RICE UNIVERSITY

An Adaptive Model of Left-Heart Function

by

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A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

Master of Science in Electrical Engineering

Thesis director's signature

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Houston, Texas

May 1972
ABSTRACT

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The methods of mathematical modeling and digital computer simulation can be beneficially applied to the development and testing of cardiac assist devices. Such methods, used in conjunction with other investigative techniques, provide the researcher with a powerful tool for approaching the many difficult problems associated with operation and control of assist devices.

Of utmost priority in this modeling application is the availability of a left-heart model which is responsive to variations in left-heart operating conditions. A left-heart model capable of realistically adapting to variations in operating conditions is necessary for evaluation of the effects which an assist device may have on the left-heart.

For this purpose a left-heart model is formulated which is responsive to variations in heart rate, left-atrial filling pressure, left-ventricular end-diastolic volume, left-ventricular end-systolic volume and aortic load. The operation of this left-heart model is demonstrated by use of a simple aortic model in computer simulations. The potential of the modeling technique in general (and of the adaptive left-heart model in particular) for use as a research tool is exhibited by the development and use in simulations of a model for an intra-aortic balloon pump assist device. Suggested enhancements to the models for research application are presented.
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CHAPTER 1 -- INTRODUCTION

Models of the arterial system, and particularly the left heart and aorta, are potentially very useful because of the widespread development and testing of cardiac assist devices. Many of these devices are designed to connect to or near the aorta, and thereby create their greatest effect at that location. Mathematical models of this part of the cardiovascular system, when combined with a suitable model for an assist device, can enable study of the effects of such a device by means of computer simulation.

Any left-heart model which is to be used to reflect the effects of a cardiac assist device must have a left ventricular component capable of adapting in a realistic fashion to pre-load (end diastolic volume) and after-load (aortic loading conditions). Cardiac assist devices commonly act so as to augment aortic pressure during diastole and maintain or reduce aortic pressure during systole, thereby raising mean aortic pressure to aid perfusion while maintaining or reducing heart work. This type operation has a direct influence on left ventricular function.

Previous modeling studies (Beneken, 1965; Suga, 1971; Robinson, 1965; Grodins, 1967) have made use of left-heart models of varying complexity and completeness. The left-heart model to be described in this study has been used by Snyder (1968), Beneken (1965) and others. Its operation requires specification of a time-course for left ventricular elastance (pressure-volume ratio). The realism of the
model's operation depends directly on this elastance time-course. In recent work, Greene, et al., have formulated an elastance generator which is a function of end-diastolic volume and time. This study has resulted in the creation of a left ventricular elastance function which is dynamically adjustable for variations in pre- and after-load, and for heart rate.

Development and verification of the ventricular elastance function is facilitated by providing an arterial load into which it can operate. For this use, an aortic model is described which serves as a means of exhibiting the operation of the left-heart model under a variety of operating conditions. The aortic model used is similar to one considered by Spencer (1963).

The potential of the adaptive left-heart model for application in the investigation of cardiac assist devices is demonstrated by formulation of a model for an intra-aortic balloon pump. This device and its control have been the object of considerable research for several years (e.g., McMahon, 1969; Brown, 1969; Kane, 1971). The device consists of a non-elastic, collapsible tubular polyurethane balloon which is connected by means of a thin catheter to a pressure/vacuum source. The balloon can be entered through a cannula into the femoral artery and moved to the vicinity of the aortic arch. By alternatively applying pressure and vacuum, in some sequence with the pumping action of the heart, the inflating and deflating balloon is
hypothesized to assist circulation, particularly the crucial coronary circulation.

The operation of the balloon pump in the aorta directly affects the left ventricle. The adaptive left ventricular model of this study will be shown to demonstrate clinically observed aspects of these effects.

Chapter two presents the fundamental mechanics of the systems to be modeled and describes the models chosen to represent each of them. Chapter three treats the computational considerations and the operation of the program for simulation with the models. Chapter four displays and explains typical output from the simulations. Chapter five serves to summarize the results and to present proposals for additional work.
CHAPTER 2 — THE MODEL

In this chapter the fundamental mechanical properties of blood flow in large arteries are described and electrical analogies are pointed-out. The electrical analogies are utilized in developing a model for blood flow in an arterial segment which adequately characterizes the important properties of that system. Using this model of an arterial segment, a lumped parameter model of a portion of the aorta is proposed. The basic mechanics of left heart function are then described, and the left heart model used to drive the arterial model is presented. Next, the mechanical properties of an intra-aortic balloon pump are described, and a simple model is formulated for this device. Finally, the left heart, arterial and intra-aortic balloon pump models are combined and the system differential equations for the complete model are given. Complete details of the development of the system differential equations may be found in Appendix A.

2.1 Fundamentals of Arterial Blood Flow and Electrical Analogies

Blood flow in an arterial segment may be considered as the flow of a viscous fluid through an elastic tube. Such a system exhibits the fundamental properties of resistance, compliance, and inertance.

Blood flowing through an artery is subject to resistance arising from internal shearing among the laminae of different velocity. This fluid resistance is analogous to electrical resistance, and is defined
as the pressure drop across the segment divided by the rate of flow through the segment. Additionally, radial movement of the arterial wall is subject to resistance resulting from the visco-elastic nature of the wall.

The ability of the arterial wall to store energy by stretching and then to later restore this energy to the system by accelerating the blood is termed compliance. Compliance is calculated as the blood volume within an arterial segment divided by the transmural pressure (pressure difference across the arterial wall). Compliance is the mechanical analogy to electrical capacitance, where blood volume is analogous to charge. The term elastance, which will be used in describing the ventricular model, is defined as the reciprocal of compliance. A vessel with high compliance hence has low elastance, and will display relatively large increases in volume with relatively small increases in transmural pressure.

The property of inertance is characteristic of the acceleration of blood, by virtue of its having mass. The inertance, which is analogous to electrical inductance, is the tendency of the blood to resist changes in velocity. It is calculated as the pressure drop across the segment divided by the time derivative of blood flow through the segment. Inertance also appears to some degree in the opposition of the arterial wall to acceleration, but this effect is considered slight, at least relative to that of the other properties of the system (Spencer, 1963).
Since the important properties of blood flow in an arterial segment have electrical counterparts whose symbols are well known and easily drawn, it is a natural simplification to deal with the electrical symbols in drawing figures. Thus the blood flow through a homogeneous segment of artery could be approximated as shown in Figure 2.1.

![Figure 2.1 Model for an Arterial Segment](image)

In Figure 2.1, L is the blood inertance; RA is the blood shearing resistance; C is the arterial wall compliance; and RR is the resistance to radial wall movement. With constant conditions, all flow is from "a" to "b", but in response to a pressure change at "a" the system will exhibit a transient flow through compliance C and the radial resistance RR. This of course, is merely a change in vessel diameter (with corresponding expenditure of energy) in response to a change in transmural pressure.

The model of Figure 2.1 demonstrates the most important features of blood flow through an arterial segment. More complicated models have been proposed (Spencer, 1963), but the four physical characteristics
included in Figure 2.1 proved adequate for the purposes of this study. Less sophisticated models, using only the elements RA and C of Figure 2.1, have also been used. This R-C model is well known as the "windkessel" arterial model; it is sufficient only when the gross characteristics of an arterial segment are needed. Robinson (1965) and Grodins (1967) have used the windkessel arterial model as a simple load for testing ventricular models.

2.2 Formulation of the Aortic Model

In order to display the effects of an intra-aortic balloon pump, a model of the aorta for the region indicated by broken lines in Figure 2.2 is required.

![Figure 2.2 Schematic of Aortic Region to be Modeled](image)

This portion of the systemic circulation has several unique properties which are significant to a modeling effort. Ignoring flow through the coronary arteries, only two possible exits for blood entering the ascending aorta from the left heart exist. It may either follow the
branch consisting of the subclavian (et. al.) arteries, or it may traverse the aortic arch and flow down the descending aorta. Of course, due to the compliant nature of the aortic wall, particularly in the ascending aorta, transient radial flows may also occur. In addition, the acceleration of the blood while being ejected by the left ventricle into the aortic root is quite high (Spencer, 1963) so that the blood inertance is likely to have a significant influence on the shape of the pressure and flow waveforms in this region.

After much consideration and testing, the model shown in Figure 2.3 was determined to adequately represent the region of the aorta depicted by broken lines in Figure 2.2.

![The Aortic Model](image)

**Figure 2.3 The Aortic Model**

In Figure 2.3, RL represents the resistive pathway consisting of the subclavian (et. al.) arteries to the upper body; CL represents the compliance of the ascending aorta and early part of the aortic arch; R1 represents the resistive nature of the aortic wall movement in the ascending and aortic arch regions; L characterizes the inertance of the
blood mass; RR represents the resistive pathway of the descending aorta; CR represents the compliance of the descending aorta; and R2 represents the resistive nature of the aortic wall movement in the descending aorta. This model, which results from combining two segmental models as shown in Figure 2.1, is essentially the same as that proposed by Spencer (1963).

The model of Figure 2.3 has three energy storage elements (CL, L and CR) and hence will require specification of three initial conditions. The desired model output for this system consists of the pressure at node A, the pressure at node B, and the flow between A and B. These quantities represent blood pressure in the aortic arch, blood pressure in the descending aorta, and blood flow between these locations, respectively. Since this model is a lumping of the actual physical distributed system, it is not possible to define an exact anatomical location for nodes A and B of Figure 2.3, but neither is it necessary. Figure 2.4 relates the nodes and branches of the lumped parameter model of Figure 2.3 to the anatomy of the physical system.

Figure 2.4 Relation of Aortic Model Elements to the Anatomy
As mentioned, the locations of points A and B are only conceptual, and serve mainly to provide a visualization of the relations between the model and the physical system.

2.3 **Fundamental Mechanics of Left Heart Operation and a Model to Simulate Them**

The problem of forming a model of the aorta is primarily one of selecting and lumping suitable segmental models. The problem of formulating a satisfactory model for the left heart is quite different, primarily as a result of the complexity of the organ. Whereas the large arteries are largely passive in nature (and completely so in the model used herein), expanding and contracting in response to internal pressure changes, the heart is an active element which undergoes changes in function in response to varying conditions of loading, filling and other parameters.

Structurally the left heart consists of two parts, an atrium and a ventricle, (see Figure 2.5) which are connected by a pressure operated flap valve called the mitral valve.

![Figure 2.5 Schematic of the Left Heart Anatomy](image-url)
The atrium functions mainly as a storage chamber for the blood arriving at the left heart from the lungs while the mitral valve is closed. The ventricle is responsible for raising the pressure of the blood obtained from the atrium to a pressure adequate for forcing the aortic valve open and ejecting the blood charge into the aorta.

The left heart has four distinct phases of operation, as shown in Figure 2.6.

Phase 1: ventricular filling
Phase 2: isovolumic contraction
Phase 3: ejection
Phase 4: isovolumic relaxation

Figure 2.6 The Four Heart Phases

The left heart model of this study is intended to function as a source of the proper pressure waveform for driving the arterial model. For this use, it must be capable of responding to variations in aortic load. For this purpose the model shown in Figure 2.7 (Beneken, 1965) is quite sufficient.
In Figure 2.7, PAS represents the left atrium pressure source; RM represents resistance of the open mitral valve; SM represents the open or closed position of the mitral valve; CV(t) is a time varying capacitance characterizing the left ventricle; SA represents the open or closed position of the aortic valve; and RA represents resistance of the open aortic valve.

Modeling the heart valves as switches is equivalent to assuming the valves operate with no leakage, and that their opening and closing is instantaneous. Note that backflow through the valves is a function of the timing of the switches, in conjunction with the pressure differentials across them, and thus can be simulated with the model shown in Figure 2.7. Representation of the atrium as a constant pressure source makes no provision for an atrial contraction, and requires ventricular filling to occur in the presence of a constant filling pressure. The effect of these assumptions is to eliminate
certain details from the model waveforms, and although it is easy to expand the heart model to eliminate the assumptions, the slight improvement in the "realism" of the waveforms is of no consequence in this study.

Operation of the heart model switches SM and SA is diagrammed in Figure 2.8. The four heart phases correspond to those shown in Figure 2.6.

During Phase 1 the left atrial pressure source PAS fills the ventricular compliance CV(t) through the mitral valve resistance RM. In Phase 2 the isolated ventricle contracts (compliance CV(t) decreases in value), thereby increasing the pressure across it. As previously mentioned, compliance is calculated as volume divided by pressure; hence, decreasing compliance with volume fixed causes an increase in pressure. In Phase 3 the higher pressure across CV(t) displaces the blood charge through the aortic valve resistance RA. In Phase 4 the isolated ventricle relaxes (compliance CV(t) increases in value) causing a decrease in
ventricular pressure to atrial source pressure, at which time the mitral valve SM reopens and Phase 1 begins for the next cycle.

Clearly the behavior of the heart model is greatly dependent on the time course of the time varying compliance CV(t). As will be discussed in Chapter 3, this time course is a rather complicated function of heart rate, end diastolic ventricular volume, and other factors. For the sake of simplification in subsequent notation, reference will be made to ventricular elastance EV(t) rather than ventricular compliance CV(t).

Elastance is computed from simultaneous measurements of ventricular volume V(t) and ventricular pressure P(t) according to the relationship

\[ EV(t) = \frac{P(t)}{V(t)} \]

Ventricular volume measurement is difficult in most experiments, and the volume is often computed from aortic flow measurements I(t) by the relation

\[ V(t) = V(0) - \int_0^t I(t) dt, \]

where V(0) is end diastolic volume (ventricular volume at the end of the filling phase).

Recent interest in characterizing ventricular elastance has resulted in a number of articles on the subject (Suga, 1969, 1970, 1971). Unfortunately most published results deal with qualitative aspects of the elastance time course rather than in quantitative terms. However, canine pressure-volume data during isovolumic contraction and ejection for a single heart rate and various end diastolic volumes were obtained from Roger R. Taylor (Dept. of Medicine, University of Western Australia). These data provided the basis for formulating the ventricular elastance function which is described in Chapter 3.
2.4 The Intra-Aortic Balloon Pump -- The fundamentals and a Model

The Intra-aortic balloon pump is basically a non-elastic, cigar-shaped, collapsible polyurethane bag to which is connected, by means of a small diameter stiff catheter, a source of gas (usually carbon dioxide) under pressure or vacuum. The balloon has a small profile when deflated and can be inserted through a cannula into the femoral artery. The balloon is then guided upward into the aorta until its tip is near the aortic arch. Operation of the balloon pump consists of switching it alternately between pressure and vacuum sources, thereby inflating and deflating it in some kind of sequence with the operation of the heart, the goal being augmentation of blood circulation, particularly to the heart and upper body. The device is generally proposed as a temporary means of assisting the heart in a patient suffering from cardiogenic shock.

