RICE UNIVERSITY

ACTIVE FILTERS FOR BIOLOGICAL SIGNAL ANALYSIS

BY

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A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

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Houston, Texas
May 1966
ABSTRACT

A way to handle biological signals by dividing the signals into bands in the frequency domain is the subject of this paper. Several filters were constructed and the results of a particular filter were analyzed.

The problem of obtaining a constant power spectrum when a signal is divided into a number of bands by bandpass filters was investigated. This is very important in applications where it is desired to have all the components of an incoming signal weighted equally.
ACKNOWLEDGEMENTS

I would like to sincerely thank everyone who has helped me in my academic career at Rice University.

I would especially like to thank all the wonderful people who work on the Rice University Computer Project, particularly the following;

Dr. Martin Graham, who was one small spark of encouragement, in what was at times an atmosphere of despair and discouragement.

Walter Orvedahl, who has valiantly tried to teach me to do things right the first time, and been at least partially successful.

All the Computer Project programmers, who managed to be of great assistance to me, in spite of myself.

Last, but not least, I would like to thank the members of my oral committee, Dr. Burrus and Professor Wischmeyer, whose advice and comments were of great assistance in the writing of this paper.
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Chapter 1

INTRODUCTION

In the analysis of medical waveforms, spectral analysis of the data obtained can be carried out by digital computer or by various filter methods. The approach investigated here is the use of active filters.

It was decided to use active, rather than passive, filters because of the low frequencies involved (10-200 cps). The size and cost of inductors and capacitors needed for filters at these low frequencies would be prohibitive. Some of the active filter designs considered are Yanagisawa's method using negative impedance converters (NIC's), and various designs using operational amplifiers.

In this thesis the design of bandpass filters by using two low-pass filters and a differential amplifier is investigated in chapter 4. Several bandpass filters were constructed and the experimental data obtained from them is shown in chapter 5.
Chapter 2
ACTIVE FILTER THEORY

Active filters were built many years before the advent of transistors. They are most useful in the low frequency range, where the physical size and cost of inductors and capacitors would be prohibitive. Even considering the extra power needed to run vacuum tube and transistor circuitry, very low frequency filters are usually practical and useful only when some sort of active device is used. The first work on active filters using vacuum tubes as the active elements was done by S. Butterworth and reported in his paper, "On the Theory of Filter Amplifiers", published in 1930.

A feedback amplifier circuit that is quite often used to give a second-order low-pass response is shown along with its transfer function in figure 1. This circuit can easily be made into a third-order filter as shown in figure 2. These two circuits can be used to obtain higher order filters by cascading two or more of them. Appendices A and B show the derivations of the voltage transfer functions of the second and third order low-pass filters shown in figures 1 and 2. Either of these circuits can be made to have a Butterworth, Chebychev, or Thompson frequency response by the proper choice of the coefficients a, b, c, and d, which are determined by the resistor and capacitor values.
Model for Second Order Low Pass Filter

\[
e_{\text{out}} = \frac{1}{as^2 + bs + c}
\]

Figure 1

Model for Third Order Low Pass Filter

\[
e_{\text{out}} = \frac{1}{as^3 + bs^2 + cs + d}
\]

Figure 2
If a, b, c, and d, are known, and the three resistors are given values, a digital computer program can be written to find the capacitor values for different cutoff frequencies.

A more recent innovation since semi-conductors became practical is the Yanagisawa method of synthesis utilizing negative impedance converters. By using this method, any open circuit voltage ratio that is a rational function of $s (s = \sigma + j\omega)$, can be realized by using one negative impedance converter and resistors and capacitors. A block diagram of the circuit is shown in figure 3. Figure 4 shows $(y)$ and $(Y)$ in the form of inverse L networks, where $y_a$, $y_b$, $Y_a$, and $Y_b$ are resistor capacitor networks.

Even though one of these circuits can synthesize any open circuit voltage ratio that is a rational function of $s$, it might be more desirable, for more complicated transfer functions, to divide the transfer function into several smaller, more easily realized, functions and cascade them as shown in figure 5. In this case the emitter follower transistors are complimentary to the negative impedance converter transistors.

