THE RICE INSTITUTE

AN APPLICATION OF THE HYDRAULIC ANALOGY
TO THE STUDY OF SUPERSONIC INLETS

by

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INTRODUCTION

The analogy which exists between water flowing with a free surface in a smooth bottomed rectangular channel and the isentropic flow of a compressible gas in two dimensions has been known for approximately forty years.1,2,3 This analogy is usually called the Hydraulic Analogy.

In the past, the analogy has been used to study such problems as the pressure and velocity distributions in a gas flowing about such two dimensional objects as cylinders and wedges.3 Most of this work has been done at simulated gas flow velocities which were below the velocity of sound. Some work has been done, however, on the study of flow about small angled wedges at gas velocities greater than that of sound.1,4 These endeavors indicate that only by a proper combination of experimental technique and method of analysis of the experimental data can useful results be obtained from the analogy.

One of the major problems in aircraft design which arises when flight speeds become supersonic (that is, greater than the velocity of sound) is that of induction system design for the propulsion units. When, in recent years, airplanes first began to fly at these supersonic speeds, it became apparent that larger performance gains could be obtained by improving the ram recovery of the induction system. By ram recovery is meant the ratio of the total pressure after the induction process to the total pressure of the free stream. A second consideration in the induction system problem which also becomes very important in the supersonic speed range is the increased aerodynamic

* Superscripts refer to references listed in the Bibliography on Pages 41 and 42.
drag which often results when an efficient design of the induction system is made on the basis of ram recovery alone. In order to achieve the highest possible speed, it is therefore necessary to consider this drag in the design of the induction system.

It is the purpose of this study to investigate the application of the hydraulic analogy to the study of induction system design for supersonic aircraft. These induction systems are usually referred to as supersonic inlets. This nomenclature will therefore be used in the following discussion. Due primarily to the difficulty of correctly obtaining the drag of the various inlet configurations, it was decided that an inlet which has no drag would be used for this work. In order to further simplify the prediction of the theoretical flow pattern and, to some extent, the pressure and velocity distributions, it was further decided to use a simple two-dimensional wedge inlet as shown schematically in Figure 5. In this inlet, the flow is deflected through two equal angles as seen in Figure 5 by the dashed lines marked as streamlines. This inlet will be discussed in detail in a later section.

Since the hydraulic analogy has previously been applied only to the supersonic flow about small angled wedges (7 - 9 deg.), an investigation was first carried out to determine the extent to which the analogy is valid for larger wedge angles. The analogy was then applied to the inlet configuration shown in Figure 5. Since this inlet is symmetric about its centerline, only half of it was studied. Both quantitative and qualitative results were compared with the corresponding air flow.
THE HYDRAULIC ANALOGY

The classical hydraulic analogy can be obtained by comparing the equations which govern the flow of an incompressible inviscid liquid with a free surface to those which govern the isentropic flow of a compressible gas in two dimensions. A complete comparison of the analogous relations is given in Table I of the Appendix. From a comparison of the potential equations, the velocity of propagation of a small free surface gravity wave in the liquid flow is seen to be analogous to the propagation velocity of a small disturbance in air (velocity of sound). Similarly, from a comparison of the continuity and energy equations, it can further be found that the specific heat ratio ($k$) for the gas which is analogous to the liquid flow must be 2.0.

If the analogy is applied to aerodynamic problems, several fundamental limitations become apparent. The foremost is the fact that the specific heat ratio ($k$) for air is 1.4 rather than 2. Other limitations also exist due to the assumptions made in the development of the analogy. These assumptions include the neglect of all vertical velocities and accelerations in the liquid flow and that the flow in each case is inviscid. The fact that the classical analogy is restricted to isentropic flow can be obtained from the following considerations. If the flow is adiabatic, but not isentropic, the total temperature is constant and the total pressure decreases. Since both the temperature and the pressure are analogous to the height of the free surface in the liquid flow according to the classical analogy, this condition cannot be satisfied. Therefore the classical analogy
cannot be used unless the flow is isentropic.

