RICE UNIVERSITY

PERFORMANCE OF AN ARC CHAMBER

by

BRUCE IRWIN HENDRICKSON

A THESIS

SUBMITTED TO THE FACULTY

IN PARTIAL FULFILLMENT OF THE

REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

Houston, Texas
April, 1962

14 May 1962
ABSTRACT

The performance of a plasma generator which used arc heating was studied with helium and hydrogen as the working gases. An optimum electrode gap and chamber diameter were determined for a stagnation pressure between 1 and 1.5 atmospheres by computing the efficiency of the unit for various values of these parameters.

Two methods of increasing the specific impulse of the test gas at sonic velocity were studied using optimum chamber conditions. First, the energy input to the gas was increased to approximately $1.5 \times 10^5$ cal/gram for hydrogen; and a maximum specific impulse of $2300 \frac{\text{grams force}}{\text{grams mass/sec}}$ was obtained. Further increases in the specific impulse resulted from removal of boundary layer gas at the throat. A theory, assuming separate regimes of flow at the throat, was developed to explain this increase.
ACKNOWLEDGMENTS

This investigation was supported in part by N.A.S.A. grant NsG - 3 - 59.

The author wishes to express appreciation to Dr. Herbert K. Beckmann for his guidance in this work.
<table>
<thead>
<tr>
<th>TABLE OF CONTENTS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>2</td>
</tr>
<tr>
<td>Acknowledgments</td>
<td>3</td>
</tr>
<tr>
<td>Table of Contents</td>
<td>4</td>
</tr>
<tr>
<td>Introduction</td>
<td>5</td>
</tr>
<tr>
<td>Symbols</td>
<td>6</td>
</tr>
<tr>
<td>Equipment</td>
<td>8</td>
</tr>
<tr>
<td>Calibration of Instruments</td>
<td>11</td>
</tr>
<tr>
<td>Test Procedure</td>
<td>12</td>
</tr>
<tr>
<td>Theory</td>
<td>15</td>
</tr>
<tr>
<td>Discussion</td>
<td>22</td>
</tr>
<tr>
<td>Summary</td>
<td>26</td>
</tr>
<tr>
<td>References</td>
<td>27</td>
</tr>
<tr>
<td>Data and Illustrations</td>
<td>28</td>
</tr>
</tbody>
</table>
INTRODUCTION

The performance of a plasma generator which uses an electric arc to heat the test gas can be studied in several ways. A determination of the over-all efficiency of a plasma generator indicates the part of total power input actually transferred to the test gas. The size of the power source for a given input energy can be determined by a heat balance on the unit, or by measurement of the energy in the gas as it leaves the converging nozzle section of the plasma unit. The specific impulse \( \frac{\text{grams force}}{\text{grams mass/sec}} \) of the gas indicates the average energy put into the gas. It is desirable to decrease the flow rate required for a given thrust thereby increasing the specific impulse of the gas.

The two goals of this investigation were to improve the over-all efficiency of a plasma generator which used arc heating, and to improve the specific impulse of the test gas at the throat of the converging nozzle section. An analytical study of flow at the throat of the nozzle was compared with the experimental results.
SYMBOLS

\( A^* \) cross sectional area at the throat
\( c \) sonic velocity in the gas
\( E \) ionization energy of the gas
\( F \) thrust
\( h \) specific enthalpy
\( i \) ratio of specific enthalpy to ionization energy
\( M_a \) atomic weight of the non-ionized gas
\( \dot{m} \) mass flow rate
\( m^* \) dimensionless critical mass flow density at the throat
\( p \) pressure
\( P \) dimensionless pressure
\( R \) universal gas constant
\( T \) absolute temperature
\( v \) velocity of the gas
\( \alpha \) weight fraction of the gas ionized
\( \gamma \) ratio of the specific heats \( = c_p/c_v \)
\( \varphi \) mass density of the gas

Superscripts and subscripts:

( )\( ^* \) referring to conditions at the throat
( )\( _0 \) referring to stagnation conditions
Numerical values of constants:

