RICE UNIVERSITY

A STUDY OF EXTERIOR SLAB-BEAM-COLUMN CONNECTIONS IN FLAT PLATE FLOORS

by

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ABSTRACT

A STUDY OF EXTERIOR SLAB-BEAM-COLUMN CONNECTIONS IN FLAT PLATE FLOORS

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The problem considered here concerns the transfer of load from the slab to the exterior columns of flat plate floor slabs. At the exterior columns, the load is transferred through a combination of shear, bending moment, and torsion. The distribution and magnitude of these forces and moments depend on the relative stiffness and strength of the spandrel beams. In general, the strength of the slab at the exterior edge is controlled by the limitations in the strength of the spandrel beams.

In order to study the load transfer problem, a series of five 0.3 scale reinforced concrete models, simulating the exterior column region of floor slabs, were tested to failure. The primary variables were the beam dimensions and reinforcement. The beam strength, which was primarily dependent upon the amount of lateral beam reinforcement, governed the amount of yielding in the slab and the extent of torsional cracking in the beam. In studying the behavior and strength of the sections, distributions of bending moments in the slab and torsional moments along the beam were determined and used in defining the failure criteria for the slab. This study indicated that spandrel beams can be designed with sufficient strength to develop the yield strength of the exterior slab section perpendicular to the spandrel beam.
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Acknowledgements</th>
<th>i</th>
</tr>
</thead>
<tbody>
<tr>
<td>List of Tables and Figures</td>
<td>iv</td>
</tr>
<tr>
<td>Chapter</td>
<td></td>
</tr>
<tr>
<td>1. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Statement of the Problem</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Object and Scope</td>
<td>1</td>
</tr>
<tr>
<td>2. Prototype and Models</td>
<td>3</td>
</tr>
<tr>
<td>2.1 Relationship of the Test Structure to Actual Structures</td>
<td>3</td>
</tr>
<tr>
<td>2.2 Design</td>
<td>4</td>
</tr>
<tr>
<td>2.3 Test Variables</td>
<td>6</td>
</tr>
<tr>
<td>3. Materials</td>
<td>9</td>
</tr>
<tr>
<td>3.1 Relationship of Test Specimens to Actual Structures</td>
<td>9</td>
</tr>
<tr>
<td>3.2 Physical Properties of the Materials</td>
<td>9</td>
</tr>
<tr>
<td>3.3 Formwork</td>
<td>11</td>
</tr>
<tr>
<td>3.4 Fabrication</td>
<td>11</td>
</tr>
<tr>
<td>3.5 Casting and Curing</td>
<td>12</td>
</tr>
<tr>
<td>4. Testing Procedure</td>
<td>13</td>
</tr>
<tr>
<td>4.1 Loading System</td>
<td>13</td>
</tr>
<tr>
<td>4.2 Instrumentation</td>
<td>13</td>
</tr>
<tr>
<td>4.3 Procedure</td>
<td>14</td>
</tr>
<tr>
<td>5. Behavior</td>
<td>15</td>
</tr>
<tr>
<td>5.1 Introduction</td>
<td>15</td>
</tr>
<tr>
<td>5.2 Behavior at Working Load</td>
<td>15</td>
</tr>
<tr>
<td>5.3 Behavior from Working Load to Ultimate Load</td>
<td>16</td>
</tr>
<tr>
<td>section</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
</tr>
<tr>
<td>6. Analysis</td>
<td>22</td>
</tr>
<tr>
<td>6.1 Moment Computation</td>
<td>22</td>
</tr>
<tr>
<td>6.2 Moment Distribution</td>
<td>23</td>
</tr>
<tr>
<td>6.3 Torsional Beam Moments</td>
<td>24</td>
</tr>
<tr>
<td>6.4 Torsional Efficiency</td>
<td>27</td>
</tr>
<tr>
<td>7. Summary</td>
<td>29</td>
</tr>
<tr>
<td>7.1 Object and Scope</td>
<td>29</td>
</tr>
<tr>
<td>7.2 Test Program</td>
<td>29</td>
</tr>
<tr>
<td>7.3 Analysis</td>
<td>30</td>
</tr>
<tr>
<td>References</td>
<td>31</td>
</tr>
<tr>
<td>Tables</td>
<td>32</td>
</tr>
<tr>
<td>Figures</td>
<td>35</td>
</tr>
<tr>
<td>Appendix: Special Finite Difference Operators</td>
<td>70</td>
</tr>
</tbody>
</table>
LIST OF TABLES AND FIGURES