3. Yanagisawa, "RC Active Networks Using Current Inversion Type Negative Impedance Converters", Pages 140-144.
Model for Realizing Voltage Transfer Function by Yanagisawa Method

Figure 3

Model for Yanagisawa Method of Realizing Voltage Transfer Functions with Admittances Shown as Inverse L Sections

Figure 4
Model for Realizing a Complex Voltage Transfer Ratio by the Yanagiasawa Method

Figure 5
Chapter 3

THE POWER SPECTRUM PROBLEM

In a filter system that uses a number of bandpass filters, as shown in figure 7, it is sometimes desired to have the sum of the output powers of the filters be a constant over the entire band passed, as shown in figure 8. If this were not true, then the output power spectrum of an actual signal could be different from the input power spectrum of that signal.

The problem of obtaining a constant power spectrum was looked into, and it was found that the sum of the output powers from a high-pass Butterworth response and a low-pass Butterworth response with the same cutoff frequency gave a constant power spectrum over the frequency range that was tested (DC to $5\omega_0$, where $\omega_0$ is the cutoff frequency). Since it is possible to design one frequency response crossover, as shown in figure 6, to have a constant power spectrum, it should be possible to design each frequency response crossover in a bandpass arrangement, such as shown in figure 7, to give a constant power spectrum over its range of influence. If each of these crossovers can be designed to give an equal power spectrum, constant over its range, then the constant power spectrum of figure 8 results.
Having Same Cutoff Frequency

Figure 6

Frequency Responses for Low Pass and High Pass Filters Having Same Cutoff Frequency

Figure 7

Frequency Responses of a Series of Bandpass Filters

Figure 8

Desired Power Spectrum of the Series of Bandpass Filters Shown in Figure 7
Chapter 4
A BANDPASS FILTER DESIGN

The first bandpass filter tested was a first order filter designed by the Yanagisawa method. With this technique the output voltage was one-third of the input voltage in the center of the pass band. This was not desirable since it might be necessary to make a band-reject filter by subtracting the frequency response of a bandpass filter from the original signal as shown in figure 10. If possible, it would be desirable to add the band-reject feature without using an adjustment potentiometer at point B in figure 10. An adjustment potentiometer would definitely be needed if the bandpass filter were constructed using the Yanagisawa technique, and thus a different approach, described below, was tried.

The next filter designed used the technique shown in figure 9. The principal idea involved is the subtraction of the frequency responses of two low-pass filters with different cutoff frequencies. The low-pass filters used were the second and third order filters shown in figures 1 and 2.

Several bandpass filters were built using two second-order low-pass filters in the scheme described above. When two second-order Butterworth filters were used the pass band was too wide. The slopes of the
Technique for Making a Bandpass Filter by Subtraction of the Frequency Responses of Two Low Pass Filters

Figure 9

Technique for Making Band Reject Filter by Subtraction of a Bandpass Filter Response from the Original Signal

Figure 10
leading and trailing edges of the pass band are not steep enough. For this experiment, the desired pass band was from 55 to 105 cps. The best that was done using two second-order low-pass Butterworth filters was a pass band of from 30 to 115 cps.

By making the low-pass filter with the higher cutoff frequency have a small damping factor, as shown in figure 11, the bandpass response shown in figure 12 results. The pass band can be made as small as needed for the purpose intended in this manner. However, as shown in figure 12, the leading and trailing edges of the pass band are not symmetrical. Because of the non-symmetrical leading and trailing edges of the pass band, this design was not acceptable.