<table>
<thead>
<tr>
<th>symbol</th>
<th>dimension</th>
<th>Helium</th>
<th>Hydrogen</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_a$</td>
<td>g/mole</td>
<td>4.003</td>
<td>1.008</td>
</tr>
<tr>
<td>$E$</td>
<td>cal/mole</td>
<td>$5.649 \times 10^5$</td>
<td>$3.747 \times 10^5$</td>
</tr>
<tr>
<td>$\sqrt{\frac{E}{M_a}}$</td>
<td>cm/sec</td>
<td>$2.435 \times 10^6$</td>
<td>$3.951 \times 10^6$</td>
</tr>
</tbody>
</table>

Note: $E$ for hydrogen contains the dissociation energy and is given in cal/atom of hydrogen.
EQUIPMENT

The test equipment used in these experiments consists of a plasma generator using arc heating, the controls necessary to operate this generator, and instruments capable of measuring the important flow parameters (Figure 1).

Power was supplied by a d-c generator with a capacity of 3000 amps at 180 volts. A high frequency oscillator was used for a short period (approximately 0.01 second) at the beginning of each test to strike an arc in the gas. A diagram of the arc chamber is shown in figure 2. The teflon section (approximately 3/4" in diameter and 1/4" in length) was lined either with quartz, or boron nitride, to decrease the sublimation of teflon and the resulting contamination of the test gas. Tangential entry of the test gas into the arc chamber was accomplished by proper orientation of the sharp edged metering orifice.

The upper and lower electrodes were made of thoriated tungsten because of its high melting temperature and relatively good electrical conductivity. The upper electrode was machined to form a conical converging nozzle section with a throat diameter of between 0.123 and 0.128 inches. Gas from the arc chamber was accelerated to sonic velocity at the nozzle throat by the pressure gradient. A 1/8" hole was drilled through the lower electrode to enable measurement of
stagnation pressure at the center of the chamber. The gap between the electrodes was maintained at 0.105 inches after the first series of tests indicated that this was the optimum spacing.

Parameters measured during the tests were: arc voltage, arc current, mass flow, line pressure, stagnation pressure, thrust, and electrode temperature. All data except mass flow rate was recorded on a 36 channel Minneapolis-Honeywell oscillograph recorder. Arc voltage was measured between the electrodes, and arc current was measured using a calibrated shunt in the d-c circuit. Line pressure and stagnation pressure were measured with pressure transducers. Thrust was measured with the thrust balance shown in figure 3. The gas lines and current leads were attached to the lever arm at the point of the knife edge to reduce the possibility of loads other than thrust from the accelerating gas, and a damping device was installed to reduce vibrations. Electrode temperatures were measured with copper-constantan thermocouples to determine heat loss to the electrodes. The copper power leads served as the copper thermocouple junction and the constantan junction was attached to the copper electrode housing. Thermocouple circuits were open circuited during the actual test to isolate the recording galvanometers for
these two circuits from voltage drop due to current flow in the copper housings.
CALIBRATION OF INSTRUMENTS

Data for calibration of the stagnation and line pressure transducers was obtained by measuring the deflections of the galvanometer traces for known pressures in the arc chamber. Simultaneously, the unit was checked for leaks. A calibration curve for thrust was obtained by measuring galvanometer deflection with known static loads on the thrust balance. All calibration loads were placed at the geometric center of the nozzle as shown in figure 3.

The mass flow metering orifice located adjacent to the arc chamber was calibrated by measuring the flow of the working gas with a commercial gas meter. Mass flow was proportional to the line pressure for the range of line pressures checked, and with atmospheric back pressure. Pressure ratios of the same order of magnitude were maintained during the tests to insure that accurate values of mass flow could be obtained from the calibration data.

Calibration of thermocouples used to measure electrode temperatures was accomplished using the melting point of ice ($0^\circ$C) as the reference temperature.
TEST PROCEDURE