<table>
<thead>
<tr>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table</td>
<td></td>
</tr>
<tr>
<td>2.1 Beam Dimensions and Stiffness Parameters</td>
<td>32</td>
</tr>
<tr>
<td>2.2 Schedule of Reinforcement</td>
<td>33</td>
</tr>
<tr>
<td>3.1 Material Properties</td>
<td>34</td>
</tr>
<tr>
<td>Figure</td>
<td></td>
</tr>
<tr>
<td>2.1 The Test Structure</td>
<td>35</td>
</tr>
<tr>
<td>2.2 Comparison of Moment Distributions in S1</td>
<td>36</td>
</tr>
<tr>
<td>2.3 Comparison of Moment Distributions in S3</td>
<td>37</td>
</tr>
<tr>
<td>2.4 Comparison of Moment Distributions in S4 and S5</td>
<td>38</td>
</tr>
<tr>
<td>2.5 Comparison of Moments for Uniform vs Concentrated Loads</td>
<td>39</td>
</tr>
<tr>
<td>2.6 Dimensions of the Test Structures</td>
<td>40</td>
</tr>
<tr>
<td>3.1 Typical Stress-Strain Curves for Reinforcement</td>
<td>41</td>
</tr>
<tr>
<td>4.1 Loading Frame</td>
<td>42</td>
</tr>
<tr>
<td>4.2 Test Setup - Loading System</td>
<td>43</td>
</tr>
<tr>
<td>4.3 Test Setup - Instrumentation</td>
<td>43</td>
</tr>
<tr>
<td>4.4 Dial Indicator Locations</td>
<td>44</td>
</tr>
<tr>
<td>5.1 Stress Distributions at Working Load</td>
<td>45</td>
</tr>
<tr>
<td>5.2 Load-Beam Rotation Curves</td>
<td>46</td>
</tr>
<tr>
<td>5.3 Load-Beam Deflection Curves (D1)</td>
<td>47</td>
</tr>
<tr>
<td>5.4 Load-Slab Deflection Curves (D7)</td>
<td>48</td>
</tr>
<tr>
<td>5.5 Crack Patterns after Load to Failure - S1</td>
<td>49</td>
</tr>
<tr>
<td>5.6 Crack Patterns after Load to Failure - S2</td>
<td>50</td>
</tr>
<tr>
<td>5.7 Crack Patterns after Load to Failure - S3</td>
<td>51</td>
</tr>
<tr>
<td>5.8 Crack Patterns after Load to Failure - S4</td>
<td>52</td>
</tr>
<tr>
<td>5.9 Crack Patterns after Load to Failure - S5</td>
<td>53</td>
</tr>
<tr>
<td>Figure</td>
<td>Title</td>
</tr>
<tr>
<td>--------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>5.10</td>
<td>Crack Patterns - University of Illinois Test Structure</td>
</tr>
<tr>
<td>5.11</td>
<td>Load-Strain Relationships, Beam Reinforcement - S4</td>
</tr>
<tr>
<td>5.12</td>
<td>Stress Distributions - Slab Reinforcement - S1</td>
</tr>
<tr>
<td>5.13</td>
<td>Stress Distributions - Slab Reinforcement - S2</td>
</tr>
<tr>
<td>5.14</td>
<td>Stress Distributions - Slab Reinforcement - S3</td>
</tr>
<tr>
<td>5.15</td>
<td>Stress Distributions - Slab Reinforcement - S4</td>
</tr>
<tr>
<td>5.16</td>
<td>Stress Distributions - Slab Reinforcement - S5</td>
</tr>
<tr>
<td>6.1</td>
<td>Typical Moment-Strain Relationships</td>
</tr>
<tr>
<td>6.2</td>
<td>Moment Distribution at Failure - S1</td>
</tr>
<tr>
<td>6.3</td>
<td>Moment Distribution at Failure - S2</td>
</tr>
<tr>
<td>6.4</td>
<td>Moment Distribution at Failure - S3</td>
</tr>
<tr>
<td>6.5</td>
<td>Moment Distribution at Failure - S4</td>
</tr>
<tr>
<td>6.6</td>
<td>Moment Distribution at Failure - S5</td>
</tr>
<tr>
<td>6.7</td>
<td>Free Body Diagram for Beam Torsional Moments</td>
</tr>
<tr>
<td>6.8</td>
<td>Efficiency of Beam in Developing Slab Reinforcement Stresses</td>
</tr>
<tr>
<td>6.9</td>
<td>Modes of Failure</td>
</tr>
</tbody>
</table>
1. INTRODUCTION

1.1 Statement of the Problem

One problem in the design of floor slabs is the provision for load transfer from the slab to the column. In flat plate construction, the benefits obtained in simplification of formwork by the omission of supporting beams, drop panels, and column capitals are directly responsible for the intensification of load transfer problems. At the exterior columns, a combination of bending, torsion, and shear provide for the load transfer from the slab to the column, subjecting the column to moments transmitted directly from the slab or through the spandrel beams. The behavior and strength of the joint depend on the relative stiffness of the members framing into the joint. The relative stiffness is controlled by the extent of cracking in the concrete and the yielding of the reinforcement.

To develop the moments specified by the ACI Code empirical moment coefficients for floor slabs, it is necessary to have stiff column connections which are not usually attained in practice. These limitations in the strengths of supporting members control the strength of the slabs at the exterior edge as indicated by a limited number of tests on actual structures and on scale models proportioned according to current design specifications. Failures in floor slab structures often may be initiated by failures at the exterior slab-beam-column joint.

1.2 Object and Scope

The object of the work reported is a study of the behavior and strength
of the exterior slab-beam-column connections of reinforced concrete floor slabs. A series of tests were performed on reinforced concrete models simulating the exterior column region of floor slab structures. The distribution of flexural and torsional moments at the joint were determined and used in defining the failure criteria of the slab.
2. PROTOTYPE AND MODELS

2.1 Relationship of the Test Structure to Actual Structures.

The model tested was essentially a slab section framing into a beam of specific stiffness, which in turn framed into an infinitely stiff column as shown in Fig. 2.1(a). Figure 2.1(b) shows the reinforcement of a typical test structure. The test model corresponded to a section of an actual structure bounded by the exterior edge and the panel centerlines.

The panel centerlines in an actual structure, which are lines of symmetry, are approximately lines of maximum positive moment and zero shear. The test structure differed from the actual case in that no shear or moment was transferred at the free edge. To study the effects of this difference, moment distributions across a section along the inside edge of the beam were determined, using three different beam stiffnesses, for both the case of zero slope at the panel centerline approximating an actual structure and the free edge case of the test structure. A finite difference solution involving an eleven by eleven grid was used. In addition to the regular operators employed for points in the slab, special operators, accounting for both torsional and flexural beam stiffness, were used.

The difference in boundary conditions was not significant as the moment distributions were essentially the same for the two cases as shown in Figs. 2.2, 2.3, and 2.4. The magnitude of the moments of the case for zero slope of the edge was approximately one-half the magnitude of the corresponding moments in the free edge case. This difference can be attributed to the moment which
is developed at the edge when rotation is restricted. Although the moment distributions were of different magnitudes, the shape of the distributions were practically identical in each case which indicated that the behavior of the test structure will be similar to the behavior of an identical section in an actual structure. The favorable agreement between the theoretical and experimental moment distributions can be seen from the curves plotted using strains measured at lower load stages when the test structure still behaved nearly elastically. Computation of moments is discussed in more detail in Chapter 6.

Although the slab designs were based upon uniformly distributed slab loads, this type of loading was not practical in these tests. Therefore, theoretical moment distributions across the inside edge of the beam were determined for concentrated loads, as applied to the test structures, and for uniform load. Comparison of the moment distributions for the two different loading conditions is shown in Fig. 2.5 for a structure with the dimensions of specimen S1. For both loading conditions the maximum unit moment will occur at the corner of the column. The magnitudes of the unit moments were greater for the concentrated loads because the point of application was farther from the beam than that of the centroid of the uniform load; however, the shape of the curve was similar for both cases.

2.2 Design

Since the boundary conditions of the test structures were different from actual structures, it was desirable to correlate the test results with
the behavior of an actual structure. Hence the design of the first two test structures was based upon a structure tested at the University of Illinois (1).

The specimens in this series of tests could have been designed to conform to the ACI Code requirements (2), except for two violations. To attain the usual dimensional relationships of concrete structures, the cover requirements in both the beam and the slab, and the slab thickness requirement could not be met.

Therefore a prototype structure with panels measuring 20 ft. center to center of columns was used and then scaled down. The prototype of specimens S1 and S2 was identical to that upon which the University of Illinois test structure was based. It was designed to carry a total design load of 155 psf and a line load over the spandrel beams of 600 lb/ft.

