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SIMULTANEOUS CONDUCTIVE AND RADIATIVE HEAT TRANSFER THROUGH MULTIPLE-LAYER INSULATION IN A SPACE ENVIRONMENT

by

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ABSTRACT

SIMULTANEOUS CONDUCTIVE AND RADIATIVE HEAT TRANSFER THROUGH MULTIPLE-LAYER INSULATION IN A SPACE ENVIRONMENT

David M. McStravick

Multiple-layer insulation consisting of alternate layers of radiation shield and spacer material, covering a surface with a fixed temperature in a space environment, is analysed. The insulation is assumed to be composed of infinite, parallel layers of opaque material and transparent spacing material. Convective heat transfer between layers is assumed to be insignificant, and the effects of simultaneous conductive and radiative transfer are studied. The conductivity of the radiation shields is assumed to be infinite compared to the spacing material. The case of a constant external radiation as well as zero external radiation is included.

The calculation of the resulting heat transfer requires the solution of a quartic equation. This equation is solved by a computer program for the various cases, and numerical results are given for some values of the various parameters effecting heat transfer.

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ACKNOWLEDGEMENT

The author wishes to thank Dr. Alan Chapman who suggested the problem and helped in its solution. I also would like to thank my wife, Kathie, who typed several rough drafts of this thesis.

NOTATION

$\mathcal{F}_{n-1,n}$	exchange factor between the n^{th}-1 surface and the n^{th} surface (dimensionless)
G	a constant external irradiation (BTU/hr - ft^2)
n	number of shields in an array
q	net heat transfer per unit area
Q_c	net heat transfer due to conduction (BTU/hr)
Q_R	net heat transfer due to radiation (BTU/hr)
q_c	net heat transfer per unit area due to conduction (BTU/hr - ft^2)
q_R	net heat transfer per unit area due to radiation (BTU/hr - ft^2)
T_b	temperature of the body surface (°R)
T_{si}	temperature of the i^{th} shield (°R)
T_{sn}	temperature of the n^{th} shield
U_i	overall thermal conductance of spacer material between i^{th} shield and the i^{th}-1 shield (BTU/°R - ft^2 - hr)
W_t	weighting factor used in variable U and ϵ program
α_{sn}	absorptivity of outermost shield (dimensionless)
γ	a ratio of conductive to radiative heat transfer for the constant U and ϵ case-- $\dfrac{U}{\sigma \epsilon T_b^3}$
γ_i	a ratio of conductive to radiative heat transfer for variable U and ϵ case-- $\dfrac{U_i}{\sigma \epsilon_{sn} T_{sn}^3}$
ϵ	emissivity of the shield surface-- constant U and ϵ case (dimensionless)
ϵ_i	emissivity of the i^{th} shield (dimensionless)

ϵ_{sn}	emissivity of the n^{th} shield (dimensionless)
θ_i	ratio of temperature of i^{th} shield to the temperature of the body surface T_{si}/T_b (dimensionless)
θ'_i	ratio of the temperature of i^{th} shield to the temperature of n^{th} shield-- T_{si}/T_{sn} (dimensionless)
θ_n	ratio of the temperature of the n^{th} shield to the temperature of the body surface-- T_{sn}/T_b (dimensionless)
θ_{nc}	temperature ratio when no conduction is present
σ	Stefan-Boltzmann constant-- 0.1714×10^{-8} BTU/hr - ft^2 - $°R^4$
Φ	angle between the incident radiation and the outermost surface of the insulation

I

INTRODUCTION

The ability to maintain a specified temperature (either hot or cold) within an enclosure subjected to a space environment is of fundamental importance for space activities. For this reason considerable work has gone into the development of an insulating material which will provide the sufficient amount of thermal protection necessary and be light enough for space applications. One type of insulation that is of current interest is the multiple-layer insulation which ideally consists of a number of opaque, parallel planes with very low emissivity which are separated by spacer material having a minimum value of conductance. This type of arrangement provides an effective reduction in heat transfer through the radiation shields, and since much of the insulation is composed of void space, the weight is small. To provide shield separation and structural integrity, some spacer material must be placed between the shields. This spacing material provides a means for conductive heat transfer to occur, and, generally it is of equal importance to the overall heat transfer as is the radiation. This thesis is concerned with a method of estimating the overall heat transfer through a multiple-layer type insulation with various values of the important parameters.

II

HEAT TRANSFER THROUGH MULTIPLE-LAYER INSULATION

The problem of heat transfer by simultaneous radiation, conduction and convection in even the most simple geometric configuration is very complex. In the case of heat transfer in multiple-layer insulation in space applications, the region between shields can be considered to be in a state of vacuum. This simplifies the analysis of heat transfer by eliminating the possibility of convection. The absence of convection makes analysis of multiple-layer insulation a problem in simultaneous conduction and radiation only. In the ideal case for multiple-layer insulation (radiation shields suspended in such a way that no conduction could take place between the shields) the heat transfer would be determined by the radiation only. In any practical case some spacing material is required to provide separation, and so there will be some conduction. Several articles have been written on methods of including conduction in the analysis of heat transfer through multiple-layer insulation such as references (4) and (5). Reference (4) also includes the case of convection for earth applications. In these approaches the modes of heat transfer are evaluated separately throughout the insulation and then combined for the total heat transfer. A more rigorous approach is taken in an article by Viskanta reference (11) in which the complex problem of scattering material between two radiating surfaces is taken into account. Only the simplest case of two surfaces separated by homogeneous, interacting, solid material is considered, and extension of this work to a large number of parallel surfaces would be very difficult. The model used in this thesis to represent the multiple-layer insulation shown in figure 1 has several simplifying assumptions. These assumptions are as follows:

1. The insulation is composed of parallel, infinite sheets (radiation shields) which are opaque to radiation.

2. The conductance of the shields is so large that there is no temperature drop across the shield.

3. The radiation between the shields is diffuse, and the surfaces obey Kirchoff's law.

4. The spacer material between the shields is transparent to radiation.

5. The emissivity is the same on both sides of the shield.

III

HEAT TRANSFER BETWEEN PARALLEL, INFINITE PLATES SEPARATED BY A TRANSPARENT CONDUCTING MATERIAL

If two constant temperature planes are separated by a transparent, solid spacing material in a vacuum (10^{-6} mm. mercury), the net heat transfer between the surfaces due to radiation can be written

$$Q_R = \sigma \mathcal{F}_{1,2}(T_1^4 - T_2^4)\, A_{1,2}$$

where

σ = Stefan-Boltzmann constant, 0.1714×10^{-8}

$\mathcal{F}_{1,2}$ = the appropriate exchange factor between surfaces 1 and 2.

T_1 = the absolute temperature of the hotter surface

T_2 = the absolute temperature of the cooler surface.

The solid spacing material is assumed to be transparent to the radiation or of such a small area so that there is no effect on the radiative interchange between the two surfaces. For the case of two parallel, infinite surfaces the exchange factor $\mathcal{F}_{1,2}$ as derived in reference (1) is

$$\mathcal{F}_{1,2} = \frac{1}{1/\epsilon_1 + 1/\epsilon_2 - 1}$$

where

ϵ_1 = the total emissivity of surface 1

ϵ_2 = the total emissivity of surface 2

When only unit areas are considered, the equation for heat transfer by radiation can be written as

$$q_R = Q_R/A = \sigma \mathcal{F}_{1,2}(T_1^4 - T_2^4)$$

The heat transfer between two surfaces due to conduction in the spacer material can be accounted for with a modified form of Fourier's law

$$Q_c = U\,A(T_1 - T_2)$$

or on a unit area basis

$$q_c = U(T_1 - T_2)$$

where

U = an overall thermal conductance for the material between the two surfaces

The value of the overall thermal conductance U includes all of the factors affecting heat conduction in the material such as conductivity of the material, contact resistance, etc. Since the space between two surfaces is in a vacuum (or the pressure $< 10^{-6}$ mm. of mercury) the convective mode of heat transfer will be insignificant. Now an expression for the overall heat transfer between the two surfaces can be written as

$$q = \sigma \mathcal{F}_{1,2}(T_1^4 - T_2^4) + U(T_1 - T_2) \qquad (1)$$

where

q = the net heat transfer between the two surfaces per unit area

If the analysis is extended to an array of surfaces at constant temperature as illustrated in figure 1, there will be n equations similar to equation (1). If the first surface is designated as the external portion of a constant temperature body then there are n shields separated by conducting spacers. The shields are assumed to have conductivities which are virtually infinite with respect to the spacing material, and thus the temperature drop across the shield can be neglected. Since the heat transferred through each pair of shields is the same, the equations for the heat transfer are as follows:

$$q = \sigma \mathcal{F}_{b,1}(T_b^4 - T_{s1}^4) + U_1(T_b - T_{s1})$$

$$q = \sigma \mathcal{F}_{1,2}(T_{s1}^4 - T_{s2}^4) + U_2(T_{s1} - T_{s2})$$

$$q = \sigma \mathcal{F}_{i-1,i}(T_{si-1}^4 - T_{si}^4) + U_i(T_{si-1} - T_{si})$$

$$q = \sigma \mathcal{F}_{i,i+1}(T_{si}^4 - T_{si+1}^4) + U_{i+1}(T_{si} - T_{si+1}) \quad (2)$$

$$q = \sigma \mathcal{F}_{n-1,n}(T_{sn-1}^4 - T_{sn}^4) + U_n(T_{sn-1} - T_{sn})$$

If this array of surfaces is assumed to be in a space environment and a heat balance is applied to the external shield, there is another relation for the heat transfer

$$q = \sigma \epsilon_{sn} T_{sn}^4 - \alpha_{sn} G \quad (2a)$$

where

ϵ_{sn} = the total emissivity of the outermost shield

G = a constant external irradiation

α_{sn} = the absorbtivity of the outermost shield.

At this point it is convenient to separate the analysis into two cases: (1) constant values for the overall thermal conductance U and the emissivities of the surfaces throughout the shielding material, and

(2) different values for conductances and emissivities in the shielding material.

