THE RICE INSTITUTE

PHOTOELASTIC ANALYSIS OF THE STRESSES AROUND THE
BOTTOM OF A CYLINDRICAL BORE HOLE DUE TO
OVERBURDEN AND FLUID COLUMN PRESSURES

by

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INTRODUCTION

The primary objective of the oil well drilling industry is that of penetrating the earth's surface to a depth at which the entrapped oil may be recovered. To accomplish this purpose, the destruction of the rock formation immediately below the drilling tool is required. In order to achieve this it is necessary for the drilling tool to produce stresses which exceed the ultimate strength of the rock in a localized area of the bottom of the bore hole, thereby producing chips which can be removed from bottom and brought to the earth's surface. Although the majority of these stresses is caused by the action of the drilling tool upon the bottom of the hole, there are in addition those stresses caused by fluid in the hole and by the weight of the overburden.

The approximate loading causing the latter two stress states can be closely represented by Figure 1(a)* where a cylindrical region of rock formation concentric with a bore hole and containing the lower portion of the bore hole is considered. Note that this approximation does not leave the body in vertical equilibrium. Therefore, the stresses at the outer boundary must somehow readjust themselves. If the radius of the bore hole is much smaller than that of the body being considered, these adjustments are distributed over a large area and actually would be very small and have a negligible effect.

By subtracting the loading of Figure 1(c) from the loading of

*Figures are arranged in sequence beginning on Page 14.
Figure 1(b), the total loading of Figure 1(a) is obtained. Since the stress condition resulting from the loading of Figure 1(b) is known to be hydrostatic, the principle of superposition may be employed to obtain the stress condition resulting from the loading of Figure 1(a), if the stress condition from the loading in Figure 1(c) is known.

Whitworth, using the two stress functions introduced by Southwell for axially symmetric problems, determined the stresses resulting from Figure 1(c) by means of a numerical network-solution employing relaxation methods.

An alternate approach to determine the stresses resulting from the loading of Figure 1(c) would be to apply the theory of elasticity in conjunction with experimental results obtained by use of the "frozen stress technique" of three-dimensional photoelasticity.

* Numbers denote items in the Bibliography on Page 27.
DESCRIPTION OF THE MODEL

The dimensions of the phthalic anhydride cured epoxy resin model are shown in Figure 2. The radius of curvature at the bottom of the cylindrical bore hole was determined after the test by viewing the longitudinal slice in a comparator. The size of the bore hole relative to the overall dimensions of the model was chosen so that the effect caused by the internal pressure would be negligible at the outside diameter of the model.
EXPERIMENTAL PROCEDURE

The model was placed inside the large photoelastic oven and, with the adapter in position, Figure 3, the system was pressure tested at 100 psi for leakage. Upon removal of the load, the temperature inside the oven was raised slowly to 300°F where it remained constant for a period of 24 hours to insure a constant temperature distribution throughout the model. At this time a load of 100 psi was applied inside the bore hole. After a period of 6 hours the oven was cooled to room temperature at a uniform rate of 3°F per hour with the previously applied load remaining constant.

A longitudinal slice was taken from the center of the model and polished to a thickness of 0.176 inches. This slice was later milled to give a nineteen degree "pie-shaped" slice whose vertex coincided with the centerline of the model.

Photographs of the isochromatics and isoclinics as viewed through a diffusion polariscope were taken to facilitate the interpretation of the experimental results
INTERPRETATION OF EXPERIMENTAL RESULTS

It is known that purely photoelastic procedures cannot solve the general three dimensional stress problem. Photoelasticity furnishes data from which only the shear stresses can be determined. These stresses, with the integration of the equilibrium equations, lead to the complete stress state. For an axially symmetric problem the only non-vanishing shear stress is $\tau_r$. It has been established previously that normal stresses parallel to the light ray produce no photoelastic effect. Therefore, if a slice is taken in such a way that the incident ray is everywhere parallel to $\phi$, the problem of the stresses around a cylindrical bore hole caused by fluid pressure could be analyzed using the much simpler techniques of two dimensional photoelasticity. By using a thin "pie-shaped" slice whose vertex coincides with the centerline of the model and viewing the slice at oblique incidence, Figure 5, a satisfactory approximation of the above requirement, that is the light ray everywhere parallel to $\phi$, is obtained.