Timing of the switching operation is usually based on the occurrence of the R-wave in the EKG, which is indicative of the onset of ventricular contraction. Based on the heart rate, the balloon is inflated after a preset delay from the occurrence of the R-wave. The delay is chosen so as to cause the balloon to inflate after the aortic valve is closed. After another preselected delay (or immediately upon encountering an unexpected R-wave) the balloon is deflated, this commonly taking place before the next ventricular ejection. Efforts to determine optimal timing strategies have been made in vivo and with mechanical circulation models (McMahon, 1969; Brown, 1969; Kane, 1971).
The problem of optimizing the effects of balloon pumping are formidable, and especially so when balloon inflation and deflation pressures and balloon volume are included as variables. The task of formulating a sufficient performance index, together with sufficient constraints on the variables, is in itself a complex problem. Results of computing a performance index according to Kane (1971) when utilizing the balloon model of Figure 2.9 are given in Chapter 4.

The balloon pump model used in this study is shown inside the dashed lines of Figure 2.9, in which the balloon pump model is shown connected to the arterial model of Figure 2.3.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.9.png}
\caption{The Intra-Aortic Balloon Pump Model}
\end{figure}

In Figure 2.9, \( RB(t) \) is the resistance to blood flow created by the presence of the balloon in the aorta; \( SB \) is a switch allowing a connection to the pressure/vacuum source to be made and broken; \( RBS \) is the resistance of the catheter connecting the pressure/vacuum source to the balloon; and \( PBS(t) \) is the pressure/vacuum source.
Clearly with direction depending on the value of the source PBS(t), flow will occur through RBS to or from the arterial model whenever switch SB is closed. To understand how this represents the action of the balloon pump, consider that when the balloon inflates in the aorta it displaces a certain volume of blood. The source of the displaced blood is the reduction in internal aortic volume caused by the inflating balloon. The effect however, is equivalent to having a fixed internal aortic volume and injecting an additional blood volume into it. This is the concept upon which the balloon model is based. When the balloon is inflating, charge flows into the arterial model. When a quantity of charge equivalent to that which the inflated balloon would displace (recall that charge is the electrical analogy to volume) has entered the arterial model, switch SB is opened. Balloon deflation involves an analogous removal of charge.

To complete the balloon pump model, it is necessary to add a resistance in the aortic model corresponding to the added resistance to blood flow created by the physical presence of the balloon in the flow path. Resistance RB(t) serves this purpose; its value is representative of the shape of the balloon, its size relative to the aorta, and its state of inflation or deflation.

Other possible considerations in formulating a balloon pump model include the inertance of the gas used to drive the balloon, the inertance of the balloon itself, and the compliance of the balloon and connecting catheter. By virtue of the (relatively) non-elastic materials
used in the balloon pump and catheter, and the slight mass of the gas accelerated to drive the balloon these factors are considered to be of negligible effect in this study.

2.5 Combination of the Models and Formulation of the Equations

The combined system, comprised of the models for the left heart, aorta, and balloon pump is shown in Figure 2.10.

![Diagram of the combined system](image)

**Figure 2.10** The Complete Model

In Figure 2.10, V1, V2, and V3 are the instantaneous volumes of compliances CV(t), CL, and CR respectively. P1, P2, and P3 are the instantaneous pressures at the locations shown. F is the flow through inertance L.

Since this system has four energy storage elements, together with switches and time varying elements, a mathematical analysis of the system may be expected to result in a set of four first-order, non-linear ordinary differential equations. The information, or output, which is desired from the model consists of the pressures P1, P2, P3,
and the flow F. Differential equations involving P_i directly would introduce a term containing the derivative of CV(t). However, CV(t) has a complicated time course, and to avoid using its derivative the system equations are formulated in terms of the compliance volumes V_1, V_2, and V_3 rather than the pressures P_1, P_2, and P_3. Since the pressures (rather than the compliance volumes) are the desired model output, the relationships necessary for translating the V's into the P's are also derived.

In writing the system equations, switches are used as multipliers, with the convention that the value of the multiplier is zero if the switch is open, and one if the switch is closed. Also, all pressures (P's), compliances volumes (V's) and flow (F) are of course functions of time, but for simplicity of notation the "(t)" will not be explicitly written. The model elements (such as ventricular elastance) which are time varying however, are so indicated by the appended "(t)".

With these considerations in mind, the system equations are as follows: [See Appendix A for details of the derivation.]

\[ \dot{V}_1 = EV(t) \cdot \{(SA/RA) \cdot [(SA \cdot R_1)/(RA \cdot K_1(t)) - 1] - (SM/RM) \} \cdot V_1 \]
\[ + \left\{ \begin{array}{c} \frac{SA}{(RA \cdot CL)} \cdot \{1 - [R_1/K_1(t)] \cdot [(SA/RA) + (SB/RBS) + (1/RL)] \} \cdot V_2 \\ \quad \frac{(SA \cdot R_1)}{[RA \cdot K_1(t)]} \cdot F \\ + \frac{(SM \cdot PAS)}{RM} + \frac{[SA \cdot R_1 \cdot SB \cdot PBS(t)]}{[RA \cdot K_1(t) \cdot RBS]} \end{array} \right\} \] \hspace{1cm} (2.1)

\[ \dot{V}_2 = \left\{ \begin{array}{c} \frac{1}{K_1(t)} \cdot [(SA/RA) \cdot EV(t) \cdot V_1] \\ -(1/CL) \cdot [(SA/RA) + (SB/RBS) + (1/RL)] \cdot V_2 \\ - F + (SB/RBS) \cdot PBS(t) \end{array} \right\} \] \hspace{1cm} (2.2)
\[ V_3 = \left[ \frac{1}{(R_2+R_2)} \right] \cdot \left[ \frac{(-1/C_R) \cdot V_3 + R_2 \cdot F}{1} \right] \] (2.3)

\[ F = \left\{ \left[ R_1 \cdot S_A \cdot E_V(t) \right]/\left[ L \cdot K_1(t) \cdot R_A \right] \right\} \cdot V_1 + \left[ \frac{1}{(L \cdot C_L)} \right] \cdot \left\{ 1 - \left[ \frac{R_1}{K_1(t)} \right] \cdot \left[ \frac{(S_A/R_A) + (S_B/R_B) + (1/R_L)}{1} \right] \right\} \cdot V_2 + \left[ \frac{1}{(L \cdot C_R)} \right] \cdot \left\{ \left[ \frac{R_2}{(R_2+R_R)} \right] - 1 \right\} \cdot V_3 \] (2.4)

\[ - \left( \frac{1}{L} \right) \cdot \left\{ \left[ \frac{R_1}{K_1(t)} \right] + \left[ \frac{(R_2 \cdot R_R)/(R_2+R_R)}{1} \right] + R_B(t) \right\} \cdot F + \left[ R_1 \cdot S_B \cdot P_B S(t) \right]/\left[ L \cdot K_1(t) \cdot R_B S \right] \]

In these equations, \( K_1(t) \) is defined as follows:

\[ K_1(t) + 1 + \frac{R_1}{R_L} + \frac{(S_A-R_1)}{R_A} + \frac{S_B \cdot R_1}{R_B S} \] (2.5)

The equations relating the pressures \( P_1, P_2, \) and \( P_3 \) to the compliance volumes \( V_1, V_2, \) and \( V_3 \) are as follows:

\[ P_1 = V_1 \cdot E_V(t) \] (2.6)

\[ P_2 = \left[ \frac{R_1}{K_1(t)} \right] \cdot \left\{ \left[ S_A \cdot E_V(t) \cdot V_1 \right]/R_A \right. \]

\[ \left. \left[ (1/C_L) \cdot \left[ \frac{(S_A/R_A) + (S_B/R_B) + (1/R_L)}{1} \right] \cdot V_2 \right. \right. \]

\[ \left. \left. - \frac{R_1}{K_1(t)} \right] \cdot \left[ \frac{(R_2 \cdot R_R)/(R_2+R_R)}{1} \right] + R_B(t) \right\} \cdot F \] (2.7)

\[ P_3 = \left( \frac{1}{C_R} \right) \cdot \left\{ 1 - \frac{R_2}{(R_2+R_R)} \right\} \cdot V_3 + \left[ \frac{(R_2 \cdot R_R)/(R_2+R_R)}{1} \right] \cdot F \] (2.8)

Models for the left-heart, aorta, and intra-aortic balloon pump have now been proposed and described, and the equations describing the combined model have been presented. The description of these models has involved a number of parameters, several of which have time-varying characteristics. Before the combined model can be used to
produce results each parameter must be specified. The appearance of the model output can be expected to be highly dependent on the choice of these parameters.
CHAPTER 3 -- CONSIDERATIONS FOR SIMULATION WITH THE MODEL

This chapter describes the procedures involved in implementing a digital computer simulation for the model developed in Chapter 2. First, determination of values for the parameters of the aortic model is discussed. The selection of values for balloon model parameters RB(t) and RBS is then described. Following this, parameter values for the left heart model are considered, with particular attention given to generation of the ventricular elastance function [EV(t)]. The critical role played by the elastance function in the model is described. [Details of the ventricular elastance calculating procedure, together with a listing of the FORTRAN routine for computing elastance, are to be found in Appendix C]. By defining the timing of model switches SM, SA, and SB, simplified equation sets for the various heart phases are formulated. Next, conventions concerning operation of the balloon pump are discussed. A brief description of the actual computer solution of the equations is then given, and this is followed by flow diagrams for the most important portions of the program logic. Finally, a complete list of the input requirements for the simulation is given and output options are described.

3.1 Aortic Model Parameter Values

The complete model of Figure 2.10 contains 14 parameters (not including switches). Of these, four are associated with the left-heart model, three with the balloon pump model, and seven with the
aortic model. The aortic model was the first to be developed, and an initial set of parameter values for it was extracted from similar models in the literature (Beneken, 1965; Snyder, 1968; Spencer, 1963). Testing of parameter values for those elements in the aortic model (Figure 2.3) was done by considering the output of the isolated aortic model in the diastolic, or filling phase of the heart cycle. In this phase, the aortic model serves merely for passive run-off of the blood ejected into it during the systolic, or ejection phase. The differential equations for the isolated aortic model are given below. [See Appendix B for the derivation of these equations.]

\[ P_2^* = \frac{1}{(R_1+R_L)} \cdot \left[ (-R_1 \cdot R_L/L + 1/C_L) \cdot P_2 + (R_1 \cdot R_L/L) \cdot P_3 - R_L \cdot F/CL \right] \] (3.1)

\[ P_3^* = \frac{1}{(R_2+R_R)} \cdot \left[ (R_2 \cdot R_R/L) \cdot P_2 - (R_2 \cdot R_R/L + 1/C_R) \cdot P_3 + R_R \cdot F/C_R \right] \] (3.2)

\[ F = \frac{1}{L} \cdot (P_2 - P_3) \] (3.3)

Note that the pressures \( P_2 \) and \( P_3 \) have been used as dependent variables rather than the compliance volumes \( V_2 \) and \( V_3 \). This simplifies the equations and is acceptable in this instance since the time varying compliance \( CV(t) \) is not involved in the isolated aortic model.

A computer program was written in FORTRAN to solve\(^1\) the three equations over an interval of 0.3 seconds, and to plot the resulting

\(^1\)The numerical method used to solve the equations is described later in this chapter.
time courses for P2, P3 and F. The plots were then visually compared with representative curves from the literature to ascertain the realism of the model output. Appropriate initial conditions for P2, P3 and F at the start of diastole were selected from data in the literature.

The initial set of parameters for the aortic model yielded completely unrealistic output, and after several "try-it-and-see" modifications, it was quite clear that a mathematical parameter identification technique would be desirable. Such a technique, given guesses at model initial conditions and parameter values, iteratively adjusts the parameter values and initial conditions so as to obtain the "best" fit of model output to supplied experimental measurements. Subsequently, an effort was made to utilize a technique described by King (1968). Unfortunately, the technique did not work well in this application, and provided no additional information about the aortic model parameters.

As experience with the effect of the aortic model parameters increased, the value of the inertance term (L) appeared to be the most critical with respect to effect on the output. Once again a mathematical procedure was used requiring experimental data — this time in an attempt to obtain a value only for the inertance parameter. If representative time courses are available for P2 and P3 over a time interval $t_0$ to $t_1$, and values for $F(t_0)$ and $F(t_1)$ are known (where $F(t)$ is the aortic flow), then equation (3.3) can be used to yield a value for L as follows:
Rearranging equation (3.3) gives
\[ \frac{dF}{dt} = \frac{1}{L} \cdot (P_2 - P_3) \]  
\[ \text{Integrating (3.4) yields} \]
\[ F(t_1) - F(t_0) = \frac{1}{L} \cdot \int_{t_0}^{t_1} (P_2 - P_3) \, dt \]  
\[ \text{Solving (3.5) for } L \text{ gives} \]
\[ L = \frac{\int_{t_0}^{t_1} (P_2 - P_3) \, dt}{[F(t_1) - F(t_0)]} \]  

The values of the experimental pressures (P2 and P3) are functions of time, but for simplicity of notation have been written in (3.4) through (3.6) without the "(t)" notation. The definite integral in equation (3.6) is evaluated by applying a numerical quadrature algorithm.

In using this procedure, it was discovered that the value obtained for L varied widely, depending on the part of the interval (over which the experimental data was available) that was used in the integration. Hence, the approach was unproductive in establishing a value for the inertance.

The reason for the difficulties in the application of this and King's (1968) method appears to be linked to the use of experimental data. King's algorithm converged nicely when the "experimental" data used was output from the model, and the initial "guesses" at parameters were the values used in generating this output but slightly perturbed. It is believed that the source of the problem lies in the fact that utilizing experimental data in essence requires model nodes A and B in Figure 2.4 to be assigned specific anatomical locations. As stated in Chapter 2, these nodes are really only conceptual points since the model
is a relatively gross lumping of the physical distributed system. The techniques used in the efforts to obtain parameter values tacitly assume that the model can (given proper parameter values and initial conditions) reproduce the experimental data. With the simple aortic model used and the experimental data available, this is not a valid assumption.

The aortic model parameters were gradually "improved" as the experience from the efforts described above provided additional insight into their effects on the model output. The parameters were finally tailored to provide acceptable aortic waveforms by systematically varying them and observing the results. The final values of the aortic model parameters are shown in Table 3.1.

3.2 Balloon Model Parameter Values

An approximation to the flow resistance $R_B(t)$ created by the presence of the intra-aortic balloon pump in the aorta was experimentally measured by constructing a mock circulatory loop. This set-up consisted of a pneumatically operated left heart with cage-constrained ball valves, compliance chambers, and a silastic mold of an aorta. Provision for inserting and removing a balloon designed for use in an intra-aortic balloon pumping experiment was made by providing a "y" at the base of the silastic aorta. By simultaneously recording flow past the balloon and differential pressure across it, the resistance created by its presence could be calculated.

\[\text{2The assistance provided by the Rice Bio-Medical Laboratory in the construction of the mock circulatory loop is much appreciated; not only did they supply most of the equipment, but invaluable advice as well.}\]
Two values for RB(t) were sought from this experiment, corresponding to the inflated and deflated states of the balloon. The resistance created by the deflated balloon was found to be very slight — on the order of one to two mm-Hg·min/liter. This value of resistance was so slight as to make measurement difficult with the available equipment. Inflated balloon resistance is dependent on the diameter of the balloon relative to the inside diameter of the aorta; with the balloon and aorta model which were available, the resistance approximately doubled when the balloon was inflated.