Case 1 constant values of U and ϵ

In this case the system of equations (2) can be rewritten and simplified considerably. With these restrictions equations (2) become

$$q = \sigma \mathcal{F}(T_b^4 - T_{s1}^4) + U(T_b - T_{s1})$$

$$q = \sigma \mathcal{F}(T_{s1}^4 - T_{s2}^4) + U(T_{s1} - T_{s2})$$

$$q = \sigma \mathcal{F}(T_{si-1}^4 - T_{si}^4) + U(T_{si-1} - T_{si})$$

$$q = \sigma \mathcal{F}(T_{si}^4 - T_{si+1}^4) + U(T_{si} - T_{si+1}) \qquad (3)$$

$$q = \sigma \mathcal{F}(T_{sn-1}^4 - T_{sn}^4) + U(T_{sn-1} - T_{sn})$$

If these equations are added together the resulting equation is

$$n\, q = \sigma \mathcal{F}(T_b^4 - T_{sn}^4) + U(T_b - T_{sn}) \qquad (4)$$

If it is further assumed the external surface is not being subjected to an external irradiation then equation (2a) becomes

$$q = \sigma\, \epsilon\, T_{sn}^4 \qquad (5)$$

Now combining equations (4) and (5) to eliminate q the final equation is

$$n\, \sigma\, \epsilon\, T_{sn}^4 = \sigma \mathcal{F}(T_b^4 - T_{sn}^4) + U(T_b - T_{sn}) \qquad (6)$$

If a non-dimensional temperature ratio θ_n is defined as $\theta_n = T_{sn}/T_b$ and also a non-dimensional quantity γ is defined as

$$\gamma = \frac{U}{\sigma\, \epsilon\, T_b^3}$$

and $\mathcal{F}$ is evaluated for this case

$$\mathcal{F} = \frac{1}{1/\epsilon + 1/\epsilon - 1} = \frac{\epsilon}{2 - \epsilon}$$

equation (6) becomes, with rearrangement,

$$\theta_n^4 + \frac{\gamma(2 - \epsilon)}{n(2 - \epsilon) + 1}\theta_n - \frac{\gamma(2 - \epsilon) + 1}{n(2 - \epsilon) + 1} = 0 \qquad (7)$$

Appendix A describes a computer program which was written to solve the above equation for θ_n as a function of n, the number of shields, with γ and ϵ as input data. It will be noted that in this instance the temperature of the body is not specified. Since γ is a function of the body temperature, this temperature must be known to use the results for a specific case. In the Numerical Results section θ_n is plotted versus the number of shields for several values of γ and ϵ.

If external radiation exists, equation (5) becomes

$$q = \sigma \epsilon T_{sn} - \alpha_{sn} G \qquad (8)$$

Using the same method as above, the equation for θ_n which corresponds to equation (7) becomes

$$\theta_n^4 + \frac{\gamma(2 - \epsilon)}{n(2 - \epsilon) + 1}\theta_n - \frac{\gamma(2 - \epsilon) + 1 + \dfrac{n(2-\epsilon)\alpha_{sn} G}{\sigma \epsilon T_b^4}}{n(2 - \epsilon) + 1} = 0 \qquad (9)$$

Again the computer program given in appendix A can be used to solve for θ_n as the function of n, but in this case values of U, ϵ, T_b, α_{sn} and G must be specified. The numerical results give θ_n for various values of U, α_{sn} and ϵ where T_b is assumed to be 530° R and G is assumed to be 443 BTU/hr/ft^2 (the intensity of solar radiation in the region of the earth's orbit).

Case II Variable Values for U and ϵ

In this case the system of equations (2) contains n equations with n values for U and n values for $\mathcal{F}$. For the first analysis the value of G in equation (2a) is assumed to be zero so that it becomes

$$q = \sigma \epsilon_{sn} T_{sn}^4 \tag{10}$$

Assuming that a value of U and $\mathcal{F}$ for each equation is specified, the system of equations can be solved by proceeding from the outermost shield and working inward. First, divide every equation in (2) by the fourth power of the temperature of the outermost shield T_{sn}^4:
If $\theta_i' = T_{si}/T_{sn}$ and $\gamma' = \dfrac{U_i}{\sigma \epsilon_{sn} T_{sn}^3}$, this division gives:

$$\frac{q}{T_{sn}^4} = \sigma \mathcal{F}_{b,1} \theta_b'^4 - \sigma \mathcal{F}_{b,1} \theta_1'^4 + \gamma_1' \sigma \epsilon_{sn} \theta_b' - \gamma_1' \sigma \epsilon_{sn} \theta_1'$$

$$\frac{q}{T_{sn}^4} = \sigma \mathcal{F}_{1;2} \theta_1'^4 - \sigma \mathcal{F}_{1,2} \theta_2'^4 + \gamma_2' \sigma \epsilon_{sn} \theta_1' - \gamma_2' \sigma \epsilon_{sn} \theta_2'$$

$$\frac{q}{T_{sn}^4} = \sigma \mathcal{F}_{i-1,i} \theta_{i-1}'^4 - \sigma \mathcal{F}_{i-1,i} \theta_i'^4 + \gamma_i' \sigma \epsilon_{sn} \theta_{i-1}' - \gamma_i \sigma \epsilon_{sn} \theta_i' \tag{11}$$

$$\frac{q}{T_{sn}^4} = \sigma \mathcal{F}_{n-1,n} \theta_{n-1}'^4 - \sigma \mathcal{F}_{n-1,n} \theta_n'^4 + \gamma_n \sigma \epsilon_{sn} \theta_{n-1}' - \gamma_n' \sigma \epsilon_{sn} \theta_n'$$

Equation (10) becomes

$$\frac{q}{T_{sn}^4} = \sigma \epsilon_{sn} \tag{12}$$

Substitution of equation (12) into the last equation of equations (11) gives

$$\sigma\, \epsilon_{sn} = \sigma\, \mathcal{F}_{n-1,n}\theta'^{4}_{n-1} - \sigma\, \mathcal{F}_{n-1,n}\theta'^{4}_{n} + \gamma'_n\, \sigma\, \epsilon_{sn}\theta'_{n-1} - \gamma'_n\, \sigma\, \epsilon_{sn}\theta'_n \qquad (13)$$

Rearranging equation (13), and noting that $\theta'_n = T_{sn}/T_{sn} = 1$, gives

$$\theta'^{4}_{n-1} + \frac{\gamma'_n\, \epsilon_{sn}}{\mathcal{F}_{n-1,n}}\theta'_{n-1} - [1 + \frac{\gamma'_n\, \epsilon_{sn}}{\mathcal{F}_{n-1,n}} + \frac{\epsilon_{sn}}{\mathcal{F}_{n-1,n}}] = 0$$

In order to get a general equation, equation (12) can be substituted into the intermediate equation in equations (11) and the result is

$$\theta'^{4}_{i-1} + \frac{\gamma'_i\, \epsilon_{sn}}{\mathcal{F}_{i-1,i}}\theta'_{i-1} - [\theta'^{4}_{i} + \frac{\gamma'_i\, \epsilon_{sn}}{\mathcal{F}_{i-1,i}}\theta'_i + \frac{\epsilon_{sn}}{\mathcal{F}_{i-1,i}}] = 0$$

The system of equations (11) can now be written in the form of a single equation as follows:

$$\theta'^{4}_{i-1} + \frac{\gamma'_i\, \epsilon_{sn}}{\mathcal{F}_{i-1,i}}\theta'_{i-1} - [\theta'^{4}_{i} + \frac{\gamma'_i\, \epsilon_{sn}}{\mathcal{F}_{i-1,i}}\theta'_i + \frac{\epsilon_{sn}}{\mathcal{F}_{i-1,i}}] = 0 \qquad (14)$$

where

$\theta'_n = 1$

and i is an integer, $1 \leq i \leq n$

and $\mathcal{F}_{i-1,i} = \dfrac{1}{1/\epsilon_{i-1} + 1/\epsilon_i - 1}$

It is important to note that γ'_i is actually an unknown now since it is a function of T_{sn}, so the values of γ'_i can be specified only in a relative sense. Since this is the case, the system of equations (14) must be solved by an iterative technique to a desired accuracy. Appendix B describes a computer program capable of giving values of θ_n with specified values of n, U_i's, ϵ_i's and T_b. The program output gives θ_i for each shield (note: this is $\theta_i = T_{si}/T_b$ not

$\theta_i' = T_{si}/T_{sn}$) so that a temperature profile for the material can be constructed from the results. This program can be used in the constant U and ϵ case if a temperature profile is desired.

If external radiation is included, equation (2a) becomes

$$q = \sigma \, \epsilon_{sn} \, T_{sn}^4 - \alpha_{sn} \, G \tag{15}$$

Equations (11) are still correct for this case. Now combining equation (15) with an intermediate equation in equations (11) gives the final result.

$$\theta_{i-1}'^4 + \frac{\gamma_i' \, \epsilon_{sn}}{\mathcal{F}_{i-1,i}} \theta_{i-1}' - [\theta_i'^4 + \frac{\gamma_i' \, \epsilon_{sn}}{\mathcal{F}_{i-1,i}} \theta_i' + \frac{\epsilon_{sn}}{\mathcal{F}_{i-1,i}} - \frac{\alpha_{sn} \, G}{\sigma \, T_{sn}^4 \, \mathcal{F}_{i-1,i}}] = 0$$

where

i = an integer, $1 \leq i \leq n$

$\theta_n' = 1.0$

$$\mathcal{F}_{i-1,i} = \frac{1}{1/\epsilon_{i-1} + 1/\epsilon_i - 1}$$

This equation can be solved in the same manner as above with the computer program given in appendix B when values of α_{sn} and G are specified in the input data.

IV

NUMERICAL RESULTS

It should be noted that throughout this analysis there has been no attempt to derive a theoretical method to evaluate U for any geometry. As a result U for a specific case must be obtained from experimental data. There is some experimental data from which values of U can be obtained in references (3) and (5). The equations in this analysis can be used to extend results of this experimental work to numerous cases. With sufficient experimental data to fix a value for U with specified geometry and materials, the computer programs can be used to determine the necessary number of shields to obtain a required temperature drop. Even without known values for U the relative merit of different configurations can be compared.