A scale factor of 0.3 was chosen for this series of tests primarily because it permitted the use of #2 reinforcing bars and required only two batches of concrete for casting. The 0.3 scale models of this series resulted in 6 ft. by 6 ft. panels and beam dimensions as shown in Table 2.1.

The slab thickness for all five tests was 2 - 1/8 in. The column dimensions of 4 - 3/4 in. by 7 - 1/4 in. were held constant for all the tests. The column reinforcement was over designed, so the column was essentially infinitely stiff. The column reinforcement consisted of 3 - #4 bars in each face and #9 gauge wire ties spaced at 3 in. cc.

The slab reinforcement ratios of specimens S1 and S2 were practically identical to the respective ratios in the University of Illinois test
structure as shown below.

<table>
<thead>
<tr>
<th>Structure</th>
<th>Column Strip</th>
<th>Middle Strip</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0.78(0.73)</td>
<td>0.31(0.31)</td>
</tr>
<tr>
<td>S2</td>
<td>0.63(0.60)</td>
<td>0.31(0.35)</td>
</tr>
</tbody>
</table>

The numbers in parentheses are the steel ratios for the University of Illinois test structure.

2.3 Test Variables

The variables in this series of tests were the beam dimensions and the beam reinforcement. For each given beam the elastic stiffness parameters \( I_b, C, H, \) and \( J \) were computed. These stiffness parameters are defined as follows.

\[ I_b = \text{moment of inertia of gross uncracked section of beam having same width as the column. Beams with webs narrower than the column are considered as L-shape beams in computing the moment of inertia.} \]

\[ I_{s_1} = \frac{(L_2/2)t^3}{12} = \text{moment of inertia of slab parallel to spandrel beam.} \]

\[ L_2 = \text{span length perpendicular to spandrel beam.} \]

\[ t = \text{slab thickness.} \]

\[ I_{s_2} = \frac{L_1t^3}{12} = \text{moment of inertia of slab perpendicular to spandrel beam.} \]
\( L_1 \) = span length parallel to spandrel beam.

\[ C = \sum \beta b_1 h_1^3 \] = torsional moment of inertia of a beam.

\( \beta = \) function of ratio \( b_1/h_1 \) (See Ref. 3).

\( b_1 = \) larger dimension of each rectangular section of beam.

\( h_1 = \) smaller dimension of each rectangular section of beam.

\[ H = I_b/I_{s1} \] = relative flexural beam stiffness.

\[ J = C/I_{s2} \] = relative torsional beam stiffness.

The beam dimensions and stiffness parameters are shown in Table 2.1. In Table 2.2 the steel arrangements are given. Figure 2.6 shows the critical beam and slab dimensions.

The area of the stirrups of the University of Illinois test was different than those of S1 and S2 and, although the stirrup spacings were not the same, comparable amounts of steel were provided in each case.

Specimens S3, S4, and S5 were designed with the same perpendicular slab reinforcement percentages as used in S1, but with specific beam dimensions such that there would be a variable range of the elastic stiffness parameters, \( I_b \), \( C \), \( H \), and \( J \). In all three specimens the slab steel parallel to the beam was overdesigned to insure a yield failure in the perpendicular steel. Specimens S4 and S5 were exactly identical, except for the stirrup spacing of the beam. Specimen S4 was designed with sufficient stirrups to carry the torsion developed in the beam when all the perpendicular slab steel
reached the yield condition. For a comparison, the stirrups of S5 were designed to comply with the shear requirements of the ACI Code (318-63).
3. MATERIALS

3.1 Relationship of Test Specimens to Actual Structures.

Before considering the material properties of the test specimens, their relationship to the prototype should be investigated. The major items of this investigation are loads, moments, and stresses.

The test structures are not models in the usual sense in which a model of a given structure is constructed on a smaller scale and is made of a different material. These test specimens were actually just scaled down from the prototype; they employed the very same materials with properties essentially the same as in full scale reinforced concrete structures.

The only violations of the ACI Code involve thickness limitations and some clearance requirements. Other code requirements regarding slenderness ratios, support dimensions, column stiffnesses, and arrangement and percentage of reinforcement are satisfied. Hence the same stress-strain properties and similar moment-rotation characteristics permit the behavior of the small scale test structures to be considered representative of full scale structures.

Loads per unit area and stresses are identical in the prototype and the test structures. Therefore, given steel stresses in the test structures represent the same stresses in the prototype. Similarly, a given load intensity in the test structure represents the same intensity of load in the prototype.

3.2 Physical Properties of the Materials.

(a) Steel
The reinforcing steel used as slab reinforcement and as beam reinforcement consisted of #2 bars of intermediate grade billet steel. Tensile tests were run on each bar in a mechanical testing machine equipped with an automatic load deformation recorder. The stress-strain curves exhibited well defined yield points and a flat top after yield as commonly assumed in yield line analysis. A representative stress-strain curve is shown in Figure 3.1. The stirrups and ties consisted of #9 gauge annealed wire. Tensile tests on this wire revealed no well defined yield point, but considerable ductility. A typical stress-strain curve is shown in Figure 3.1.

(b) Concrete.

The small size of the test specimens required a mix using small aggregate. The design strength was set at 4000 psi at 15 days, and in addition, the mix had to be workable for placement. By trial batching, a mix with a 2-3 in. slump was determined to be adequate. The aggregate consisted of 50% sand and 50% pea gravel with a maximum size of 3/8 in. Type III Portland cement was used.

The concrete was mixed in 3 cu. ft. batches in a horizontal drum counter-current mixer. Each specimen required two batches, the first of which comprised the column, beam, and the critical section of the slab near the column, while the second completed the slab.

Three 6 x 12 in. control cylinders and three unreinforced modulus of rupture beams 3 in. wide by 2-1/8 in. deep and 21 in. long were cast for each batch. The 2-1/8 in. depth was chosen so that the control beams would
be the same depth as the slabs. The tensile and compressive strengths at the test dates are given in Table 3.1.

3.3 **Formwork**

Most of the form was fabricated from 3/4 in. plastic coated plywood, rigidly supported on a framework of 2 x 4 and 2 x 6 lumber. Steel angles milled straight were used for the outside edges of the slab to establish a uniform slab thickness. The beam form was made to the maximum width and maximum depth of all beams in this series of tests, with fillers used to provide beams with any required dimensions.