The value selected for use when the balloon is other than fully inflated was 1.0 mm-Hg·min/liter. The value representing inflated balloon resistance is varied in the runs of the simulation to display the effect which this parameter has on the results of balloon pumping; the value is varied from 2 to 200 mm-Hg·min/liter.

The use of only two values for RB(t) (one for the fully inflated balloon, and one for all other conditions) is done to simplify the model simulation. It would be relatively easy to make RB(t) a continuous function of balloon volume, but the simplification resulting from using only two values is considered justification for the slight loss of realism. Also, it is believed that the resistance created by the inflating balloon increases much more rapidly as the balloon nears the aortic wall, and this lends justification to the decision to maintain balloon resistance at its low value until fully inflated. The effect of the step increase in resistance is most noted in the aortic flow
waveform when the value used for the fully inflated condition is high; the result is a precipitous drop in aortic flow when the high value of resistance goes into effect. This feature can be observed in the aortic flow waveform shown in Figure 4.9 of Chapter 4.

There are two additional parameters associated with the balloon pump model -- RBS and PBS(t). PBS(t) is an input variable in the simulation, the effect of which is one of the results exhibited in Chapter 4. PBS(t) assumes two values, a high value for inflation of the balloon (160 to 260 mm-Hg) and a lesser value (0 to 60 mm-Hg) for deflation. The resistance RBS was selected to be 3.0 mm-Hg·min/liter. This choice gives a flow rate into the balloon of 20 liters/min when the pressure difference across RBS is 60 mm-Hg. At this flow rate a 10 cc balloon would inflate in 30 milliseconds. The choice of RBS is a determining factor in the balloon inflation and deflation rates, but the same rates can equally well be obtained for different values of RBS by adjusting PBS(t); hence the value of RBS is of great importance only if one wishes to specify deflation and inflation time together with PBS(t). The purposes of this study do not necessitate this, and the value chosen for RBS provides reasonable inflation and deflation times for the range of inflation and deflation pressures used.

3.3 Left-Heart Model Parameter Values

Four parameters are associated with the left heart model (Figure 2.7). The atrial source pressure (PAS) is not preset in the simulation, but is an input variable. The value of PAS (along with other factors)
determines the extent of ventricular filling and resulting stroke volume and therefore has a pronounced effect on the output waveforms, as is demonstrated in Chapter 4. The results presented in Chapter 4 make use of atrial source pressures in the range 6 to 14 mm-Hg; the model can tolerate values of PAS from about 4 to 16 mm-Hg.

Initial estimates for the ventricular valve resistances RM and RA were obtained from the literature (Beneken, 1965). Adjustment of these values was later made in tuning the model parameters to achieve more "realistic" output waveforms. The final values set for these two parameters were RM = 0.341 and RA = 0.45, both in units of mm-Hg·min/liter.

Table 3.1 contains a summary of the parameter values (or ranges of values in the case where parameters are input variables) used in the computer simulation from which results are presented in Chapter 4.

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>ventricular model PAS</td>
<td>4. to 14.</td>
<td>mm-Hg</td>
</tr>
<tr>
<td>RM</td>
<td>0.341</td>
<td>mm-Hg·min/liter</td>
</tr>
<tr>
<td>RA</td>
<td>0.45</td>
<td>&quot;</td>
</tr>
<tr>
<td>aortic model L</td>
<td>0.0055</td>
<td>mm-Hg·min²/liter</td>
</tr>
<tr>
<td>CL</td>
<td>0.00023</td>
<td>&quot;</td>
</tr>
<tr>
<td>CR</td>
<td>0.00013</td>
<td>&quot;</td>
</tr>
<tr>
<td>RL</td>
<td>140.</td>
<td>mm-Hg·min/liter</td>
</tr>
<tr>
<td>RR</td>
<td>95.</td>
<td>&quot;</td>
</tr>
<tr>
<td>RI</td>
<td>1.0</td>
<td>&quot;</td>
</tr>
<tr>
<td>R2</td>
<td>5.0</td>
<td>&quot;</td>
</tr>
<tr>
<td>balloon model RB (balloon not full)</td>
<td>1.0</td>
<td>mm-Hg·min/liter</td>
</tr>
<tr>
<td>balloon model RB (balloon full)</td>
<td>2. to 200.</td>
<td>&quot;</td>
</tr>
<tr>
<td>RBS</td>
<td>3.0</td>
<td>&quot;</td>
</tr>
<tr>
<td>PBS (inflating balloon)</td>
<td>160. to 260.</td>
<td>mm-Hg</td>
</tr>
<tr>
<td>PBS (deflating balloon)</td>
<td>0. to 60</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

Table 3.1 Model Parameter Values
3.4 The Ventricular Elastance Function

Of all the parameters in the model, the ventricular elastance $EV(t)$ is by far the most difficult to characterize, and is also the most critical. The time course of this parameter directly influences rate and amount of ventricular filling and ejection, and the length of each of the heart phases.

The apparent simplicity of the left heart model (Figure 2.7) is somewhat misleading, in that generation of a sufficient elastance function can be a difficult (though challenging) problem. Other cardiac modelers (Grodins, 1967; Robinson, 1965; Beneken, 1965) have formulated ventricular models based on details of cardiac muscle mechanics. In these models one typically deals in terms such as length-tension relationships, and rate and duration of muscle fiber shortening. This type model is necessary for investigation of the effects on the ventricle which variation in the muscle characteristics creates. However, as was pointed-out in Chapter 2, the left heart model of this study exists to provide a proper driving waveform for the aortic model. For this purpose it is sufficient to characterize the left ventricle as a time varying elastance. In fact, this characterization is in no way restrictive on the nature of the waveforms which can be generated by the ventricular model; it merely requires that any desired deviation in ventricular function be expressed directly as variation in ventricular elastance, rather than variation in (say) a time-tension diagram for contracting cardiac muscle.
The ventricular elastance function is obtained by curve-fitting experimental pressure-volume data. The number of parameters which may be permitted to influence the elastance calculation is unlimited, and the range of effects which can be included is restricted only by the availability of experimental pressure-volume measurements. In the most simple (and uninteresting) case, the elastance function could be a single curve, unalterably repeating itself each heart cycle -- a true function-of-time only. The elastance function created in this study has a time course which is a function of heart rate, aortic load, end-diastolic volume, end-systolic volume, and atrial source pressure.

A significant factor in the successful operation of the ventricular elastance function used in this study is the separation of the elastance time course calculation into four parts, which coincide with the four heart phases described in Chapter 2 (see Figure 2.6). Figure 3.1 shows a "typical" ventricular elastance curve, drawn to illustrate the general form for such curves, and the relation of the shape of the curve to the four heart phases.

![Figure 3.1 A Typical Ventricular Elastance Curve](image)

1 - isovolumic contraction
2 - ejection
3 - isovolumic relaxation
4 - filling
Note the change in slope which occurs at the start of ejection; this anomaly is usually quite distinct -- the extent of the slope change varying with end-diastolic volume and other factors. Variation in aortic load should effect the location of this break in slope occurring at the transition from isovolumic contraction to ejection. As an example, if aortic pressure increases (perhaps as a result of the inflation of an intra-aortic balloon), the duration of isovolumic contraction (in which the ventricle contracts without change in volume to raise the internal blood pressure to aortic pressure) will increase. This simply reflects the additional time required to raise the ventricular pressure to the elevated aortic pressure. By characterizing the ventricular elastance with four distinct phases, such a modification in duration (or shape) of any phase, with or without effecting the shape of the remaining phases, is readily possible. The elastance time-course is made continuous at the changes in calculating method by using the terminal value of elastance in a completing heart phase as the initial value for the next beginning heart phase.

Utilizing the single heart rate (150 beats per minute) canine pressure-volume data from Taylor (mentioned in Chapter 2), a four part elastance function was created which would be responsive to variations in aortic loading and atrial filling pressure (PAS). (See Appendix C

3This feature is well illustrated in the data supplied by Dr. Rodger Taylor. Figure C.1 (Appendix C) depicts three elastance curves computed from his data, together with curves calculated using the elastance function created in this study.
for details of the derivation of the function from Taylor's data, and for
a description of how the resulting function operates.) Unfortunately,
no pressure-volume data for variations in heart rate were available. It
was believed desirable however, to demonstrate the feasibility of
creating an elastance generating function for variable heart rate, even
though quantitative data on which to base the behavior of the elastance
function were not available. Some data (albeit human data) concerning
the effect of heart rate on duration of isovolumic contraction, ejection,
and isovolumic relaxation phases were found in the literature (Kumar,
1970; Metzger, 1970; Leighton, 1971). With this information,
modifications to the elastance function were made so that heart rates in
the approximate range of 90 to 200 could be tolerated by the model. Due
to the nature of the data on which the heart rate modifications were
based, it should be emphasized that the effects of variation in heart
rate as reflected by the calculation of elastance are of necessity
primarily qualitative. Figures 3.2 and 3.3 depict actual model generated
ventricular elastance time courses for several heart rates and atrial
filling pressures (PAS). These figures are presented at this point to
demonstrate the basic appearance of the curves computed by the elastance
function. Figures showing the effect which the variation in these curves
has on the model waveforms output in the course of the simulation, and
correlations with experimentally observed phenomena, are reserved for
Chapter 4.
Figure 3.2 Model Generated Left Ventricular Elastance Time-Course for Heart Rate of 120 bpm and Several Values of Atrial Filling Pressure (PAS).
Figure 3.3 Model Generated Left Ventricular Elastance Time-Course for Atrial Filling Pressure of 8 mm-Hg and Several Values of Heart Rate.
3.5 **Definition of Switch Operation to Simplify the Equations**

In the previous chapter a set of four ordinary, non-homogeneous, non-linear differential equations with time varying coefficients were presented as defining the complete model mathematically. Having determined the parameter values, it remains to discuss the procedures used in solving the equations to produce the desired model output. Rather than dealing with the equations in the form presented in (2.1) through (2.5), advantage shall be made of restrictions imposed on the operation of switches SM, SA, and SB to form sets of equations which have considerably simplified coefficients. These sets of equations will, not surprisingly, correspond to the previously mentioned phases of the cardiac cycle. Operation of the balloon pump in the simulation is to be restricted to isovolumic contraction, ejection and isovolumic relaxation phases of the heart cycle. This restriction is based on experimental evidence (McMahon, 1969) that balloon pumping is most effective when constrained to these heart periods.

With this stipulation, the value combinations assumed by the switches as a function of heart phase are shown in Table 3.2.

<table>
<thead>
<tr>
<th>HEART PHASE</th>
<th>SWITCH and VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>iso. contraction</td>
<td>0 0 1 or 0</td>
</tr>
<tr>
<td>ejection</td>
<td>0 1 0</td>
</tr>
<tr>
<td>iso. relaxation</td>
<td>0 0 1 or 0</td>
</tr>
<tr>
<td>filling</td>
<td>1 0 1 or 0</td>
</tr>
</tbody>
</table>

Table 3.2 Model Switch Values for Each Heart Phase
The switch values corresponding to each heart phase can be substituted into equations (2.1) through (2.5) to obtain the reduced form of the equations for that phase. Note that the switch settings are identical for both isovolumic phases so the result will be only three unique sets of equations.

The equations for the ejection phase shall be considered first. During this phase, the ventricular model is coupled by switch SA to the aortic model. The ejection equations, derived from equations (2.1) through (2.5) by substituting switch values $SM = SB = 0$ and $SA = 1$, are given below:

\[
\begin{align*}
V_1 &= \left[ \frac{EV(t)}{RA} \right] \cdot [R_1 \cdot (RA \cdot K2) - 1] \cdot V_1 \\
&\quad + \left[ \frac{1}{(RA \cdot CL)} \right] \cdot [1 - (R1/K2) \cdot (1/RA + 1/RL)] \cdot V_2 - \left[ \frac{R1}{(RA \cdot K2)} \right] \cdot F \\
V_2 &= \left[ \frac{EV(t)}{(RA \cdot K2)} \right] \cdot V_1 - \left[ \frac{1}{(CL \cdot K2)} \right] \cdot (1/RA + 1/RL) \cdot V_2 - F/K2 \\
V_3 &= -\frac{V_3}{[CR \cdot (R2 + RR)]} + \left[ \frac{RR}{(R2 + RR)} \right] \cdot F \\
\dot{F} &= \left[ \frac{R1 \cdot EV(t)}{(L \cdot RA \cdot K2)} \right] \cdot V_1 \\
&\quad + \left[ \frac{1}{(L \cdot CL)} \right] \cdot [1 - (R1/K2) \cdot (1/RA + 1/RL)] \cdot V_2 \\
&\quad - \left[ \frac{RR}{[L \cdot CR \cdot (R2 + RR)]} \right] \cdot V_3 - \left( \frac{1}{L} \right) \cdot \left[ \frac{R1/K2 + (R2 \cdot RR)/(R2 + RR) + RB(t)}{} \right] \cdot F
\end{align*}
\]

where $K2 \triangleq 1 + R1/RL + R1/RA$

**EQUATIONS FOR EJECTION**

Since the balloon is inoperative during the ejection phase by definition, balloon resistance $RB(t)$ is constant, and assumes the value corresponding to the deflated balloon condition.
For the isovolumic phases and the filling phase the equations shall first be formulated for the case with the balloon pump inactive; this is equivalent to switch SB having a value of zero. Note that this situation can represent either a fully deflated balloon or a fully inflated one -- the distinction being the value used for RB(t).

In the isovolumic phases, all switches have value zero (with the balloon assumed inactive). Utilizing these zero values in equations (2.1) through (2.5) provides the following equation set for isovolumic phases with inactive balloon.

\[ \dot{V}_1 = 0 \] (3.12)

\[ \dot{V}_2 = \left[-1/(R_1 + R_L)\right] \cdot \left(V_2/C_L + R_L \cdot F\right) \] (3.13)

\[ \dot{V}_3 = \left[1/(R_2 + R_R)\right] \cdot \left(-V_3/C_R + R_R \cdot F\right) \] (3.14)

\[ \dot{F} = \left[1/(L \cdot C_L)\right] \cdot \left[R_L/(R_1 + R_L)\right] \cdot V_2 - \left[1/(L \cdot C_R)\right] \cdot \left[R_R/(R_2 + R_R)\right] \cdot V_3 \]

\[ - \left(1/L\right) \cdot \left[R_1 \cdot R_L/(R_1 + R_L) + R_2 \cdot R_R/(R_2 + R_R) + R_B(t)\right] \cdot F \] (3.15)

**EQUATIONS FOR ISOVOLUMIC PHASES WITH BALLOON INACTIVE**

The equations for the filling phase with the balloon inactive are obtained by using switch values SA = SB = 0 and SM = 1 in equations (2.1) through (2.5). The result of this procedure is the following set of equations.
3.18

\[ \dot{V}_1 = -\left[\frac{E(t)}{RM}\right] \cdot V_1 + \frac{PAS}{RM} \]  

(3.16)

\[ \dot{V}_2 = -\left[\frac{1}{(R_1+R_L)}\right] \cdot \left(\frac{V_2}{CL + RL \cdot F}\right) \]  

(3.17)

\[ \dot{V}_3 = \left[\frac{1}{(R_2+R_R)}\right] \cdot \left(-\frac{V_3}{CR + RR \cdot F}\right) \]  

(3.18)

\[ \dot{F} = \left[\frac{1}{(L \cdot CL)}\right] \cdot \left[\frac{RL}{(R_1+R_L)}\right] \cdot V_2 \]

\[- \left[\frac{1}{(L \cdot CR)}\right] \cdot \left[\frac{RR}{(R_2+R_R)}\right] \cdot V_3 \]  

(3.19)

\[- \left(\frac{1}{L}\right) \cdot \left[\frac{R_1 \cdot RL}{(R_1+RL)} + \frac{(R_2 \cdot RR)}{(R_2+RR)} + RB(t)\right] \cdot F \]

\[EQUATIONS FOR FILLING WITH BALLOON INACTIVE\]

Note that the equations for the filling phase are identical to those for the isovolumic phases with the exception of the equation for \( \dot{V}_1 \).

