ be one unit of length, we obtain the expression

$$f_N = n^2/(2\pi d) \ (x \ 10^{-3} \ \text{dynes/cm}).$$

These units are consistent with the electric fields used, which are in volts per centimeter.

In order to simplify the calculations, equation (1-12) will be used, bearing in mind that the equations derived using it will be valid only for $d = x_n - x_e$ less than approximately $s/2$, or one half the stopping distance of the ionizing particle. After this point has been reached, the motions of the electrons and holes are more accurately described by the equations of motion derived from the case of no charge interaction.

Using equation (1-12), we obtain for the field between the lines of charge (produced by charge interaction)

$$E_N = n/(2\pi d) \ \text{volts/cm}.$$
so that the total field actually seen by the lines of charge at each point in the depletion layer becomes (by superposition)

\[ E = E_{\text{bias}} - \frac{E_N}{2} = -\left(\frac{2V}{x_p}\right)(1 - x/x_p) - n/(2\pi\varepsilon d), \]

where \( d = (x_n - x_e) \), and the unsubscripted \( x \) may be either \( x_e \) or \( x_n \). This equation and all those following it in this section assume the model of Figure 4.

Using equation (1-14), equations (1-15) and (1-16) are derived in a manner exactly similar to that used in obtaining equations (1-8) and (1-9), although the integration is slightly more complicated. We obtain

(1-15)
\[ t_e = K_1 \ln \left[ \frac{x_e^2 + b_e x_e + c_e}{x_o^2 + b_e x_o + c_e} \right]^{1/2} \]

\[ \left( \frac{2x_e x_o + b_e (x_o + x_e) - (x_o - x_e)\sqrt{b_e^2 - 4c_e}}{2x_e x_o + b_e (x_o + x_e) + (x_o - x_e)\sqrt{b_e^2 - 4c_e} + 2c_e} \right)^{K_2} \]

where \( b_e = -(x_p + x_n) \), \( c_e = (x_n x_p + \frac{n x_p^2}{4\pi\varepsilon V}) \), \( K_1 = -\frac{x_p^2}{(2\pi\varepsilon V)} \), and

\( K_2 = \frac{-(b_e/2 + x_n)}{\sqrt{b_e^2 - 4c_e}} = \frac{x_p - x_n}{2\sqrt{b_e^2 - 4c_e}} \)

(1-16)
\[ t_h = K_3 \ln \left[ \frac{x_o^2 + b_h x_o + c_h}{x_h^2 + b_h x_h + c_h} \right]^{1/2} \]

\[ \left( \frac{2x_o x_h + b_h (x_o + x_h) + (x_o - x_h)\sqrt{b_h^2 - 4c_h}}{2x_o x_h + b_h (x_o + x_h) - (x_o - x_h)\sqrt{b_h^2 - 4c_h} + 2c_h} \right)^{K_4} \]

where \( b_h = -(x_p + x_e) \), \( c_h = (x_e x_p + \frac{n x_p^2}{4\pi\varepsilon V}) \), \( K_3 = -\frac{x_p^2}{(2\pi\varepsilon V)} \), and
\[ K_4 = - \frac{(b_h/2 + x_e)}{\sqrt{b_h^2 - 4c_h}} = \frac{(x_p - x_e)}{2 \sqrt{b_h^2 - 4c_h}} \]

It might seem that the above equations were left in a singularly uninformative form. However, they are of such a degree of complexity that it would be difficult to tell by inspection anything about their behavior, regardless of their form. In addition, an explicit solution of equations (1-15) and (1-16) for \( x_e \) and \( x_h \) would be quite difficult and time consuming. Because of the availability of the IBM 1620 computer, it was decided to program the computer to obtain the hole and electron positions as a function of time. The solution was obtained by a "cut and try" method, and the above form of the equations is quite satisfactory for this purpose. It should be noted that equations (1-15) and (1-16) reduce to equations (1-8) and (1-9) when \( n = 0 \). In the solutions of (1-15) and (1-16), the specific ionizations used were as follows: \( 2.2 \times 10^6 \) hole-electron pairs per cm, corresponding approximately to 10 mev alpha particles; \( 5.5 \times 10^7 \), corresponding approximately to 2.5 mev protons; and \( 1.0 \times 10^6 \). This gives values of \( n \) as follows: \( n_{\alpha} = 3.52 \times 10^{-11} \), \( n_p = 8.79 \times 10^{-12} \), \( n_e = 1.60 \times 10^{-13} \) coulombs per centimeter.

Equations (1-15) and (1-16) were first equated, and the resulting expression was solved for \( x_h \) at specified values of \( x_e \). Corresponding values of \( x_e \) and \( x_h \) were then used to find the transit times of holes and electrons to these points. The hole and electron positions at a given time were used to find (by means of equation (1-14)) the field seen by the holes and electrons at each point in the depletion layer. The effective internal potential traversed by the holes or electrons is given by the integral of the field obtained above. The integration was accomplished by planimetering the graphs of the field in the depletion layer. Graphs of the field seen by the holes and electrons, and the internal potential traversed by them for different values of detector doping and bias, and different ionizing particles, are shown on pages 17 through 26. Fields and potentials for different ionizing particles are denoted, logically enough, by subscript \( \alpha \), \( p \), and \( e \), for alphas, protons, and electrons, respectively.
It should be noted on the curves of field and potential that there are cases in which the field is shown as zero for a (in some cases fairly large) distance about the point where the ionizing particle enters the detector. Actually, according to the equations used in obtaining the curves, the fields should go negative at these points. They were not so shown for two reasons. First, because we know that the situation is not initially as it appears in the equations. The charges are initially in two concentric cylinders, not two lines. This would tend to reduce the apparent field at first. A second and more pragmatic reason is that there is only about a two percent difference in the potential obtained in the worst case between the graphs as shown and as the equations would have them shown. Hence, there is little cause for worry, even if it is not known exactly what the shape of the potential curve is for a short distance about $x_0$.