Consider the flow of shallow water with a free surface along a flat-bottomed channel. Let the flow encounter a wedge with one side parallel to the original flow direction and the other at an angle \( \delta \) with the flow. This flow situation is shown schematically in the following sketch, as viewed from above.

If the flow velocity \( v_1 \) before the wedge is larger than the propagation velocity of the disturbance, a wave caused by the disturbance will be formed as shown above, making an angle \( \sigma \) relative to the initial flow direction. Since there is no characteristic length involved, each streamline will be parallel to the one at the wall and will change direction when passing through the wave. The continuity equation therefore requires that the height \( h_2 \) after the wave be constant. The height ratio \( h_2/h_1 \) will thus have a unique value. This ratio as well as the wave angle \( \sigma \) are uniquely determined by the Froude number and the deflection angle \( \delta \) of the wedge. The Froude number is defined as the ratio of the flow velocity to the local propagation
velocity of small waves. This relation is given in Equation 1.

\[ F_1 = \frac{V_1}{\sqrt{g h_1}} \]

\( V_1 \) = flow velocity
\( g \) = acceleration of gravity
\( h_1 \) = local water height

If we consider the supersonic (faster than sound) flow in two dimensions of air past a similar wedge, exactly the same flow situation will be obtained as for the case of the shallow water flow, qualitatively speaking. The wave which is formed in air is called a shock wave. The wave angle can be determined from the deflection angle and the Mach number. The Mach number is defined as the ratio of the flow velocity to the velocity of sound. This relation is given in Equation 2.

\[ M_1 = \frac{V_1}{\sqrt{k R T}} \]

\( k \) = specific heat ratio
\( R \) = specific gas constant
\( T \) = static temperature of the gas

From the equations which govern the two above situations, Equation 3, shown below, can be obtained by setting the height ratio (\( h_2/h_1 \)) equal to the density ratio (\( \rho_2/\rho_1 \)) and eliminating the wave and deflection angles.¹

\[ \frac{F_1}{h_1} = \frac{\rho_2/\rho_1 + 1}{2} \sqrt{1 - k \left( \frac{\rho_2/\rho_1 - 1}{\rho_2/\rho_1 + 1} \right)} \]

A graphical presentation of this equation is given in Figure 1 of the Appendix. From this figure it can be seen that if the Froude and Mach numbers are to be analogous (i.e., \( F_1/h_1 = 1 \)), then the water flow must be considered analogous to the flow of a gas with a specific heat
ratio \((k)\) obtained from Equation 3. The value of \(k\) will thus vary with the density or height ratio. If, on the other hand, we wish the water flow to be analogous to the flow of the air \((k = 1, k)\), then we must allow an inequality between the initial Mach and Froude numbers. From Figure 1, this means a maximum difference of ten per cent over the range of density ratios from 1 to 5. Reasonably accurate correlation should therefore be obtained experimentally for the case of non-isentropic flows simply by comparing only the density ratio of the air flow with the height ratio of the water flow and by using a modified Froude number.

The temperature and pressure ratios on the other hand are no longer analogous and must be calculated using the density ratio in their respective gas equations. These equations are given in Equations 4 and 5.

\[
\frac{P_2}{P_1} = \frac{k + 1}{k - 1} \frac{\rho_2/\rho_1}{\rho_2/\rho_1} = 1
\]

\[
\frac{T_2}{T_1} = \frac{k + 1}{k - 1} \frac{\rho_1/\rho_2}{\rho_1/\rho_2}
\]