Striking and maintaining an arc in the test gas were two of the major test problems, especially in the case of hydrogen. When helium was used, 65-100 volts d-c was sufficient to strike the arc with the aid of the high frequency oscillator. Tests with hydrogen, however, indicated that a potential of approximately 150 volts d-c (plus the high frequency voltage) was necessary to strike an arc in the gas at a pressure of approximately one-tenth of an atmosphere. The large current corresponding to this supply voltage caused excessive damage to the arc chamber, so it was necessary to develop a different voltage adjustment. First, a resistor was placed in series with the arc chamber to reduce the voltage between the electrodes after the current began to flow. This was possible since a much smaller voltage (approximately 25 volts) was required to maintain the arc at one atmosphere than was required to strike it at the low initial pressure. Helium gas was introduced immediately prior to the hydrogen test gas to strike the arc. Once the arc was established, it could be maintained in a hydrogen atmosphere. A system of check valves and pressure regulators was used to regulate the flow and to switch from helium to hydrogen gas so that pure hydrogen was flowing during the actual tests. Tests with hydrogen gas could be performed satisfactorily after these two corrections.
were made. A supply voltage of 80 volts was found to be most suitable for hydrogen with the prevailing test conditions. With the resistor (0.1 ohms) in series with the arc circuit, a voltage of 25-35 volts and a current of 1000-1500 amps was available for the electric arc.

Several changes in arc chamber size and configuration were made during the test period to improve the efficiency. First, the effect of the electrode spacing on the efficiency of the unit was studied by changing the spacing while holding all other test variables constant. A spacing of 0.105 inches was found to give the highest efficiency for these tests. Radiation losses were reduced by lining the cylindrical part of the arc chamber with a silver coated quartz ring. This quartz was later replaced with boron nitride which was found to have good resistance to heat and was easily machined.

The original design of the plasmajet unit used in these tests depended on the vortex motion of the test gas in the arc chamber to rotate the arc in a circular path on the electrode surface. A continuous motion of the arc was necessary to protect the surface of the electrode from melting. For tests with very high energy input, the vortex motion of the gas proved insufficient to protect the electrodes; it was therefore necessary to develop another means of inducing rotation of the arc. A coil of 7 turns was wound around
the upper electrode and wired in series with the arc. The field of this coil produced an electro-magnetic force on the stream of electrons and ions composing the arc in the direction of the vortex and thus augmented the existing rotation. (See figure 2). Reference 2 explains this application in detail.

Provisions were made to investigate experimentally the prediction of a cold boundary layer of gas at the throat. A schematic diagram of the apparatus developed to remove boundary layer gas is shown in figures 1 and 2. Before each test using boundary layer removal, the bottle shown in figure 1 was evacuated to approximately 1.5 mm mercury. At the start of the test (when arc actually struck) the solenoid valve was automatically opened, thereby connecting the suction chamber shown around the throat of the nozzle to the evacuated bottle. Gas in the boundary layer at the throat flowed into the surrounding suction chamber and then into the vacuum bottle due to the pressure gradient present. The change in pressure in the bottle caused by this gas was measured with a MacLeod vacuum gauge. Using the change in bottle pressure and the time the valve remained open, an average flow rate was calculated. (See reference 4). At the end of the test the valve also closed automatically. The length of time that the valve was open was recorded with a galvanometer trace.
Several factors in the operation of a plasma generator which uses arc heating suggest the possibility that a relatively cold, outer region and a hot, highly ionized center region exist in the gas flow at the nozzle throat. First, in the arc chamber, the gas emerging directly from the arc is very hot, probably fully ionized, while regions of gas away from the arc remain relatively cool. The vortex motion of the gas in the chamber and in the converging section of the nozzle causes a centrifugal force on the gas which could tend to cause the colder, more dense gas to move to the outside and concentrate the very hot gas along the axis of the nozzle.

In actual flow of the type suggested, the change from the hot core gas to the cold, boundary layer gas at the nozzle throat would probably be gradual with a continuous energy gradient. Several assumptions and simplifications are made in order to allow a mathematical study of this flow. It is assumed that the flow at the throat can be separated into two regions: a fully ionized region in the center and a non-ionized region surrounding it. Both regions are assumed to have uniform properties.

The following symbols represent the flow parameters for these two regions with subscript (1) referring to the fully
ionized region, subscript (2) referring to the non-ionized region, and subscript (*) referring to the throat. One can derive three equations to determine the amount of gas and the specific impulse of the gas flowing in each region.