1-5. Rise Time Determination

The depletion region of a detector may be considered as a parallel plate capacitor, the capacitance varying with the detector dimensions, doping, and bias. The time rate of energy delivered to a charged particle (a hole or electron) moving in the depletion region is $(dW/dt) = qvE = q(dx/dt)(dY/dx) = q(dY/dt)$. Motion of a charged particle corresponds to a current, $i$, so we also have $(dW/dt) = i$ where $Y$ is the potential traversed by a particle moving completely across the depletion layer. Hence, $i = (q/\mu_{max})(dY/dt)$. Now the motion of a particle through a potential $d$ gives a change of the charge on the detector capacitance, $C$, of $dQ = \int i dt = (q/\mu_{max})dY$; and thus a voltage change of $dV = dQ/C = (q/C\mu_{max})dY$, if there is no leakage of charge during the motion of the particle.

Thus, for a hole created at a point $x_0$ and moving to a point $x_h$, we get a potential change of

$$\Delta V_h = \left(\frac{q}{C\mu_{max}}\right)(\mu_{x_h} - \mu_{x_0}).$$
and for an electron created at $x_0$ and moving to $x'_e$ we get

$$(1-18) \Delta V_e = \left(\frac{q}{C_{max}}\right)(\psi_e' - \psi_e),$$

In the model we are using, all holes and electrons are created at the same point and travel through the depletion layer in lines parallel to its edges. Thus, we see from equations (1-17) and (1-18) that the change of potential due to the motion of the charges is of the form $K(\psi_h' - \psi_e)$, where $K$ is a constant. It is a simple matter, then, to obtain rise times from the potential curves and the time versus distance data obtained from the computer. The computer data is not included in this work, since it is quite lengthy, and all we are interested in is rise times.

A table of rise times for the different detector dopings, biases, and particle types is given on page 13. In the case of no charge interaction, the rise times were calculated by means of equations (1-2), (1-9), (1-10), (1-11), (1-17) and (1-18) in a straightforward way, as well as graphically. In only one case did the graphically calculated rise time differ from the numerically calculated rise time by more than ten percent. This error must be attributed to the graphical methods employed. In most cases, the error was less than five percent. A curve of rise time versus detector doping is shown on page 16. It should be noted that the charge collection time is independent of detector bias.

Rise time calculations for the case in which the ionizing particle enters the detector perpendicular to the depletion region edges have been carried out (neglecting charge interaction) elsewhere, using essentially the same methods used in this paper. Charge collection times for this case are taken from a nomograph giving the charge collection time as a function of resistivity and the ratio of particle range to depletion layer width, when such data is needed for comparison with experimental results in a later section.
1-6. Results of Theoretical Calculations

In the table below, the rise times shown are for the case in which the ionizing particle passes through the depletion layer parallel to its edges. Unless otherwise noted, the particle through the center of the depletion layer. (i.e., \( x_0 = x_p/2 \). See figure 4.) The column containing numerically calculated rise times is denoted with an asterisk. All other rise times were calculated graphically.

<table>
<thead>
<tr>
<th>Bias (volts)</th>
<th>Res. (A-cm)</th>
<th>No Charge(^\dagger) Interaction</th>
<th>No Charge(^\dagger) Interaction</th>
<th>Alpha (sec)</th>
<th>Proton (sec)</th>
<th>Electron (sec)</th>
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<td>1000</td>
<td>( 4.4 \times 10^{-10} )</td>
<td>( 4.5 \times 10^{-10} )</td>
<td>-----</td>
<td>-----</td>
<td>( 4.5 \times 10^{-10} )</td>
</tr>
<tr>
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<td>&quot;</td>
<td>&quot;</td>
<td>( 4.6 )</td>
<td>-----</td>
<td>-----</td>
<td>( 4.6 )</td>
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<td>&quot;</td>
<td>( 4.4 )</td>
<td>-----</td>
<td>-----</td>
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<tr>
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<td>&quot;</td>
<td>( 4.3 )</td>
<td>-----</td>
<td>-----</td>
<td>( 4.5 )</td>
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<td>&quot;</td>
<td>( 4.3 )</td>
<td>-----</td>
<td>( 4.5 \times 10^{-10} )</td>
<td>-----</td>
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<td>&quot;</td>
<td>&quot;</td>
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<td>-----</td>
<td>( 4.4 \times )</td>
<td>-----</td>
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<td>( 3.1 )</td>
<td>( 3.1 \times 10^{-10} )</td>
<td>( 3.2 )</td>
<td>-----</td>
</tr>
<tr>
<td>-100</td>
<td>500</td>
<td>2.2</td>
<td>2.2</td>
<td>-----</td>
<td>2.4</td>
<td>-----</td>
</tr>
<tr>
<td>-200</td>
<td>500</td>
<td>2.2</td>
<td>2.5</td>
<td>2.4</td>
<td>2.5</td>
<td>-----</td>
</tr>
</tbody>
</table>

The following three rows are rise times for 50 -cm silicon at -200 volts bias for different values of \( x_0 \). All column headings are the same as above, with Bias and Resistivity replaced by \( x_0 \), the point at which the ionizing particle enters the depletion region.

\[
x_0 (cm)
\[
0.4x_p \quad 3.2 \times 10^{-11} \quad ----- \quad 3.3 \times 10^{-11} \quad ----- \quad ----- \\
0.5x_p \quad 2.2 \quad ----- \quad 2.0 \quad ----- \quad ----- \\
0.6x_p \quad 9.4 \times 10^{-12} \quad ----- \quad 8.0 \times 10^{-12} \quad ----- \quad ----- \\
\]