In considering the application of the analogy to the study of flow problems, the question arises as whether to use a stationary or moving model. Each of these methods has its own advantages and disadvantages depending on the nature of the model and the type of results which are being sought. Generally, water heights are difficult to determine for the moving model method, and the boundary layer on the floor of the channel introduces many complications in the fixed
model method. The size of the model must also be determined from a consideration of the effects of surface tension and boundary layer effects on the wave formation. Another important requirement is the use of a water depth which will cause the propagation speed of all waves to be equal to the ideal value of \( \sqrt{gh} \). For water, this requires a constant height of approximately 0.25 inches.\(^5\)
THE SUPERSONIC INLET

The primary function of a supersonic inlet is to supply air to the propulsive unit of an aircraft flying at a speed greater than that of sound. However, in general, the inlet must supply the air at a speed which is much less than the speed of sound. This requires that the air be compressed (i.e., slowed down) by the inlet in the induction process. Since any energy which is lost in this process must appear as drag on the aircraft, this process must occur as efficiently as possible. This is equivalent to saying that the inlet process must occur with a minimum loss in total pressure. The total pressure is defined as the sum of the static and dynamic pressures. This relation is given below in Equation 6:

\[ P_0 = P_0 + \frac{\rho v^2}{2} \]

Therefore, the ratio of the total pressure after the inlet process to the total pressure of the free stream will represent the efficiency of the inlet. This ratio is called the total pressure recovery.

The simplest type of supersonic inlet is obtained by reversing an ideal Laval nozzle. Such an inlet is shown schematically in Figure 2. This type of inlet is referred to as a full internal compression inlet. It is capable of giving good pressure recovery even when operating at very high supersonic speeds as long as it is operating at its design values of speed and mass flow rate. However, it gives very poor recovery at off design speeds or flow rates. Since its
external surfaces are parallel to the free stream, this inlet configuration does not produce any external drag when operating at design conditions. It is therefore best suited to high speed aircraft which operate at nearly constant speeds.

A second type of inlet is the full external compression inlet. A simple inlet of this type is shown schematically in Figure 3. The compression from supersonic to subsonic speeds is accomplished by means of external shock waves. These inlets are usually either two-dimensional or axially symmetric. The geometry of the simple type shown in Figure 3 is defined by the cone or wedge angle and the design Mach number. These two values dictate the required cowl lip position as shown in Figure 3. This type of inlet gives good recovery in the low supersonic range \((M = 1.4 \text{ to } M = 2.0)\). Its outstanding feature is that the recovery does not decrease at speeds and flow rates below the design values. This is due to the fact that the shock waves are all external. This type of inlet has the disadvantage, however, of having a larger external drag due to the shock wave produced by the cowl lip. This drag becomes very large for Mach numbers greater than 2. This configuration is therefore best suited to aircraft which operate at various speeds up to Mach number 2.

A third type of inlet can be used when a fixed geometry inlet which will operate efficiently over a wider range of Mach numbers is required. It is called a partial internal - partial external inlet because part of the supersonic compression occurs externally and part internally. A simple inlet of this type is shown schematically in Figure 5. This inlet type has the basic characteristics of each of
the two previously mentioned types. The combination of these characteristics must be made so as to best perform the required function. The resulting configuration therefore depends on these requirements.

For the purpose of this study, a simple inlet constructed essentially as shown in Figure 5 was chosen. This configuration has the advantage of being easily constructed. Also, it incorporates most of the features of each type of inlet. A cowl lip which was parallel to the free stream was used to eliminate external drag considerations. The reason for the third shock wave shown in Figure 5 not being normal to the flow direction is that, due to stability considerations, the minimum cross-sectional area must be greater than the ideal value so as to pass the required flow rate. This condition requires that an expansion take place in the region prior to the intersection of the shock wave from the cowl lip at point $a$ in Figure 5. Stability requirements also require that this shock wave from the cowl lip must be reflected from the upper surface at point $a$. 
THE EXPERIMENTAL APPARATUS

The basic water table was of the fixed model type and consisted of the following: an open rectangular channel to contain the flow, a means of obtaining a variable height - variable velocity water flow which was constant across the channel, and a means of measuring the depth of the water at any location.