Activation of the balloon pump modifies the equations for the isovolumic and filling phases. In this situation, switch SB has value 1. Whether the balloon is inflating or deflating is determined by the value of PBS(t). The value of RB(t) will be a constant while the balloon is active (as described earlier in this chapter), corresponding to the deflated balloon resistance. The equations for the isovolumic phases with the balloon pump active are given below.

\[ \dot{V}_1 = 0 \]  

(3.20)

\[ \dot{V}_2 = \left(\frac{1}{K_3}\right) \cdot \left[-\left(\frac{1}{CL}\right) \cdot \left(\frac{1}{RBS} + \frac{1}{RL}\right) \cdot V_2 - F + PBS(t) / RBS\right] \]  

(3.21)

\[ \dot{V}_3 = \left[\frac{1}{(R_2+R_R)}\right] \cdot (-V_3/CR + RR \cdot F) \]  

(3.22)

\[ \dot{F} = \left[\frac{1}{(L \cdot CL)}\right] \cdot \left[\frac{-\left(\frac{1}{R_1/K_3}\right) \cdot \left(\frac{1}{RBS} + \frac{1}{RL}\right)}{V_2} - \left[\frac{1}{(L \cdot CR)}\right] \cdot \left[\frac{RR}{(R_2+R_R)}\right] \cdot V_3 \right] \]

\[- \left(\frac{1}{L}\right) \cdot \left[\frac{R_1/K_3 + (R_2 \cdot RR)/(R_2+RR) + RB(t)}{F} + \frac{R_1 \cdot PBS(t)}{(L \cdot K_3 \cdot RBS)}\right] \]

where \( K_3 \triangleq 1 + R_1/RL + R_1/RBS \)  

(3.24)

\[EQUATIONS FOR ISOVOLUMIC PHASES WITH BALLOON ACTIVE\]
The equations for filling with the balloon active result from substitution of the switch values $SM = SB = 1$ and $SA = 0$ into equations (2.1) through (2.5). The equations thus obtained are shown below.

\[
\dot{V}_1 = -\left[\frac{EV(t)}{RM}\right] \cdot V_1 + \frac{PAS}{RM} \tag{3.25}
\]

\[
\dot{V}_2 = \left\{-\frac{1}{(K3 \cdot CL)} \cdot \left(\frac{1}{RBS} + \frac{1}{RL}\right) \cdot V_2 - \frac{F}{K3} + \frac{PBS(t)}{RBS}\right\} \tag{3.26}
\]

\[
\dot{V}_3 = \left[\frac{1}{(R2+RR)}\right] \cdot \left(-\frac{V_3}{CR} + RR \cdot F\right) \tag{3.27}
\]

\[
\dot{F} = \left[\frac{1}{(L \cdot CL)}\right] \cdot \left[1 - \left(\frac{R1}{K3}\right) \cdot \left(\frac{1}{RBS} + \frac{1}{RL}\right)\right] \cdot V_2
- \left[\frac{1}{(L \cdot CR)}\right] \cdot \left[\frac{RR}{(R2+RR)}\right] \cdot V_3
- \left(\frac{1}{L}\right) \cdot \left[\frac{R1}{K3} + \left(\frac{R2 \cdot RR}{(R2+RR)} + RB(t)\right)\right] \cdot F
+ \left[\frac{R1 \cdot PBS(t)}{(L \cdot K3 \cdot RBS)}\right] \tag{3.28}
\]

where $K3$ is defined according to equation (3.24).

**EQUATIONS FOR FILLING WITH BALLOON ACTIVE**

In summary, there is a single set of equations for the ejection phase, and two sets each for the isovolumic and filling phases. For these later phases, which equation set is used is determined by the status of balloon switch SB. These equation sets are the ones implemented in the simulation. Logic within the program determines which set is to be used to represent the model at any instant during the heart cycle.

3.6 **Conventions Concerning Operation of the Balloon Pump Model**

Operation of the balloon pump can be regarded as superimposing four pumping phases on the three (isovolumic relaxation, filling, and
These pumping phases correspond to the following balloon conditions:
1) fully deflated (either awaiting inflation or just after deflation);
2) inflating; 3) full (awaiting deflation); and 4) deflating. The current pumping phase is determined by the status of balloon switch SB, and by the values of the balloon model parameters RB(t) and PBS(t). The relation of these parameters to the pumping phases is shown in Table 3.3 below.

<table>
<thead>
<tr>
<th>PUMPING PHASE</th>
<th>PARAMETERS and VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SB</td>
</tr>
<tr>
<td>balloon empty</td>
<td>0</td>
</tr>
<tr>
<td>balloon inflating</td>
<td>1</td>
</tr>
<tr>
<td>balloon full</td>
<td>0</td>
</tr>
<tr>
<td>balloon deflating</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3.3 Relation of Balloon Pumping Phases to Balloon Model Parameters

Timing of the pumping events is determined directly by two time values input to the simulation, and indirectly by the values of balloon volume and balloon driving pressures. The time values correspond to the times, relative to the start of isovolumic contraction, at which balloon inflation and balloon deflation are to begin; the values are referred to as TON and TOFF respectively. The value of TON must be less than the total heart period if the balloon is to operate. Values of TON greater than the heart period are taken as an indication that
the model simulation is to proceed with the balloon remaining deflated, which is useful in establishing "control" situations. The indirect effect resulting from balloon volume and driving pressures consists of the variation in the times required to fully inflate and deflate the balloon, which are a consequence of the values of these variables.

An effort has been made in the simulation to provide reasonable operation of the balloon without the necessity of knowing specifically when the various heart phases begin and end during the heart cycle. This is done by dynamically adjusting any values of TON and TOFF which do not conform to the operating conventions mentioned. For instance, values of TON specifying inflation to begin before the onset of isovolumic relaxation (e.g. during ejection) are dynamically reset to the start of isovolumic relaxation. Of course, TOFF must have a larger value than TON (deflation must occur later than inflation).

Usual operation of the balloon consists of beginning inflation shortly after the end of ejection (either during isovolumic relaxation or early filling); the value of TOFF is normally specified so that the balloon is allowed to fill and remain full until near the end of the heart filling phase, at which time deflation is begun; deflation may carry-over into the next cycle's isovolumic contraction phase, but must be complete before the beginning of ejection. In the event that the balloon is not fully deflated at the start of ejection, the remaining volume is ignored and TOFF (the time at which balloon deflation is
begun) for the subsequent cycle is reduced by 10 milliseconds -- this procedure continuing with each cycle until either the balloon is fully deflated prior to ejection or the value of TOFF has been reduced to the value of TON, in which event the simulation is terminated.

The balloon is allowed to be in any of the four pumping phases during the heart's isovolumic relaxation and filling phases (of course inflation must precede deflation). However, the balloon must either be deflating or already empty at the start of isovolumic contraction, to help assure that deflation is complete at the start of ejection. If isovolumic contraction commences with the balloon full or still inflating, TOFF for the next cycle is reset to the current time for the start of isovolumic contraction, and balloon deflation begins immediately. No requirement that the balloon be fully inflated is made, so that deflation can occur anytime after inflation, regardless of the current volume of the balloon. The maximum volume obtained by the balloon is output for each cycle.

In order for the balloon pump model to operate, it is necessary that balloon volume be computed as the balloon inflates and deflates. The computation of balloon volume in the simulation is done by making use of the equation given for balloon flow in Appendix A [equation (A.17)]. As noted in Chapter 2, volume can be computed as the time integral of flow, and this is the procedure used to compute balloon volume. Whenever switch SB is closed (SB = 1), the integral of balloon flow is accumulated at each time step to yield the current balloon
volume. Logic within the program monitors this volume and signals the opening of switch SB when the desired balloon capacity is achieved.

As a means of demonstrating the capabilities of the model developed in this work to produce meaningful results, a measurement of balloon pumping effectiveness as proposed by Kane (1971) has been computed for simulation runs involving use of the balloon pump. Kane's balloon performance index, which is described in Appendix D, includes several objectives of balloon pumping. Most significantly, the index is a measure of the augmentation of aortic root pressure during diastole and the reduction of heart work during ejection. Kane's efforts were directed toward using the performance index to optimize balloon timing (equivalent to finding optimal values for TON and TOFF); hence, his work did not include balloon volume or driving pressures as variables, although these parameters certainly contribute to the effectiveness of balloon pumping. Chapter 4 contains results of the calculation of Kane's performance index for variations in balloon timing, balloon volume, inflation and deflation pressures, and balloon resistance.

3.7 Solving the Equations

The numerical integration method which is used in the computer simulation to "solve" the various sets of differential equations is an Adams-Moulton predictor-corrector method with a Runga-Kutta starter. This method gives good results in single precision (about 11 decimal digits) on a Burroughs B-5500 computer; subsequent work on a Xerox
Sigma-7 required the use of double precision. An appropriate time step for the integration method was determined by first selecting some integer multiple of 1/2 that seemed reasonable and using it in solving the desired equations over a specified time interval. (A multiple of 1/2 is used for the step size since it is necessarily exactly representable in the computer as a binary number.) The step size was then either doubled or halved, used in solving the equations again, and the results compared with those obtained using the previous step size. This technique permits choosing the largest step size consistent with desired accuracy. For simulations involving the complete model, the time step used was smaller than actually required for accuracy of integration in order to keep the change in solution values at each step suitably small. This is necessary because the operation of the switches (and subsequent choice of which equation set to use) is determined by tests on the pressure differences between various nodes. If these pressures are changing substantially with each time step, it is not possible to operate the switches just when the pressure differential reverses. In particular, ventricular pressure changes quite rapidly during isovolumic contraction, and care must be taken that a time step which permits accurate operation of switch SA is used. The step size selected for the runs with the complete model was 0.0005 seconds. With this time step, ventricular pressure typically overshoots aortic pressure by one mm-Hg or less at the end of isovolumic contraction.
In order to solve the four differential equations describing the model it is necessary to specify four initial conditions. The initial conditions specified for the model are in terms of the pressures $P_1$, $P_2$, and $P_3$ and the flow $F$. Since the equations are in terms of compliance volume rather than node pressure, the initial conditions are converted to volumes utilizing equations (2.5) through (2.8). The heart cycle was chosen to begin with isovolumic contraction, so the initial conditions are specified for the beginning of that phase. The progression of heart phases in a complete cycle is thus isovolumic contraction, ejection, isovolumic relaxation, and filling.

Since the heart is periodic, in a "steady state" situation the values of $P_1$, $P_2$, $P_3$ and $F$ at the end of the filling phase are the same as those at the start of the isovolumic contraction phase for that cycle. The initial conditions specified are usually only estimates, and it is necessary to let the model run through several complete cycles to attain a steady state condition. This is accomplished by using the final values of $P_1$, $P_2$, $P_3$ and $F$ from the filling phase as initial conditions for the next isovolumic contraction phase, and allowing the model to cycle until it satisfies some designated steady state criterion. In the results presented in Chapter 4, the steady state criteria used were 1% or less variation in pressures $P_1$ through $P_3$, and 2% or less variation in flow $F$, as recorded at the onset of isovolumic contraction and at the end of filling.

3.8 Program Logic

Figure 3.4 shows the overall logic flow of the computer program
for the model simulation, excluding the logic associated with balloon pump operation. Note that once steady state is achieved, new parameter values (including balloon timing, heart rate, atrial pressure and others) may be specified. These new values, together with the steady state initial conditions currently existing, will be utilized to bring the model to a new steady state. Hence the model can undergo changes in parameters and provide continuous waveforms for all variables of interest. This permits (for instance) a run to be made without utilizing the balloon, followed by one which does use the balloon, and this in turn followed by a run with a larger balloon, etc., and the results can be displayed as a continuous series of waveforms with no discontinuities at points where new parameter values are specified.

The programs for the isovolumic and filling phases contain additional logic to determine such things as when balloon inflation starts, when the balloon reaches full volume, when to deflate the balloon, etc. The logic required for this is similar in all three heart phases, and hence it will be illustrated in only one. Figure 3.5 shows the basic logic utilized in the isovolumic relaxation phase, in which the balloon pump may be operational.
read param. values and init. conds.

evaluate equation coefs. using param. values

take time step using isovolumic equations

does ventricular pressure exceed aortic pres.?

no

yes

take time step using ejection equations

does ventricular pressure exceed aortic pres.?

no

yes

take time step using isovolumic equations

does ventricular pressure exceed atrial pres.?

no

yes

take time step using filling equations

end of time for this cycle?

no

yes

may more cycles be taken?

no

yes

steady state?

no

yes

read new parameters

is run to be continued with new parameters?

no

yes

output solutions for complete cycle

stop

Figure 3.4 Overall Program Logic Disregarding Balloon Operation
Figure 3.5 Logic for Isovolumic Relaxation Phase
(continued on next page)
Figure 3.5 (continued from previous page)
3.9 Input and Output Description

There are two distinct times at which input is requested by the simulation program -- initially, before any calculations have been performed, and each time steady state is achieved. The items included in the initial input request are shown in Table 3.4 below. The request for input at the end of each occurrence of steady state constitutes a selected subset of the values initially input. Those items that are included in this subset are so indicated in Table 3.4 by an asterisk in the column labeled "May Be Redefined."

Output from the model consists of waveforms and related information which can be obtained for each cycle taken, or only after steady state is achieved, as desired. The program includes a line-printer plot routine which may be invoked to plot the time-courses. A tabular printing of the solution values is also optional. The output options desired are specified as part of the input requested by the program, and these options can be modified each time new parameter values are input following a steady state occurrence.

Waveforms normally included in the model output include ventricular pressure (P1), ventricular volume (V1), ventricular elastance (EV), aortic root pressure (P2), downstream aortic pressure (P3), and aortic flow (F). Other standard output items include valve operation times, balloon operation times, maximum balloon volume achieved, balloon performance index, average aortic root pressure, and stroke volume. Modification to the program to cause the outputting of other information can be readily made.
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<td>performance index</td>
<td></td>
</tr>
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<td>equation</td>
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</tr>
</tbody>
</table>

Table 3.4 List of Model Input Items
CHAPTER 4 -- SIMULATION RESULTS

This chapter serves to demonstrate the capabilities of the model to produce meaningful results under a variety of operating conditions through presentation of output from computer runs of the simulation program. Correlation of model behavior with experimentally observed phenomena is pointed-out when it exists -- when it does not, explanations are provided. The first portion of the chapter is devoted to runs of the model in which the intra-aortic balloon pump was not used. The second part consists of results from a variety of balloon pumping "experiments," including calculation of a balloon pumping performance index as described in Appendix D. All results are for the steady state conditions unless indicated otherwise.