Here, \( x_p \) is the depletion layer width, \( 3.26 \times 10^{-3} \) cm.
CALCULATED RISE TIME VS DETECTOR RESISTIVITY

\( x_0 = \frac{x_P}{2} \)

P-TYPE DETECTOR
INTERNAL FIELD & POTENTIAL IN DEPLETION REGION

BIAS = -100 VOLTS, RESISTIVITY = 500 Ω-cm
DEP. LAYER WIDTH (XP) = 7.28 X 10^-3 cm
INTERNAL FIELD & POTENTIAL IN DEPLETION REGION

BIAS = -200 VOLTS, RESISTIVITY = 500 Ω·cm

DEP. LAYER WIDTH ($\chi_p$) = 1.03 $\times$ 10^{-2} cm
INTERNAL FIELD & POTENTIAL IN DEPLETION REGION
BIAS = -500 VOLTS, RESISTIVITY = 500 Ω·CM
DEP. LAYER WIDTH (X₀) = 1.63×10⁻² CM

\[ E(\text{VOLTS/cm}) \quad \psi(\text{VOLTS}) \]

\[ E_e \quad \psi_e \quad \psi_p \quad \psi_\alpha \]

DEP REGION POSITION

0.0 0.4 0.8 1.2 1.6×10⁻² (CM)
INTERNAL FIELD & POTENTIAL IN DEPLETION REGION
BIAS = 200 VOLTS, RESISTIVITY = 750 $\Omega \cdot \text{CM}$
DEP. LAYER WIDTH ($x_p$) = $1.26 \times 10^{-2}$ CM
INTERNAL FIELD & POTENTIAL IN DEPLETION REGION

BIAS = -50 VOLTS, RESISTIVITY = 1000 \, \Omega \cdot \text{cm}

DEP. LAYER WIDTH (x_p) = 7.28 \times 10^{-3} \, \text{cm}
INTERNAL FIELD & POTENTIAL IN DEPLETION REGION

BIAS = -200 VOLTS, RESISTIVITY = 1000 Ω·cm

DEP. LAYER WIDTH (Xp) = 1.46 X 10⁻² cm
II. Video Amplifier Design

2-1. General Design Considerations

In order to observe rise times on the order of magnitude of those calculated previously, it was necessary to construct a pulse amplifier, the rise time of which was to be approximately one nano-second, less, if possible. A small rise time necessarily implies a large bandwidth. An investigation of the literature showed that there are five major circuit configurations used today to construct wideband (sometimes called "video") amplifiers: distributed amplifier techniques, series peaking, shunt peaking, emitter degeneration, and collector-to-base feedback. It was the desire of the project advisor that the amplifier be transistorized, and that as large a bandwidth as possible be obtained using conventional circuitry, as opposed to distributed amplifier techniques. Furthermore, with present day high-frequency transistors, it appears that the only advantage the distributed amplifier offers over the feed-back amplifier is the ability to produce large power outputs. Since the last two methods mentioned above offer one the opportunity of trading mid-band gain for bandwidth, it was decided to employ them in the construction of the amplifier. The remaining question then was what the gain-bandwidth curve should be for best pulse response.

There are two kinds of video amplifiers. One is an amplifier designed to give maximally flat gain vs. frequency response for the desired bandwidth, and the other is designed for pulse amplification with no (or a very small amount of) overshoot for desired rise time. Pulse inputs in the first case result in considerable amounts of overshoot unless the bandwidth is in the neighborhood of \(1/2T_r\), cps where \(T_r\) is the rise time of the input pulse. As a matter of fact, it is possible to materially improve amplifier rise time (on the order of 50% improvement) over the maximally flat case while introducing only approximately two percent overshoot.

A complete and detailed analysis of the emitter feedback case and the collector-to-base feedback case may be found elsewhere. These
analyses however, result in equations which are too cumbersome to use in designing these circuits. The following analyses follow closely those of Reddil, and result in equations which are sufficiently simple to be used in design, but which are also accurate enough to give good results when so used.

2-2. Transistor Equivalent Circuits

An equivalent circuit of a transistor is shown in Figure 5. Note that collector body resistances and stray capacitances from emitter to collector and base are not shown.

The symbols used in the circuit above are as follows:

\[ r_e = \frac{kT}{qI_{Eдc}} \text{ ohms at } T=290^\circ K \]

\[ r_b = \text{Base spreading resistance.} \]

\[ h_{fe} = \beta / (1 + j \omega C \tau) ; \beta = \frac{\Delta f}{1 - \alpha} \]

\[ h_{fc} = \omega \beta \left(1 - \alpha\right) \omega + \]

\[ \beta = \text{Low frequency short circuit current gain.} \]

\[ r_e = \text{Emitter body and contact resistance.} \]

\[ C_s = \text{Stray capacitance from collector to base.} \]

\[ C_{pa} = \text{Capacitance from collector to base which is not directly under emitter dot. This capacitance is important in mesa type transistors.} \]

\[ C_c = \text{Collector capacitance under emitter dot.} \]

The transistor model shown in Figure 5 above, is too complicated to use for amplifier analysis. A simpler model is shown in Figure 6 below. The neglected quantities are taken into account in an approximate way later.

The h-parameters for the circuit of Figure 6 are as follows:
\[ R = \frac{r_e'}{1-d} = (\beta+1)r_e' \]

**FIGURE 5**

\[ \frac{1}{\omega R} = \frac{1}{\omega R} \]

**FIGURE 6**

\[ C_{bc}, C_{bc} \& r_e'' \]

NEGLECTED
\[ h_{11e} = r_b + \frac{R_w}{s + \omega_p}, \quad h_{21e} = h_{re} = \frac{\beta w_p}{s + \omega_p}, \quad h_{22e} = h_{le} = 0. \]

2-3. Analysis of Amplifier With Emitter Degeneration

Using the transistor equivalent circuit of Figure 6, we obtain the circuit of Figure 7 for a one stage amplifier with emitter feedback.