The floor of the channel was formed by a 4 x 10 feet glass plate which was leveled around its perimeter to within ± 0.005 inches. The maximum sag at the center was 0.015 inches. The table could be tilted up or down in the direction of the flow by means of jack screws which were located at the downstream end of the supporting structure. In order to obtain a uniform velocity across the channel, a suppressor gate was used. The complete flow situation is shown schematically in Figure 6. The primary suppressor gate could be adjusted to provide a water depth up to 2 inches. With a given height, the flow rate could be varied to obtain the desired velocity in the test section. Froude numbers up to approximately seven could be obtained from this arrangement. An auxiliary suppressor gate was used to control the back pressure on the model.

Height measurements of the free surface were obtained by means of a simple pointed probe which was attached to a dial indicator. This apparatus was transported by a carriage which moved on rails that were mounted above the table.

The model was constructed of two 4 x \( \frac{1}{8} \) inch strips of polyethylene. They were attached to support brackets on the sides of the
channel. An inlet configuration of the type shown in Figure 5 was selected for purposes of this study. It was selected for the following reasons: it is simple to construct, it incorporates most of the basic features of each of the other types, and it represents very nearly the inlet configurations which are in current use. This configuration also has the advantage of providing an exit flow which is parallel to the free stream direction. Since it is symmetric about its centerline, only half of the configuration was used. The model was constructed so that the initial deflection angle could be varied from 10 to 25 degrees. The straight wall which formed the lip of the inlet could also be varied so as to have the correct position for each value of the initial deflection angle \( \theta \). The entire curved portion of the contour could also be varied. The complete model can be seen in Figure 11.
The purpose of this experiment was to investigate the application of the hydraulic analogy to the study of supersonic inlets. Since the flow configuration to be studied consisted basically of three individual shock waves, two oblique with respect to the flow and one approximately normal to it, it was first necessary to justify the assumption that the height ratio across a hydraulic jump in water is analogous to the density ratio for a similar shock wave in air as discussed earlier. Therefore, the normal and oblique shock waves were investigated separately to determine if this assumption was correct.

A normal shock wave (hydraulic jump) was formed across the channel by means of the auxiliary suppressor gate shown in Figure 6 which increased the pressure downstream of the primary suppressor gate. The shock wave was positioned about six inches downstream of the primary suppressor gate so that the effect of the boundary layer on the floor of the channel would be small. The height of the free surface was measured before and after the shock for various initial Froude numbers. In all cases, the table was tilted down at various angles from 0 to 3 degrees; the results of these tests are given in Figures 7 and 8.

A single oblique shock wave was obtained by placing a polyethylene strip at the required angle to the flow direction. The apparatus and the resulting shock wave are shown in Figure 10. Height measurements were made for Froude numbers of 2, 3, and 4 for various deflection angles $\theta$. Several readings were taken whenever results were not consistent. It was found that an average value for the water
height behind the shock wave could be obtained by taking the measurement midway between the shock wave and the wall. For all oblique tests, the table was tilted at an angle which caused the wave angle to correspond to the calculated value for air. This tilting was necessary in order to compensate for the boundary layer effects on the floor of the channel. This required approximately 2 degrees for most of the points. The results of these tests are presented in Figure 9.

The analogy was then applied to an inlet configuration of the type shown in Figure 5 with various equal deflection angles $\theta_1 = \theta_2$. All tests were made at an initial Froude number of 3.0 and with the table tilted down so that the initial shock wave had the correct wave angle $\sigma_1$. This Froude number was chosen for two reasons. First, it gave the closest agreement in Figure 9. Also, it represents the Mach number at which most of the present work on inlets is being done. A total of three sets of data was taken.