The shear stress $\tau_r$ is given by

$$\tau_r = nF \sin 2\phi$$

where $n$ is the fringe order, $F$ is the model fringe value and $\phi$ is the isoclinic parameter.

The fringe order $n$ is a measure of the relative retardation of one component of the light ray which has been broken into two components along the principal stress directions. This retardation is proportional to the magnitude of the principal stress difference, the thickness, and
a material constant. The fringe order \( n \) was determined from examination of the photograph of the longitudinal slice before the "pie-shaped" slice was taken. This photograph, Figure 6, was taken in a dark field of a circular polariscope using monochromatic light. Each dark fringe represents an integral number of wave length retardations in the "frozen stress" slice. It is a simple process to count the fringes after an initial value has been obtained. By utilizing the fact that at a free square corner the stress state is zero and therefore the fringe order is also zero, the values of \( n \) as shown in Figure 6 were determined. A maximum fringe order of 15 was observed in the vicinity of the corner of the hole by means of an E. Leitz Wetzlar - 30 power binocular. To facilitate the computation of the stresses, the field of interest was divided into a square network as shown in Figure 7. Since the fringe order at a majority of the nodes was not an integer, it was necessary to use the Tardy Method \(^5\) of compensation to determine the corresponding fractional values of the fringe order.

With a plane polariscope, dark bands also occur for the locus of points where the principal stress directions coincide with the axes of the polarizer and analyzer. It is therefore possible to determine the angle \( \Theta \) in equation (1) by rotation of polarizer and analyzer together. If a body contains an axis of symmetry and is loaded axially symmetrically, then this axis of symmetry is an isoclinic of zero parameter. \(^4\) The initial setting of the polariscope was made to give the zero isoclinic. By turning the polarizer and analyzer in 10° increments
and photographing the resulting fringe patterns, the values, Figure 8, of the isoclinic parameters throughout the stress field were determined.

$F$ may be defined as the stress in psi shear required to change the model fringe order by one.

Let $F = \frac{f}{t}$

where $f$ is the "effective material fringe value" determined from a calibration test, Figure 9, and $t$ is the thickness of the "frozen stress" slice at the point in question. For the longitudinal slice taken from the center of the cylindrical bore hole, $t$ remained constant at 0.176 inches and combined with a value of 1.2 pounds per inch per fringe for $f$ to give

$F = 6.82$ pounds per in$^2$ per fringe.

The thickness of the "pie-shaped" slice varied along a radial line extending from the centerline of the hole. Hence the value of $F$ changed for each vertical line in the square network of Figure 7 from a minimum of $F = 6.82$ to a maximum of 28.6. The introduction of this variable did not overly complicate the problem because the variation in $F$ was proportional to its distance from the centerline.
CALCULATION OF THE STRESSES

When the experimental data has been obtained photoelastically at each node of the network, the stresses may be calculated.

Examination of equation (1) reveals that $\tau_{r_2}$ can be calculated directly at every node from the previously determined values of the fringe order $n$, the model fringe value $F$, and the isoclinic parameter $\Theta$.

Consider a vertical line in the positive $Z$ direction passing through the model, Figure 7. At all points along this line one of the partial differential equations of equilibrium for an axially symmetric problem, with body forces neglected, is

$$\frac{\partial \sigma_x}{\partial z} + \frac{\partial \tau_{r_3}}{\partial r} + \frac{\tau_{r_2}}{r} = 0. \quad (2)$$

This is for a point "a"

$$\left( \frac{\partial \sigma_x}{\partial z} \right)_a = -\left( \frac{\partial \tau_{r_3}}{\partial r} \right)_a - \left( \frac{\tau_{r_2}}{r} \right)_a. \quad (3)$$

Using Taylor's series and expanding about "a", Figure 10a (neglecting third order and higher terms), equation (3) written in finite difference form is

$$\sigma_{20} - \sigma_{33} = -\frac{1}{q_h} \left( \tau_{r_2} + \tau_{r_6} - \tau_{r_4} - \tau_{r_8} \right) - \frac{1}{q_0} \left( 6 \tau_{r_2} - \tau_{r_2} + 3 \tau_{r_3} \right).$$

Solving for $\sigma_{20}$,

$$\sigma_{20} = \sigma_{33} - \frac{K}{q_h} \left( \tau_{r_2} + \tau_{r_6} - \tau_{r_4} - \tau_{r_8} \right) - \frac{K}{q_0} \left( 6 \tau_{r_2} - \tau_{r_2} + 3 \tau_{r_3} \right). \quad (4)$$
Since the shear stresses are known, the normal stress $\sigma_2$ could be determined if the normal stress $\sigma_3$ were also known. At the upper boundary $\sigma_3 = 0$, and on the hole bottom $\sigma_2 = -P$. Thus for each vertical line, equation (4) may be numerically integrated in the positive $z$ direction.