The following notation is used:

\[ A = \text{cross-sectional area (cm}^2\text{)} \]
\[ m = \text{mass-flow density at the throat (}\frac{\text{grams}}{\text{cm}^2 \cdot \text{sec}}\text{)} \]
\[ h = \text{energy added to the gas (cal/gram).} \]

The cross-sectional area occupied by the fully ionized flow plus the cross-sectional area occupied by the non-ionized flow must equal the total area of the throat.

1) \[ A_1 + A_2 = A^* \]

Also, the mass flow rate of the fully ionized region plus the mass flow rate of the non-ionized region must equal the measured mass flow rate to satisfy the condition of continuity. This can be written as:

2) \[ m_1 A_1 + m_2 A_2 = m_{\text{ave}} A^* \]

Finally, the sum of the energy added to the gas in the fully ionized region and the energy added to the gas in the non-ionized region must equal the total energy added to the gas in the stagnation chamber. If no energy losses between the
stagnation chamber and the throat occur, this can be written as:

3) \[ m_1 h_{01} A_1 + m_2 h_{02} A_2 = m_{ave} h_{ave} A^* \]

Either \( A_1 \) or \( A_2 \) can be eliminated from equations (2) and (3) using equation (1). \( A_2 \) will be eliminated for the case considered in this analysis. Equations (2) and (3) thus become:

2)' \[ m_1 A_1 + m_2 A^*_2 - m_2 A_1 = m_{ave} A^* \]

3)' \[ m_1 h_{01} A_1 + m_2 h_{02} A^* - m_2 h_{02} A_1 = m_{ave} h_{ave} A^* \]

It is helpful to non-dimensionalize the terms in these two equations by introducing the non-dimensionalized mass flow density \( (m^*) \), pressure \( (P_0) \), and specific enthalpy \( (i) \) from reference 1 in the following form.

\[
\frac{m^*}{P_0} = \frac{m}{P_0} \sqrt{E/M_a}
\]

\[
i = \frac{h}{E}
\]

Multiplication of equation (2)' by \( \frac{1}{P_0 A^*} \sqrt{E/M_a} \) \( \frac{h_{02}}{E} \) and equation (3)' by \( -\frac{1}{P_0 A^*} \sqrt{E/M_a} \) \( \frac{i_{02}}{E} \) gives the following two expressions.

4) \[ \frac{m_1}{P_0} i_{02} A_1 A^* - \frac{m_2}{P_0} i_{02} A_1 A^* + \frac{m_2}{P_0} i_{02} = \frac{m_{ave}^*}{P_0} i_{02} \]
Assuming that the relationships for isentropic, one dimensional flow from reference 1 hold separately for the expansion process of the ionized and non-ionized gas, the only unknown quantities in these two equations are \( \frac{A_1}{A^*} \), \( \frac{m_2}{P_0} \), and \( i_{o2} \). The term \( \frac{m_2}{P_0} \) can be expressed as a function of \( i_{o2} \) so two equations containing two variables exist. The following two expressions are used to simplify these equations:

6) \[
\frac{m_1}{P_0} = \left( \frac{2}{3+1} \right)^{\frac{1}{8-1}} \left( \frac{\gamma+1}{\gamma-1} \right)^{\frac{1}{2}} \frac{\gamma}{\sqrt{2}} \frac{1}{\sqrt{i_{o1} - 1}} = \frac{a_1}{\sqrt{i_{o1} - 1}}
\]

7) \[
\frac{m_2}{P_0} = \left( \frac{2}{3+1} \right)^{\frac{1}{8-1}} \left( \frac{\gamma+1}{\gamma-1} \right)^{\frac{1}{2}} \frac{\gamma}{\sqrt{2}} \frac{1}{\sqrt{i_{o2} - 1}} = \frac{a_2}{\sqrt{i_{o2} - 1}}
\]

Addition of equations (4) and (5) then gives a relationship for \( \frac{A_1}{A^*} \) in terms of \( i_{o2} \).

8) \[
\frac{A_1}{A^*} = \left( \frac{i_{ave} - i_{o2}}{i_{o1} - i_{o2}} \right) \frac{m_2}{P_0} \frac{\frac{m_{ave}}{P_0}}{a_1/\sqrt{i_{o1} - 1}}
\]

Substitution of equations (6), (7), and (8) into equation (4) gives an expression containing only one variable \( i_{o2} \).
This equation is of the general quadratic type, \( ax^2 + bx + c = 0 \), and can easily be solved for \( i_{02} \).