The first set of data was taken using an initial water height of 0.50 inches. The energy equation for the water flow can be written either in the form of Equation 7 or 8.

\[
\begin{align*}
(7) & \quad h_{01} = h_2 + \frac{v_1^2}{2g} \\
(8) & \quad h_{01} = h_1 \left[1 + \left(\frac{p_1}{\rho_1}\right)^{\frac{1}{2}}\right]
\end{align*}
\]

Substituting $F_1 = 3.0$ and $h_1 = 0.50$ inches, gives a value of 2.75 inches for the initial total head. The flow rate for this first set of data was adjusted to a value which gave this reading of 2.75 inches for the initial total head. A simple pitot tube was used for this.
The second set of data was taken using an initial water height of 0.25 inches. The flow rate was set at a value which gave an average velocity across the channel which was determined for a Froude number of 3. The total head of the resulting flow was measured using the pitot tube and was found to be 1.78 inches. However, Equation 8 gives a value of 1.36 inches using \( F_1 = 3.0 \) and \( h_1 = 0.25 \) inches. An attempt was then made to run a third set of tests at \( h_{01} = 1.36 \) inches, but it became obvious that this value was too low and gave incorrect results. Therefore, a third set of data was taken using a flow rate which was approximately ten per cent less than the calculated value in an attempt to obtain the correct initial Froude number. An initial height of 0.25 inches was also used for this set of tests. The results of these three sets of data are shown in Figures 12 and 13.

In order to show the qualitative aspects of the analogy, photographs were taken of various flow patterns. These photographs are shown in Figures 14 through 17.
RESULTS

The data from the normal shock wave tests which is presented in Figure 7 shows that the height ratio compares well with the density ratio for air over the complete range tested. However, if the results are analyzed more carefully, two major discrepancies can be noted. The first is the dip in the data in the region from $M = 2$ to $M = 3$ as seen in Figure 7. This error is due to the difference in the equations which govern the water and air flow. It can be eliminated by using a modified Froude number as discussed on Page 5 using Equation 3. The effect of this correction can be seen by comparing the data to the height ratios predicted by Reference 7. This comparison is shown in Figure 8. However, when this correction is applied, another discrepancy appears in Figure 8 in the range $M_1 = 1$ to $M = 2$. This error did not appear in Figure 7 because it has a canceling effect on the first error. It is probably due to the larger boundary layer thickness on the floor of the channel for the lower flow velocities. These two errors therefore explain the experimental data very well.

The results of the oblique shock tests presented in Figure 9 are very good. The experimental points are within 4 per cent of the theoretical values of air. Some difficulty was encountered, however, in obtaining consistent values for the high Mach numbers when the deflection angle was greater than 15 degrees. This is not very important because the region in which this inconsistency occurred is not often encountered in actual flow problems. It is felt that the reason
for the accurate correlation of height ratio to density for these shock waves was that the two errors mentioned previously in relation to the normal shock waves had an opposite sign and approximately canceled each other in the region in which these tests were run. The effect of surface tension on the waves obtained in these tests can be seen in Figure 10 near the leading edge of the polyethylene strip. As long as the model is large, however, this effect is not important. Another non-ideality which can be seen in this photograph is the fact that the wave is actually a group of waves. Since this group becomes larger as the size of the model increases, there may be an optimum size model which should be used.

The complete flow pattern for the application of the analogy to the inlet configuration is shown in Figure 11. The height ratios across each shock wave are presented in Figure 12 for the three sets of data taken. They are compared with the density ratios across each corresponding shock wave of the ideal shock configuration obtained by the Oswatitsch method. It must be noted, however, that this shock configuration could not be obtained for an air inlet and therefore should not be considered as the expected results for air especially for the third shock wave. This is primarily due to the stability condition which dictates that an expansion must occur before the interaction of the second oblique shock wave with the opposite surface at point a on Figure 5. The effects due to friction also cause these values to be incorrect.