A design using two third-order low-pass filters was tried next. This gave the best results, although the leading and trailing edges of the pass band were still not symmetrical. The reason that the leading and trailing edges of the passband are not symmetrical is because the phase shift of the low-pass filters causes the subtraction to be of the form

\[ V_{\text{out}}(\omega) = V_1 \angle \theta_1 - V_2 \angle \theta_2. \]

Where \( V_1 \) and \( V_2 \) are the output voltages of the low-pass filters and \( \theta_1 \) and \( \theta_2 \) are the respective phase angles. If the phase angles were equal at all frequencies, the leading and trailing edges of the pass band would be
Figure 11

Frequency Responses of Two Low Pass Filters

\[ \frac{e_{\text{out}}}{e_{\text{in}}} \]

Frequency response of second order low pass Butterworth filter

Frequency response of second order low pass filter with small damping factor

Figure 12

Frequency Response Obtained by Subtracting the Two Low Pass Responses Shown in Figure 11

\[ \frac{e_{\text{out}}}{e_{\text{in}}} \]

frequency

frequency
symmetrical.

It was decided to use this filter design since it was possible to obtain the desired pass band and the leading and trailing edges of the pass band were close enough to being symmetrical to make this design usable. A schematic of the filter that was tested is shown in figure 13.

For each third-order low-pass filter it was neccessary to determine the values of three resistors and three capacitors. However, there are only three non-linear algebraic equations to work with (see Appendix B). Therefore, three of the six elements have to be assigned values. It would have been more practical, from the viewpoint of obtaining the components, to make the three capacitors take on values that are easily obtained. However, when this is done, the three equations do not always have a solution. Due to the nature of the three equations, if the three resistors are given values, it is quite easy to solve for the values of the capacitors. Futhermore, the three equations always have a solution if the three resistors are given positive, non-zero values and the amplifier has a gain of +1 ($K = +1$).
Figure 13
Schematic of Third Order Bandpass Filter
Chapter 5

TESTING THE THIRD ORDER FILTER

The frequency and phase responses of the three bandpass filters that were constructed using the scheme of figure 9 are shown in figures 14, 15, and 16. The only filter that had a narrow enough pass band and also had the leading and trailing edges of the pass band fairly symmetrical was the third order filter whose frequency response and phase shift are shown in figure 16. The third order filter was subjected to a series of test signals and the results were recorded.

Figure 17 shows the step response of the third-order bandpass filter. This result is explained by considering the step input as the summation of all frequencies. The bandpass filter will pass only the frequencies in its pass band. Therefore, at each step input, the filter will pass only that part of the step input that is in the pass band.

Figure 18 shows the response of the third-order bandpass filter to a square wave in the pass band of the filter. The output is an almost perfect sine wave. This can be explained by dividing the square wave into its harmonic components. It has a large first harmonic component. This large first harmonic component is multiplied by the value of the frequency response of the filter at the square wave frequency. The value of the frequency response of the filter at the next component of
Phase frequency (cps)

Frequency Response and Phase Shift of Second Order Bandpass Filter Using Two Butterworth Low Pass Filters in Scheme of Figure 9 (input voltage = 8 volts)

Figure 14
Frequency Response of Bandpass Filter Using Two Low Pass Filters in Scheme of Figure 9 with the Higher Cutoff Frequency Filter Having a Small Damping Factor and the Lower Cutoff Frequency Filter Having a Butterworth Characteristic (Input Voltage = 2 volts)

Figure 15

-17-
Frequency Response and Phase Shift of Third Order Bandpass Filter Using Two Butterworth Low Pass Filters in Scheme of Figure 9 (input voltage = 4 volts)

Figure 16
Input
0.5 volts/div.

Output
0.5 volts/div.

Step Response of Third Order Bandpass Filter
Square Wave Frequency = 10 cps

Figure 17

Response of Third Order Bandpass Filter to
90 cps Square Wave

Figure 18
the square wave, which is the third harmonic, is quite small compared to its value at the first harmonic. In addition, the third harmonic of the square wave is only one-third of the magnitude of the first harmonic. When the third harmonic component that results is added to the first harmonic, very little distortion results. Any harmonics beyond the third would be negligible.

Figure 19 shows the response of the bandpass filter to a 20 cps triangular waveform. The output would contain primarily the first and third harmonics. The phase shift between these harmonics would cause them to subtract and thus cause the flattened output waveform.

