THE RICE INSTITUTE

THE EFFECT OF HYDROSTATIC STRESS ON THE DRILLING RATES OF ROCK FORMATIONS

by

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NOTATION

\( x, y \) Rectangular coordinates
\( r, \Theta \) Polar coordinates
\( \sigma_x', \sigma_y \) Normal component of stress parallel to \( x \) and \( y \) axes
\( \sigma_n \) Normal component of stress parallel to \( n \)
\( \sigma_r, \sigma_\theta \) Radial and tangential normal stresses in polar coordinates
\( \tau_{xy} \) Shearing stress component in rectangular coordinates
\( \tau \) Shearing stress
\( \tau_r, \tau_\theta \) Shearing stress components in polar coordinates
\( \tau_c \) Maximum shearing stress at rupture
\( \alpha \) Angle between direction of maximum principal stress and plane of shear failure
\( \phi \) Rake angle
\( h \) Depth of cut
\( L \) Load on bit
\( d \) Diameter of bit
\( N \) Revolutions per minute
\( V \) Drilling rate
\( V_a, V_m \) Drilling rates at atmospheric pressure and high pressure, respectively
\( p \) Pressure
\( p_o \) Formation constant
\( \sigma_{Rc} \) Stress normal to cutter to produce rupture
\( \sigma_R \) Stress normal to cutter
\( \sigma_c \) Compressive stress at rupture, atmospheric pressure
\( \sigma_{tu}, \sigma_{cu} \) Stress at rupture, in tension and compression respectively
\( \sigma \) A formation physical factor
\( K, K', K', m, n, c, \) Constants
\( h \) Formation constant
INTRODUCTION

The process of drilling an oil well bore hole in the crust of the earth presents many problems in mechanics and hydraulics. The immediate purpose of drilling is to produce rock chips within a confined area which can be raised to the earth's surface and there either used or discarded. These chips are produced by exceeding the ultimate strength of the main body of the rock. This fracture is dependent upon the physical properties of the formation, the mechanical action of the drilling tool, and the stress conditions resulting from sources other than the drilling tool.

The term "drilling", as used here, refers to the actual process of producing a hole from the earth's surface to some hoped-for oil bearing formation beneath the surface. "Drilling rate" is the average velocity at which the hole producing tool advances into the material being drilled.

In the past, one drilling tool manufacturer made laboratory tests using rock samples brought up from the borehole. These tests were simple, easily performed tests from which empirical estimates could be made of the probable drilling rates to be encountered under a given set of field conditions. Rock properties were assumed to be the same in the laboratory as at the bottom of the hole. These procedures were successful in many cases, but in many others they could not be correlated with field results. For
instance, laboratory tests indicated that some shales* could be drilled very rapidly. However, when encountered in place, the actual drilling rates were quite slow — in many cases as little as one-tenth that indicated by laboratory tests.

Works by Adams,1** Griggs,2 and Handin3 have shown that the stress necessary to produce rupture of a rock cylinder increases if the cylinder is subjected to a uniform hydrostatic stress in addition to the axial compressive loading stress. These works also indicate that some rocks which behave like brittle materials under atmospheric conditions assume properties of ductile materials under high confining pressures. These results suggest a possible reason for the difference in behavior of rocks in the laboratory and in the field. Field conditions are generally such that the formation being drilled is influenced by a column of fluid equal to the depth of the hole.

One uncertainty with the tests referred to above is that the hydrostatic pressures used were many times greater than those encountered in oil well drilling. Tests by the Bureau of Reclamation,4 Jones,5 and Bredthauer6 have been made using procedures similar to those of Griggs,2 and Handin3 but using hydrostatic pressures more nearly in

*Geological names are described in Appendix A
**Numbers denote items in the Bibliography Page
keeping with those in oil well drilling. Here again the compressive strengths were increased greatly by the application of a hydrostatic stress. In some few cases Bredthauer found a combination of brittle and plastic failure of materials which are normally brittle.

These known changes of physical properties thus become important in the study of the drilling process. As pointed out by Kühne, one quantity which has been ignored in the study of drilling is that of the effect of a hydrostatic stress within the formation on drilling rates. He further states that one possibility of testing the effect of this stress is to drill various rock samples under a "specified hydrostatic pressure shut off from the outside world". This method can produce empirical results only, but results which are very necessary in helping to complete the study of the variables involved in drilling.

Another factor which is largely an unknown quantity is the mechanical action of the drilling tool. Different drilling tools exhibit different actions. For instance, one of the earliest rotary tools is known as a drag bit. This bit scrapes the material loose from the hole bottom and is suitable only in soft unconsolidated formations. By far the most commonly used tool in the drilling industry today is that of the rolling cutter rock bit. Basically this tool consists of rolling elements with suitable projections which are pushed against the formation to be removed while the
rock bit is available. A machine has been built to use this bit. The design provides for drilling the specimen while it is subjected to hydrostatic stress.

It is the author's purpose to present many of the drilling tests which illustrate the effects of hydrostatic stress on drilling rate. These results, together with Kühne's analytical work, may provide a better understanding as to how and why hydrostatic pressure affects drilling rate.
The purpose of oil well drilling is to provide an opening into the earth's crust so that oil bearing formations may be located. Once located, the hole is then used to conduct crude oil products to the surface of the earth. An examination of the equipment and techniques used in drilling a hole will illustrate the position of the formations encountered and will show the purposes of the component parts of a drilling rig.

Fig. 1 represents the cross section of a bore hole with drilling equipment in place. The rig consists basically of a cutting tool known as a rock bit, a long string of tubular members which serve to transmit a rotating drive from the surface to the rock bit, and a fluid circulating system.

At the bottom of the hole is the rock bit. The purpose of the rock bit is to fracture or otherwise disrupt the formation immediately beneath it. The most commonly used type of rock bit is a rolling cutter rock bit similar to the one shown in Fig. 2a. Basically this tool consists of one or more rolling elements which are provided with wedge-shaped teeth on its outer periphery. The bit is pushed against the formation and rotated about its axis. The rolling elements rotate about their axes while forcing the teeth into the formation, causing concentrated loading and fracture. If these cutters are not "true rolling", the teeth are also forced to "plow" or scrape across the formation causing
FIGURE 1
SCHEMATIC DIAGRAM OF THE PERTINENT PARTS OF A ROTARY DRILLING RIG.
additional stresses which tend to cause fracture. This action is quite complicated to analyze but nevertheless effective. The hole drilled by this rock bit is ideally a cylindrical hole having the same diameter as the rock bit.

The connecting members between the surface equipment and the rock bit consist primarily of tubular members connected to each other to form a long rotating shaft. These tubular members are generally of three types:

1. Immediately above the bit are heavy, thick-walled tubes called drill collars. Drill collars are approximately thirty feet long and have threaded connections at each end so that they may be coupled together to form a long section. The rock bit is attached on the lower end of this section and another portion of the drill string is attached to the upper end. In addition to forming a connecting link, the purpose of the drill collars is to provide a load on the bit. This load is concentrated as close to the bit as possible.

2. Above the drill collars is the drill pipe, usually thin-walled tubing. The drill pipe sections are approximately thirty feet long and have threaded connections known as tool joints permanently attached to each end. The tool joints make it possible to couple any number of drill pipe sections together to form a string of the required length.
3. The topmost section of the drill stem is called the kelly. The kelly is a tubular member about forty feet long with suitable projections on the outside surface which serve as a splined shaft. A rotating member fits these projections to provide rotation to the drill string and yet allow free movement of the string in the vertical direction.

At the surface, a derrick with suitable handling and power equipment produces the necessary power to rotate the drill string and provides for removing the drill string from the hole.