In order to analyze the circuit of Figure 7, we use the \( h \) parameters. The \( h \) parameters of this circuit are:

\begin{align*}
(1) & \quad h'_{11e} = h_{11e} + (h_{21e} + 1)z_e, \quad h'_{21e} = h_{21e} \\
(2) & \quad h'_{22e} = h'_{12e} = 0
\end{align*}

Using (1) and (2) we obtain

\begin{align*}
(3) & \quad \frac{\varepsilon_o}{\varepsilon_s} = \frac{-h'_{21e}R_L}{h'_{11e} + R_s}.
\end{align*}

When \( Z_e = R_e \),

\begin{align*}
(4) & \quad \frac{\varepsilon_o}{\varepsilon_s} = \frac{-\beta w_p R_L}{s(R_s + r_b^' + R_e) + \omega_p[R_s + R + r_b^' + (\beta + 1)R_e]}
\end{align*}

and the low frequency voltage gain,

\begin{align*}
(5) & \quad A_{vo} = \frac{\beta R_L}{R_s + R + r_b^' + (\beta + 1)R_e}.
\end{align*}

The 3 db bandwidth is seen to be

\begin{align*}
(6) & \quad B_v = \omega_T \left[ \frac{R_e + r_b^' + (R_s + r_b^')/(\beta + 1)}{R_s + r_b^' + R_e} \right].
\end{align*}

When \( Z_e = R_e w_e/(s + w_e) \), i.e., the feedback circuit is a parallel combination of \( R_e \) and \( C_e \), and \( w_e = 1/(R_e C_e) \),

\begin{align*}
(7) & \quad \frac{\varepsilon_o}{\varepsilon_s} = \frac{-\beta w_p R_L(s + \omega_e)}{s^2(R_s + r_b^') + 2s \omega_e [(\omega_e(R_s + R + r_b^') + \omega_p R_e (R_s + R + r_b^') + \omega_e R_s + R + r_b^') \omega_e R_s + R + r_b^' + R_e (\beta + 1)]}
\end{align*}
FIGURE 7

FIGURE 8
\[
\frac{e_o}{e_s} = \frac{K(s + \omega e)}{as^2 + bs + c}
\]

When the above transfer function is driven by a step function, time response will contain only damped exponentials which combine to give a monotonic rise to the final value, provided the denominator quadratic has real roots; i.e., \(b^2 \geq 4ac\). This is a sufficient, but not a necessary condition for no overshoot.

With the condition \(b^2 = 4ac\), we get the value of \(\omega_0\) as:

\[(8) \omega_0 = \frac{4R_v}{1 - \frac{R_e}{R_s + r'_b + R_e}}\]

using the approximation \((R_s + R + r'_b) \ll (R_s + R_e + r'_b)\).

This approximation will be satisfied if \(B_v \gg 5f\) where \(B_v\) is the bandwidth with only the resistance \(R_e\) connected.

The double pole location will lie at 

\[-\frac{b}{2a} = 2B_v\]  

Now,

\[(9) \frac{e_o}{e_s} = \frac{K}{a} \left[ \frac{s + \frac{4R_v}{1 + R_s + r_b'} R_e}{(s + 2B_v)^2} \right] \]

The rise-time improvement with capacitive peaking is potted as a function of \(R_e/(R_s + r_b')\) in figure (9). This graph is taken from Reddi, who, in his turn, took it from class notes at Stanford. It is apparently derived by actually calculating the transient response of the amplifier, with and without peaking.

The rise-time improvement factor, \(\eta\), is defined as

\[\eta = \frac{\text{One-stage ampl., rise-time, only } R_e \text{ connected}}{\text{Rise-time of the stage with } R_e \text{ and } C_e \text{ in the feedback where } C_e \text{ is chosen for no overshoot in pulse response}}\]

\[= \frac{2\pi \times 0.35/B_v}{\text{Rise-time of the stage with } R_e \text{ and } C_e \text{ in the feedback where } C_e \text{ is chosen for no overshoot in pulse response}}\]

The values of \(\eta\) are not useful when \(R_e/(R_s + r_b')\) is less than 0.05, since these values of \(R_e\) correspond to the small bandwidth case and the approximations used in the analysis are not valid.
\[ \frac{R_e}{R_s + r'_b} \quad \text{(EMITTER FB.) OR} \quad \frac{r_b + R_L}{R_f} \quad \text{(C-B FB.)} \]
2-4. Analysis of Amplifier With Collector-to-Base Feedback

We now proceed with the analysis of the collector-to-base single-stage amplifier. The circuit of Figure 8 is used in this analysis. Here, the $y$ parameter representation is more useful. The parameters of the circuit including feedback are:

$$y_{1le} = \frac{s + \omega f}{s r_b + \omega f (R + r_b') + y_f}$$

(10)

$$y_{22e} = -y_{12e} = y_f$$

(11)

$$y_{21e} = \frac{s + \omega f}{s r_b + \omega f (R + r_b')}$$

(12)

When $y_f = G_f = 1/R_f$,

$$\frac{i_2}{i_s} = \frac{G_L}{(y_{11} + G_s)(G_f + G_L) + G_s y_{21}} = \frac{A_{io} B'_1}{s + B'_1}$$

(13)

Where $A_{io}$ = low frequency current gain

$$A_{io} = \frac{\beta G_L}{(G_f + G_s) \left[ 1 + (G_s + G_f) (R + r_b') \right] + G_f \beta}$$

(14)

$B'_1$ = 3 dB bandwidth

$$B'_1 = \frac{\beta G_L}{A_{io} (G_f + G_s) \left[ 1 + (G_s + G_f) (R + r_b') \right]}$$

(15)

The current gain-bandwidth product is then equal to

$$A_{io} B'_1 = \frac{\beta G_L}{(G_f + G_s) \left[ 1 + (G_s + G_f) (R + r_b') \right]}$$

(16)

This is a maximum when $G_s = 0$; let us consider only this case.