Equation (7) provides the basis for calculating the temperature drop across a configuration of radiation shields which have constant values of conductance and emissivity and when no external irradiation is present. To compare the effects of the parameters γ and ϵ, θ_n is plotted versus n in figures 2. In these figures γ = 10.0, 1.0, 0.1 and ϵ = 0.05. The quantity γ indicates the relative importance of conductive to radiative heat transfer so the above values cover a wide range of conditions. Experimental data gives the value of the emissivity of aluminized mylar (a common material used in this type of insultation) as equal to 0.05. Also plotted in this graph for comparison is the theoretical limiting value of θ_n which represents a minimum value for any given value of n and ϵ. The derivation of this function is given in appendix C. As can be seen from the curves in figure 2, for any given set of parameters (γ and ϵ) the temperature drop per shield is greater for a smaller number of shields. To show this effect more clearly two other functions are plotted versus n in figures 3 and 4. The first function ($\theta_n^{-4}/[n(2-\epsilon) + 1]$) gives the ratio of heat transferred by a conductionless shield array to that transferred by the shield under consideration. The derivation of θ_n^4 for a conductionless shield is given in appendix C. The second function ($\theta_{n-1}^4/\theta_n^4$) is the ratio of the heat transferred by a given number of shields to the heat transferred by a configuration with one additional shield. The limit for this function is 1.0. Both of these functions indicate that the addition of a shield has a larger effect on decreasing the heat transfer when the total number of shields is small. The function $\theta_{n-1}^4/\theta_n^4$ becomes very flat in all cases for n greater than 15. In figure 5 the function θ_n^4 is plotted versus n for the previously mentioned values of γ and ϵ to present the relative decrease in heat transfer for different numbers of shields.

For the case of variable values of U and ϵ, solved with the computer program listed in appendix B, several specific cases were considered. The first situation considered was the effect of the order of a spacing material with various values of conductance. In case (I) the conductance of the spacing material nearest the body surface is the smallest and largest in the last shield, and case (II) uses the same values of conductance but in reverse order. Figure 6 is a plot of the two temperature profiles. The values used in this comparison are $n = 6$, $\epsilon = 0.05$ and $\gamma_i = 1.0, 0.9, 0.8, 0.7, 0.6$ and 0.5. This plot indicates that there is a slight difference in the overall θ_n and the lowest value of θ_n is achieved in case (I) where the largest value of conductance is at the body surface. The next situation considered is the decrease in θ_n with a decrease in conductance for some of the spacing material. Figure 7 is a plot of the temperature profile for six cases. The values used were $n = 5$, $\epsilon = 0.05$ and the various values of γ_i are listed on the graph.

For the case of external irradiation the results of the program in appendix A are plotted in figure 8. For these curves G was assumed to be equal to 443 BTU/hr and the body temperature 530° R. The value of the Stefan-Boltzmann constant σ is taken to be $.1714 \times 10^{-8}$ BTU/hr - ft^2 - °R^4. It should be noted that in special cases an equilibrium can be attained between the heat flux from the external irradiation and from the body. This possibility can be shown by setting q equal to zero in equation (17):

$$q = 0 = \sigma\, \epsilon_{sn}\, T_{sn}^4 - \alpha_{sn}\, G \tag{17}$$

It is clear that if the body temperature and the external shield temperature are the same then the net heat transfer will be zero. Setting $T_b = T_{sn}$ and rearranging equation (17) becomes

$$\left.\frac{\alpha}{\epsilon}\right|_{sn} = \frac{\sigma\, T_b^4}{G} \tag{18}$$

So for specific values of T_b and G a corresponding value of $\left.\frac{\alpha}{\epsilon}\right|_{sn}$ can be calculated which will give $q = 0$, and this condition is independent of the number of shields. If T_b = 530°R and G = 443 BTU/hr - ft, $\frac{\alpha}{\epsilon}\Big|_{sn}$ = 0.309 for equilibrium. This is a special condition since G must be fixed at a constant value, and this is a function of orientation of the surface, which in general would vary. This consideration is important because the requirements for the insulation in the case of external irradiation are virtually the opposite of the requirements in the zero external irradiation case. As can be seen in figure 8 for all values of α_{sn} the larger the number of shields added, the larger the net heat transfer. For the case of zero external irradiation in figure 2 the larger the number of shields, the smaller the net heat transfer. This difference is because in the external irradiation case the optimum value of θ_n $(q = 0)$ is $\theta_n = 1.0$ and in the zero external irradiation case the optimum value $(q = 0)$ is $\theta_n = 0.0$. Due to this difference in design criteria and the likelihood of encountering both conditions the best approach would be to design the insulation for no external irradiation then attempt to provide the external shield surface with an $\frac{\alpha}{\epsilon}\Big|_{sn}$ ratio which will give the best results over a range of values for effective external irradiation. To aid in finding a value of $\frac{\alpha}{\epsilon}\Big|_{sn}$ which is the best, figure 9 is a plot of q versus the angle of orientation Φ (where Φ is the angle between the incident radiation and the surface) for various values of $\frac{\alpha}{\epsilon}\Big|_{sn}$. For this plot, the values used were $\epsilon = 0.05$, $\gamma = 1.0$, $n = 10$, $\frac{\alpha}{\epsilon}\Big|_{sn}$ = 0.309, 0.437, 0.60, 0.8, 1.0, and θ varied from 0° to 90°. The value of $\frac{\alpha}{\epsilon}\Big|_{sn}$ desired would depend on the method in which the temperature of the body is maintained. If $\frac{\alpha}{\epsilon}\Big|_{sn}$ = 0.309 as calculated through equation (18) (case (I) in figure 9), the net heat transfer is the lowest for any condition requiring only heat input to maintain the body temperature. Other values of $\frac{\alpha}{\epsilon}\Big|_{sn}$ might be more appropriate depending on the environment expected.

Since the heat transfer is very sensitive to the $\frac{\alpha}{\epsilon}\Big|_{sn}$ ratio of the outermost shield in the case of external irradiation, the case of variable U and ϵ was not investigated thoroughly. In figures 6 and 7 the effect of variable U and ϵ is significant but in general the $\frac{\alpha}{\epsilon}\Big|_{sn}$ ratio will predominate in determining the heat transfer with external irradiation. Figure 10 is a plot of θ_n vs. n for some of the arrays used in figure 7 except with external irradiation. The variation in θ_n in figure 10 is not nearly as great as in the case of zero external irradiation (figure 7).

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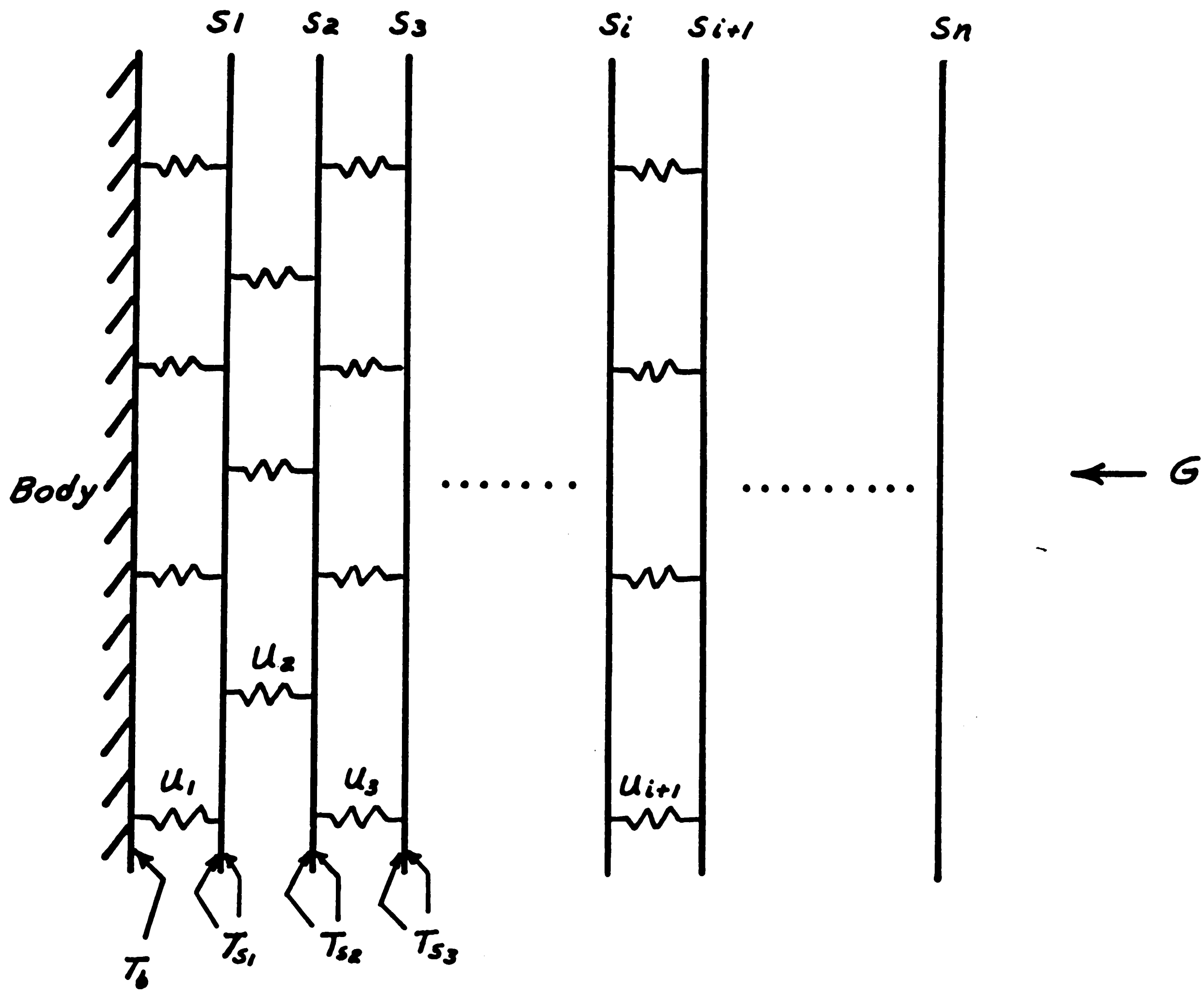
FIGURE 1