The chips formed by the rock bit as it is rotated must be removed or they become so numerous that they interfere with the forward progress of the bit. A fluid circulating system is provided to keep the bottom of the hole and the teeth of the rock bit clean. This fluid is moved by pumps (not shown) through the mud hose, kelly, drill pipe, drill collars and finally through the bit. The bit is designed to allow the fluid to wash against the cutters and/or the bottom of the hole. The fluid velocity must be great enough to carry the chips up the annular area between the drill pipe and the bore hole to the earth's surface.

There are many types of drilling fluids. These fluids, called "muds", are usually water or oil base liquids
with carefully controlled solid additives which impart certain desirable properties to the fluid. One of these properties is density. Density is necessary to control high gas pressure encountered when drilling through gas producing strata. The density of the mud is adjusted so that hydrostatic pressure at the gas zone is greater than the gas pressure. This prevents combustible gases from rising to the surface during drilling operations and creating safety hazards.

This briefly illustrates the equipment and the operating procedures used in drilling practices.
GENERAL TEST AND EQUIPMENT SPECIFICATIONS

Assuming that either water or "mud" is circulated to remove the chips in the process of drilling an oil well bore hole, then the long column of fluid in the hole exerts a hydrostatic pressure on the wall of the hole and the surface of the portion being drilled. For the purpose of this paper it will be assumed that the portion of rock (a small fraction of an inch beneath the hole bottom) disturbed by the rock bit is under a state of hydrostatic stress due to the fluid pressure. This assumption greatly simplified equipment design and test procedures needed.

The actual state of stress in the formation at some depth below the earth's surface in the vicinity of the bottom of the hole is not known. This state of stress is in part a function of the difference between the densities of the fluid and the formation itself. At present this problem is under study.

If one considers a hole 10,000 feet deep in which water is used as a circulating fluid, calculation will show that the hydrostatic pressure on the bottom is approximately 4,400 psi. Using "mud" with a density twice that of water, the pressure against the bottom is approximately 8,800 psi.

In the consideration of constructing equipment for testing the effect of this hydrostatic pressure, the first prerequisite was to use pressures of the order of magnitude encountered in bore holes. The equipment was designed to
contain a maximum of 10,000 psi.

A second condition required the use of a miniature rock bit with actions similar to those of the full size bits. Such a bit, having a 1-1/4 inch diameter was in existence. Fig. 2b shows this bit and Fig. 2 gives a comparison of the microbit with the field bit, both in size and design. The drilling machine was built around the microbit and the specimen to be drilled.

Other design problems included provisions for:

1. A circulation system capable of cleaning the rock chips from the drilled sample while the sample is subjected to the test pressure and is drilled.
2. A measurable load between the bit and the rock.
3. A rotary motion to the bit.
4. A means of measuring the depth of penetration.

A more detailed description of the design and operation of this equipment is given in Appendix B. Appendix C gives additional information about the calibration of the equipment.
TEST PROCEDURES

Several considerations are necessary for reliable results while testing rock samples. By far the most important, and the only one to be discussed here, is the variation of the rock properties. Rock is not a uniform substance. The strength varies markedly even from point to point in the rock. As a result, a given element of rock might drill with several times the velocity of an adjacent element.

In each rock specimen 2-1/4 inches long, one hole 1-3/4 inches deep can be drilled. Starting from a flat surface, 1/8 of an inch is needed to let the bit set up its "bottom hole pattern" and assume a semblance of a uniform drilling rate. This leaves a total of 1-5/8 inches of hole in each specimen from which tests can be made.

This is an example of the system used. The 1-5/8 inches may be divided into six 1/4 inch increments leaving an extra 1/8 of an inch. If eight rock samples are used with eight holes, each containing six increments, a total of forty-eight increments or individual tests can be made. Fig. 3a shows how one of these holes is divided for test purposes. Each increment is tested using a different pressure. Load on bit is held constant throughout the complete tests of the forty-eight increments.

If the increments are divided as shown and numbered 1 to 6, then six hydrostatic stress states can be investigated. That is, one stress state is used for each increment.
<table>
<thead>
<tr>
<th>TEST NO.</th>
<th>DISCARDED</th>
<th>PSi</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEST NO. 1</td>
<td>DISCARDED</td>
<td>0 psi</td>
</tr>
<tr>
<td>TEST NO. 2</td>
<td>500 psi</td>
<td></td>
</tr>
<tr>
<td>TEST NO. 3</td>
<td>1000 psi</td>
<td></td>
</tr>
<tr>
<td>TEST NO. 4</td>
<td>2000 psi</td>
<td></td>
</tr>
<tr>
<td>TEST NO. 5</td>
<td>3000 psi</td>
<td></td>
</tr>
<tr>
<td>TEST NO. 6</td>
<td>4000 psi</td>
<td></td>
</tr>
</tbody>
</table>

**SAMPLE A**

**CONSTANT HYDROSTATIC STRESS**

DIVISION OF SAMPLES FOR TESTING PURPOSE

FIG 3

---

FIG 4

SAMPLE DATA SHOWING INDIVIDUAL AND AVERAGE DATA OF DRILLING RATE AS A FUNCTION OF CONFINING PRESSURE

CONFINING PRESSURE PSI X 10^*

SAMPLE MATERIAL: WYOMING RED BED

1/4 INCH DIAMETER BIT
1000 LB BIT LOAD
50 RPM

* INDIVIDUAL TESTS
X AVERAGE OF INDIVIDUAL TESTS
If sample "A" is divided and tested as shown in Fig. 3a, sample "B" will shift the 500 psi stress test to increment 1, 1000 psi to increment 2, 2000 psi to increment 3, and so forth to 0 psi to increment 6. In sample "C" these pressures will be shifted again with increment 1 tested at 1000 psi, increment 2 at 2000, etc. This procedure means that during the forty-eight individual tests, six stress states are investigated. For each stress state, eight individual tests are made.

A hypothetical hole can be constructed as in Fig. 3b with each increment representing a different sample tested at the same pressure. These eight individual tests are averaged and only this average drilling rate plotted.

Fig. 4 is a plot of drilling rate as a function of pressure. Here each individual test is plotted to illustrate the scatter obtained. John Eenink used a statistical procedure to evaluate test data. His analysis indicated that in most cases an average of six or more tests would give reliable points for plotting. A comparison of the average with the individual tests seem to justify his conclusions.
RESULTS OF TESTS ON ROCK

A large number of drilling tests have been performed on rock using the apparatus designed for drilling oil field core samples subjected to a hydrostatic stress. These cores represent many varieties of earth formations encountered in drilling; as examples, soft sedimentary shales, medium hard dolomites and hard basalt. Tests are yet to be made on hard, tightly cemented quartzites. A geological classification of many of the formations described in the following pages is given by R. O. Bredthauer.6

Of the formations tested thus far, the physical properties of all but a very few were altered in a manner which caused the drilling rate to decrease as the hydrostatic stress in the formation was increased. All tests were made with the drilling rate measured as a function of load on bit and hydrostatic stress in the formation.

Clarification of the term "hydrostatic stress" as used here is needed at this point. For a material to assume a true state of hydrostatic stress when subjected to a hydrostatic pressure, the material must be homogeneous and isotropic, i. e., having properties the same from point to point and the same in all directions. The physical properties of rock very often do not fulfill these conditions. Therefore, in describing test results, the term "confining pressure" will be used to describe the hydrostatic pressure of the fluid pressing against the surface of the rock.
Drilling Rate as a Function of Confining Pressure

Fig. 5 shows drilling rate as a function of confining pressure from tests made on seven different formations. Of the seven formations shown, four are actual oil field cores taken during coring operations while three are taken from quarry rocks used in laboratory testing. These samples represent shales, limestone, dolomites and basalts, all of which have very different physical and chemical properties. These are plotted together because of the apparent similarity of reaction to pressure. Also, this emphasizes the type and extent of the reaction of most formations subjected to confining pressures.