On the wall of the hole the shear stress $\tau_{1z} = 0$. Therefore the normal stresses $\sigma_r$ and $\sigma_\theta$ become the principal stresses $\sigma$ and $q$ respectively and the principal stress difference is related to the fringe order $n$ and the model fringe value $F$ by $\frac{\sigma - q}{2} = nF$. Thus

$$\sigma_2 = 2nF + \sigma_\theta \quad .$$

Boundary conditions make $\sigma_r = -\cos n$ along the wall of the cylindrical bore hole. Furthermore, since the values of $n$ and $F$ are known, $\sigma_2$ along the wall of the hole can be determined.

Finally attention is given to equation (2) as $r$ approaches zero. Equation (2) may be written

$$\frac{\partial \sigma_2}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \tau_{1z} \right) = 0 . \quad (5)$$

In the limit, $r$ and $\tau_{1z}$ approach zero as $r$ approaches zero so that equation (2) approaches

$$\frac{\partial \sigma_2}{\partial z} + 2 \frac{\partial \tau_{1z}}{\partial r} = 0 . \quad (6)$$

In finite difference form, Figure 10b, equation (6) becomes

$$\sigma_{2_1} = \sigma_{2_3} - 4 \tau_{1z} .$$
The normal stress component $\sigma_r$ can be determined for each node from the relationship, derivable from Mohr's circle,

$$
\sigma_r = \sigma_z + \frac{\tau_{r\theta}}{\tan 2\theta}.
$$

(7)

where $\sigma_z$ and $\tau_{r\theta}$ are the two previously calculated stresses and $\Theta$ is the isoclinic parameter at the corresponding node.

Equation (7) is valid everywhere except along the bottom of the hole. In a manner similar to that used in the determination of $\sigma_z$ along the wall of the hole, $\sigma_r$ can be determined along the bottom of the hole.

The normal stresses $\sigma_r$ and $\sigma_z$ and the shear stress $\tau_{r\theta}$ having been calculated at each of the nodes of the network, the remaining normal stress $\sigma_\theta$ may be determined.

The other partial differential equation of equilibrium for an axially symmetric problem, with body forces neglected, is

$$
\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{r\theta}}{\partial z} + \sigma_r - \sigma_\theta = 0.
$$

(8)

Referring to Figure 10c for nodal orientation, equation (8) in finite difference form is

$$
\frac{\sigma_z - \sigma_r}{2h} + \frac{\tau_{r\theta} - \tau_{r\theta}}{z h} + \frac{\sigma_r - \sigma_\theta}{r o} = 0.
$$

Solving for $\sigma_\theta$,

$$
\sigma_\theta = \frac{r o}{2 h} (\sigma_z - \sigma_r + \tau_{r\theta} - \tau_{r\theta}) + \sigma_r o.
$$

(9)
Equation (9) is valid throughout the stress field except along the wall and bottom of the hole where either node 3 or node 4 is fictitious. Using Taylor's series and expanding about "0", Figure 10e (neglecting fifth order and higher terms), equation (8) along the wall of the hole, written in finite difference form, is

\[
\frac{-25\delta r_0 + 48\delta r_2 - 36\delta r_{10} + 16\delta r_6 - 36\delta r_b}{12h} + \frac{6\phi_0 - 6\phi_o}{r_o} = 0.
\]

Solving for \(\delta\phi_0\),

\[\delta\phi_0 = \frac{r_o}{12h} \left( -25\delta r_0 + 48\delta r_2 - 36\delta r_{10} + 16\delta r_6 - 36\delta r_b \right) + \delta r_o.\]

Similarly equation (8) along the bottom of the hole, Figure 10d, may be written in finite difference form as

\[
\frac{\delta r_4 - \delta r_1}{2h} + \frac{(-25\tau_{r_0} + 48\tau_{r_2} - 36\tau_{r_10} + 16\tau_{r_6} - 36\tau_{r_12})}{12h} + \frac{6\phi_0 - 6\phi_o}{r_o} = 0.
\]