The column form also was made from 3/4 in. plywood. The bottom of the column consisted of a 3/4 in. thick steel plate recessed at the center to fit on a 3/4 in. steel ball seated in the loading frame. The plate had shear connectors welded to it to provide for horizontal thrusts from the column to the plate.

The framework was carefully aligned and leveled prior to casting each test specimen to provide accurate dimensional control. Upon completion it was coated with oil to prevent bonding to the form and to retard evaporation of moisture.

3.4 **Fabrication.**

All bars, stirrups, and ties were cleaned and bent where required. Wire resistance strain gages having a 0.4 in. gage length were applied to both beams and slab bars and stirrups, short leads were attached, and each gage was water-proofed with Mobil Wax 2300.
After gage placement had been completed, column cages were assembled for the portion of the column below the beam, beam cages were completely assembled, and slab steel was tied together. The order of assembling these components in the form was column cage, beam cage, slab steel, and then the remaining column ties were positioned. The beam cages were supported by special notched rectangular chairs, while the slab steel was supported on standard 1-1/2 in. chairs. Spacers prevented the slab steel mats from lateral displacement during the placement of the concrete.

3.5 Casting and Curing.

Two 3 cu. ft. batches of concrete were required for each test specimen. The casting operation required approximately 1-1/2 hours for completion. The concrete was placed by hand and vibrated internally while the form was being filled and the form was vibrated when it appeared full. Care was taken to keep all the strain gage leads above the surface while casting. The concrete was struck with a wooden screed and later troweled smooth. It was then covered with a polyethylene sheet and was cured in this manner for one week. The control cylinders and beams were also covered with polyethylene sheets and placed near the test structure.

Then the test structure was removed from the form and set in the test frame and allowed to cure for one additional week prior to testing. At the same time the control specimens were removed from their forms and again placed near the test structure.
4. TESTING PROCEDURE

4.1 Loading System.

Load was applied to the test structure by a single hydraulic jack. The load was distributed equally between two load points on the slab by a 3 x 3 box section. The remainder of the load assembly consisted of two steel rods extending up through 2 in. diameter holes in the slab with each bearing on a steel plate 6 inches square. A 10 kip capacity load cell, placed between the hydraulic ram and the box section, measured the loads applied to the structure. Figure 4.1 is a schematic view of the loading frame. Figure 4.2 shows the loading frame with the test structure and the loading equipment in place.

The load was applied as a concentrated load, but to overcome the high shear around a concentrated load, the load was distributed over a 6 in. square plate. The exact positions of the load points (Fig. 2.6) were symmetrical points 3/8 of the span distance from the column centerline; these points are approximately points of contraflexure in a continuous slab structure.

4.2 Instrumentation.

Twenty strain gages were located to provide data at various critical sections. Instrumentation included a digital strain indicator with automatic balance unit, and an X-Y recorder which provided a constant record of load and deflection at the middle of the slab. The photograph in Figure 4.3 shows the test set up. Dial indicators to measure deflection and rotation were located as shown in Fig. 4.4 for all the tests.
4.3 Procedure.

Calculations of the probable failure load were made to determine the magnitude of each load increment so that there would be a sufficient number of load stages to follow the behavior carefully. For each load stage, after the load increment had been applied, the strains were read and the dial indicators were read. The slab was then examined for cracks and the progression of cracking was marked.

The length of time required for each load stage was from five to ten minutes. The total time of testing was 1-1/2 to 1-3/4 hours for the first three structures which failed at lower loads, and 2 to 2-1/2 hours for the last two specimens which carried greater loads and thus required a greater number of load stages.
5. **BEHAVIOR**

5.1 **Introduction**

The behavior of the test specimens will be described in this chapter in terms of load-deflection and load-rotation curves, formation of cracks and resulting crack patterns, and measured steel strains and their resulting stresses. In addition the sequence of yielding of the reinforcement as the ultimate load carrying capacity of the structure is attained will be discussed. Since both strength and serviceability must be considered, the behavior at working load as well as at ultimate load should be examined.

5.2 **Behavior at Working Load**

Because an effective working load is difficult to determine for these structures, the working load considered here is that load at which the maximum stress in the slab reinforcement perpendicular to the edge beam is approximately 20,000 psi. Plots of the stress distributions in the slab reinforcement across the inside edge of the beam at the assumed working load are given in Fig. 5.1 for the five specimens. Examination of these curves shows the effect of the torsional stiffness on the stress distributions and on the assumed working loads.

Specimens S1 and S2, which had nearly equal torsional beam stiffnesses, had similar stress distributions, as well as approximately equal applied working loads of 571 lbs. and 543 lbs. respectively. The stress distributions for S3 revealed that the two middle strip bars and one of the column strip bars were practically unstressed as the applied working load
of 320 lbs. was reached. This was to be expected as the elastic torsional
stiffness of S3 was very small.

Specimens S4 and S5 had similar stress distributions at working
load, since their elastic torsional stiffnesses were identical. The middle
strip bars of both structures were stressed higher than the corresponding
bars of S1, S2, and S3. Since the elastic torsional stiffnesses were greater,
the applied working loads also were greater, with S4 and S5 carrying 1081 lbs.
and 975 lbs. respectively. However, in all the tests the middle strip bars
were stressed to less than 5000 psi.

The load-rotation curves at working loads remained linear for all
the test structures except S3 as shown in Fig. 5.2. The deflections at the
end of the beam (point D1) for each specimen were still small at the working
load as shown in Fig. 5.3. At the designated working loads, the behavior
of all the test structures was characterized by small deflections and, with
the exception of S3, by small rotations. This may be attributed in part to
the high strength of the concrete.

5.3 Behavior from Working Load to Ultimate Load.

After working load, for which serviceability was the primary
requirement, strength becomes the foremost consideration. The strength
of the structure is in turn determined by the mode of failure.

(a) Beam Rotations and Deflections

Beam rotation was computed by dividing the difference between the
horizontal deflections at two points on one lateral plane through the beam by
the distance between those points. In the same manner the rotation of the
column at the level of the beam was computed. The relative beam rotation
was determined as the rotation of the beam with respect to the column. The
load-beam rotation curves for each structure were plotted in Fig. 5.2. The
beams with greater torsional stiffness experienced smaller rotations and the
slab deflections at the middle of the slab (point D7) were also smaller at
corresponding loads. The load-beam rotation curves remained linear and
all the beams had the same stiffness until torsional cracking occurred; the
only major difference was the load at which the concrete first cracked.

Further rotation was then governed by the amount of steel in the
beam, and, particularly, by the number of stirrups. Analysis of the beams
showed that no additional longitudinal steel was needed to meet the torsional
requirements in beams designed for flexure and shear considerations. How¬
ever, the number of stirrups required for shear was not sufficient to carry
torsion in all cases.