In order to study this data, consider the shock waves one by one. The first shock wave is essentially the same case as the single oblique
shock waves tested before, and since the downstream height $h_2$ was taken in the supersonic region before the occurrence of the expansion, the results should be representative of the actual error involved in the analogy. Figure 12a shows that the results of the first set of data, for which $h_1 = 0.50$ inches, are poor. The results of the second and third sets are better, and they indicate that a value of initial Froude number exists which will give almost exact correlation for this shock wave. Therefore an initial height of 0.25 inches is necessary to obtain the correct shape, and a flow rate approximately 15 - 20 per cent lower than the calculated value is required to give exact quantitative correlation to the density ratio. The above conclusion regarding the initial height is further emphasized by the results for the second shock wave which are given in Figure 12b. The initial heights for the second shock waves were approximately one inch for the first set of data and approximately one-half inch for the second and third sets. Note that each curve has the same shape and approximately the same level and that they resemble very closely the curve for the first set in Figure 12a. This indicates that the height effect is present but that it does not appear to increase with height. The height ratios for the normal portion of the shock wave pattern are given in Figure 12c. The results cannot be compared with the solid line which is for the ideal normal shock wave because of the reasons discussed previously concerning the expansion which occurs. In this internal region, there is no way of comparing the data with any predicted values. As can be seen in Figure 16, the initial shock wave was actually a group of waves and interacted with the lip to form
another group. It can also be seen from this photograph that the normal portion of the shock wave occurs only over about half of the duct. This occurs also in air inlets and is necessary so that the shock configuration of the inlet will remain stable. This configuration is identical to the corresponding shock wave pattern which would be expected for an air inlet of this type as shown schematically in Figure 5. The total pressure ratios were calculated using Equation 9 from Table I and by noting that, for air flow at these velocities, the total temperature is constant. Thus the total pressure ratio is equivalent to the total density ratio. The results of these calculations are given in Figure 13. The solid line represents the calculated shock losses for air using the ideal shock configuration obtained by the Goussitch method. The upper dashed line is obtained by subtracting the turning loss due to stability requirement, which is obtained from Reference 10, from the solid curve. Actually, this could only be done at the maximum point of the curve. The rest of the curve was drawn from this point assuming that the maximum turning loss occurs at the maximum point of the curve. The lower dashed curve was drawn in an attempt to include the additional losses due to friction and turbulence. Since no data was available to determine these additional losses, a value of ten per cent was assumed for purposes of comparison.

The fact that the curves for the second and third set of data did not have the same slope as the predicted curve is perhaps due to the greater effects of the friction losses for the shallower depths. No conclusion can be drawn regarding the absolute values of these pressure recoveries except that they are within about ten per cent of the
expected values in air. The curves do, however, predict very nearly the optimum wedge angle which should be used.

Figure 14 shows the throat and subsonic section of the inlet. Separation can be seen at the interaction of the second shock wave at the right-hand wall. In Figure 15, the first shock wave can be seen reflecting off the lip to form the second shock wave. The non-ideal wave group and the resulting complex interaction can also be seen in this photograph. The complete internal shock wave system can be seen in Figure 16. Figure 17 shows the inlet operating below the critical mass flow. In other words, the flow in the model duct was restricted to a value lower than the critical mass flow. This type of shock wave pattern occurs in the same manner for the air inlet and is shown schematically in Figure 4.
CONCLUSION

The hydraulic analogy as applied to supersonic inlets gives a good picture of the flow situation. The numerical results show that the analogy predicts the density ratio for a normal shock wave within 10 per cent and for oblique shock wave within 4 per cent if an initial water height of 0.25 inches is used. For the more complex geometry of the inlet configuration, the results of the comparison of the water height ratio to the air density ratio varied from about 4 per cent to about 20 per cent. The total pressure recovery for the complete inlet was within about 10 per cent of expected value for each case. The major reasons for the inaccuracies in the complicated system are: errors in the first regions are carried over and produce larger errors in the following regions, the ideal water height which gives equal wave velocities of $\sqrt{gh}$ cannot be maintained when successive shock waves are present, and friction effects from the boundary layer on the floor can be quite large for large models.