Solving for \(\delta\phi_0\),

\[\delta\phi_0 = \frac{r_o}{2h} (\delta r_4 - \delta r_1) + \frac{r_o}{12h} \left( -25\tau_{r_0} + 48\tau_{r_2} - 36\tau_{r_10} + 16\tau_{r_6} - 36\tau_{r_12} \right) + \delta r_o.\]
RESULTS AND CONCLUSIONS

The stresses calculated for a fluid pressure of \( P = 100 \) psi are given in Figures 11 - 13. The area presented is that which is of particular interest in the vicinity of the bottom of the hole. At greater distances from the bottom, stresses are either of nearly negligible value or have substantially the values given by the thick wall cylinder equations. The curves shown superimposed over the area were obtained by means of cross-plotting using the values of the stresses calculated at each individual node.

The values calculated for \( \sigma_\phi \) by the method given in the previous section were not consistent enough to be plotted. The fault does not lie in the mathematics but in the values available for the determination of \( \sigma_\phi \). In the single slice method, \( \sigma_\phi \) is dependent on the values of the other three stresses. Throughout the entire calculation of the stresses there is a pyramiding of errors. Equation (4) used to determine \( \sigma_z \) is dependent not only upon \( \sigma_z \) but also on the previously determined value of \( \sigma_z \). Thus any error in \( \sigma_z \) will induce an error in \( \sigma_z \) which will accumulate throughout the calculations. Although \( \sigma_r \) and \( \sigma_\phi \) can be calculated without using previously determined values of \( \sigma_r \) and \( \sigma_\phi \), both are dependent upon \( \sigma_z \) and \( \sigma_z \). Thus it becomes apparent that the shear stress \( \sigma_z \) must be determined very accurately. \( \sigma_z \), Equation (1), is a function of the fringe order \( n \), the model fringe value \( F \), and the isoclinic parameter \( \Theta \). Since the value of \( f \) used in the determination of \( F \) agreed with that given by Leven\(^2\) for the material, any errors in
\( \gamma_{r2} \) must arise from the readings of the fringe orders and isoclinics. When reading the fringe order and isoclinic in a dark field of the polariscope it is necessary to determine the point at which the intensity of the light is at a minimum. Using only a diffusion polariscope the determination of the point at which minimum intensity occurs is left to the experimenter's discretion. A light meter or photometric device might be used to increase the accuracy.

It should be noted further that in applying the results from the three-dimensional photoelastic analysis to the rock formation the effect of Poisson's ratio must be considered. According to Frocht\textsuperscript{6} theoretical solutions have shown the effect to be small but complete justification must await more comprehensive investigations.
Approximate loading on cylindrical region of formation containing lower portion of bore hole.

Figure 1
MODEL

FIGURE 2
Direction Of Light Ray

"PIE SHAPED" SLICED

FIGURE 5
CALIBRATION CURVE

FIGURE 9
NODE DESIGNATION

FIGURE 10
RADIAL NORMAL STRESS $\sigma_r$

FIGURE 11
VERTICAL NORMAL STRESS $\sigma_z$

FIGURE 12


APPENDIX
LABORATORY EQUIPMENT

Oven for Casting, Annealing, and Stress Freezing:

The photoelastic oven, Figure 4, used during the experiment had been designed and built prior to the start of this investigation. The oven chamber measured 44 inches wide, 56 inches high and 24 inches deep. It was heated and controlled electrically. The automatic temperature controller was cam operated so as to heat and cool in the required linear fashion and could do so at practically any desired rate. To allow continuous inspection of the oven contents without disturbing the temperature level, double-walled glass panels 16-1/2" by 24-1/2" were provided in both the front and back. The oven was equipped with an integral loading frame.

To facilitate working inside the oven the entire front portion of the oven was mounted on casters and could be opened by the loosening of 8 screw type clamps.

Casting Oven:

In addition to the main oven, a smaller one was used for preheating the epoxy resin during the casting cycle. It was the conventional type of laboratory drying oven and was electrically heated and manually controlled.

Mold:

Figure 1A* shows a typical aluminum mold and a 6 inch diameter, 6 inch long casting made in it. The body of the mold was made of 6

* Figures are arranged in sequence beginning on Page 39.
inch Standard Aluminum pipe Schedule 40. A flat plate was placed in the bottom and sealed by an "O" ring. To facilitate removal of the casting from the mold a half degree of draft was used on the inside diameter.