Plots of the load-deflection curves for the five specimens are
given in Figs. 5.3 and 5.4 for points at the end of the beam (D1) and at the
middle of the slab (D7) respectively. Neither curve is representative of
actual structures which are continuous across these points. However, the
curves demonstrate the ductility of the slab-beam-column connection.

The beams of specimens S1 and S2 had elastic torsional moments
of inertia that were approximately equal. However, shear requirements
resulted in S1 having twice as many stirrups as S2. These additional
stirrups provided greater rotational beam capacity after cracking as evidenced by the greater ductility of S1 in comparison with S2.

In comparing the failure loads of S1 and S2, note that the column strip steel percentages were 0.78 and 0.63 respectively, while the middle strips were identical. Therefore the maximum loads of the two structures were comparable.

Specimen S3, which had the smallest torsional beam capacity, had cracked and showed torsional distress by the time it reached its working load and, following cracking, beam rotation increased very rapidly. Structures S4 and S5, which had identical elastic torsional beam stiffnesses, exhibited similar behavior until cracking. After torsional cracking occurred, the beam for specimen S4, which had been designed for torsion, was stiffer than that of S5, which had been designed only for shear.

Specimen S4 never reached its maximum rotational capacity even at loads 10% over the calculated yield load, while S5 attained its maximum rotational capacity before reaching its calculated yield load. After cracking, structure S5 behaved similarly to S1 and S2, but its ultimate load was greater because of the greater beam torsional stiffness.

Since the failure in specimen S4 was by yielding of the slab steel, the ultimate rotational capacity of the beam was not attained. All the other test structures showed severe torsional beam distress at ultimate load.

(b) Development of Cracking

The final crack patterns for each of the structures are shown in
Figs. 5.5 to 5.9, A negative moment crack across the column face appeared first in each test. Next diagonal cracks began to radiate out from the edge of the column; at the same time, the negative moment crack extended further along the inside edge of the beam, and negative moment cracks began to form in the slab perpendicular to the beam. In the beams with greater torsional stiffnesses, there was less widening of the cracks until the final load stages. The diagonal cracks were observed to form square spirals around the beam.

In specimen S4, which had been designed for torsion, there was practically no widening of the torsional cracks, but the negative moment crack along the inside edge of the beam widened considerably as all the slab steel perpendicular to the beam reached yield. In direct contrast, S3, which had a beam depth equal to the slab depth, showed severe torsional distress and developed a negative moment crack along the inside edge of the beam for only a small distance from the edge of the column.

Specimens S1 and S2 developed only a few torsional cracks and most of the rotation was concentrated in one major crack. The torsional strength of each specimen was sufficient to enable the slab to develop a negative moment crack along the entire length of the slab, but was not sufficient to yield all the bars in the slab. The crack patterns of S1 and S2 compare very closely with those of the continuous structure tested at the University of Illinois (Fig. 5.10) and the three-quarter scale structure with 15 ft. square panels tested in the Portland Cement Association Labora-
Specimen S5 behaved similarly to S1 and S2, but its torsional stiffness was greater and severe rotational distress did not develop until the final load stages. Therefore, most of the siab bars yielded, or were stressed to higher levels than were reached in S1 and S2.

(c) Load-Strain Curves

Figure 5.11 shows the load-strain relationship of the top and bottom longitudinal bars and the stirrups in the beam of specimen S4. The relationship is representative of all the other structures tested.

At loads prior to torsional cracking, the top longitudinal bars were in tension and the lower bars were in compression, while the stirrups were practically unstressed. This indicated that the stirrups did not materially affect the torsional capacity of the beam prior to torsional cracking.

With increasing load, torsional cracks developed in the beam and began to widen accompanied by increasing rotations. At the same time the tensile strains in the top bars rapidly increased and the bottom bars went from compression to tension as shown in Fig. 5.11. The strains in the stirrups, which had been negligible prior to the development of the beam torsional cracks, also increased rapidly as the torsional cracks widened in the vicinity of the stirrup.

(d) Stress Distributions

The distribution of stresses for succeeding loads shows the effectiveness of the respective beams in carrying the loads. Figures 5.12-
5.16 show the distribution of stresses across the inside edge of the beam corresponding to given loads. These plots show the progression of yielding and the manner in which each section develops yield stresses.

Although the shapes of the corresponding stress distributions throughout the progression of yielding were similar for all the structures, the extent of yielding was a function of the torsional stiffness of the beams. As the beam torsional stiffness increased, the middle strip stresses also increased.

In specimen S4 (Fig. 5.15), which was designed for torsion, the entire slab section reached yield. In contrast, S3 (Fig. 5.14) reached yield in only four of the ten column strip bars, while the middle strip bars remained at very low levels of stress throughout loading. In specimens S1, S2, and S5, most of the column strip bars yielded, and the magnitudes of the stresses in the middle strip bars depended on the torsional stiffness of the spandrel beams.
6. ANALYSIS

6.1 Moment Computation

Slab moments along the inside edge of the beam are determined from the strains measured in the slab steel perpendicular to the beam. The usual procedure involves conversion of strains to unit moments using a bilinear moment strain curve as shown in Fig. 6.1. The cracking moment is determined by

\[ M_{cr} = f_r I_t / C \]

where \( f_r \) = rupture stress (from beam tests)

\( I_t \) = transformed uncracked section

\( c \) = distance to outermost fiber from centroid of transformed section

The yield moment is calculated by

\[ M_y = A_s f_y j d \]

where \( A_s \) = area of steel in the section

\( f_y \) = yield stress of the steel

\( j d \) = moment arm

The bilinear curve assumes no tension in the concrete after cracking.

Substantial differences were detected between calculated and measured unit moments corresponding to strains immediately after cracking. Since the steel percentages of these tests were very small, tensile stresses in the concrete may contribute significantly to the moments.
even after cracking is initiated. To overcome this discrepancy and to determine
a more realistic moment-strain relationship, an analysis which considered
tension in the concrete was developed. The analysis was based on satisfying
equilibrium of forces, including concrete tensile forces, for any strain at the
level of the reinforcement. To facilitate these calculations a short computer
program was developed.

Computation of unit moments in this manner was equivalent to using
the moment-strain relationship represented by the broken line in Fig. 6.1
for the conversion of strains to moments. Since the steel percentage was
very low, the tensile force of the concrete up to cracking was greater than
that of the steel, and immediately after cracking, there was an abrupt de-
crease in moment. As the steel percentage increased, the drop in moment
became less pronounced and the moment-strain relationships approached the
bilinear curve as a limiting case.