-26-
\[
\begin{align*}
(17) \quad \frac{i_2}{I_s} &= \frac{y_{21}G_L}{y_{11}(y_{22} + y_L) - y_{12}y_{21}} \\
\text{When } y_f = 1/Z_f = 1/(sL + R_f), \text{ i.e., resistance and inductance in series in the feedback path, then} \\
(18) \quad \frac{i_2}{I_s} &= \frac{K'(s + \omega_f)}{s^2 + Bs + C}, \text{ where} \\
B &= R_L\omega_f(G_f + G_L + r_b'G_fG_f) \\
C &= R_L\omega_f\omega_n\left[G_f(B + 1) + G_LG_f(R + r_b')\right] \\
K' &= \beta \omega_n \\
\text{Here the approximations used in obtaining the expressions are well satisfied if the bandwidth is greater than } 5\omega_n. \\
\text{The gain function, equation (18), is of the same form as in the case of emitter feedback. For no overshoot the value for } \omega_f = R_f/L_f \text{ is given by} \\
(19) \quad \omega_f &= 4B_1/(1 + \frac{r_b' + R_L}{R_f}) \\
\text{where } B_1 \text{ is the bandwidth with only resistance } R_f \text{ in the feedback path (see equation 15, with } G_z = 0), \\
(20) \quad B_1 &= \frac{1}{\omega_n}\left[\frac{(G_f + G_L)[1 + G_f(R + r_b')] + G_f\beta}{(G + G_L)(1 + G_fr_f')}\right] \\
\text{The expression for current gain is of the form:} \\
(21) \quad \frac{i_2}{I_s} &= \frac{K'[s + 4B_1/(1 + \frac{r_b' + R_L}{R_f})]}{(s + 2B_1)^2}. \\
\text{This current gain equation is in the same form as the voltage gain expression in the emitter feedback case (equation (9)). The rise-time-}
\end{align*}
\]
improvement factor in this case can be defined in the same way as before, and the Figure 9 can be used except that the variable in this case is \((r_b' + R_L)/R_T'\).

2-5. Corrections for Factors Neglected in Analysis

Having completed the analysis of the amplifiers using both types of feedback, we now proceed to take into account in an approximate way the factors in the transistor equivalent circuit which were neglected in order to simplify the analysis. The presence of \(r_c''\) (which varies from 1-5 ohms depending on the type of transistor) has the effect of built-in emitter feedback in the transistor. In the case of an emitter feedback circuit without capacitive peaking, \(r_e''\) adds to the externally connected \(R_e\) and hence the calculated value of \(R_e\) should be reduced by \(r_e''\). In the case of capacitive peaking, \(r_e''\) is not included in peaking and the correction in the wideband case \((R>>r_e'')\) is not very important. In the collector-to-base feedback case, \(r_e''\) can be taken into account by writing \(R = (r_e' + r_e'')/(\beta + 1)\) instead of \(r_e'/(\beta + 1)\). At high frequencies \(C_e\) and \(C_{bc}\) are the primary causes of the non-unilateral characteristic of a transistor. \(C_{bc}\) varies with collector-to-base reverse bias voltage \((V_{CB})\); it is lower for high voltages and is independent of emitter bias current \((I_E)\). \(C_c\) varies with \(V_{CB}\) and \(I_E\). It decreases with higher values of \(V_{CB}\) and increases with higher values of \(I_E\). Accounting exactly for \(C_c\) and \(C_{bc}\) in amplifier analysis would be quite complicated. An approximate way of taking into account \(C_c\) and \(C_{bc}\) is to add a Miller type capacitance across the emitter diffusion capacitance in the equivalent circuit and modify the value of \(\omega_p\) or \(\omega_T\). Such a correction is dependent on stage voltage gain. The type of correction to \(\omega_p\) or \(\omega_T\) is to reduce these quantities by a factor \(1/(1+K\omega_{R.C})\).

In this expression, \(C = C_c + C_{bc}\) and \(K\) is a constant depending on the transistor, and on the type of feedback used. In the emitter feedback case, the effect of \(C_c\) is more pronounced. For transistors similar to the one used here, (Philco T-2351), a value of \(K = 0.7\) for the collector-to-base feedback case have been found useful. An effective beta cutoff frequency \(\omega_p'\) should be used in all the above derived equations.
in place of \( \omega_p \), where

\[
(22) \quad \omega_p' = \frac{\omega_p}{1 + B \omega_p R_e C}.
\]

Similarly, \( \omega_T \) should be replaced by \( \omega_T' \), where

\[
(23) \quad \omega_T' = (\beta + 1) \omega_T.
\]

We may now use equations (22) and (23) to put equation (6) into a more convenient form.

\[
(24) \quad R_g + r^* + R_e + (1 + B) R_e.
\]

From this and equation (5) we obtain the voltage gain bandwidth product of the single-stage amplifier with emitter feedback.

\[
(25) \quad \text{VGBP} = A_v(\omega_p R_e - R_s + r^* + R_e).
\]

The rise time for pulse response is

\[
(26) \quad r.t. = \frac{0.35}{B_v} \text{ seconds (} B_v \text{ in cps)}.
\]

From equation (25) it is seen that the VGBP is a maximum for \( R_s = 0 \). Equation (24) shows that if the amplifier is driven by a current source, \( B_v \) is independent of feedback to a first order approximation. Thus we may deduce that for a low impedance driving source, an emitter feedback stage allows wide gain-bandwidth trading and gives good VGBP.