Model Loading Fixture:

Figures 3 and 4 show the model loading fixture with and without the model in position. The adapter was sealed in the model with a silicone rubber "O" ring, since a standard Hycar "O" ring could not withstand the 300°F encountered during the "stress freezing" cycle. The adapter was connected to a nitrogen bottle outside the oven by 1/4" tubing, and was restrained from any movement by rigid fastening to a horizontal angle iron which in turn was securely clamped to the integral loading frame.

Calibration Device:

The calibration device, Figure 2A, consisted of a very simple arrangement whereby the specimen, Figure 3A, could be subjected to gradually increasing tensile forces. The tension model was mounted between two vertical holders. The upper holder was attached to the integral loading frame and the lower one extended through the floor of the oven. To the lower holder was attached a pan on which various weights could be applied. The holders provided complete freedom of movement of the specimen to insure that upon application of the load to the pan the test piece would be subjected to pure tension.
Diffusion Polariscope:

Figure 4A shows a schematic diagram of the diffusion circular polariscope used during the analysis. The light source consisted of a bank of seventeen 15 watt florescent tubes which were arranged behind a diffusing glass to provide a light source 18 inches square. Each polarizer and quarter-wave plate was mounted in a plastic ring and retained in pairs in the outer steel ring. This ring carried degree calibrations around the rim and could be rotated by means of gear teeth on its periphery. The polarizing unit was pivoted on its base so that it could be used either as a linear polarizer or as a circular polarizer without the necessity of removing the quarter-wave plates. The driving pinions of the two units could be coupled together so that the polarizer and analyzer could be rotated as a unit.

Photographic Facilities:

The equipment used for taking photographs of the isochromatics and isoclinics as viewed through the diffusion polariscope was made available by the Hughes Tool Company. A 4 x 5 graphic with 127 millimeter Ektar lens photographed the isochromatics while a 35 millimeter Leica IIIC with 50 millimeter Elmar lens was used to record the isoclinics. The film was processed by the laboratory of Hughes Tool Company.
SELECTION OF A PHOTOELASTIC MATERIAL

For three-dimensional "frozen stress" tests, the properties which an ideal model material must possess may be summarized as follows:

1. **Birefringence**: When under stress, the material must polarize and transmit light along the principal planes with velocities dependent upon the principal stress differences.

2. **Castability in Large Sizes**: This implies that the curing agent can be added to the resin at a sufficiently high temperature to permit the escape of air bubbles caused by stirring without excessive heat liberation during the process of curing. Large castings permit the making of more accurate and intricate models. More important, the slices removed from the model will be thinner in proportion to the overall dimensions, approaching the ideal case of a plane through the model.

3. **High Figure of Merit**: The figure of merit, Q, is defined as

\[ Q = \frac{E_{\text{eff}}}{f_{\text{eff}}} \]

where \( E_{\text{eff}} \) and \( f_{\text{eff}} \) are respectively the modulus of elasticity and the material fringe value obtained in a "frozen stress" test. Thus Q is a relative measure of the number of fringes of retardation produced by a unit strain.

4. **Freedom from Time-edge Effect Stresses**: Due to the emission or absorption of volitile components (usually water content),
most plastics exhibit the phenomena of developing extraneous stresses in the edges after "stress freezing" and/or slicing. These time-edge effect stresses can very drastically reduce the accuracy of the photoelastic results, especially in more complicated models where a considerable length of time can elapse between "stress freezing," slicing and photographing of the stress pattern.

5. **Machinability and Cementability:** It should be possible to machine the material into complicated models by means of turning, boring, drilling, and milling. Also cementing of parts should be feasible.

6. **Freedom from Initial Stresses:** Residual stresses due to casting of the material should be low or else annealable, in order that the optical effects obtained should reflect only those due to the applied loads.

7. **Transparency**

8. **Linear Stress-Strain and Stress-Fringe Relationship**

9. **Isotropy**

It is rather difficult to assign an order of importance to the requirements listed above since the lack of any one of them is sufficient to render a material useless for photoelastic purposes.