Fig. 5 illustrates the decrease in drilling rate with added confining pressure. With pressure as a variable, a value is reached above which added pressure does not appreciably reduce drilling rate. The pressure at which this occurs is different for each formation.

Another result is shown in Fig. 5. Consider the drilling rate \( V_a \) of a given formation at atmospheric conditions and the minimum drilling rate \( V_m \) which occurs on the near horizontal portion of the curve. An examination of the maximum percentage decrease in drilling rate \( \left( \frac{V_a - V_m}{V_a} \right) \) shows that as the formation becomes easier to drill, \( \left( \frac{V_a - V_m}{V_a} \right) \) becomes larger. For instance, \( \left( \frac{V_a - V_m}{V_a} \right) \) for the seven curves of Fig. 5 are shown to decrease as follows: Rifle
DRILLING RATE FT/HR

1. RIFLE SHALE
2. SPRABERRY SHALE
3. RED BEDS WYOMING
4. PENNSYLVANIAN
5. RUSH SPRING SAND STONE
6. BASALT
7. ELLENBERGER DOLOMITE
8. DIAMETER OF ROCK BIT
1000 LB. BIT LOAD
50 RPM

CONFINING PRESSURE (X 10^3 PSI)

DRILLING RATE AS A FUNCTION OF CONFINING PRESSURE

FIG. 5

CONFINING PRESSURE (X 10^3 PSI)

DRILLING RATE AS A FUNCTION OF CONFINING PRESSURE

FIG. 6

LEA PARK SHALE

1. WHITE DOLOMITE
2. CHICO LIMESTONE
3. CANADIAN DOLOMITE

CONFINING PRESSURE (X 10^3 PSI)

DRILLING RATE AS A FUNCTION OF CONFINING PRESSURE

FIG. 7

CONFINING PRESSURE (X 10^3 PSI)

DRILLING RATE AS A FUNCTION OF CONFINING PRESSURE

FIG. 8
shale, 78 per cent; Spraberry shale, 76 per cent; Wyoming Red Beds, 63 per cent; Pennsylvanian limestone, 50 per cent; Rush Springs sandstone, 33 per cent; Basalt, 33 per cent; Ellenberger dolomite, 22 per cent. Fig. 6 further illustrates this. This shale was drilled at 21.2 Ft./Hr. under atmospheric conditions with a minimum rate of less than 1.4 Ft./Hr. This gives

\[
\frac{V_a - V_m}{V_a} = 93.4 \text{ per cent}
\]

Fig. 7 shows a formation having the characteristic decrease in drilling rate and then apparently no further decrease above some confining pressure. Here only 500 psi is sufficient to reduce the drilling rate 77 per cent.

Some formations react to confining pressure at variance with the ways described above. Fig. 8 is given to illustrate three different reactions.

1. The White dolomite shows a characteristic reaction except that it is an easily drilled formation which has a small percentage of reduction in drilling rate.

2. The drilling rate of Chico limestone, under some selected conditions, will decrease until the pressure has reached a value above which the drilling rate begins to increase again.
3. A dolomite from Canada apparently shows no effect when subjected to pressures.

These three formations are shown here only for contrast. A discussion of the possible reasons for their behavior is in "Discussion of Results".

**Drilling Rates as a Function of Confining Pressure and Bit Load**

Results of tests in which drilling rate is a function of confining pressure while using bit load as a parameter are shown in Fig. 9 and Fig. 10. From these tests it is possible to see other trends. Results given below are representative of most tests.

Fig. 9 is the result of drilling tests made on Ellenberger dolomite (see curve 7, Fig. 5) showing drilling rate as a function of confining pressure using bit loads of 1000 pounds and 2000 pounds as parameters. A comparison of these two curves reveal the following:

1. Doubling the bit load more than doubled the drilling rate at any given pressure. This is in line with field experience when drilling some of the harder formations.

2. With 1000 pounds bit load, the drilling rate became uniform above 1000 psi confining pressure. With 2000 pounds load the drilling rate became almost uniform above 4500 psi.
DRILLING RATE AS A FUNCTION OF CONFINING PRESSURE

FIG. 9

DRILLING RATE AS A FUNCTION OF CONFINING PRESSURE

FIG. 10

ROCK BIT LEAD-POUNDS
DRILLING RATE AS A FUNCTION OF CONFINING PRESSURE

FIG. 11

FIG. 12
3. The maximum decrease in drilling rate with 1000 pound bit load is 22 per cent as compared to approximately 50 per cent with 2000 pounds bit load.

The same general results as mentioned above can be seen in Fig. 10. These were obtained using Wyoming Red Beds which is a medium hard shale. Bit loads of 500, 1000, 1500 and 2000 pounds were used. One additional characteristic is seen using a bit load of 500 pounds. At this bit load, confining pressure seems to affect drilling rates very little.

The results on Wyoming Red Beds, shown in Fig. 11, are the same as those in Fig. 10. The difference is that in Fig. 11, drilling rate is given as a function of bit load, with confining pressure as a parameter. Additional confining pressure has the same effect as strengthening the rock.

One other result is worth mentioning. The results shown in Fig. 12 were obtained by one rock bit manufacturer using a "drag bit". A "drag bit" in no way resembles a rolling cutter bit. Its action consists of scraping across the rock's surface like a grader blade. This type of action is very effective in soft unconsolidated formation. These bits are used to drill soft top hole formations. A decrease in drilling rate is obtained using this drag bit similar to that obtained using the rolling cutter bit.
DISCUSSION OF TEST RESULTS

Because of the many unknown variables in the action of a rotary rock bit, a valid mathematical analysis of these test results seems out of the question at this time. As pointed out by Kuhne, tests made under conditions such as these would yield results of an empirical magnitude about whose basic causes one could state little or nothing. However, these results can be studied qualitatively. Using these results, some efforts can be made to predict drilling rate as a function of known variables. Also, additional unpublished results of one rock bit manufacturer can be used in considering drilling rate as a function of variables not studied in these pressure tests.

A study will be made of the problem of drilling as follows:

1. Reviewing unpublished empirical works and their origin.
2. Using dimensional analysis to determine dimensionless groups and then using laboratory and field results to relate these groups.
3. Studying certain idealized, previously solved problems.
4. Relating these idealized problems to the process of drilling.
5. Discussing certain miscellaneous phenomena.
Review of Unpublished Empirical Works

Laboratory tests by one rock bit manufacturer were made drilling four rocks of widely different physical properties. These tests considered drilling rate as a function of bit load, rpm, and bit diameter. All tests were made under atmospheric conditions. The rock bits in this case were full size. Using these test results, H. B. Woods proposed an empirical relationship as follows:

\[ V = 24 N \left( \frac{L}{3 \times 10^4 d} \right)^k \]  

where:  
\( 24 \) and \( 3 \times 10^4 \) are not dimensionless  
\( V \) = drilling rate - Ft./Hr.  
\( N \) = rpm of bit  
\( L \) = load on bit - Pounds  
\( d \) = diameter of bit - inches  
\( k \) = presumed to be a constant for a given formation

As \( k \) increases, \( V \) decreases if \( L < 3 \times 10^4 d \) which is generally true. This equation can be used if \( k \) is calculated from one test and \( L, d \) and \( N \) are varied relatively small amounts from the test condition.

Although equation (1) takes into account rock strength, it does not reflect any effect of hydrostatic pressure. Recent correlative work between laboratory pressure drilling tests and field tests indicates that hydrostatic pressure can
be accounted for by replacing k with \( K_0 + \frac{P}{P_0} \). \( K_0 \) is a formation constant determined at atmospheric pressure; \( \frac{1}{P_0} \) is another constant determined with one pressure drilling test; \( p \) is the hydrostatic pressure against the formation. This equation does not indicate an asymptotic approach of drilling rate to a constant value greater than zero as indicated by laboratory results. It does indicate a decided decrease in drilling rate as hydrostatic pressure is increased.