6.2 Moment Distribution

The calculated unit moments were plotted at the points at which
strains were measured in the section along the inside edge of the beam. In
order to compare the measured moment distribution with the theoretical
moment distribution (as discussed in Chapter 2), unit moment coefficients
were calculated using strains measured before cracking occurred. The
unit moment coefficients were computed by dividing the unit moments by
the load on the structure producing those moments. As stated previously in
Chapter 2, there was excellent agreement between the experimental and the
theoretical results which are presented in Figs. 2.2, 2.3, and 2.4.

To determine the maximum moment the slab section developed, a moment distribution was calculated for the final load stage. The area beneath this final unit moment curve was equal to the maximum total slab moment on the section. The unit moment distribution at maximum load was plotted as shown in Part (a) of Figs. 6.2-6.6 for each test structure. As a comparison the calculated yield unit moments for the column strip and middle strip are also shown.

6.3 Torsional Beam Moments

As indicated in Fig. 6.7, the unit slab moment $m_s$ is resisted by a unit torsional moment $t_B$ in the beam. Moreover, the total slab moment at any arbitrary point $d$ must be resisted by the torsional beam moment $T_B$ at that point as shown in the following expression

$$T_B = \int_0^d m_s \, dx$$

The slab moment $m_s$ will continue to increase until either the slab yields or the beam fails in torsion. Therefore the ultimate torsional capacity of the beam determines the mode of failure. The torsional capacity of the beams was determined using a procedure suggested by Cowan (5). The torsional capacity was considered as the sum of the torsional capacity of a plain concrete section, with the same dimensions as the beam, and an additional torsional capacity provided by the lateral reinforcement in the beam.
For any plain cross section the torsional moment of inertia is given by

\[ C = \sum \beta x^3 y \]

where

- \( x \) = length of shorter side of each component rectangle.
- \( y \) = length of longer side of each component rectangle.
- \( \beta \) = a function of the ratio \( y/x \) (See Ref. 3)

The angle of twist of the section is

\[ \tau = \frac{f_r}{\gamma b^1 G} \]

where

- \( \tau \) = angle of twist in radians per unit length
- \( f_r \) = rupture stress (from beam tests)
- \( \gamma \) = a function of \( y/x \)
- \( b^1 \) = width of web
- \( G \) = shearing modulus of elasticity

The torsional moment

\[ M_t = CG \tau \]

which may be written

\[ M_t = \frac{f_r}{\gamma b^1} \sum \beta x^3 y \]

The value of \( \gamma \) is essentially equal to unity for the usual ratios \( y/x \) in flanged beams, so for an L-shape flanged beam

\[ M_t = \frac{f_r}{b^1} \sum \beta x^3 y \]
For a rectangular cross section with \( b^1 = x \), the moment equation becomes

\[
M_t = \int_{x}^{\beta/\gamma} x^3 y \, \frac{f_{y}}{r}
\]

and by designating \( a = \beta/\gamma \) (See Ref. 3)

\[
M_t = \int_{r}^{a} x^2 \, y
\]

The additional torsional moment due to the lateral steel provided in the beam is given by the empirical expression

\[
M_{ts} = \frac{1.6 \ A_{sv}}{s} \ f_y \ x_1 y_1
\]

where \( A_{sv} \) = cross-sectional area of reinforcement in the closed hoop

\( f_y \) = yield stress of lateral steel

\( s \) = pitch of the hoops

\( x_1 \) and \( y_1 \) = smaller and larger dimensions of the hoop, respectively, measured center to center.

An equal amount of longitudinal steel is required to resist the horizontal component of the diagonal tension and must satisfy

\[
A_{sl} > 2 A_{sv} \ \frac{x_1 + y_1}{s}
\]

where \( A_{sl} \) = cross-sectional area of all the longitudinal bars.

Using the equations above, the ultimate torsional capacity \( T_{ult} \) is given by

\[
T_{ult} = M_t + M_{ts}
\]
In order to develop yield in all the slab bars perpendicular to the beam, 
\[ T_{ult} \geq T_B \]. The total slab moment and the ultimate torsional capacity of each structure are plotted in Part (b) of Figs. 6.2 - 6.6.

The total slab moment and the beam torsional capacity were equal only in specimen S4. As a result this was the only test structure that developed yield in every slab bar perpendicular to the beam. Torsional cracking (Part (c) of Figs. 6.2-6.6) was most severe in the region between the edge of the column and the point at which the torsional capacity was surpassed. The worst rotational distress was observed at the cracks represented by heavy lines.

6.4 Torsional Efficiency.

The efficiency of these structures in carrying load was correlated according to the torsional beam stiffness of each specimen. Efficiency was determined by computing the ratio of steel stresses developed at failure to the yield stresses of the steel; the efficiency was based upon the development of a yield moment \( M'_y \) along the clear span of the spandrel beam. To study the extent of the development of the steel, the efficiency factor was plotted against the dimensionless parameter \( T_{ult}/M'_y \) (Fig. 6.8) for each of the five test specimens.

The straight line shown represents an approximate relationship between the development of the slab steel in a structure and the beam torsional strength. The figure implies that the perpendicular slab steel will be developed when \( T_{ult} > M'_y \). In effect the parameter \( T_{ult}/M'_y \) distinguishes
between a yield failure in the slab and a torsional beam failure and provides a basis for decreasing the required slab strength if the beam is incapable of developing sufficient torsional strength.

The two kinds of failures mentioned above are important considerations in yield line analysis. Yield lines provide a convenient method of determining the strength of a section. This approach requires the assumption of a yield line in various possible locations and computing the load corresponding to a given yield pattern.

The commonly assumed yield line for a slab is along the inside edge of the beam as shown in Fig. 6.9(a) and the yield load is determined on the basis of this assumed pattern. However, the results obtained in this study point out that such a yield line is incorrect for any structure that does not have sufficient beam torsional strength. The valid yield line for such a structure is shown in Fig. 6.9(b). Only if the beam has sufficient torsional capacity to yield all the slab bars is the yield line in Fig. 6.9(a) valid.
7. **SUMMARY**

7.1 **Object and Scope.**

This paper describes the results of tests on five 0.3 scale models of the exterior column region of reinforced concrete floor slabs. The behavior and results of the tests to failure are discussed. The test structure (corresponding to the section of an actual structure bounded by the exterior edge and the panel centerlines) was loaded by two symmetrical concentrated loads applied at points that were approximately points of contraflexure in a continuous slab structure.