The best material available today for three-dimensional "frozen stress" tests is a phthalic anhydride cured epoxy resin, in particular, an epoxy resin consisting of 100 parts by weight of Ciba Araldite No. 6020 and 45 parts by weight of phthalic anhydride Ciba Araldite No. 901.
Epoxy resins offer great promise for use in three-dimensional photoelasticity not only because of their high figure of merit, but also because they can readily be cast in large sizes and meet all the requirements of a good photoelastic material. Because of their high adhesive qualities, epoxy parts can be cemented together to simulate welded structures or to facilitate model making.

Epoxy resins cured with phthalic anhydride display the property of elimination of time-edge effect stresses after storage of models or slices for a period of time at reasonably constant temperature and humidity.
CASTING PROCEDURE

After careful cleaning with acetone, the mold was coated with a mold release compound, Dow Corning 20, Silicone Mold Release Agent. Due to its low viscosity, several coats were brushed on to insure that all parts of the mold were coated. After 2 hours of drying at room temperature the coated mold was placed in the photoelastic oven and heated to 320°F for a period of 24 hours.

Figure 5A shows the complete temperature-time chart for the epoxy resin casting. The Araldite 6020 was first heated to about 168-176°F in the original container received from the manufacturer. During this period the temperature in the photoelastic oven was reduced to 208°F. This was to insure that there would not be any drastic change in the temperature of the resin when it was poured into the mold. At this temperature 5.125 pounds of the resin were easily poured into the mixing container and heated to the pre-heat temperature of 320°F, Tpr, at which the previously weighed 2.3125 pounds of Araldite 901 were added and stirred. The resulting temperature of the mixture, Tm, was 220°F. After slowly raising the temperature to 234°F, Ts, the anhydride was completely dissolved. The solution was cooled to 218°F. After pouring the solution into the mold its temperature was further reduced to 208°F, Tg, where it remained for two days. It was imperative during this portion of the casting cycle that the temperature not drop below 203°F or the phthalic anhydride would have precipitated out of solution. The temperature was then raised to 302°F at a uniform rate of 4°F per
hour where it remained for two days and was cooled to room temperature at a uniform rate of 3°F per hour. The casting was removed by simply upsetting the aluminum mold.

After the model was completely machined, it was annealed at 302°F for four days and cooled uniformly to room temperature.
MACHINING OF PHOTOELASTIC MATERIAL

Epoxy resins have very high abrasive effects on cutting tools and very quickly dull the ordinary tool steels. Carbide-tipped tools were used in all of the machining processes. In the turning, facing, and sawing of the model very high rotating speeds were combined with extremely slow feed speeds to prevent chipping on the edges and excessive heat.

The cylindrical bore hole was drilled to within 1/8 inch of the final depth with a 13/32 inch drill after which it was reamed to full size, Figure 2, with a flat bottom drill. Plenty of coolant and RPM of 440 were used during the operation.

The longitudinal "frozen stress" slice was cut on a Do-All band saw with an eighteen pitch blade and finished to the desired thickness with number 280 and 400 wet or dry paper. The "pie-shaped" slice was machined from the longitudinal slice by successively fine passes with a milling cutter. Both slices were then polished with Dupont Compound No. 45.
CALIBRATION OF PHOTOELASTIC MATERIAL

The "effective material fringe value" is a measure of the number of fringes that will result from a given principal stress difference at the critical temperature in a "frozen system."

The "effective material fringe value" \( f_{\text{eff}} \) was computed from the expression given in the literature for the material fringe value for a tensile specimen

\[
\frac{f}{2nd} = \frac{P}{2nd}
\]

where \( P \) is the load in pounds on the model at the time when the fringe at the shank is of order \( n \) and \( d \) is the width of the shank in inches.

The calibration tests were kept at a moderate stress level to allow the simple tension specimen shown in Figure 3A to be used. The specimen was placed in the calibration device, Figure 2A, and heated to the critical temperature and allowed to remain at that temperature for several hours before the loads were applied. The results which are plotted in Figure 9 show \( f_{\text{eff}} = 1.2 \) pounds shear per fringe per inch.
MOLD AND CASTING

FIGURE 1A
CALIBRATION DEVICE

FIGURE 2A
TENSION SPECIMEN

FIGURE 3A
SCHEMATIC DIAGRAM OF A DIFFUSION CIRCULAR POLARISCOPE

FIGURE 4A
COMPLETE TEMPERATURE - TIME CHART FOR EPOXY RESIN CASTING

FIGURE 5A