An early equation used to approximately represent the laboratory results where drilling rate is a function of hydrostatic pressure is

\[
V = V_m + \left( V_a - V_m \right) e^{-\frac{-P}{P_0}}
\]

\( V_m, V_a \) are constants (see Fig. 5)

\( p_0 \) - formation constant

Three tests are necessary to establish the constants of this equation. This equation does predict an asymptotic approach of drilling rate to a constant value greater than zero.

These equations are presented here for the purpose of showing the small amount of effort which has been made in trying to correlate drilling rate with its many variables. It must be remembered that these are strictly empirical in nature and should be used under highly restricted circumstances. No one equation gives drilling rate as a function of all known variables.
Dimensional Analysis

All of the important variables must be considered if a dimensional analysis is to be satisfactory. There are certain physical properties of the material being drilled which are not precisely known but can be handled as described later. Using all variables known to be important, certain relations can be derived which can be used to advantage.

Consider an equation of the form

$$f \left( L, d, N, p, \sigma, V \right) = 0 \quad (3)$$

where:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>L = load on bit</td>
<td>$F$</td>
</tr>
<tr>
<td>d = diameter of bit</td>
<td>$L$</td>
</tr>
<tr>
<td>N = radians per minute of bit</td>
<td>$T^{-1}$</td>
</tr>
<tr>
<td>p = hydrostatic stress in the formation</td>
<td>$F L^{-2}$</td>
</tr>
<tr>
<td>V = drilling rate</td>
<td>$L T^{-1}$</td>
</tr>
<tr>
<td>$\sigma$ = some physical strength factor of the formation, i.e., tensile or shear strength, modulus of elasticity or combination thereof. It is given the units of tensile strength.</td>
<td>$F L^{-2}$</td>
</tr>
</tbody>
</table>

Six variables involving three dimensions should yield three dimensionless groups.
Using Buckingham's $\pi$ notation and solving the following relations

$$\pi_1 = L^a d^b N^c V$$
$$\pi_2 = L^a d^b N^c p$$
$$\pi_3 = L^a d^b N^c \sigma.$$  

Equation (3) may now be rewritten in terms of dimensionless groups as

$$f \left[ \left( \frac{V}{dN} \right), \left( \frac{d^2 p}{L} \right), \left( \frac{d^2 \sigma}{L} \right) \right] = 0 \quad (4)$$

Four variables were used in the pressure drilling tests, i.e., $L$, $p$, $\sigma$ and $V$. These four appear in the three dimensionless groups so that any one group can be varied by varying one of the variables. Once these dimensionless groups are obtained, any arrangement into a final equation becomes empirical. The data which is plotted with $V$ as a function of $p$ is in effect the same as $\left( \frac{V}{dN} \right)$ as a function of $\left( \frac{d^2 p}{L} \right)$ since $L$, $d$ and $N$ were held constant. The data for the formation shown in Fig. 5 and Fig. 6 can be represented fairly well by altering equation (2) as

$$V_m + \left( V_a - V_m \right) \tau$$

where $\tau > 1 \quad (5)$

$V_a$, $V_m$ = constants of Fig. 5

$$\tau = \text{constant calculated using one point on the curve other than } V_a \text{ and } V_m$$
The following shows the approximate values of C, K and c.

<table>
<thead>
<tr>
<th>FORMATION</th>
<th>$V_a - V_m$</th>
<th>$V_m$</th>
<th>$\bar{c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 6</td>
<td>21.2</td>
<td>1.40</td>
<td>2.62</td>
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<tr>
<td>Fig. 5-1</td>
<td>4.7</td>
<td>1.05</td>
<td>2.05</td>
</tr>
<tr>
<td>-2</td>
<td>3.5</td>
<td>.85</td>
<td>2.05</td>
</tr>
<tr>
<td>-3</td>
<td>2.15</td>
<td>.80</td>
<td>2.23</td>
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<tr>
<td>-4</td>
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<td>.90</td>
<td>.60</td>
<td>3.09</td>
</tr>
<tr>
<td>-6</td>
<td>.66</td>
<td>.47</td>
<td>3.51</td>
</tr>
<tr>
<td>-7</td>
<td>.53</td>
<td>.40</td>
<td>5.60</td>
</tr>
</tbody>
</table>

The data which is plotted with $V$ as a function of $L$ is the same as $\left(\frac{V}{d N}\right)$ as a function of $\left(\frac{L}{\sigma d^2}\right)$. Fig. 11 shows drilling rate $V$ as a function of bit load $L$ with $p$ as a parameter. The relation of these two groups suggest rewriting equation (1) as

$$\left(\frac{V}{d N}\right) = \left(\frac{L}{\sigma d^2}\right)^k.$$

Data are not sufficient to warrant calculating these constants.

Still another relationship can be written as

$$V = N d \left(\frac{L}{d^2 \sigma}\right)^n + N d C \left(\frac{D}{\sigma}\right)^m \quad (6)$$

This empirical relationship is obtained as a result of using dimensionless groups rather than individual variables. No attempt will be made to evaluate $N$, $c$ and $m$ because adequate
data are not available. This equation is not given here as a justifiable equation. However, it is worth noting the variation of drilling rate as a function of each of the quantities involved and comparing this with laboratory and field experience. If \( 1 < n < 2, m \approx 1 \) and \( C > 1 \):

1. Drilling rate \( V \), decreases and approaches a constant value as pressure \( p \) is increased. This is in agreement with laboratory test results.

2. As load on bit \( L \), increases, drilling rate increases as \( L^n \). This is in rough agreement with laboratory and field results.

3. Drilling rate increases linearly with increased \( N \). This is in close agreement with field results of soft formations. Harder formations differ slightly.

4. Drilling rate decreases (not linearly) as bit diameter \( d \) increases. This is in good agreement with field results.

5. Drilling rate decreases as rock strength \( \sigma \), increases. This is in agreement with field and laboratory results.

6. Drilling rate is affected more by a given amount of hydrostatic pressure as \( \sigma \) decreases. This is indicated by the laboratory pressure tests.
STRESS STRAIN DIAGRAM FOR STEEL
FIG. 13

STRESS STRAIN DIAGRAM FOR STONE
FIG. 14

IDEALIZED STRESS STRAIN RELATIONSHIP
FIG. 15

MOHR'S CIRCLE
FIG. 16

MOHR'S ENVELOPE
FIG. 17
Kühne discusses many separate phases of the drilling problem. Only those dealing with the actual failure mechanism of the rocks are outlined in the following pages.

In deep rock drilling, materials are fractured whose properties differ widely from those of metals. Many metals behave elastically if a well-defined yield strength is not exceeded. Also the tensile and compressive yield strengths are usually of equal magnitude. On the other hand, rocks usually behave in an inelastic manner, and the tensile strength is very small when compared to the compressive strength. Fig. 13 and Fig. 14 make this clear.

The mathematical treatment of problems involving the theory of elasticity generally requires the assumption of Hooke's law. The stress-strain relationship of Fig. 14 is assumed to be elastic for stresses below a critical stress \( P \) and then plastic at the critical stress \( P \) as shown in Fig. 15.

If the idealized material of Fig. 15 is stressed beyond the point \( P \), plastic flow begins. Here no investigation is made of the process within the material which causes plastic flow. For the end result of failure, it does not matter if the individual rock particles are separated from each other after extensive plastic deformation or whether the separation is the result of brittle fracture.
For a material which deforms from the action of normal and shear stresses, there are a number of failure theories which predict the limits of elastic strength. For consideration here, two of these theories appear significant.

1. Maximum Shearing Stress Theory: For three principal stresses $\sigma_1$, $\sigma_3$ and $\sigma_2$ there exists

$$\tau_{\text{max}} = \frac{\sigma_1 - \sigma_2}{2} \text{ where } \sigma_1 > \sigma_3 > \sigma_2.$$ (7)

Failure occurs when $\tau_{\text{max}}$ reaches a critical value $\tau_c$. $\tau_c$ can be determined from a simple tension or compression test.