7.2 **Test Program**

The primary variables of each test were the size and stiffness of the spandrel beam. Two of the test structures had beams scaled down directly from a typical prototype structure, while the other three structures had beams of such dimensions that there was a wide range in beam stiffness parameters (See Table 2.1).

Each structure was tested to failure in one test in which incremental loads were applied. For each load increment various deflections and reinforcing steel strains were read. At an effective working load, based on maximum stresses of 20 ksi in the slab steel, the behavior of the structures was characterized by small deflections and small rotations. This was attributed in part to the high strength of the concrete. At ultimate load, four of the structures showed severe torsional beam distress while structure S4 failed by yielding of the slab steel. Specimens S1 and S2 developed cracks.
comparable to those developed in corresponding slab sections of two continuous structures (1, 4). Beam strength, which was primarily dependent upon the amount of lateral beam reinforcement, governed the amount of yielding in the slab and the extent of torsional cracking in the beams.

7.3 Analysis

Measured strains were converted to unit moments by using a moment-strain relationship which considered tension in the concrete (See Chapter 6). The torsional capacity of the beams was determined using a procedure suggested in Reference 5. The ultimate beam torsional moment was compared to the yield moment in the slab in computing the efficiency of the structure (ratio of steel stresses developed to yield stresses of the steel), and in determining the mode of failure (development of a yield line in the slab at the face of the beam or a torsional beam failure).

Although the extent of this series of tests is not sufficient to develop a quantitative method of analysis for the stiffness of a slab-beam-column joint, the study points out that spandrel beams can be designed in a manner such that the joint will have sufficient stiffness to develop the yield strength of the exterior slab section perpendicular to the spandrel beam.
REFERENCES


6. Mayes, G. T., M. A. Sozen, and C. P. Siess, "Tests on a Quarter-Scale Model of a Multiple-Panel Reinforced Concrete Flat Plate Floor," Structural Research Series No. 181, Department of Civil Engineering, University of Illinois, September, 1959.

TABLE 2.1  BEAM DIMENSIONS AND STIFFNESS PARAMETERS

<table>
<thead>
<tr>
<th>Structure</th>
<th>Beam Dimensions</th>
<th>Elastic Stiffness Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Depth, in.</td>
<td>Width, in.</td>
</tr>
<tr>
<td>S1</td>
<td>3.25</td>
<td>4.75</td>
</tr>
<tr>
<td>S2</td>
<td>6.25</td>
<td>2.375</td>
</tr>
<tr>
<td>S3</td>
<td>2.125</td>
<td>4.75</td>
</tr>
<tr>
<td>S4</td>
<td>4.75</td>
<td>4.75</td>
</tr>
<tr>
<td>S5</td>
<td>4.75</td>
<td>4.75</td>
</tr>
</tbody>
</table>
### TABLE 2.2 SCHEDULE OF REINFORCEMENT

<table>
<thead>
<tr>
<th>Structure</th>
<th>Slab Steel</th>
<th>Beam Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Perpendicular Bars*</td>
<td>Parallel Bars*</td>
</tr>
<tr>
<td></td>
<td>Middle Strip</td>
<td>Column Strip</td>
</tr>
<tr>
<td>S1</td>
<td>4 @ 9</td>
<td>10 @ 3.6</td>
</tr>
<tr>
<td>S2</td>
<td>4 @ 9</td>
<td>8 @ 4.5</td>
</tr>
<tr>
<td>S3</td>
<td>4 @ 9</td>
<td>10 @ 3.6</td>
</tr>
<tr>
<td>S4</td>
<td>4 @ 9</td>
<td>10 @ 3.6</td>
</tr>
<tr>
<td>S5</td>
<td>4 @ 9</td>
<td>10 @ 3.6</td>
</tr>
</tbody>
</table>

* Number of bars and spacing in inches, #2 bars
** Spacing in inches from column, #9 gauge wire
<table>
<thead>
<tr>
<th>Structure</th>
<th>Steel f_y, ksi</th>
<th>Batch</th>
<th>W/C By wt.</th>
<th>Curing in form-days</th>
<th>Age at Testing-days</th>
<th>f'_c, psi</th>
<th>f_r, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>43.4</td>
<td>A</td>
<td>0.67</td>
<td>7</td>
<td>15</td>
<td>4805</td>
<td>585</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>0.67</td>
<td></td>
<td></td>
<td>5005</td>
<td>609</td>
</tr>
<tr>
<td>S2</td>
<td>42.4</td>
<td>A</td>
<td>0.71</td>
<td>7</td>
<td>16</td>
<td>4335</td>
<td>466</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>0.71</td>
<td></td>
<td></td>
<td>4285</td>
<td>460</td>
</tr>
<tr>
<td>S3</td>
<td>45.6</td>
<td>A</td>
<td>0.71</td>
<td>7</td>
<td>16</td>
<td>4505</td>
<td>481</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>0.71</td>
<td></td>
<td></td>
<td>4665</td>
<td>535</td>
</tr>
<tr>
<td>S4</td>
<td>43.2</td>
<td>A</td>
<td>0.71</td>
<td>7</td>
<td>14</td>
<td>4035</td>
<td>445</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>0.71</td>
<td></td>
<td></td>
<td>4290</td>
<td>458</td>
</tr>
<tr>
<td>S5</td>
<td>47.0</td>
<td>A</td>
<td>0.71</td>
<td>7</td>
<td>14</td>
<td>3875</td>
<td>524</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>0.71</td>
<td></td>
<td></td>
<td>4015</td>
<td>450</td>
</tr>
</tbody>
</table>
FIG. 2.1(a) THE TEST STRUCTURE

FIG. 2.1(b) REINFORCEMENT IN PLACE
FIG. 2.2 COMPARISON OF MOMENT DISTRIBUTIONS IN S1
FIG. 2.3 COMPARISON OF MOMENT DISTRIBUTIONS IN S3

- Theoretical (free edge)
- Experimental (free edge)
- Theoretical (zero slope)

Section had cracked under dead load

Unit Moment Centres, \( \frac{p}{M} \)
FIG. 2.4 COMPARISON OF MOMENT DISTRIBUTIONS IN S4 AND S5
FIG. 2.5 COMPARISON OF MOMENTS FOR UNIFORM VS. CONCENTRATED LOADS
FIG. 2.6 DIMENSIONS OF THE TEST STRUCTURE
FIG. 3.1 TYPICAL STRESS-STRAIN CURVES FOR REINFORCEMENT

- #2 Plain Bars
  - σult = 65.5 ksi
  - εult = 2.52" in 2"