The quantities \( i_{\text{ave}} \) and \( m^* \) are evaluated from test data, and values for \( a_1 \), \( a_2 \), and \( i_{01} \) are known constants for a given gas. A numerical value of \( i_{02} \) can therefore be determined. Equation (8) can then be used to determine \( \frac{A_1}{A^*} \), \( \frac{A_2}{A^*} \) and the thrust produced by flow in these two regions can be calculated.

Let \( \frac{F_{1*}}{m_1} \) and \( \frac{F_{2*}}{m_2} \) represent the specific impulse at the throat of the fully ionized and non-ionized flow respectively.

Reference 1 gives the following expression for specific impulse in terms of the velocity, temperature and degree of ionization of the gas:

9) \[
\left[ \frac{a_2}{a_1} \sqrt{i_{01}^{-1}} - \frac{a_2}{m^*} \right] i_{02} + \left[ i_{\text{ave}} - i_{01} \right] \sqrt{i_{02}} + \left[ \frac{a_2 i_{01}}{m^*} - \frac{a_2}{a_1} i_{01}^{-1} i_{\text{ave}} \right] = 0
\]

10) \[
\frac{F}{m} = \frac{v^2 + (1 + \alpha) \frac{RT}{Ma}}{v}
\]

In terms of the dimensionless specific enthalpy of the gas, this equation for specific impulse at the throat becomes:

11) \[
\frac{F_{1*}}{m_1} = \sqrt{\frac{2g}{Ma}} \frac{(i_{01} - i_*) + \frac{8-1}{2^3} (i_{1*} - 1)}{(i_{01} - i_*)^{1/2}} \quad \alpha = 1
\]

and
Equations (11) and (12) can be reduced to functions of \( i_{01} \) and \( i_{02} \) respectively using the relationships for specific enthalpy at the throat \( i^* \) in terms of the stagnation specific enthalpy \( i_o \).

The total thrust at the throat produced by the gas is:

\[
\bar{F}^* = m_1 A_1 \frac{F_1^*}{\dot{m}_1} + m_2 A_2 \frac{F_2^*}{\dot{m}_2}
\]

Non-dimensionalizing this equation using the procedure described for equations (4) and (5) gives:

\[
\bar{F}^* = \frac{P_o A^*}{\sqrt{1/\gamma /M_0}} \left[ \frac{m_1^* A_1 \frac{F_1^*}{\dot{m}_1}}{P_o A^* \dot{m}_1} - \frac{m_2^* A_2 \frac{F_2^*}{\dot{m}_2}}{P_o A^* \dot{m}_2} + \frac{m_2^* F_2^*}{P_o \dot{m}_2} \right]
\]

An expression for total thrust at the throat in terms of the stagnation pressure can be obtained for monatomic gases utilizing equations (6), (7), (11), and (12).

\[
\bar{F}^* = \left( \frac{2}{\gamma + 1} \right)^{\frac{1}{\gamma + 1}} \left( \gamma + 1 \right) P_o A^*
\]

This is the same expression that was obtained for isentropic flow of a partially ionized monatomic gas in reference 1.
For the case of hydrogen which is assumed to be diatomic in the non-ionized region and monatomic in the fully ionized region, equation (14) cannot be reduced to the form of equation (15). Analysis of equation (14) and equation (15) for hydrogen does show that equation (15) can be used as a good approximation with the error in the thrust always being less than two per cent. This is sufficiently accurate to compare with the test data.
DISCUSSION

A study of the performance of the plasma generator used in these experiments requires consideration of the specific impulse of the test gas at the nozzle throat and the over-all efficiency of the unit. The specific impulse is important because it is a measure of the energy level obtained in the gas; a large specific impulse indicates a large energy input per unit mass of test gas. The over-all efficiency is important because it indicates the amount of power that must be supplied to the arc to maintain a given energy level in the gas at the nozzle exit; or, in other words, it indicates the power necessary to produce a given specific impulse.