The hydraulic analogy may be very useful in comparing various proposed inlet configuration for the purpose of eliminating the worst ones before more expensive wind tunnel tests are made. In this respect, however, it will be necessary to have a good understanding of the errors in the analogy which are involved in each case so that a true comparison can be made.

In conclusion, it can be said that the hydraulic analogy, when properly used, can be very valuable as an aid towards understanding inlet problems and in many cases will give usable quantitative results.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>Specific gas constant</td>
</tr>
<tr>
<td>$g$</td>
<td>Acceleration of gravity</td>
</tr>
<tr>
<td>$k$</td>
<td>Specific heat ratio</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Specific heat at constant pressure</td>
</tr>
<tr>
<td>$C_v$</td>
<td>Specific heat at constant volume</td>
</tr>
<tr>
<td>$h$</td>
<td>Height of free surface of water</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density of air flow</td>
</tr>
<tr>
<td>$T$</td>
<td>Static Temperature of air flow</td>
</tr>
<tr>
<td>$p$</td>
<td>Static pressure of air flow</td>
</tr>
<tr>
<td>$T_0$</td>
<td>Total or stagnation temperature</td>
</tr>
<tr>
<td>$p_0$</td>
<td>Total or stagnation pressure</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Wedge angle or cone half-angle</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Wave angle</td>
</tr>
<tr>
<td>$v$</td>
<td>Flow velocity</td>
</tr>
<tr>
<td>$v_x$</td>
<td>x-component of velocity</td>
</tr>
<tr>
<td>$v_y$</td>
<td>y-component of velocity</td>
</tr>
<tr>
<td>$c$</td>
<td>Velocity of sound</td>
</tr>
<tr>
<td>$M$</td>
<td>Mach number</td>
</tr>
<tr>
<td>$F$</td>
<td>Froude number</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Potential function</td>
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<td>$\phi_x$</td>
<td>Partial derivative of $\phi$ with respect to $x$</td>
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<tr>
<td>$\phi_y$</td>
<td>Partial derivative of $\phi$ with respect to $y$</td>
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<tr>
<td>$\phi_{xx}$</td>
<td>Second partial derivative of $\phi$ with respect to $x$</td>
</tr>
<tr>
<td>$\phi_{yy}$</td>
<td>Second partial derivative of $\phi$ with respect to $y$</td>
</tr>
</tbody>
</table>
\[ \frac{T}{T_0} \quad \text{Temperature ratio} \]
\[ \frac{\rho}{\rho_0} \quad \text{Density ratio} \]
\[ \frac{P}{P_0} \quad \text{Pressure ratio} \]
\[ \frac{P_{02}}{P_{01}} \quad \text{Total pressure recovery} \]
TABLE I

SUMMARY OF ANALOGOUS EQUATIONS FOR THE HYDRAULIC ANALOGY

Continuity Equation for Air:
(1) \( \frac{\partial (\rho v_x)}{\partial x} + \frac{\partial (\rho v_y)}{\partial y} = 0 \)
\( \rho \) = density
\( v_x \) = x-component of velocity
\( v_y \) = y-component of velocity

Continuity Equation for Liquid:
(2) \( \frac{\partial (hv_x)}{\partial x} + \frac{\partial (hv_y)}{\partial y} = 0 \)
\( h \) = liquid height

Energy Equation for Air:
(3) \( v^2 = 2gC_p(T_0 - T) \)
\( g \) = acceleration of gravity
\( C_p \) = specific heat
\( T_0 \) = total temperature
\( T \) = static temperature

Energy Equation for Liquid:
(4) \( v^2 = 2g(h_0 - h) \)
\( h_0 \) = total head
\( h \) = local liquid height