4.1 Results of Simulation Runs Without Balloon Pumping

Figure 4.1 shows model generated time courses for five commonly observed variables created under steady state conditions of 8mm-Hg atrial filling pressure (PAS) and heart rate of 120 bpm, stroke volume was 18.2 ml and mean aortic root pressure had a value of 126 mm-Hg. These time-courses are similar in shape to corresponding experimental measurements. Certain details, such as the "dicrotic notch" a short duration dip in aortic pressure created by the closing aortic valve and the "bump" in ventricular pressure caused by atrial contraction are not present due to the nature of the model; these details are not considered significant. Note that descending aortic pressure (which is the pressure
Figure 4.1 Typical Time-Courses of Relevant Variables
P3 of Figure 2.10) reaches a higher peak than aortic root pressure (which is the pressure P2 of Figure 2.10) and the peak is delayed in time. Both of these phenomena are commonly seen in the literature (Luchsinger, 1964; Spencer, 1963). In the model, these effects result from the decreased compliance "downstream," and the blood iner-tance term.

Figures 4.2 and 4.3 display ventricular pressure-volume relationships under various steady state conditions. These curves are traversed in a counter-clockwise direction, with isovolumic contraction beginning at the extreme lower right corner. Points are shown at 10 millisecond intervals along the curves. Considerable information about the operation of the ventricular model is available from these figures. In Figure 4.2, heart rate is fixed at 120 bpm and pressure-volume relationships are plotted for three values of atrial filling pressure (PAS). Note that end-diastolic volume increases substantially with increasing PAS, but that stroke volume also increases. This is consistent with the Frank-Starling relationship, and also with published data concerning end-diastolic volume, end-systolic volume and stroke volume as functions of atrial filling pressure (Kamiya, 1971). The increase in pressure at which ejection occurs for higher values of PAS is a consequence of increased aortic pressure (load). Since aortic resistance is constant, increased cardiac output from the larger stroke volume results in increased mean aortic pressure. In vivo, neural regulatory processes would usually act to reduce arterial resistance, thereby eliminating or at least reducing this increase in aortic pressure. Figure 4.3 shows
Figure 4.2 Ventricular Pressure - Volume Relationship for Constant Heart Rate and Three Atrial Filling Pressures
Figure 4.3 Ventricular Pressure - Volume Relationship for Constant Atrial Filling Pressure and Three Heart Rates
ventricular pressure-volume curves for a constant filling pressure of 8 mm-Hg and three values of heart rate. Notice that the increase in heart rate from 90 to 150 has no appreciable effect on end-diastolic volume, but does cause a reduction in stroke volume. The reduced stroke volume is a consequence of the reduction in ejection duration. Despite the smaller stroke volume, cardiac output (stroke volume times heart rate) increases — from 1.87 liters/min at 90 bpm to 2.43 at 150 bpm. Further increase in heart rate to 200 bpm produces a reduction in end-diastolic volume due to incomplete filling. In this case stroke volume is reduced proportionately more than heart rate is increased, so that cardiac output decreases somewhat, to 2.38 liters/minute. The elastance curves of Figures 3.2 and 3.3 (Chapter 3) correspond to the pressure-volume relationships shown in Figures 4.2 and 4.3 respectively. It is informative to compare the two pairs of figures; although they contain the same basic information, they display different aspects of the operation of the left-heart model.

More explicit information concerning the effect which heart rate has on cardiac output is shown in Figure 4.4. For a constant atrial filling pressure of 8 mm-Hg, the cardiac output first increases with increasing heart rate, peaking at about 160 bpm, then begins a gradual decline. This curve is characteristic of experimental observations of cardiac output as a function of heart rate (Selkurt, 1966). Figure 4.5 displays cardiac output for a constant heart rate of 120 bpm as a function of

1 The reduction in ejection duration results from the change in ventricular elastance as a function of heart rate, which is discussed in Appendix C.
Figure 4.4 Cardiac Output for Constant Filling Pressure as a Function of Heart Rate

Figure 4.5 Cardiac Output for Constant Heart Rate as a Function of Atrial Filling Pressure
atrial filling pressure. Since at this heart rate filling is complete before the end of the filling phase, increases in atrial filling pressure have a pronounced effect on end-diastolic volume, and in-turn cardiac output.

Figure 4.6 shows ventricular pressure-volume curves for two steady state conditions (curves "S" and "7") and for the first two non-steady state cycles in the transition from the steady state "S" to the steady state at "7." Curve "S" corresponds to steady state operating conditions of 8mm-Hg atrial filling pressure and heart rate of 120 bpm. The curves labeled "1" and "2" are the first two cycles following the specification of a new atrial filling pressure of 10 mm-Hg and a new heart rate of 180 bpm. The new conditions are specified at the start of isovolumic contraction, so that the principal effect of the increased filling pressure (augmentation of end-diastolic volume) is delayed until the second cycle. Note that the maximum and minimum excursion of volume is virtually unchanged after cycle "2," but that the pressure during ejection rises considerably (as a result of increased cardiac output) to reach the new steady state shown as curve "7."

A parameter of considerable interest (Metzger, 1970; Mullins, 1971) is the rate of change of ventricular pressure with respect to time. Figure 4.7 illustrates such a curve, superimposed on the ventricular pressure curve to which it applies. The curves are for steady state conditions of 6 mm-Hg atrial filling pressure and heart rate of 150 bpm. The flat plateau and valley in the derivative curve during the isovolumic
Figure 4.6 Ventricular Pressure-Volume Relationships for Two Steady States with Transitional Cycles
Figure 4.7 Rate of Change of Ventricular Pressure and Ventricular Pressure
phases is created by the constant slope of ventricular elastance during these phases. With that exception, the two curves compare quite well with a corresponding curve displayed by Metzger (1970) as being "typical" for dogs. One source of qualitative data which was considered in formulating model response to heart rate variation was that of Metzger (1970). His findings demonstrated an inverse relationship between duration of isovolumic contraction and maximum rate of left-ventricular pressure rise. Figure 4.8 displays model generated values for these variables that illustrates the presence of Mullin's inverse relationship. The source of the relationship is the modification made to the slope of ventricular elastance during isovolumic contraction for increased heart rates. The increased slope shortens the duration of isovolumic contraction, producing the relationship shown in Figure 4.8.

4.2 Simulation Results With Balloon Pumping Implemented

The results presented in this section, for the case where the intra-aortic balloon pump model was active, serve to provide further verification of meaningful model response, and in addition demonstrate the feasibility of the model for use in investigating the effects of the intra-aortic balloon assist device. Figure 4.9 shows a number of relevant time courses in the steady state condition corresponding to that of Figure 4.1 except that balloon pumping has been implemented. In this case pumping was implemented with a 15ml balloon having driving pressures [PBS(t)] of 220 mm-Hg (inflation) and 30 mm-Hg (deflation). Balloon inflation began at 0.197 seconds; the balloon was fully inflated at 0.234 seconds and
Figure 4.8 Maximum Rate of Change of Left-Ventricular Pressure as a Function of Isovolumic Contraction Time
Figure 4.9 Pressure & Flow Time-Courses with Balloon Pump Operating at Near Optimal Timing
remained so until deflation began at 0.499 seconds. Deflation continued to the very end of isovolumic contraction, at 0.033 seconds into the next cycle.

The balloon pump has some rather pronounced effects on the waveforms of Figure 4.9. The most conspicuous of these are the modified aortic pressure and flow time-courses. The initiation of balloon inflation immediately begins elevating aortic root pressure. This pressure continues to increase until slightly after the balloon has reached maximum volume, then falls off steadily until, just at the end of the cycle, balloon deflation begins. Aortic-root pressure drops very rapidly in conjunction with deflation of the balloon. (This is observed on the left-hand side of Figure 4.9). Descending aortic pressure also begins to rise (after a slight delay) as the balloon inflates. When the balloon reaches its limiting volume, the balloon resistance term $R_B(t)$ changes from $1 \text{ mm-Hg} \cdot \text{min/liter}$ to $100 \text{ mm-Hg} \cdot \text{min/liter}$. This causes the aortic flow to drop precipitously (the effect mentioned in Chapter 3), and as it falls the descending aortic pressure also decreases rapidly. The flow soon stabilizes at a small positive value and descending aortic pressure decays in a passive fashion.

Of perhaps more significance, though less obvious, is the effect on the ventricular pressure curve. Note that the deflating balloon (left-hand side of Figure 4.9) drops aortic pressure so that isovolumic contraction terminates (and ejection proceeds) at a much lower pressure than in the non-pumped case shown in Figure 4.1. This clearly demonstrates
the responsiveness of the left-heart model (particularly the elastance function) to aortic loading. The effect of the balloon in this case has been to increase mean aortic pressure from 122.9 mm-Hg to 138.4 mm-Hg while reducing heart work and maintaining stroke volume. This is nearly ideal balloon operation and resulted in a value of the balloon performance index of 20.7 -- the maximum achieved in any of the simulation runs.

The effects of pumping on the ventricle are more clearly shown by the pressure-volume plots of Figure 4.10. The area enclosed by the pressure-volume curve represents heart work, and the effective reduction in this area by the action of the balloon pump is obvious. Note also that stroke-volume is virtually unchanged. These results correlate exactly with a similar diagram from Mullins (1971).

The effect which the inflated balloon resistance value \([RB(t)\text{-high}]\) has on the aortic root pressure waveform is shown in Figures 4.11 and 4.12. Both are steady state cycles under identical conditions except for the value of \(RB(t)\text{-high}\), which is 10 mm-Hg·min/liter in Figure 4.11 and 220 mm-Hg·min/liter in Figure 4.12.

Figure 4.13 exhibits the effect of less severe driving pressure \([PBS(t)]\) on the aortic root pressure time-course. Balloon inflation pressure \([PBS(t)\text{-high}]\) was 160 mm-Hg while balloon deflation pressure \([PBS(t)\text{-low}]\) was 60 mm-Hg. This figure should be compared to Figure 4.12, for which inflation and deflation pressures were 220 mm-Hg and 30 mm-Hg respectively. The greatly prolonged balloon inflation time is quite apparent in Figure 4.13. As will be shown, this increased inflation time is detrimental to the effectiveness of balloon pumping.
Figure 4.10 Ventricular Pressure-Volume Relationship for Pumped and Non-Pumped Cases.
Figure 4.11 Balloon Pumped Aortic Root Pressure with Low Balloon Resistance

RB(t)-high = 10 mm-Hg·min/liter
Figure 4.12. Balloon Pumped Aortic Root Pressure with High Balloon Resistance
Figure 4.13 Effect of Balloon Driving Pressure on Aortic Root Pressure

PBS(t)-high = 160 mm-Hg
PBS(t)-low = 60 mm-Hg
The remaining figures in this chapter are displays of the effects of a number of pumping variables on a balloon performance index computed as described in Appendix D. To understand the results shown in these figures it is only necessary to know that the performance index is increased by decreasing aortic root pressure during ejection and increasing aortic root pressure during all other phases. In these figures, the timing of balloon inflation and deflation will be given in terms of "delay" and "duration." "Delay" is the elapsed time from start of isovolumic contraction to beginning of balloon inflation. (This is the time TON described in Chapter 3). "Duration" is the elapsed time from the beginning of inflation to the beginning of deflation (in the terms of Chapter 3, duration is TOFF-TON). The "control" condition for the runs from which results are displayed was: heart rate of 120 bpm, atrial filling pressure of 8 mm-Hg, balloon inflation pressure of 220 mm-Hg, balloon deflation pressure of 30 mm-Hg, inflated balloon resistance 100 mm-Hg·min/liter, and balloon volume of 15 ml. Where one or more of these conditions is changed it is so noted on the figure which displays the results of those changes.

Figure 4.14 shows the performance index for a constant balloon duration of 0.15 seconds as a function of delay. This figure illustrates that pumping effectiveness is decreased by increased delay. In effect, the less time given for the blood to drain off, the greater will be the aortic root pressure augmentation while the balloon is inflated.
The performance index for the condition of constant delay (0.2 sec) and variable duration is displayed in Figure 4.15. The increase in effectiveness with increased duration results from prolonging the augmentation to aortic root pressure during filling. A sharp increase in performance index occurs as balloon deflation extends into isovolumic contraction, lowering the pressure at which ejection occurs. As duration extends beyond this, the balloon fails to fully deflate by the start of ejection. When this occurs the performance index drops off sharply, reflecting the effect of the penalty term in the performance index calculation. The relationship shown in Figure 4.15 is similar to the result of Kane (1971) for fixed delay, variable duration.

When the time to start deflation (TOFF) is fixed, but the delay (time to start inflation) varies, a quite linear relationship results for the performance index. This case, for deflation time fixed at 0.39 seconds, is shown in Figure 4.16. Since duration also varies in this situation, duration values are given below the figures for delay.

Balloon inflation pressure directly effects the time required to inflate the balloon. For fixed delay and duration, the more rapidly the balloon inflates the greater should be the aortic pressure augmentation. This hypothesis is verified by the results shown in Figure 4.17. Note that the performance index reaches a plateau for inflation pressures above approximately 240 mm-Hg. This is an indication that balloon effectiveness is no longer limited by balloon inflation time. Figure 4.18 illustrates the effect of balloon volume on the pumping performance.
Figure 4.14 Balloon Performance for Variable Delay, Fixed Duration

Figure 4.15 Balloon Performance for Fixed Delay, Variable Duration
Figure 4.16  Balloon Performance for Fixed Deflation Time, Variable Delay

Figure 4.17  Balloon Performance for Variable Balloon Inflation Pressure
index with all other factors constant. The positive correlation is anticipated and simply reflects the ability of a larger balloon to produce more pronounced effects. Figure 4.19 displays the relationship between the inflated balloon resistance \([RB(t) - \text{high}]\) and the performance index, all other conditions remaining fixed. Resistance values above about 100 mm-Hg·min/liter are seen to be effectively occlusive, and further resistance increase is of no advantage.
Figure 4.18 Effect of Balloon Volume on Performance Index

Figure 4.19 Effect of Inflated Balloon Resistance on Performance Index
A left-heart model used previously in several modeling studies (Beneken, 1965; Snyder, 1968) has been described (Chapter 2, section 3; Figure 2.7). This model requires specification of a time-course for left-ventricular elastance (pressure-volume ratio). An elastance function has been developed for this purpose (Chapter 3, section 4 and Appendix C) which is responsive to ventricular pre-load (end-diastolic volume) and after-load (aortic loading) and which can operate over a range of heart rates from about 90 to 200. This elastance function is unique in that each heart phase (isovolumic contraction, ejection, isovolumic relaxation, and filling) has its own computational section. This feature permits dynamic variation in the duration of each heart phase in response to changing pre- and after-load conditions while maintaining total heart period constant. In addition, the elastance function creates time courses which are tailored to reflect changing relationships among the heart phases, both in time and "contractility," as a function of heart rate.