The over-all efficiency of the plasma unit thus is a comparison of total energy in the gas at the nozzle exit to the total power consumed in the arc. There were two possible methods of evaluating this efficiency. One method involved measuring the heat loss to the electrodes and arc chamber. The difference between the total power input (computed from the arc current and arc voltage) and this heat loss is the energy in the gas as it leaves the nozzle. With the total power input and the heat loss both expressed in cal/gram of test gas, the efficiency was easily calculated. The major difficulty with this method was obtaining accurate thermocouple data for the electrode temperatures. This
method only allowed a determination of the average efficiency since only average values of heat loss were obtained.

The second method of calculating the efficiency is based on the equations derived in reference 3. It is possible to determine the dimensionless specific enthalpy at the throat ($i^*$) using figure 4, which presents the results of this paper. A comparison of $i^*$ to the total power input per unit mass gives a measure of the efficiency. This was the efficiency used to determine an optimum electrode spacing and an optimum chamber diameter. A specific impulse of 2300 (gram force/gram mass/sec) was obtained with hydrogen using the optimum electrode spacing and chamber diameter. Contamination of the test gas with teflon was reduced from as high as 20 per cent to less than 2 per cent by lining the arc chamber with a quartz or boron nitride ring.

The equations developed under theory for two regions of flow at the throat indicate the possibility of using boundary layer suction at the throat to increase the measured specific impulse of the test gas. Measured specific impulse is defined as the total measured thrust divided by the mass flow through the nozzle. (The mass flow through the nozzle is the total mass flow measured with the metering orifice minus the mass flow removed with the suction apparatus.) Calculation of the specific impulse of the
fully ionized gas \(\frac{F_{i}^{*}}{m_{1}}\) and the specific impulse of the non-ionized gas \(\frac{F_{n}^{*}}{m_{2}}\) using equations (11) and (12) indicates that, for the range of energy inputs obtained in these tests, \(\frac{F_{i}^{*}}{m_{2}}\) is much less than \(\frac{F_{n}^{*}}{m_{1}}\). Therefore, removal of gas from the boundary layer with specific impulse \(\frac{F_{i}^{*}}{m_{2}}\) would tend to make the measured specific impulse larger. The magnitude of this increase depends on the relative mass flow rates in the two regions and the amount of gas removed. Theory suggests that all the boundary layer could be removed leaving only the ionized gas with specific impulse \(\frac{F_{i}^{*}}{m_{1}}\). This was not done because of the possibility of disturbing the flow pattern at the throat. When only a portion of the boundary layer is removed, the measured specific impulse should fall between \(\frac{F_{i}^{*}}{m_{1}}\) and the value of the specific impulse obtained without removal. A series of test data, taken for the same stagnation pressure, indicated that removing approximately one-tenth of mass flow at the throat increased the measured specific impulse by a relatively small amount. Calculated values of \(\frac{A_{x}}{A_{0}}, i_{02},\) and \(\frac{m_{2}^{*}}{F_{0}}\) as shown in table 2 imply that most of the flow was in the non-ionized region and thus predict this small increase in specific impulse. Table 3 gives a comparison of the experimental and theoretical increases obtained. The fact that as much as 30 per cent of the total mass flow could be removed at the throat through a relatively
A small area further supports the assumption of a relatively cold, dense boundary layer at the throat.

Verification of equation (15) is shown by the test data plotted in figure 7. Both hydrogen and helium test data are included. The difference between measured \( \frac{F^*}{A^*} \) and calculated \( \frac{F^*}{A^*} \) is probably due to energy losses between the stagnation chamber and the nozzle throat.
SUMMARY

The over-all efficiency of the arc heated plasma generator was improved by determining an optimum electrode spacing. A magnetic field was used to induce rapid rotation of the arc in the arc chamber. This allowed a high energy input without excessive damage to the electrodes.

A procedure for initiating ionization in hydrogen was developed, and a maximum specific impulse of $2300 \frac{\text{gram force}}{\text{grams mass/sec}}$ was obtained with this gas. Boundary layer removal was used to increase the measured specific impulse for tests with low energy input, and the increase obtained agreed with a suggested analytical approach to the problem.
REFERENCES


FIGURE 3  THRUST BALANCE
Figure 4
Electrode spacing (inches)

Over-all efficiency of the plasma generator vs. electrode spacing.