Equation (7) states that for a uniaxial stress state $\tau_{\text{max}} = \frac{\sigma_{\text{tu}}}{2} = \frac{\sigma_{\text{cu}}}{2}$ where $\sigma_{\text{tu}}$ and $\sigma_{\text{cu}}$ are ultimate tensile and compressive strengths respectively. This is not verified for brittle materials.

2. Mohr's Theory of Strength: This is a generalized version of the maximum shearing stress theory. Both assume that only the largest and smallest principal stresses ($\sigma_1$ and $\sigma_2$ as above) affect impending failure. According to Mohr, a material may fail through plastic slip or brittle fracture in one of two ways:
a. When the shearing stress $\tau$ in the planes of slip has reached a given value, which depends on the normal stress $\sigma$ acting across these shear planes.

b. When the largest normal tensile stress has reached a limiting value.

The type of failure depends on the properties of the material. This theory will be discussed later in terms of Mohr's circle representation of a stress state.

The above theories are stated for three dimensional stress states. As pointed out, both theories assume that the intermediate stress $\sigma_3$ does not influence failure if $\sigma_1 > \sigma_3 > \sigma_2$. Therefore, we consider the two dimensional stress state in the plane of $\sigma_1$ and $\sigma_2$. Further, we assume these maximum stresses are in the vertical plane.

Consider the small element of material shown in Fig. 16. From equilibrium considerations

$$\sigma \cos \alpha - \tau \sin \alpha = \sigma_x \cos \alpha + \tau_{xy} \sin \alpha$$  \hspace{1cm} (8)

$$\sigma \sin \alpha + \tau \cos \alpha = \sigma_y \sin \alpha + \tau_{xy} \cos \alpha.$$  \hspace{1cm} (9)

By multiplying these by $\sin \alpha$ and $\cos \alpha$ respectively and changing to double angle formulas these become
\[
\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\alpha + \tau_{xy} \sin 2\alpha \quad (10)
\]

\[
\tau = -\frac{\sigma_x - \sigma_y}{2} \sin 2\alpha + \tau_{xy} \cos 2\alpha . \quad (11)
\]

Equation (10) can be rewritten as

\[
\left[\sigma - \left(\frac{\sigma_x - \sigma_y}{2}\right)\right]^2 + \tau^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2 . \quad (12)
\]

This is in the form of the equation of a circle. As such it can be represented as in Fig. 16 using rectangular coordinates with \(\sigma\) as the abscissa and \(\tau\) the ordinate. The displacement of the circle is \(\frac{\sigma_x + \sigma_y}{2}\) and the radius \(\pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\). On the planes of minimum shear (\(\tau = 0\)), the normal stress has maximum and minimum values

\[
\begin{pmatrix}
\sigma_1 \\
\sigma_2
\end{pmatrix} = \frac{1}{2} \left(\sigma_x + \sigma_y\right) \pm \frac{1}{2} \sqrt{\left(\sigma_x - \sigma_y\right)^2 + 4\tau_{xy}^2} . \quad (13)
\]

The maximum and minimum shear stresses are

\[
\tau = \pm \frac{1}{2} \sqrt{\left(\sigma_x - \sigma_y\right)^2 + 4\tau_{xy}^2} . \quad (14)
\]

with the direction \(\alpha = 45^\circ\) from the principal stress direction.

When the principal stresses \(\sigma_1\) and \(\sigma_2\) are given and \(\alpha\) is the angle between \(\sigma_1\) and the x-axis then
\begin{align*}
\sigma_x &= \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\alpha \\
\sigma_y &= \frac{\sigma_1 + \sigma_2}{2} - \frac{\sigma_1 - \sigma_2}{2} \cos 2\alpha \\
\tau_{xy} &= -\frac{\sigma_1 - \sigma_2}{2} \sin 2\alpha
\end{align*}

Further clarification of Mohr's theory is possible with the use of the circle representation of a stress state. Let Fig. 17 represent a Mohr stress circle in the plane of the maximum and minimum principal stresses. Let \( \sigma \) and \( \tau \) represent the components of stress in the plane ready to slip. Suppose, according to Mohr, that the shearing stress \( \tau \) in a plane of slip depends only on the value of the normal stress \( \sigma \) acting on that plane. Then \( \tau = f(\sigma) \) is represented by a curve drawn in the \( \sigma-\tau \) coordinate plane as a locus of points \( P(\sigma, \tau) \). This curve defines the limiting values of both stress components for an infinite number of limiting stress conditions. This limiting curve \( \tau = f(\sigma) \) forms an envelope tangent to all of the possible limiting conditions.

In the Mohr \( \sigma-\tau \) plane of Fig. 18, a stress state for a uniaxial compression test of rock is represented by a circle passing through the origin. The largest principal stress \( \sigma_c \) corresponds to the compressive strength of the rock. At failure, slip occurs at an angle \( \alpha_1 \) to the direction of the load.
If the rock specimen is subjected to a uniform pressure of \( p \), then \( P_1 \) moves along Mohr's envelope until a circle can be drawn with the stress \( p \) on the \( \sigma \) axis. \( R_1 \) moves to \( R_2 \), \( P_1 \) to \( P_2 \) and angle \( 2\alpha_1 \), changes to \( 2\alpha_2 \). \( \sigma_p \) equals the stress now necessary to cause failure to occur. When Mohr's envelope is determined experimentally \( \sigma_c, p, \sigma_p \), and the angle \( 2\alpha_1 \) and \( 2\alpha_2 \) may be determined.

When the envelope is not determined experimentally but \( p \) is small compared to \( \sigma_c \), the envelope can be approximated by the straight lines, \( \tau = \pm (k - \sigma \tan \delta) \). Here \( k \) is the intercept on the \( \tau \) axis and \( \delta \) the angle between the line and the \( \sigma \) axis. Also, \( \tan \delta = \cot 2\alpha \), and \( 2\alpha \) is constant. From this approximation, shown in Fig. 19, it follows that

\[
\cos 2\alpha = \frac{1}{2} \frac{\sigma_c}{\sigma_c + p'} = \frac{1}{2} \left( \frac{\sigma_p - p}{p' + p + \frac{1}{2}(\sigma_p - p)} \right)
\]

\[
p' = \frac{\sigma_p - p}{2 \cos 2\alpha} - \frac{\sigma_p - p}{2} - p
\]

\[
\sigma_p = \sigma_c + p \left( \frac{1 + \cos 2\alpha}{1 - \cos 2\alpha} \right)
\]

(18)

The value of \( \frac{1 + \cos 2\alpha}{1 - \cos 2\alpha} \) changes rapidly for smaller values of \( 2\alpha \) as follows:

\[
2\alpha = 90^\circ \quad 80^\circ \quad 70^\circ \quad 60^\circ \quad 50^\circ \quad 40^\circ \quad 30^\circ
\]

\[
\frac{1 + \cos 2\alpha}{1 - \cos 2\alpha} = 1.00 \quad 1.42 \quad 2.04 \quad 3.00 \quad 4.60 \quad 7.55 \quad 13.00
\]
For Blair dolomite, a laboratory test rock,
\( \sigma_p = 48,000 \) psi and \( 2\alpha = 45^\circ \) for \( p \leq 15,000 \) psi.

In the following, a Prandtl analysis of the question of penetration strength is considered. Fig. 20 represents the cross section of a semi-infinite solid loaded over a width AB with a uniform stress \( \sigma_R \). The length of this loading is infinite and is perpendicular to the plane of the paper.

The solution to this problem will require considering the stress states in the three sectors ABC, BCE and BEG. Only these three will be referred to since AFD and ACD are mirror images of BEG and BCE respectively.