An arterial model was necessary for testing and verification of the left-heart model. For this purpose an aortic model was described (Chapter 2, section 2; Figure 2.3) which characterizes the major properties of the aortic arch region. This aortic model permits measurements of pressure corresponding to locations in the ascending (aortic root) and descending (thoracic) aorta and of flow between these two locations. Though simplistic, this aortic model is shown to produce physiological pressure and flow waveforms.
A computer program was written (Chapter 3, Sections 7-9) to simulate the continuous operation of these two models under conditions which could be varied while the simulation was in process. The output from these experiments (Chapter 4, Section 1; Figures 4.1-4.8) indicated realistic response and operation of the ventricular model under varying physiological circumstances.

To serve as additional verification of the realistic reaction of the left-heart model to changing conditions (particularly afterload) and to provide an example of the potential use of this model for research, a simple model was described for the intra-aortic balloon pump assist device (Chapter 2, Section 4; Chapter 3, Section 6; Figure 2.9). This model was then used in conjunction with the left-heart and aortic models (Figure 2.10) in a number of computer simulations. The output from these simulations (Chapter 4, Section 2; Figures 4.9-4.19) was found to exhibit many of the characteristics observed in clinical studies of balloon pumping.

The results presented in Chapter 4 indicate the fundamental validity of the left-heart model, and display its ability to react meaningfully to normal physiological circumstances and to those brought on by use of an intra-aortic balloon assist device. These results are believed to establish the feasibility of the adaptive left-heart model for use in research applications involving cardiac assist devices.

For purposes of such research, a number of enhancements could be made to both the left-heart and aortic models. The filling phase of
the left-heart model could be significantly improved by utilizing the model shown in Figure 5.1.

![Figure 5.1 Proposed Left-Heart Model](image)

The atrial pressure source PAS (see Figure 2.3) has been replaced with three elements. Compliance CA(t) would represent the left atrium, and could be given a simple time-course to enable simulation of atrial contraction. Resistance RV would represent the small resistance at the connection of the pulmonary artery to the atrium. The pressure source VRP would represent venous return pressure. An addition improvement would result from making valve resistances RM and RA functions of time. This would permit simulating the finite opening and closing times of these valves in vivo.

Improvement could be made to the ventricular elastance function with regard to yet more realistic response and broader operating ranges if sufficient left-ventricular pressure-volume data were available. Further, pressure-volume data for various kinds of ventricular failure could be used to produce an elastance function which would adequately characterize the behavior of the ventricle in one or more stages of failure. For the while, such efforts must await the availability of experimental results.
There are a number of additions to the aortic model which would provide more physiologic waveforms while adding some useful features to the model. An algorithm to select values of resistances RL and RR (see Figure 2.3) based on desired mean aortic pressure, for instance, would provide the capability of maintaining a relatively constant mean aortic pressure for changing cardiac output. The aortic inertance parameter, L, (Figure 2.3) reflects the mass of the blood in the system and hence should also vary with cardiac output. An algorithm could be developed to sample cardiac output and adjust this parameter accordingly.

Another potentially useful addition to the aortic model would be a coronary flow pathway. This could be included as an additional model resistance as shown in Figure 5.2.

![Figure 5.2 Possible Coronary Artery Addition to Aortic Model](image)

Coronary resistance RC(E) could be a function of ventricular elastance to simulate the increased coronary artery resistance during ventricular contraction.
REFERENCES


Appendix A
DERIVATION OF THE SYSTEM EQUATIONS

The complete model for which the describing differential equations are sought is shown in Figure A.1.

In this development, the model switches (SM, SA, SB) shall be treated as multipliers with values of 1 if closed and 0 if open. Pressures (P's), compliance volumes (V's) and flows (f's and F) are, of course, functions of time. For brevity of notation these terms shall be written without the "(t)" appended to them. Model elements however, which are time varying (such as the balloon pump driving source PBS(t)) will be written with the "(t)".

The following basic relationships are used in the development of the system equations. These relations can be found in any elementary circuit theory text.

\[ P = F \cdot R \]  
Ohm's law

\[ V = C \cdot P = P/E \]  
Characteristic of a linear, time invarient capacitor
\[ F = C \cdot \dot{P} = \dot{P}/E \]

Derivative of expression above for \( V \)

\[ P = L \cdot \dot{F} \]

Characteristic of linear, time invariant inductor

\[ F = \dot{V} \]

Definition of current

where: \( P \) is pressure

\( F \) is flow

\( R \) is resistance

\( V \) is volume (charge)

\( C \) is compliance (capacitance)

\( L \) is inertance (inductance)

\( E \) is elastance

Dots above the symbols indicate time derivatives.

To begin analysis, consider the flows at node N1 shown in Figure A.2.

![Figure A.2 Flows at Node 1](image)

From reference to Figure A.1 it is clear that

\[ f_1 = SM \cdot (PAS - P_1)/RM \]  

(A.1)

but

\[ P_1 = V_1 \cdot EV(t) \]  

(A.2)
where \( EV(t) \) is defined as \( 1/CV(t) \). Thus (A.1) can be written as

\[
f_1 = SM \cdot (P_2 - V_1 \cdot EV(t)) / RM
\]  

(A.3)

It is obvious that

\[
f_2 = \dot{V}_1
\]  

(A.4)

[f_2 could also be expressed as \( CV(t) \cdot P_1 + CV(t) \cdot P_1 \) but this would involve the derivative of \( CV(t) \) which is to be avoided.]

Analogously to \( f_1 \), note that

\[
f_3 = SA \cdot (P_1 - P_2) / RA
\]  

(A.5)

which, by substituting (A.2) becomes

\[
f_3 = SA \cdot (V_1 \cdot EV(t) - P_2) / RA
\]  

(A.6)

\( P_2 \) is eliminated from (A.6) by considering the situation at node N2 as shown in Figure A.3. Labels \( f_41 \) and \( f_42 \) have been assigned to the individual flows which comprise \( f_4 \).

Figure A.3 The Flows at Node 2

From Figure A.3 and the definition of \( V_2 \),

\[
V_2 = CL \cdot (P_2 - P_4)
\]  

(A.7)
Also, clearly,
\[ f_{42} = V_2 = P_4 / R_1 \]  \hspace{1cm} (A.8)

Equation (A.8) can be rearranged to give
\[ P_4 = R_1 \cdot V_2 \]  \hspace{1cm} (A.9)

Substituting (A.9) into (A.7) and solving for \( P_2 \) yields
\[ P_2 = R_1 \cdot V_2 + V_2 / \text{CL} \]  \hspace{1cm} (A.10)

This expression can be substituted into (A.6) to eliminate \( P_2 \), giving
\[ f_3 = (S_4 / R_4) \cdot (V_1 \cdot E(t) - R_1 \cdot V_2 - V_2 / \text{CL}) \]  \hspace{1cm} (A.11)

Equations have thus been obtained for \( f_1 \), \( f_2 \), and \( f_3 \) of Figure A.2 in terms of the variables \( V_1 \) and \( V_2 \). By applying Kirchoff's current law to node N1 one obtains
\[ f_1 = f_2 + f_3 \]  \hspace{1cm} (A.12)

Substituting (A.3), (A.4), and (A.11) into (A.12) and solving for \( \dot{V}_1 \) yields the following tentative equation for \( \dot{V}_1 \):
\[ \dot{V}_1 = (S_4 / R_4) \cdot [P_4 - V_1 \cdot E(t)] + (S_4 / R_4) \cdot [R_1 \cdot \dot{V}_2 + V_2 / \text{CL} - V_1 \cdot E(t)] \]  \hspace{1cm} (A.13)

Note that (A.13) contains the derivative term \( \dot{V}_2 \). An expression for \( \dot{V}_2 \) will now be obtained from further consideration of Figure A.3; this expression will then be substituted into (A.13) to eliminate \( \dot{V}_2 \).

Refering to Figure A.3, an equation has already been written for \( f_3 \). \( f_4 \) is obtained by noting that
\[ f_{41} = P_2 / R_L \]  \hspace{1cm} (A.14)

Combining (A.10) and (A.14) to eliminate \( P_2 \) gives
\[ f_{41} = (R_1 \cdot \dot{V}_2 + V_2 / \text{CL}) / R_L \]  \hspace{1cm} (A.15)

Since \( f_4 = f_{41} + f_{42} \), equations (A.8) and (A.15) may be added.
to give

\[ f_4 = \dot{V}_2 + \frac{(R_1 \cdot \dot{V}_2 + V_2/CL)}{RL} \]  

(A.16)

Balloon flow, \( f_B \), is given by

\[ f_B = \frac{S_B \cdot (P_BS(t) - P_2)}{R_BS} \]  

(A.17)

Substituting (A.10) into (A.17) to eliminate \( P_2 \) yields

\[ f_B = \frac{(S_B/R_BS) \cdot (P_BS(t) - R_1 \cdot \dot{V}_2 - V_2/CL)}{} \]  

(A.18)

Applying Kirchoff's current law to node \( N_2 \) gives

\[ f_3 + f_B = f_4 + f_5 \]  

(A.19)

Substituting (A.11), (A.16), and (A.18) into (A.19), and noting that \( F = f_5 \), yields

\[ \frac{(S_A/R_A)}{\cdot V_1 \cdot EV(t)} - \frac{R_1 \cdot \dot{V}_2 - V_2/CL}{(S_B/R_BS)} \cdot \frac{P_BS(t) - R_1 \cdot \dot{V}_2 - V_2/CL}{(S_B/R_BS)} \]  

= \dot{V}_2 + \frac{(1/RL) \cdot (R_1 \cdot \dot{V}_2 + V_2/CL)}{F} \]  

(A.20)

Solving (A.20) for \( \dot{V}_2 \) gives

\[ \dot{V}_2 = \left[ \frac{1}{K_1(t)} \right] \cdot \left[ \frac{(S_A/R_A) \cdot EV(t) \cdot V_1}{(S_B/R_BS)} \right] - \frac{(1/CL) \cdot [(S_A/R_A) + (S_B/R_BS) + (1/RL)] \cdot V_2}{F + (S_B/R_BS) \cdot P_BS(t)} \]  

(A.21)

where

\[ K_1(t) = 1 + \frac{R_1}{RL} + \frac{(S_A \cdot R_1)}{RA} + \frac{(S_B \cdot R_1)}{RBS} \]  

(A.22)

\( K_1(t) \) is a function of time due to the inclusion of switches \( S_A \) and \( S_B \).

Equation (A.21) can be substituted into (A.13) to eliminate \( \dot{V}_2 \), thereby providing the final differential equation for \( V_1 \).

After rearranging, the result of this substitution is
\[ \dot{V_1} = E(t) \cdot \left\{ \frac{(SA)}{(RA)} \cdot \left[ \frac{(SA)}{(RA-K_1(t))} - 1 \right] - \frac{(SM)}{(RM)} \right\} \cdot V_1 \\
+ \left\{ \frac{(SA)}{(RA-CL)} \cdot \left[ 1 - \left[ \frac{R_1}{K_1(t)} \right] \cdot \left[ \frac{(SA)}{(RA)} + \frac{(SB)}{(RBS)} + \frac{(I)}{(RL)} \right] \right\} \cdot \dot{V_2} \\\n- \left\{ \frac{(SA)}{(RA-K_1(t))} \right\} \cdot F \\
+ \frac{(SM \cdot PAS)}{RM} + \left\{ \frac{(SA)}{(RA-K_1(t) \cdot RBS)} \right\} \\
\]

Differential equations have thus far been obtained for \( V_1 \) and \( V_2 \); the equation for \( V_3 \) is obtained by considering the flows at node N3 as shown in Figure 2.4.

\[\text{Figure A.4 The Flows at Node 3}\]

Analysis at node N3 is similar to that at node N2. Note that

\[ f_{62} = \frac{P_3}{RR} \]  \hspace{1cm} (A.24)

\( P_3 \) must be replaced with an expression involving \( V_3 \). From the definition of \( V_3 \) one can write

\[ V_3 = CR \cdot (P_3 - P_5) \]  \hspace{1cm} (A.25)

Furthermore,

\[ f_{61} = \dot{V_3} = \frac{P_5}{R_2} \]  \hspace{1cm} (A.26)
Solving (A.26) for \( P_5 \) gives

\[
P_5 = R_2 \cdot V_3
\]  \hspace{1cm} (A.27)

Substituting (A.27) into (A.25) and solving for \( P_3 \) yields

\[
P_3 = R_2 \cdot V_3 + \frac{V_3}{CR}
\]  \hspace{1cm} (A.28)

The desired expression for \( f_{62} \) is obtained by substituting (A.28) into (A.24), which gives

\[
f_{62} = \frac{(R_2 \cdot V_3)}{RR} + \frac{V_3}{(CR \cdot RR)}
\]  \hspace{1cm} (A.29)

Adding (A.26) to (A.29) yields the following expression for \( f_6 \):

\[
f_6 = V_3 + \frac{1}{RR} \cdot (R_2 \cdot V_3 + \frac{V_3}{CR})
\]  \hspace{1cm} (A.30)

Applying Kirchoff's current law to node N3 gives

\[
f_5 = f_6
\]  \hspace{1cm} (A.31)

By definition \( F = f_5 \); substituting this and (A.30) into (A.31) and solving for \( \dot{V}_3 \) provides the third differential equation for the system:

\[
\dot{V}_3 = \left[ \frac{1}{(RR + R_2)} \right] \cdot \left[ \frac{-1}{CR} \cdot V_3 + RR \cdot F \right]
\]  \hspace{1cm} (A.32)

Finally, an equation must be obtained for \( \dot{F} \). The flow path between nodes N2 and N3 is shown in Figure A.5.

![Figure A.5: The Flow Between Node 2 and Node 3](image)

From Figure A.5 it is evident that

\[
L \cdot \ddot{F} + F \cdot RB(t) = P_2 - P_3
\]  \hspace{1cm} (A.33)
Solving equation (A.33) for $\dot{F}$ gives

$$\dot{F} = \left(\frac{1}{L}\right) \cdot [P_2 - P_3 - RB(t) - F] \quad (A.34)$$

$P_2$ and $P_3$ can be eliminated from (A.34) by substituting (A.10) and (A.28) respectively. This substitution yields the following tentative equation for $\dot{F}$:

$$\dot{F} = \left(\frac{1}{L}\right) \cdot [R_1 \cdot V_2 + V_2/CL - R_2 \cdot V_3 - V_3/CR - RB(t) \cdot F] \quad (A.35)$$

Equations (A.21) and (A.32) can now be substituted into (A.35) to eliminate $V_2$ and $V_3$; this produces the final equation for $\dot{F}$:

$$\dot{F} = \left[\frac{[R_1 \cdot SA \cdot EV(t)]}{[L \cdot K_1(t) \cdot RA]}\right] \cdot V_1$$
$$+ \left[\frac{1}{(L \cdot CL)}\right] \cdot \left\{1 - \frac{[R_1/K_1(t)] \cdot [SA/RA + SB/RBS + 1/RL]}{[R_2/(R_2+RR)] - 1}\right\} \cdot V_2$$
$$+ \left[\frac{1}{(L \cdot CR)}\right] \cdot \left\{\frac{R_2/(R_2+RR)}{[R_2/(R_2+RR)]} + RB(t)\right\} \cdot V_3$$
$$- \left(\frac{1}{L}\right) \cdot \left\{\frac{[R_1/K_1(t)]}{[R_1/K_1(t)]} + \frac{[(R_2 \cdot RR)/(R_2+RR)] + RB(t)}{[R_1 \cdot SB \cdot PBS(t)]/[L \cdot K_1(t) \cdot RBS]} \right\}$$

Equations (A.23), (A.21), (A.32), and (A.36), with $K_1(t)$ defined in (A.22), comprise the set of describing differential equations for the system. The solution to this set yields values for compliance volumes $V_1$, $V_2$, and $V_3$ and flow $F$. The equations required to convert $V_1$, $V_2$, and $V_3$ into $P_1$, $P_2$, and $P_3$ are developed below.