Potential Equation for Air:
(5) \( \varphi_{xx} \left(1 - \frac{\varphi_x^2}{c^2}\right) - \frac{\varphi_x \varphi_y}{c^2}(2\varphi_{xy}) + \varphi_{yy} \left(1 - \frac{\varphi_y^2}{c^2}\right) = 0 \)

Potential Equation for Liquid:
(6) \( \varphi_{xx} \left(1 - \frac{\varphi_x^2}{gh}\right) - \frac{\varphi_x \varphi_y}{gh}(2\varphi_{xy}) + \varphi_{yy} \left(1 - \frac{\varphi_y^2}{gh}\right) = 0 \)
### Table I: Continued

**Summary of Analogous Quantities for the Hydraulic Analogy**

<table>
<thead>
<tr>
<th>Quantities in Gas Flow</th>
<th>Analogous Quantities in Liquid Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Temperature Ratio:</strong></td>
<td><strong>Liquid Height Ratio:</strong></td>
</tr>
<tr>
<td>$(7) \quad \frac{T}{T_0} = \left[1 + \frac{k-1}{2}(\mathcal{M})^2\right]^{-1}$</td>
<td>$(8) \quad \frac{h}{h_0} = \left[1 + \frac{k}{2}(\mathcal{F})^2\right]^{-1}$</td>
</tr>
<tr>
<td><strong>Pressure Ratio:</strong></td>
<td><strong>Square of Liquid Height Ratio:</strong></td>
</tr>
<tr>
<td>$(9) \quad \frac{P}{P_0} = \left[1 + \frac{k-1}{2}(\mathcal{M})^2\right]^{-\frac{k}{k-1}}$</td>
<td>$(10) \quad \left[\frac{h}{h_0}\right]^2 = \left[1 + \frac{k}{2}(\mathcal{F})^2\right]^2$</td>
</tr>
<tr>
<td><strong>Density Ratio:</strong></td>
<td><strong>Liquid Height Ratio:</strong></td>
</tr>
<tr>
<td>$(11) \quad \frac{\rho}{\rho_0} = \left[1 + \frac{k-1}{2}(\mathcal{M})^2\right]^{-\frac{1}{k-1}}$</td>
<td>$(12) \quad \frac{h}{h_0} = \left[1 + \frac{k}{2}(\mathcal{F})^2\right]^{-1}$</td>
</tr>
<tr>
<td><strong>Velocity of Sound:</strong></td>
<td><strong>Velocity of Surface Waves:</strong></td>
</tr>
<tr>
<td>$(13) \quad c = \sqrt{kRT}$</td>
<td>$(14) \quad c = \sqrt{gh}$</td>
</tr>
<tr>
<td><strong>Mach Number:</strong></td>
<td><strong>Froude Number:</strong></td>
</tr>
<tr>
<td>$(15) \quad M = \frac{v}{c}$</td>
<td>$(16) \quad F = \frac{v}{\sqrt{gh}}$</td>
</tr>
</tbody>
</table>
FIGURE 1

FIGURE 2
COMPARISON OF THE HEIGHT RATIO ACROSS A NORMAL WAVE IN WATER TO THE DENSITY RATIO ACROSS A NORMAL SHOCK WAVE IN AIR

FIGURE 7
COMPARISON OF THE HEIGHT RATIO ACROSS A NORMAL WAVE IN WATER TO THE DENSITY RATIO ACROSS A NORMAL SHOCK WAVE IN AIR

FIGURE 8
COMPARISON OF HEIGHT RATIO ACROSS AN OBLIQUE WATER WAVE TO DENSITY RATIO ACROSS AN OBLIQUE SHOCK WAVE IN AIR

FIGURE 9
COMPARISON OF HEIGHT RATIO ACROSS EACH WAVE TO DENSITY RATIOS OF IDEAL SHOCK WAVES

FIGURE 12
TOTAL PRESSURE RECOVERIES OBTAINED FROM HYDRAULIC ANALOGY

FIGURE 13
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