Consider first the sectors ABC and BEG. For a rectangular coordinate system, the equilibrium equations in two dimensions in the absence of body forces are

\[
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0
\]

\[
\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0.
\]

These conditions and the compatibility conditions are satisfied if \( \sigma_x, \sigma_y \) and \( \tau_{xy} \) are constant. Therefore, a solution is obtained for the sectors ABC and BEG if the state of stress is constant within each region and the boundary conditions are satisfied.
For the present, assume the stress normal to the surface BG is zero. The boundary condition along BG is satisfied if the vertical principal stress in the sector BEG is zero. The horizontal principal stress is maximum and determines failure within this sector. Failure occurs when the stress condition is described by a Mohr circle passing through the origin and tangent to Mohr's envelope at $P_1$, Fig. 21. $\sigma_c$ represents the horizontal principal stress to produce failure. Failure occurs along slip planes which intersect the surface (and the direction of the maximum principal compressive stress) at angles $\alpha_1$ (Fig. 20). This sector must fail and be removed before sectors ABC and BCE can displace enough to be considered free chips.

In sector ABC the vertical principal stress is $\sigma_R$. The horizontal principal stress, $\sigma_H$, is unknown. However, if failure exists in this sector at the same instant failure occurs in the sector BEG, the stress condition just prior to failure is shown in Fig. 21 by a circle tangent to Mohr's envelope at $P_2$. According to Mohr the envelope defines the limiting stress conditions and further, slip occurs along planes which make angles $\alpha_2$ with the direction of maximum compressive principal stress. Therefore, $\sigma_R$ and the limiting envelope are enough to determine the stress state at failure. The horizontal principal stress, $\sigma_H$, can then be determined from the circle construction as shown in Fig. 21.
For the stress state in sector BCE the most convenient representation is with a polar coordinate system. For this system the equilibrium equations in two dimensions in the absence of body forces are

\[
\begin{align*}
\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{r\theta}}{\partial \theta} + (\sigma_r - \sigma_\theta) &= 0 \\
\frac{\partial \sigma_\theta}{\partial \theta} + r \frac{\partial \tau_{r\theta}}{\partial r} + 2 \tau_{r\theta} &= 0. \tag{20}
\end{align*}
\]

If \( \sigma_r, \sigma_\theta \) and \( \tau_{r\theta} \) are constant with respect to \( r \), varying only with \( \theta \), the equations (20) reduce to

\[
\begin{align*}
\frac{\partial \tau_{r\theta}}{\partial \theta} &= \sigma_\theta - \sigma_r \quad \text{(a)} \\
\frac{\partial \sigma_\theta}{\partial \theta} &= -2 \tau_{r\theta} \quad \text{(b)}.
\end{align*}
\]

Dividing equation (21a) by equation (21b)

\[
\frac{\partial \tau_{r\theta}}{\partial \sigma_\theta} = - \frac{\sigma_\theta - \sigma_r}{2 \tau_{r\theta}}. \tag{22}
\]

Noting the Mohr stress circle of Fig. 16, equation (22) represents the slope of the stress circle at point A, where \( \sigma_\theta \) corresponds to \( \sigma_x, \sigma_r \) to \( \sigma_y \) and \( \tau_{r\theta} \) to \( \tau \). If point A lies on Mohr's limiting envelope then "the shearing stress \( \tau \) in a plane of slip depends only on the value of the normal stress \( \sigma \) on that plane" (see page 30). Therefore, on a plane parallel to \( r \), \( \tau_{r\theta} = f (\sigma_\theta) \) and on a plane perpendicular to \( r \), \( \tau_{r\theta} = f (\sigma_r) \). For any given stress
state represented by a Mohr circle construction, all points on the circle correspond to the state of stress. However, failure occurs on a plane which forms an angle $\alpha$ with the maximum compressive principal stress. Then

$$\tan 2\alpha = \frac{2 \tau_{r\theta}}{\sigma_{\theta} - \sigma_r} = \frac{d \sigma_{\theta}}{d \tau_{r\theta}}.$$  \hspace{1cm} (23)

Since for a given material the envelope is fixed, each point on the envelope defines a stress state just prior to failure. Progressing along the limiting envelope one gets new stress states represented by new circles. If failure occurs at the same instant in sectors ABC, BEG and throughout BCE, then the slip planes described by the Mohr's envelope correspond to the radial planes and spiral surfaces shown in Fig. 20. The slip planes are at an angle $\alpha$ with the maximum compressive principal stress direction. Then the spiral surfaces make an angle $2\alpha$ with $r$, while the principal stress directions bisect the angles formed by the two slip planes.

If the limiting envelope $\tau_{r\theta} = f(\sigma_{\theta})$ then from equation (21b)

$$d \Theta = \pm \frac{d \sigma_{\theta}}{2 \tau_{r\theta}} = \pm \frac{d \sigma_{\theta}}{2f(\sigma_{\theta})},$$

$$\Theta = \pm \frac{1}{2} \int \frac{d \sigma_{\theta}}{f(\sigma_{\theta})} + c.$$ \hspace{1cm} (24)

This gives the expression for the state of stress in the sector BCE.
If the stress states in all three sectors reach critical stresses which produce failure at the same instant, then the stress states in each sector may be described in Fig. 22. The stress state in sector BEG is represented by the circle tangent to the envelope at P and passing through the origin. Failure occurs along planes which intersect the surface at an angle $\alpha_1$. At boundary BE between BCE and BEG the stress state is common to both sectors. Moving into sector BCE the stress states are described by stress circles tangent to the envelope. The tangency point moves from P at the boundary BE to Q at the boundary BC. At the boundary BC between ABC and BCE the stress condition is described by a stress circle tangent to the envelope at Q. Thus the boundary conditions are satisfied between adjacent sectors. The angle between the slip plane and the maximum principal stress changes from $\alpha_1$ to $\alpha_2$ while progressing through sector BCE.

A similar analysis is given by Kühne for the action of a machine shop cutting tool. This analysis will not be described here. Fig. 23 and Fig. 24 are pictured to illustrate the cutting action of the tool as it moves horizontally into the material. The stress states of the material being disturbed are shown in outline and are lettered to correspond to similar stress states in Fig. 20 and Fig. 22.

The "rake angle" of Fig. 23 is positive. The "rake angle" of Fig. 24 is negative. The negative "rake angle"
GLIDE LINES AND CUTTING FORCES WITH POSITIVE RAKE ANGLE

FIG. 23

\( \sigma' = \text{normal stress} \)

\( \tau = \text{friction stress} \)

\( \sigma_R = \text{resultant stress} \)

---

GLIDE LINES AND CUTTING FORCES WITH NEGATIVE RAKE ANGLE

FIG. 24

\( \sigma' = \text{normal stress} \)

\( \tau = \text{friction stress} \)

\( \sigma_R = \text{resultant stress} \)
more nearly represents the condition which exists on a drag bit. Even if the cutter has a positive "rake angle", action of hard, abrasive formations soon reduces it to the condition of Fig. 24.

As seen in the foregoing discussions, the stress conditions for both the penetrator and the cutter can be represented by Mohr's stress circles as shown in Fig. 22. If this system is now subjected to a hydrostatic stress, the entire system will move to the left. The radii of the stress circles remain the same since for any one circle

\[ \tau_{\text{max}} = \frac{(\sigma_1 + p) - (\sigma_2 + p)}{2} = \frac{\sigma_1 - \sigma_2}{2} \quad (25) \]

In moving to the left, these stress circles are no longer tangent to the envelope. For failure to occur now, \( \frac{\sigma_1 - \sigma_2}{2} \) must increase until the stress circles are again tangent to the envelope. For this to happen, the loading stress \( \sigma_R \) must be increased.

**Idealized Problems as Applied to Drilling**

A full analysis of the process of drilling is clouded by the many unknown quantities of the rock bit. In addition, other errors are introduced by reducing a three dimensional problem to one of two dimensions. The idealized two dimensional problems were discussed in the foregoing section. In this section attempts will be made to analyze the action of a rock bit in terms of the idealized problems. In the
following, the assumption is made that the total action of a rock bit consists of two components.