Figure 5
Thrust per unit area at the throat as a function of stagnation pressure.

Figure 6
Specific impulse vs. ave energy input

Figure 7
<table>
<thead>
<tr>
<th>Test No.</th>
<th>Gas</th>
<th>$P_0$ (atm)</th>
<th>E.I (KW)</th>
<th>$F^<em>/A^</em>$ (atm)</th>
<th>$F^*/m$ (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>He</td>
<td>1.11</td>
<td>48.0</td>
<td>1.09</td>
<td>617</td>
</tr>
<tr>
<td>3</td>
<td>He</td>
<td>1.12</td>
<td>45.0</td>
<td>1.03</td>
<td>626</td>
</tr>
<tr>
<td>4</td>
<td>He</td>
<td>1.37</td>
<td>52.3</td>
<td>1.34</td>
<td>890</td>
</tr>
<tr>
<td>16</td>
<td>He</td>
<td>1.21</td>
<td>49.2</td>
<td>1.05</td>
<td>704</td>
</tr>
<tr>
<td>21</td>
<td>He</td>
<td>1.30</td>
<td>44.3</td>
<td>1.16</td>
<td>1030</td>
</tr>
<tr>
<td>31</td>
<td>He</td>
<td>1.18</td>
<td>43.9</td>
<td>1.02</td>
<td>685</td>
</tr>
<tr>
<td>51</td>
<td>H</td>
<td>0.87</td>
<td>48.4</td>
<td>1.00</td>
<td>1022</td>
</tr>
<tr>
<td>105</td>
<td>H</td>
<td>1.21</td>
<td>50.0</td>
<td>1.60</td>
<td>1740</td>
</tr>
<tr>
<td>111</td>
<td>H</td>
<td>1.20</td>
<td>46.3</td>
<td>1.62</td>
<td>1775</td>
</tr>
<tr>
<td>112</td>
<td>H</td>
<td>1.40</td>
<td>56.1</td>
<td>1.95</td>
<td>2300</td>
</tr>
<tr>
<td>115</td>
<td>H</td>
<td>1.38</td>
<td>58.8</td>
<td>1.60</td>
<td>1825</td>
</tr>
<tr>
<td>140</td>
<td>H</td>
<td>1.56</td>
<td>63.5</td>
<td>1.85</td>
<td>1910</td>
</tr>
<tr>
<td>1008</td>
<td>H</td>
<td>1.20</td>
<td>52.7</td>
<td>1.40</td>
<td>1430</td>
</tr>
<tr>
<td>1009</td>
<td>H</td>
<td>1.44</td>
<td>60.5</td>
<td>1.47</td>
<td>1560</td>
</tr>
<tr>
<td>1010</td>
<td>H</td>
<td>1.09</td>
<td>50.7</td>
<td>1.16</td>
<td>1190</td>
</tr>
<tr>
<td>1011</td>
<td>H</td>
<td>1.14</td>
<td>48.6</td>
<td>1.20</td>
<td>1230</td>
</tr>
<tr>
<td>1013</td>
<td>H</td>
<td>1.34</td>
<td>50.5</td>
<td>1.60</td>
<td>1640</td>
</tr>
<tr>
<td>1101</td>
<td>H</td>
<td>0.82</td>
<td>46.3</td>
<td>0.95</td>
<td>1050</td>
</tr>
<tr>
<td>1102</td>
<td>H</td>
<td>0.84</td>
<td>54.4</td>
<td>0.99</td>
<td>1090</td>
</tr>
<tr>
<td>1103</td>
<td>H</td>
<td>0.81</td>
<td>61.0</td>
<td>0.99</td>
<td>1090</td>
</tr>
<tr>
<td>1104</td>
<td>H</td>
<td>0.81</td>
<td>51.0</td>
<td>0.93</td>
<td>1020</td>
</tr>
<tr>
<td>1105</td>
<td>H</td>
<td>0.84</td>
<td>48.6</td>
<td>0.99</td>
<td>1090</td>
</tr>
<tr>
<td>1118</td>
<td>H</td>
<td>0.64</td>
<td>49.2</td>
<td>0.78</td>
<td>874</td>
</tr>
</tbody>
</table>
### TABLE II: CALCULATED DATA