Volume $V_1$ is related to pressure $P_1$ by equation (A.2), which is

$$P_1 = V_1 \cdot EV(t) \quad (A.2)$$

Equation (2.21) may be substituted into (2.10) to provide an expression for $P_2$; the result of this substitution is
P2 = \[\frac{R_1}{K_1(t)} \cdot \{[SA \cdot EV(t) \cdot V_1]/RA \cdot \}
\]
\[- (1/CL) \cdot \{(SA/RA)+(SB/RBS)+(1/RL)-(K_1(t)/R_1)\} \cdot V_2 \] (A.37)
\[- F + [SB \cdot PBS(t)]/RBS} \]

P3 may be obtained in terms of \(V_3\) and \(F\) by substituting (A.32) into (A.28). The result of this substitution is

\[P_3 = \left(\frac{1}{CR}\right) \cdot [1 - \frac{2}{(R_2+RR)}] \cdot V_3 + \left[\frac{(R_2 \cdot RR)/(R_2+RR)}\right] \cdot F \] (A.38)

This completes the formulation of the system equations. The result has been a set of four first-order, non-linear, ordinary differential equations with time-varying coefficients. In addition three auxiliary equations were obtained for translating the compliance volumes which result from solving the equations into pressures.
Appendix B
DERIVATION OF THE EQUATIONS FOR THE ISOLATED AORTIC MODEL

The isolated aortic model, for which a mathematical description is sought is shown in Figure B.1.

![Isolated Aortic Model Diagram](image)

Figure B.1 Isolated Aortic Model

For brevity, the "(t)" notation indicating that pressures (P's) and flows (F and f's) are functions of time are omitted from the notation. No time varying parameters are involved in the isolated aortic model.

Applying Kirchoff's Current Law to node N2 of Figure B.1 yields

\[ f_1 + f_2 + F = 0 \quad (B.1) \]

Equation (B.1) can be rearranged to yield

\[ f_2 = -f_1 - F \quad (B.2) \]

\( f_1 \) however, is given by

\[ f_1 = \frac{P_2}{R_L} \quad (B.3) \]

Equation (B.3) can now be substituted into (B.2) to eliminate \( f_1 \), the result being

\[ f_2 = -\frac{P_2}{R_L} - F \quad (B.4) \]

This expression can be differentiated (with respect to time) to give the following expression for \( \dot{f}_2 \):
\[ \dot{f_2} = -\frac{P_2}{RL} - \dot{F} \]  
(B.5)

Noting Figure B.1, the voltage (pressure) \(P_2\) can be written as the sum of the voltage drops across \(R_1\) and \(C\),

\[ P_2 = R_1 \cdot f_2 + \frac{1}{C} \int f_2 \, dt \]  
(B.6)

Differentiating (B.6) gives

\[ \dot{P}_2 = R_1 \cdot \dot{f}_2 + \frac{f_2}{C} \]  
(B.7)

Equations (B.4) and (B.5) can now be substituted into (B.7) to eliminate \(f_2\) and \(\dot{f}_2\) respectively. The result is

\[ \dot{P}_2 = R_1 \cdot (-\frac{P_2}{R_1} - \dot{F}) + \frac{1}{C} \cdot (-\frac{P_2}{R_1} - F) \]  
(B.8)

Equation (B.8) is solved for \(\dot{P}_2\) to give

\[ \dot{P}_2 = \frac{1}{(R_1 + R_L)} \cdot (-R_1 \cdot \frac{R_L}{L} - \dot{F} - P_2 - \frac{R_L}{CL} - \frac{F}{CL}) \]  
(B.9)

But from Figure B.1, \(\dot{F}\) can be written immediately as

\[ \dot{F} = \frac{1}{L} \cdot (P_2 - P_3) \]  
(B.10)

Substituting this expression into (B.9) yields the following equation for the voltage at node N2:

\[ \dot{P}_2 = \frac{1}{(R_1 + R_L)} \cdot [-(R_1 \cdot \frac{R_L}{L} + \frac{1}{C}) \cdot P_2 + R_1 \cdot \frac{R_L}{L} \cdot P_3 - R_L \cdot \frac{F}{CL}] \]  
(B.11)

The situation at node N3 is now considered. Writing Kirchoff's Current Law at that node gives

\[ F = f_3 + f_4 \]  
(B.12)
Noting that

\[ f_3 = \frac{P_3}{R_R} \quad (B.13) \]

Equation (B.12) can be rewritten as

\[ f_4 = F - \frac{R_3}{R_R} \quad (B.14) \]

Differentiating (B.14) gives

\[ f_4' = F' - \frac{P_3'}{R_R} \quad (B.15) \]

Analogously to (B.6), an equation can be written for \( P_3 \).

\[ P_3 = R_2f_4 + \frac{1}{CR}\int f_4 dt \quad (B.16) \]

Equation (B.16) can be differentiated to give

\[ P_3' = R_2f_4' + f_4'CR \quad (B.17) \]

Equations (B.14) and (B.15) can be substituted into (B.17) to eliminate \( f_4 \) and \( f_4' \) respectively, giving

\[ P_3' = R_2\left(F - \frac{P_3}{R_R}\right) + \frac{1}{CR}\left(F - \frac{P_3}{R_R}\right) \quad (B.18) \]

Equation (B.18) can be solved for \( P_3 \) to give

\[ P_3 = \left[\frac{1}{(R_2+R_R)}\right]\left[R_2\cdot F - \frac{P_3}{CR} + R_R\cdot F/CR\right] \quad (B.19) \]

Equation (B.10) is now used to eliminate \( F \) from (B.19) to yield the following equation for the voltage at node N3:

\[ P_3 = \left[\frac{1}{(R_2+R_R)}\right]\left[R_2\cdot R_R\cdot P_2/L - (R_R\cdot R_2/L + 1/CR)\cdot P_3 + R_R\cdot F/CR\right] \quad (B.20) \]
The equations (B.11), (B.20) and (B.10) comprise the set of linear, homogeneous, first-order ordinary differential equations which describe the isolated aortic model of Figure B.1. These are the equations presented as equations (3.1) through (3.3) in Chapter 3.
Appendix C

THE VENTRICULAR ELASTANCE FUNCTION

The left-heart model described in Chapter 2 requires the specification of a function to generate left-ventricular elastance values during the course of the heart cycle. It was desired to make this function responsive to variations in atrial filling pressure (PAS), aortic load, and heart rate -- all factors which influence ventricular elastance in vivo. Canine pressure-volume data for the ejecting left ventricle were obtained for a range of end-diastolic volumes and a constant heart rate (150 bpm) from Dr. Roger Taylor of the Department of Medicine, University of Western Australia. These data covered only isovolumic contraction and ejection, with no measurements available for isovolumic relaxation and filling phases. Hence, although the isovolumic contraction and ejection portions of the elastance function to be described were based on Taylor's experimental data, the isovolumic relaxation and filling calculations had to be created without specific data on which to base them. This is not as adverse as it might seem, however, because the ventricle is "disconnected" from the aorta during isovolumic relaxation and filling. The critical part of the elastance function in terms of overall model operation is the ejection phase. Elastance calculation for ejection determines how much and how quickly blood is forced into the aorta, and this in turn determines aortic pressures and flow for the remainder of the heart cycle.
After computing and plotting the elastance time courses for Taylor's data, it was noted that a clearly defined transition in the shape of the elastance curve occurred upon the start of ejection. This observation prompted the idea that a four-part elastance calculating function, in which each heart phase has a corresponding elastance calculating procedure, would be a good approach to the problem. The concept of computing elastance separately for each heart phase has several merits. First, it simplifies the job of fitting curves to the experimental data — it being substantially easier to fit the complicated elastance time-courses with a series of relatively simple equations than with a single more complicated one. Further, variations in the elastance curve which effect primarily a single phase can be easily handled, without necessitating refitting the whole curve. Finally, and perhaps most significantly, the technique readily lends itself to incorporating any new features of the elastance time-course which may be discovered by further experimentation. Of course, weight is given to these arguments by the ability of a four-part elastance calculation to yield satisfactory curve fits to Taylor's experimental data. Figure C.1 shows elastance curves for three values of end-diastolic volume (EDV) calculated from Taylor's experimental data and as computed for these same values of EDV by the elastance function. The break in slope occurring at about 0.04 to 0.05 seconds on these curves corresponds to the opening of the aortic valve, as described. Note that the slope change at this point is greater for greater values of EDV.
Figure C.1 Experimental and Calculated Elastance Curves

EDV = 30.3 ml
EDV = 38.3 ml
EDV = 48.0 ml

LEFT VENTRICULAR ELASTANCE (mm-Hg/ml)

TIME (from start of isovolumic contraction in secs)
The effect which variation in heart rate exerts on the elastance curve in vivo is not at all well documented. As indicated, the data obtained from Taylor was for a single heart rate. Some information regarding the effect of heart rate on the duration of the isovolumic and ejection phases in humans was found (Leighton, 1971; Metzger, 1970; Kumar, 1970), but considerable difference exists between dogs and humans with regard to the percent of the heart cycle spent in each phase (Ruch, 1966), so this information could not be directly utilized. It did serve however, to provide a qualitative feel for how heart rate might effect some aspects of the elastance curve. Experimental data displaying the effect of heart rate on ventricular elastance with other parameters (end-diastolic volume, aortic load) held constant would permit a more extensive (and realistic) modeling of the influence on elastance which variation in heart rate has.

Figure C.2 shows a typical elastance curve with the four heart phases defined on it. The characteristic shape of each part of this curve was used in determining the form of the equations to represent that part in the elastance calculation.

Figure C.2 Characteristic Shape of Each Phase of the Elastance Curve
Formulation of the isovolumic contraction portion of the elastance function was begun by fitting straight lines to Taylor's elastance curves for isovolumic contraction. From this procedure was obtained a series of slopes, each corresponding to the rate of change of elastance during isovolumic contraction for a particular value of end-diastolic volume. These slopes were found to be a reasonably linear function of end-diastolic volume. Figure C.3 shows the experimental slopes and the straight line chosen to predict them, as a function of end-diastolic volume. Ventricular elastance during isovolumic contraction is thus calculated according to a linear equation in time, the slope of which is computed from the value of end-diastolic volume at the onset of isovolumic contraction. In accord with the observation from Metzger (1970) that isovolumic contraction time shortens with increased heart rate, a term was added to the elastance slope calculation to increase slope at heart rates above 150 bpm. Increasing the slope of the elastance curve during isovolumic contraction effectively shortens isovolumic contraction time since ventricular pressure is more quickly brought to aortic pressure.

It may be noted from Figure C.1 that the elastance time course during ejection essentially follows a straight line until near the end of that phase, at which time the curve bends downward. The function chosen to represent this curve consists of the sum of a straight line and a hyperbola. The basic equation is of form \( E = S \cdot T + A/T + C \), where \( S \) is the straight line slope, \( A \) is the hyperbolic constant, \( C \) is the constant term, \( T \) is time, and \( E \) is elastance. \( T \) must be negative in
this equation, which requires a translation of axes. By properly choosing the value of A, the straight line may be made to dominate the initial part of the curve generated by this equation, with the contribution of the hyperbolic term causing the curve to bend downward as the end of the phase is reached.

Figure C.3 Elastance Slope for Isovolumic Contraction as a Function of End-Diastolic Volume
The slope of the straight line used in this relation is a function of end-diastolic volume, and was obtained in the same way as with isovolumic contraction. However, in this case, a cubic equation in end-diastolic volume was chosen to predict slope. Figure C.4 shows the slopes of the experimental elastance data during the initial part of ejection as a function of end-diastolic volume, and the cubic equation used to predict these slopes. The hyperbolic parameter, A, was selected to give the best fit of the overall computed elastance curve to the experimental data. It was found that a constant value for this parameter produced satisfactory results.
One additional factor is required to characterize the elastance for ejection — the time at which the curve must begin to bend downward. Duration of the heart ejection phase in the model is directly influenced by this characteristic. From examination of Taylor's data it was found that duration of ejection was a function of end-diastolic volume; in addition, it is also known to be a function of heart rate. These factors are incorporated into the calculation of elastance for ejection by adjusting the time at which the hyperbolic component begins to bend the elastance curve downward. This time ("anticipated ejection duration") is determined as a percentage of the total cycle duration — the percentage being based on the value of end-diastolic volume and adjusted for variation in heart rate. Figure C.5 shows anticipated ejection duration as a function of heart period for three values of end-diastolic volume (EDV). In actual model operation, the duration of ejection is a function of the pressure differential across the aortic valve resistance RA, but since ventricular elastance is one of the factors controlling this differential, the actual duration follows the anticipated duration rather closely.
Figure C.5 Anticipated Ejection Duration
No elastance data was available for the isovolumic relaxation phase, but the general behavior of the curve in this phase was known from the literature (Suga, et. al). A straight line was chosen to characterize the elastance during this phase, with its slope a linear function of end-diastolic volume. As with the isovolumic contraction calculation, the slope is increased (to reduce the duration of the phase) for heart rates in excess of 150 bpm.

End-diastolic volume (EDV) has entered into the elastance calculation for each phase; it plays a significant role in determining the shape of the elastance time-course, which in turn is a major factor in the operation of the entire model. Hence the proper control of EDV is important. Three parameters may immediately be identified as directly effecting EDV in vivo; these are end-diastolic volume, atrial filling pressure, and duration of filling. All of these parameters have been included in the elastance calculating function for the filling phase. The manner in which they are utilized will now be described.

At normal heart rates (100 to 150 bpm for dogs) it is believed that ventricular filling is virtually complete. This means that prolonging the filling phase would not result in appreciably increased EDV. EDV in such a situation should be primarily a function of filling pressure. Based on this concept, EDV was obtained from Taylor's data as a function of atrial filling pressure \(^1\) (PAS). A quadratic equation in PAS was found to predict EDV closely; the experimental and calculated curve are shown in Figure C.6.