First, consider a rock bit designed so that the teeth contact the formation without sliding. As the teeth contact the formation they push in a direction perpendicular to the drilled surface. This action is similar to the idealized problem of the penetrator in Fig. 20, as presented by Kühne.

At any given instant there may be many teeth in contact with the formation. The total bit load is equal to the sum of the individual loads on the many teeth. The surface of the formation drilled is rough. For this reason, the loads on the teeth may vary widely from tooth to tooth. An analysis of the drilling process of this bit would require a knowledge of all the failure conditions developed per unit of time and a study of the number of stress conditions which are reduced to non-failure conditions by the application of an increment of hydrostatic stress. For instance, as an individual tooth approaches bottom, a certain maximum load $\Delta L$ is available to push against the formation. The remaining part of the total load $L$ will be supported by the other teeth. $\Delta L$ may be sufficient to produce fracture if a small hydrostatic stress exists in the rock. If the hydrostatic stress is increased, $\Delta L$ may no longer be enough to cause failure. As an illustrative example, if 20 per cent of all individual penetrations are reduced to non-failure conditions by this added hydrostatic stress, then the drilling rate
would be reduced a specific amount. An evaluation of this type will require a rock bit with "true rolling" characteristics. Since no such bit exists at present, this analysis cannot be attempted.

Second, consider a rock bit which is designed to impart only scraping of the teeth across the formation. As the cutter rolls, the teeth approach the formation, cut arcuate grooves and then leave the formation. For the purpose of this analogy, the cutter shown in Fig. 24 represents each tooth as it cuts its way through the formation.

Let \( h \) (Fig. 25) be the depth of the cut made by the cutting edge. As \( h \) varies, the shape of the disturbed area remains the same (Fig. 24). The area disturbed varies as \( h^2 \). Since the rock bit tooth is wedge-shaped in the plane perpendicular to the paper, the width of the cut varies as \( h \). As each tooth cuts a groove, the volume removed varies as \( h^3 \). (Here is considered a three dimensional cut. The idealized problem was for two dimensions). The drilling rate varies with the volume of material removed per unit of time. The drilling rate varies as \( h^3 \), if the number of cuts remains constant. From Fig. 25, if friction is neglected

\[
L = \frac{\sigma_R}{\tan \phi} h
\]

(26)

where

\[
L = \text{load per unit length}
\]

\[
\sigma_R = \text{stress normal to cutter}
\]

\[
h = \text{depth of cut}
\]

\[
\phi = \text{rake angle}
\]
FIG. 25

FIG. 26

CONFINING PRESSURE (X 10^5 PSI)
COMPARISON OF LABORATORY RESULTS
WITH RESULTS FROM THEORETICAL EQUATIONS
FIG. 26

CONFINING PRESSURE (X 10^5 PSI)
COMPARISON OF LABORATORY RESULTS
WITH RESULTS FROM THEORETICAL EQUATIONS
FIG. 27
If the hydrostatic stress is applied to the system, the loading stress $\sigma_{Rc}$, necessary to cause failure is given as $\sigma_{Rc} = \sigma_p - p$ (See Fig. 19). Then equation (18) becomes

$$\sigma_{Rc} = \sigma_c + p \left( \frac{1 + \cos^2 \alpha}{1 - \cos^2 \alpha} \right) - p. \quad (27)$$

$\sigma_c$ is the compressive strength of the formation at atmospheric conditions. Combining equations (26) and (27) and setting drilling rate proportional to $h^3$

$$v = K \ h^3$$

$$= \frac{KL^3}{\left[ \sigma_c + p \left( \frac{1 + \cos^2 \alpha}{1 - \cos^2 \alpha} \right) - 1 \right]^3 \tan^3 \phi}. \quad (28)$$

For a given bit design and a constant load, $\phi$ and $L$ are constant.

$$\therefore \ v = \frac{K'}{\left[ \sigma_c + p \left( \frac{1 + \cos^2 \alpha}{1 - \cos^2 \alpha} \right) - 1 \right]^3}. \quad (29)$$

For a given formation $\sigma_c$ and $\alpha$ are constant. Then equation (29) may be rewritten as

$$vK = \frac{(\sigma_c)^3}{\left[ \sigma_c + p \left( \frac{1 + \cos^2 \alpha}{1 - \cos^2 \alpha} \right) - 1 \right]^3}. \quad (30)$$
Knowing the compressive strength of the formation, the slope of the limiting envelope and one drilling rate test at atmospheric conditions, the drilling rate as a function of pressure can be calculated by use of equation (30).

Bredthauer\(^6\) gives the compressive strength and Mohr's envelope for many formations. Drilling tests were made on two of these formations at various pressures. A comparison of the actual drilling rates obtained with those calculated by equation (30) are shown in Fig. 26 and Fig. 27.

**Miscellaneous Discussion**

As previously mentioned under Results of Tests on Rocks, one formation showed no effect on drilling rate when subjected to confining pressures. Another formation showed a decrease in drilling rate up to a point and then an increase in drilling rate as the pressure was further increased.

Compression tests were made on cylindrical specimens of a permeable nature by Bredthauer\(^6\) under two different conditions. For one series of tests the cylinder was jacketed so that fluid producing the pressure could not enter the interstices of the material. For the second test the specimen was saturated with fluid and exposed to the fluid pressure so that the internal fluid was under the same pressure as the external fluid. In the first case the compressive strength of the cylinder increased with increasing pressure. In the second case, no effect on the
compressive strength was noted. No attempt will be made to explain this. In the first case, the fluid pushes against the outer surface of the cylinder and the entire hydrostatic loading is constrained by the lattice work of the formation. With the jacket removed, the fluid presses against each individual piece of the lattice work within the rock.

Most of the test rocks drilled are, for all practical purposes, impermeable to the fluids at the pressures used. The impermeable rock specimen in effect has a jacket which restrains the fluid from entering the pores of the specimen. In these cases the drilling rate was noticeably decreased as pressure was increased as shown by the majority of the tests.

The drilling rate of one rock tested is known to have shown no reaction to increased pressure. When this rock was broken open it was found to be wet inside, indicating a condition similar to that described for the unjacketed cylinder above.

The drilling rates of two rocks showed a tendency to decrease with an increase in confining pressure up to a certain pressure. The drilling rate then increased with further applied confining pressure provided the tests were performed in a certain specified manner. If a dry specimen were placed in the pressure vessel, drilled at atmospheric pressure and the pressures increased, then the drilling rate would decrease. At a pressure of about 2500 to 3000 psi the drilling rate would increase to the same as that at atmospheric condition. If the dry rock were placed in the
apparatus, drilling started at 5000 psi and then the pressure decreased, no change was noted in drilling rates. Again when broken open the rock was wet. Apparently when pressures were applied in ascending order, the bit drilled faster than the fluid penetrated, simulating the condition of the jacketed cylinder. When the pressure reached about 2500 to 3000 psi the fluid moved ahead of the bit causing the condition of the unjacketed cylinder.

Since no way has been devised to jacket these specimens and drill them without rupturing the jacket, the investigation of this phenomena has been limited.
CONCLUSIONS

1. The strength properties of rock which affect drilling rate increase with added hydrostatic stress.

2. Part of the mechanism of decrease in drilling rate can be described by the use of Mohr's theory as applied to a facsimile of a rock bit tooth.

3. The laboratory tests are similar enough to field operations to provide important results necessary to help evaluate theoretical problems.

4. The drilling rate of a permeable formation whose interstices are filled with a fluid whose pressure is dependent on and equal to the external pressure on the rock, will not be affected by varying the external pressure.
APPENDIX A

EXPLANATORY NOTES

Certain geological terms are used in this work which may not be familiar to the average mechanical engineer. In an effort to define these terms, they are described here as in Dana's Textbook of Mineralogy, Fourth Edition.