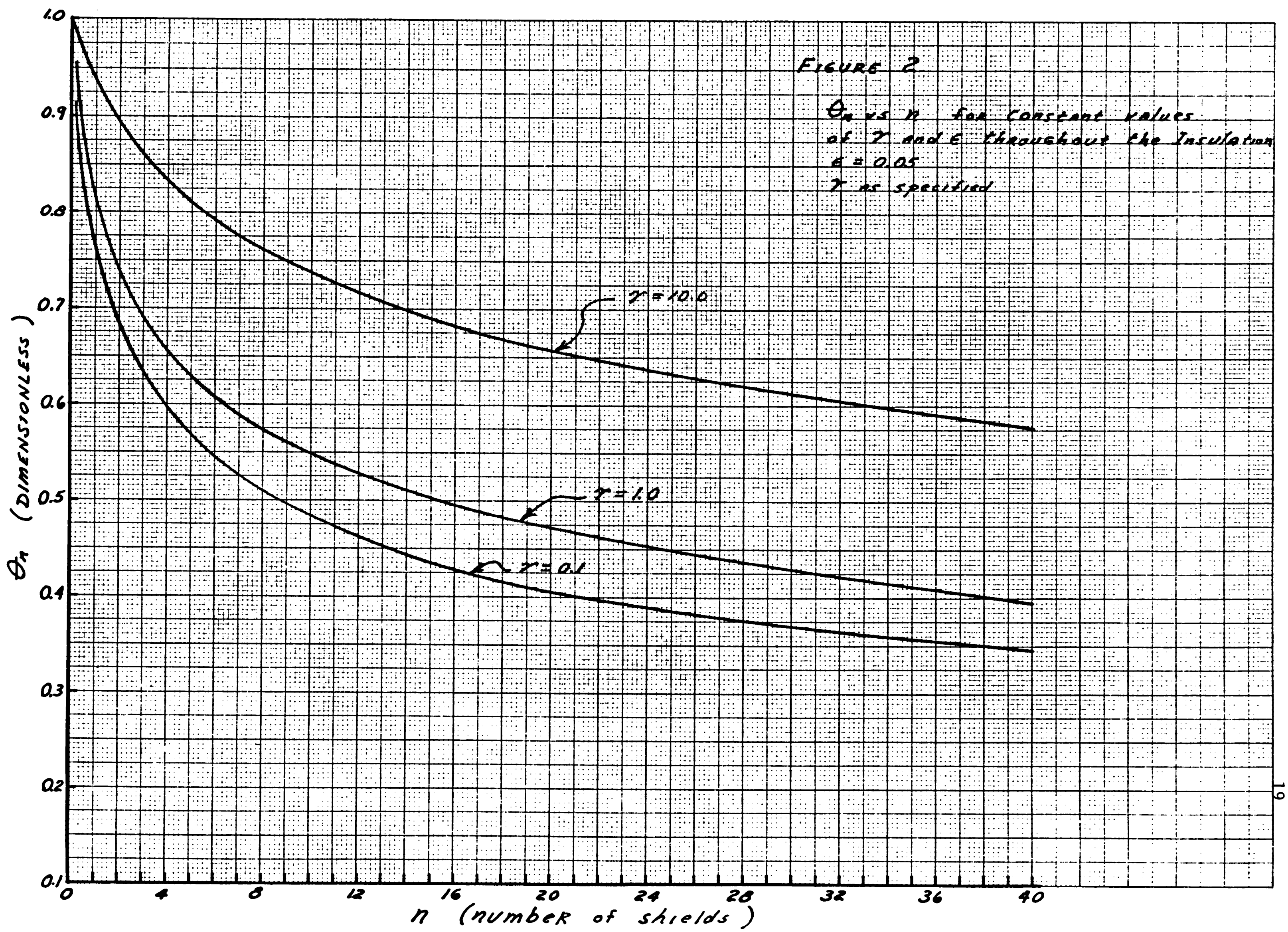


FIGURE 2

θ_n vs n for constant values of γ and ϵ throughout the Insulation
ϵ = 0.05
γ as specified

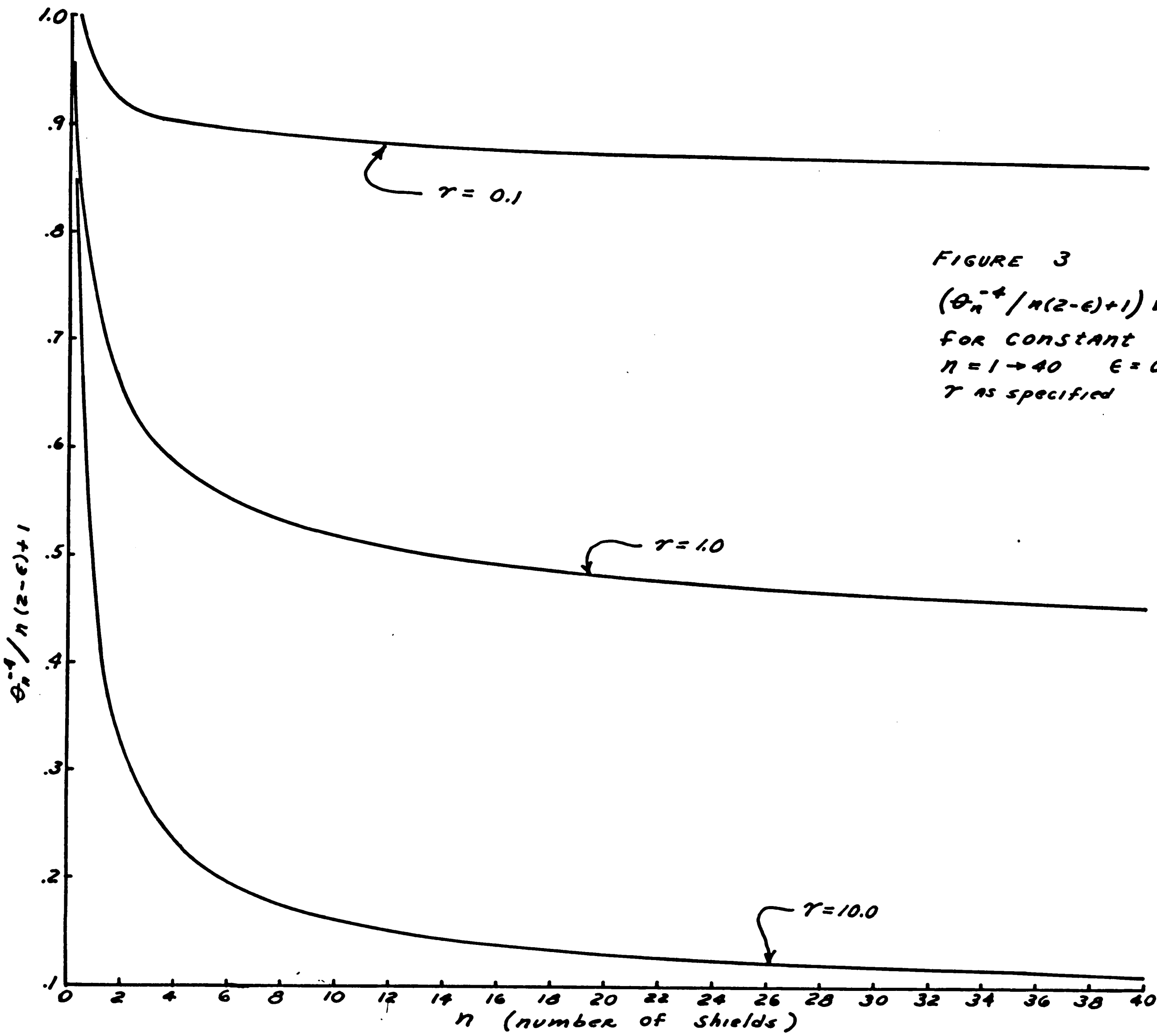


FIGURE 3

$(\theta_n^{-4}/n(2-\epsilon)+1)$ vs. n

for constant U and ϵ

$n = 1 \rightarrow 40$ $\epsilon = 0.05$

τ as specified

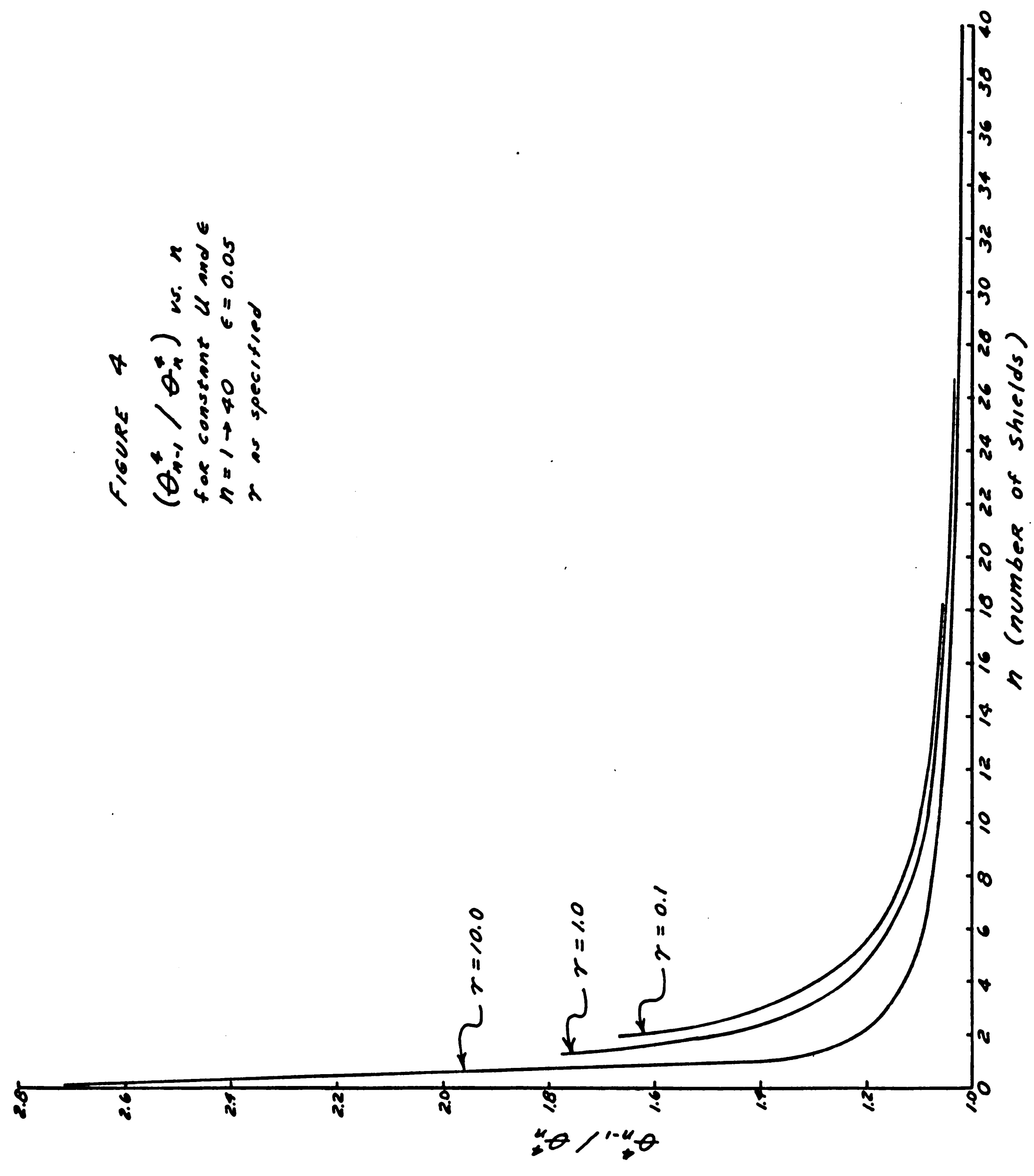


FIGURE 4

$(\phi^4_{n-1} / \phi^4_n)$ vs. n
for constant U and ϵ
$n = 1 \rightarrow 40$ $\quad \epsilon = 0.05$
r as specified