\(^1\)The value used to represent atrial filling pressure was the ventricular pressure at the end of filling.
Figure C.6 End-Diastolic Volume as a Function of Atrial Filling Pressure
The elastance curve computed during filling is dependent upon the value of EDV predicted by this equation. The elastance curve is generated in such a way that, given a heart rate of less than 150 bpm, the ventricle will fill to (or near) the predicted value of EDV. The elastance time-course during filling is represented by two straight lines, as shown in Figure C.7.

![Diagram](image)

**Figure C.7 Elements of the Elastance Calculation for Filling**

In Figure C.7, $EV_4$ is the value of elastance at start of filling; $S_4$ and $S_5$ are the slopes of the two lines; $EMIN$ is the minimum value of elastance obtained; $TMIN$ is the time at which elastance is a minimum (and also the time at which the second straight line begins).

Slopes $S_4$ and $S_5$ are based on the initial elastance value $EV_4$, and on the values computed for $EMIN$, $TMIN$, $EEND$ and $TEND$. The value of $EEND$ is chosen as the elastance for which ventricular pressure would equal $0.99$ of atrial filling pressure if ventricular volume equaled the predicted EDV. $TEND$ is computed from a quadratic equation in atrial
filling pressure (PAS); for a particular value of PAS, TEND is the time which would ordinarily be spent in the filling phase for a heart rate of 150 bpm. Note that the value of TEND is only used in obtaining the slopes S4 and S5; the actual duration of filling is controlled by the program logic. The value for TMIN is computed as 0.3·TEND. The location of TMIN within the filling interval determines when the rate of filling begins to decrease. A value equal to 1/3 that of TEND was arbitrarily selected to represent the observation that the majority of filling occurs in the earlier stages of the filling phase. The value of EMIN is chosen so as to cause the stated goal of filling the ventricle to the predicted EDV at time TEND to be achieved if the heart rate is less than 150 bpm. Calculation of EMIN is performed by computing empirical approximations to ventricular pressure and volume at time TMIN.

With values for EV4, EMIN, TMIN, EEND and TEND the slopes S4 and S5 are established. The elastance during filling is computed using the first straight line until time exceeds TMIN; thereafter the second line is used. Note that heart rate was not included as a variable in the calculation of the filling elastance. Heart rates of less than 150 bpm have no effect on the end-diastolic volume. For heart rates less than 150 bpm, the actual filling time will exceed TEND. In this case, elastance continues to be computed by the straight line with slope S5 until ventricular pressure equals atrial filling pressure (PAS), after which time the elastance remains constant until the start of the subsequent isovolumic contraction. For heart rates in excess of 150 bpm,
the duration of filling will be less than TEND, and hence the end-diastolic volume will be reduced. The greater the heart rate, the smaller the value of end-diastolic volume, providing atrial pressure remains constant.

In operation, the particular phase for which elastance is computed is controlled by a variable assuming values 1 through 4 -- corresponding to the four heart phases. The variable is given the proper value by the simulation program before the elastance function is "called" to compute an elastance value. The terminal value for the elastance in each phase becomes the initial value for the elastance calculation during the next phase. The result is a "smooth" curve, characterizing ventricular elastance at every step during the heart cycle. This elastance function, when used in the left-heart model to drive the aortic model, is capable of operating with heart rates in the approximate range of 90 to 200 bpm and with atrial filling pressures in the approximate range of 4 mm-Hg to 16 mm-Hg. A listing of the FORTRAN program used to compute elastance is provided in Appendix E.
Appendix D

THE BALLOON PERFORMANCE INDEX

The measure of balloon pumping effectiveness used in this work is based on a performance index described by Kane (1971). Balloon effectiveness is considered maximized when the performance index is maximized. The index (J) used is computed according to the expression

\[
J = K1 \cdot MDP + K2 \cdot MSP + K4 \cdot S \cdot (K3 - EDP)^2
\]  

(D.1)

where: K1 and K3 are positive constants
K2 and K4 are negative constants
MDP is computed as \((1/TC) \int_{TC}^{TO} P1 \cdot dt + \frac{1}{(TD - TS)} \int_{TS}^{TD} P1 \cdot dt\)
MSP is computed as \((1/(TS - TC)) \int_{TC}^{TS} P1 \cdot dt\)
EDP is aortic pressure at the end of isovolumic contraction
S is defined as 1, if \((K3 - EDP) \geq 0\); and 0 otherwise
P1 is aortic pressure as measured at model mode N1 in Figure A.1 (Appendix A)
TC is the time at start of ejection
TS is the time at end of ejection
TD is the time at end of filling

The term \(K1 \cdot MDP\) in equation (D.1) represents the balloon pumping objective of increasing aortic root pressure to augment the coronary circulation. The remaining two terms in equation (D.1) represent the objective of decreasing heart work.
The constants $K_1$ through $K_4$ are computed from the control situation (when the balloon is not used) according to the following expressions:

$$
K_1 = 100 \cdot MDP_0 \\
K_2 = -100/MSP_0 \\
K_3 = EDP_0 \\
K_4 = -500 \cdot (1/EDP_0)^2
$$

where the $0$ subscript indicates that these values are to be obtained with the balloon deflated and inactive. The terms $MDP$, $MSP$, and $EDP$ retain the definition given above.

Increasing the mean aortic pressure during diastole contributes positively to the index, while elevating either mean aortic pressure during ejection or the aortic pressure value at the start of ejection contributes negatively. This performance index was computed for variation in a number of balloon pumping parameters; the results are displayed in Chapter 4.
THE ELASTANCE FUNCTION PROGRAM

FUNCTION EV(IPER)

THIS FUNCTION COMPUTES A VALUE OF LEFT VENTRICULAR
ELASTANCE FOR ONE OF THE FOLLOWING FOUR HEART PHASES,
DETERMINED BY THE VALUE OF IPER:

ISOVOLUMIC CONTRACTION        IPER = 1
EJECTION                        IPER = 2
ISOVOLUMIC RELAXATION          IPER = 3
FILLING                         IPER = 4

LOGICAL VARIABLE 'FIRST' IS .TRUE. ON THE INITIAL PASS
THROUGH EACH PHASE SIGNALING THE NEED TO INITIALIZE THE
PARAMETERS FOR THAT PHASE.

CONVERSION FACTORS

THE ELASTANCE FUNCTION WAS INITIALLY FORMULATED USING
PRESSURE IN MM-HG
VOLUME IN MILLILITERS
TIME IN SECONDS
THE SIMULATION REQUIRES THE FOLLOWING UNITS
PRESSURE IN MM-HG
VOLUME IN LITERS
TIME IN MINUTES
THE FOLLOWING CONSTANTS ARE USED IN THIS CONVERSION:

\[ \text{CONSL} = 6 \times 10^4 \quad \text{MM-HG/(ML*SEC)} \quad \text{TO} \quad \text{MM-HG/(L*SEC)} \]
\[ \text{CONA} = 1000 \times 60 \quad \text{MM-HG*SEC/ML} \quad \text{TO} \quad \text{MM-HG*MIN/L} \]
\[ \text{CONV1} = 1000 \quad \text{L TO ML} \]
\[ \text{CONV2} = 0.001 \quad \text{ML TO L} \]
\[ \text{C0NTM} = 1 \times 60 \quad \text{SEC TO MIN} \]

THIS ROUTINE WAS 'WRITTEN FOR USE OF A XEROX SIGMA 7.'

THE FOLLOWING 'COMMON' VARIABLES ARE USED IN THIS ROUTINE:
(UNITS ARE MINUTES, MM-HG, AND LITERS)

PERIOD - TOTAL HEART CYCLE DURATION
FIRST - INITIALIZATION SWITCH
T - ELASPED TIME SINCE THE BEGINNING OF THE
CURRENT HEART PHASE
TM - ELASPED TIME SINCE THE BEGINNING OF
ISOVOLUMIC CONTRACTION
TMIC - ELASPED TIME FROM START OF ISOV. CON.
TO END OF ISOV. CON.
TMIR - ELASPED TIME FROM START OF ISOV. CON., TO
END OF ISOVOLUMIC RELAXATION
EV1 TO EV4 - ELASTANCE VALUES AT START OF ISOV. CON.,
EJECTION, ISOV. REL., AND FILLING
EDV - VENTRICULAR VOLUME AT END OF LAST FILLING PHASE
VAS - ATRIAL FILLING PRESSURE
COND(S(4,1) - VENTRICULAR VOLUME AT START OF FILLING PHASE (I.E., END-SYSTOLIC VOLUME)

COMMON
1 COND(S(5/4), TM, TMIC, TMSYS, TMI, TFULL, TOFF, TEMP,
2 TPEP, SYSLIM, PERIOD, EV1, EV2, EV3, EV4, EV5,
3 SOLN(150/6), DIALIM, EDV, NTIMES, FIRST

LOGICAL FIRST
COMMON /SOLVE/ DELT, DEP(4), T0N
COMMON /PARAM/ VAS, RM, RA, RL, C, CL, R3, XL, CR, CR, R1, R2, R8M, R8S, V8S1, V8S2, V8L

DEFINE CONVERSION FACTORS

DATA CONSL9 CONNA CONV1 CONV2, CONTM / 6*E04, 16.6666666667, 1 1000, 0.001, 0.01666666667 /

GET HEART RATE (HR) FROM HEART PERIOD.
HR = 1 /PERIOD

SELECT CALCULATIONS FOR PROPER PHASE BASED ON VALUE OF IPER.
GO TO (100, 110, 120, 130) IPER

ISOVOLUMIC CONTRACTION

100 IF (.NOT. FIRST) GO TO 105
C INITIALIZE PARAMETERS FOR FIRST EVALUATION IN THIS PHASE.
FIRST = .FALSE.
C COMPUTE SLOPE OF STRAIGHT LINE.
VOL = EDV*CONV1
S1 = R1*54.5563*VOL
S1 = S1*CONSL9
C INCREASE SLOPE FOR HR GREATER THAN 150.
IF (HR *LE 150.) GO TO 105
S1 = S1+0.4*51*(HR/150.-1.)
C COMPUTE ELASTANCE
105 EV = EV1+S1*T
RETURN

EJECTION

110 IF (.NOT. FIRST) GO TO 115
C INITIALIZE PARAMETERS FOR FIRST EVALUATION IN THIS PHASE.
FIRST = .FALSE.
C COMPUTE SLOPE OF STRAIGHT LINE PORTION.
VOL = EDV*CONV1
E.3

\[ ES = VBL*VGL \]
\[ S2 = 567.1-36.97*VGL+8388*ES-.006307*ES*VGL \]
\[ S2 = S2*CONS \]

C A IS THE HYPERBOLIC CURVE PARAMETER.
A = .010
A = A*CONS

C COMPUTE PERCENT OF PERIOD ALLOCATED FOR EJECTION.
PRCNT = 0.436*CONS1*EV

C ADJUST PERCENT TO REFLECT VARIATION DUE TO HR CHANGES.
PRCNT = PRCNT + 40.40*EXP((50.-HR)/150.)

C COMPUTE DURATION OF EJECTION.
XT = .01*PRCNT*PERIOD

C COMPUTE CONSTANT TERM IN ELASTANCE EQUATION.
YTB = EV2+A/XT+S2*XT
113 TT = TMMIC+XT
C COMPUTE ELASTANCE.
EV = S2*TT+A/TT+YTB
RETURN

C-------------------------------
C IS EVOLUMIC RELAXATION.
C-------------------------------
120 IF (*NET.FIRST) GO TO 125
C INITIALIZE PARAMETERS FOR FIRST EVALUATION IN THIS PHASE.
FIRST = .FALSE.
C COMPUTE SLOPE OF STRAIGHT LINE.
VOL = EDV*CONS1
S3 = 2.*(VOL-30.)-120.
IF (S3 .LT. -120.) S3 = -120.
S3 = S3*CONS1
C INCREASE SLOPE FOR HR GREATER THAN 150.
IF (HR .LE. 150.) GO TO 125
S3 = S3+0.4*S3*(HR/150.-1.)
C COMPUTE ELASTANCE.
125 EV = S3*T+EV3
RETURN

C-------------------------------
C FILLING.
C-------------------------------
130 IF (*NET.FIRST) GO TO 134
C INITIALIZE PARAMETERS FOR FIRST EVALUATION IN THIS PHASE.
FIRST = .FALSE.
C COMPUTE TARGET EDV.
VTARGET = 14.56+4.536*VA3-0.1226*VAS*VA3
VTARGET = CONS2*VTARGET
C COMPUTE TARGET FILLING TIME AS FUNCTION OF VAS BASED ON HEART RATE OF 150 BPM.
TEND = .2260-.007357*VAS+.5357*VAS*VAS
TEND = CONTM*TEND
C COMPUTE TIME TO ACHIEVE MINIMUM ELASTANCE.
TMIN = 0.3*TEND
C SEE IF ANY FILLING IS TO BE DONE.
IF (CONS(4,1) .GE. VTARGET) EMIN=EEND=EV4; S4=S5=0.; GO TO 134
C COMPUTE TARGET PRESSURE DROP (VAS-MIN. VEN. PRES.)
\[ DR6P = 2 \cdot \text{RM} \cdot (V\text{TARGET} - \text{COND}_S(4,1)) / T\text{END} \]
\[ DR6P = DR6P + 0.25 \cdot \text{AMAX} \cdot (V\text{AS} - 4.0) \cdot \text{AMAX} \cdot (T\text{END} - 0.1 \cdot \text{COND}_T, 0) \cdot \]

\[ 1 \text{ DR6P} \]
\[ \text{IF} \ (DR6P \times GT \times 0.9 \times V\text{AS}) \text{ DR6P} = 0.9 \times V\text{AS} \]

\[ C \text{ COMPUTE ANTICIPATED FINAL ELASTANCE} \]
\[ E\text{EEND} = 0.99 \times V\text{AS} / V\text{TARGET} \]

\[ C \text{ COMPUTE ANTICIPATED MINIMUM ELASTANCE} \]
\[ E\text{MIN} = (V\text{AS} - DR6P) / ((T\text{MIN} / T\text{END}) \times (V\text{TARGET} - \text{COND}_S(4,1))) \]

\[ 1 \text{ CONDS}(4,1) \]

\[ C \text{ COMPUTE SLOPES OF STRAIGHT LINES} \]
\[ S_4 = (E\text{MIN} - EV4) / T\text{MIN} \]
\[ S_5 = (E\text{EEND} - E\text{MIN}) / (T\text{END} - T\text{MIN}) \]

\[ C \text{ DETERMINE WHICH STRAIGHT LINE TO USE} \]
\[ 134 \text{ IF} \ ((T\text{M} - T\text{MIR}) \times LT \times T\text{MIN}) \text{ GO TO} 135 \]

\[ C \text{ COMPUTE ELASTANCE} \]
\[ EV = E\text{MIN} + S_5 \cdot (T\text{M} - T\text{MIR} - T\text{MIN}) \]
\[ \text{GO TO} 136 \]

\[ C \text{ COMPUTE ELASTANCE} \]
\[ 135 \text{ EV} = EV4 + S_4 \cdot (T\text{M} - T\text{MIR}) \]
\[ 136 \text{ RETURN} \]

\[ \text{END} \]