Figure 20 shows the response of the bandpass filter to a triangular waveform in its pass band. This can be explained by using the same reasoning that applied to the square wave response in the pass band. The triangular waveform contains a large first harmonic component. The next harmonic present, the third, is only one-ninth of the first harmonic. Therefore, since the first harmonic is larger than any other and the frequency response is larger at the first harmonic than at any other harmonic, a good representation of a sine wave is formed.
Response of Third Order Bandpass Filter to 20 cps Triangular Waveform

Figure 19

Response of Third Order Bandpass Filter to 80 cps Triangular Waveform

Figure 20
Chapter 6

CONCLUSIONS

In this paper a scheme for designing a bandpass filter by the subtraction of the frequency responses of two low-pass filters was investigated. The main drawback of this method is that the slopes of the leading and trailing edges of the pass band are dependent on each other. As the pass band is made wider, the leading and trailing edges of the pass band would have less effect on each other. This can be seen by looking at figure 21. The phase and magnitude of low-pass filter number 1 would change appreciably only between points A and B, and the phase and magnitude of low-pass filter number 2 would change appreciably only between points C and D. Therefore, the leading edge of the pass band, shown in figure 22, would be of the form

\[
\text{Leading Edge} = \text{Constant} - \frac{V_1}{\theta_1},
\]

and the trailing edge would be of the form

\[
\text{Trailing Edge} = \frac{V_2}{\theta_2} - 0.
\]

If \( V_1 \) and \( V_2 \) both have Butterworth responses and are of the same order, then the leading and trailing edges of the pass band would be very close to symmetrical. Therefore, this method can be used to obtain a symmetrical pass band for wide bands. This would not be true if the low-pass filters did not have a constant slope in the regions between A and B, and C and D in figure 21. Work needs to be done on making a narrow band filter of this type with
Magnitude Responses of Two Low-pass Filters

Figure 21

Bandpass Response Obtained by Subtracting the Two Low-pass Responses Shown in Figure 21

Figure 22
symmetric leading and trailing edges.

The power spectrum problem needs to be investigated more thoroughly. Specifically, several bandpass filters with overlapping pass bands, as shown in figure 7, should be constructed or simulated on a digital computer. Then the sum of the output powers should be found to see if it is possible to have a constant power spectrum, and if so, under what conditions this is true.
Appendix A
Second Order Low Pass Filter Transfer Function
Derivation

Circuit Model

Mathematical Model

\[ I_1 = I_2 + I_3 \]  
\[ \frac{e_1 - e_2}{R_1} = \frac{e_2}{R_2 + \frac{1}{C_2s}} + \frac{e_2 - e_4}{\frac{1}{C_1s}} \]  
\[ \frac{e_1 - e_2}{R_1} = \frac{C_2se_2}{R_2C_2s + 1} + C_1se_2 - C_1se_4 \]  
\[ e_1 = e_2 \left( \frac{1}{R_1} + \frac{C_2s}{R_2C_2s + 1 + C_1s} \right) - C_1se_4 \]  
\[ e_3 = \frac{e_2}{R_2C_2s + 1} \]
\( \frac{e_4}{e_3} = \frac{K}{e_4} \)

From 5 and 7

\[ \frac{e_4}{K} = \frac{e_2}{R_2C_2s + 1} \]

From 8

\[ e_2 = \frac{R_2C_2s + 1}{K} e_4 \]

From 4 and 9

\[ e_1 = \frac{R_2C_2s + 1 + R_1C_2s + R_1R_2C_1C_2s^2 + R_1C_1s}{K} e_4 \]

\[ - R_1C_1se_4 \]

From 10

\[ \frac{e_4}{e_1} = \frac{K}{R_1R_2C_1C_2s^2 + (R_1C_1 - KR_1C_1 + R_1C_2 + R_2C_2)s + 1} \]

If \( K = 1 \)

\[ \frac{e_4}{e_1} = \frac{1}{R_1R_2C_1C_2s^2 + C_2(R_1 + R_2)s + 1} \]
Appendix B