<table>
<thead>
<tr>
<th>Test No.</th>
<th>$i_{ave}$</th>
<th>$i_{o2}$</th>
<th>$A_1/A_t$</th>
<th>$A_2/A_t$</th>
<th>$m_1/P_o$</th>
<th>$m_2/P_o$</th>
<th>$F_1/m_1$ (sec)</th>
<th>$F_2/m_2$ (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.23</td>
<td>0.079</td>
<td>0.236</td>
<td>0.764</td>
<td>1.41</td>
<td>4.12</td>
<td>2300</td>
<td>790</td>
</tr>
<tr>
<td>4</td>
<td>0.22</td>
<td>0.163</td>
<td>0.073</td>
<td>0.927</td>
<td>1.41</td>
<td>2.86</td>
<td>2300</td>
<td>1136</td>
</tr>
<tr>
<td>26</td>
<td>0.22</td>
<td>0.163</td>
<td>0.073</td>
<td>0.927</td>
<td>1.41</td>
<td>2.86</td>
<td>2300</td>
<td>1136</td>
</tr>
</tbody>
</table>

**Helium**

<table>
<thead>
<tr>
<th>Test No.</th>
<th>$i_{ave}$</th>
<th>$i_{o2}$</th>
<th>$A_1/A_t$</th>
<th>$A_2/A_t$</th>
<th>$m_1/P_o$</th>
<th>$m_2/P_o$</th>
<th>$F_1/m_1$ (sec)</th>
<th>$F_2/m_2$ (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1013</td>
<td>0.24</td>
<td>0.178</td>
<td>0.087</td>
<td>0.913</td>
<td>1.53</td>
<td>3.04</td>
<td>3420</td>
<td>1690</td>
</tr>
<tr>
<td>1026</td>
<td>0.14</td>
<td>0.103</td>
<td>0.068</td>
<td>0.932</td>
<td>1.53</td>
<td>3.98</td>
<td>3420</td>
<td>1220</td>
</tr>
<tr>
<td>1101</td>
<td>0.12</td>
<td>0.079</td>
<td>0.079</td>
<td>0.921</td>
<td>1.53</td>
<td>4.55</td>
<td>3420</td>
<td>1130</td>
</tr>
<tr>
<td>1102</td>
<td>0.15</td>
<td>0.069</td>
<td>0.151</td>
<td>0.849</td>
<td>1.53</td>
<td>4.87</td>
<td>3420</td>
<td>1050</td>
</tr>
<tr>
<td>1103</td>
<td>0.15</td>
<td>0.066</td>
<td>0.167</td>
<td>0.833</td>
<td>1.53</td>
<td>4.98</td>
<td>3420</td>
<td>1035</td>
</tr>
<tr>
<td>1115</td>
<td>0.15</td>
<td>0.035</td>
<td>0.256</td>
<td>0.744</td>
<td>1.53</td>
<td>6.84</td>
<td>3420</td>
<td>742</td>
</tr>
<tr>
<td>1118</td>
<td>0.12</td>
<td>0.037</td>
<td>0.201</td>
<td>0.799</td>
<td>1.53</td>
<td>6.66</td>
<td>3420</td>
<td>772</td>
</tr>
</tbody>
</table>

### TABLE III

Specific impulse with boundary layer removal = $F^{*'}/m$

Specific impulse without boundary layer removal = $F^{*}/m$

<table>
<thead>
<tr>
<th>Test No.</th>
<th>26-He</th>
<th>1115-H</th>
<th>1116-H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculated $\frac{F^{*'}}{m}$</td>
<td>1.02</td>
<td>1.03</td>
<td>1.02</td>
</tr>
<tr>
<td>Calculated $\frac{F^{*}}{m}$</td>
<td>1.02</td>
<td>1.02</td>
<td>1.02</td>
</tr>
<tr>
<td>Measured $\frac{F^{*'}}{m}$</td>
<td>1.03</td>
<td>1.05</td>
<td>1.04</td>
</tr>
<tr>
<td>Measured $\frac{F^{*}}{m}$</td>
<td>1.02</td>
<td>1.02</td>
<td>1.02</td>
</tr>
</tbody>
</table>