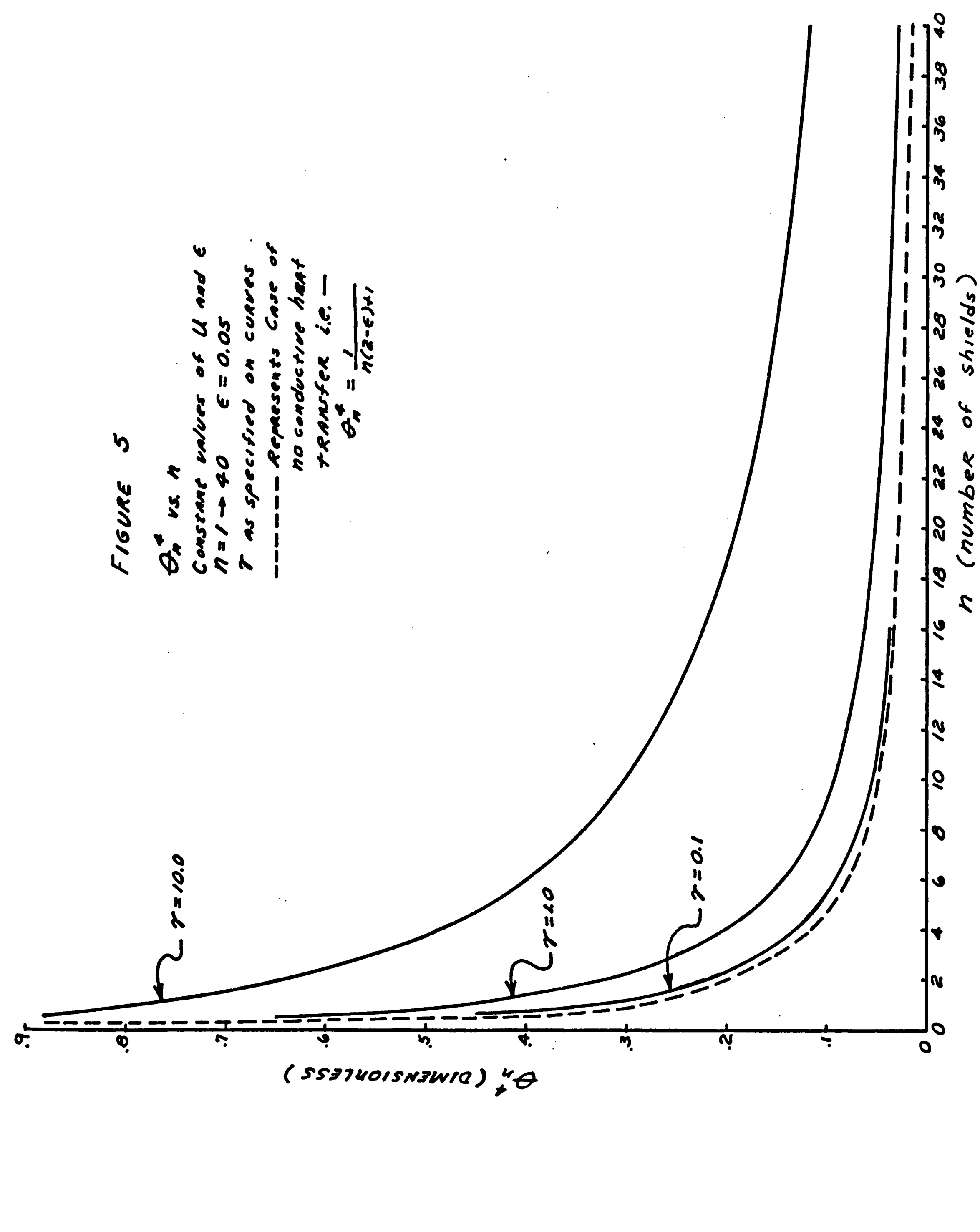


FIGURE 5
θ_n^4 vs. n
Constant values of U and ε
$n = 1 \rightarrow 40$ $\epsilon = 0.05$
γ as specified on curves
------ Represents case of no conductive heat transfer i.e. — $\theta_n^4 = \frac{1}{n(2-\epsilon)+1}$
γ=10.0
γ=1.0
γ=0.1
θ_n^4 (DIMENSIONLESS)
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
n (number of shields)

FIGURE 6

θ_i vs Shield number for two arrays of r_i where $G = 0$

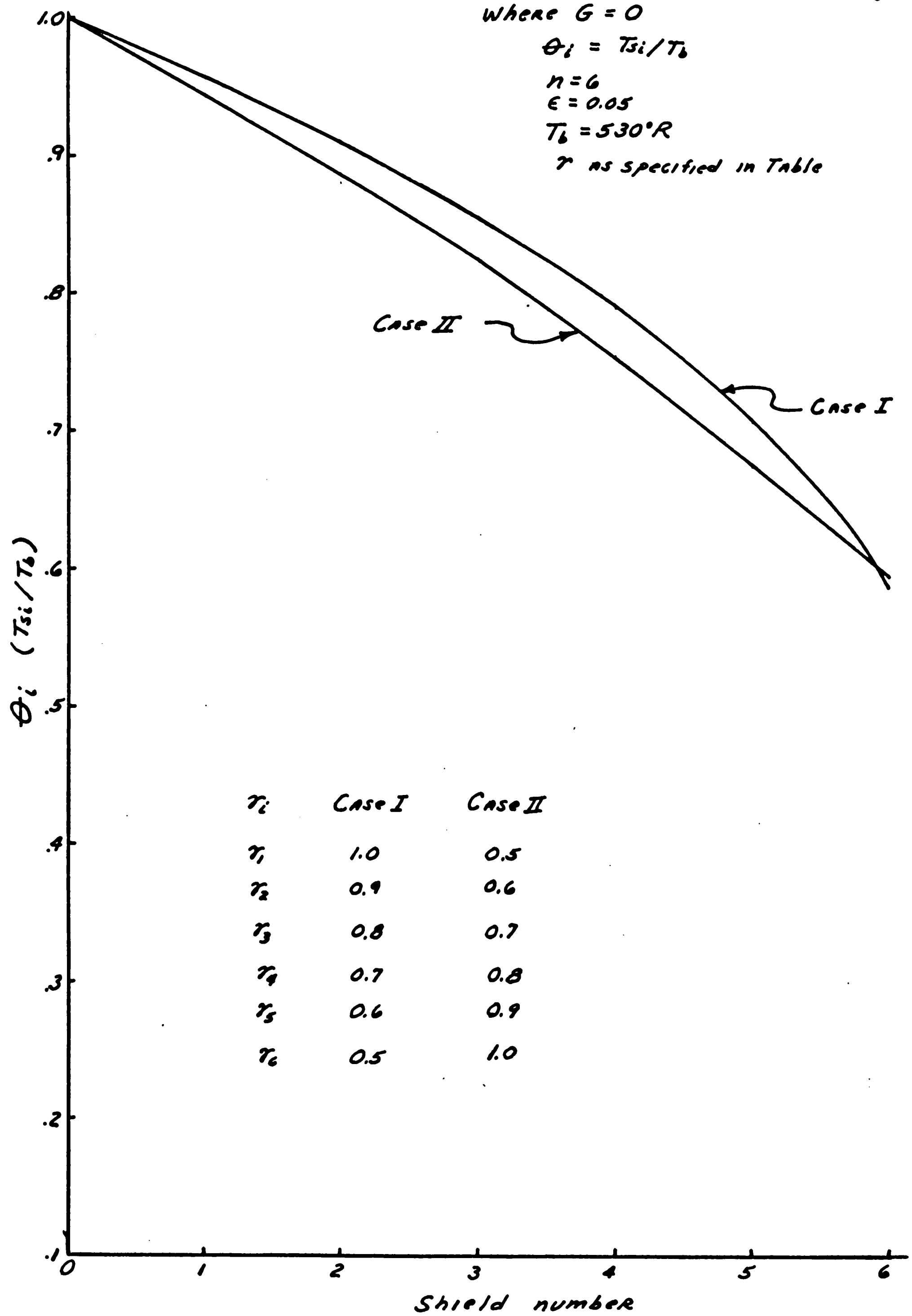
$\theta_i = T_{si}/T_b$

$n = 6$

$\epsilon = 0.05$

$T_b = 530°R$

r as specified in Table



r_i	Case I	Case II
r_1	1.0	0.5
r_2	0.9	0.6
r_3	0.8	0.7
r_4	0.7	0.8
r_5	0.6	0.9
r_6	0.5	1.0