We may find the value of capacitance which, when connected across \( R_e \), gives maximum rise time with no overshoot from equation (8) as follows:

\[
(27) \quad \omega_c = \frac{1}{R_c C} = \frac{1}{B_v} \left( 1 + \frac{R_e}{R_s + r^*} \right)
\]

This form is particularly convenient for use with the rise time improvement curve of Figure 5. The low frequency gain remains the same as the
gain of the stage without peaking.

For the single-stage amplifier with collector-to-base feedback, it is seen from equations (14) and (15) that if the driving source is a voltage source, i.e., $G_g = \infty$, the bandwidth is less than or equal to $\omega_p (r_b' + R)/r_b'$, and is independent of the feedback impedance to a first order approximation. Hence, a low impedance source will not allow wide bandwidth variations with this type of feedback. The current gain bandwidth product is found to be

$$CGBP = A_{\text{lo}}B_i = \frac{\beta \omega_p^2 G_L}{(G_F + G_L) [1 + (G_F + G_L) r_b']}.$$  

Since the emitter feedback stage with a fairly large (about $kK$ or larger) collector resistor will closely resemble an ideal current source, let $B$ be the bandwidth obtained by letting $G_g = 0$ in equation (15). This gives

$$B_i = \left[ \frac{1 + G_F (R + r_b') + \beta G_F / (G_F + G_L)}{1 + G_F r_b'} \right] \omega_p.'$$

The rise time for pulse response is: $r.t. = 0.35/B_i$ (seconds).

The value of peaking inductance, $L_p$, may be computed from equation (19) by setting $\omega_p = R_F/L_p$. The values of $\eta$, the rise-time improvement factor, for $(r_b' + R_L)/R_F$ in the neighborhood of or less than 0.2 are not useful, since this corresponds to higher values of $R_F$, or smaller bandwidths, where the approximations used in deriving the above equations cannot be justified.

It is beyond the scope of this paper to prove rigorously that the best combination for a two stage amplifier driven by a low impedance source is a stage with emitter feedback followed by a stage of collector-to-base feedback. However, it is felt that the preceding analysis has given indication that this is indeed the case. It suffices to note that this fact has been verified experimentally. The next best combination is two stages of collector-to-base feedback, and the worst is two stages with emitter degeneration.
Two important properties of multi-stage pulse amplifiers are that if individual stages are designed for monotonic pulse response, the overall rise time is the root mean square of the individual rise times of the stages, i.e., \( T_R = (t_1^2 + t_2^2 + \ldots + t_n^2)^{1/2} \), where \( T_R \) is the overall rise time and \( t_i \) is the rise time of the \( i^{th} \) stage. Further, for given gain the best overall rise time results when all stages are designed to have the same rise time.

Assuming the desired overall rise time, source impedance, load impedance, and the transistor parameters are known, starting from the last stage (the c-b fb. stage), the design of the feedback circuitry for the pulse amplifier is accomplished as follows: (1) Estimate the number of stages necessary to produce the desired gain, calculate the desired rise time per stage using the formulae above. (2) Calculate the reduced values \( \omega' \) of \( \omega_n \) and \( \omega' \). These reduced values are to be used in the design equations in place of the original values. (3) Assume a rise-time improvement factor, \( \gamma \). (4) Using this value of \( \gamma \), calculate the necessary rise time and bandwidth of the uncompensated stage. (Note: \( \gamma \) will in general be different in the emitter and collector-base feedback stages.) (5) Using this bandwidth in equation (29) or (24), whichever applies to the stage under consideration, calculate \( R_f \) and the ratio \( (r_b' + R_e)/R_f \), or \( R_e \) and the ratio \( R_e/(R_e + r_b') \), respectively. (6) If the calculated ratio when checked with Figure (9), the rise-time improvement factor curve, gives the value of \( \gamma \) assumed in step (3), you are finished. If it does not, pick another value of \( \gamma \) and start again at step (4). Once \( R_f \) (or \( R_e \)) is determined, the calculations are straightforward.

When \( R_f \) and hence \( L_f \) (or \( R_e \) and \( C_e \)) have been determined, the appropriate stage gain may be calculated. It is necessary to calculate the current gain of the collector-to-base feedback stage before going on to the emitter stage, since \( A_{10} \) is used in finding the low-frequency input impedance of the c-b stage, which is in turn used
as the load impedance of the preceding stage. The input impedance of the collector-base feedback stage is given by

\[
\frac{A_{i0}(R + r')}{\beta(1 + R_L/R_2)}.
\]

It should be noted as a final remark on the design procedure (for the feedback elements) that all calculations are carried out using equations derived for the uncompensated stages. We are actually designing uncompensated stages having a bandwidth equal to the desired bandwidth divided by the rise time improvement factor.

The low frequency cutoff point of the amplifier (which will be determined by the choice of the coupling capacitors) is relatively unimportant for the purposes of this paper. In fact, it would be desirable to have it as high as one or two megacycles, in order to eliminate some of the low frequency noise. The design of the d-c portions of the circuit is completely conventional, and will not be discussed here.

Before purchasing expensive uhf transistors, two two-stage amplifiers were designed using the above procedure. In both cases, the performance was within 15% of the calculated values. As with most design procedures, the results obtained may be improved upon by laboratory testing and adjustment after construction. In this case, the amplifier response may be adjusted to give the desired degree of overshoot by varying \(C_e\) and \(L_p\), increasing either tending to increase the amount of overshoot.