Third Order Low Pass Filter Transfer Function

Mathematical Model

1. \[ I_1 = I_2 + I_3 \]
2. \[ I_3 = I_4 + I_5 \]
3. From 1 and 2
   \[ I_1 = I_2 + I_4 + I_5 \]
4. \[ e_5 = Ke_4 \]
5. \[ e_4 = \frac{e_3}{R_3C_3s + 1} = \frac{e_5}{K} \]
6. \[ e_3 = e_5 \frac{R_3C_3s + 1}{K} \]
From 3

\[ \frac{e_1 - e_2}{R_1} = \frac{e_2}{C_1 s} + \frac{e_3}{R_3 + \frac{1}{C_3 s}} + \frac{e_3 - e_5}{C_2 s} \quad \text{7} \]

\[ \frac{e_1}{R_1} = e_2 \left( \frac{R_1 C_1 s + 1}{R_1} \right) + e_3 \left[ \frac{C_3 s + (C_2 s)(R_3 C_3 s + 1)}{R_3 C_3 s + 1} \right] - e_5 (C_2 s) \quad \text{8} \]

From 1

\[ \frac{e_1 - e_2}{R_1} = e_2 C_1 s + \frac{e_2 - e_3}{R_2} \quad \text{9} \]

\[ \frac{e_1}{R_1} = e_2 \left( \frac{1}{R_1} + \frac{1}{R_2} + C_1 s \right) - \frac{e_3}{R_2} \quad \text{10} \]

\[ \frac{e_1}{R_1} = e_2 \left( \frac{1}{R_1} + \frac{1}{R_2} + C_1 s \right) - e_5 \left( \frac{R_3 C_3 s + 1}{KR_2} \right) \quad \text{11} \]

\[ e_2 = \frac{KR_2 e_1 + e_5 (R_1 R_3 C_3 s + R_1)}{K(R_2 + R_1 R_2 C_1 s) + 1} \quad \text{12} \]

From equations 6, 8, and 12

\[ \frac{e_1}{R_1} = \left[ KR_2 e_1 + e_5 \left( R_1 R_3 C_3 s + R_1 \right) \right] (R_1 C_1 s + 1) \]

\[ \frac{e_1}{R_1} = \frac{KR_1 \left( \frac{R_1 + R_2 + R_1 R_2 C_1 s}{R_1} \right)}{KR_2 \left( \frac{R_1 + R_2 + R_1 R_2 C_1 s}{R_1} \right)} + \frac{e_5 (C_2 s + C_3 s + R_3 C_2 C_3 s^2)}{K} - e_5 C_2 s \quad \text{13} \]
Simplifying

\[ \frac{e_5}{e_1} = \frac{K}{as^3 + bs^2 + cs + K} \]

Where

\[ a = R_1 R_2 R_3 C_1 C_2 C_3 \]

\[ b = K( R_1 R_3 C_1 C_3 - R_1 R_2 C_1 C_2 ) + R_1 R_3 C_2 C_3 + R_2 R_3 C_2 C_3 \]
\[ + R_1 R_2 C_1 C_2 + R_1 R_2 C_1 C_3 \]

\[ c = K( R_1 C_1 + R_3 C_3 - R_1 C_2 - R_2 C_3 ) + R_1 C_2 + R_1 C_3 \]
\[ + R_2 C_2 + R_2 C_3 \]

If \( K = 1 \) then

\[ \frac{e_5}{e_1} = \frac{1}{R_1 R_2 R_3 C_1 C_2 C_3 s^3 + ( R_1 R_3 C_1 C_3 + R_1 R_3 C_2 C_3 + R_2 R_3 C_2 C_3 \]
\[ - - + R_1 R_2 C_1 C_2 ) s^2 + ( R_1 C_1 + R_3 C_3 + R_1 C_3 + R_2 C_3 ) + 1 \]
BIBLIOGRAPHY


Yanagiasawa, Takesi, "RC Active Networks Using Current Inversion Type Negative Impedance Converters", IRE Transactions on Circuit Theory, Volume CT-4, Number 3, September, 1957, Pages 140-144.