FIGURE 7

θ_i vs. Shield number for six Cases for γ_i

Where $G = 0$

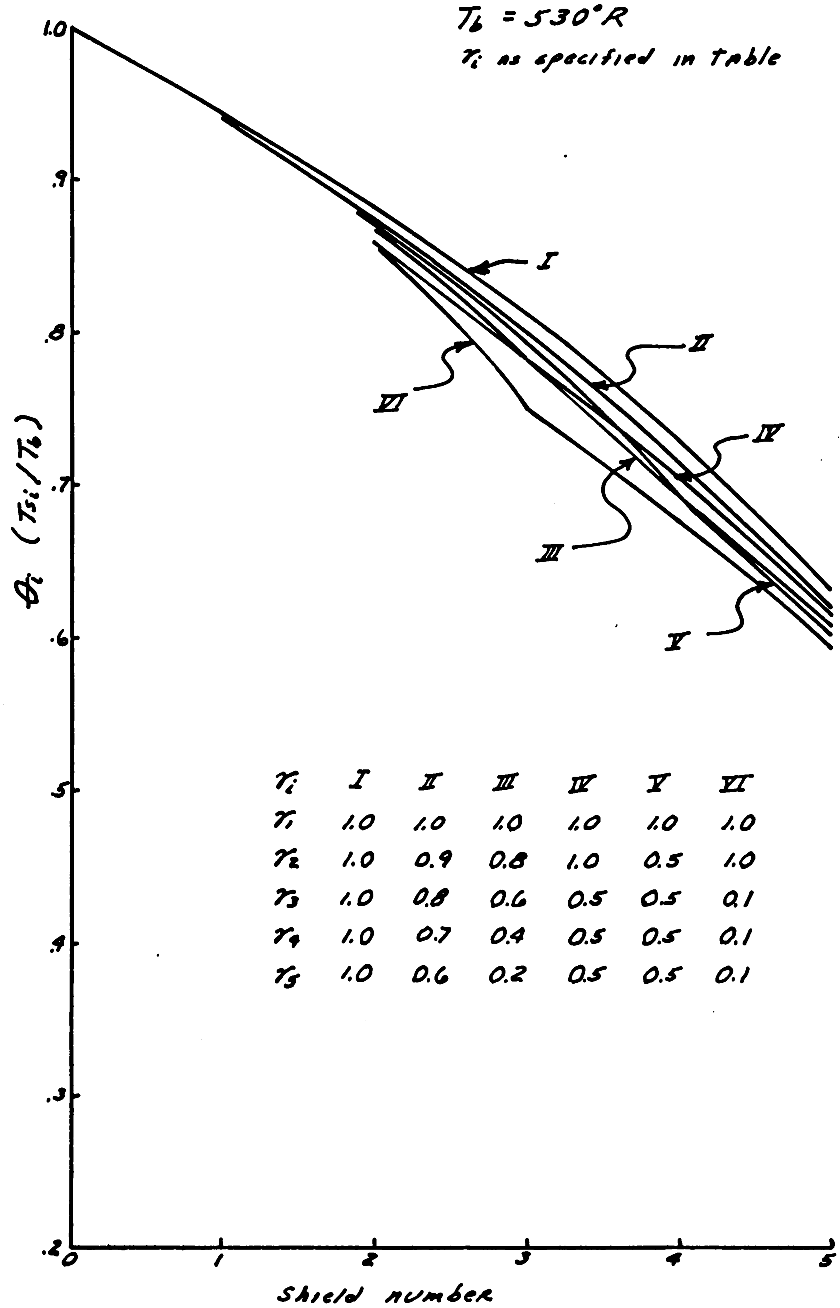
$$\theta_i = T_{si}/T_b$$

$n = 5$

$\epsilon = 0.05$

$T_b = 530°R$

γ_i as specified in table



γ_i	I	II	III	IV	V	VI
γ_1	1.0	1.0	1.0	1.0	1.0	1.0
γ_2	1.0	0.9	0.8	1.0	0.5	1.0
γ_3	1.0	0.8	0.6	0.5	0.5	0.1
γ_4	1.0	0.7	0.4	0.5	0.5	0.1
γ_5	1.0	0.6	0.2	0.5	0.5	0.1

FIGURE 8

θ_n vs n for constant U and ϵ
with EXTERNAL IRRADIATION
$T_b = 530°R$ $\epsilon = 0.05$ $G = 443$ BTU/hr.-ft^2
α_{sn} and τ as specified in table

Curve	α_{sn}	τ
1	0.05	1.0
2	0.01	1.0
3	0.05	10.0
4	0.01	10.0
5	0.05	0.1
6	0.01	0.1