As far as actual construction is concerned, since we are interested here in bandwidths of approximately 400 mc., the usual uhf construction practices must be adhered to, such as judicious use of interstage shielding; using extreme care in component placement to minimize stray capacitance, lead inductance; etc. The reported stability difficulties encountered in amplifiers having feedback loops around more than one transistor stage was another reason for using the circuit configuration discussed above.
Using Philco T-2351 uhf transistors ($f_T = 2.5$ kmc., typical), a two stage amplifier was designed to give a risetime of 0.9 nanoseconds at a gain of 12 db. The design was carried out using the manufacturer's specifications, since none of the transistors were available so that actual measurements could be made of their parameters. "Educated guesses" were made about parameters for which no data was given. Experimental data on the amplifier performance is given in Section III.
III. Experimental Results

3-1. Amplifier Performance Data

A video amplifier was designed as outlined in section II, using the Philco T-2351 transistors. Three transistors were obtained, and the amplifier was designed initially to have three stages; the first, an emitter feedback stage, followed by two collector-base feedback stages. The amplifier was designed to have a gain of 21 db and a rise time of 0.9 nano-seconds. In testing the completed amplifier, it appeared that the performance of one of the transistors was considerably worse than that given in the manufacturer's specifications as typical. This was, perhaps, to be expected, since this transistor is just out of the experimental stage, and Philco states that "Due to the small number produced, no degree of statistical quality control may be exercised." After considering the probable rise times of the available detectors, it was decided to operate the amplifier at a rise time of 2 nano-seconds and a gain of approximately 20 db. Unfortunately, the amplifier worked well for a few days, and then, for some undetermined reason, one of the transistors went out. The amplifier was then re-designed with only two stages. The schematic diagram of the two stage amplifier as used in the experiments is shown in Figure 10.

Fixed biasing was used so that the operating point of the transistors could be varied to find the point at which the gain-bandwidth product was a maximum. It was found, as was expected, that the optimum operating point was obtained for $V_{CE}$ slightly less than 0.75$V_{CC}$, where $V_{CC}$=20 v. It was also found that as the amplifier rise time approached 1.5 nano-seconds, the gain-bandwidth product fell off quite rapidly. It is assumed that this was due to the lumped parameter circuitry used. A rise time of 1.5 nano-seconds corresponds to a bandwidth of approximately 230 mc., and it would probably be necessary to use distributed circuitry to get an efficient amplifier at frequencies higher than this. The two-stage amplifier was adjusted to operate at a gain of approximately 11 db at a rise time of 2 nano-seconds, Figure 11 shows the amplifier gain and time...
Gain = 11 dB
RISE TIME = 2 NANO SEC

FIGURE 10
Amplifier gain and time delay.
Horizontal scale 5 nsec/div;
Vertical scale, 20 mv/div.

Figure 11
delay, and Figure 12 shows the rise time of the amplifier. These measurements were made using a Tektronix 561 oscilloscope with sampling plug-ins having a rise time of 0.4 nano-seconds in conjunction with a Tektronix 110 mercury pulser with a rise time of 0.25 nano-seconds.

In order to provide a long decay time for the output pulse from the detectors and to provide impedance matching, an emitter follower was also constructed. The emitter follower unit also contained provisions for biasing the detectors. The rise time of the emitter follower and biasing unit was measured as 2.3 nano-seconds. This gives a total system rise time of 3 nano-seconds, as shown in Figure 13. A schematic diagram of the emitter follower and biasing unit is shown in Figure 14.

3-2. Detector Characteristics

There are three factors influencing the characteristics of the output voltage of the detector: (1) the charge collection time, (2) the equivalent circuit of the detector, and (3) the characteristics of the amplifier and display system. (1) and (3) have already been discussed. (2) will now be considered. It has been shown\(^\text{15}\) that, when the charge collection time is sufficiently fast (< 1/10 of the circuit rise time), the charge on the detector may be considered as a δ-function charge distribution at time \( t = 0 \), and that the response of the detector to a δ-function charge distribution at \( t = 0 \) can be approximated from the equivalent circuit of Figure 15. \( C_D \) is the capacitance associated with the depletion region, \( R_D \) is the series (ohmic) resistance of the detector, \( R_L \) is the input resistance of the emitter follower unit, and \( C_A \) is the input capacitance of the emitter follower.

For \( R_L \) sufficiently large (\( \simeq 1 \, \text{k} \)) and \( C_A \gg C_D \), the 10% to 90% rise time of the detector output pulse is approximately equal to \( 1.25 R_D C_D \). In our case, both criteria are satisfied. It will be possible to observe the charge collection time only when it is greater

-35-
Amplifier rise time. Horizontal scale, 2 nsec/div.

Figure 12
System rise time

Horizontal scale: 2 nano-sec./division.

Figure 13
EMITTER FOLLOWER & DETECTOR BIASING CIRCUIT

FIGURE 14

DETECTOR EQUIVALENT CIRCUIT

FIGURE 15
than the rise time due to \( R_D \) and \( C_D \). An investigation of the detectors available showed that the majority of them had large effective junction areas and resistivities supposedly ranging from 1000 to about 6000 ohm-centimeters. These two conditions imply a large junction capacitance and hence a large \( R_D C_D \) rise time. The capacitance of the smaller detectors was measured on a bridge, and their series resistance was measured on a Tektronix transistor curve tracer. This proved to be an inaccurate method for measuring the series resistance, but was as good as any other method available. The results of the capacitance and resistance measurements on the two small detectors are shown in Figures 16 and 17.

The Hughes 8I is a p-type detector with an effective junction area of 2 mm\(^2\) and a maximum reverse bias of 200 volts. Its resistivity is not given in the manufacturer's specifications but may be calculated from the capacitance measurements. For a step junction, we have

\[
C_s = \frac{\varepsilon}{(x_p - x_n)} = \frac{\varepsilon}{x_p} \quad \text{for a p-type detector.}
\]

Here \( C_s \) is the capacitance per unit area of junction, \( x_p \) is approximately equal to the depletion layer width. At 30 volts reverse bias, Figure 16 shows that \( C_D = 3.6 \text{ pf} \). Therefore, \( C_s = 1.8 \times 10^{-10} \text{ ffd/cm.} \), which gives \( x_p = 5.9 \times 10^{-3} \text{ cm} \). From equations (1-4) and (1-5) we find \( \rho_h = 1100 \text{ ohm-cm.} \) The charge collection time for this detector for 8 mev alpha particles with a range of about 3.5 \( \times 10^{-3} \text{ cm} \) is approximately 2.3 nano-seconds.