- #9 Annealed Wire
  - σult = 63.8 ksi
FIG. 4.1 LOADING FRAME

- Free to rotate
- 27" 
- 2-1/8"
- 3" 
- Steel Ball
- 4-3/4"
FIG. 4.2 TEST SETUP - LOADING SYSTEM

FIG. 4.3 TEST SETUP - INSTRUMENTATION
FIG. 5.1 STRESS DISTRIBUTIONS AT WORKING LOAD
FIG. 5.2 LOAD-BEAM ROTATION CURVES
FIG. 5.3 LOAD—BEAM DEFLECTION CURVES (D1)
FIG. 5.4 LOAD -- SLAB DEFLECTION CURVES (D7)
FIG. 5.5 CRACK PATTERNS AFTER LOAD TO FAILURE - S1
FIG. 5.6 CRACK PATTERNS AFTER LOAD TO FAILURE - S2
FIG. 5.7 CRACK PATTERNS AFTER LOAD TO FAILURE - S3
FIG. 5.8 CRACK PATTERNS AFTER LOAD TO FAILURE - S4
FIG. 5.9 CRACK PATTERNS AFTER LOAD TO FAILURE - S5
FIG. 5.10 CRACK PATTERNS - UNIVERSITY OF ILLINOIS
TEST STRUCTURE
FIG. 5.11 LOAD - STRAIN RELATIONSHIPS, BEAM REINFORCEMENT - S4
FIG. 5.12 STRESS DISTRIBUTIONS - SLAB REINFORCEMENT - S1
FIG. 5.13 STRESS DISTRIBUTIONS - SLAB REINFORCEMENT - S2
FIG. 5.14 STRESS DISTRIBUTIONS - SLAB REINFORCEMENT - S3
FIG. 5.15 STRESS DISTRIBUTIONS - SLAB REINFORCEMENT - S4
FIG. 5.16 STRESS DISTRIBUTIONS - SLAB REINFORCEMENT - S5
Typical Moment - Strain Relationships

\[ M_y = A_s f_y \]

\[ M_{cr} = \frac{f_y h_1}{c} \]
FIG. 6.2 MOMENT DISTRIBUTION AT FAILURE - S1
FIG. 6.3 MOMENT DISTRIBUTION AT FAILURE - S2
(a) SLAB UNIT MOMENTS

(b) SLAB TOTAL MOMENT

(c) CRACKS IN SIDE OF BEAM

FIG. 6.4 MOMENT DISTRIBUTION AT FAILURE - S3
FIG. 6.5 MOMENT DISTRIBUTION AT FAILURE - S4
FIG. 6.6 MOMENT DISTRIBUTION AT FAILURE - S5
FIG. 6.7  FREE BODY DIAGRAM FOR BEAM TORSIONAL MOMENTS
FIG. 6.8 EFFICIENCY OF BEAM IN DEVELOPING SLAB REINFORCEMENT STRESSES
FIG. 6.9 MODES OF FAILURE

(a) FLEXURAL FAILURE

(b) TORSIONAL FAILURE
APPENDIX: SPECIAL FINITE DIFFERENCE OPERATORS

The special difference operators used here account for both the torsional stiffness and the flexural stiffness of spandrel beams. The model leading to the difference operators is shown below, where the beam includes torsional rods as well as flexural springs. This model is used in specifying the deflection at the nodes. As shown, a uniform grid spacing in both directions is used in the slab, while the beam spacing is $h$ along the centerline with a beam width of $2b$. Poisson's ratio, $\mu$, is assumed equal to zero. The parameters used in the following patterns are

$$k = \frac{h}{b}$$

$$\gamma = \frac{GJ}{Db}$$
\[ H = \frac{EI}{Dh} \]

where \( G \) = shearing modulus of elasticity

\[ J = \text{torsional moment of inertia of beam}, \quad \sum \theta x^3 y \]

\[ D = \text{flexural plate stiffness}, \quad Et^3/12(1-\mu^2) \]

\[ E = \text{modulus of elasticity} \]

\[ I = \text{moment of inertia of beam} \]

Moments at the beam-slab interface are given by the following:

\[ M_y = \frac{2D}{h} \begin{bmatrix} -k & 1+k & -1 \end{bmatrix} \]

\[ M_x = \frac{D}{h} \begin{bmatrix} -1 & 2 & -1 \end{bmatrix} \]

MOMENT ACROSS INTERFACE

MOMENT ALONG INTERFACE

The beam moment is given by

\[ M_B = \frac{D}{h} \begin{bmatrix} 0 & -H & 2H & -H \end{bmatrix} \]

The following deflection patterns are for a uniformly distributed load \( p \) and for a concentrated load \( P \) applied at the node points.

* See page 25
\[
\frac{D}{h^2} = p h^2 + P
\]

GENERAL SLAB POINT ADJACENT TO BEAM

\[
\frac{D}{h^2} = p h^2 + P
\]

SLAB POINT ADJACENT TO BEAM AND FREE EDGE
SLAB POINT ADJACENT TO BEAM AND FREE EDGE

\[ \begin{array}{c}
\frac{D}{h^2} \\
\frac{8.5}{-6} \\
\frac{-4}{-1} \\
\text{free edge} \\
\frac{0.5}{2}
\end{array} \begin{array}{c}
k \\
-4 -k \\
2 \\
1 \\
\text{beam } \phi
\end{array} = p h^2 + P
\]

GENERAL POINT ON BEAM - SLAB INTERFACE

\[ \begin{array}{c}
\frac{D}{h^2} \\
\frac{0.5}{-4} \\
\frac{-4}{-4} \\
\frac{-8}{-2k} \\
\frac{2}{1}
\end{array} \begin{array}{c}
k \gamma \\
-2k(1+k+\gamma) \\
k \gamma \\
-2k(2+k+\gamma) \\
-2k \\
\text{beam } \phi
\end{array} = \frac{p h^2}{2} + P \]
\[ \frac{D}{h^2} = \frac{p h^2}{2} + P \]

POINT ON BEAM - SLAB INTERFACE ADJACENT TO FREE EDGE

\[ \frac{D}{h^2} = \frac{p h^2}{4} + P \]

POINT ON BEAM - SLAB INTERFACE AND FREE EDGE
\[
\frac{D}{h^2} = 2 \text{pbh} + P
\]

**GENERAL POINT ON BEAM CENTERLINE**

\[
\frac{D}{h^2} = 2 \text{pbh} + P
\]

**POINT ON BEAM CENTERLINE ADJACENT TO FREE EDGE**

\[
\frac{D}{h^2} = \frac{\text{pbh}}{2} + P
\]

**POINT ON BEAM CENTERLINE AND FREE EDGE**