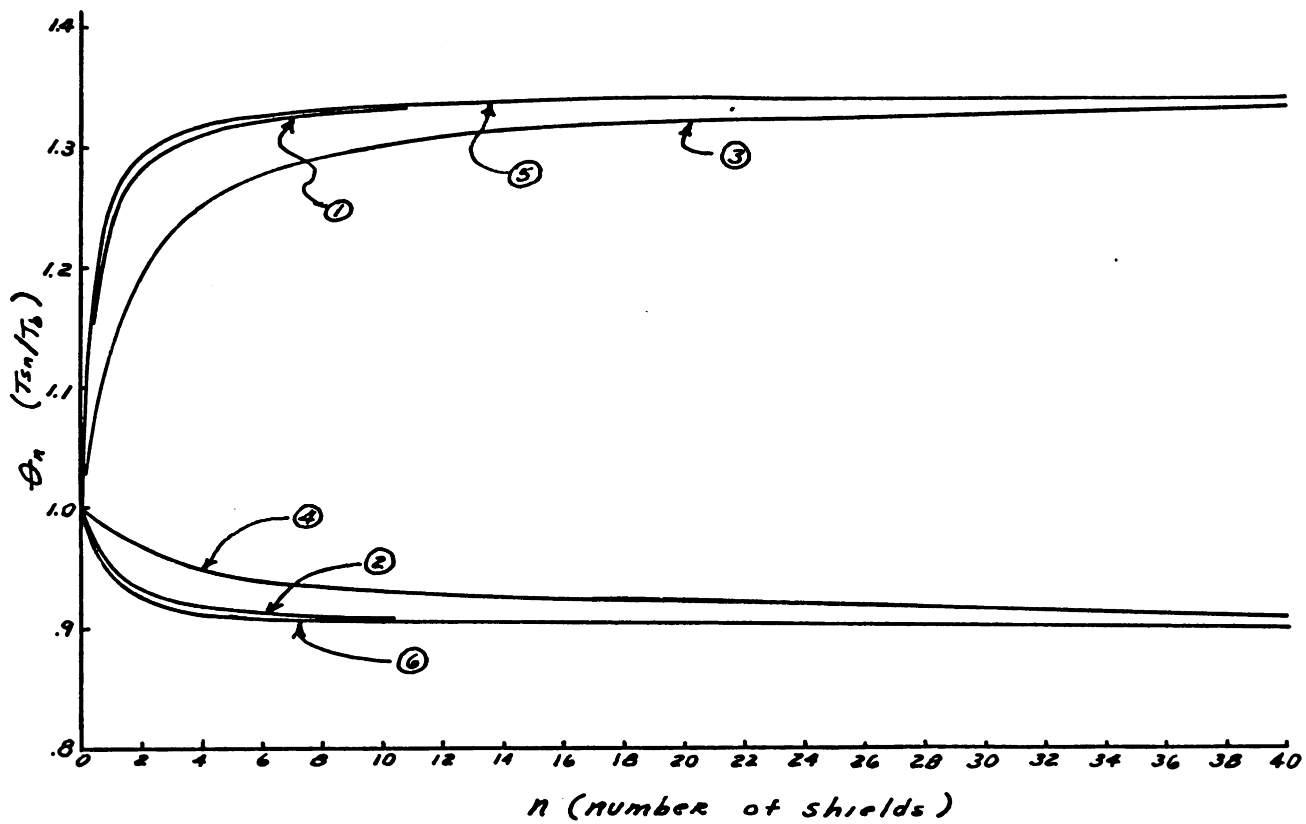


FIGURE 9

Q vs ϕ

Where

Q is the net heat transfer between the external surface and the environment

ϕ angle between incident irradiation and the surface

$n = 10$, $\epsilon = 0.05$, $U = 0.01276$

$T_b = 530°R$, $G = 443$ $BTU/hr\text{-}ft^2$

$\alpha/\epsilon|_{sn}$ as specified in table

Case	$\alpha/\epsilon\|_{sn}$
I	0.309
II	0.437
III	0.60
IV	0.80
V	1.00

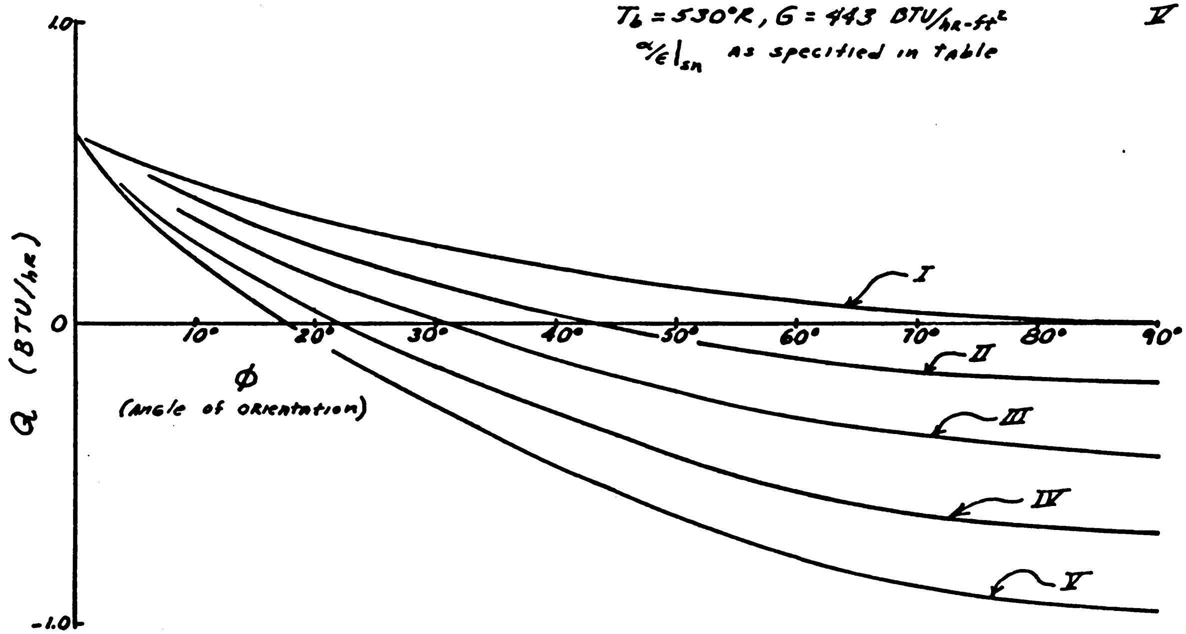


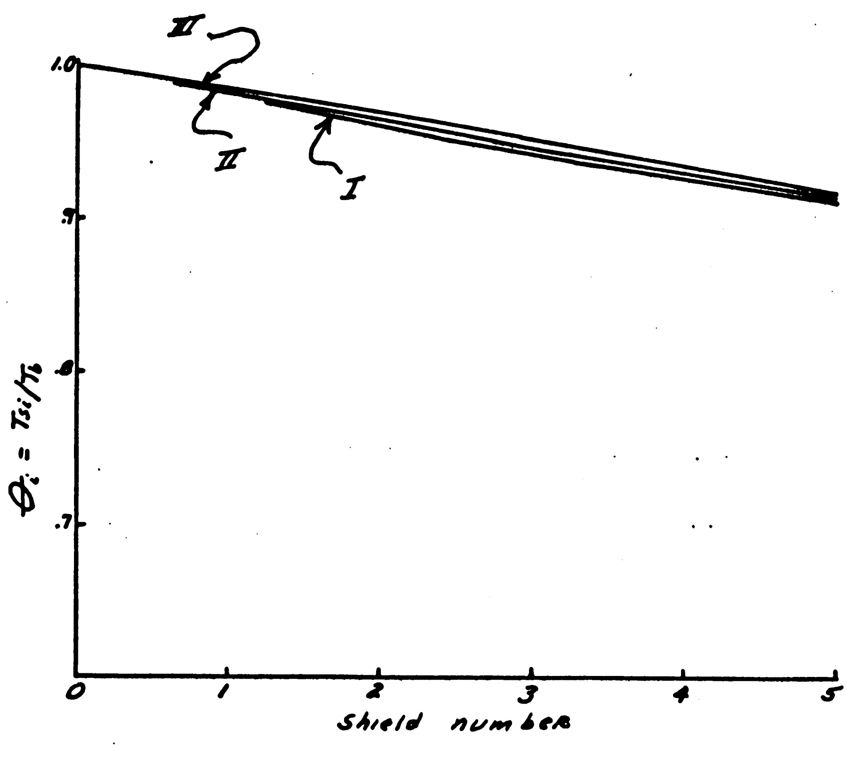
FIGURE 10

θ_i vs shield number for three arrays of γ_i with external irradiation

$n = 5$, $\epsilon = 0.05$, $\alpha = 0.01$ $G = 443$ BTU/hr-ft²

$T_b = 530°R$

γ_i	I	II	III
γ_1	0.1	0.5	1.0
γ_2	0.1	0.5	1.0
γ_3	0.1	0.5	1.0
γ_4	1.0	1.0	1.0
γ_5	1.0	1.0	1.0



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APPENDIX A

Solution for θ_n by computer program with input data for n, U, and ϵ in the case of constant U and ϵ where the equation for θ_n is

$$\theta_n^4 + \frac{\gamma(2 - \epsilon)}{n(2 - \epsilon) + 1}\theta_n - \frac{\gamma(2 - \epsilon) + 1 + \frac{\alpha G}{\sigma \epsilon T_b^4}}{n(2 - \epsilon) + 1} = 0 \qquad \text{(A-1)}$$

can be solved by Newton's method. Equation (A-1) is of the form

$$\theta_n^4 + B\,\theta_n - C = 0$$

where A and B are constants. Let D equal

$$D = \theta_n^4 + B\,\theta_n - C$$

The derivative of D is

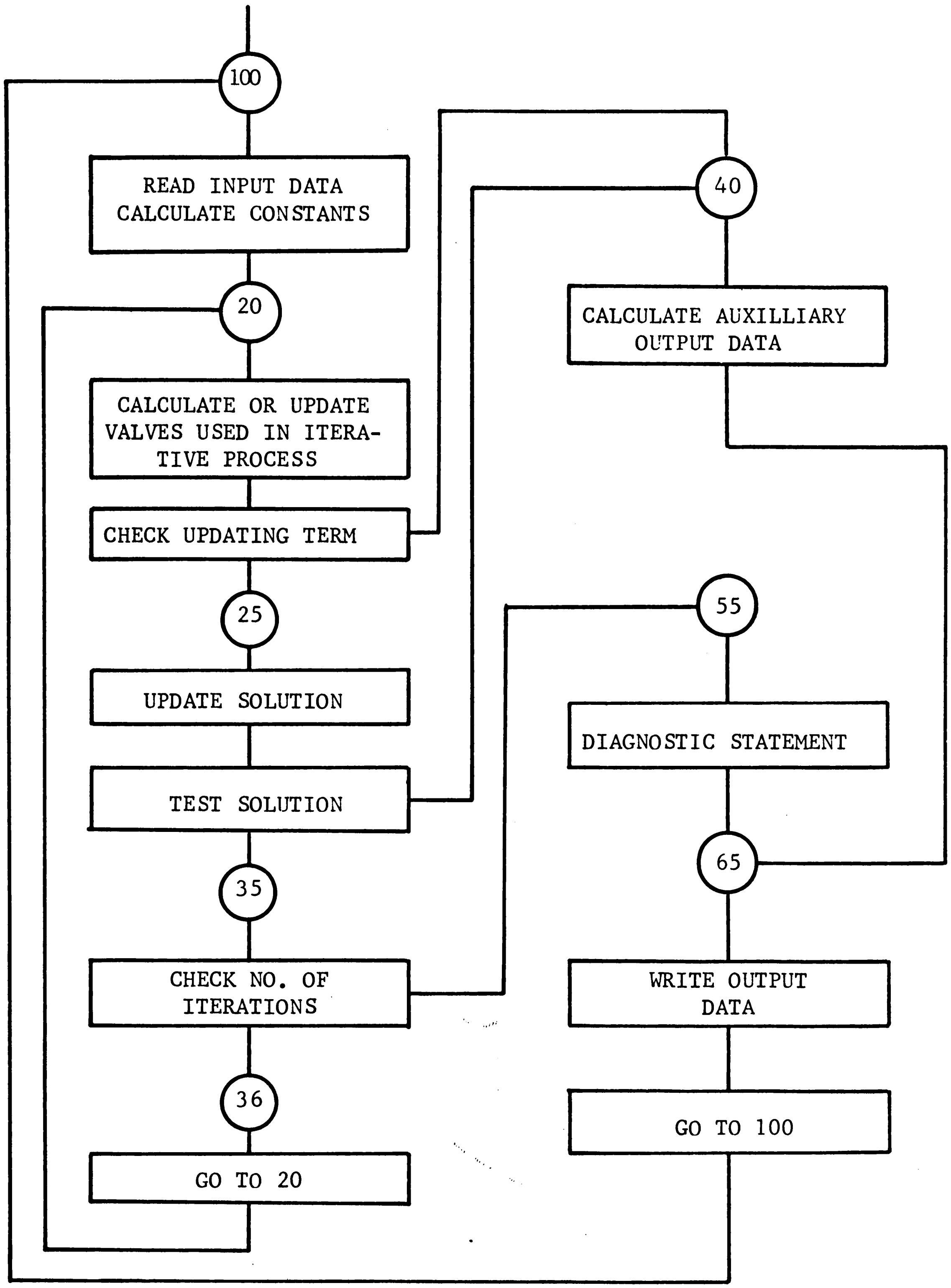
$$\frac{dD}{d\theta_n} = 4\theta_n^3 + B$$

An initial value for θ_n is assumed ($\theta_n = 1.0$) and a new value of θ_n is calculated by the formula

$$\theta_n(\text{new}) = \theta_n(\text{previous}) - D/\frac{dD}{d\theta_n}$$

This process is repeated until the new value of θ_n differs from the old value of θ_n by a specified amount (.00001 was used as the criterion in this work). In this program values of θ_n can be calculated without input for the body temperature, but in this case the values for U are assumed to be the values of gamma. So to use the program in this manner the input data for the body temperature should be left blank and values of gamma should be placed in the U input. To use the program with external radiation a value of body temperature must be specified. A flow diagram for the computer program used to solve the case of constant U and ϵ is shown in figure A-1. The program stops when the machine runs out of input data cards.

Figure A-1: COMPUTER PROGRAM FLOW DIAGRAM FOR CASE OF CONSTANT U AND ε



APPENDIX B

The solution of the equations for θ_n in the case of variable conductance U_i and emissivity ϵ_i can also be accomplished by Newton's method. The equations are of the form

$$\theta'^4_{i-1} + \frac{\gamma'_i \epsilon_{sn} \theta'_{i-1}}{\mathcal{F}_{i-1,i}} - \theta'^4_i + \frac{\gamma'_i \epsilon_{sn} \theta'_i}{\mathcal{F}_{i-1,i}} + \frac{\epsilon_{sn}}{\mathcal{F}_{i-1,i}} - \frac{\alpha G}{\sigma T^4_{sn} \mathcal{F}_{i-1,i}} = 0 \qquad \text{(B-1)}$$

where

i is an integer $1 \leq i \leq n$

$\theta'_n = 1.0$

$$\mathcal{F}_{i-1,i} = \frac{1}{1/\epsilon_{i-1} + 1/\epsilon_i - 1}$$

The program starts with the outermost shield (the n^{th}), and equation (B-1) becomes

$$\theta'^4_{n-1} + \frac{\gamma'_n \epsilon_{one}}{\mathcal{F}_{n-1,n}} \theta'_{n-1} - 1.0 + \frac{\gamma'_n \epsilon_{one}}{\mathcal{F}_{n-1,n}} + \frac{\epsilon_{one}}{\mathcal{F}_{n-1,n}} - \frac{\alpha G}{\sigma T_{sn} \mathcal{F}_{n-1,n}} = 0 \qquad \text{(B-2)}$$

Initially the body temperature is used in place of T_{sn} to calculate γ' and the external irradiation term $(\alpha G/\sigma T^4_{sn} \mathcal{F}_{n-1,n})$ since T_{sn} is the unknown. The method used to solve equation (B-2) is the same as listed in appendix A. The value calculated for θ_{n-1} is then used to calculate θ_{n-2} by equation (B-1). After n equations have been solved the θ'_b calculated is the overall temperature ratio (θ'_b = T_b/T_{sn}). This value of θ'_b is used to "update" γ' and other terms involving T_{sn}, and the set of equations (B-1) is solved again with the new values. This process is repeated until the value of θ_b' differs from the previous value of θ_b' by a specified amount. Then the program takes the inverse of the value for overall theta ($1/\theta_b' = \theta_n = T_{sn}/T_b$) and prints it out.

When the values are "updated", a weighting function is used to calculate the new overall theta (θ_b') used for this purpose. The method is as follows

$$\theta_i' = W_t \, \theta_{i-2}' + (1 - W_t) \, \theta_{i-1}' \qquad \text{(B-3)}$$

where

W_t = the weight function.