The Hughes 5C-1 is also a p-type detector with a effective junction area of 5 mm\(^2\) and a maximum reverse bias of 60 volts. Its resistivity was calculated to be 100 ohm-cm. This figure is probably somewhat low, due to two factors: (1) it was not possible to determine the lead and case capacitance, which in this case might cause the measured capacitance to be high by as much as 5 pf. This would change the calculated resistivity to 140 ohm-cm. (2) the junction is not exactly a step junction. These comments also apply to the calculations made on detector 8I, where the capacitance might be off by as much as 1 pf due to the case and lead capacitance. This would approximately double both the calculated resistivity and charge collection time of detector 8I.
RISE TIMES AND JUNCTION CAPACITANCE VS. DETECTOR BIAS

DETECTOR: HUGHES 8I
SERIES RESISTANCE: ≤ 900 Ω
MAX. BIAS: 200 VOLTS
RESISTIVITY: ≤ 1100 Ω·CM

FIGURE 16
RISE TIMES AND JUNCTION CAPACITANCE VS. DETECTOR BIAS

DETECTOR: HUGHES 5C-1
SERIES RESISTANCE: ≈ 0.3 kΩ
RESISTIVITY: ≈ 100 Ω·cm
MAX. BIAS: 60 VOLTS

FIGURE 17
The charge collection time for detector 5C-1 for an 8 mev alpha particle is approximately 0.6 nano-seconds.

3.3 Experimental Determination of Rise Times

The rise times of the Hughes 5C-1 and 8I detectors were observed at different values of bias for 8 mev alpha particles. The amplifier and emitter follower previously described were used, their output being displayed on the sampling oscilloscope. The particle source actually produced both 6.6 and 8 mev alphas, but the oscilloscope was adjusted so that it triggered only on the 8 mev particles. The larger area of detector 5C-1 and its correspondingly higher counting rate made it possible to measure the rise times for this detector directly on the oscilloscope screen. A typical output pulse from this detector is shown in Figure 18. The rise time of the pulse of Figure 18 is approximately 13 nano-seconds; the decay time, approximately 100 nano-seconds.

The small area of detector 8I made it necessary to measure the rise time from time exposure oscilloscope pictures. One of these pictures is shown in Figure 19. The results of the rise time measurements are shown in Figures 16 and 17. The rise time due to the detector $R_D$ and $C_D$ are shown in the same figures. The curves of $1.25 R_D C_D$ for both detector 5C-1 and detector 8I are somewhat in error, values shown for detector 5C-1 being high, and values for detector 8I being low. As mentioned previously, this is the result of the measurements of $R_D$.

Results obtained with detector 8I were quite good, considering its low counting rate.

It should be noted here that the observed rise times are not exactly the detector output pulse rise times. The actual output pulse rise time, $T_{act} = \sqrt{T_{obs}^2 - T_{syst}^2}$, where $T_{obs}$ is the observed rise time, which is always greater than 10 nano-seconds, and $T_{syst} = 3$ nano-seconds. The maximum possible difference between $T_{obs}$ and $T_{act}$ is $10 - (T_{act})^{1/2} = 0.45$ nano-seconds. For this reason, it was not felt necessary to plot the actual output pulse rise times.

It is to be noted that the rise times of the output pulses of
Hughes detector 5C-1, output pulse. 60 volts bias

Horizontal scale: 20 nsec./div. Vertical scale 5 mv/div.

Figure 16
Hughes Detector 81 output pulses.

Upper pulse: 60 volts bias. Lower pulse: 50 volts bias.

Horizontal scale: 5 nsec./div. Vertical scale: 5 mv/div.

Figure 19
detector \( \delta I \) remained constant at approximately 10.5 or 11 nano-seconds for large reverse biases. Among other things, this indicates that the calculated resistivity and charge collection time for this detector was indeed low, as indicated previously, since this phenomenon has been observed by others only at the point where the rise time due to \( R_D \) and \( C_D \) approaches the charge collection time. However, the calculated charge collection time would have to be off by a factor of five for it to be the cause of the observed "saturation" of the rise time. This is impossible. The most the calculated charge collection time could be off is a factor of three, which would still leave it approximately one half the observed rise time.

Thus it seems that there is a real effect causing a "saturation" of the rise time at some value about a factor of two larger than the charge collection time for the detector. This phenomenon has been observed by others\(^{19}\). Unfortunately, the highest energy electron detector \( \delta I \) will stop is approximately 300 kev. Since the maximum detector output pulse with 8 mev alpha particles was approximately 6 milli-volts, this means the maximum will be about 0.2 mv. for electrons. An attempt was made to look at the output pulse due to electrons, but the system gain was not high enough. No heavy fission fragment sources were available, and time limitations made it impossible to use the Van de Graff accelerator as a proton source. Thus it is not possible to state definitely on the basis of experimental work whether or not the observed effect is due to the specific ionization of the ionizing particle. It can be stated, however, that Raymo and Mayer\(^{20} \), who also observed this effect were able to look at particles from a Cs\(^{137} \) source, which produced the same effect on the rise time, to within experimental accuracy.
IV. CONCLUSIONS

The theoretical calculations indicate that the charge collection rise time of a semiconductor nuclear particle detector is not influenced by the differences in the magnitude of the forces of charge interaction caused by different types of ionizing particles. However, the experimental results have shown that there is definitely some effect on the charge collection time which is not taken into account in the usual methods of rise time calculation. The observed charge collection times were more than twice as large as the calculated times. Raymo and Mayer claim to have shown experimentally that this effect is not due to charge interaction, and, as mentioned above, the calculations made in this paper bear them out.
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