**Basalt**  
Fine grained rocks composed of microscopic grains of a soda-lime feldspar with pyroxeme, iron ore and often olivine. These are volcanic in origin.

**Dolomite**  
Carbonate rocks composed usually of CaMg(CO$_3$)$_2$.

**Limestone**  
Carbonate rocks usually composed chiefly of calcite (CaCO$_3$) although dolomite may also be an important constituent. The carbonate has in the great majority of cases been extracted from the sea water by the agency of minute organisms and then deposited in beds which ultimately are consolidated into rock. These rocks are usually fine and even-grained in structure and sometimes quite dense.

**Sandstone**  
Quartzitic particles usually cemented together loosely such that a fracture occurs between grains rather than through grains.

**Shale**  
Very fine-grained sedimentary rocks formed by the consolidation of beds of mud, clay or silt. They have usually a thinly laminated structure. Their color is commonly some tone of gray although they may be white, yellow, brown, green or black.

**Quartzite**  
A compact rock composed essentially of Quartz (SiO$_2$), which is firm, compact and breaks with an uneven splintery or conchoidal fracture. Derived from sandstone by intense metamorphism.
APPENDIX B

EQUIPMENT

Equipment for drilling rock samples under pressure was designed and constructed to utilize a two cutter 1-1/4 inch diameter miniature rock bit (Fig. 2b). In this design, conditions as nearly like those in actual drilling were provided while placing the core in a state of hydrostatic stress. An understanding of the important parts of the mechanism will aid in understanding the drilling problems and the significance of the results.

Pressure Chamber

Fig. 28 shows a cross section view of the pressure chamber with its included parts. The drilling fluid is confined within a volume bounded by a circular top (2) and a cylindrical container with one end closed (1).

A short drill stem (4) enters this chamber through suitable packing. The inner end is provided with threads which hold the rock bit (3). The outer end of the drill stem is provided with a thrust bearing which in turn is mounted in a U-shaped member attached to the chamber (1). The thrust bearing withstands all loads applied against the bit plus loading due to the pressure in the vessel against the drill stem. Immediately above the thrust bearing is a drive sprocket keyed to the shaft. This sprocket provides
linkage from a motor and gear reduction unit to rotate the drill stem. The only motion allowed the drill stem is rotational.

In the actual oil well bore hole the bit and drill string advance into the formation drilled. In this machine the formation advances against the rock bit. This is managed with a rock core holder (5) which is movable in the longitudinal direction.

The rock specimen (9) is prepared from cores 2 inches to 5 inches in diameter. These cores have the ends cut perpendicular to the axis of the core to a length of approximately 2-1/4 inches.

The movable carriage (5) is raised until the lower plate is above the bit. A core is placed with one of its faces against the lower plate. A second plate is placed against the upper end of the core and tightened lightly against the end of the core by means of bolts. This holds the core firmly in place while exposing part of one end to the bit. The bit enters the holder through a small hole in the specimen holder.

The holder is restrained from rotating by two tongue-in-groove systems attached to the outside wall of the specimen carriage and the inside wall of the chamber. This allows the holder to move vertically only. The longitudinal control of the specimen is maintained by two small diameter rods (6). These connect the specimen holder to a yoke (7).
SCHEMATIC DIAGRAM OF THE HIGH PRESSURE DRILLING APPARATUS USED BY THE AUTHOR

FIG. 28
Schematic Diagram of Circulation System for High Pressure Drilling Apparatus
Fig. 29

Assembly of High Pressure Drilling Apparatus
Fig. 30
The air cylinder (8) provides tension in the rods, which in turn pull the rock against the bit with any desired force.

This general design was chosen because it was reasoned that the moving small diameter rods would produce small friction losses. Also, the fluid pressure is in a direction to help load the bit such that the only place pressure has to be overcome is at the drill stem. This is accomplished with the thrust bearing.

Circulating System

Circulation similar to that in an oil well bore hole has been provided. Fluid is introduced through a series of holes and annular areas into the drill stem and then through the bit. Holes in the bit direct the fluid against the cutters and the rock surface to remove the cuttings. Circulation is indicated by arrows shown in Fig. 28. The fluid exits through a hole in the side of the pressure vessel.

Fig. 29 shows a schematic diagram of the entire circulation system external of the drilling chamber. A filter is provided just outside of the drilling unit to trap as many cuttings as possible to prevent damage to pumps and valves.

Two pumps are used. One is a high pressure, low volume chemical injection pump operated by air, capable of providing 14,000 psi. This pump simply "charges" the entire system with the desired fluid pressure.
The circulating pump is basically the same as the one above except it is capable of circulating four gallons per minute within the closed system. The circulating pump's intake pressure is equal to the system pressure. Its exhaust pressure equals the system pressure plus the pressure required to move the circulating fluid.

This arrangement allows the pumping of a relatively large volume of fluid under high pressure with relatively little power output.

Miscellaneous

A depth indicator measures the movement of one of the pull rods (6) of Fig. 28. The movement is magnified approximately four times so that a 1/4 inch increment equals approximately one inch.

An electric clock is connected to the switch that operates the rotary motor. Drilling time is automatically recorded as rotation proceeds.

Fig. 30 shows the complete setup ready for operation. Here can be seen the pressure vessel, filter pumps, gages, handling equipment, etc. The controls are mounted for easy access and operation. Fluid pressures and load on formation can be regulated or changed in a matter of a few seconds with testing time reduced to a minimum.
APPENDIX C

CALIBRATION OF EQUIPMENT

The important quantities to be measured are drilling rate, load on bit, and hydrostatic pressure.

The drilling rate is obtained by measuring the time required to drill a given depth of hole. Time is measured with an electric clock. Depth drilled is measured by visual reference to the depth indicator. Each measured portion of depth is made large enough to minimize the errors likely in a visual measurement.

Hydrostatic pressure is measured by means of a Bourdon hydraulic gage which was checked by a dead weight tester to within ± one-half per cent of indicated reading.

Load on bit requires a more elaborate means of calibration. Load is applied to the bit by pulling the rock into the bit (see Fig. 3) by means of the specimen holder (5), pull rod (6), yoke (7) and air cylinder (8). In this arrangement the parts likely to have frictional resistance are the piston in the air cylinder, pull rods through the bottom of the pressure chamber seals and the tongue-in-groove arrangement between the specimen holder and inner wall of the pressure chamber. (See section on Equipment, Appendix B). The total friction was measured for the three parts. This friction varied with hydrostatic pressure in the pressure chamber and was calibrated over the pressure range to be used.
Sufficient air pressure, \( P_a \), was introduced into the air cylinder to move the mechanism to the extreme "up" position and hold it there while the hydrostatic pressure, \( P_h \), was maintained in the pressure chamber. \( P_a \) was slowly reduced until the carriage started a slow downward movement. At this point \( P_a \) was recorded. Then

\[
F_f = F_h - F_a
\]

where \( F_f \) = Frictional Force

\[
F_h = P_h \times A_r \quad (A_r = \text{area of rods})
\]

\[
F_a = P_a \times A_c \quad (A_c = \text{area of air cylinder})
\]

Except for very light bit loads and very high hydrostatic pressures, this friction force was found to be less than 2 per cent of bit load. These exceptions were outside the ranges in which tests were conducted. Frictional losses in the rotary system are of no interest in these tests. It is for this reason that the rotary member was designed as it is.

The fluid pumps are positive displacement piston pumps. The volume of fluid circulating is computed by multiplying the number of strokes per minute by the volume per stroke. For these tests the primary purpose of the fluid was to keep the loose chips removed from the area being drilled.

The errors encountered and outlined above are felt to be small in comparison to the variations in physical properties encountered in the rock.