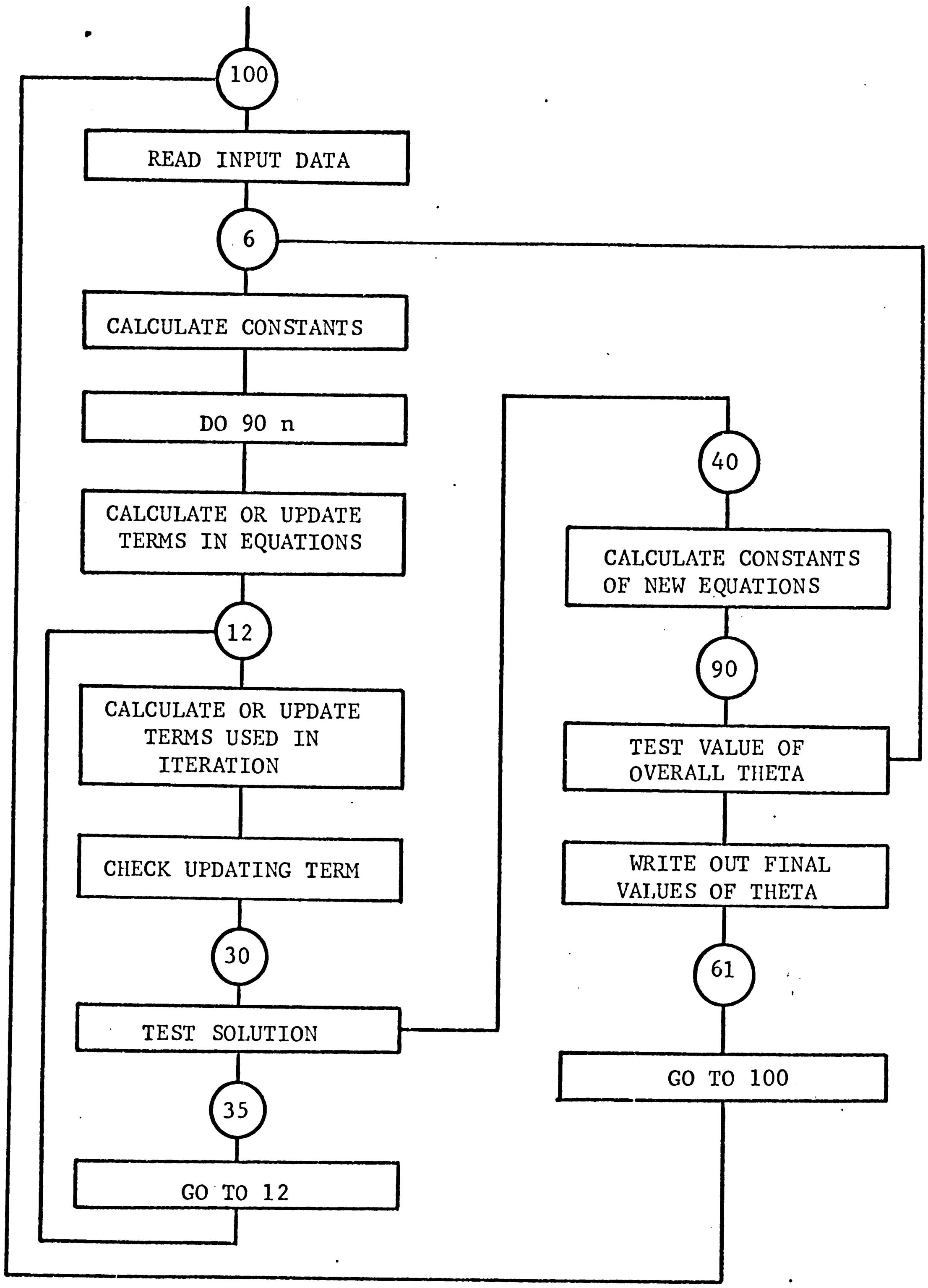
i = number of iterations

If $G = 0$ (there is no external irradiation), a value need not be specified for W_t, but if $G \neq 0$, W_t should be close to 1.0 (usually 0.9 is sufficient).

A flow diagram of this program is shown in figure B-1. The program stops when all of input data cards have been processed.

Figure B-1: COMPUTER PROGRAM FLOW DIAGRAM

FOR CASE OF VARIABLE U AND ε



APPENDIX C

The ratio of θ_n^4 of n shields under consideration to θ_{nc}^4 of a conductionless array of n shields is a measure of the quality of shielding material. θ_{nc}^4 is defined here as the fourth power of the temperature ratio when no conduction is present. With no conduction, equation (1) becomes

$$q = \sigma \mathcal{F}(T_b^4 - T_{sn}^4)$$
$$q = \sigma \mathcal{F}(T_{sn-1}^4 - T_{sn}^4) \qquad \text{(C-1)}$$

and equation (2a) is

$$q = \sigma \epsilon T_{sn}^4 \qquad \text{(C-2)}$$

Adding equations (C-1) and eliminating q with equation (C-2) the result is

$$n \epsilon T_{sn}^4 = \mathcal{F}(T_b^4 - T_{sn}^4)$$

and letting $\theta_{nc} = T_{sn}/T_b$, when conductive heat transfer is absent, gives

$$\theta_{nc}^4 (n + \mathcal{F}/\epsilon) = \mathcal{F}/\epsilon$$

or

$$\theta_{nc}^4 = \frac{1}{n(2 - \epsilon) + 1} \qquad \text{(C-3)}$$

FORTRAN SOURCE LIST
Program A

```
ISN          SOURCE STATEMENT

 0 $IBFTC KADIHO
 1     100 READ(5,5) TN,UEFF,EPSILN,TEMPB,ALPHA,G
 2       5 FORMAT(6E10.3)
 3         IF (TEMPB)10,10,6
 4       6 SIGMA= 0.1714
 5         EXRAD = (TN*(2.0-EPSILN)*ALPHA*G)/(SIGMA*EPSILN*
           (TEMPB/100.)**4)
 6      12 GAMMA=UEFF/(SIGMA*0.01*EPSILN*(TEMPB/100.)**3)
 7         GO TO 15
10      10 GAMMA =UEFF
11      15 B=GAMMA*(2.0-EPSILN)/(TN*(2.0-EPSILN)+1.0)
12         C=-(GAMMA*(2.0-EPSILN)+1.0+EXRAD)/(TN*(2.0-
           EPSILN)+1.0)
13         TH=1.0
14         I=1
15      20 DIF=TH**4 B*TH+ C
16         I=I+1
17         DIRDIF=4.0*TH**3+B
20         IF (DIF) 25,40,25
21      25 THN=TH-(DIF/DIRDIF)
22         DIFTH=THN-TH
23      30 IF (ABS(DIFTH)-0.00001) 40,40,35
24      35 TH=THN
25         IF(I-100) 36,55,55
26      36 GO TO 20
27      40 THETA=THN
30         THFOUR=THETA**4
31         IF(TEMPB)50,50,45
32      45 Q = EPSILN*SIGMA*THFOUR*(TEMPB/100.)**4 - (ALPHA*G)
33      50 QRATIO=(1.0/THFOUR)*(1.0/(2.0*TN+1.0))
34         IF (EXRAD) 501,501,502
35     502 IF (TN-1.0) 503,503,504
36     503 QB = SIGMA*EPSILN*(TEMPB/100.)**4-ALPHA*G
37         SRATIO = QB/Q
40         GO TO 505
41     504 SRATIO = QPREV/Q
42     505 QPREV = Q
43         GO TO 65
44     501 IF (TN-1.0) 51,51,52
45      51 SRATIO=1.0/THFOUR
46         GO TO 53
47      52 SRATIO=PTHFOR/THFOUR
50      53 PTHFOR=THFOUR
51         GO TO 65
52      55 WRITE(6,60)
53      60 FORMAT(10X,13HMAXIMUM COUNT)
54      65 WRITE(6,70) THETA,THFOUR,Q,QRATIO,SRATIO
55      70 FORMAT(10X,8HTHETA IS,E20.8/10X,9HTHFOUR IS,E20.8/
           10X,4HQ IS,E20.8 X/10X,9HQRATIO IS,E20.8/10X,
           9HSRATIO IS,E20.8)      GO TO 100        END
```

MC STRAVICK FORTRAN SOURCE LIST

ISN SOURCE STATEMENT

Program B

```
$IBFTC KAHOMC
  100 READ (5,5) TN,EONE,ALPHA,GEXT,TEMPB,WT
    5 FORMAT (6E10.3)
      N = TN
      SIGMA = 0.1714
      J = 0
      DIMENSION GA(100),EPS(100),THT(100)
    6 J=J+1
      IF (TEMPB) 4,4,3
    3 EXRAD = ALPHA*GEXT/(SIGMA*(TEMPB/100.)**4)
    4 IF (J-1) 7,7,8
    7 THM = 1.0
      READ(5,10) (GA(I),EPS(I),I=1,N)
   10 FORMAT (2E10.3)
      GO TO 13
    8 THM = WT*THM+(1.0-WT)*TH
   13 IF (J-50) 14,95,95
   14 EXRAD = EXRAD*(THM)**4
      DO 90 I=1,N
      GAMMA = GA(I)*(THM)*(THM)*(THM)
      IF(I-1) 15,15,20
   15 F=1.0/((1.0/EPS(I))+(1.0/EONE)-1.0)
      H=1.0
      GO TO 25
   20 F=1.0/((1.0/EPS(I))+(1.0/G)-1.0)
   25 B=GAMMA*EONE/F
      C = -(H**4+GAMMA*EONE*H/F+EONE/F-EXRAD/F)
      K = 0
      IF (I-1) 9,9,11
    9 TH=1.0
   11 TH=H
   12 DIF=TH**4+B*TH+C
      K = K+1
      DIFPR = (4.0*(TH)*(TH)*(TH))+B
      IF (K-700) 29,95,95
   29 IF(DIF) 30,40,30
   30 THN=TH-(DIF/DIFPR)
      DIFT=THN-TH
      IF (ABS(THN-TH)-0.00001) 40,40,35
   35 TH=THN
      GO TO 12
   40 TH=THN
      THT(I)=TH
      G=EPS(I)
      H=TH
   90 CONTINUE
      IF (ABS(THM-TH)-0.0001) 50,50,6
   50 M=N-1
      THETA=1.0/THT(N)
      GO TO 65
   55 DO 61 I=1,M
      THET=THETA*THT(I)
      WRITE(6,60) THET
   60 FORMAT(10X,8HTHETA IS,E20.8)
   61 CONTINUE
```

```
    ISN       SOURCE STATEMENT

     75          GO TO 100
     76       65 WRITE(6,70) THETA
     77       70 FORMAT(10X,16HOVERALL THETA IS,E20.8)
    100          GO TO 55
    101       95 DIFTH=THM-TH
    102          WRITE (6,96) DIFT,J
    103       96 FORMAT(10X,13HDIFFERENCE IS,E20.8/10X,7HSTEP IS,I3)
    104          GO TO 100
    105          END
```